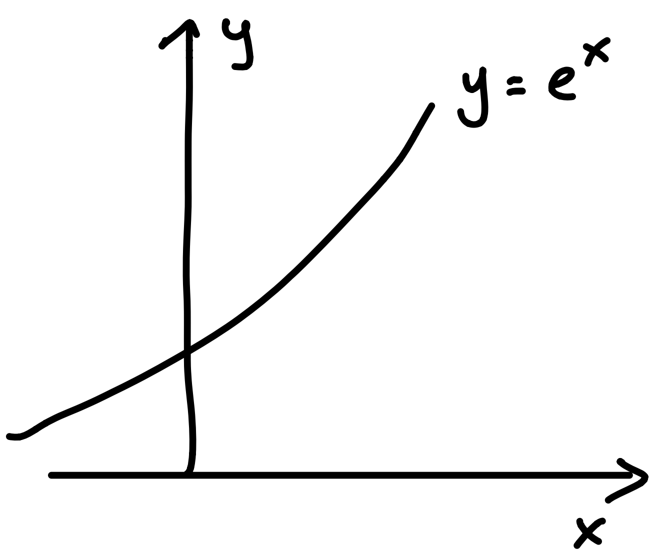
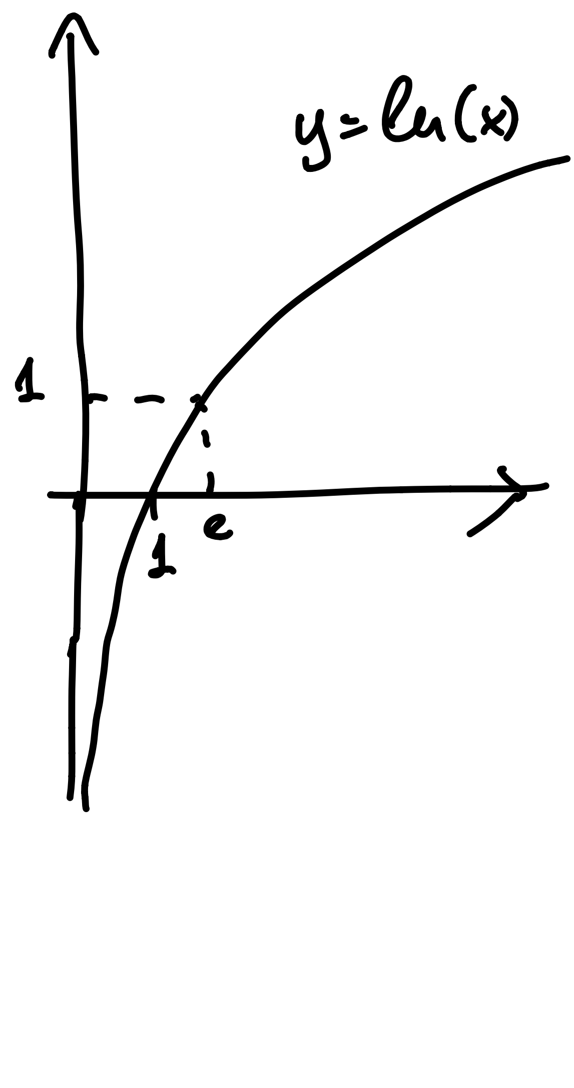
Last time: Logarithms

