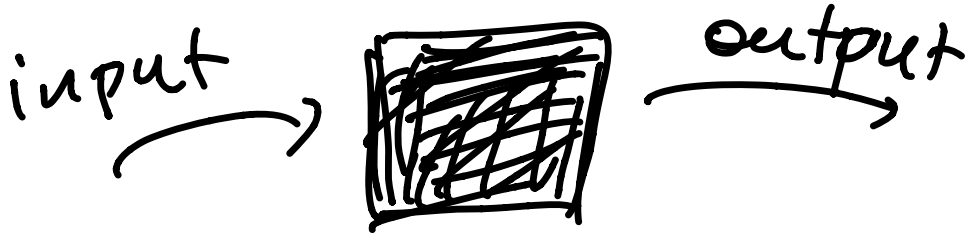
Functions (Ch.5)

What is a fct:

A procedure that returns a unique output for any allowable input.

"black box"



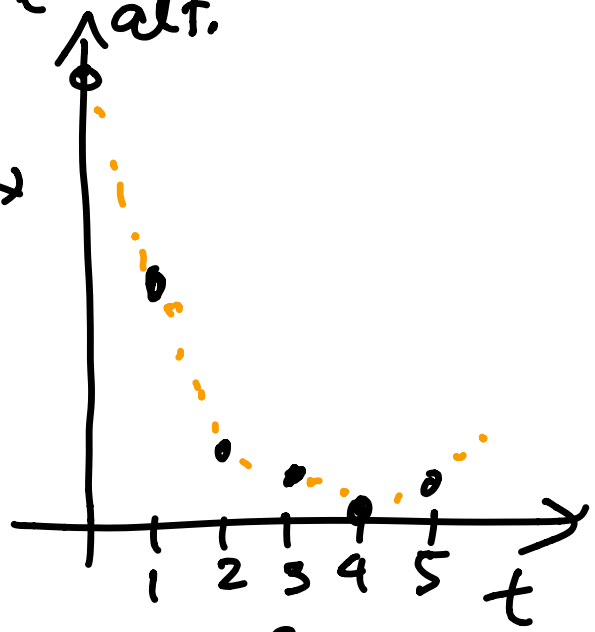
$y = 2x$

Understand functions in math:

Ex: gull diving to eat your sandwich.
Measure altitude at different times, make table

s	m
t=0	8
t=1	4.5
t=2	2
t=3	0.5
t=4	0
t=5	0.5
t=6	2

plot



Use eq'ns to describe functions.

Defn: A function is a package of 3 things:

→ The "rule"
$y =$ "some expr. of x" $= f(x)$

→ The domain: the allowable input. Set of x values, call it D.

→ The range: the output we find once we feed the function with all x values in D. Call this R.

Ex: ① $y = 2x$, for all real numbers x.

rule

Domain: all real numbers.
Range: all real numbers.

② $y = x^2$ rule

Domain: all real numbers
Range: all non-negative numbers
same: also write: $0 \le y < \infty$, $[0, \infty)$

Note:
$$[a,b) = \{x : a \le x < b\}$$
$$(a,b] = \{x : a < x \le b\}$$

③ ! $y = x^2$ rule.

Domain: all non-negative real numbers $[0, \infty)$

! As a function this is different from ②: different domain, same rule.

Range: all non-neg. reals $[0, \infty)$

④ $y = \frac{1}{x}$

Domain: all real numbers for which this makes sense: everything but 0

$(-\infty, 0) \cup (0, \infty)$

↑ union

$-\infty < x < 0$ or $0 < x < \infty$

Range: everything but 0.

Note: When finding largest domain where a function makes sense, look for

denominators and roots.

- denominators can't be 0

- something inside a root can't be negative.

