Last time: functions
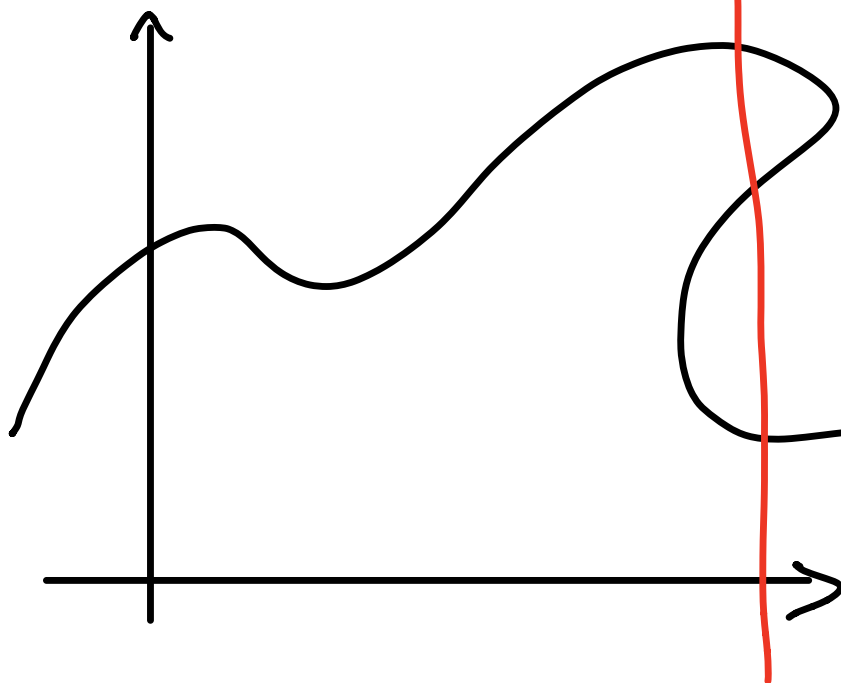→ Rule $y = f(x)$

→ Domain D : x values
