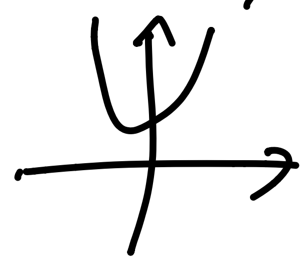
Still ch. 7.

Yesterday: From $y = x^2$ get all parabolas:

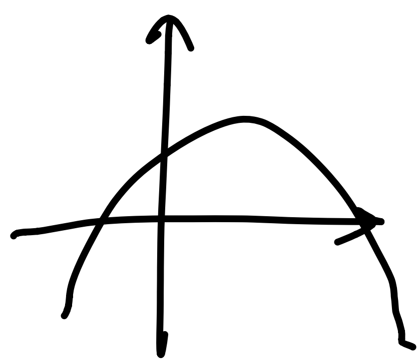
$$y = a(x-h)^2 + k$$

h, k, a const. $a \neq 0$

$a > 0 \rightarrow$ parabola opens up



$a < 0$ opens down.



Today: Quadratic functions

$$f(x) = ax^2 + bx + c$$

Idea: If we can rewrite $y = ax^2 + bx + c$ as

$$y = a(x-h)^2 + k$$

then we know how to draw

the graph of $f(x) = ax^2 + bx + c$.

<u>How</u>: Complete the square!

Remember: did that to write circles in standard form:

$$x^2 + 2x + y^2 = 10$$

$$\to x^2 + 2x + 1 + y^2 = 11$$

$$(x+1)^2 + y^2 = (\sqrt{11})^2$$

$$y = ax^2 + bx + c$$

$$y = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$$y = a\left(x^2 + 2x\frac{b}{a}\frac{1}{2} + \frac{c}{a}\right)$$

$$\boxed{(t+s)^2 = t^2 + 2ts + s^2}$$

$$y = a\left(x^2 + 2\cdot x\cdot\frac{b}{2a} + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$$

$$y = a\left(\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right)$$

$$\Rightarrow \quad y = a\left(x + \frac{b}{2a}\right)^2 + c - a\,\frac{b^2}{4a^2}$$

$$y = a\Big(x - \underbrace{\left(-\frac{b}{2a}\right)}_{h}\Big)^2 + \underbrace{c - \frac{b^2}{4a}}_{k}$$

Found: $y = ax^2 + bx + c$ can be written as $y = a(x-h)^2 + k$

$$h = -\frac{b}{2a}$$

$$k = c - \frac{b^2}{4a}$$

Now we can use shifting, reflection & rotation to draw the graph of a quadr. fct

$$y = f(x) = ax^2 + bx + c$$

Ex: $\quad y = 4x^2 + 9x + 2$

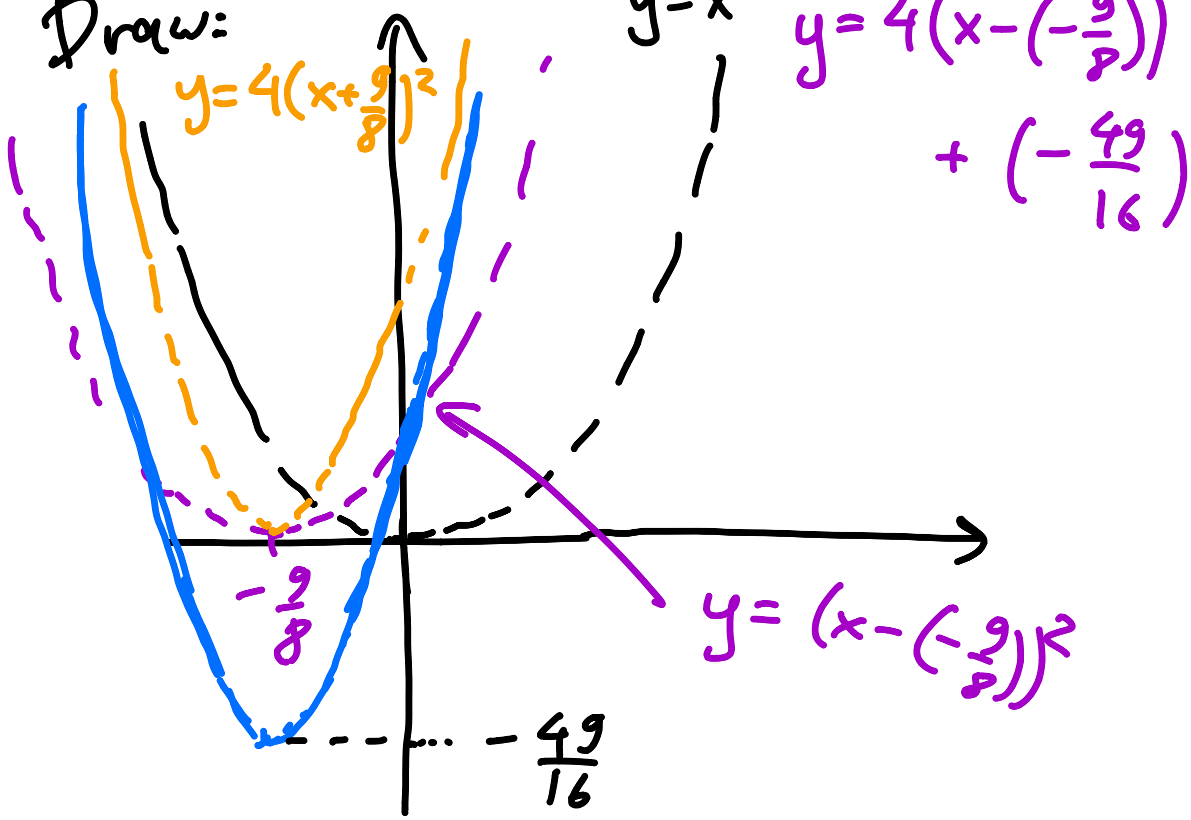
$$h = -\frac{9}{2\cdot 4} = -\frac{9}{8}, \quad k = 2 - \frac{9^2}{4\cdot 4}$$

