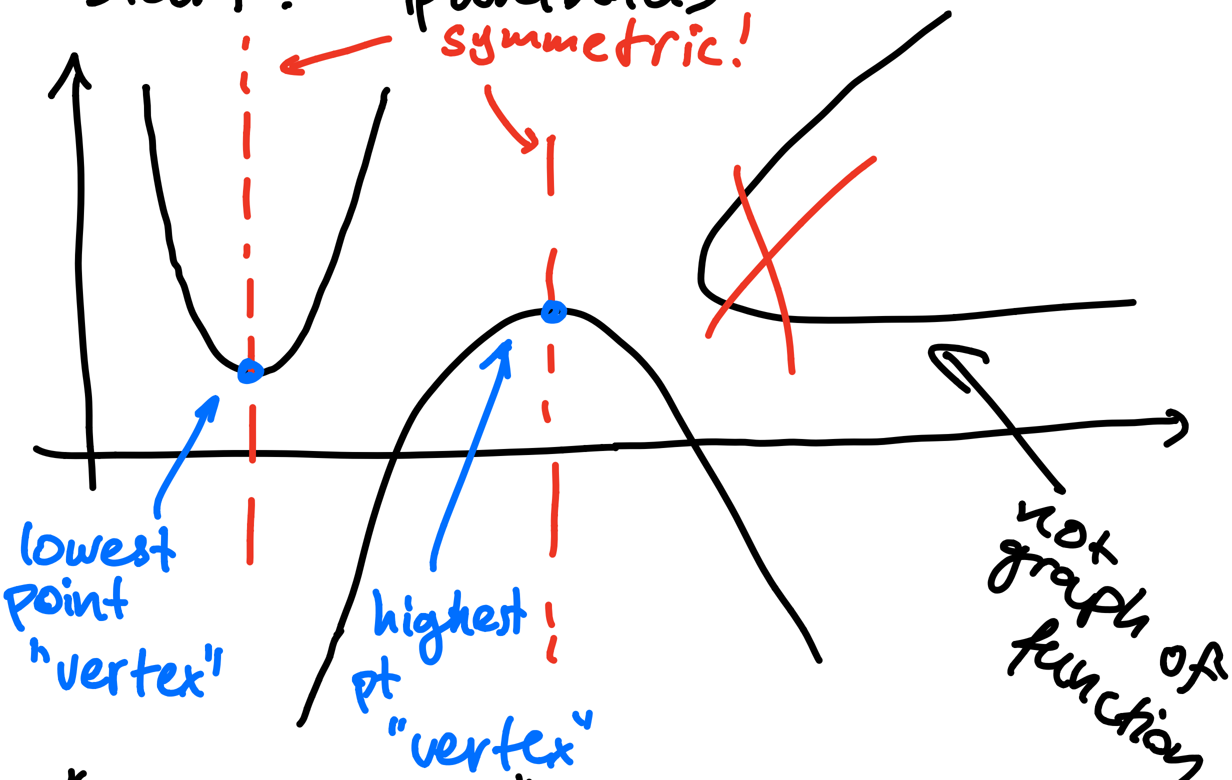
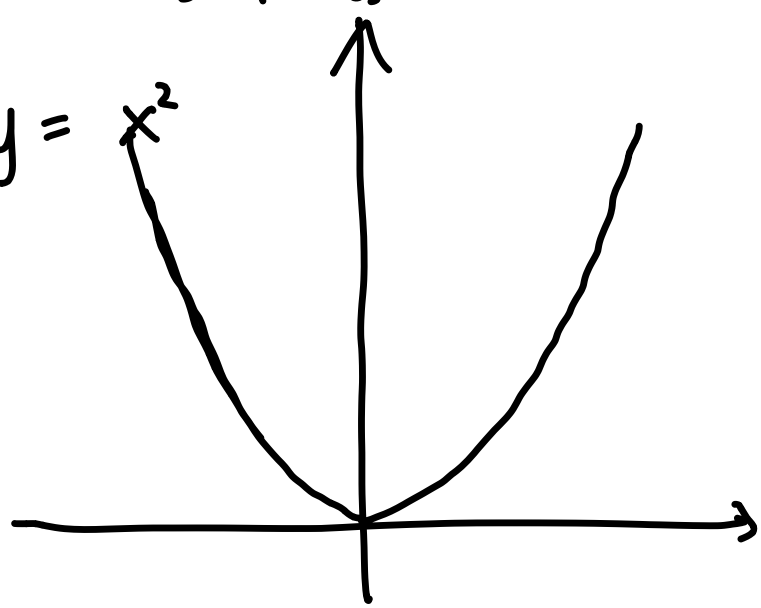
Chapter 7 Quadratic Modeling

Start: parabolas



"conic sections"