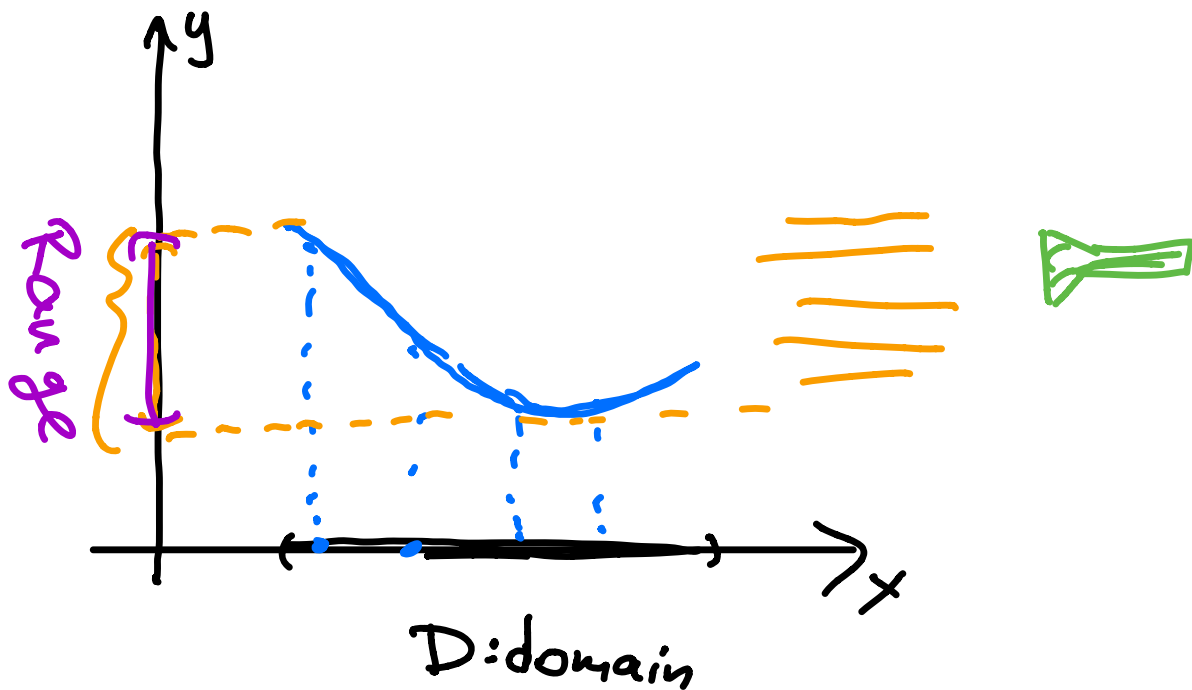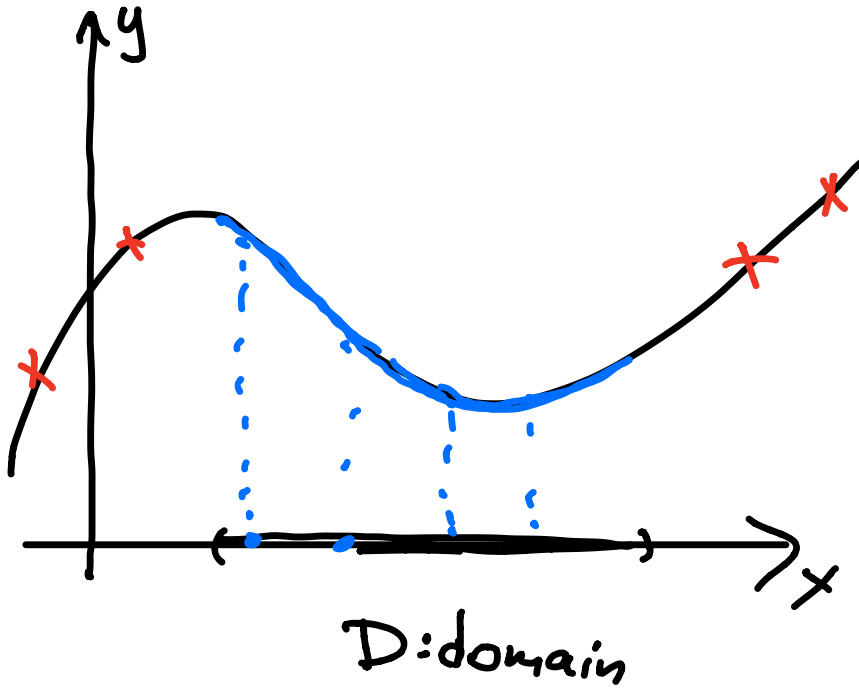
→ Range R : y values

Look at graph of a function.