$\Rightarrow k = 2 - \frac{81}{16}$

$k = -\frac{49}{16}$

$a = 4$

Draw:



Found: graph of

$$y = f(x) = ax^2 + bx + c$$

is parabola w/ vertex

$$\left(\overset{x}{-\frac{b}{2a}},\ \overset{y=f(x)}{c - \frac{b^2}{4a}}\right)$$

- If $a > 0$ it opens up, has a minimum, $f\left(-\frac{b}{2a}\right) = c - \frac{b^2}{4a}$

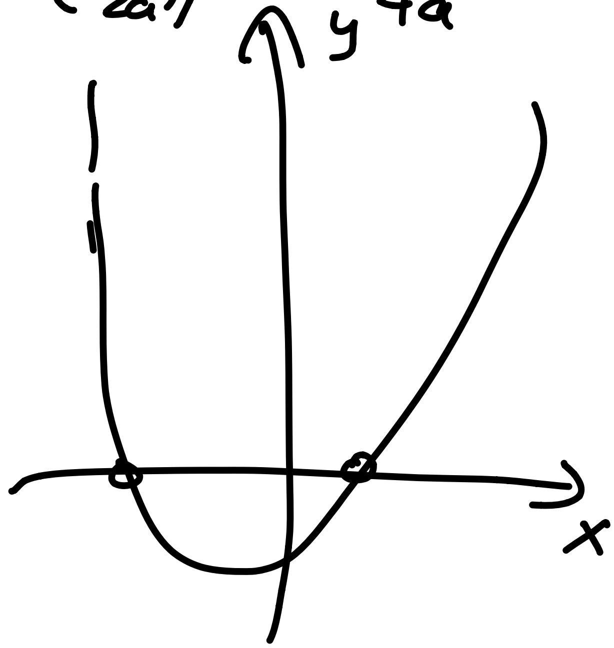
- If $a < 0$, parabola opens down, has maximum

$$f\left(-\frac{b}{2a}\right) = c - \frac{b^2}{4a}$$

Side note: Find x intercept of parabola

$$y = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + c - \frac{b^3}{4a}$$

Set $y = 0$



$$a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c\cdot 4a}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{\pm 2a}$$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Q. How many points determine parabola?

Recall: $y = ax + b$
needed 2 pieces of info to determine line :
- 2 pts
- pt & slope
- slope & interc.

$y = ax^2 + bx + c$
3 unknown quantities a, b, c
- Need 3 pts, not all on

the same line

- Vertex + a point.

e.g. know vertex $(1,2)$

pt. $(3,4)$

$y = a(x-h)^2 + k$

Find a, h, k so that has vertex $(1,2)$ & passes through $(3,4)$.

x coord. of vertex

$4 = a(3-1)^2 + 2$ y coord. of vertex

y coord. of pt.

x coord. of pt

$4 = a\,2^2 + 2$

$4 = 4a + 2$

$4a = 2$

$a = \frac{1}{2}$

So:

$y = \frac{1}{2}(x-1)^2 + 2$

Parabola from 3 pts: $(0,0)$
$(1,4)$
$(-1,3)$

All satisfy $y = ax^2 + bx + c$

$$0 = a \cdot 0^2 + b \cdot 0 + c$$
$$4 = a \cdot 1^2 + b \cdot 1 + c$$
$$3 = a \cdot (-1)^2 + b(-1) + c$$

$$c = 0$$
$$a + b = 4$$
$$a - b = 3 \quad (+)$$

$$2a = 7 \Rightarrow a = \frac{7}{2}$$

$$b = 4 - a \Rightarrow b = 4 - \frac{7}{2} \Rightarrow b = \frac{1}{2}$$

Eq:

$$y = \frac{7}{2}x^2 + \frac{1}{2}x + 0$$