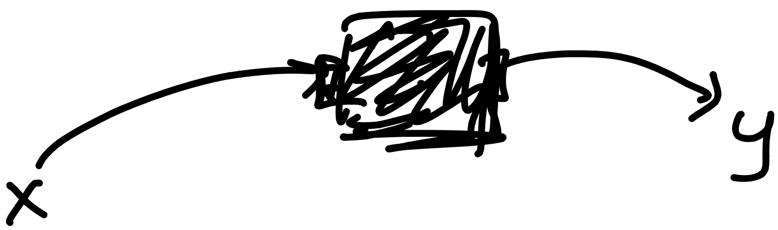
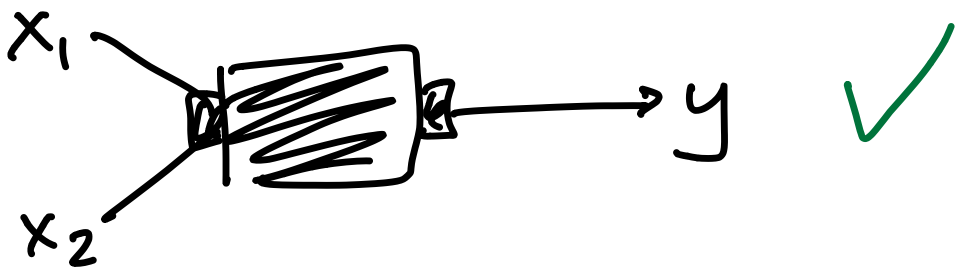
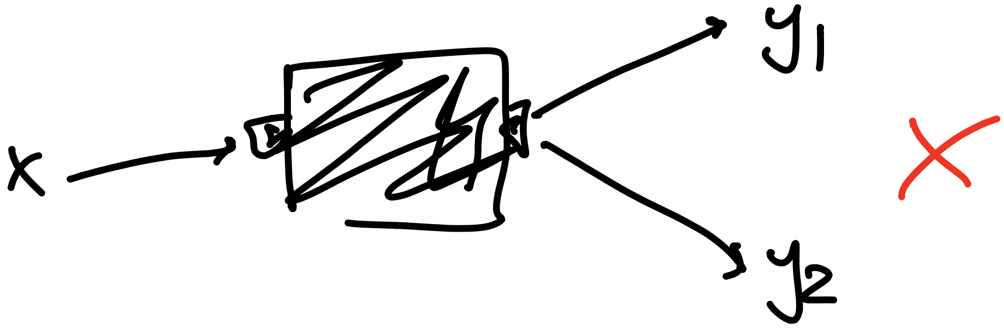
Inverse fcts

Recall: fnct is a procedure that assigns to each input exactly one output.



Possible to feed a fct w/ 2 different inputs, find same output





Q: Given a function $y = f(x)$

(e.g) $y = 2x + 3$

or

$y = x^2$

Given a y value, say $y = 1$, can we determine what inputs to the function give output $y = 1$?