(5) $f(x) = \sqrt{1-x}$

Domain: want $1-x \geq 0$

$1 \geq x$

Range: $y \geq 0$

(6) $f(x) = \frac{1}{\sqrt{1-x}}$

Want: $\sqrt{1-x} \neq 0 \Rightarrow x \neq 1$

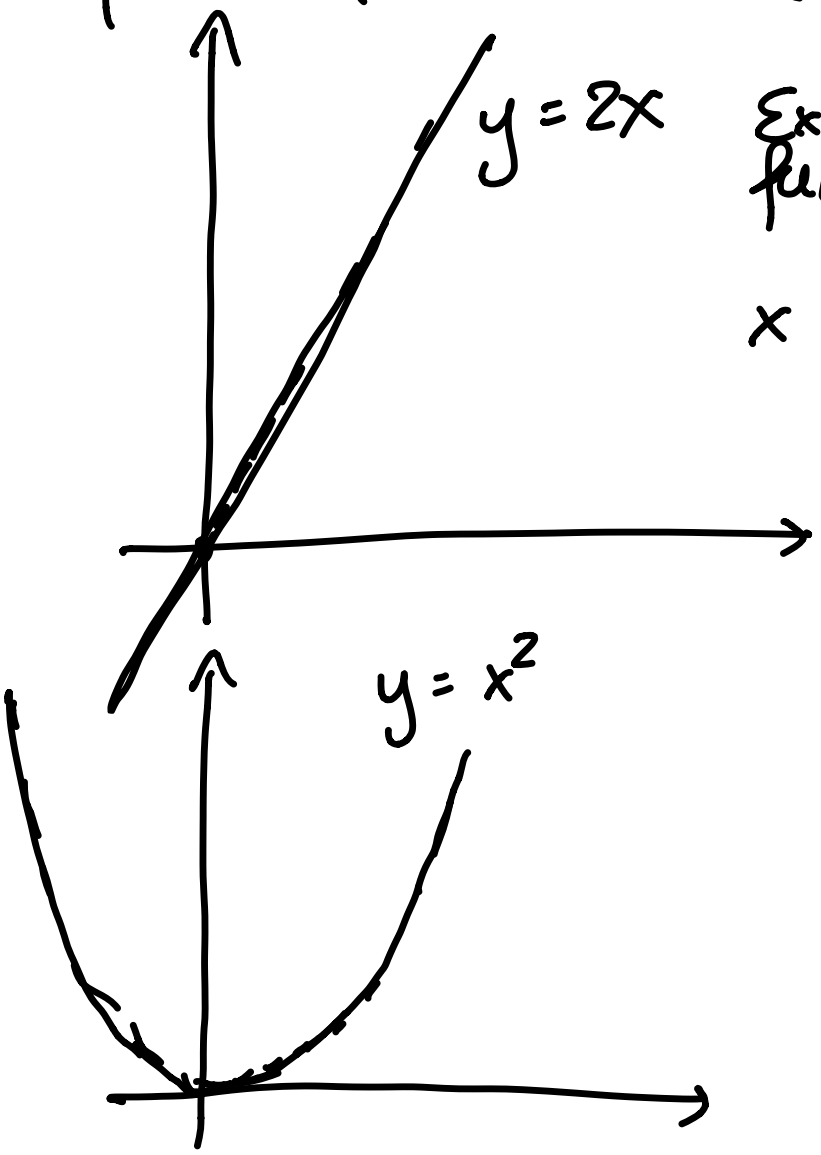
AND:

$1-x \geq 0 \Rightarrow x \leq 1$

Finally: $x < 1$

Graph of function $y = f(x)$

Plot the solutions of $y = f(x)$ for allowable x (x in D)



Ex: linear functions

$y = 2x$

x all real numbers

Something that is not a function.

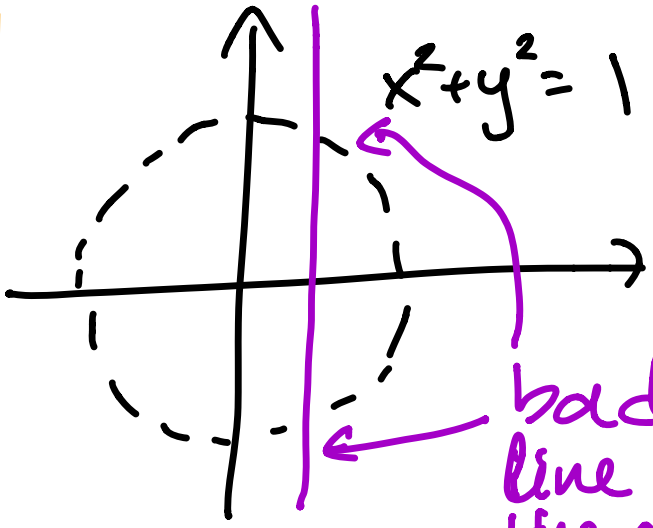
- $x = 5$. Can't write as $y = f(x)$
- $x^2 + y^2 = 1$

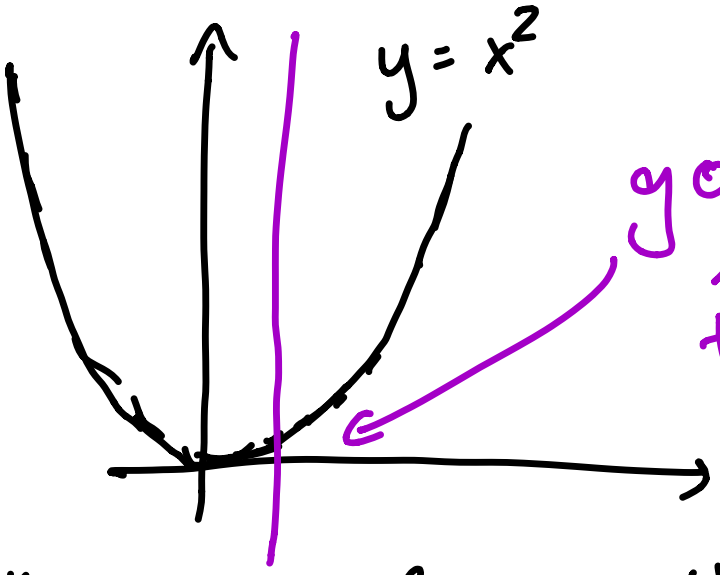
$$y^2 = 1 - x^2 \Rightarrow y = \pm\sqrt{1-x^2}$$

Plug in $x = 0$, find $y = \pm 1$

Found 2 outputs for 1 input. Not a function!

But: $y = \sqrt{1-x^2}$ is a function





good: vertical line intersects the graph once.

"Vertical line test": Any vertical line can intersect the graph of a function <u>at most</u> once.

The graph of a function $f(x)$ with domain D is:

$$\{(x, f(x)) : x \in D\}$$

$y = f(x)$

Vertical line: $x = k$ (const.)

Intersection of $x = k$, $y = f(x)$ $\Big\}$ $y = f(k)$ ← exactly one y

if $k \in \bar{D}$