Chapter 17 Circular functions

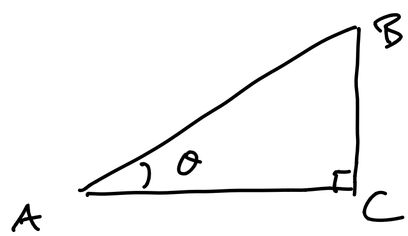
Past 2 times: angles
arc lengths
linear speed
angular speed.

In previous lectures we had to describe the position of an object on a circle and used angle for that. How can we find its x-y coordinates?

How can we find the x-y coordinates of the person given an angle?

Remember trigonometric ratios

Right triangle ABC



θ has to be an acute angle.
Define

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{|BC|}{|AB|}$$

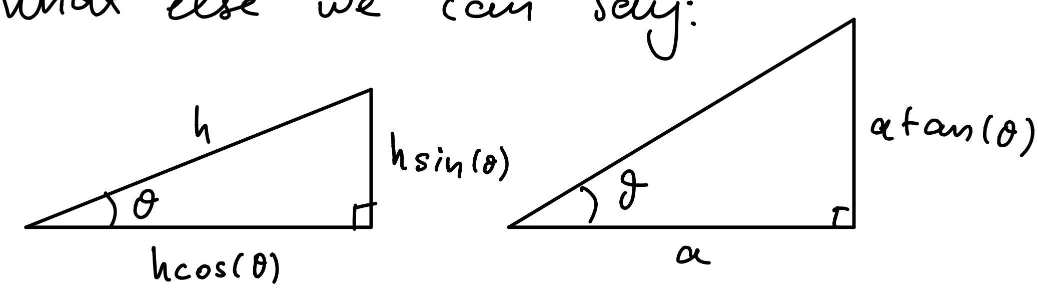
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{|AC|}{|AB|}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\text{opposite}}{\text{hypotenuse}}}{\frac{\text{adjacent}}{\text{hypotenuse}}} = \frac{\text{opposite}}{\text{adjacent}}$$

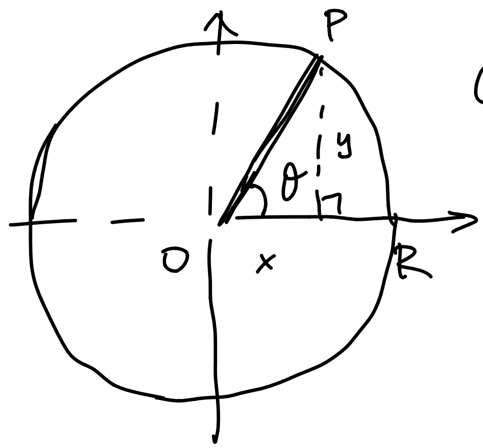
Note: By Pythagorean thm:

$$\sin^2\theta + \cos^2\theta = \frac{|BC|^2 + |AC|^2}{|AB|^2} = \frac{|AB|^2}{|AB|^2} = 1$$

What else we can say:



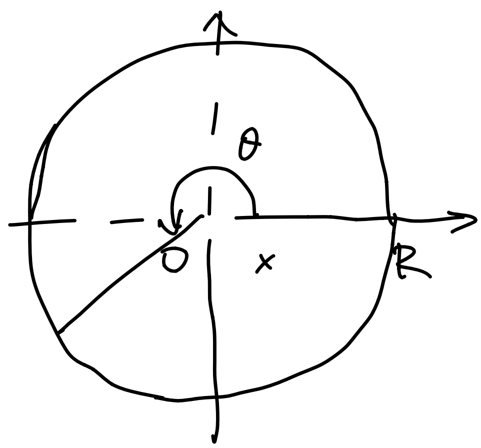
Want to define $\sin\theta$ and $\cos\theta$ for angles that aren't acute. We can't use right triangles anymore!
Draw unit circle:



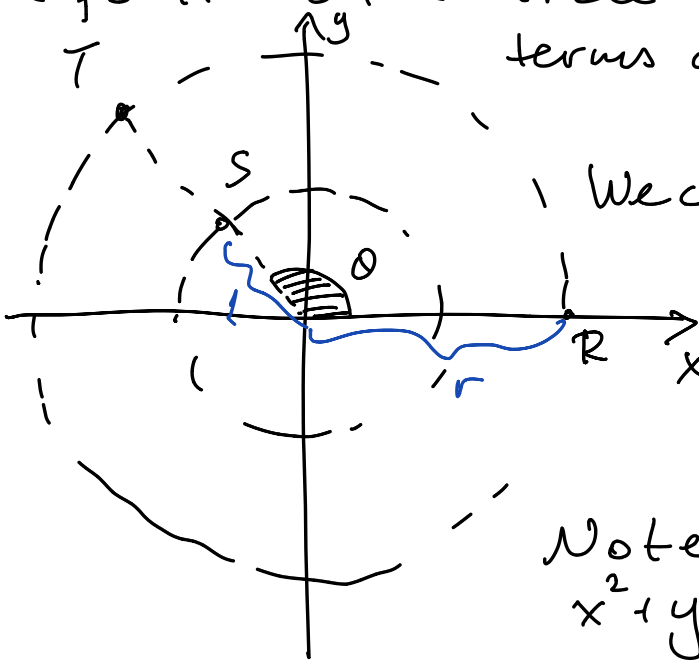
Observe that for the x and y coordinate of P we have
$y = |OP| \sin\theta = \sin\theta$
$x = |OP| \cos\theta = \cos\theta$.

Now, for any angle θ we can find a point P such that $\theta = \angle ROP$, swept counterclockwise, and define

$\sin\theta :=$ y coord of P
$\cos\theta :=$ x coord of P
(called circular functions)



Now let's go back and answer a question we had earlier: How do we find the coordinates of a point T on a circle of radius r in terms of angle $\theta = \angle ROT$?



We can find that the coords of T satisfy

$x = r\cos\theta$

$y = r\sin\theta$

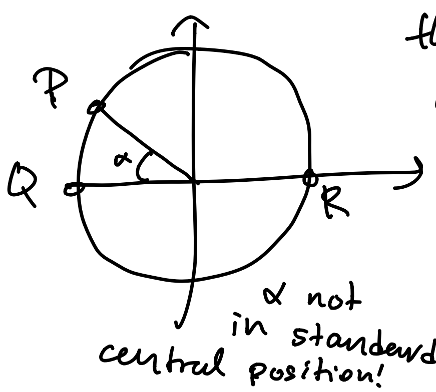
Note that

$$x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2(\cos^2\theta + \sin^2\theta) = r^2$$

Important Remarks:

→ Be careful to use degrees / radians on calculator!

→ To say that $\sin\theta, \cos\theta$ correspond to the y and x coords of a point P such that $\theta = \angle POR$ it is important that one side of the angle agrees with x axis. "standard central position"



Here we have to compute $\cos(\pi - \alpha)$ to find x coordinate of P.

The signs of cosine and sine are as follows, for θ in standard central position.

	$\cos(\theta)$	$\sin(\theta)$
1st quadrant	+	+
2nd "	−	+
3rd "	−	−
4th "	+	−

Out of the basic trigonometric functions we can create more:

$$\tan(\theta) := \frac{\sin(\theta)}{\cos(\theta)} \qquad \sec(\theta) := \frac{1}{\cos(\theta)}$$

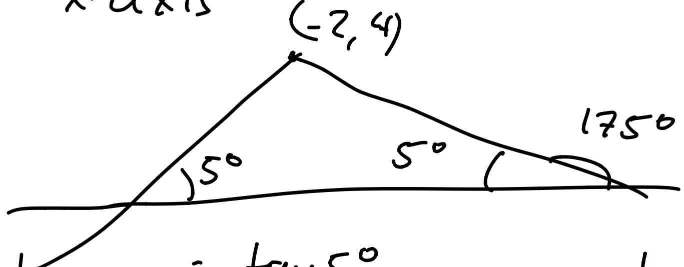
$$\cot(\theta) := \frac{\cos(\theta)}{\sin(\theta)} \qquad \csc(\theta) := \frac{1}{\sin(\theta)}$$

most important.

Note that $\tan(\theta)$ is <u>not</u> defined when $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ etc, whenever $\cos\theta = 0$.

Why is $\tan(\theta)$ important: If a line has slope $m = \tan\theta$ then it forms angle θ with the x axis.

Ex: Find eq'n of lines passing through $(-2,4)$ and making angle of 5° with x-axis



slope: $m = \tan 5^\circ$ or $m = \tan 175^\circ$

$y - 4 = \tan 5^\circ(x-(-2))$ or $y - 4 = \tan 175^\circ(x-(-2))$ $\boxtimes$

circular functions of some standard angles (good to know!)

θ(rad)	θ(°)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
$\frac{\pi}{6}$	30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{6}$	60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	90	1	0	not defined

increasing for θ between 0 and $\frac{\pi}{2}$

decreasing for θ between 0 and $\frac{\pi}{2}$

Note that you don't need to remember $\tan\theta$, can find it easily by dividing!