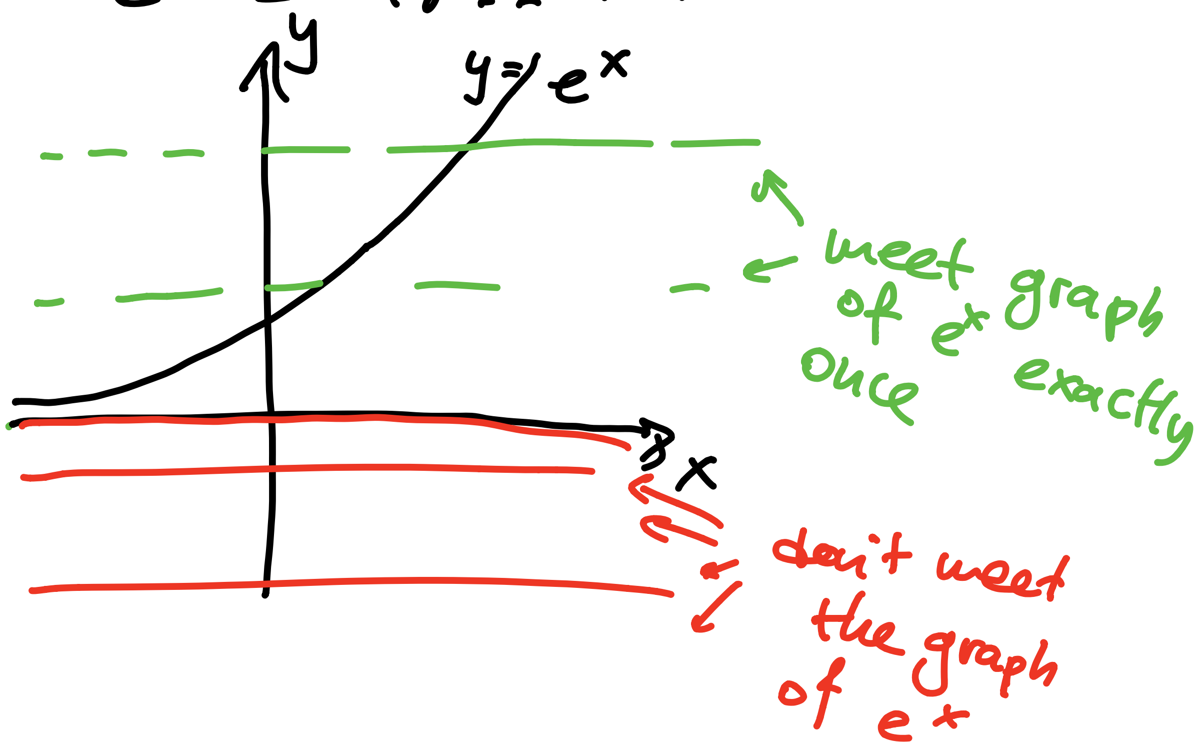
Chapter 12.

Logarithms

Goal: Invert $y = e^x$

$e \sim 2.718\ldots > 1$



Range of $e^x \to 0 < y < \infty$

Domain of inverse?

Range of original fct!

so domain of inverse

$0 < y < \infty$

Call $y = f(x) = e^x$

$f^{-1}(y) =$ sol. of $f(x) = y$, for x in the domain of f, and y in the range of f

Inverse of e^x:

