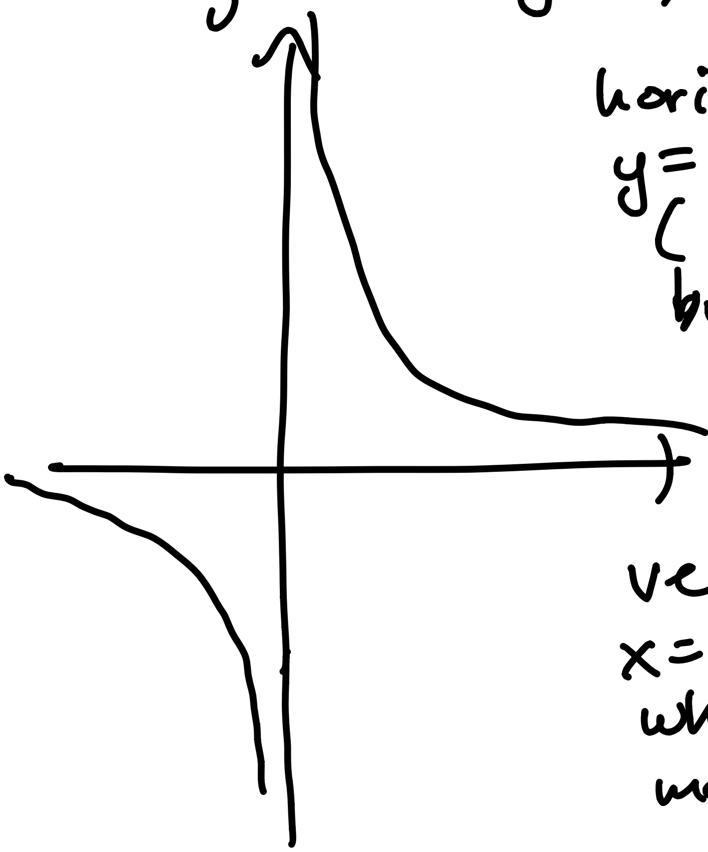
Last time: Rational fcts

linear-to-linear $\frac{ax+b}{cx+d}$

Only saw $y = \frac{1}{x}$



horizontal asymptote $y=0$ (value approached by $y=\frac{1}{x}$ when x becomes large)

vertical asymptote $x=0$ where $\frac{1}{x}$ doesn't make sense.

Q: How about other rational fcts?

Ex: $f(x) = \frac{2x+3}{5x+7}$

$$= \frac{\frac{1}{5}\cdot 2\cdot 5x+3}{5x+7}$$

$$= \frac{\frac{2}{5}(5x+7-7)+3}{5x+7}$$

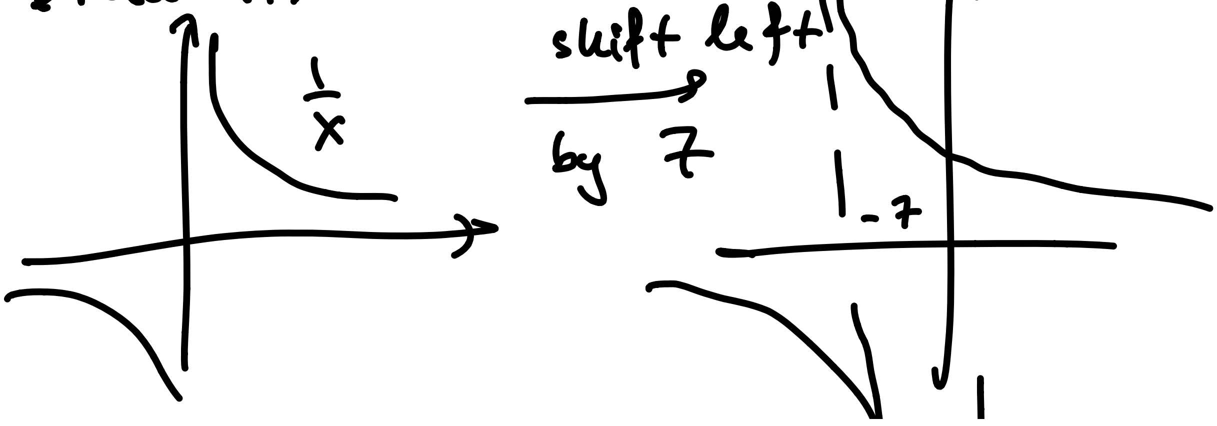
$$= \frac{\frac{2}{5}(5x+7)-7\cdot\frac{2}{5}+3}{5x+7}$$

