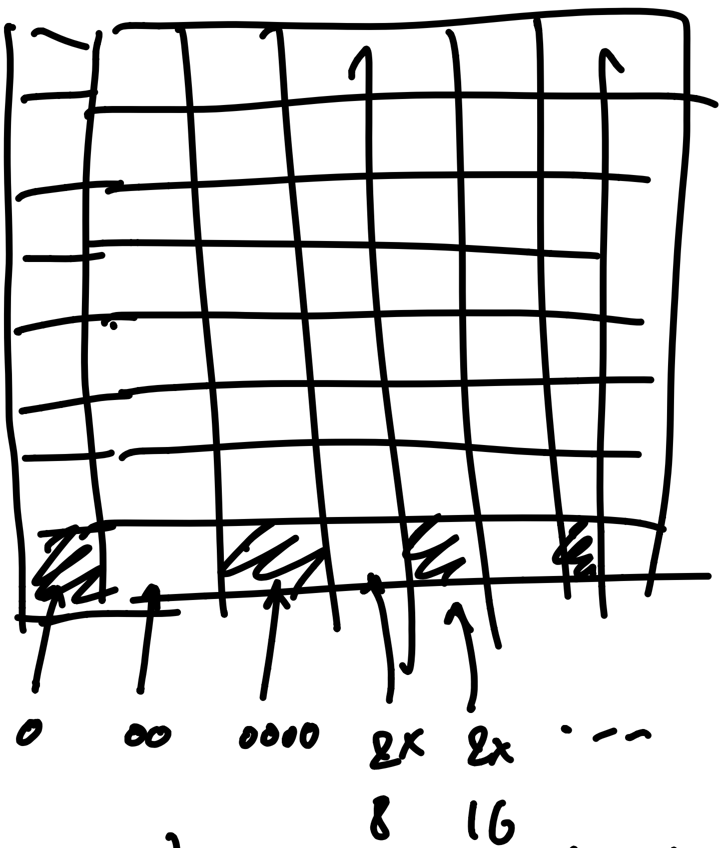


Square	number of wheat seeds
1	1
2	2 $\rightarrow 2^1 = 2^{2-1}$
3	4 $\rightarrow 2^2 = 2^{3-1}$
4	8 $\rightarrow 2^3 = 2^{4-1}$
...	...
n	2^{n-1}

64th sq: $9.2 \cdot 10^{18}$

Exponential functions

$f(x) = b^x$ contrast with $f(x) = x^b$

$b = 2$

x	2^x
1	2
2	4
3	8
4	16
5	32
6	64

x	x^2
1	1
2	4
3	9
4	16
5	25
6	36

What sense does it make to write b^x when x is not an integer?

$\to -1, 0, 1, 2, 3,$

$2^{2.414317}$

Start w/ simple case, assume $b > 0$

What is $b^{\frac{1}{3}}$?

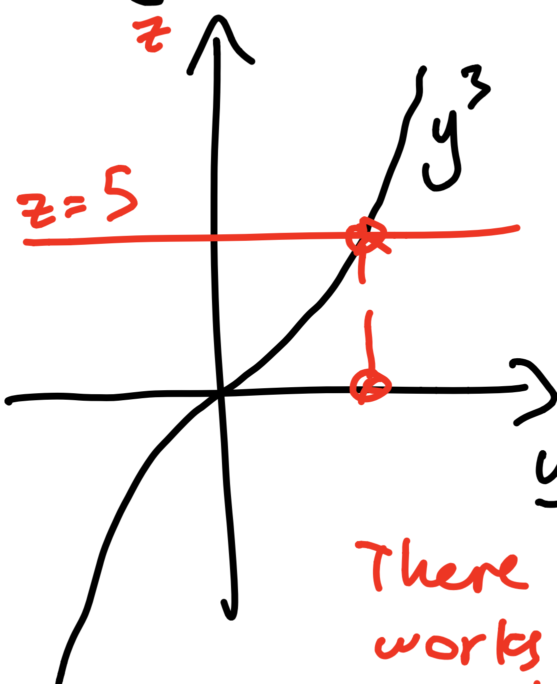
If $y = b^{\frac{1}{3}}$ then $y^3 = (b^{\frac{1}{3}})^3$

$$\Rightarrow y^3 = b^{\frac{1}{3}\cdot 3}$$

$$\Rightarrow y^3 = b$$

Can we find such a y for any positive b?

