Math 324 C - Winter 2017
Midterm 2A
Friday, February 24, 2017

Name: ______________________________

Student ID Number: ______________________________

Problem 1	8	
Problem 2	6	
Problem 3	10	
Problem 4	9	
Problem 5	8	
Problem 6	9	
Total	50	

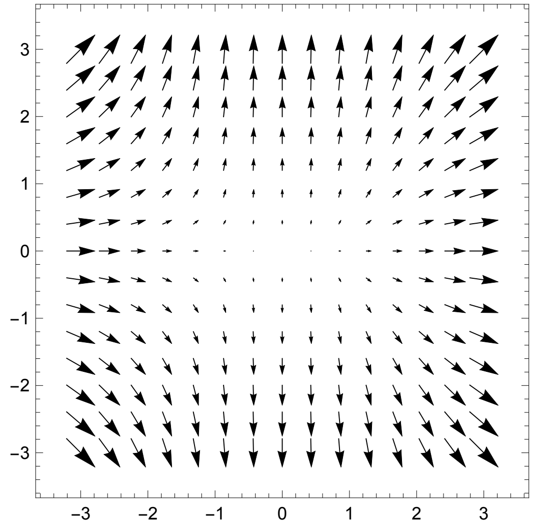
- There are 6 problems spanning 6 pages (your last page should be numbered as 6). Please make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems (unless explicitly instructed otherwise). The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the back of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely. **Do not spend too much time on an individual problem, unless you are done with all the rest.**
- You are not allowed to discuss this exam with other people until 2.00 pm today.

GOOD LUCK!

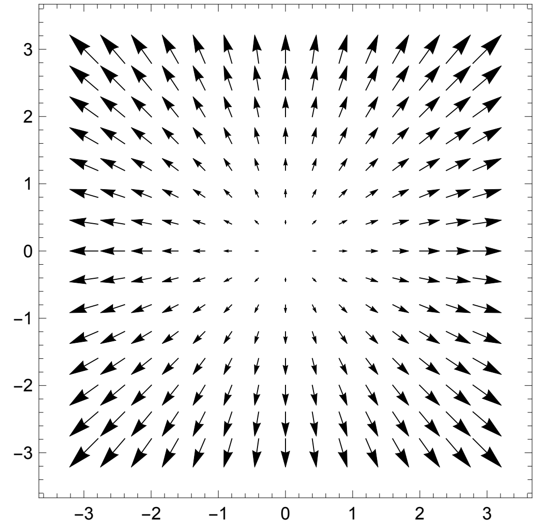
1. (8 pts) **You do not need to explain your answers for this problem.**

(a) Choose the plot that corresponds to the vector field $f(x,y) = \langle x^2, 3y \rangle$.

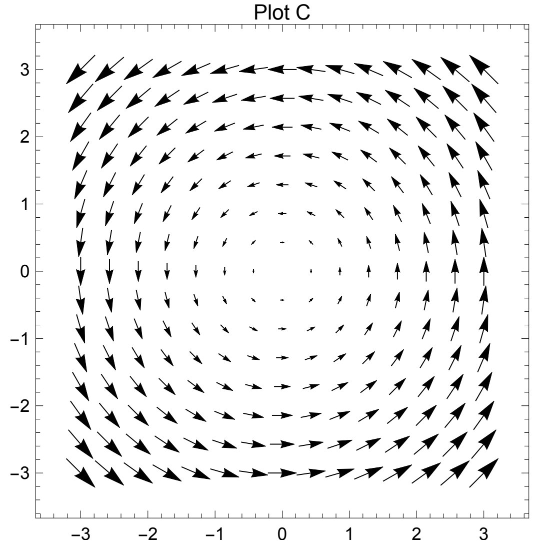
Plot A



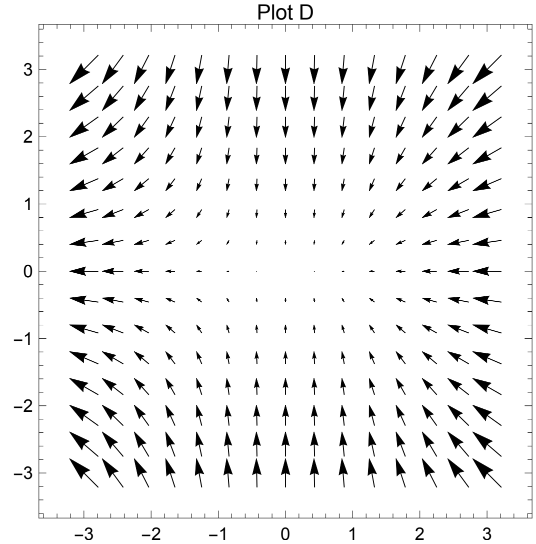
Plot B



Plot C



Plot D



(b) Mark the following sentence as **true** or **false**. Let c be the unit circle in $\mathbb{R}^2$ parametrized counterclockwise, so that $-c$ is the unit circle parametrized clockwise. Then for every scalar valued continuous function $f(x,y)$ we have

$$\int_{-c} f(x,y)ds = -\int_{c} f(x,y)ds.$$

True **False**

*****THIS PROBLEM IS CONTINUED IN THE NEXT PAGE*****

(c) Mark the correct answer: Let $\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z)\rangle$ be a vector field in $\mathbb{R}^3$, where P, Q, R have continuous second partial derivatives. Then

$$(\mathrm{div}(\mathrm{curl}\,\vec{F}))\vec{F}$$

is

a. A vector field **b.** A scalar field **c.** Undefined (nonsense)

