Math 324 C - Winter 2017
Midterm 1
Friday, January 27, 2017

Name: ______________________________

Student ID Number: ______________________________

Problem 1	12	
Problem 2	8	
Problem 3	18	
Problem 4	12	
Total	50	

- There are 4 problems spanning 4 pages (your last page should be numbered as 4). Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems (unless explicitly instructed otherwise). The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely.
Do not spend too much time on an individual problem, unless you are done with all the rest.

GOOD LUCK!

1. (12 pts) The two parts are not related.

   (a) Determine whether the following statement is **true** of **false**, and explain your answer: The set in $\mathbb{R}^3$ described in cartesian coordinates as $A = \{(x, y, z) \in \mathbb{R}^3 : z^2 = x^2 + y^2\}$ is the same as the set in $\mathbb{R}^3$ described in spherical coordinates as $B = \{(\rho, \theta, \phi) : \phi = \frac{3\pi}{4}\}$, under the usual convention $\rho \geq 0$, $\theta \in [0, 2\pi)$ and $\phi \in [0, \pi]$.

   (b) A thin lamina occupies the region

$$D = \{(x, y) : 1 \leq x^2 + y^2 \leq 16 \text{ and } y \geq |x|\}.$$

   If the density function ρ at each point (x, y) is inversely proportional to the square of the distance of the point to the origin, find the y coordinate of the center of mass of the lamina (the $\bar{y}$).

2. (8 pts) Let $f(x, y, z) = xy$. Set up **but do not evaluate** $\iiint_E f(x, y, z)dV$ in **cylindrical coordinates**, where E is the solid that lies inside the sphere $x^2 + y^2 + z^2 = 9$, under the cone $z = -\sqrt{x^2 + y^2}$ and satisfies $y \leq 0$.

3. (18 pts) [You should be able to answer each part regardless of whether you have answered the other one] Let $f(x, y, z) = z$.

(a) Set up **but do not evaluate** $\displaystyle\iiint_E f(x, y, z)dV$ **in the order** $dxdydz$, where E is the solid **in the first octant** bounded by the coordinate planes, the circular cylinder $x^2 + y^2 = 4$ and circular cylinder $x^2 + z^2 = 4$. (make sure to involve the given function in your formula!)

(b) **Evaluate** $\iiint_E f(x, y, z)dV$ using cylindrical coordinates.

4. (12 pts) Let R be the trapezoid in the xy plane defined by the points (1,1), (2,2), (2,0) and (4,0), as in the picture, and you are given the transformation $x = u + v$ and $y = u - v$.

   (a) Compute the Jacobian determinant $\frac{\partial(x,y)}{\partial(u,v)}$.

   (b) Find the inverse transformation T^{-1} (that is, $u = u(x,y)$ and $v = v(x,y)$).

   (c) Find the image S of R under T^{-1} in the uv plane (that is, the set $S = T^{-1}(R)$) and **draw a picture of it.**

   (d) Use your work in the parts (a)-(c) to calculate $\iint_R e^{\frac{x-y}{x+y}}\, dA$ (you can use the back of the page if you run out of space).