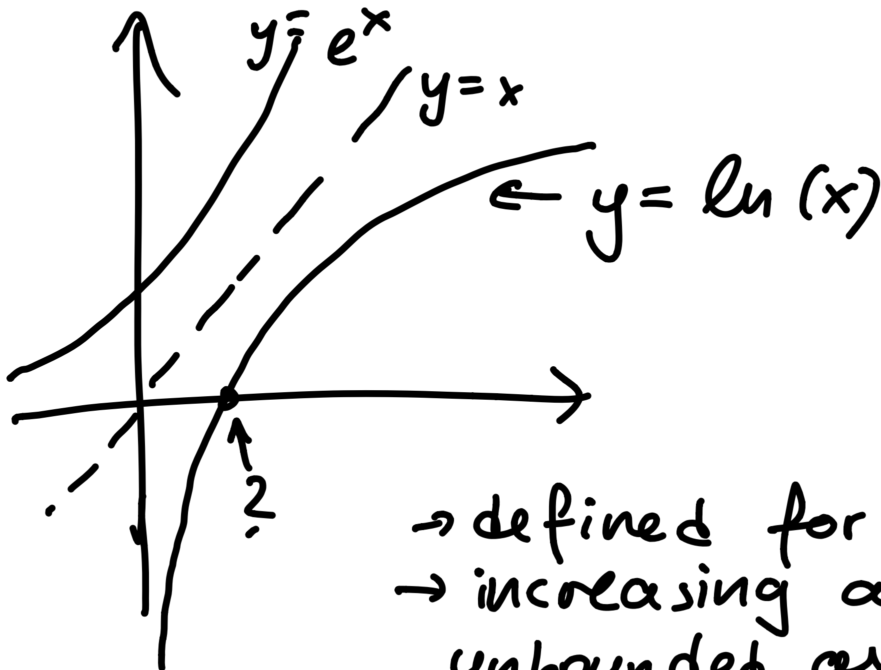
$$\ln(y) := \begin{cases} \text{the solution of } e^x = y, & \text{if } 0 < y < \infty \\ \text{undefined}, & \text{if } y \leq 0 \end{cases}$$

(bec. range of e^x is $y > 0$)

natural logarithm

(Napier)

Graph of $y = \ln(x)$



$\rightarrow$ defined for $x > 0$

$\rightarrow$ increasing and unbounded as x grows large

$\rightarrow$ approaching y axis as $x \to 0$

$\rightarrow$ Range is all of $\mathbb{R}$

x intercept: $y = 0 \Rightarrow \ln(x) = 0$

$\Rightarrow \quad f^{-1}(x) = 0 \;, \; f(x) = e^x$

$\Rightarrow \quad f(f^{-1}(x)) = f(0)$

$\Rightarrow \quad x = e^0$

$\Rightarrow \quad x = 1$

Found $\ln(1) = 0$

Side note: In questions of domain (what is the largest domain where a fct makes sense)

Restriction: - denominator $\neq 0$
- inside a root ≥ 0
- inside a logarithm > 0

Ex: largest domain where $\ln(2x+3)$ makes sense:

want: $2x+3 > 0$

$\Leftrightarrow 2x > -3$

$\Leftrightarrow x > -\frac{3}{2}$

Remark: - logarithms increase slowly.
- exponentials increase fast
- polynomials like $x^2, x^4 \ldots$ are in between.

Properties of ln.

a) $\ln e^x = f^{-1}(f(x)) = x$

so

$$\ln e^x = x$$

b) $e^{\ln(x)} = f(f^{-1}(x)) = x$

from a) $\ln e = \ln e^1 = 1$

c) $\ln b^t = t \ln b, \quad b > 0, \; t \in \mathbb{R}$

d) $\ln(ba) = \ln b + \ln a \quad a, b > 0$

e) $\ln\left(\frac{b}{a}\right) = \ln(b \cdot a^{-1})$

$$\overset{d)}{=} \ln(b) + \ln(a^{-1})$$

$$= \ln(b) + (-1)\ln(a)$$

$$= \ln(b) - \ln(a)$$

!!! $\ln(a+b) \to$ nothing

$\ln(a) \cdot \ln(b) \to$ nothing

Definitely $\ln(a+b) \neq \ln(a) + \ln(b)$

Ex: solve $6 \cdot 4^{2x+3} = 7$.

$$\ln(6 \cdot 4^{2x+3}) = \ln(7)$$

$$\overset{b)}{\Rightarrow} \quad \ln(6) + \ln(4^{2x+3}) = \ln(7)$$

$$\overset{c)}{\Rightarrow} \quad \ln(6) + (2x+3)\ln(4) = \ln(7)$$

$$\Rightarrow \quad (2x+3)\ln(4) = \ln(7) - \ln(6)$$

$$\Rightarrow \quad 2x+3 = \frac{\ln(7)-\ln(6)}{\ln(4)}$$

$$\Rightarrow \quad 2x = \frac{\ln(7)-\ln(6)}{\ln(4)} - 3$$

$$\Rightarrow \quad x = \frac{1}{2}\left(\frac{\ln\left(\frac{7}{6}\right)}{\ln(4)} - 3\right)$$

can't write as $\ln\left(\left(\frac{7}{6}\right)/4\right)$