Math 120 B - Spring 2018
Midterm 1
Thursday April 19, 2018

Name: ______________________________

UW email address: ______________________________

Problem 1	12	
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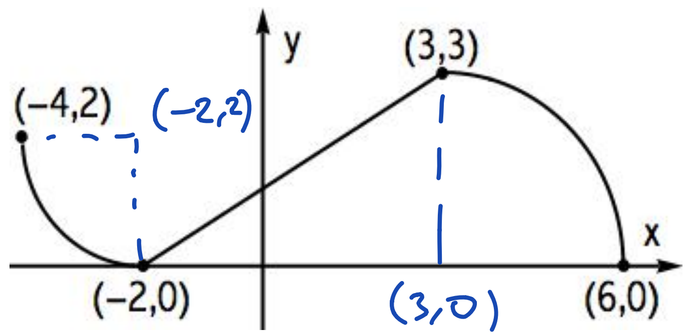
- There are 5 problems spanning 5 pages (your last nonempty page should be numbered as 5). Please make sure your exam contains all these questions.
- You are allowed to use a TI-30x IIS Calculator and one double sided hand-written 8.5 by 11 inch page of notes.
- You must show your work on all problems (unless explicitly instructed otherwise). The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you use a trial-and-error or guess-and-check method when an algebraic method is available, you will not receive full credit.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 80 minutes to complete the exam. Budget your time wisely. **Do not spend too much time on an individual problem, unless you are done with all the rest.**
- You may leave your answers in exact form or round them to 2 decimal places; your choice.

GOOD LUCK!

1. (12 pts.) Below is the graph of a multipart function, $f(x)$, composed of two quarter-circular arcs and one straight line. Its domain is $[-4, 6]$.



(a) What is the range of f?

$[0, 3]$

(b) Write the multipart rule for this function

1st circle: $(x+2)^2 + (y-2)^2 = 4 \Rightarrow (y-2)^2 = 4-(x+2)^2$

$y = 2 \pm \sqrt{4-(x+2)^2}$

$y = 2 - \sqrt{4-(x+2)^2}$

2nd circle: $(x-3)^2 + y^2 = 9 \Rightarrow y = \pm\sqrt{9-(x-3)^2} \Rightarrow y = \sqrt{9-(x-3)^2}$

Line: $y - 3 = \frac{3-0}{3-(-2)}(x-3) \Rightarrow y = 3 + \frac{3}{5}(x-3)$

$$f(x) = \begin{cases} 2-\sqrt{4-(x+2)^2} & -4 \le x \le -2 \\ 3+\frac{3}{5}(x-3) & -2 < x \le 3 \\ \sqrt{9-(x-3)^2} & 3 < x \le 6 \end{cases}$$

(c) Find all values of x for which $f(x) = 1$.

$1 = 2 - \sqrt{4-(x+2)^2} \Rightarrow 4-(x+2)^2 = 1 \Rightarrow (x+2)^2 = 3$

$\Rightarrow x+2 = \pm\sqrt{3}$

$\Rightarrow x = -2 \pm \sqrt{3}$

$\Rightarrow \boxed{x = -2-\sqrt{3}}$

bec. of domain

$1 = 3 + \frac{3}{5}(x-3) \Rightarrow -2 = \frac{3}{5}(x-3) \Rightarrow x-3 = -\frac{10}{3} \Rightarrow x = 3 - \frac{10}{3}$

$\Rightarrow \boxed{x = -\frac{1}{3}}$

$1 = \sqrt{9-(x-3)^2} \Rightarrow 9-(x-3)^2 = 1 \Rightarrow (x-3)^2 = 8$

$\Rightarrow x = 3 \pm \sqrt{8} \Rightarrow \boxed{x = 3+\sqrt{8}}$

2. (9 pts.) (Ostroff W16) Phileas is going on a hot air balloon ride. His height above the ground after t minutes is a quadratic function of t.

He begins from a platform at a height of 76 meters.

The balloon reaches its maximum height after 30 minutes.

