Math 120 B, Spring 2018
Final Exam
Wednesday, June 6, 2018

Name: ____________________

UW email address: ____________________

Problem 1	12	
Problem 2	12	
Problem 3	12	
Problem 4	10	
Problem 5	12	
Problem 6	12	
Total	70	

- There are 6 problems spanning 6 pages (your last nonempty page should be numbered as 6). Please make sure your exam contains all these questions.
- You are allowed to use a TI-30x IIS Calculator and one double sided hand-written 8.5 by 11 inch page of notes.
- You must show your work on all problems (unless explicitly instructed otherwise). The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 110 minutes to complete the exam. Budget your time wisely. **Do not spend too much time on an individual problem unless you are done with all the rest.**

GOOD LUCK!

1. (12 pts.) **All parts are independent.**

(a) Write the equation of the line perpendicular to $2x+3y-1=0$ that passes through the point $(2,3)$.

$2x+3y-1=0 \Rightarrow 3y=1-2x \Rightarrow y=\frac{1}{3}-\frac{2}{3}x$

slope of perp. $= \frac{3}{2}$

$(y-3)=\frac{3}{2}(x-2)$

(b) Write an equation for the lower semicircle of radius 2 centered at (4,3).

$(x-4)^2+(y-3)^2=4 \Rightarrow (y-3)^2=4-(x-4)^2$

$\Rightarrow y-3=\pm\sqrt{4-(x-4)^2}$

$\Rightarrow y=3-\sqrt{4-(x-4)^2}$

(c) Find a linear-to-linear rational function whose graph passes through the point (2,3) and has horizontal asymptote $y=2$ and vertical asymptote $x=-4$.

$y=\frac{Ax+B}{x+C}$ hor. as: $y=A \Rightarrow A=2$

v. as. $x=-C \Rightarrow C=4$

$y=\frac{2x+B}{x+4}$

$3=\frac{2\cdot 2+B}{2+4} \Rightarrow B=18-4$

$\Rightarrow B=14$

$f(x)=\frac{2x+14}{x+4}$

2. (12 pts.) **The parts are independent.**

(a) Find a formula for the inverse of the function $f(x) = \dfrac{2x+4}{x-3}$.

$y = \frac{2x+4}{x-3} \Rightarrow y(x-3) = 2x+4 \Rightarrow yx - 3y = 2x+4$

$\Rightarrow x(y-2) = 3y+4$

$\Rightarrow x = \frac{3y+4}{y-2}$

So $f^{-1}(x) = \frac{3x+4}{x-2}$

(b) Solve the equation:

$$\ln\left(\frac{2x}{x-5}\right) = 3.$$

$\ln\left(\frac{2x}{x-5}\right) = 3 \Rightarrow \frac{2x}{x-5} = e^3 \Rightarrow 2x = e^3 x - 5e^3$

$\Rightarrow x = \frac{e^3 x - 5e^3}{2}$

(c) Solve the equation:

$$3 \cdot 4^{3x+7} = 1$$

$3 \cdot 4^{3x+7} = 1 \Rightarrow \ln 3 + \ln 4^{3x+7} = \underset{0}{\overset{}{\ln 1}}$

$\Rightarrow (3x+7)\ln 4 = -\ln 3$

$\Rightarrow 3x+7 = -\frac{\ln 3}{\ln 4}$

$\Rightarrow 3x = -7 - \frac{\ln 3}{\ln 4}$

$\Rightarrow x = -\frac{1}{3}\left(7 + \frac{\ln 3}{\ln 4}\right)$

2. (12 pts.) You are given the functions $g(x) = 2x - 4$ and $f(x) = \begin{cases} 5 - \sqrt{9 - (x-3)^2}, & 3 \le x \le 6 \\ 2 - (x-3)^2, & x \le 3 \end{cases}$.