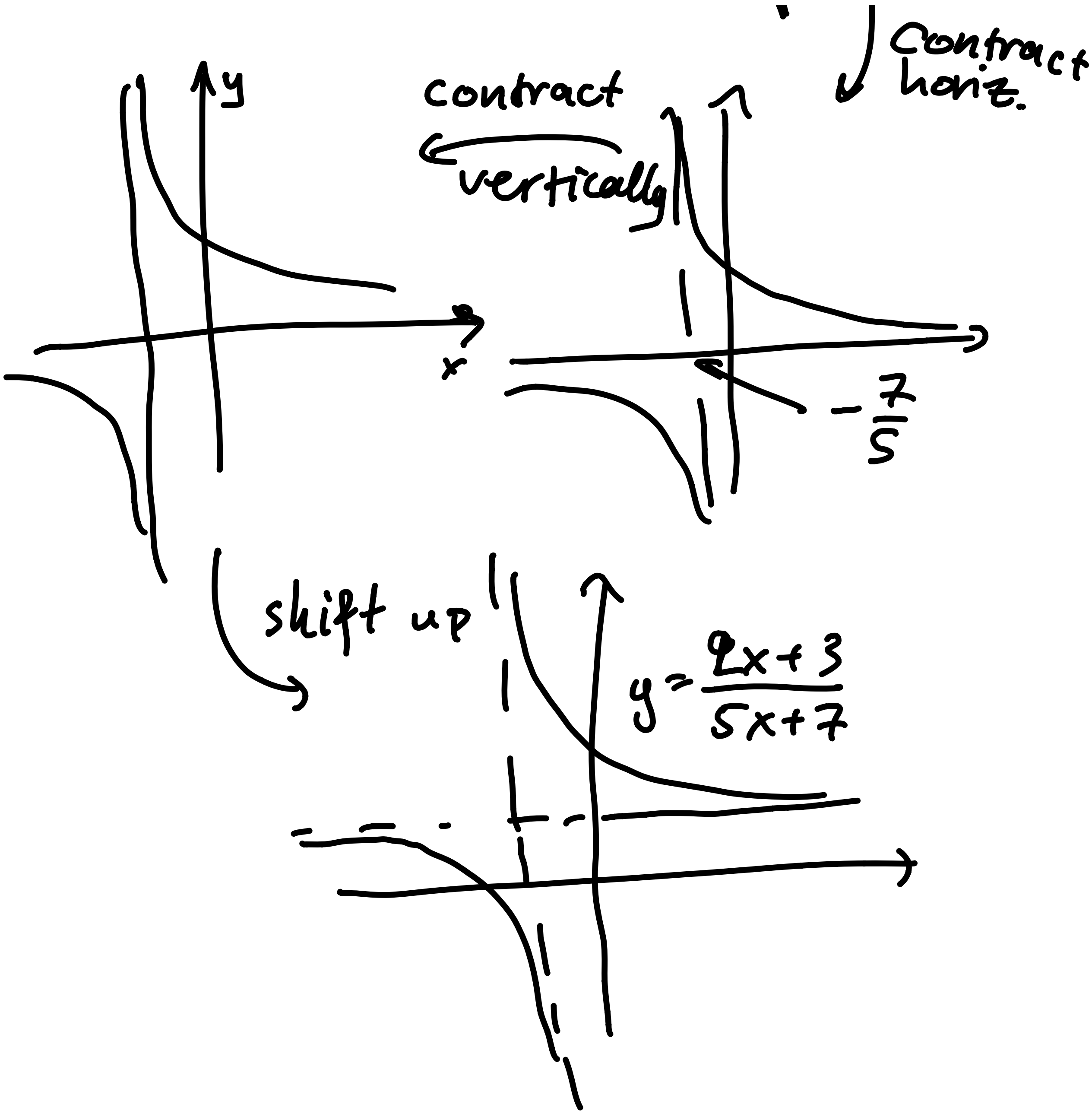
$$= \frac{2}{5}\,\frac{5x+7}{5x+7}+\frac{-\frac{14}{5}+3}{5x+7}$$

