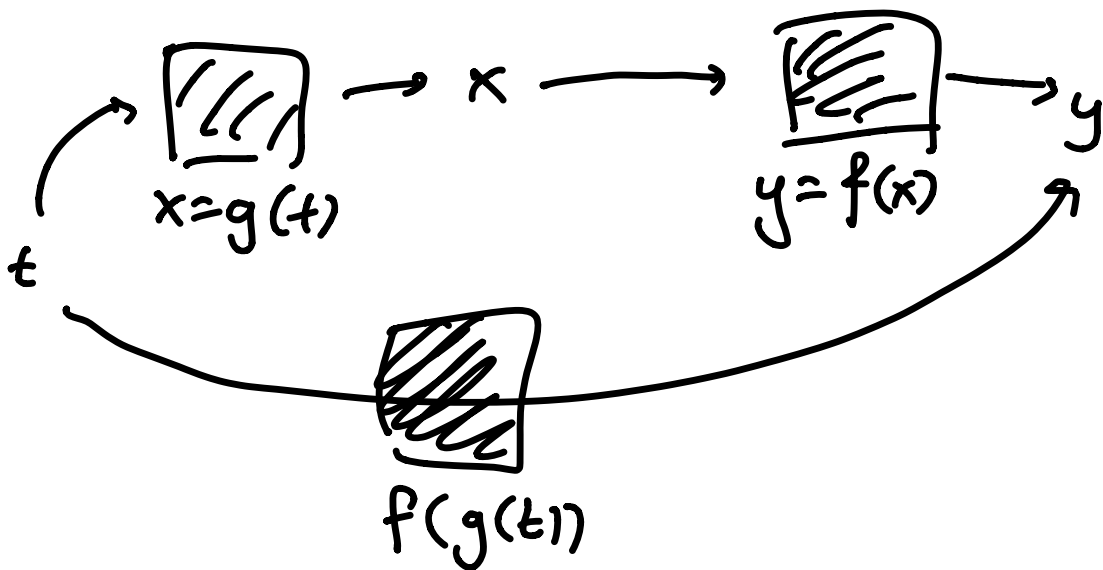


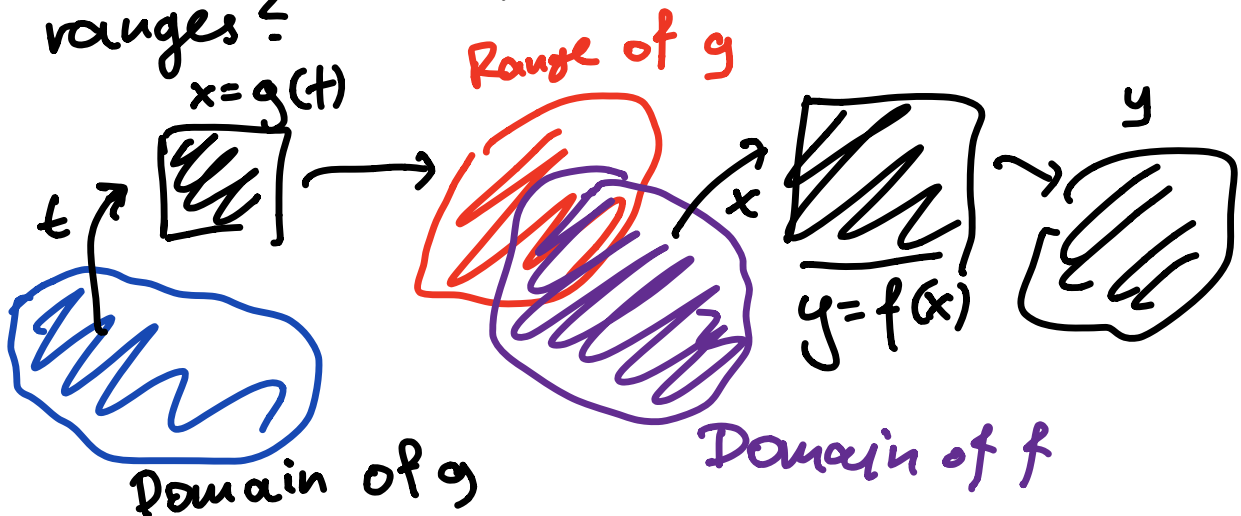
Chapter 8 (cont.)



Said: $y = f(x)$ $y = g(x)$

$f(g(x))$, $g(f(x))$ different,

Q: What happens with domains and ranges?