(a) Write a multipart rule for the function $g(f(x))$

$$g(f(x)) = \begin{cases} 2\left(5 - \sqrt{9-(x-3)^2}\right) - 4, & 3 \le x \le 6 \\ 2(2-(x-3))^2 - 4, & x \le 3 \end{cases}$$

(b) Write a multipart rule for the function $f(g(x))$

$$f(g(x)) = \begin{cases} 5 - \sqrt{9 - (2x-4-3)^2}, & 3 \le 2x-4 \le 6 \\ 2 - (2x-4-3)^2, & 2x-4 \le 3 \end{cases}$$

$$= \begin{cases} 5 - \sqrt{9-(2x-7)^2}, & \frac{7}{2} \le x \le \frac{10}{2} \\ 2 - (2x-7)^2 & x \le \frac{7}{2} \end{cases}$$

(c) Solve the equation $f(g(x)) = 3$

$$5 - \sqrt{9-(2x-7)^2} = 3 \Rightarrow$$

$$\Rightarrow 9 - (2x-7)^2 = 2^2$$

$$\Rightarrow (2x-7)^2 = 5 \Rightarrow 2x - 7 = \pm\sqrt{5}$$

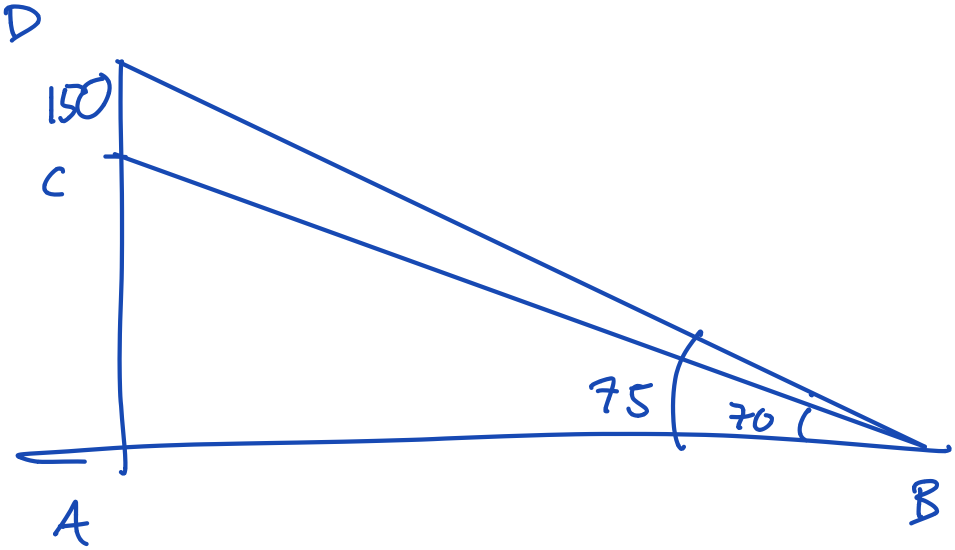
$$\Rightarrow x = \frac{7 \pm \sqrt{5}}{2}$$

$$\boxed{x = \frac{7+\sqrt{5}}{2}}$$ bec. $\frac{7-\sqrt{5}}{2} < \frac{7}{2}$

$$2 - (2x-7)^2 = 3 \Rightarrow$$

$$-(2x-7)^2 = 1 \quad \text{no sol.}$$

4. (10 pts.) You are standing on flat ground some distance away from a skyscraper. Climbing up the skyscraper, 150 feet from the top, is a gorilla. From where you stand, you measure the angle of elevation from the ground to the gorilla, and you find it to be $70°$. Then you measure the angle of elevation from the ground to the top of the skyscraper. It's $75°$. How tall is the skyscraper?



$$\frac{AD}{AB} = \tan 75 \qquad \frac{AC}{AB} = \tan 70$$

$$AB = \frac{AD}{3.73} \qquad AB = \frac{AC}{2.747}$$

$$\frac{AD}{3.73} = \frac{AC}{2.747}$$

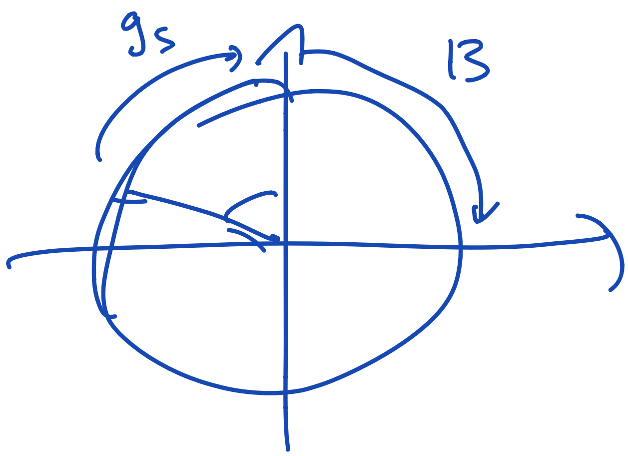
$AD - AC = 150\text{ft.}$ $\quad - 2.747A$

$$AD - \frac{2.747}{3.73}AD = 150$$

$$0.263\, AD = 150$$

$$AD \sim 570\text{ft}$$

5. (12 pts.) [Autumn 2016] Essun is running 3 meters per second clockwise around a circular track. From her starting point, it takes her 9 seconds to reach the northernmost point of the track, and then an additional 13 seconds to reach the easternmost point of the track. After 2 minutes, how far east is Essun from the westernmost point of the track?



