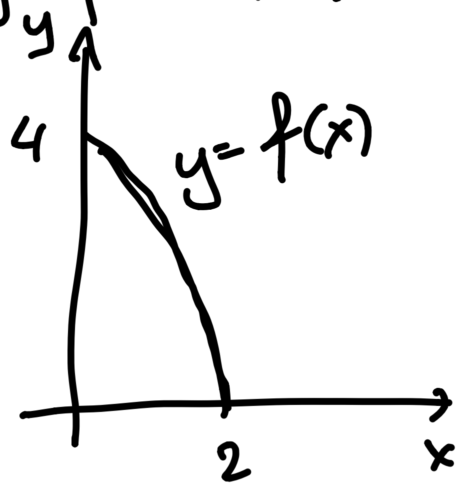
Recall: graph of $y = f(x)$

→ Reflection about x & y axis

→ Shifting up & down

Today: dilation

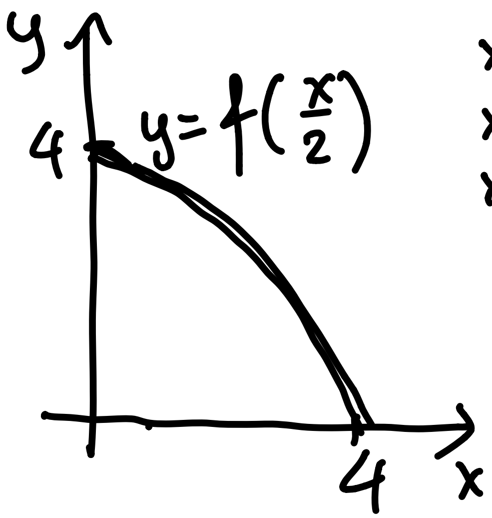
$y = f(x) = 4 - x^2 \quad 0 \le x \le 2$



Replace $x \to \frac{x}{2}$

$y = f\left(\frac{x}{2}\right) = 4 - \frac{x^2}{4}$

Domain: $0 \le \frac{x}{2} \le 2 \Rightarrow 0 \le x \le 4$



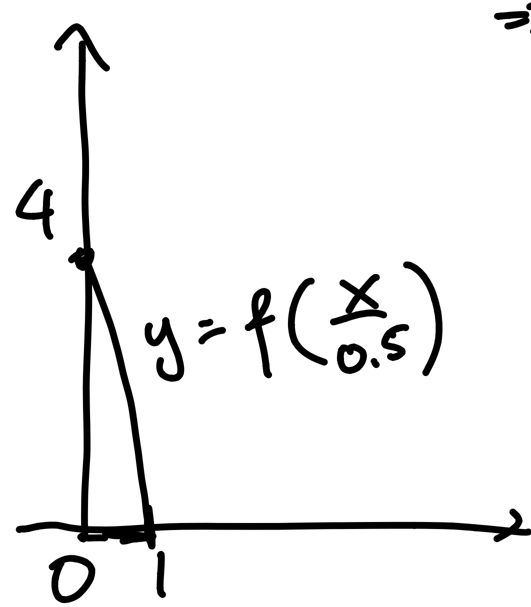
$x=0 \to y=4$
$x=2 \to y=3$
$x=4 \to y=0$

graph is stretched horizontally.

Replace x with $\frac{x}{0.5}$ instead.

$$y=f\left(\frac{x}{0.5}\right) = 4-\left(\frac{x}{0.5}\right)^2$$
$$= 4-4x^2$$

$$0 \le x \le 2 \Rightarrow 0 \le \frac{x}{0.5} \le 2$$
$$\Rightarrow 0 \le x \le 1$$



graph is contracted horizontally.

Cond. 5.

Replace x by $\frac{x}{c}$ in $y=f(x)$:

→ graph is stretched horiz. if $c>1$

→ " " contracted horiz. if $0<c<1$

→ Nothing happens if $c=1$ (duh)

→ Domain: If domain for $f(x)$ was $a \le x \le b$, domain for $f(\frac{x}{c})$ is

$$a \le \frac{x}{c} \le b \Rightarrow ac \le x \le bc$$

→ Range: Same.