Ex. of this situation

$g(t) = 1 - t^2$ defined for all t in \mathbb{R}

$$f(x) = \sqrt{x}, \quad x \geq 0$$

Look at $f(g(t)) = \sqrt{1 - t^2}$

What happens if $t = 5$?

$$g(5) = -24$$

$\sqrt{-24}$ doesn't make sense!

→ How we compose 2 functions, $f(x), g(x)$
To find rule of $f(g(x))$:

Replace any occurrence of x
in expression of $f(x)$ with $g(x)$.

To find domain of $f(g(x))$:

$g(x)$ has some domain in
terms of x (e.g. $x \leq 2$) ①

$f(x)$ has some domain
in terms of x (e.g. $x > 1$) ②

Replace occurrences of
"x" in ② with $g(x)$. ③

The domain of $f(g(x))$ is the x satisfying ① and ③

Ex: $g(x) = 1 - x^2, \quad x \geq 0$

$$f(x) = \sqrt{x}, \quad x \geq 0$$

Rule: $f(g(x)) = \sqrt{1 - x^2}$

Domain: ① : $x \geq 0$

what g understands ② : $x \geq 0$

Replace "x" in ② by $g(x) = 1 - x^2$

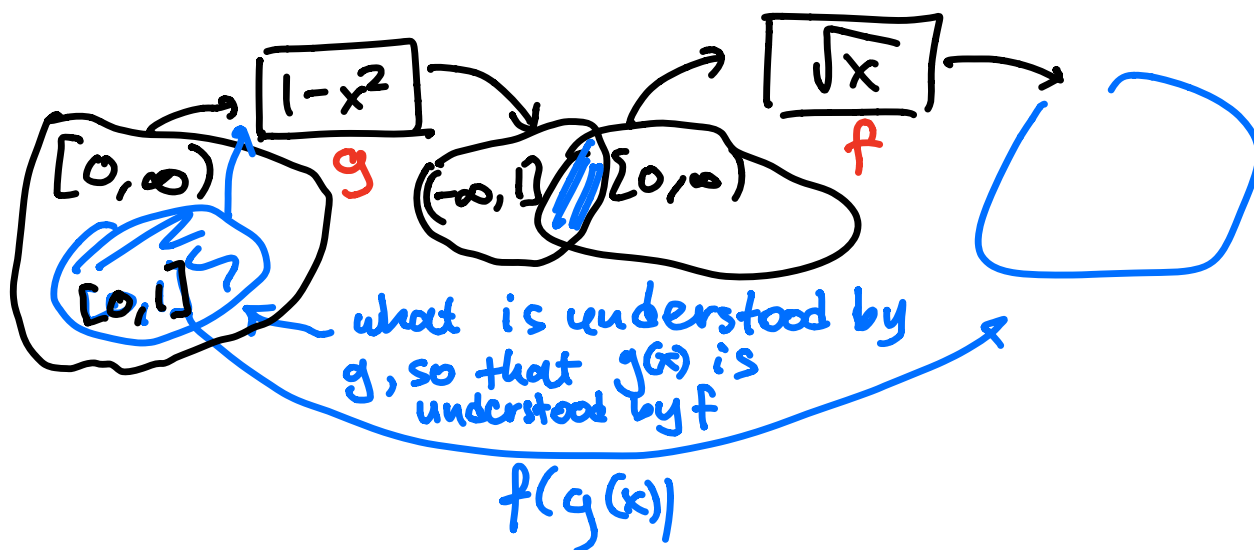
$$1 - x^2 \geq 0$$

$$x^2 \leq 1 \Rightarrow |x| \leq 1 \Rightarrow -1 \leq x \leq 1$$

This is finding what outputs of g are understood by f .

Want: ① & ③ to be true at the same time!

$$\left. \begin{array}{l} x \geq 0 \\ -1 \leq x \leq 1 \end{array} \right\} \Rightarrow 0 \leq x \leq 1 \Rightarrow x \in [0, 1] \quad \text{domain of } f(g(x))$$



Ex: It's possible that no x works!

$$f(x) = \sqrt{x}, \quad x \geq 0$$

$$g(x) = -1 - x^2, \quad x \in \mathbb{R}$$

Rule $f(g(x)) = \sqrt{-1 - x^2}$

Domain: $g(x) \geq 0 \Leftrightarrow -1 - x^2 \geq 0$

$$\Leftrightarrow x^2 \leq -1$$

can't happen!

$f(g(x))$ doesn't make sense for any x !