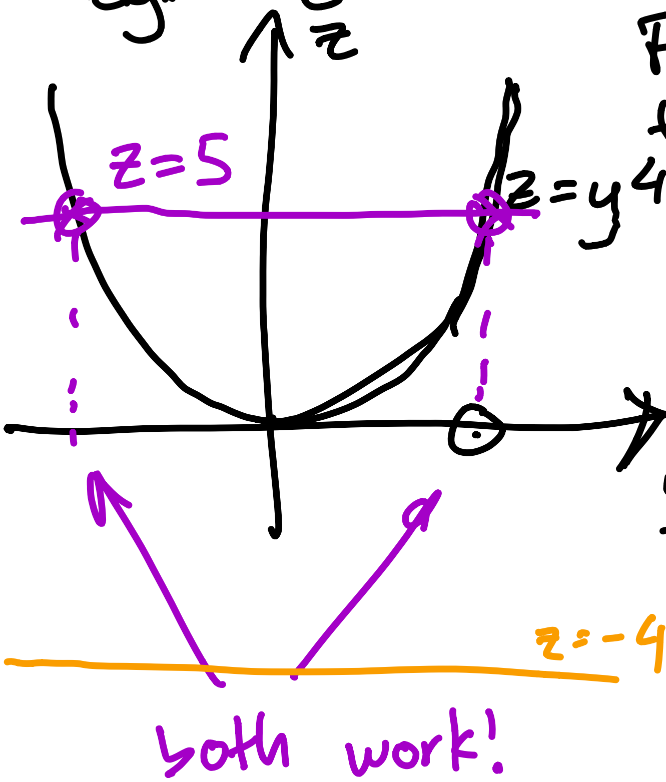
Say $b = 5$. Is there a number y so that $y^3 = 5$?



There is a y that works, and there's exactly one!

This y we'll call $5^{\frac{1}{3}}$

Another example:
Find what $b^{\frac{1}{4}}$ is, say
eg. $5^{1/4}$



Find y so that
$$y^4 = 5$$

We'll call $5^{\frac{1}{4}}$ the largest number y such that
$$y^4 = 5$$

In general: we <u>define</u> $b^{\frac{1}{n}}$ to be the largest solution of $y^n = b$, we assume always $b > 0$.

Why $b > 0$: sup. $b = -4$: then

$y^4 = (-4)$ has no solutions.

A few facts on manipulation of expressions:

$\to b^{p/q} = \left(\sqrt[q]{b}\right)^p = \sqrt[q]{b^p}$

ex. $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

$8^{2/3} = \left(8^2\right)^{\frac{1}{3}} = (64)^{\frac{1}{3}} = 4$ bec. $2^3 = 8$

bec. $4^3 = 64$

$\to b^r b^s = b^{r+s}$

$\to b^{rs} = (b^r)^s$

$\to (ab)^r = a^r b^r$

ex: $(2 \cdot 3x)^2 = 4 \cdot 9x^2$

$\to b^0 = 1$

$\to b^{-r} = \frac{1}{b^r}$

Now can make sense of $b^{m/n}$, m, n are integers

$b > 0$ $(m, n = -2, -1, 0, 1, \ldots)$

$$b^{m/n} = \Big(\underbrace{b^{\frac{1}{n}}}_{\text{made sense of this}}\Big)^m$$

Can't write all numbers as $\frac{m}{n}$ for m, n integers.

