Math 120 B

Midterm 2

Thursday, May 10, 2018

Name: ______________________________

UW email address: ______________________________

Problem 1	8	
Problem 2	6	
Problem 3	8	
Problem 4	10	
Problem 5	8	
Problem 6	10	
Total	50	

- There are 6 problems spanning 5 pages (your last nonempty page should be numbered as 5). Please make sure your exam contains all these questions.
- You are allowed to use a TI-30x IIS Calculator and one double sided hand-written 8.5 by 11 inch page of notes.
- You must show your work on all problems (unless explicitly instructed otherwise). The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- Scratch paper is available. Please do not use your own.
- You have 80 minutes to complete the exam. Budget your time wisely. **Do not spend too much time on an individual problem, unless you are done with all the rest.**

GOOD LUCK!

1. (8 pts.) For each of the following questions, solve for x. If there are multiple solutions, find all of them. Leave your answers in exact form.

   (a) $\log_4(2x+4) = 3$

$$\log_4(2x+4) = 3 \Rightarrow \quad 4^{\log_4(2x+4)} = 4^3$$

$$\Rightarrow 2x+4 = 64$$

$$\Rightarrow 2x = 60$$

$$\Rightarrow x = 30$$

   (b) $e^{5|x+2|+1} = 12$

$$e^{5|x+2|+1} = 12 \Rightarrow \ln\left(e^{5|x+2|+1}\right) = \ln 12$$

$$\Rightarrow 5|x+2|+1 = \ln 12$$

$$\Rightarrow |x+2| = \frac{\ln 12 - 1}{5}$$

$$\Rightarrow x+2 = \pm \frac{\ln 12 - 1}{5}$$

$$\Rightarrow x = -2 \pm \frac{\ln 12 - 1}{5}$$

2. (6 pts.) Let $f(x) = x^4 + 5^{x-1}$. Suppose we take the graph of $y = f(x)$ and do three things:

- First, shift it 3 units down.
- Then, stretch it horizontally by a factor of 2
- Then, reflect it with respect to the $x-$axis

Write a function $g(x)$ for the new transformed graph.

$$y = x^4 + 5^{x-1} \xrightarrow{y \to y+3} y = x^4 + 5^{x-1} - 3$$

$$\xrightarrow{x \to \frac{x}{2}} \quad y = \left(\frac{x}{2}\right)^4 + 5^{\frac{x}{2}-1} - 3 \xrightarrow{y \to -y}$$

$$y = -\left(\frac{x}{2}\right)^4 - 5^{\frac{x}{2}-1} + 3$$

3. (8 pts) Write a multipart rule for the function $f(g(x))$, where $f(x) = |4x - 3|$ and $g(x) = 2x + 1$

$$f(x) = \begin{cases} 4x-3, & 4x-3 \geq 0 \\ -4x+3, & 4x-3 < 0 \end{cases}$$

$$\Rightarrow f(g(x)) = \begin{cases} 4(2x+1)-3, & 4(2x+1)-3 \geq 0 \\ -4(2x+1)+3, & 4(2x+1)-3 < 0 \end{cases}$$

$$\Rightarrow f(g(x)) = \begin{cases} 8x+1, & 8x+1 \geq 0 \\ -8x-1, & 8x+1 < 0 \end{cases}$$

$$= \begin{cases} 8x+1, & x \geq -\frac{1}{8} \\ -8x-1, & x < -\frac{1}{8} \end{cases}$$

4. (10 pts.) You are given the function $f(x) = \sqrt{4 - x^2}$, $-2 \leq x \leq 0$.

(a) Draw the graph of $y = f(x)$ on Grid 1.

$y = \sqrt{4-x^2} \Rightarrow y^2 + x^2 = 4$

(b) Find a formula for $y = f^{-1}(x)$.

$y = \sqrt{4-x^2} \Rightarrow y^2 = 4 - x^2 \Rightarrow x^2 = 4 - y^2$

$\Rightarrow x = \pm\sqrt{4-y^2}$

$\Rightarrow x = -\sqrt{4-y^2}$ bec. $x \leq 0$.

So $y = -\sqrt{4-x^2}$

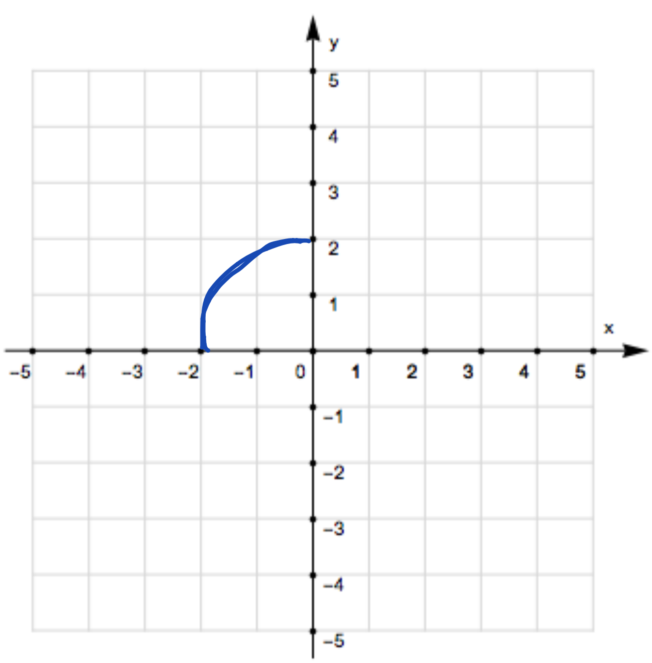
(c) Find the domain of the function $y = f^{-1}(x)$