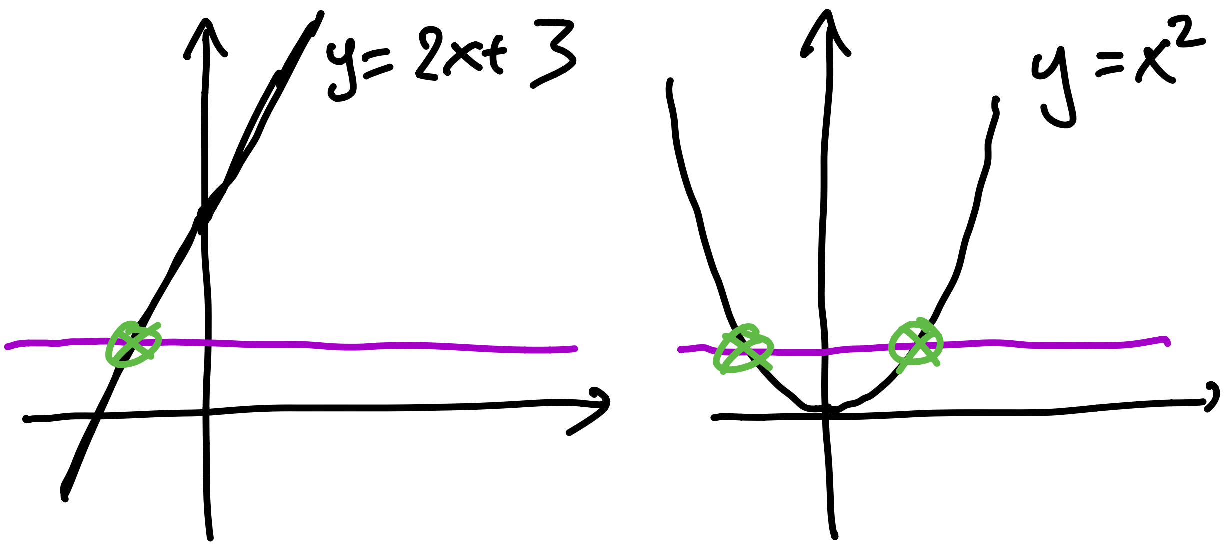
Done this! $2x + 3 = 1$

$2x = -2$

$x = -1$

$x^2 = 1 \Rightarrow$

$\Rightarrow x = 1$ or $x = -1$



What we did before was finding intersection of graphs with horizontal line $y = 1$

We say that a function $y=f(x)$ is <u>one-to-one</u> <u>on its domain</u>

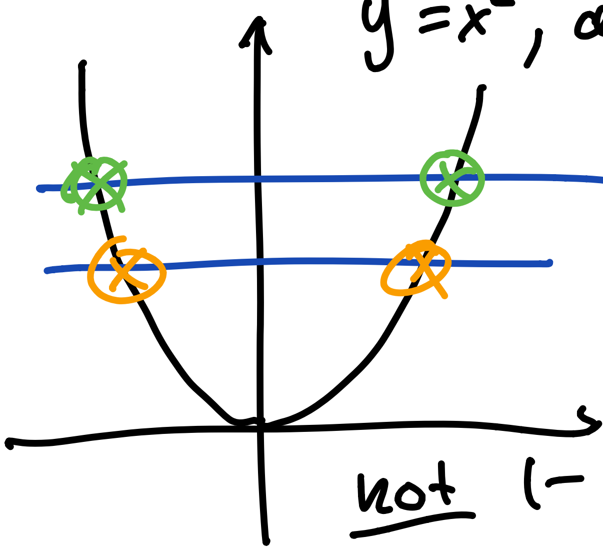
if for any value $y=c$ there is at most one solution to $f(x)=c$, for x in the domain.

Determine if a function is one to one in a domain by <u>horizontal line test</u>:

Graph $f(x)$ for all x in domain, if f is one-to-one then any horizontal line intersects the graph at most once

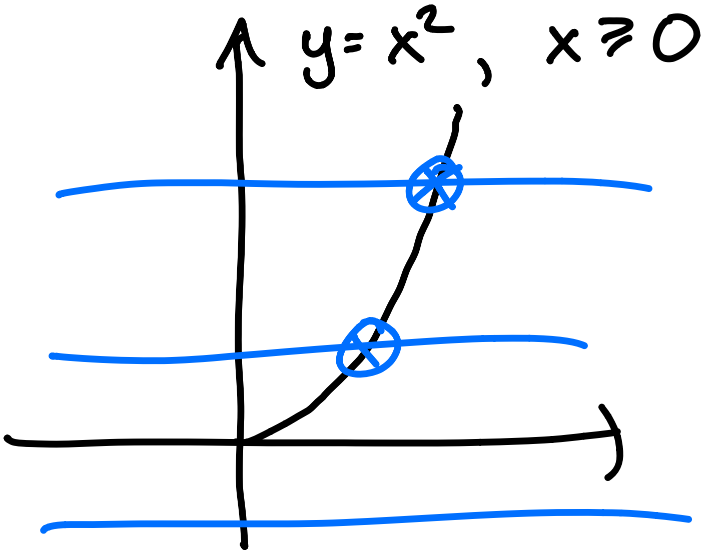
! Why we say one-to-one on the domain

$y = x^2$, all $x \in \mathbb{R}$



<u>not</u> 1-1 !

$y = x^2$, $x \geq 0$



this <u>is</u> 1-1 !

The rule alone can't determine if a function is 1-1!

Inverse function.

Given $y = f(x)$, with domain D, that is **one to one**.

Want: a new fct that tells us what input we should give to f to get a desired output.

Write $f^{-1}(y) =$ the x value so that $f(x) = y$

"f inverse"

The domain of $f^{-1}(y)$ is the range of f.