(d) Mark the correct answer: Let $\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z)\rangle$ be a vector field in $\mathbb{R}^3$, where P, Q, R have continuous third partial derivatives. Then

$$\vec{F} \cdot \mathrm{curl}(\nabla(\mathrm{div}\,\vec{F}))$$

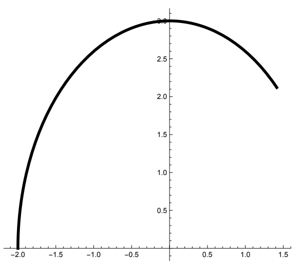
is

a. A vector field **b.** A scalar field **c.** Undefined (nonsense)

2. (6 pts) The mass of a wire lying on a curve c on the $xy-$plane and having density function $\rho(x,y)$ is given by

$$m = \int_c \rho(x,y)ds.$$

If c is the part of the ellipse $9x^2 + 4y^2 = 36$ between the points $(\sqrt{2}, \frac{3\sqrt{2}}{2})$ and $(-2, 0)$ satisfying $y \geq 0$, as in the picture, and $\rho(x,y) = y$, **set up but do not evaluate** an integral calculating the mass of the wire.



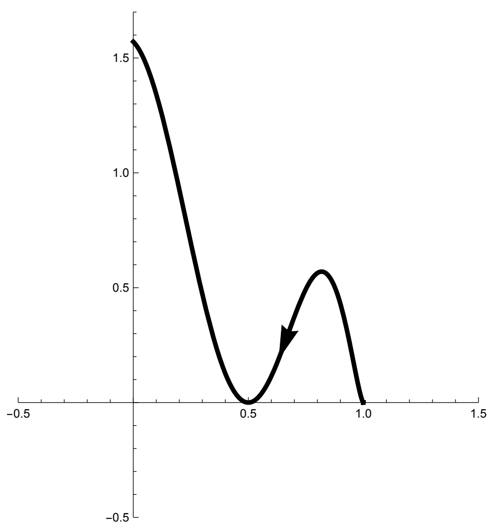
3. (10 pts) You are given the vector field

$$\vec{F}(x, y) = \langle 6\cos(y) + 2xy^3, -6x\sin(y) + 3x^2y^2 + 1\rangle$$

in $\mathbb{R}^2$ and the curve c paramatrized by $\vec{r}(t) = \langle \cos(t), t\sin^2(3t)\rangle$, $t \in [0, \frac{\pi}{2}]$ as in the picture.

a) Show that $\vec{F}$ is conservative.

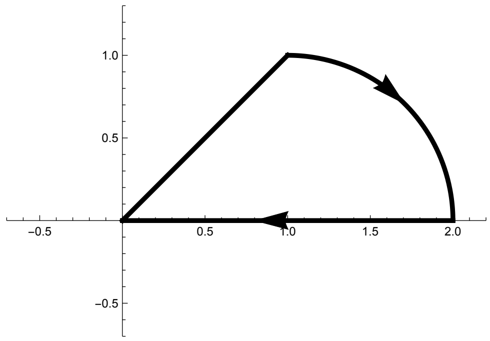
b) Evaluate $\int_c \vec{F} \cdot d\vec{r}$ (show your work clearly).



4. (9 pts) Let c be the curve of the picture, consisting of the following pieces:

- A line segment from (0,0) to (1,1)
- An arc of the circle $(x-1)^2 + y^2 = 1$ from (1,1) to (2,0) (the one that satisfies $y \geq 0$).
- A line segment from (2,0) to (0,0).

Evaluate $\int_c f(x,y)dx$, where $f(x,y) = xy^2$.



5. (8 pts) The temperature at a point (x, y) of the plane is given in degrees Celcius by

$$T(x, y) = x^2y^3 + 2\cos(3x\pi + y\pi),$$

where x and y are in meters. You are standing at the point (1,2) and you want to move towards the direction in which the temperature **drops** most rapidly (that is, the direction in which you have the minimum net rate of change of temperature).

(a) **Find a vector** that gives this direction.

(b) **Find the directional derivative of** T in the direction determined by this vector. **Make sure to include units in your answer.**

6. (9 pts) Let $z = z(x, y)$ be a twice differentiable function with continuous second partial derivatives and $x = x(t)$, $y = y(t)$ be differentiable with continuous first partial derivatives. You are given the following table with data. Use the chain rule to evaluate $\dfrac{d^2z}{dt^2}(0)$.

$x(0) = 1$	$y(0) = -1$	$z(1, -1) = -1$
$\dfrac{\partial z}{\partial x}(1, -1) = -2$	$\dfrac{\partial z}{\partial y}(1, -1) = 3$	$\dfrac{dx}{dt}(0) = 1$
$\dfrac{dy}{dt}(0) = 0$	$\dfrac{d^2x}{dt^2}(0) = 0$	$\dfrac{d^2y}{dt^2}(0) = 2$
$\dfrac{\partial^2 z}{\partial x^2}(1, -1) = -2$	$\dfrac{\partial^2 z}{\partial y^2}(1, -1) = -6$	$\dfrac{\partial^2 z}{\partial x \partial y}(1, -1) = 6$

Hint: Note that some of the partial derivatives given are 0; this may simplify your calculation.