Range of $f(x)$, so $0 \leq x \leq 2$

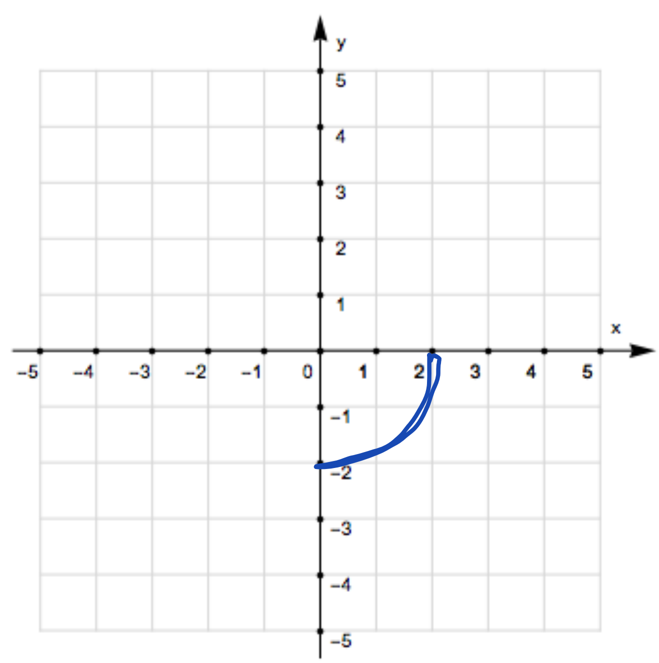
(d) Find the range of the function $f^{-1}(x)$

Domain of $f(x)$, so $-2 \leq x \leq 0$

(e) Draw the graph of the function $y = f^{-1}(x)$ on Grid 2.



Grid 1



Grid 2

5. (8 pts.) Lucky's owner (from quiz 4) decided to use a linear-to-linear rational function to model Lucky's weight. She notices that he weighed 1 lb when he was born, 9 lb when he was 18 months old and 11 lb when he was 2 years old.

(a) Find a linear-to-linear rational function $f(t)$ giving Lucky's weight when he is t years old.

Lucky

$f(t) = \frac{At+B}{t+C}$

(1) $f(0)=1 \Rightarrow \frac{B}{C} = 1 \Rightarrow B = C$

(2) $f(1.5) = 9 \Rightarrow \frac{A \cdot 1.5 + B}{1.5 + C} = 9 \Rightarrow A \cdot 1.5 + B = 13.5 + 9C$

(3) $f(2) = 11 \Rightarrow \frac{A \cdot 2 + B}{2 + C} = 11 \Rightarrow 2A + B = 22 + 11C$

(1),(2) $\Rightarrow A \cdot 1.5 - 8C = 13.5 \Rightarrow 3A - 16C = 27$ (4)

(1),(3) $\Rightarrow A \cdot 2 - 10C = 22 \Rightarrow 3A - 15C = 33$ (5)

(4)−(5) $\Rightarrow -C = -6 \Rightarrow C = 6$ so $B = 6$, $A = \frac{33 + 15 \cdot 6}{3}$

$\Rightarrow A = 41$

$f(t) = \frac{41t + 6}{t + 6}$

(b) What is Lucky's weight going to be when he is 10 years old according to her model?

$f(10) = \frac{41 \cdot 10 + 6}{10 + 6} = 26$ lb

(c) What weight is Lucky approaching as he gets older (assuming that he lives for a very long time)?

Find hor. asymptote:

$y = \frac{41}{1}$ so approaches 41 lbs

6. (10 pts.) (Ostroff Aut. 16) The rent for a one-bedroom apartment in Beattle is growing exponentially (Even though the city is filled with bees).

(a) In the year 2000, the rent in Beattle was \$1020, and it increases by 2.3% per year. Write a function $f(t)$ for the rent in Beattle t years after 2000.

$$f(t) = 1020 \cdot (1.023)^t$$

(b) The average monthly rent in Tickoma is also growing exponentially.

In the year 2007, the rent in Tickoma was \$500 less than the rent in Beattle.

In the year 2016, the rent in Tickoma is \$1000.

Write a function $g(t)$ for the rent in Tickoma t years after 2000.

$$f(7) = 1020 \cdot (1.023)^7 = 1{,}196$$

$$g(7) = 1{,}196 - 500 = 696$$

$$g(16) = 1000$$

$$g(t) = A_0 b^t$$

$$\left.\begin{array}{l} g(7) = 696 \Rightarrow A_0 b^7 = 696 \\ g(16) = 1000 \Rightarrow A_0 b^{16} = 1000 \end{array}\right\} \frac{A_0 b^{16}}{A_0 b^7} = \frac{1000}{696} \Rightarrow b^9 = 1.436$$

$$\Rightarrow b = 1.436^{\frac{1}{9}}$$

$$\Rightarrow b = 1.041$$

so $A_0 = 696 \cdot b^{-7} \Rightarrow A_0 = 696 \dfrac{1}{(1.436)^{7/9}} = 525.$

$$\boxed{g(t) = 525 \cdot (1.041)^t}$$

(c) When will the rents in Beattle and Tickoma be equal? (Round your answer to the nearest year.)

$$g(t) = f(t) \Rightarrow$$

$$1020 \cdot (1.023)^t = 525(1.041)^t$$

$$\Rightarrow \ln(1020) + t\ln(1.023) = \ln 525 + t\ln(1.041)$$

$$\Rightarrow t(\ln(1.023) - \ln(1.041)) = \ln(525) - \ln(1020)$$

$$\Rightarrow t\ln\left(\frac{1.023}{1.041}\right) = \ln\frac{525}{1020} \Rightarrow t\ln(0.983) = \ln(0.514)$$

$$t = \frac{-.66}{-.017} \Rightarrow t = 38 \text{ years}$$

so in 2038.