How we compute it: solve for x, write in terms of y.

Had $y = f(x) = 2x + 3$, $x \in \mathbb{R}$

Said that this is 1-1.

Find inverse:

$$y = 2x + 3 \Rightarrow$$

$$\Rightarrow 2x = y - 3$$

$$\Rightarrow x = \frac{y-3}{2}$$

$$\Rightarrow x = \frac{y}{2} - \frac{3}{2}$$

Write $f^{-1}(y) = \frac{y}{2} - \frac{3}{2}$.

Often: flip the roles of x, y at this stage

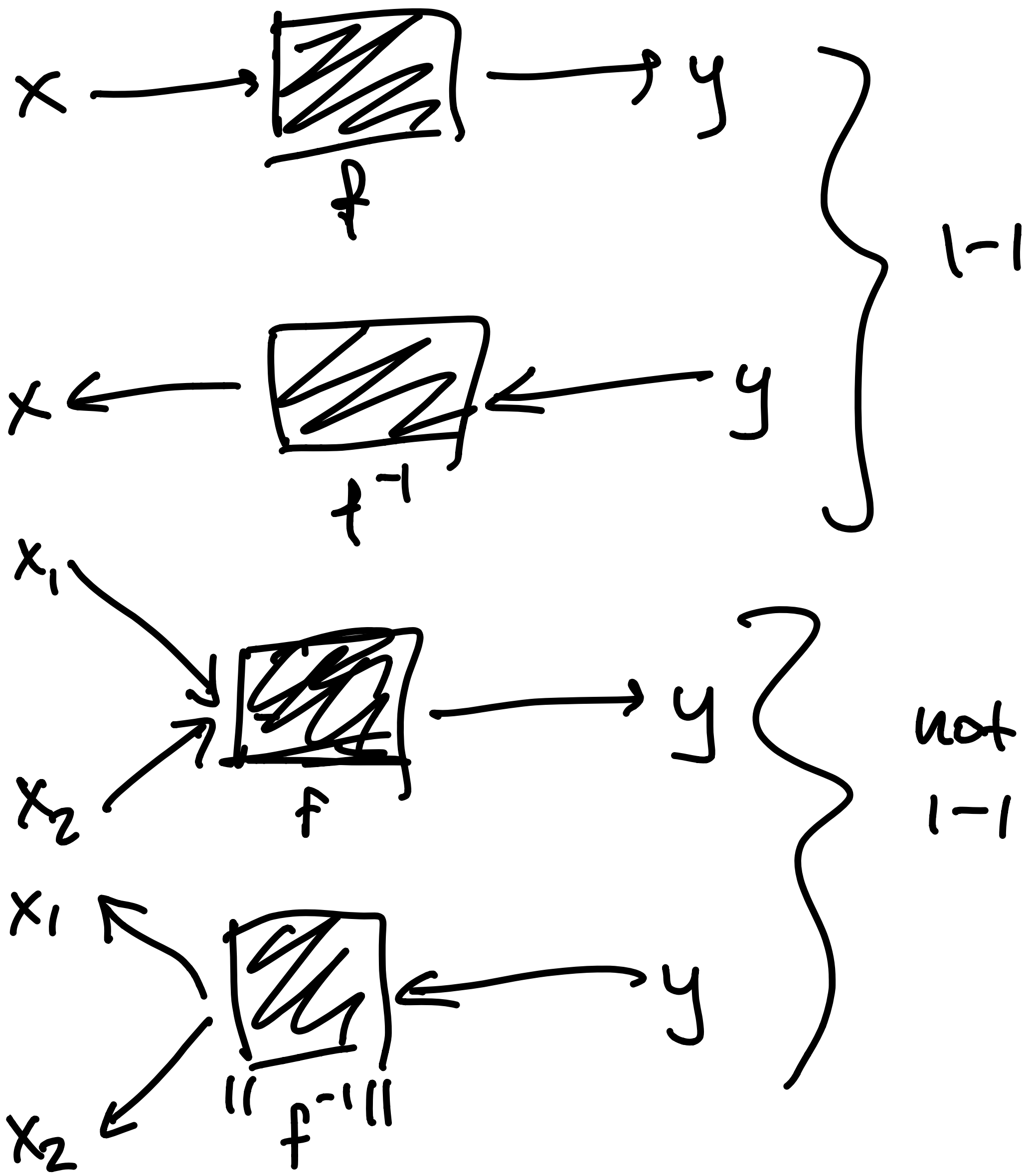
$$f^{-1}(x) = \frac{x}{2} - \frac{3}{2}$$

This only makes sense for 1-1 functions!

$$y = x^2 \Rightarrow x = \pm\sqrt{y}$$

Need to make a choice.

