Worksheet

October 16, 2017

1. Make a change of variables to evaluate the integral

$$\iint_R e^{\frac{y-x}{y+x}} dA,$$

where R is the trapezoidal region with vertices (1,0), (2,0), (0,2) and (0,1).

2. Set up an integral $\iiint_E f(x,y,z)dV$, where E is bounded above by the plane $z = 3 - x - y$, below by the xy plane, and also bounded by the planes $x = -1$, $x = 1$, $y = 0$ and $y = 1$ **in the order** $dxdzdy$.

3. Compute the volume of the bounded domain lying between the cones $z^2 = x^2 + y^2$ and $z^2 = 3(x^2 + y^2)$, and under the plane $z = 3$.

4. If a transformation T is written as $x = x(u,v)$ and $y = y(u,v)$ and given a point (u_0, v_0) on the uv plane, we can produce an affine transformation dT called the **differential** or **pushforward** of T at (u_0, v_0) that is the best affine approximation of T near (u_0, v_0) and is given by

$$x = \frac{\partial x}{\partial u}_{|(u,v)=(u_0,v_0)}(u - u_0) + \frac{\partial x}{\partial v}_{|(u,v)=(u_0,v_0)}(v - v_0) + x(u_0, v_0)$$
$$y = \frac{\partial y}{\partial u}_{|(u,v)=(u_0,v_0)}(u - u_0) + \frac{\partial y}{\partial v}_{|(u,v)=(u_0,v_0)}(v - v_0) + y(u_0, v_0).$$

For the transformation $T(u,v) = (\frac{u^2}{v}, u^2 v)$ defined on $\{(u,v) : u > 0, v > 0\}$:

(a) Find the transformation dT relative to the point (1,1).

(b) Find and draw the image of the box $[1,2] \times [1,2]$ under T and dT.