Chapter 4

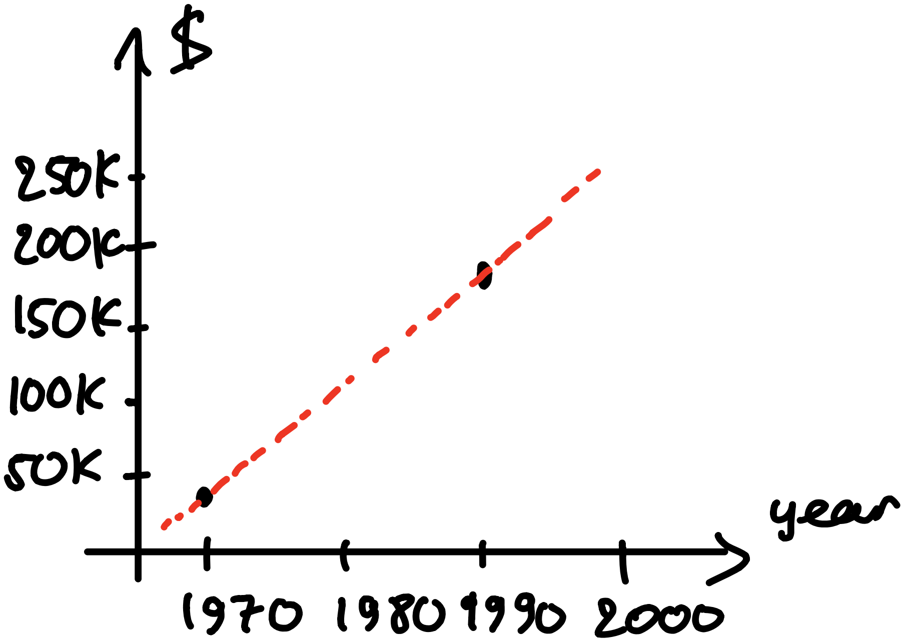
Modeling
- describe
- predict

using maths.

Linear Modeling (simplest)

Ex: 1970 avg price of a single family house in Seattle: $38,000

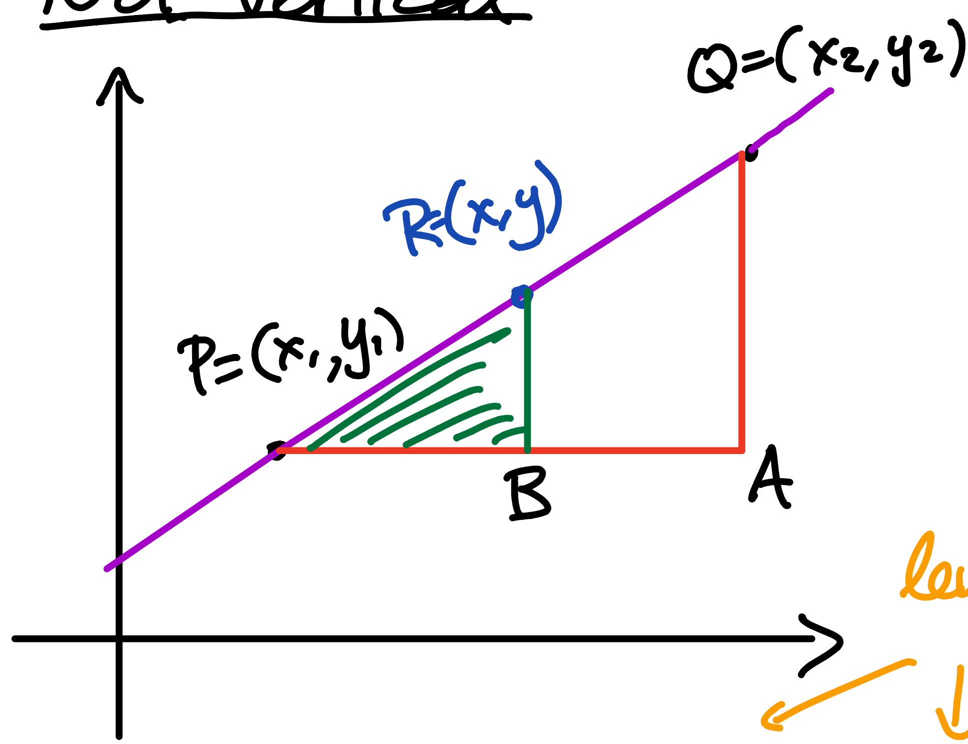
1990: $175 000



Can we find a formula that would predict the price of a house in any year?

Want equation in x, y for a line that might not be vertical or horizontal.