After 80 minutes, he finally lands (at a height of 0 meters)

Give a function $f(t)$ for Phileas's height above the ground after t minutes.

Use vertex form

$f(t) = a(t-h)^2 + k$

vertex at $(30, f(30))$

$f(t) = a(t-30)^2 + k$

$f(0) = 76$ so $a(0-30)^2 + k = 76$

$\Rightarrow a \cdot 900 + k = 76$ (1)

$f(80) = 0$ so $a(80-30)^2 + k = 0$

$\Rightarrow a \cdot 2500 + k = 0$ (2)

(1) $-$ (2) $\Rightarrow a(900-2500) = 76 \Rightarrow a = -\frac{76}{1600}$

$-\frac{76}{1600} \cdot 2500 + k = 0 \Rightarrow k = \frac{25 \cdot 76}{16}$

So $f(t) = -\frac{76}{1600}(t-30)^2 + \frac{1900}{16} = -\frac{19}{400}t^2 + \frac{57}{20}t + 76$

OR

$f(t) = at^2 + bt + c$

$f(0) = 76 \Rightarrow a \cdot 0^2 + b0 + c = 76 \Rightarrow c = 76$

$f(80) = 0 \Rightarrow a \cdot 80^2 + b\,80 + c = 0 \Rightarrow a\,80^2 + b\,80 = -76$

$-\frac{b}{2a} = 30 \Rightarrow b = -60a$

$a \cdot 6400 - 60a \cdot 80 = -76 \Rightarrow$

$a \cdot 1600 = -76 \Rightarrow a = -\frac{76}{1600}$

$b = -\frac{76}{1600} \cdot (-60) = \frac{4560}{1600}$

3. (6 pts.) Let $g(x) = 3 + x + \left|1 - \frac{1}{2}x\right|$. Find all solutions to the equation

$$g(x) = \frac{x}{3}.$$

$$g(x) = \begin{cases} 3 + x + 1 - \frac{1}{2}x, & 1 - \frac{1}{2}x \geq 0 \Leftrightarrow x \leq 2 \\ 3 + x - 1 + \frac{1}{2}, & 1 - \frac{1}{2}x < 0 \Leftrightarrow x > 2 \end{cases}$$

$$3 + x + 1 - \frac{1}{2}x = \frac{x}{3}$$

$$4 = \left(\frac{1}{3} - \frac{1}{2}\right)x \Rightarrow x = -24 \leq 2$$ so <u>it works</u>

$$3 + x - 1 + \frac{1}{2}x = \frac{x}{3}$$

$$\Rightarrow 2 = \frac{x}{3} - \frac{3x}{2} \Rightarrow 2 = \frac{2 - 9}{6}x$$

$$\Rightarrow x = -\frac{12}{7}$$ since $-\frac{12}{7} < 2$

it's <u>not</u> a solution

so only solution: $x = -24$

4. (12 pts.) There is a perfectly circular lake with 1 km radius. Pippin sits on the easternmost point on the shore when he sees a tower located exactly 2 km west and 2 km north of the center of the lake. He immediately begins swimming on a straight line towards the tower at a rate of 3 m/s.

(a) Set coordinates. Mark the coordinates of Pippin, the tower, and also draw Pippin's path.

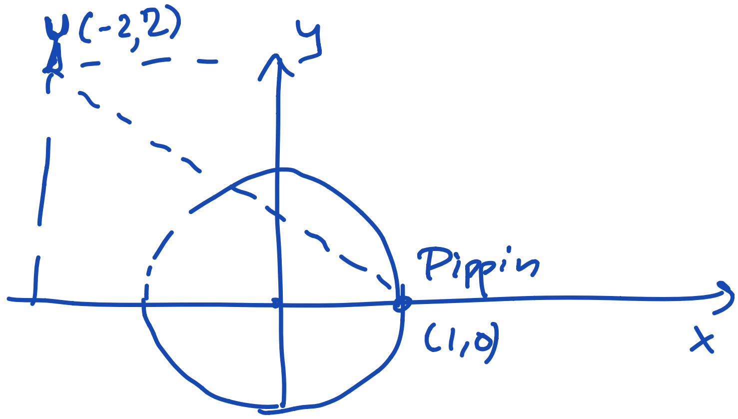


Figure 1: The tower

(b) Where does Pippin exit the lake? (that is, find the coordinates of the point where he exits with respect to the coordinate system you imposed in part (a))

