

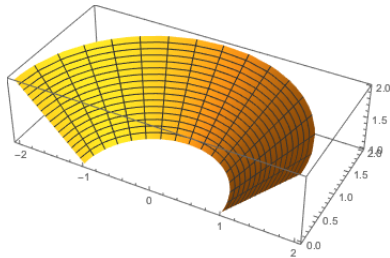
## Quiz 7

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Let  $S$  be the surface that consists of a part of the cone  $z^2 = x^2 + y^2$  that lies between the planes  $z=1$  and  $z=2$  and it satisfies  $y \geq 0$ . Find a parametrization for  $S$  and use it to compute the surface integral

$$\iint_S y dS.$$



write

$$\vec{r}(u,v) = \langle u, v, \sqrt{u^2 + v^2} \rangle, \quad v \geq 0, \quad 1 \leq \sqrt{u^2 + v^2} \leq 2$$

$$\vec{r}_u \times \vec{r}_v = \left\langle -\frac{1}{2\sqrt{u^2 + v^2}} \cdot 2u, -\frac{1}{2\sqrt{u^2 + v^2}} \cdot 2v, 1 \right\rangle$$

$$\Rightarrow |\vec{r}_u \times \vec{r}_v| = \sqrt{\frac{u^2}{u^2 + v^2} + \frac{v^2}{u^2 + v^2} + 1} = \sqrt{2}$$

$$\text{So } \iint_S y dS = \iint_{\substack{v \geq 0 \\ 1 \leq \sqrt{u^2 + v^2} \leq 2}} v \sqrt{2} dA \stackrel{\text{polar}}{=} \int_0^{\pi} \int_1^2 r \sin \theta \cdot \sqrt{2} r dr d\theta$$

$$= \sqrt{2} \cdot 2 \left. \frac{r^3}{3} \right|_1^2 =$$

$$= 2\sqrt{2} \cdot \frac{7}{3}$$