Assume:
<u>Not Vertical</u>



length of side

Similar triangles: $$\frac{|AQ|}{|AP|} = \frac{|BR|}{|BP|} \quad (*)$$

(bec. $\triangle AQP$, $\triangle BRP$ similar)

$$(\ast) \Rightarrow \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \boxed{y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1}$$

2 point formula!

$P(x_1, y_1)$, $Q(x_2, y_2)$, 2 point formula gives the equation of the line connecting P, Q.

Note: there is exactly one such line!

Define:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

Write: Point-slope formula:

$$\boxed{y = m(x - x_1) + y_1}$$

Nice if you know 1 point & slope.

Look at "y intercept" : where line meets y axis

$$y = m(x - x_1) + y_1$$

$x = 0$

y axis

If line and y axis meet at $(0, b)$ how we can find b:

$$b = m(0 - x_1) + y_1$$

So: $b = -mx_1 + y_1$

Rewrite point slope formula:

$$y = m(x - x_1) + y_1$$

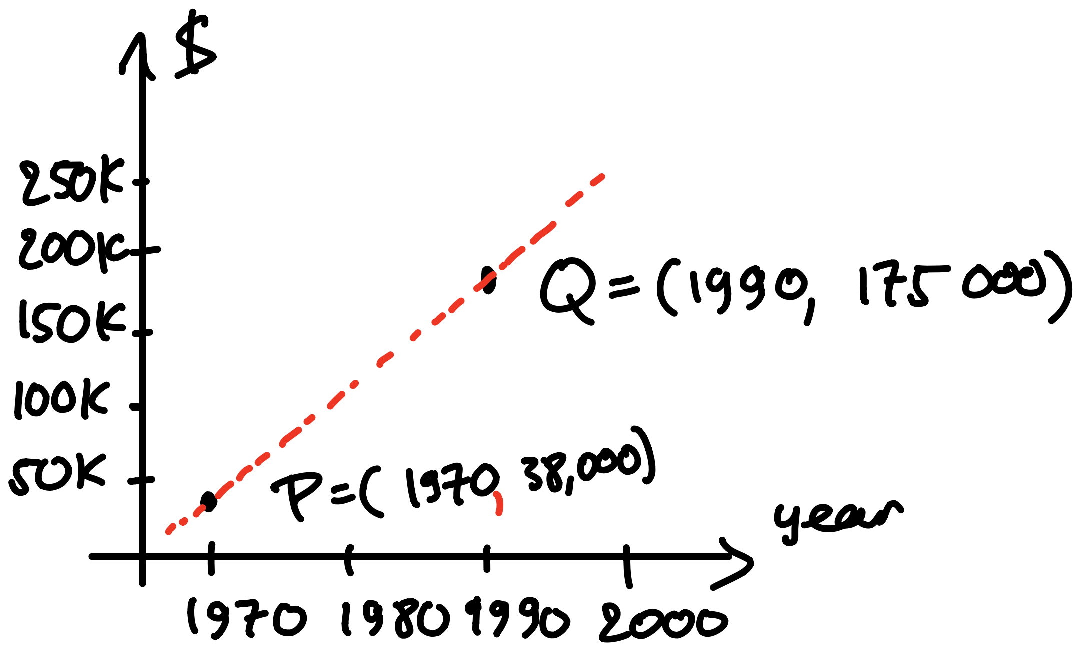
$$\Rightarrow y = mx \underbrace{-mx_1 + y_1}$$

$$\Rightarrow \boxed{y = mx + b}$$

slope-intercept formula.

Note: A line is determined by 2 pieces of info.

Back to example:



$$y = \underbrace{\frac{175000-38000}{1990-1970}}_{\text{Rate of change! units \$/year.}}(x - \underset{\substack{\uparrow \\ x_1}}{1970}) + 38000$$

All 3 formulas above were for non-vertical lines.

A general expression for all lines:

$$\boxed{Ax + By + C = 0}, \quad A, B, C \text{ const.}$$

Why:

slope-int.: $y = mx + b$

$$\Rightarrow (-m)x + 1 \cdot y + (-b) = 0$$

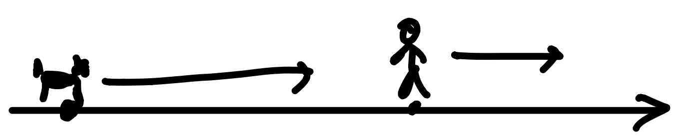
vertical: $x = k$

$$1 \cdot x + 0y - k = 0$$

Another example

John in 10 m in front of Lucky and runs 4 m/s.
Lucky runs 7 m/s.