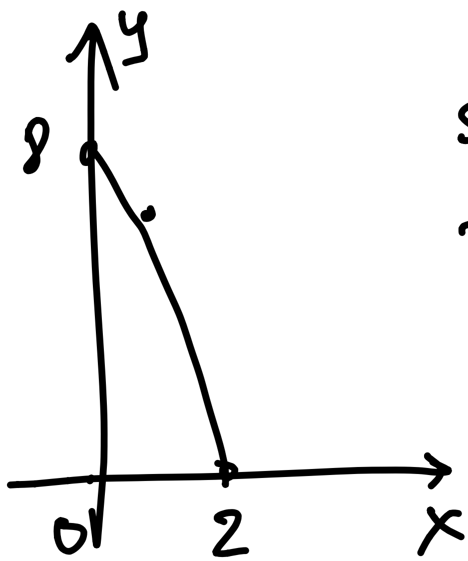
<u>Finally</u>: $y = 4 - x^2$, $0 \le x \le 2$

Range: $0 \le y \le 4$

Replace $y \to \frac{y}{2}$

$$y = 2(4-x^2) \Rightarrow y = 8 - 2x^2$$

$$0 \le x \le 2$$



stretch vertically

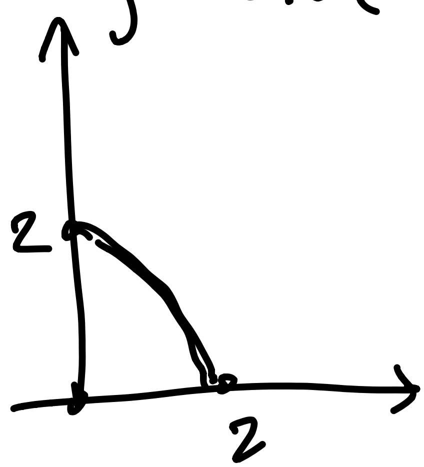
Range: $0 \le \frac{y}{2} \le 4$

$\Rightarrow 0 \le y \le 8$

Same with replacing $y \to \frac{y}{0.5}$

$y = 0.5(4 - x^2) \Rightarrow y = 2 - 0.5x^2$

$0 \le x \le 2$



contracts vertically.

Conclusion:

Replace $y \to \frac{y}{c}$ in $y = f(x)$

Graph contracts vertically if $0 < c < 1$

stretches vertically if $c > 1$

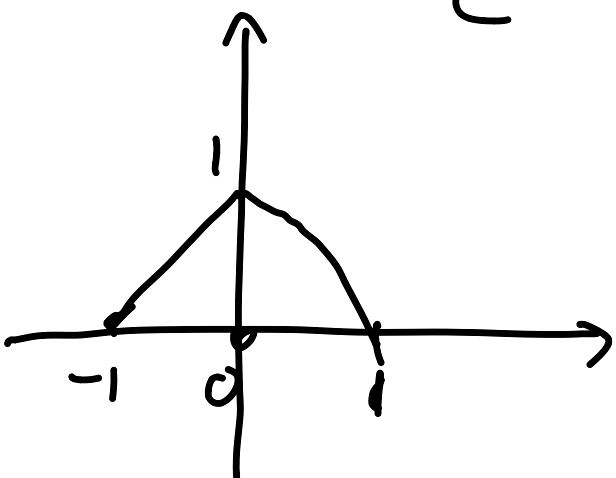
Domain: same.

Range: if $y=f(x)$ has range
$a \le y \le b$

then $\frac{y}{c} = f(x)$ has range
$ca \le y \le cb$
$(c > 0)$

Ex.

$$f(x)=\begin{cases} 1-x^2 & 0 \le x \le 1 \\ x+1 & -1 \le x \le 0 \end{cases}$$



Q: Graph $2f\left(\frac{x-3}{4}\right)-1$ $\ast$

One option: write multipart rule (exercise)

Another: move graph of f

around.

Want to build (*) by reflections, dilations, shiftings.

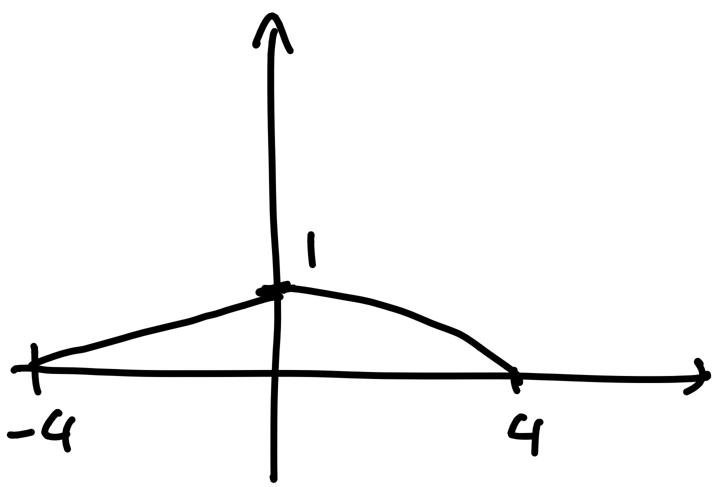
$$y = 2f\left(\frac{x}{4} - \frac{3}{4}\right) - 1$$