$$= \frac{2}{5}+\frac{1}{5}\,\frac{1}{5x+7}$$

$$= \frac{2}{5}+\frac{1}{5}f(5x+7), \quad f(x)=\frac{1}{x}$$

Draw it!





<u>Faster way</u>: Find the asymptotes!

Vertical asymptote: where the denominator becomes 0 (and fraction is not defined)

$$f(x) = \frac{2x+3}{5x+7}$$

Not defined when $5x+7 = 0$

$\Rightarrow x = -\frac{7}{5}$

In general: $f(x) = \frac{ax+b}{cx+d}$, vertical as. is $x = -\frac{d}{c}$

Hor. asymptote: y value when x becomes large.

$$f(x) = \frac{2x+3}{5x+7} = \frac{\frac{1}{x}(2x+3)}{\frac{1}{x}(5x+7)}$$

$$= \frac{2+\frac{3}{x}}{5+\frac{7}{x}}$$

As x becomes larger, $f(x) \to \frac{2}{5}$

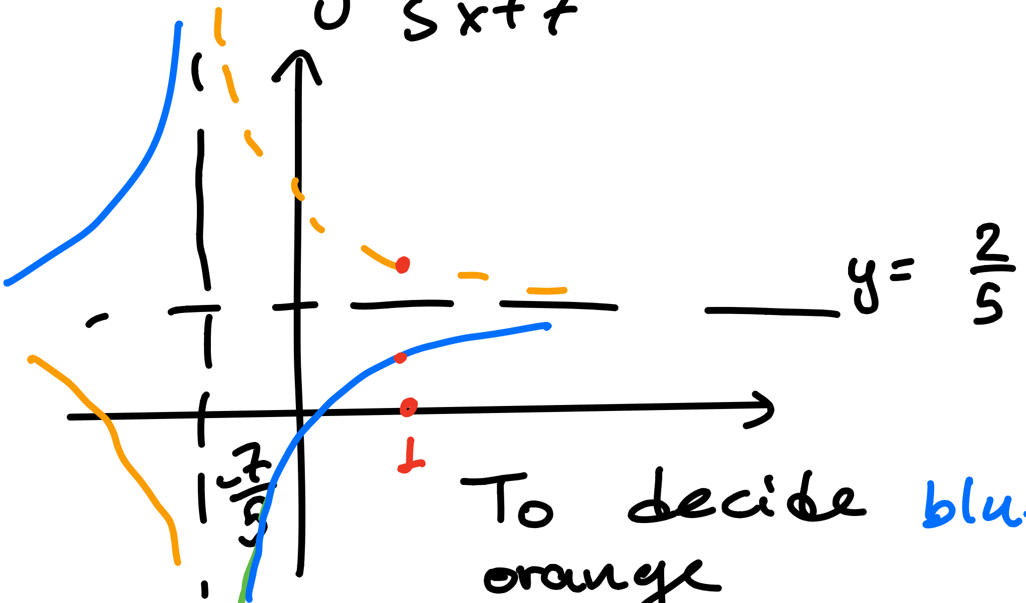
Horizontal asymptote: $y = \frac{2}{5}$

(as expected)

