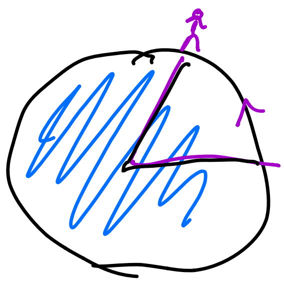
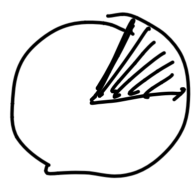
Measuring angles

What is an angle: Union of 2 rays starting at 1 point (the vertex)

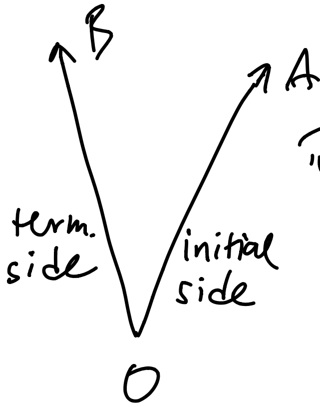
Want to answer questions like



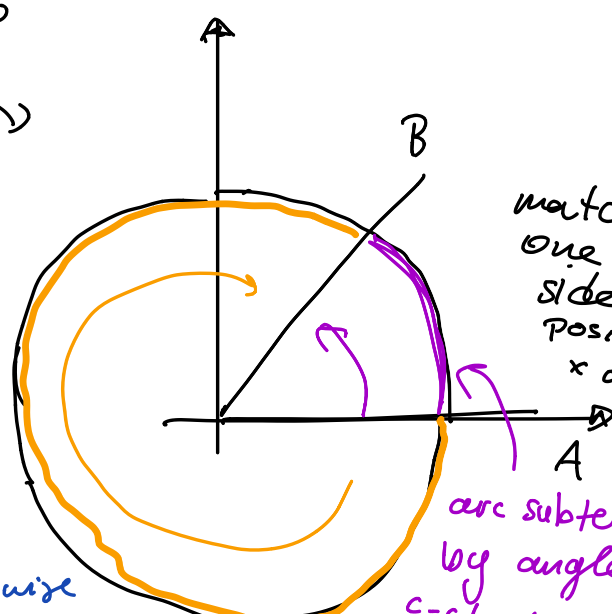
distance walked along the shore of a lake or area of part of pizza



How we typically work w/ angles



slide to "standard position"



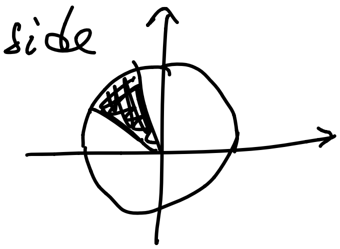
match one side w/ positive x axis

arc subtended by angle swept clockwise still arc(AB)

arc subtended by angle swept c-clockwise arc(AB

Central angle: its vertex is at center of circle, initial side may not be pos. x-axis.

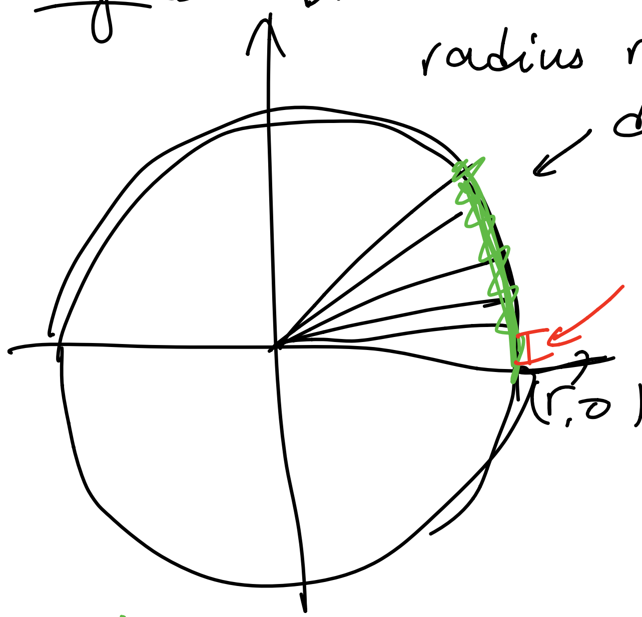
Ex:



Measuring: use 2 different units — degrees
— radians

In both cases: come up with a wedge of fixed opening we can use as a reference.

Degrees: Draw circle as before, with radius r.



divide into 360 equal wedges.

