

Math 324 C - Winter 2017
Final Exam
Wednesday, March 15, 2017

Name: _____

Student ID Number: _____

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Total	60	

- There are 7 problems spanning 7 pages (your last page should be numbered as 7). Please make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems (unless explicitly instructed otherwise). The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the back of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 110 minutes to complete the exam. Budget your time wisely.
Do not spend too much time on an individual problem, unless you are done with all the rest.
- You are not allowed to discuss this exam with other people until 5.00 pm today.

GOOD LUCK!

1. (6 pts.) **You do not need to explain your answers for this problem.**

- (a) Mark the following sentence as **true** or **false**. Let c be the unit circle in \mathbb{R}^2 parametrized counterclockwise, so that $-c$ is the unit circle parametrized clockwise. Then for every scalar valued continuous function $f(x, y)$ we have

$$\int_{-c} f(x, y) dx = - \int_c f(x, y) dx.$$

True False

- (b) Mark the following sentence as **true** or **false**. Let S denote the unit ball in \mathbb{R}^3 with positive(outward) orientation and \tilde{S} the unit ball with negative (inward) orientation. Then, for any vector field $\vec{F}(x, y, z)$ with continuous coefficients

$$\int_S \vec{F}(x, y, z) \cdot d\vec{S} = - \int_{\tilde{S}} \vec{F}(x, y, z) \cdot d\vec{S}.$$

True False

- (c) Mark the following sentence as **true** or **false**. Let S denote the upper hemisphere of the unit ball centered at the origin in \mathbb{R}^3 (the one that satisfies $z \geq 0$), with **upward** orientation, and \tilde{S} the lower hemisphere of the unit ball centered at the origin (the one that satisfies $z \leq 0$), again with **upward** orientation. Then, for any vector field $\vec{F}(x, y, z)$ with differentiable coefficients

$$\iint_S \text{curl } \vec{F}(x, y, z) \cdot d\vec{S} = \iint_{\tilde{S}} \text{curl } \vec{F}(x, y, z) \cdot d\vec{S}.$$

True False

2. (6 pts.) Show the following version of the product rule: Let $\vec{F}(x, y) = \langle P(x, y), Q(x, y) \rangle$ be a vector field, where P, Q are differentiable scalar valued functions, and let $g(x, y)$ be a differentiable scalar valued function. Then

$$\operatorname{div}(g\vec{F}) = g \operatorname{div}(\vec{F}) + (\nabla g) \cdot \vec{F}.$$

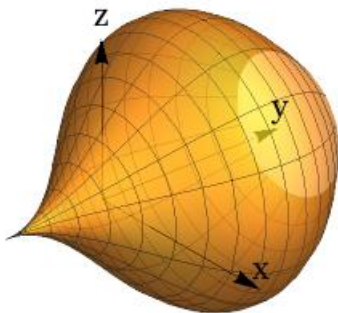
Make sure that each step follows clearly from the previous one, otherwise you may not receive full credit.

3. (9 pts) Find the mass of a thin piece of aluminum foil occupying the part of the paraboloid $x = y^2 + z^2$ that satisfies $x \leq 4$, assuming that its density at the point (x, y, z) is

$$\rho(x, y, z) = \sqrt{\frac{x}{4x + 1}}.$$

4. (10 pts.) Let S be the onion-like surface obtained from the revolution of the graph of the function $z = \sin(y) + 1$, $-\frac{\pi}{2} \leq y \leq \pi$, around the y -axis (look at the picture).

Compute $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = \langle 0, 1, 0 \rangle$, assuming that the orientation is such that the unit normal vector field is pointing away from the origin.

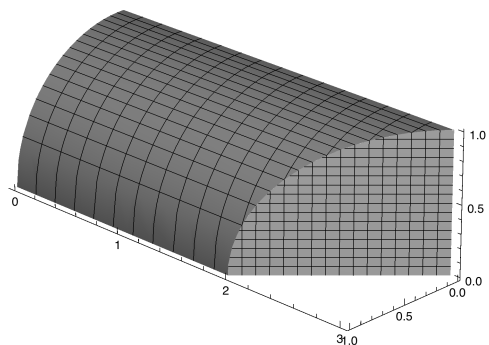


5. (10 pts) (The two parts are not related)

(a) Find the tangent plane to the surface described implicitly by $z^3 = x^2 - y^4 + zxy$ at $(1, 1, 1)$

(b) (10 pts.) Let E be the solid **in the first octant** bounded by the coordinate planes, the cylinder $x^2 + z^2 = 1$ and the plane $y = 3 - x$, as in the picture. For a function $f(x, y, z)$, set up an integral $\iiint_E f(x, y, z) dV$ in the following way:

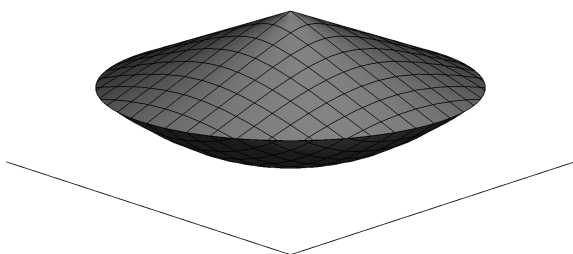
$$\int_{\underline{\hspace{1cm}}}^{\underline{\hspace{1cm}}} \int_{\underline{\hspace{1cm}}}^{\underline{\hspace{1cm}}} \int_{\underline{\hspace{1cm}}}^{\underline{\hspace{1cm}}} f(x, y, z) dy dz dx$$



6. (9 pts) Let E be the solid of the picture below, bounded below by the paraboloid $z = 4x^2 + 4y^2$ and bounded above by the cone $z = 8 - 4\sqrt{x^2 + y^2}$.

(a) Compute the volume of E .

- (b) If $\vec{F} = \langle y+x, y, 4z \rangle$ and S is the boundary of E with **inward orientation**, compute $\iint_S \vec{F} \cdot d\vec{S}$.
(Hint: Use the divergence theorem).



7. (10 pts.) Let S be the unit sphere centered at the origin. Let c be the path consisting of the following curves, as in the picture at the bottom of the page:

- An arc of the intersection of S with the plane $y = x$, from $(0,0,1)$ to $(\frac{\sqrt{3}}{2\sqrt{2}}, \frac{\sqrt{3}}{2\sqrt{2}}, -\frac{1}{2})$ (the one satisfying $x \geq 0$).
- An arc of the intersection of S with the plane $z = -\frac{1}{2}$, from $(\frac{\sqrt{3}}{2\sqrt{2}}, \frac{\sqrt{3}}{2\sqrt{2}}, -\frac{1}{2})$ to $(\frac{\sqrt{3}}{2\sqrt{2}}, -\frac{\sqrt{3}}{2\sqrt{2}}, -\frac{1}{2})$ (the one satisfying $x \geq 0$).
- An arc of the intersection of S with the plane $y = -x$, from $(\frac{\sqrt{3}}{2\sqrt{2}}, -\frac{\sqrt{3}}{2\sqrt{2}}, -\frac{1}{2})$ to $(0,0,1)$ (the one satisfying $x \geq 0$).

Let $\vec{F}(x, y, z) = \langle -yx, x^2, z \rangle$. Compute $\int_c \vec{F} \cdot d\vec{r}$ (you may do it directly, or use one of the theorems of chapter 16; if you do so, clearly state which theorem you are using).