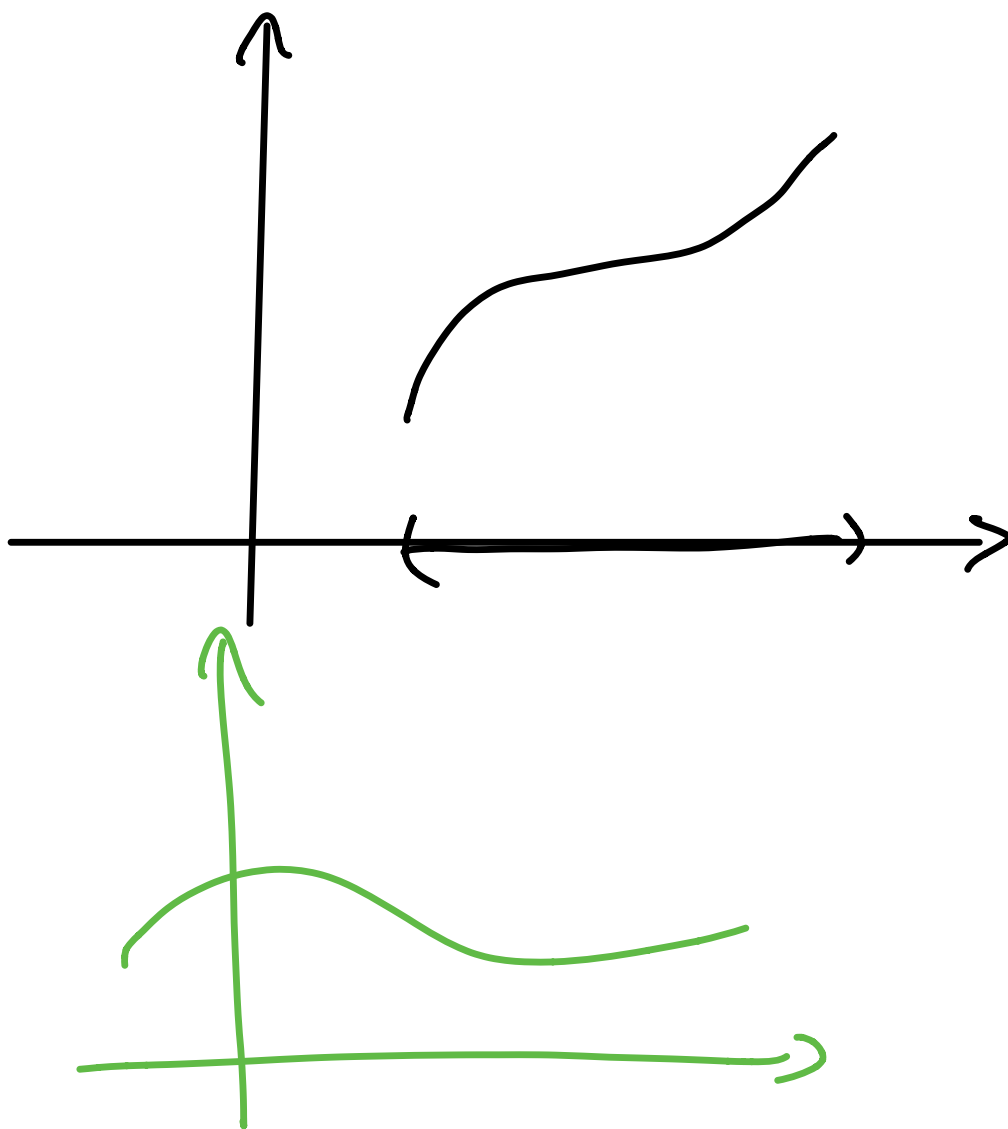$\{(x, y) : y = f(x)\}$

✓ Not
graph
of fct !!

D:domain

Range

D:domain

Height of point $P(x) = (x, f(x))$ controlled by value of f at x:

If $f(x) > 0 \rightarrow P(x)$ above y axis

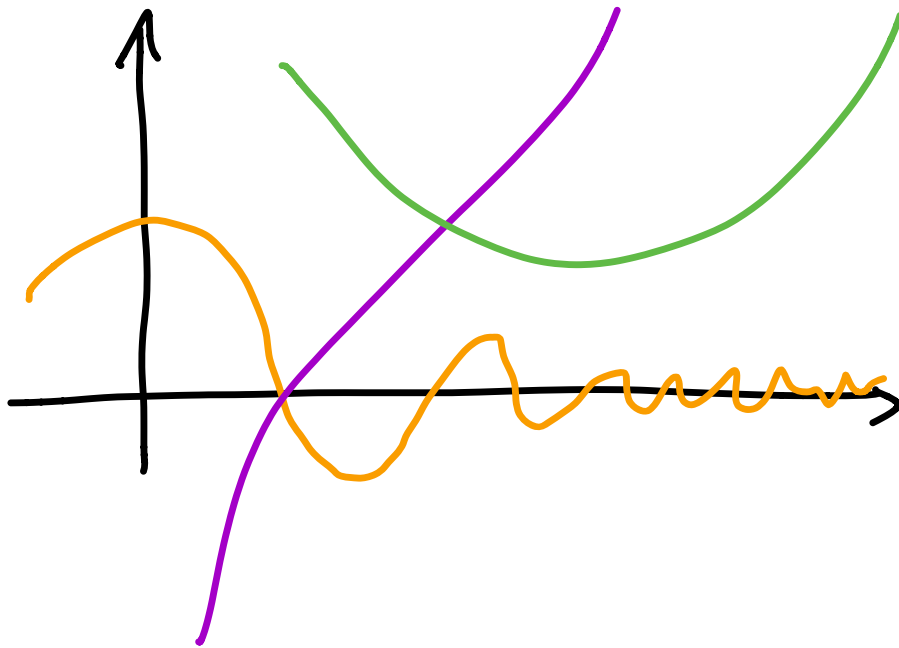$f(x) < 0 \rightarrow P(x)$ below y axis.

