Ecological Models

1 Predator-Prey

Our goal is to model two species, one of which (the predator) feeds on the other (the prey). We denote the population of the prey as a function of time by $x(t)$ and the one of the predator by $y(t)$. If you took MA 266, for one of the computer projects you had to study a population of ladybugs (predators) interacting with a population of aphids (prey). The main assumptions of the model are the following:

1. In the absence of predators, the prey population grows at natural rate $dx/dt = ax$, $a > 0$

2. In the absence of prey, the population of predators declines at natural rate, $dy/dt = -by$, $b > 0$

3. Encounters between predators and prey result in an increase of the growth rate of predators which is proportional to the product xy and a decrease of the growth rate of the prey which is also proportional to xy.

We have the general **predator-prey system**

$$\frac{dx}{dt} = ax - pxy = x(a - py)$$
$$\frac{dy}{dt} = -by + qxy = y(-b + qx)$$

with $a, b, p, q > 0$. Notice where the signs are! It is the signs that make x stand for the population of a prey and y the population of a predator.

1. Find the

2 Competing Species

In this case we have two species with populations $x(t)$ and $y(t)$ competing for the resources available in their environment. Here we assume the following:

1. in absence of interaction between the two populations, they both follow logistic models
$$\frac{dx}{dt} = a_1 x - b_1 x^2, \qquad \frac{dy}{dt} = a_2 y - b_2 y^2,$$
with $a_j, b_j > 0$, $j = 1, 2$.

2. Interaction between the populations results in a decrease of their growth rates which is proportional to xy.

We have the **competition system**

$$\begin{aligned} \frac{dx}{dt} &= a_1 x - b_1 x^2 - c_1 xy = x(a_1 - b_1 x - c_1 y) \\ \frac{dy}{dt} &= a_2 y - b_2 y^2 - c_2 xy = y(a_2 - b_2 x - c_2 x) \end{aligned}$$

with $a_j, b_j, c_j > 0$, $j = 1, 2$.