natural logarithm: inverse function of $y = e^x$

denoted nat. log. by $y = \ln(x)$, $x > 0$

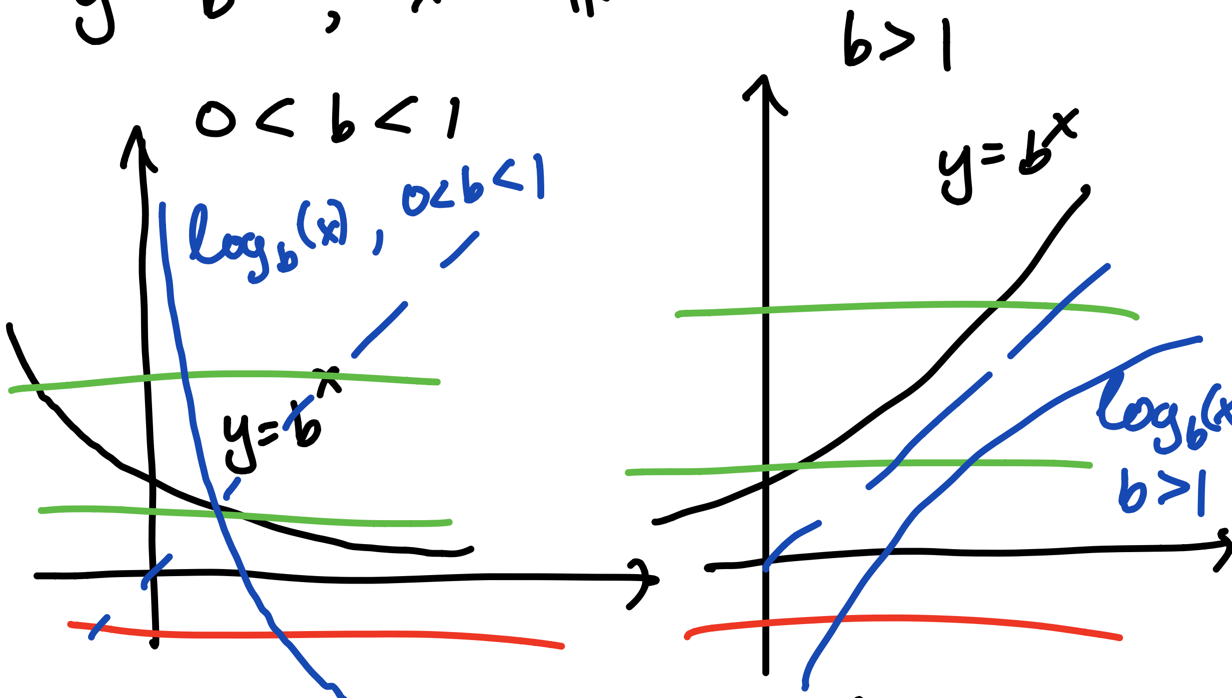
$e \sim 2.718 > 1$





<u>Today:</u> Inverse function of

$y = b^x$, $x \in \mathbb{R}$



Find inverse of $y = b^x$

$$\ln(y) = \ln(b^x)$$

$$\ln(y) = x \ln b$$

$$\Rightarrow x = \frac{\ln(y)}{\ln b}$$

Swap role of x, y

$\nearrow y = \frac{\ln(x)}{\ln(b)}$, makes sense for $x > 0$

Inverse of $y = b^x$.
Give it special name

$$y = \log_b(x)$$

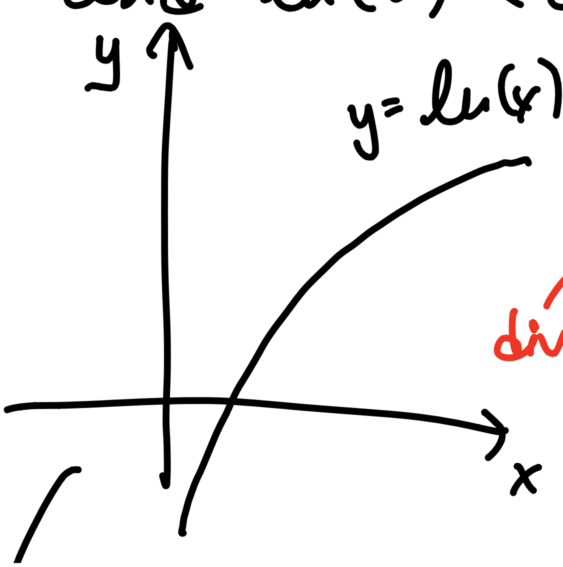
So e.g $\ln(x) = \log_e(x)$

Bad notation: log usually means $\log_{10}$

In more advanced books, might mean ln.