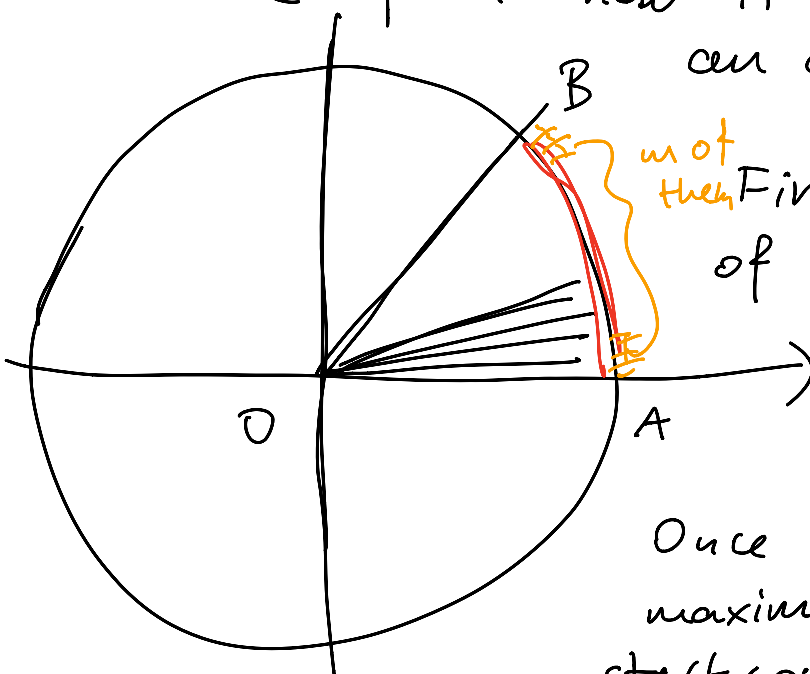
Divides circumference of circle into 360 equal arcs. Call each one of them a 1° arc.

Each degree is split into 60 minutes (also write $60'$)

and each minute in 60 seconds (also write 60")

so $1^\circ = 60' = 3600''$

How we measure: Bring angle in standard form: now it subtends an arc (red)



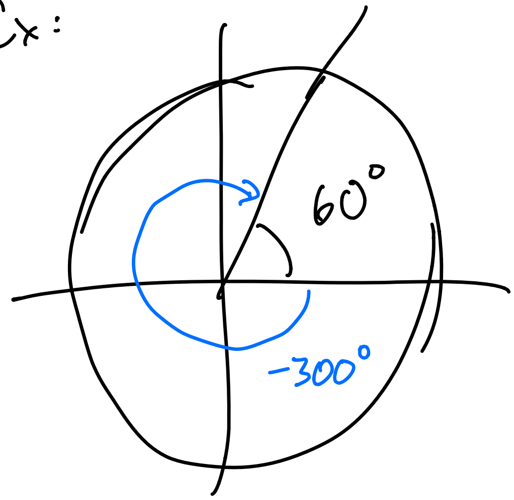
Find number m of 1° arcs that fit inside given arc.

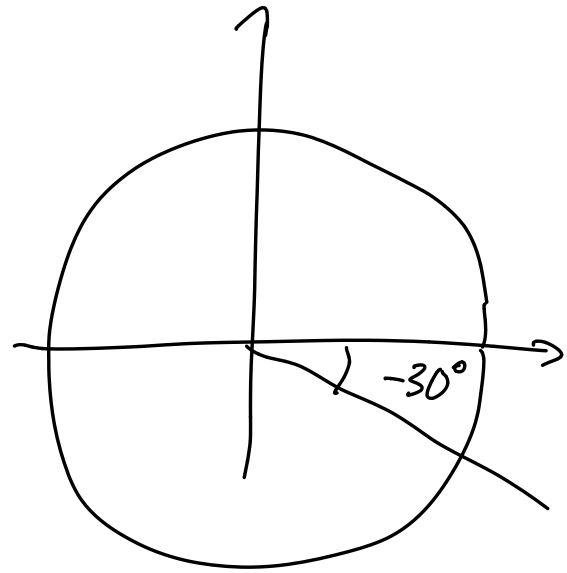
Once reach maximum number, start counting minutes & seconds

If sweeping counterclockwise and have m°, n', ℓ'' then angle measure $= m^\circ n' \ell''$

If sweeping clockwise angle measure $-m^\circ n' \ell''$

Ex:





Conversion exercises:

$(5.32)°$

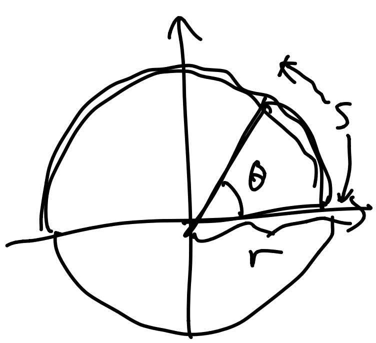
$$= (5.32)° = 5° + 0.32°$$
$$= 5° + 0.32 \cdot 1° \cdot \frac{60'}{1°}$$
$$= 5° + 19.2'$$
$$= 5° 19' + 0.2'$$
$$= 5° 19' + 0.2 \cdot 1' \cdot \frac{60''}{1'}$$
$$= 5° 19' 12''$$

Other way: $15° 17' 20''$

$$= 15° + 17' \cdot \frac{1°}{60'} + 20'' \cdot \frac{1°}{3600''}$$
$$= 15° + 0.283° + 0.005°$$

$= 15.288°$

Now on a circle of radius r, want to relate the measure of an angle $\angle AOB$ with length of arc of circle subtended by angle, s.