$y = x^2$

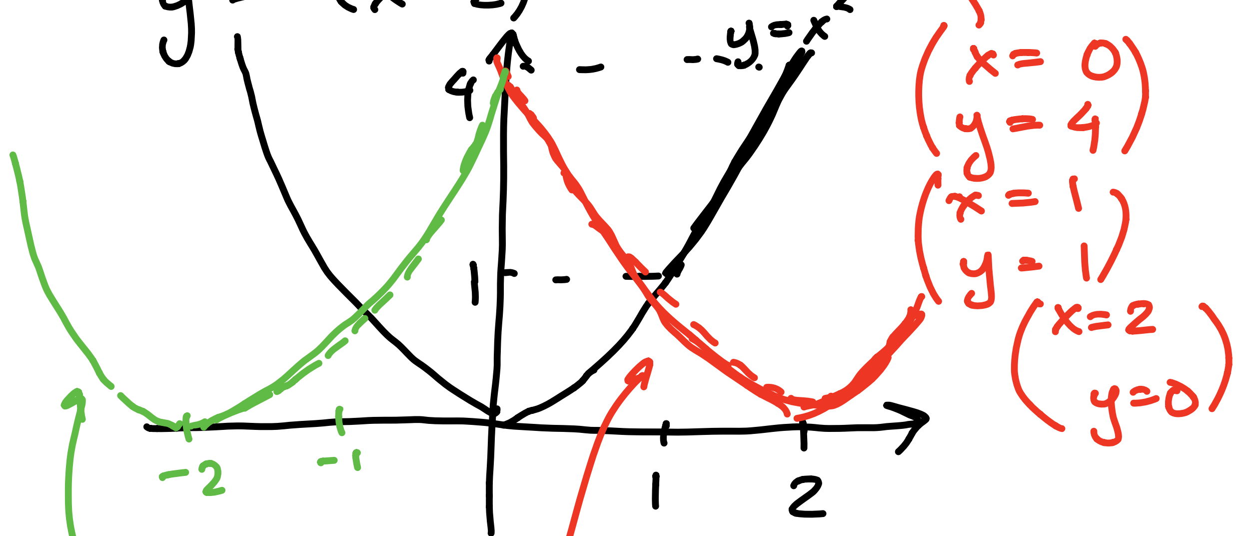


$(0,0)$ satisfies $y = x^2$

1st goal: take $y = x^2$ and move it around.
Want: to produce all parabolas by moving $y = x^2$ around.

1st operation: shifting

$y = (x-2)^2$



$y = (x-2)^2$

$y = (x+2)^2$
$= (x-(-2))^2$

1st fact: Graph of $y = (x-h)^2$

is same as graph of $y=x^2$
shifted to the right by h
Note: If $h<0$ (e.g. -2)
then we're shifting to the right by sth negative, or to the left by sth positive.

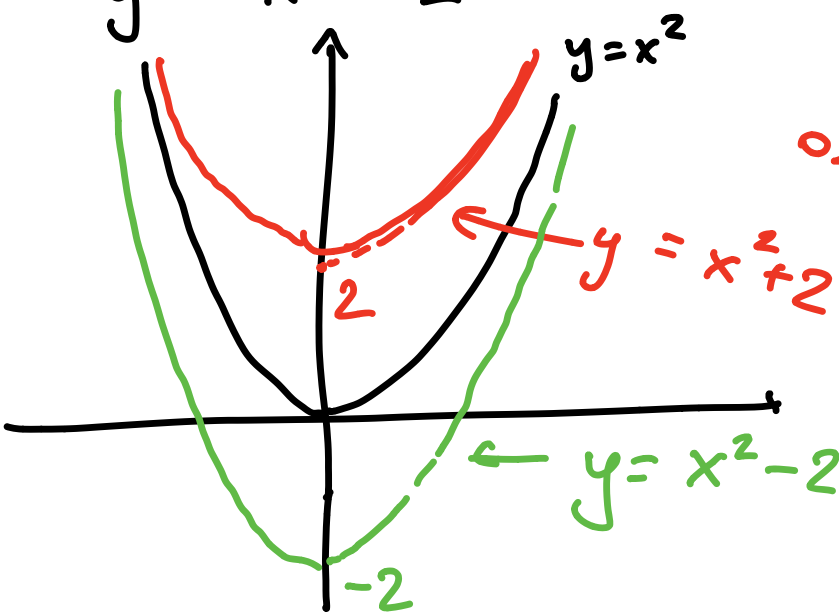
$$y=(x+2)^2=(x-(-2))^2$$

shifting to right by -2, shifting to left by 2

Shift up and down:

$$y=x^2+2$$

"adds 2 to everything on graph of $y=x^2$"



