

# Worksheet 3

December 7, 2017

1. Decide which of the three plots below corresponds to the parametrization

$$\vec{r}(u, v) = \langle (2 + \cos(v)) \cos(u), (2 + \cos(v)) \sin(u), \sin(v) \rangle, \text{ for } (u, v) \in [0, 2\pi] \times [0, 2\pi]$$

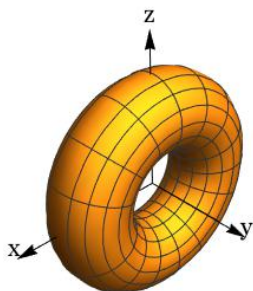


Figure 1: Plot 1

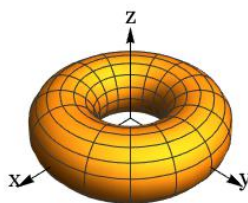


Figure 2: Plot 2

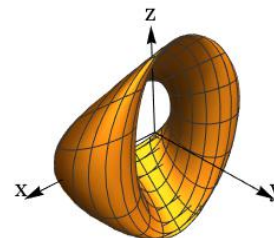


Figure 3: Plot 3

2. Prove the first step towards showing Green's First Identity: Show that for any twice differentiable functions  $u(x, y, z)$  and  $v(x, y, z)$  we have

$$\operatorname{div}(u \nabla v) = \nabla u \cdot \nabla v + u \Delta v \quad (1)$$

3. Compute the line integral  $\int_c \vec{F} \cdot d\vec{r}$ , where  $\vec{F} = \langle -y, x, 2 \rangle$  and  $c$  is the path that consists of the following line segments, as in Figure 5:

- A line segment from  $(0, 0, 1)$  to  $(-1, 1, 0)$ .
- A line segment from  $(-1, 1, 0)$  to  $(1, 1, 0)$ .
- A line segment from  $(1, 1, 0)$  back to  $(0, 0, 1)$ .

4. Let  $S$  be the surface that consists of the part of the cylinder  $x^2 + y^2 = 1$  lying between the planes  $z = 0$  and  $z = -1$ , together with the part of the sphere

$$x^2 + y^2 + (z + 1 + \sqrt{3})^2 = 4$$

that lies below the plane  $z = -1$ , and let  $S$  have orientation pointing away from the origin, as in picture 4.

- (a) Compute  $\int_S \vec{F} \cdot d\vec{S}$ , where  $\vec{F}(x, y, z) = \langle y + x, x + z, -z + y^2 \rangle$ .  
 Hint: Modify the surface accordingly so you can use divergence theorem.
- (b) \*Find the surface area of  $S$ .

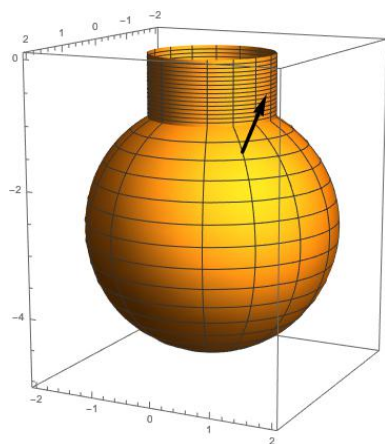


Figure 4: Problem 4

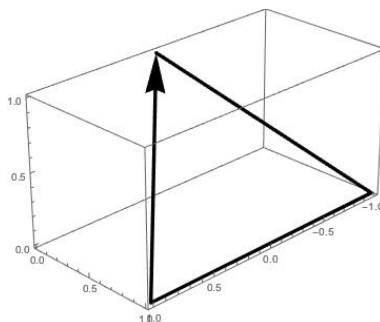


Figure 5: Problem 3