Intercepts

x intercept: intersection
w/ x axis

y intercept: int. w/ y axis

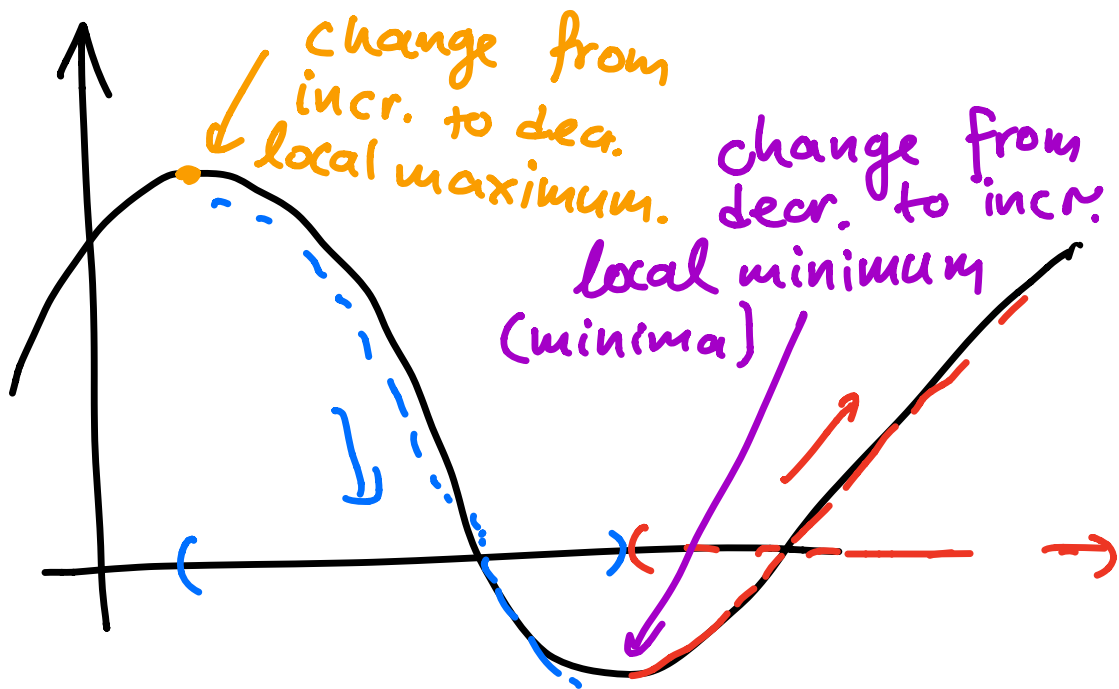! We can have at most 1
  y intercept.

As many   x intercepts as
we wish

Can also take intercepts with
lines   y = k
   To find them: solve eq'n
        k = f(x)

change from
incr. to decr.
local maximum.

change from
decr. to incr.
local minimum
(minima)

values become smaller
as x moves to right:
Function is decreasing here.

values become larger →increasing

Ex: $f(x) = x^2$, $x \in (-2, 4]$

$-2 < x \leq 4$

✓ incl. this point

not including this pt.

$y = 4$

$y = 1$

(-2)