Total perimeter of circle $= 2\pi r$. So

$$\frac{s}{2\pi r} = \frac{\theta^\circ}{360^\circ}$$

$$\Rightarrow \quad s = \frac{2\pi}{360} r\theta$$

(Arc length in degrees).

! Fact

Central angle measuring θ degrees in circle of radius r subtends arc of length

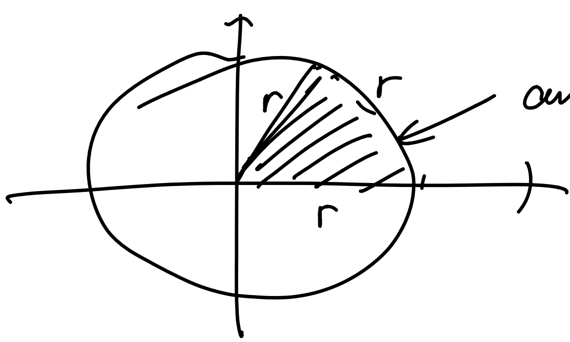
$$\boxed{s = \frac{2\pi}{360} r\theta}$$

Measuring with radians

Degrees are completely arbitrary as a way of measuring angles (why 360?)

We see a more natural way to do it, using "radians"

We define 1 radian by looking at a circle of radius 1 and considering the angle that corresponds to an arc of length r



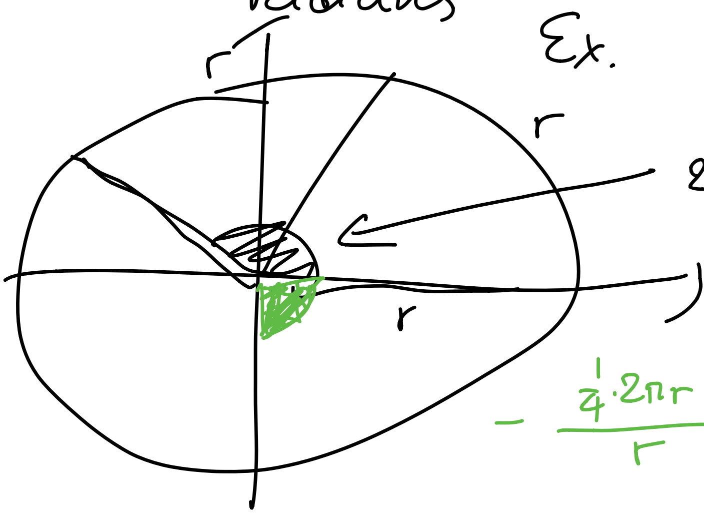
! Measure of θ in radians if it subtends arc of length s on circle of radius r:

$$\begin{cases} \theta = \frac{s}{r} & \text{if swept counterclockwise} \\ \theta = -\frac{s}{r} & \text{if swept clockwise} \end{cases}$$

Note: a) If $r=1$ $\theta = \pm s$!

b) Entire circle has circumference $2\pi r$, so it corresponds to 2π radians

Ex.



$$-\frac{\frac{1}{4}\cdot 2\pi r}{r} = -\frac{\pi}{2}$$

Standard angles:

$$45^\circ = \frac{1}{8}\cdot 360^\circ = \frac{1}{8}2\pi = \frac{\pi}{4}$$

$$60^\circ = \frac{1}{6}\,360^\circ = \frac{\pi}{3}$$

$$30^\circ = \frac{1}{2}\cdot\frac{\pi}{3} = \frac{\pi}{6} \text{ rad.}$$

$$90^\circ = \frac{\pi}{2}$$

$$180^\circ = \pi.$$

Why prefer radians: easy to compute areas and arc lengths.

Areas of wedges:

Use principle

area of part = (fraction of part)(area of whole)

So if have angle θ in rad, disk of radius r

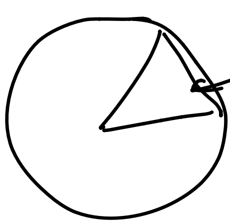
$$\text{area}_\theta = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{\theta r^2}{2}$$

Ex: find area of a pie of radius 5 inches corresponding to angle 2 rad:

$$\text{area} = \frac{2 \cdot 5^2}{2}$$

Chord approximation

When angle is small, length of arc it subtends is very similar to the length of the corresponding chord.