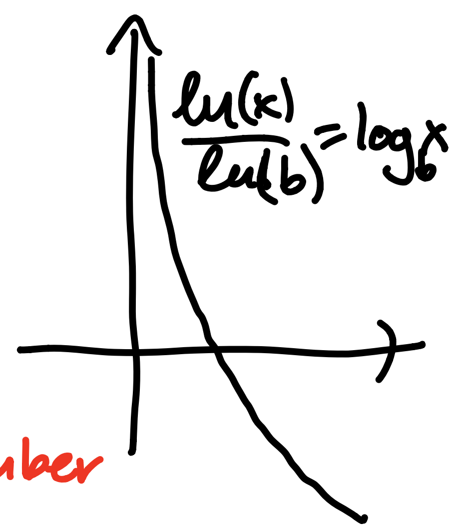
If $b < 1$, $y = \log_b(x) = \dfrac{\ln(x)}{\ln(b)}$

and $\ln(b) < 0$



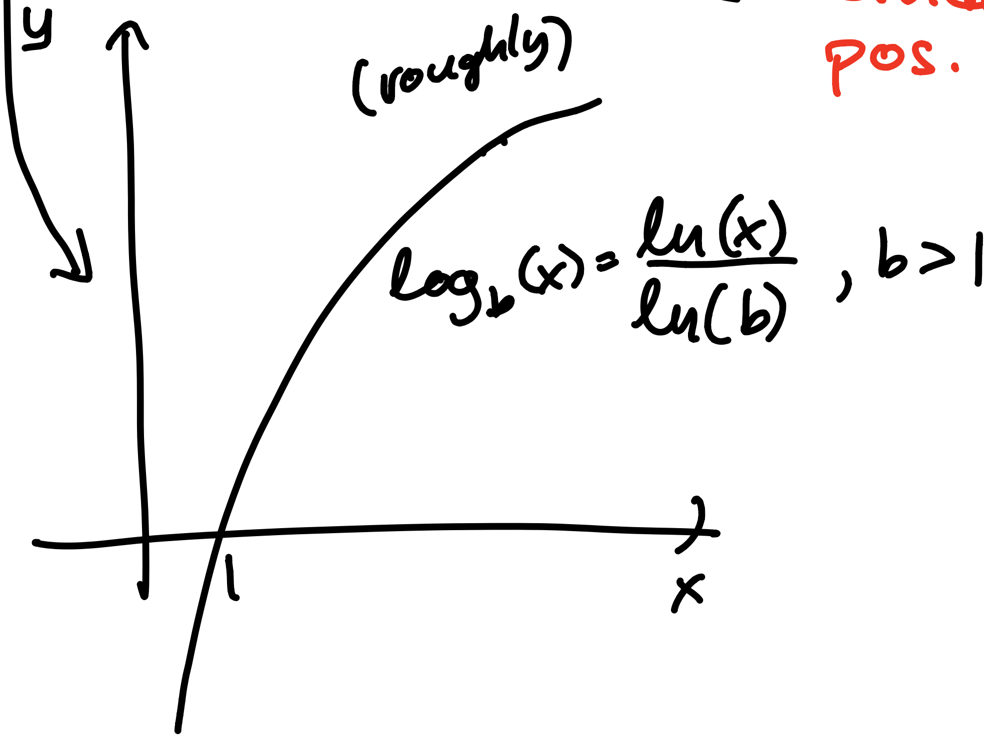
$\dfrac{\ln(x)}{\ln(b)}$

divide by neg. number



If $b>1$, $\ln(b)>0$

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$ divide by pos. number



Properties: $\underbrace{b \neq 1,\ b>0}_{\text{either } 0<b<1 \text{ or } b>1}$,

- $\log_b(b^x) = f^{-1}(f(x)) = x$,
- $b^{\log_b x} = f(f^{-1}(x)) = x$ $\quad f(x) = b^x$ $\quad$ 2
- $\log_b(r^t) = t\log_b(r)$, $r>0$

- $\log_b(rs) = \log_b(r) + \log_b(s)$
- $\log_b\left(\frac{r}{s}\right) = \log_b(r) - \log_b(s)$

Again: $\log_b(r+s) \to$ nothing

$\log_b(r) \cdot \log_b(s) \to$ nothing

Ex: Solve

$$\log_5(4) = \log_2(x)$$

$$2^{\log_5(4)} = 2^{\log_2(x)}$$

$$x = 2^{\log_5(4)}$$

Fun exercise. Use logarithmic table.

