Quiz 4

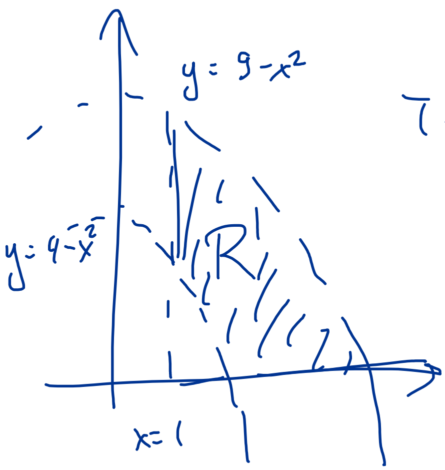
Name: ____________________

Student ID Number: ____________________

Let R be the domain in the **first quadrant**, bounded by the curves $y = 4 - x^2$, $y = 9 - x^2$, $x = 1$ and $y = 0$. **Use the transformation** $x = v$, $y = u - v^2$ to compute the integral

$$\iint_R x dA.$$

You do not need to show that the transformation is invertible, but show the rest of the steps clearly.



$$T: \begin{cases} x = v \\ y = u - v^2 \end{cases}$$

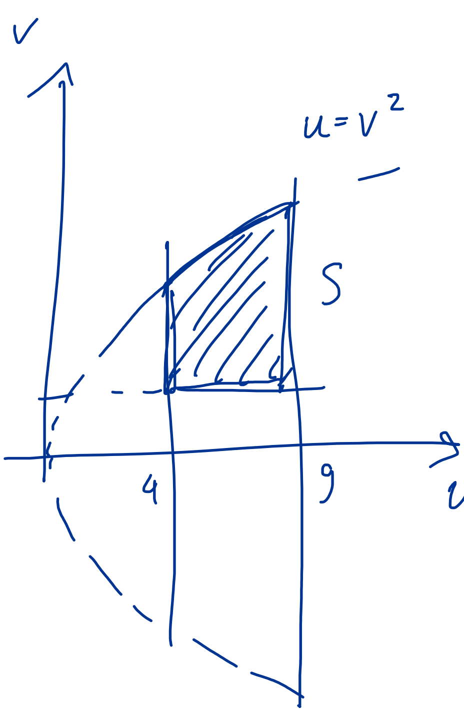
Find $S = T^{-1}(R)$:

$y = 4 - x^2 \Rightarrow u - v^2 = 4 - v^2 \Rightarrow u = 4$

$y = 9 - x^2 \Rightarrow u - v^2 = 9 - v^2 \Rightarrow u = 9$

$x = 1 \Rightarrow v = 1$

$y = 0 \Rightarrow u - v^2 = 0 \Rightarrow u = v^2$



Jacobian:

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 0 & 1 \\ 1 & -2v \end{vmatrix} = -1 \neq 0, \text{ so } T \text{ is } C^1; \; T^{-1} \text{ is } C^1$$

Then

$$\iint_R x\,dA = \iint_S v\,|-1|\,du\,dv = \int_4^9 \int_1^{\sqrt{u}} v\,dv\,du$$

$$= \int_4^9 \frac{v^2}{2}\Big|_1^{\sqrt{u}} du = \int_4^9 \frac{u-1}{2} du = \frac{u^2}{4} - \frac{u}{2}\Big|_4^9$$

$$= \frac{81}{4} - \frac{9}{2} - \frac{16}{4} + 2 = \frac{55}{4}$$