In general: $f(x) = \frac{ax+b}{cx+d}$

Hor. asymptote: $y = \frac{a}{c}$

Draw $y = \frac{2x+3}{5x+7}$



To decide blue or orange

Test at one point!

e.g. at $x = 1$

$$y = \frac{2+3}{5+7} = \frac{5}{12}$$

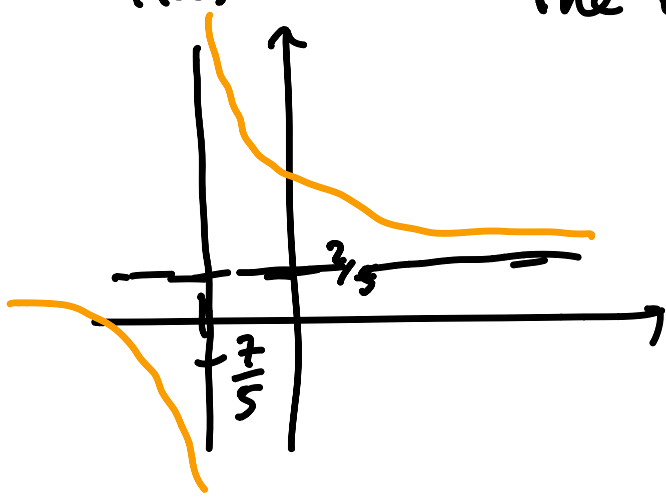
Is that above or below horizontal asymptote?

$$\frac{5}{12} \lessgtr \frac{2}{5}$$

$$\frac{25}{60} > \frac{24}{60}$$

So our point $\left(1, \frac{5}{12}\right)$ is above the hor. asymptote.

Has to be the orange one!



Use data to find linear-to-linear model.

$$f(x) = \frac{ax+b}{cx+d} = \frac{\frac{1}{c}(ax+b)}{\frac{1}{c}(cx+d)}$$

$$= \frac{\frac{a}{c}x + \frac{b}{c}}{x + \frac{d}{c}} = \frac{Ax+B}{x+D}$$

$A = \frac{a}{c}$ $B = \frac{b}{c}$

$D = \frac{d}{c}$

3 pieces of info
- 3 pts.
- 2 pts & asymptote
- 1 pt & 2 asymptotes

$$f(x) = \frac{Ax+B}{x+D}$$

asymptotes:
horizontal: $y = A$
vertical: $x = -D$

Ex:

25 yo → 85 kg

27 yo → 95 kg

28 yo → 105 kg

Find a linear-to-linear model that describes weight in terms of age.

Find $f(t) = \dfrac{At+B}{t+D}$ such that

$f(25) = 85$ kg
$f(27) = 95$ kg
$f(28) = 105$ kg

$$\frac{25A+B}{25+D} = 85 \Rightarrow 25A+B = 2125+85D \quad (1)$$

$$\frac{27A+B}{27+D} = 95 \Rightarrow 27A+B = 2565+95D \quad (2)$$

$$\frac{28A+B}{28+D} = 105 \Rightarrow 28A+B = 2940+105D \quad (3)$$

$(1)-(2)$ $25A-27A+\cancel{B}-\cancel{B} = 2125-2565+85D-95D$

$$-2A = -440-10D$$

$$10D-2A = -440 \quad (*)$$

$(3)-(2)$ $28A+\cancel{B}-27A-\cancel{B} = 2940-2565+105D-95D$

$$A = 375+10D$$

$$-10D+A = 375 \quad (**)$$

$(*)+(**) \Rightarrow -A = -65$

$$\boxed{A=65}$$

$$10D = 65-375 \Rightarrow \boxed{D=-31}$$

So: $28 \cdot 65 + B = 2940 + 105(-31)$

$B = -1{,}820 + 2{,}940 - 3{,}255$

$\boxed{B = -2{,}135}$

Finally:

$$f(t) = \frac{65t - 2135}{t - 31}$$

It ended up not being a very good model! It has vertical asymptote at $t = 31$, so my weight becomes infinitely large then!