$$y - 0 = \frac{2-0}{-2-1}(x-1) \Rightarrow y = -\frac{2}{3}(x-1)$$

$$\begin{cases} x^2+y^2=1 \\ y=-\frac{2}{3}(x-1) \end{cases} \Rightarrow \frac{4}{9}(x-1)^2 + x^2 = 1 \Rightarrow \frac{4}{9}(x^2-2x+1)+x^2=1$$

$$4x^2-8x+4+9x^2=9$$

$$13x^2-8x-5=0$$

$$x = \frac{8 \pm \sqrt{64-4\cdot5\cdot13}}{2\cdot13}$$

$$x = \frac{8\pm18}{26} \Rightarrow x = 1$$

or $x = -\frac{5}{13}$

Find y: $y = -\frac{2}{3}\left(-\frac{5}{13}-1\right)$

$\Rightarrow y = \frac{12}{13}$

(c) For how many minutes is Pippin swimming?

distance $= \sqrt{\Delta x^2 + \Delta y^2}$

$$= \sqrt{\left(\frac{12}{13}-0\right)^2 + \left(-\frac{5}{13}-1\right)^2}$$

$$= \sqrt{\frac{144}{169}+\frac{324}{169}} = \sqrt{\frac{468}{169}} = \frac{6}{\sqrt{13}} \text{ km}$$

$$\text{so time} = \frac{\frac{6}{\sqrt{13}}\cdot 1000m}{3m/s} = \frac{2000}{\sqrt{13}}s = \frac{200}{6\sqrt{13}}\text{min}$$

5. (10 pts.) Aiman and Hue are walking in the xy-coordinate plane, with units in kilometers. They move at constant speeds and they move linearly (i.e., their paths are straight lines). Aiman starts walking from the origin. His parametric equations of motion are

$$x = \frac{5}{2}t, \qquad y = t.$$

Hue started walking at the same time as Aiman. She started from the point (-3,-4) and will reach the point (6,1) after walking for 2.5 hours.

(a) Write parametric equations for the coordinates of Hue, t hours after she starts walking.

$\begin{cases} x = a+bt \\ y = c+dt \end{cases}$

At time $t=0$ she's at $(-3,-4)$

$\left.\begin{array}{l} -3 = a+b\cdot 0 \\ -4 = c+d\cdot 0 \end{array}\right\} \Rightarrow \begin{array}{l} a=-3 \\ c=-4 \end{array}$

At time $t=2.5$: $\begin{cases} 6 = -3+b\cdot 2.5 \\ 1 = -4+d\cdot 2.5 \end{cases} \Rightarrow \begin{cases} b = \frac{9}{2.5} \\ d = \frac{5}{2.5} \end{cases} \Rightarrow \begin{cases} b = 3.6 \\ d = 2 \end{cases}$

$\begin{cases} x = -3+3.6t \\ y = -4+2t \end{cases}$

(b) What is the smallest distance between them (the closest that they get to each other)? Include units.

$\text{dist}(t) = \sqrt{(-3+3.6t-2.5t)^2 + (-4+2t-t)^2}$

$= \sqrt{(-3+1.1t)^2 + (-4+t)^2}$

$= \sqrt{9-6.6t+1.21t^2+16-8t+t^2}$

$= \sqrt{2.21t^2-14.6t+25}$

Find min of $f(t) = \text{dist}^2(t) = 2.21t^2-14.6t+25$

$\min(f) = f\left(\frac{14.6}{2\cdot 2.21}\right) = f(3.3) = 24.11 - 48.18 + 25$

$= 0.93$

So minimum distance $= \sqrt{0.96} \sim .96\,\text{km}$