Story Problems

Difficulties/sources of errors

- Not reading problem
- Knowing definition (e.g. "speed")
- ALGEBRA.

Common Mistakes

Deadly sins

$$\cancel{(a+b)^2 = a^2 + b^2}$$

$$\cancel{\sqrt{a+b} = \sqrt{a} + \sqrt{b}}$$

$$\cancel{\frac{\sin x}{x} = \sin x = 6}$$

<u>Ex.</u> $\sqrt{2x-1} - \sqrt{x} = 1$. solve for x.

$$\left(\sqrt{2x-1} - \sqrt{x}\right)^2 = 1^2$$

$$(2x-1) - 2\sqrt{x}\sqrt{2x-1} + (x) = 1$$

(identity : $(a+b)^2 = a^2 + 2ab + b^2$

$\rightarrow (a-b)^2 = a^2 - 2ab + b^2$)

$$3x - 2\sqrt{x}\sqrt{2x-1} - 2 = 0$$

OR:

$$\sqrt{2x-1} = 1+\sqrt{x}$$

$$\Rightarrow \quad 2x-1 = 1+2\sqrt{x}+x$$

$$x-2\sqrt{x}-2=0 \quad (*)$$

Set $y=\sqrt{x}$

$$(*) \Rightarrow \quad y^2-2y-2=0$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)\cdot(-2)}}{2\cdot 1}$$

$$y = \frac{2 \pm \sqrt{4+8}}{2}$$

$$y = \frac{2 \pm \sqrt{12}}{2}$$

$$y = \frac{2+\sqrt{12}}{2} \qquad y = \frac{2-\sqrt{12}}{2} \leftarrow$$

Negative

Side note: $y = \frac{2 \pm \sqrt{12}}{2} \Rightarrow$

$\Rightarrow y = \frac{2 \pm \sqrt{4 \cdot 3}}{2}$

$\Rightarrow y = \frac{2 \pm 2\sqrt{3}}{2}$

$\Rightarrow y = 1 \pm \sqrt{3}$

$y = \frac{2 + \sqrt{12}}{2} \Rightarrow \sqrt{x} = \frac{2 + \sqrt{12}}{2}$

$\Rightarrow x = \left(\frac{2 + \sqrt{12}}{2}\right)^2$

<u>Moral:</u> Be careful when squaring.

Squaring generates sol's:

$x = 1 \Rightarrow x^2 = 1^2 \Rightarrow x^2 = 1$

$\Rightarrow x = 1, \overbrace{x = -1}$

When squaring, go back and check solution makes sense.

<u>Ex:</u> Story Problem.

A B767 needs 22,300 kg of jet fuel for a trip from Montreal to Edmonton. In Montreal, Pilot checks that that there are 7,682 lt of fuel in tank. Known: Density

of fuel = $1.77 \frac{lb}{lt}$.

How many additional liters of fuel does he need?

<u>Sol:</u>

<u>1st step:</u> Find mass of fuel in tank.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\underset{kg}{\text{mass}} = \underset{?\ \frac{kg}{lt}}{\text{density}} \cdot \underset{lt}{\text{volume}}$$

$$\underline{1\ kg} = \underline{2.2\ lb}$$

$$\text{density} = 1.77 \frac{\cancel{lb}}{ft} \cdot \left(\frac{1\,kg}{2.2\,\cancel{lb}} \right)$$

conversion

$$\text{density} = \frac{1.77}{2.2} \frac{kg}{ft} \sim 0.8 \frac{kg}{ft}$$

$$\text{mass in tank} = \text{density} \cdot (\text{volume in tank})$$

$$= 0.8 \frac{kg}{ft}\, 7{,}682\,ft$$

$$= 6{,}121.6\ kg$$

$$\text{mass needed} = 22{,}300\,kg - 6{,}121.6\,kg$$

$= 16,178.4 \text{ kg}$

$$\text{density} = \frac{\text{mass}}{\text{volume}} \Rightarrow$$

$$\Rightarrow \text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{volume} = \frac{16,178.4 \text{ kg}}{0.8 \text{ kg/lt}}$$

$$\boxed{\text{fuel needed} = 20,223 \text{ lt}}$$

Rate of Change

How a quantity changes with time.