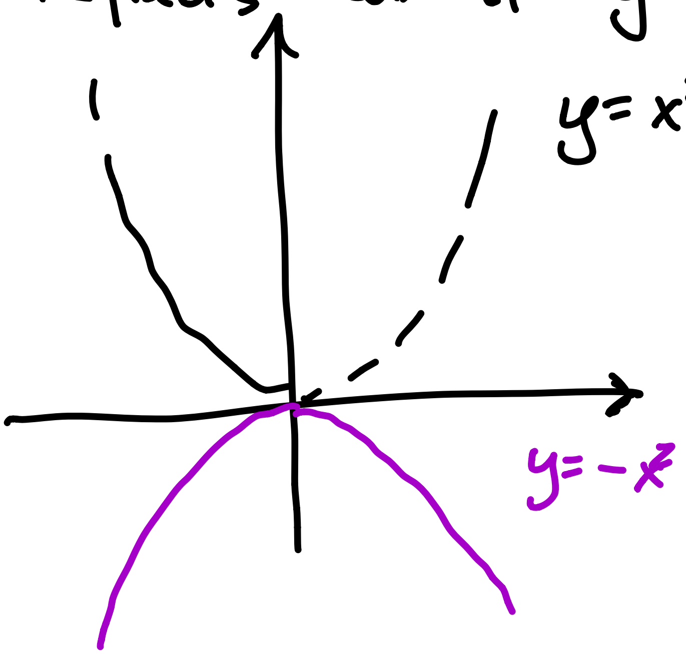
2nd fact: $y = x^2 + k$ shifts graph of $y = x^2$ <u>up</u> by k

Combine those:

$$y = (x-h)^2 + k.$$

<u>Another operation</u> <u>Reflection</u>

$y = f(x)$, taking $y = -f(x)$ reflects about y axis.



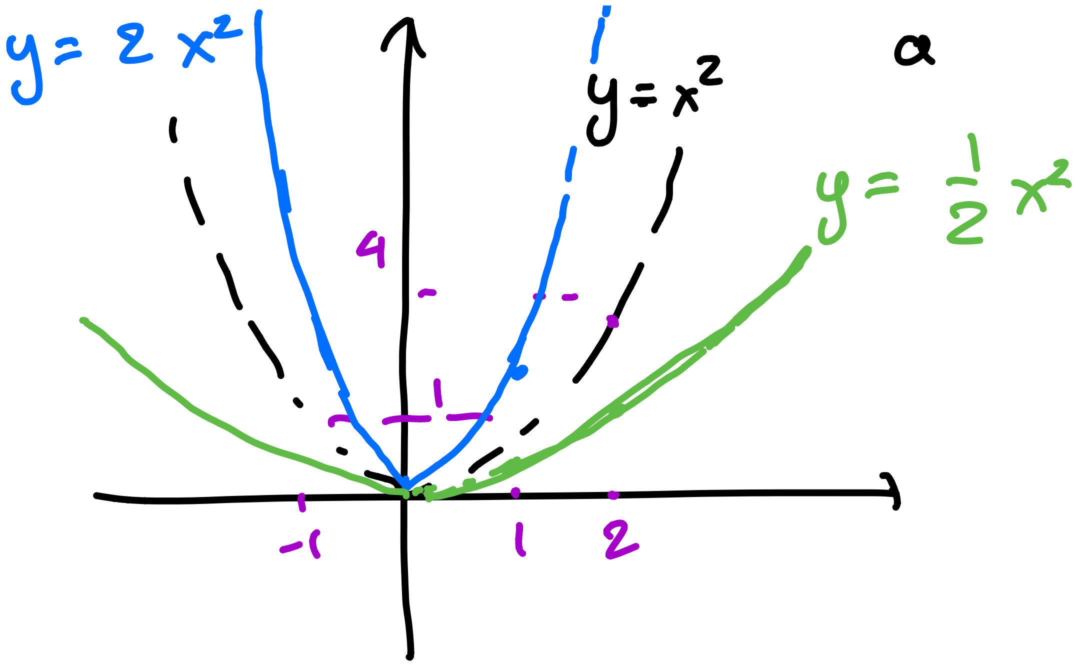
Last one: Vertical dilation:

$$y = a x^2$$

$a >$ (if $a = 0$ not parabola!)

Taking $a > 1$: expands vertically

$0 < a < 1$: contracts vertically



Conclusion: A parabola is the graph of

$$y = f(x) = a(x-h)^2 + k$$

h, k, a const.
$a \neq 0$

if $a > 0$ then we're opening up

if $a < 0$ we're opening down (first dilating by $|a|$, then reflecting)

$y = a(x-h)^2 + k$ is the vertex

form of the parabola.

why called like that: can easily read vertex off of this formula

Vertex: (h,k).

Ex: with what operations we can get from $y=x^2$ to

$$y=-3(x-1)^2+2$$

$$y=x^2 \xrightarrow[\text{1 to right}]{\text{shift by}} y=(x-1)^2$$

$$y=(x-1)^2 \xrightarrow{\text{dilate (shift later)}} y=3(x-1)^2$$

$$y=3(x-1)^2 \xrightarrow{\text{reflect}} y=-3(x-1)^2$$

$$y=-3(x-1)^2 \xrightarrow{\text{shift up by 2}} y=-3(x-1)^2+2$$