'Replace $x \to \frac{x}{4}$ in (*)

$$y = f\left(\frac{x}{4}\right) = g_1(x)$$

$-1 \le \frac{x}{4} \le 1$

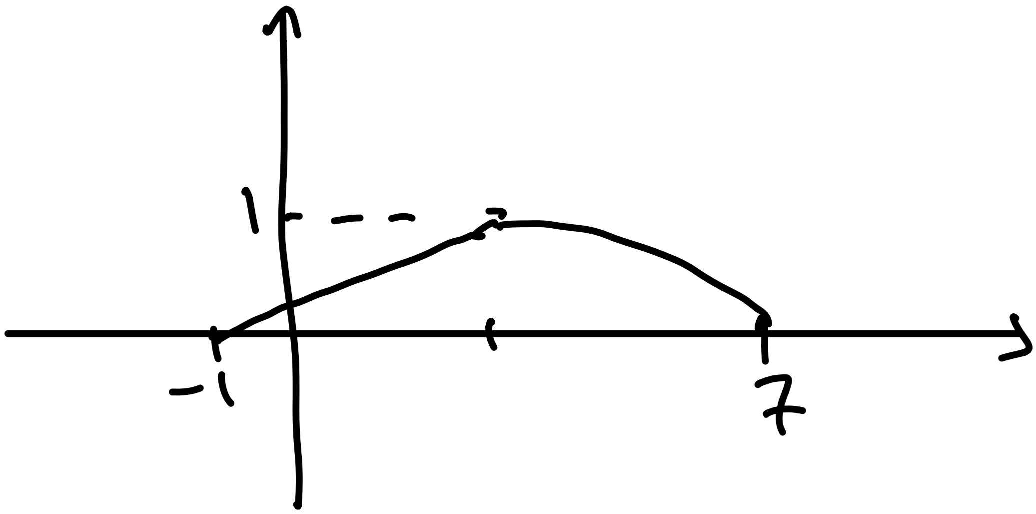
$\Rightarrow -4 \le x \le 4$



Shift by 3 to the right

$$y = f\left(\frac{x-3}{4}\right) = g_1(x-3)$$

$-4 \le x - 3 \le 4 \Rightarrow -1 \le x \le 7$



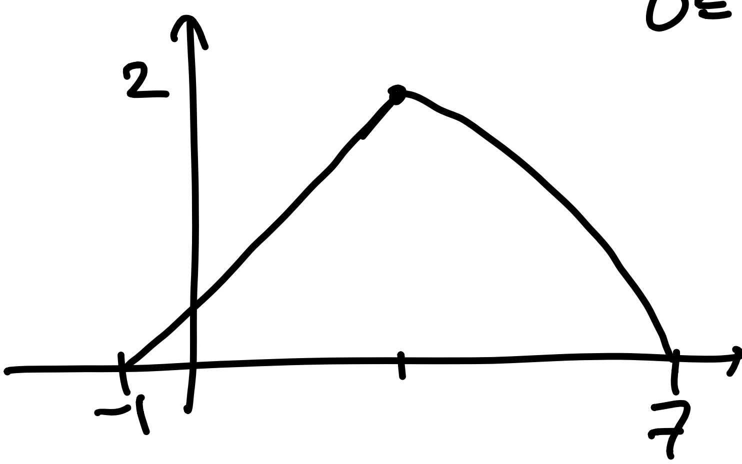
Replace $y \to \frac{y}{2}$

$$y = 2f\left(\frac{x-3}{4}\right)$$

New range:

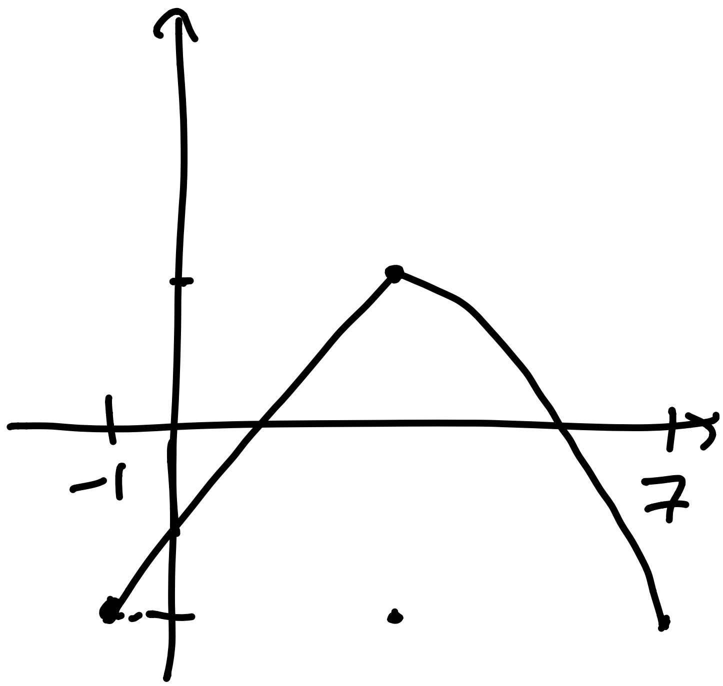
$0 \le \frac{y}{2} \le 1$

$0 \le y \le 2$



Replace $y \to y+1$

$$y = 2f\left(\frac{x-3}{4}\right) - 1$$



Ch. 14.

Rational fcts

$$\frac{\text{polynomial}}{\text{polynomial}}$$

$$\frac{x^2+3x+1}{2x-2}$$

or $\dfrac{3x}{3x^2+4}$

only linear-to-linear:

$$\frac{3x+2}{4x-5}$$

$y = \frac{1}{x}$

Makes sense for $x \neq 0$

Plug in large values:

$x = 1000 \Rightarrow y = 0.001$

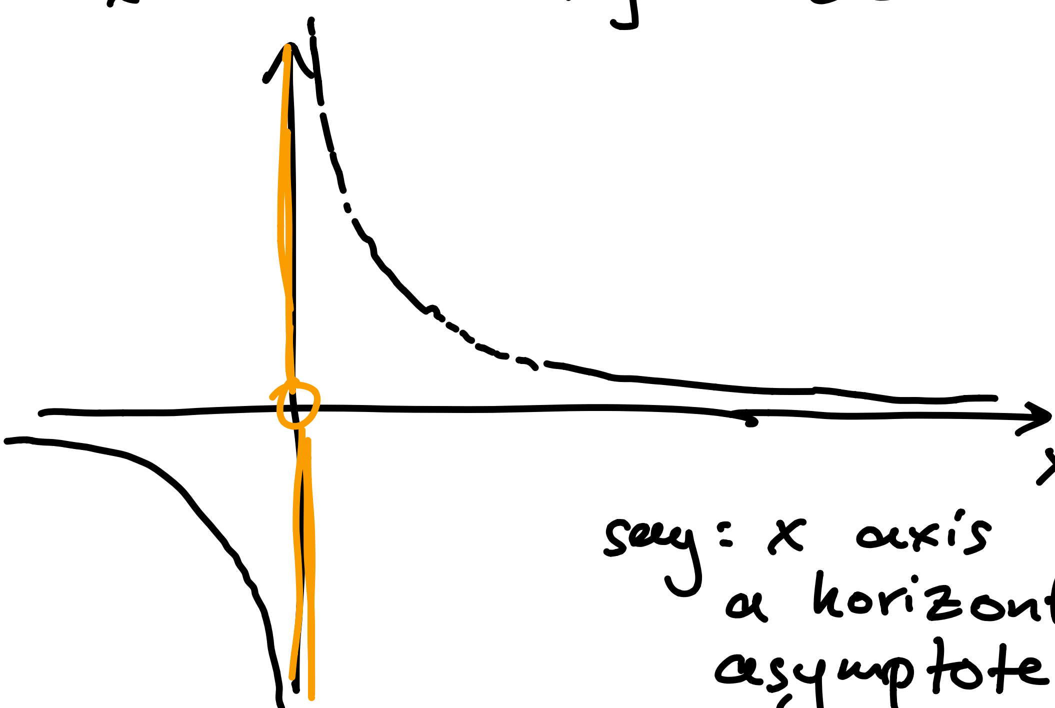
$x = 10^6 \Rightarrow y = \frac{1}{10^6} = 10^{-6} = 0.000001$

$x = -1000 \Rightarrow y = -0.001$

$x = -100{,}000 \Rightarrow y = -0.00001$

$x = 0.001 \Rightarrow y = 1000$

$x = -0.001 \Rightarrow y = -1000$



say: x axis is a horizontal asymptote.

y axis is a vertical asymptote.

Range: $y \neq 0$ ($y < 0$ or $y > 0$)

Note: horizontal as. is $y=0$
What is missing from the range is 0 and hor. asymptote is $y=0$.

vertical as.: $x=0$
domain is $x \neq 0$

Fun fact: graph of $y=\frac{1}{x}$ hyperbola

same picture as $x^2-y^2=1$ after rotating.