decreasing,   increasing

4

a) Find x intercept:
Set $y = 0$

$0 = x^2 \Rightarrow x = 0$

$\rightsquigarrow (0, 0)$
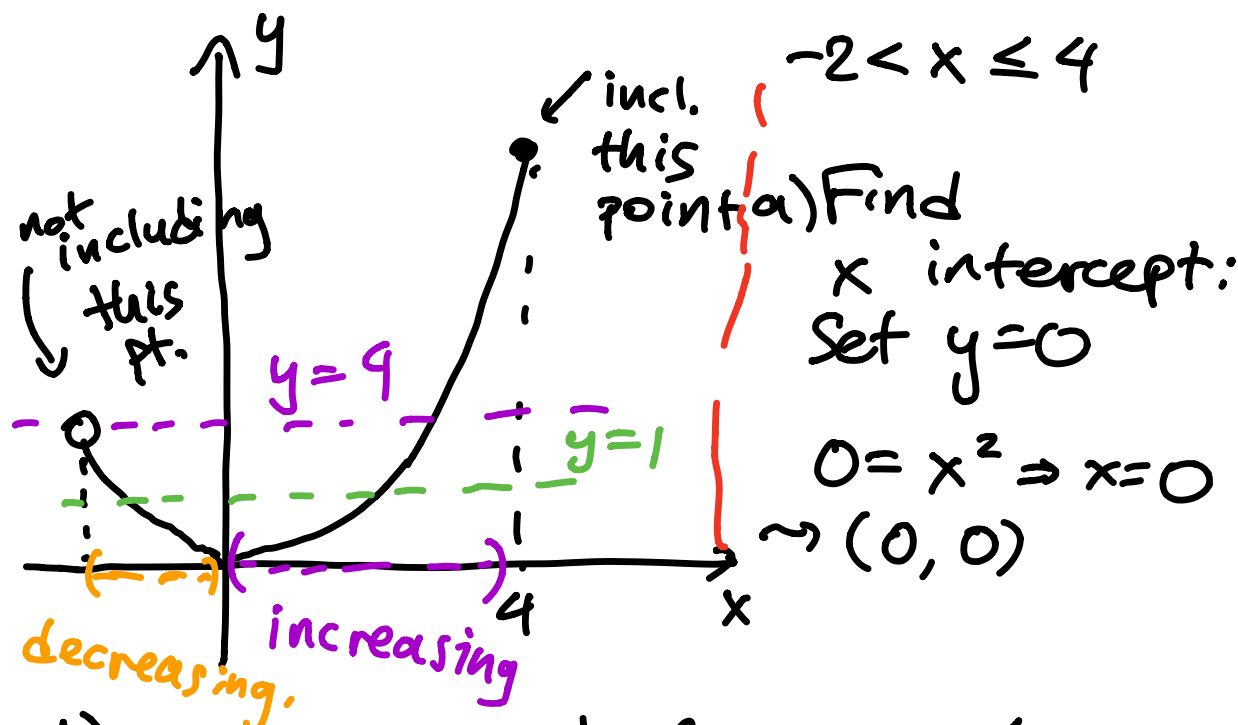
b) Intercept w/ line $y = 4$.
   Solve $f(x) = 4$
      $x^2 = 4 \Rightarrow x = \pm 2$
      Int: $(2, 4)$
               $(-2, 4)$ ✗ $x > -2$

c) Intercept w/ line $y = 1$
   Solve $f(x) = 1$
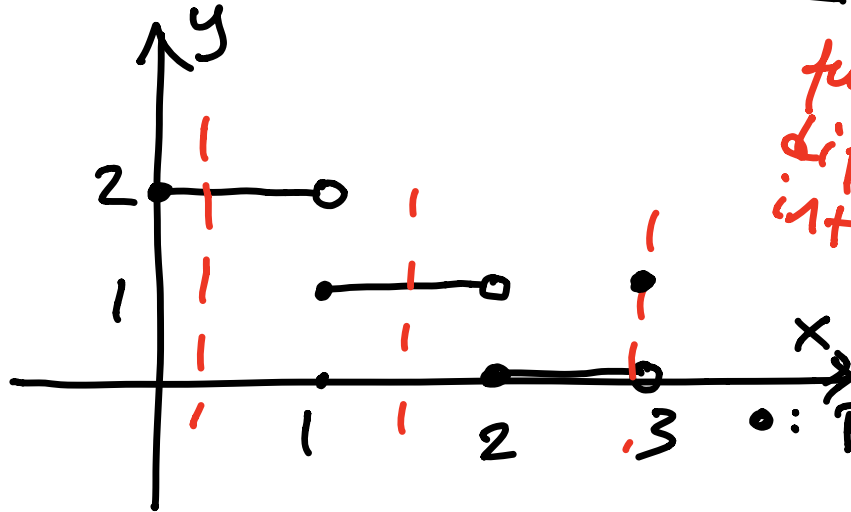 $x^2 = 1 \Rightarrow x = \pm 1$
               $(1, 1)$
               $(-1, 1)$

Increasing   on $(0, 4]$ ← intervals
decreasing   on $(-2, 0)$ ↙
local min at $x = 0$, point $(0, 0)$

Multipart Functions

✓ need different expression for values of function on different intervals.