$13s \to \frac{\pi}{2}$

$\omega = \frac{\frac{\pi}{2}}{13} = \frac{\pi}{26}$ rad/s

radius

$\frac{3 \text{ m/s}}{\frac{\pi}{26} \text{ rad/s}} = \frac{78}{\pi}$ m

Initial angle:

$\theta_0 - \frac{\pi}{26} \cdot 9 = \frac{\pi}{2} \Rightarrow \theta_0 = \frac{22\pi}{26}$

After 2min:

$$\frac{22\pi}{26} - \frac{\pi}{26} \cdot 120 = -\frac{98\pi}{26}$$

So x coord: $\frac{78}{\pi} \cos\left(-\frac{98\pi}{26}\right)$

$= 18.58$

Distance east $\frac{78}{\pi} + 18.58$

~ 43.4 m

6. (12 pts.) Stanley checked the concentration of glucose in his blood several times during the day today and found out that it was at its lowest level of 65 mg/dl at 8 am and at its highest level of 125 mg/dl at 2pm. Assume that the concentration of glucose in Stanley's blood is modeled by a sinusoidal function and that there was no other highest or lowest value between 8 am and 2 pm.

(a) Determine a sinusoidal function that gives Stanley's blood glucose today, t hours after midnight.

$$A = \frac{\max - \min}{2} = \frac{125-65}{2} = 30$$

$$D = \frac{\max + \min}{2} = \frac{125+65}{2} = 95$$

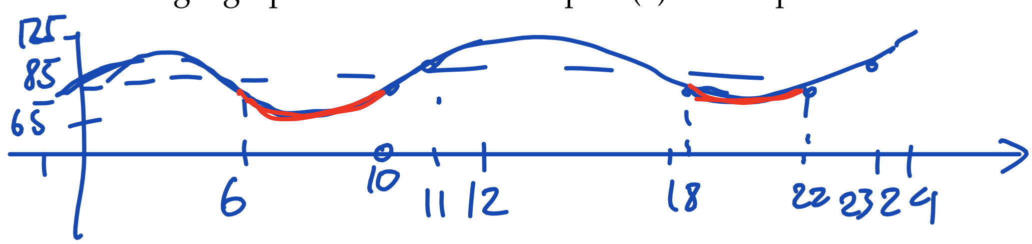
$$\frac{B}{2} = \underset{\substack{\uparrow \\ 2pm}}{14} - 8 = 6 \Rightarrow B = 12$$

$$C = x \text{ of } \max - \frac{B}{4} = 14 - \frac{12}{4} = 11$$

$$f(t) = 30\sin\left(\frac{2\pi}{12}(t-11)\right) + 95$$

(b) Stanley feels dizzy when his blood glucose is below 80 mg/dl. For how long did Stanley feel dizzy today, in the time interval between 6 am and 11 pm?

Hint: drawing a graph of the function in part (a) can help.



$$30\sin\left(\frac{2\pi}{12}(t-11)\right) + 95 = 80$$

$$\Rightarrow \sin\left(\frac{2\pi}{12}(t-11)\right) = -\frac{15}{30} = -\frac{1}{2}$$

$$\frac{2\pi}{12}(t-11) = 2k\pi + \sin^{-1}\left(-\frac{1}{2}\right) \Rightarrow t - 11 = \frac{6}{\pi}\left(2k\pi - \frac{\pi}{6}\right)$$

$$t = 11 + 12k - 1 \Rightarrow t = 12k + 10$$

$$\frac{2\pi}{12}(t-11) = 2k\pi + \pi - \sin^{-1}\left(-\frac{1}{2}\right) \Rightarrow t - 11 = \frac{6}{\pi}\left(2k\pi + \pi + \frac{\pi}{6}\right)$$

$$\Rightarrow t - 11 = 12k + 6 + 1$$

$$t = 12k + 18$$

Sol's: 10, 22, 6, 18

Total: 8 hours