e.g. π can't be written like this

Functions of exponential type

$$f(x) = A_0 b^x$$

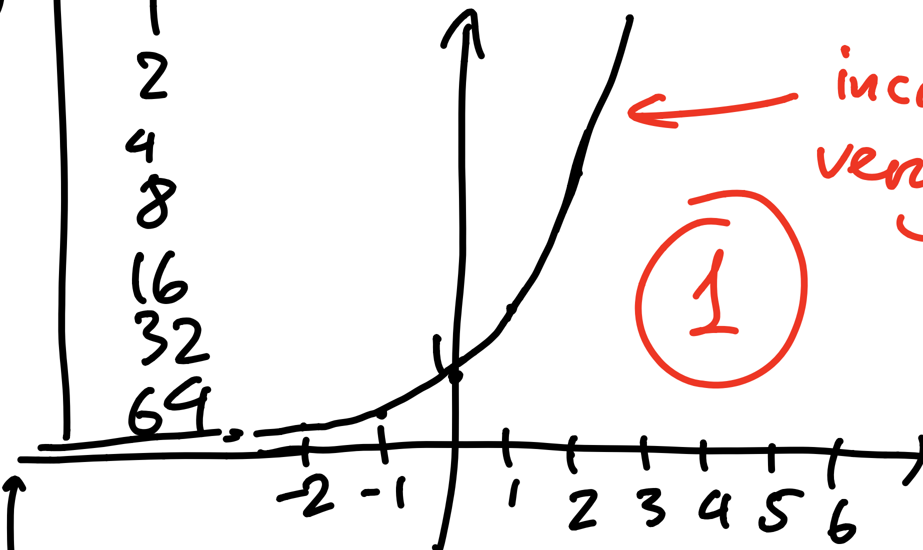
$$A_0 \neq 0, \quad b \neq 1, \quad b > 0$$

How do their graphs look like?

x	2^x
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8
4	16
5	32
6	64

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$



increasing very fast

1

getting close to x axis, never touch x axis ("approach asymptotically")

Look at graph of $f(x) = \left(\frac{1}{2}\right)^x$

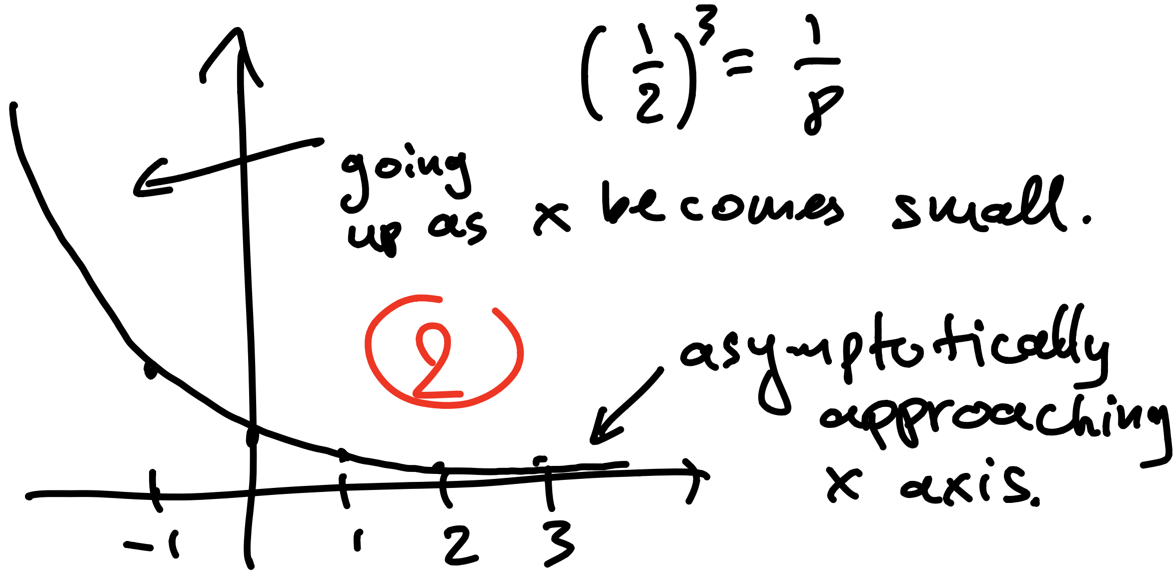
x	$\left(\frac{1}{2}\right)^x$
-1	2
0	1
1	0.5
2	0.25
3	0.125

$$\left(\frac{1}{2}\right)^{-1} = \frac{1}{\left(\frac{1}{2}\right)^1} = 2$$

$$\left(\frac{1}{2}\right)^0 = 1$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$