• : point included

○ : point excluded

Write:

$$f(x) = \begin{cases} 2, & \text{if } 0 \le x < 1 \\ 1, & \text{if } 1 \le x < 2 \\ 0, & \text{if } 2 \le x < 3 \\ 1, & \text{if } x = 3 \end{cases}$$

How we evaluate:

$$f(2.1342) = 0 \text{ bec.}$$
$$2 \le 2.1342 < 3$$

The Most Important Multipart
function: The absolute value.
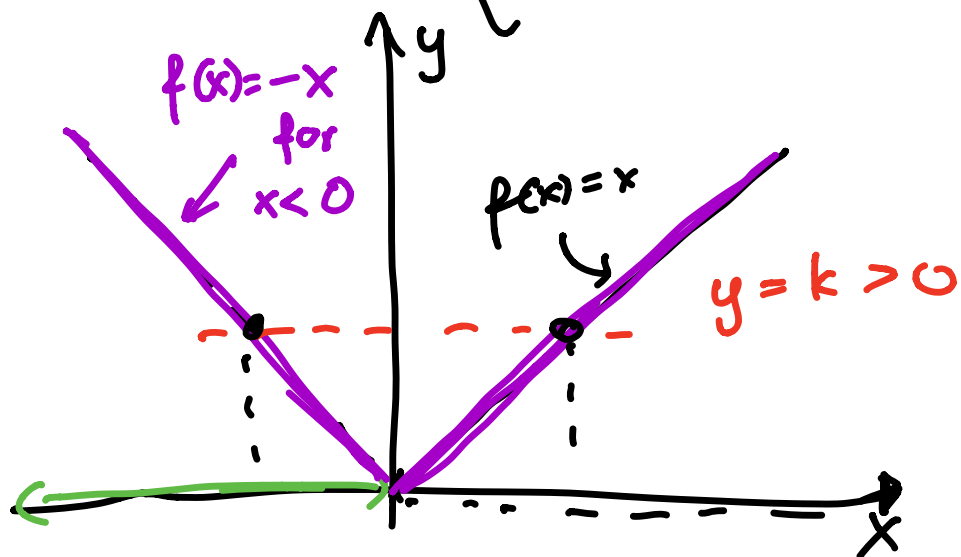
$$|x| = x \text{ if } x \geq 0$$
$$|x| = -x \text{ if } x < 0$$

Abs. value gives distance of number
from
0
on x axis!
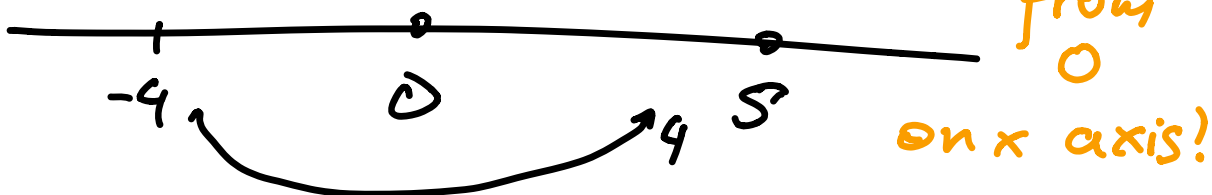
$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$f(x) = -x$ for $x < 0$

$f(x) = x$

$y = k > 0$

Intersection with: $y = k, k > 0 \rightarrow 2$ sol's
$y = h, h < 0 \rightarrow$ None

↳ Example: Find intercept w/th
y=4
  Check both expressions.
     1st expr: x=4.
        does this satisfy
                  x≥0?
         Yes!   (4,4) is a sol.
     2nd :   -x=4
            ⇒ x = -4
    Is x in the interval that
    corresponds to f(x) = -x ?
       Yes!
         (-4,4) is another sol.

Exercise: Find intercept w/ y=4
of the function
$$f(x)= \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$$

If a graph has a corner (cusp)
hint that you need multipart
function