Math 324 C - Winter 2017

Midterm 1

Friday, January 27, 2017

Name: _____

Student ID Number: _____

| Problem 1 | 12 | |
|:---:|:---:|:---:|
| Problem 2 | 8 | |
| Problem 3 | 18 | |
| Problem 4 | 12 | |
| Total | 50 | |

- There are 4 problems spanning 4 pages (your last page should be numbered as 4). Make sure your exam contains all these questions.

- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.

- You must show your work on all problems (unless explicitly instructed otherwise). The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.

- If you need more room, use the backs of the pages and indicate to the grader that you have done so.

- Raise your hand if you have a question.

- Any student found engaging in academic misconduct will receive a score of 0 on this exam.

- You have 50 minutes to complete the exam. Budget your time wisely.
  **Do not spend too much time on an individual problem, unless you are done with all the rest.**

GOOD LUCK!

1. (12 pts) The two parts are not related.

   (a) Determine whether the following statement is **true** of **false**, and explain your answer: The set in $\mathbb{R}^3$ described in cartesian coordinates as $A = \{(x, y, z) \in \mathbb{R}^3 : z^2 = x^2 + y^2\}$ is the same as the set in $\mathbb{R}^3$ described in spherical coordinates as $B = \{(\rho, \theta, \phi) : \phi = \frac{3\pi}{4}\}$, under the usual convention $\rho \geq 0$, $\theta \in [0, 2\pi)$ and $\phi \in [0, \pi]$.

   (b) A thin lamina occupies the region

   $$D = \{(x, y) : 1 \leq x^2 + y^2 \leq 4 \text{ and } y \geq |x|\}.$$

   If the density function $\rho$ at each point $(x, y)$ is inversely proportional to the square of the distance of the point to the origin, find the moment about the $x$ axis (the $M_x$) of the lamina.

2. (8 pts) Let $f(x, y, z) = xy$. Set up **but do not evaluate** $\iiint_E f(x, y, z)dV$ in **cylindrical coordinates**, where $E$ is the solid that lies inside the sphere $x^2 + y^2 + z^2 = 9$, under the cone $z = -\sqrt{x^2 + y^2}$ and satisfies $y \leq 0$.

3. (18 pts) [You should be able to answer each part regardless of whether you have answered the other one] Let $f(x, y, z) = z$.

(a) Set up **but do not evaluate** $\iiint_E f(x, y, z)dV$ **in the order** $dxdydz$, where $E$ is the solid **in the first octant** bounded by the coordinate planes, the circular cylinder $x^2 + y^2 = 4$ and circular cylinder $x^2 + z^2 = 4$. (make sure to involve the given function in your formula!)

(b) **Evaluate** $\iiint_E f(x, y, z)dV$ using cylindrical coordinates.

4. (12 pts) Let $R$ be the trapezoid in the $xy$ plane defined by the points $(1,1)$, $(2,2)$, $(2,0)$ and $(4,0)$, as in the picture, and you are given the transformation $x = u + v$ and $y = u - v$.

(a) Compute the Jacobian determinant $\frac{\partial(x,y)}{\partial(u,v)}$.

(b) Find the inverse transformation $T^{-1}$ (that is, $u = u(x,y)$ and $v = v(x,y)$).

(c) Find the image $S$ of $R$ under $T^{-1}$ in the $uv$ plane (that is, the set $S = T^{-1}(R)$) and **draw a picture of it.**

(d) Use your work in the parts (a)-(c) to calculate $\iint_R e^{\frac{x-y}{x+y}} dA$ (you can use the back of the page if you run out of space).



4