

$$\text{Assume: } y_j(x,t) = X(x)T(t)$$

$$\textcircled{1} \Rightarrow X(0)T'(t) = \alpha^2 X''(x)T(t) \quad \text{for } t > 0$$

$$\Rightarrow \frac{T''(t)}{\alpha^2 T(t)} = \frac{X''(x)}{X(x)} = -\lambda$$

$$\text{Solve: } \begin{cases} T''(t) = -\lambda T(t) \\ X''(x) = -\lambda X(x) \end{cases}$$

Saw: non-trivial sols exist only when $\lambda = \left(\frac{n\pi}{L}\right)^2$

$$X(x) = \sin\left(\frac{n\pi}{L}x\right)$$

Solve PDE w/ b.c.

$$\text{Want: } y_A(x,t) = \sum_{j=1}^{\infty} c_j y_j(x,t)$$

Here: y_j : building blocks

$$\text{Want: } \begin{cases} \partial_t y_j = \tilde{\alpha}^2 \partial_x^2 y_j \\ y_j(0,t) = y_j(L,t) = 0 \\ \partial_t y_j(x,0) = 0 \end{cases}$$