Want: $2362 \cdot 1768$

$$\log_{10}(2362 \cdot 1768)$$

$$= \log_{10}(2362) + \log_{10}(1768)$$

$$= \log_{10}(2.362 \cdot 10^3) + \log_{10}(1.768 \cdot 10^3)$$

$= \log_{10}(2.362) + \log_{10} 10^3 +$

$\log_{10}(1.768) + \log_{10}(10^3)$

$= \log_{10}(2.362) + 3 + \log_{10}(1.768) + 3$

$= 0.373 + 3 + 0.247 + 3$

$= 0.620 + 6$

Found: $\log_{10}(\cdot) = 6 + 0.620$

$2362 \cdot 1768 = 10^{\log_{10}(2362 \cdot 1768)}$

$= 10^{6 + 0.62}$

$= 10^{6 + \log_{10}(4.18)}$

$= 10^6 \cdot 10^{\log_{10}(4.18)}$

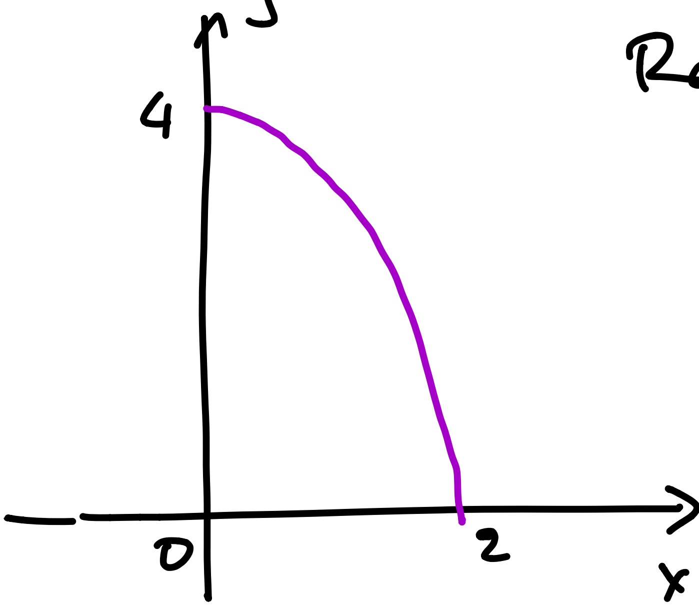
$= 10^6 \cdot 4.18$

$= 4{,}180{,}000.$

Chapter 13

Manipulating graphs, relating them to composition.

$f(x) = 4 - x^2, \quad 0 \le x \le 2$



Range: $0 \le y \le 4$

3 main techniques: - reflect
- swift
- dilate

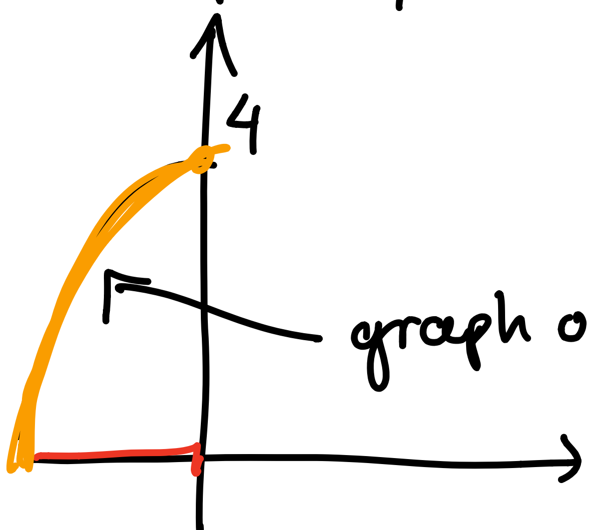
$$f(\overset{y=-x}{-x}) = 4-(-x)^2 = 4-x^2$$

Domain: $0 \le (-x) \le 2 \Rightarrow 0 \le -x \le 2$

!!! $\underbrace{0 \ge x \ge -2}$

when multiplying by negative, change direction of inequality.