$y = x^2$ is not one to one if defined for all $x \in \mathbb{R}$, so we can't find its inverse.



doesn't make sense

Important fact: $f^{-1}(f(a)) =$

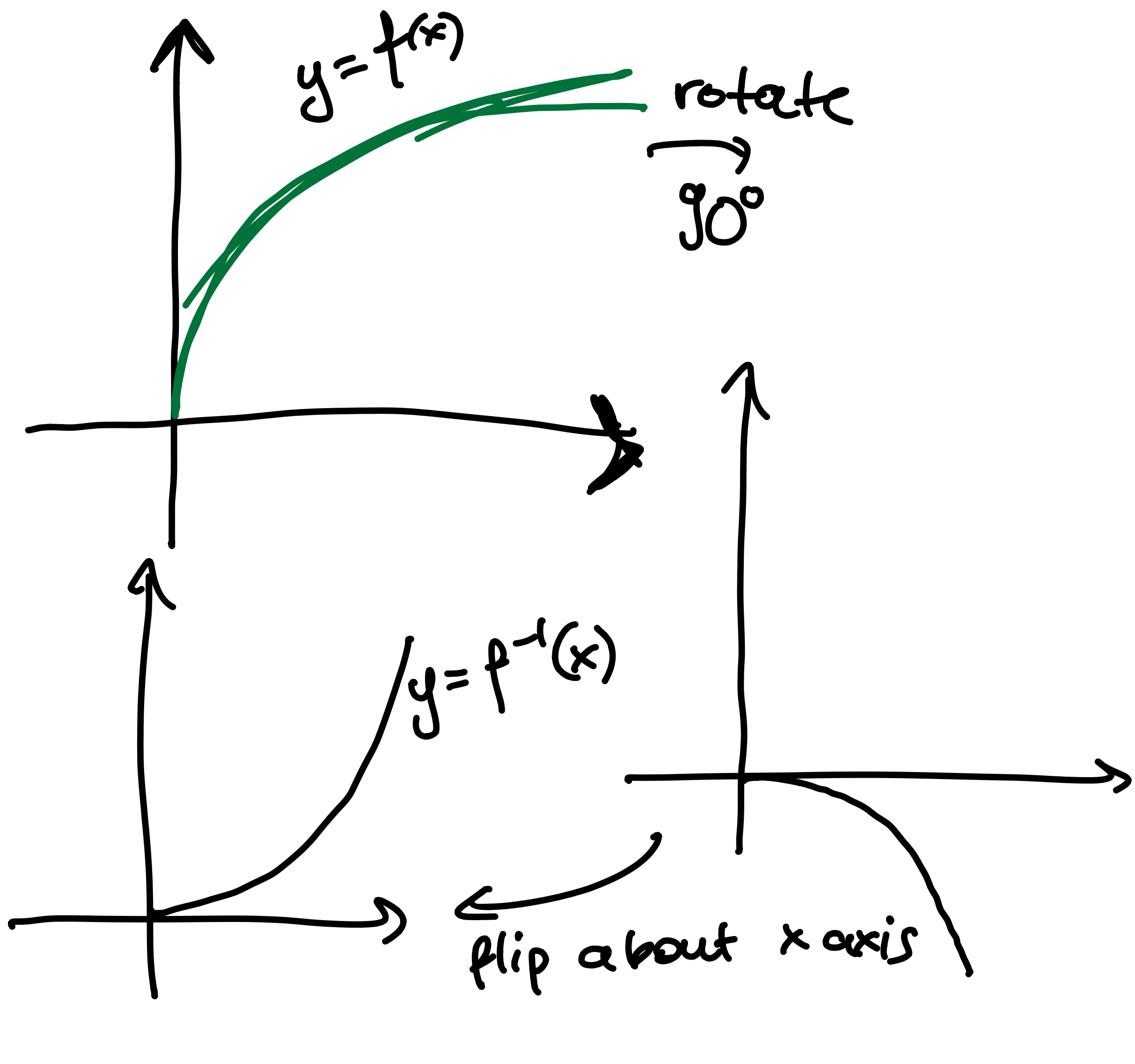
= the value we should plug into f to find $f(a)$

