Worksheet 2

November 15, 2017

1. Among the following vector fields, one has constant upwards pointing curl. Which one?

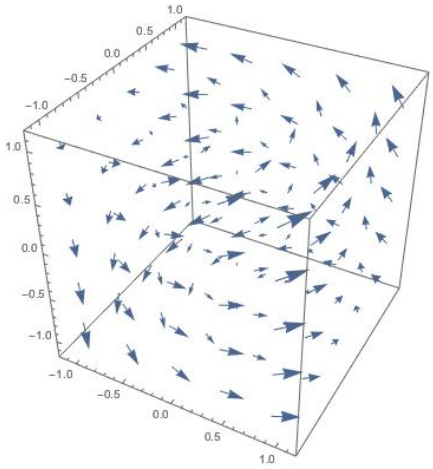


Figure 1: Plot A

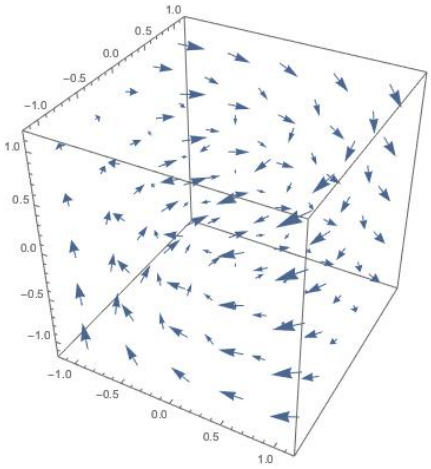


Figure 2: Plot B

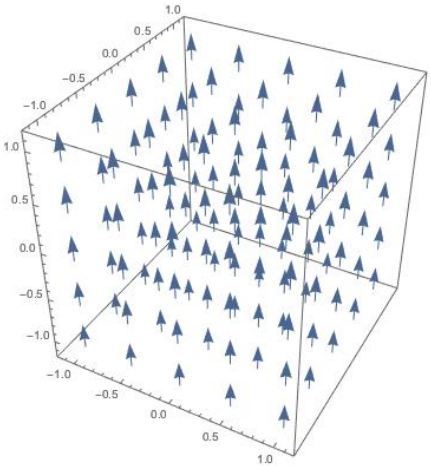


Figure 3: Plot C

2. One of the vector fields below has always negative divergence. Which one?

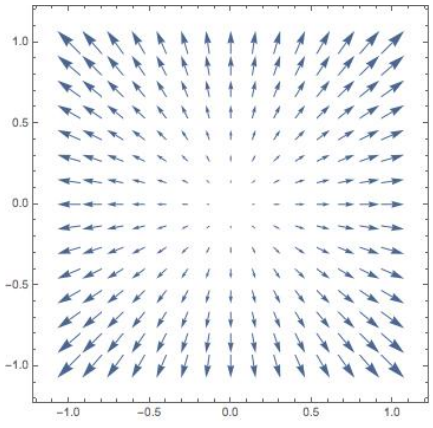


Figure 4: Plot A

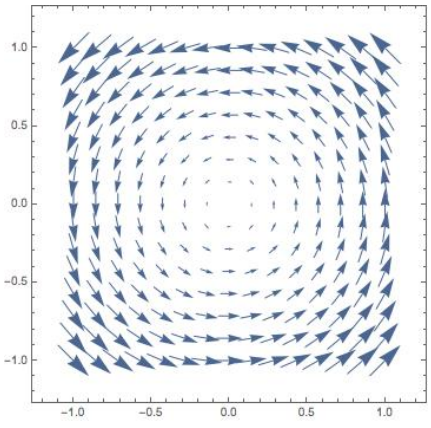


Figure 5: Plot B

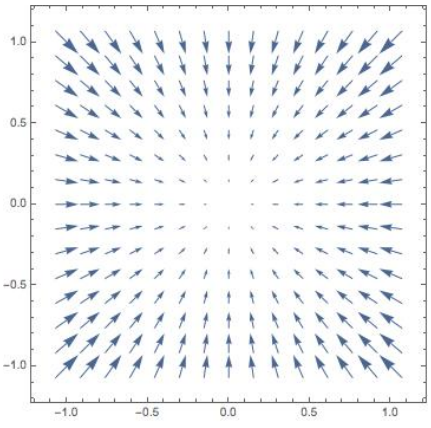
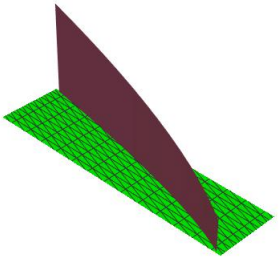


Figure 6: Plot C

3. A fence lies over the curve $y = x^2$ for $x \in [1, 3]$ and under the graph of the function $f(x, y) = x$, where x, y are in meters. Find the area of the fence, including units.



4. A fly flies in a room along the curve

$$c(t) = (2\sin(t), \cos(t), 2t),$$

where t is measured in seconds. The temperature at the point (x, y, z) of the room is given by the function

$$T(x, y, z) = x^2 + 2e^z y,$$

in $^\circ$ C.

(a) Find the gradient of T.

(b) As the fly moves, it experiences a different temperature at each point. Find the instantaneous rate of change of the temperature that the fly experiences at time π seconds (include units).

5. You are given the vector field

$$\vec{F}(x, y, z) = \langle 2xy + 2e^z, x^2, 2xe^z \rangle,$$

defined on $\mathbb{R}^3$. Show that it is conservative and find a potential function for it.

6. An object moves along the boundary of the set

$$D = \{(x, y) : x^2 + y^2 \geq 1,\ |y| \leq \sqrt{3}x,\ x \leq 3\}$$

in counterclockwise direction. Find the work produced by the force field

$$\vec{F}(x, y) = \langle 3, \ln(x^2 + y^2) \rangle$$

during the movement of the object.