Graph $f(-x)$:



$f(-x) = 4 - x^2$

1. For a function $y = f(x)$, replacing "x" with "$-x$" reflects graph about y axis.

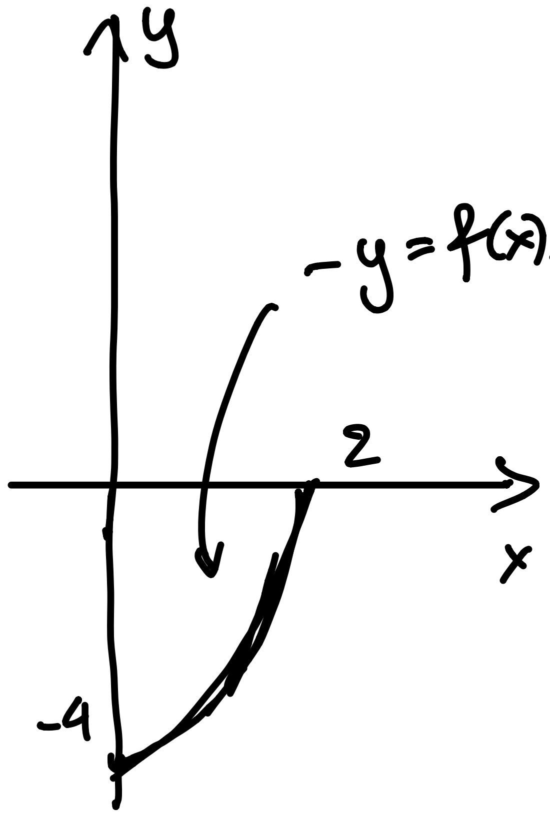
Domain: If $f(x)$ has $a \le x \le b$
then $f(-x)$: $-a \ge x \ge -b$

Range: same.

Replace y w/ $-y$ in $y = f(x) = 4 - x^2$

$-y = 4 - x^2 \Rightarrow y = x^2 - 4$

Domain: $0 \le x \le 2$



$-y = f(x)$, flips graph of $y = f(x)$ over x axis

Range:

$0 \leq (-y) \leq 4$

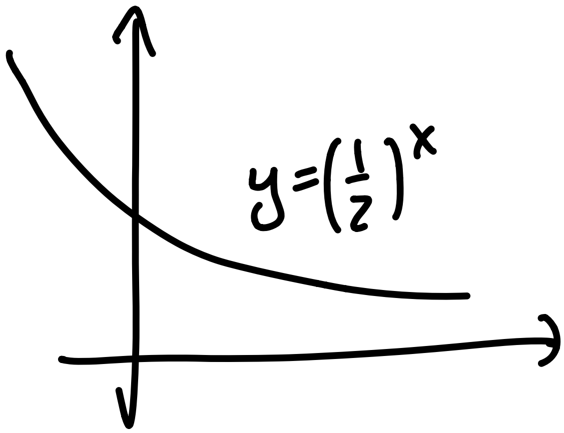
$\Rightarrow -4 \leq y \leq 0$

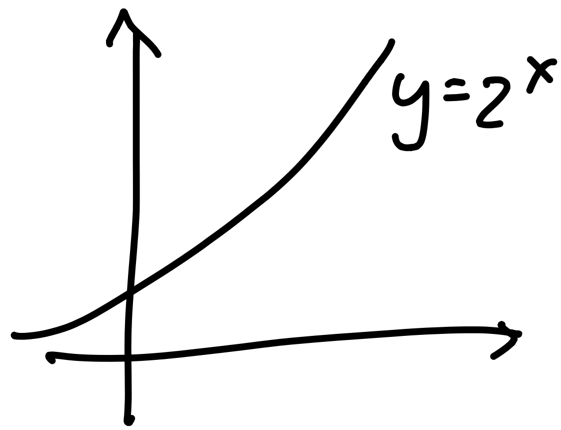
2. For $y = f(x)$, replace y w/ $-y$, the graph of $y = -f(x)$ (or $-y = f(x)$) is the graph of $y = f(x)$ reflected about the x axis.