$= a$

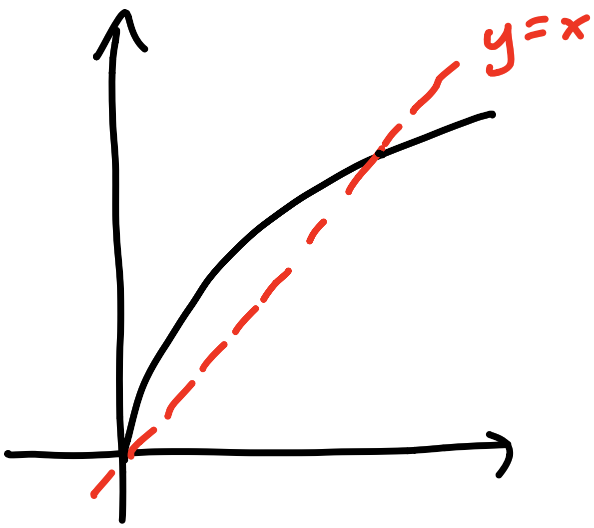
So $f^{-1}(f(a)) = a$, a any number in domain of f

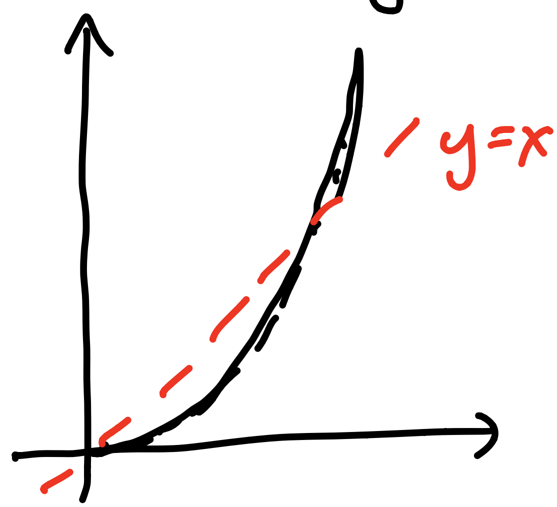
f is one to one on the domain.