$$\text{Rate} \underbrace{:=}_{\text{by definition}} \frac{\text{change in quantity}}{\text{change in time}}$$

$$= \frac{(\text{final quantity}) - (\text{initial quant})}{(\text{final time}) - (\text{initial time})}$$

$$\overset{\text{notation}}{=} \frac{\Delta\,\text{quantity}}{\Delta\,\text{time}}$$

"Delta"

<u>Ex:</u> Temperature at 8 am

$\rightarrow 20°C$

at 10 am

$\rightarrow 23°C$

$$\text{rate} = \frac{23°C - 20°C}{10h - 8h} =$$

$$= \frac{3°C}{2h} = 1.5\ °C/h$$

<u>Another ex</u>: Temp. → 23°C at 8am

20°C at 10am

$$\text{rate} = \frac{20°C - 23°C}{10h - 8h} = \frac{-3}{2}\ °C/h$$

$$= -1.5°C/h$$

→ Rate of change may or may not be const.

When it <u>is</u> constant:

Recover change

Total change = rate · (time elapsed)

<u>Ex</u>: 50's Seattle dumps 20

million gallons of sewage/day in lake Washington.

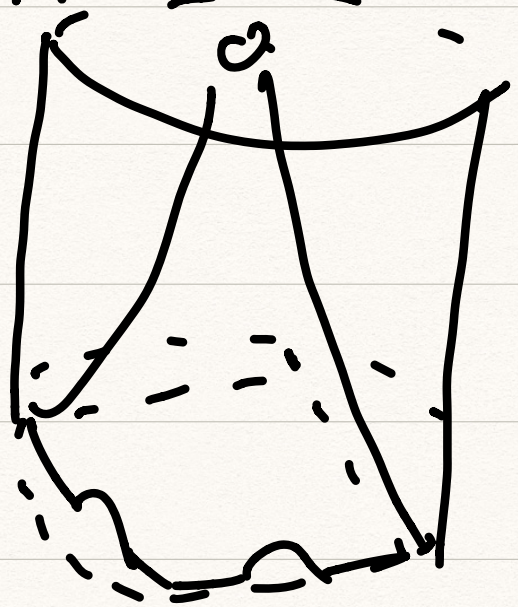
Sewage dumped in 1 week:

$$20 \text{ million gallons}/\text{day} \cdot 1 \text{ week}$$

$$= 20 \text{ million gallons}/\text{day} \cdot 7 \text{ days}$$

$$= 140{,}000{,}000 \text{ gallons.}$$

Problem 19

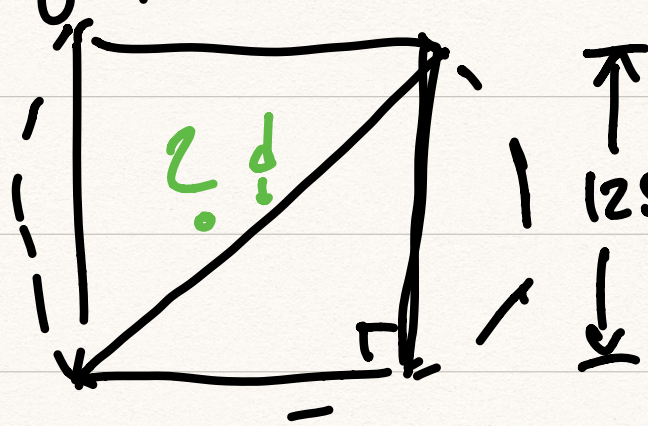


Find mass of air in cylinder.

Know: density of air.

Enough to find volume of air!

To find volume: Volume of cylinder - Volume of tower.



Vol. of cylinder

$V = \pi r^2 h$

$\leftarrow 125 \rightarrow$

radius ↗ ↑ height

Pyth. Thm

$$d^2 = 125^2 + 125^2$$

$$\Rightarrow d = \sqrt{2 \cdot 125^2}$$

$$\Rightarrow d = \sqrt{2}\, 125$$

$$\text{Radius} = \frac{d}{2} = \frac{125}{2}\sqrt{2}\, m$$

$$\text{Vol. of cylinder} = \left(\frac{125}{2}\sqrt{2}\, m\right)^2 \cdot \pi \cdot \underset{\substack{\uparrow \\ \text{height}}}{324\, m}$$

$$= 7{,}948{,}125\ m^3$$

Vol. of air: (Vol. of cyl) − (Vol. tower)

$$= 7{,}948{,}125 - 930\ m^3$$

$= 7,947,195 m^3$