Ex: $y = \left(\frac{1}{2}\right)^x$

$$y = \left(\frac{1}{2}\right)^{-(-x)} = \left(\left(\frac{1}{2}\right)^{-1}\right)^{-x} = 2^{-x}$$

Graph of $y = 2^{-x}$ is reflection about y axis of $y = 2^x$





Shifting $y = 4-x^2, \quad 0 \le x \le 2$

range $0 \le y \le 4$

Replace $x \longrightarrow x-2$

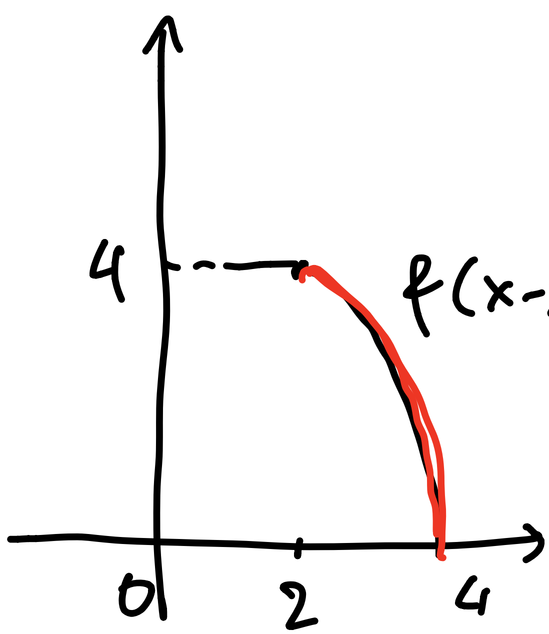
$y = 4-(x-2)^2$

vertex at $(2,4)$

Domain?

$0 \le x-2 \le 2$

$\Rightarrow 2 \le x \le 4$



$f(x-2) = 4-(x-2)^2$

Range: $0 \le y \le 4$

3. $y=f(x)$, replace x by $x-h$, shift graph by h <u>to the right</u>

If $h<0$: eg., $h=-2$
Shift graph by -2 <u>to the right</u>
or, shift by 2 <u>to the left</u>

Domain: If f has domain $a \le x \le b$

$f(x-h)$: $a \le x-h \le b$
$\Rightarrow a+h \le x \le b+h$

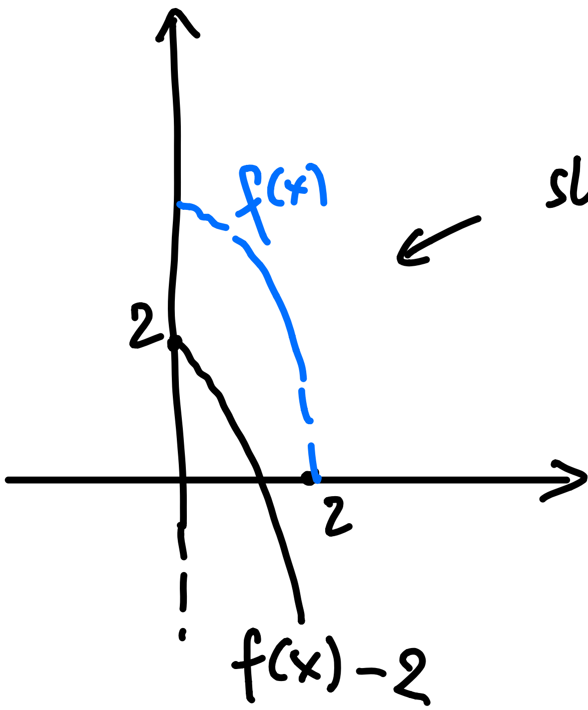
Range of $f(x-h)$: same as range of f.

<u>Vertical shift:</u> $y = 4-x^2$, $0 \le x \le 2$

Replace $y \to y+2 = y-(-2)$

$y+2 = 4-x^2 \Rightarrow y = 2-x^2$

$0 \le x \le 2$



shifted "up" by −2

4: Replace $y \to y-k$ in $y=f(x)$
graph of $y-k=f(x)$ is
graph of $y=f(x)$ shifted up
by k.