"Conclude": graph of $f(x) = A_0 b^x$

looks roughly like 1 if $b > 1$

2 if $b < 1$

and $A_0 > 0$

number, not 0

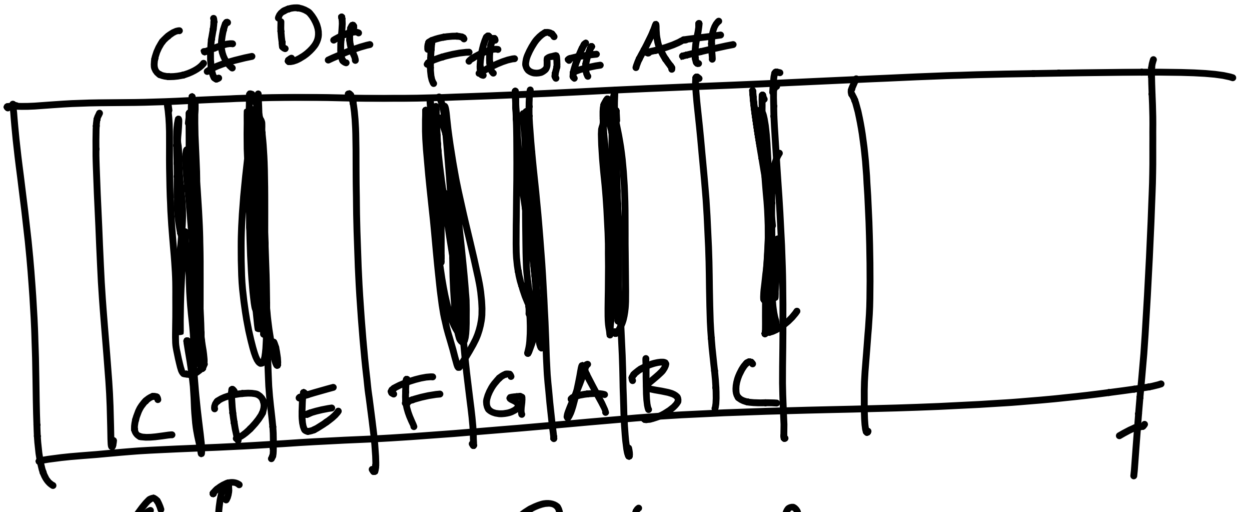
$f(x) = \boxed{A} b^x$ "standard form"

ex: $f(x) = 2\left(\frac{1}{4}\right)^{3x+2}$ $\quad a^{r+s} = a^r \cdot a^s$

$= 2\left(\frac{1}{4}\right)^{3x} \cdot \left(\frac{1}{4}\right)^2$

$= 2\left(\frac{1}{4}\right)^2 \cdot \left[\left(\frac{1}{4}\right)^3\right]^x$

$= \frac{2}{16}\left(\frac{1}{64}\right)^x$ standard form



frequencies

Ratio of frequencies is $2^{\frac{1}{12}}$

A $\quad 220 \text{ Hz}$

A# $\quad 220 \cdot 2^{\frac{1}{12}}$

B $\quad 220 \cdot 2^{\frac{1}{12}} \cdot 2^{\frac{1}{12}} = 220 \cdot 2^{\frac{2}{12}}$

A $\quad 220 \cdot \left(2^{\frac{1}{12}}\right)^{12} = 220 \cdot 2$

$= 440 \text{ Hz}$