Graph of inverse function



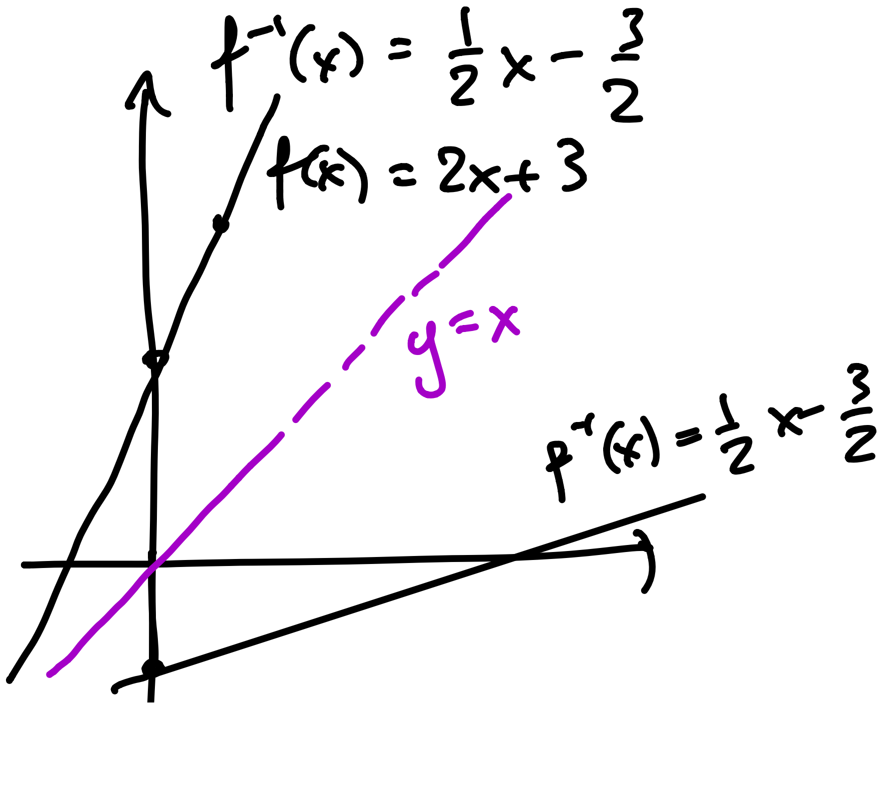
Faster: reflect about line $y = x$



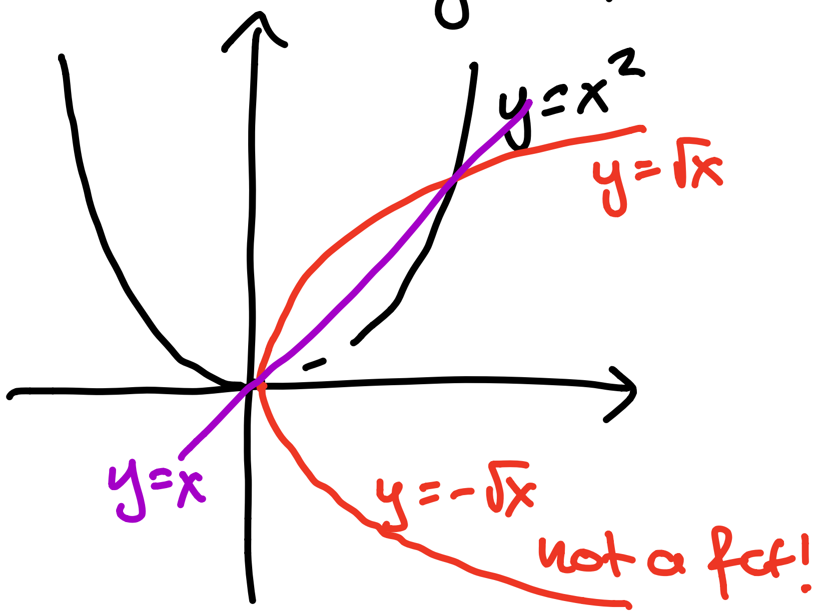


graphs of $y = f(x)$, $y = f^{-1}(x)$ are symmetric about the line $y = x$

Ex: $f(x) = 2x + 3$

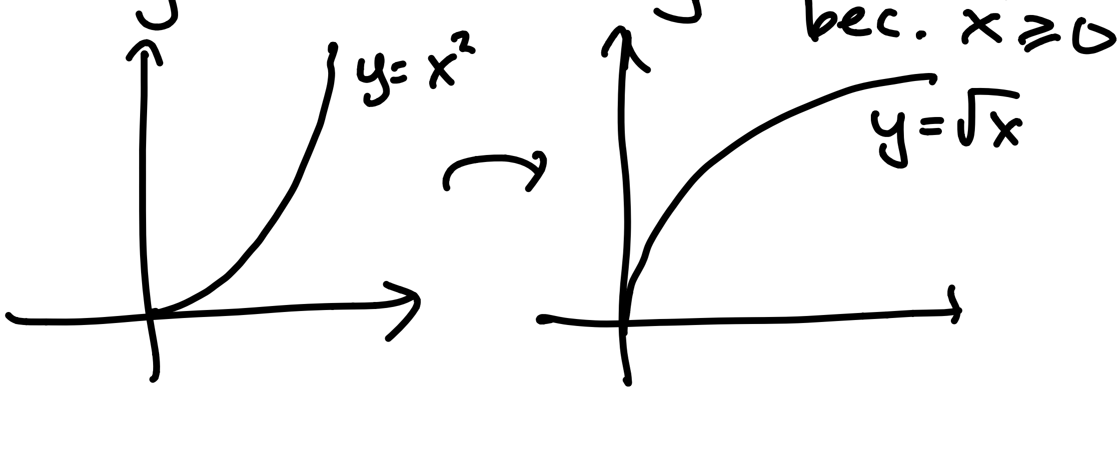


What happens when we try to invert $y = x^2$, $x \in \mathbb{R}$



If we restrict $y = x^2$ to $x \geq 0$ then we can find the inverse:

$y = x^2 \Rightarrow x = \pm\sqrt{y} \Rightarrow x = \sqrt{y}$
bec. $x \geq 0$



Or if we restrict $y = x^2$ to $x \leq 0$

$y = x^2 \Rightarrow x = \pm\sqrt{y} \Rightarrow x = -\sqrt{y}$

bec. $x \leq 0$

$y = -\sqrt{x}$