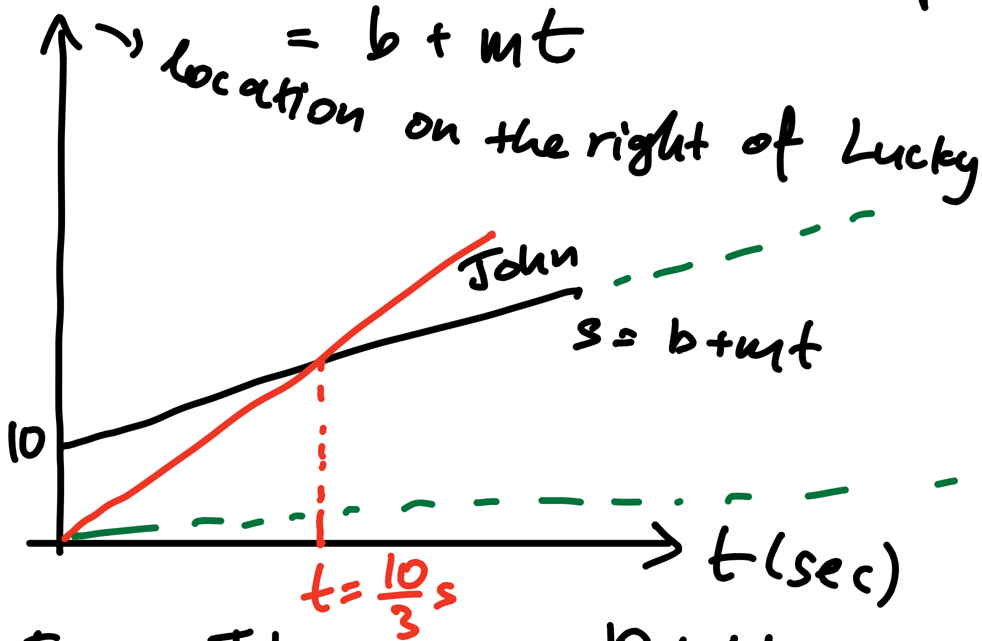
Find linear eq'ns for the distance between them.



For John: s location
b initial location
m speed

$s = (\text{initial loc.}) + (\text{time travelled}) \cdot \text{speed}$

$= b + mt$



For John: $s = 10 + 4t$

For Lucky $s = 0 + 7t$

Dist: Loc John - Loc. Lucky

$$= (10 + 4t) - (0 + 7t)$$
$$= 10 - 3t.$$

Lucky catches John when

dist $= 0$ or $t = \frac{10}{3}$ s

If Lucky had speed 2 m/s (smaller than John): less steep line (smaller slope)

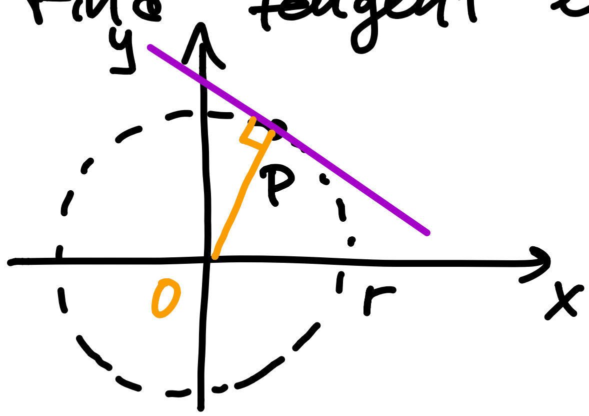
Fact: Parallel lines have the same slope.
Non-vertical Perpendicular lines have slopes with product -1: If m_1, m_2 are slopes of 2 perp. lines

$$m_1 \cdot m_2 = -1$$

Use this fact to find an equation for tang. line to circle.
Assume that a circle is centered @ origin, radius r.

$$x^2 + y^2 = r^2$$

Let $P = (x_0, y_0)$ be a point on the circle. not O!
Find tangent line at P.



Note: Line between O, P and tang. at P are perp.

2 pt formula to find line through
O, P :

$$y = \frac{y_0 - 0}{x_0 - 0}(x - 0) + 0$$

"x naught"

$$y = \underbrace{\frac{y_0}{x_0}} x$$

slope $m_1 = \frac{y_0}{x_0}$

Slope of tang m_2: $m_1 \cdot m_2 = -1$

$$\Rightarrow m_2 = -\frac{x_0}{y_0}$$

Point-slope formula for tang. line

(x_0, y_0) $\quad y = -\frac{x_0}{y_0}(x - x_0) + y_0$ (multiply by y_0)

$$y_0 y = -\cancel{y_0}\frac{x_0}{\cancel{y_0}}(x - x_0) + y_0 \cdot y_0$$

$$y_0 y = -x_0 x + x_0^2 + y_0^2$$

P is on circle! So $x_0^2 + y_0^2 = r^2$

Finally: $\boxed{x x_0 + y y_0 = r^2}$

Tang. line on circle $x^2+y^2=r^2$
at the point $P=(x_0, y_0)$