

Ex: Find a linear-to-linear function w/ hor. asymptote $y=3$, vertical asymptote $x=-2$ and such that $f(2)=1$.

Sol: Set $f(x) = \frac{Ax+B}{x+C}$

Hor. asymptote at $y = \frac{A}{1} = A$

ver. asymptote at $x+C=0$
 $\Rightarrow x=-C$

So set $3=A$

$-C = -2 \Rightarrow C=2$.

To find B :

$$f(2)=1 \Rightarrow \frac{3 \cdot 2 + B}{2+2} = 1$$

$$\Rightarrow 6+B=4 \Rightarrow B=-2$$

So $f(x) = \frac{3x-2}{x+2}$

Ex: Find a linear-to-linear function w/ hor. asymptote $y=3$ such that $f(2)=1$ and $f(1)=-1$

Sol: Set $f(x) = \frac{Ax+B}{x+C}$

Hor. asymptote at $y = \frac{A}{1} = A$

So set $3=A$

Then: $f(2)=1 \Rightarrow \frac{3 \cdot 2 + B}{2+C} = 1$

$$\Rightarrow 6+B = 2+C \quad (1)$$

$$f(1)=-1 \Rightarrow \frac{3 \cdot 1 + B}{1+C} = -1 \Rightarrow$$

$$3+B = -1-C \quad (2)$$

$$(1) + (2) \Rightarrow 9 + 2B = 1 \Rightarrow B = -4$$

So $3-4 = -1-C \Rightarrow C=0$

$$f(x) = \frac{3x-4}{x}$$