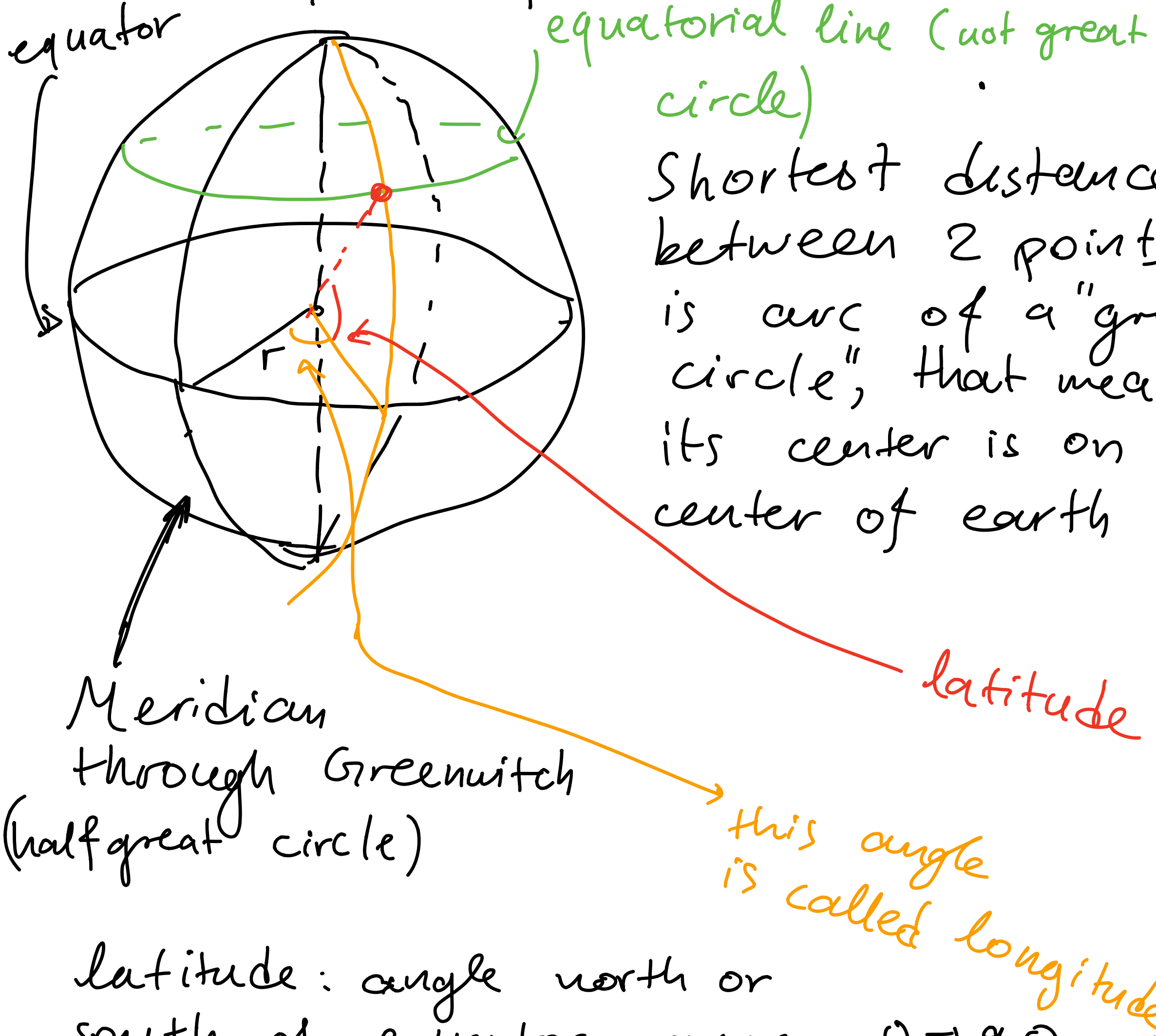
chord.

As of now we don't know how to find the length of a chord

(we'll need sines and cosines)
but if angle is small we can approximate it using arc length.
Ex: chord corresponding to $1°$ on circle of radius 20 in is about $\frac{1°}{360°} \cdot 2 \cdot 20 \cdot \pi$

Computations on the Earth

Earth: sphere of radius $r = 3{,}960$ miles



Shortest distance between 2 points is arc of a "great circle", that means its center is on center of earth

latitude: angle north or south of equator, ranges $0 \to 90$ north or $0 \to 90$ south

longitude: angle east or west of
Greenwich meridian

This imposes a coordinate system and
we can express any point on the
earth using 2 angles.