Math 324 C - Winter 2017

Midterm 1

Friday, January 27, 2017

Name: ______________________________

Student ID Number: ______________________________

Problem 1	12	
Problem 2	8	
Problem 3	18	
Problem 4	12	
Total	50	

- There are 4 problems spanning 4 pages (your last page should be numbered as 4). Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems (unless explicitly instructed otherwise). The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely.
Do not spend too much time on an individual problem, unless you are done with all the rest.

GOOD LUCK!

1. (12 pts) The two parts are not related.

   (a) Determine whether the following statement is **true** of **false**, and explain your answer: The set in $\mathbb{R}^3$ described in cartesian coordinates as $A = \{(x, y, z) \in \mathbb{R}^3 : z^2 = x^2 + y^2\}$ is the same as the set in $\mathbb{R}^3$ described in spherical coordinates as $B = \{(\rho, \theta, \phi) : \phi = \frac{3\pi}{4}\}$, under the usual convention $\rho \geq 0$, $\theta \in [0, 2\pi)$ and $\phi \in [0, \pi]$.

   (b) A thin lamina occupies the region

$$D = \{(x, y) : 1 \leq x^2 + y^2 \leq 4 \text{ and } y \geq |x|\}.$$

   If the density function ρ at each point (x, y) is inversely proportional to the square of the distance of the point to the origin, find the moment about the x axis (the M_x) of the lamina.

2. (8 pts) Let $f(x, y, z) = xy$. Set up **but do not evaluate** $\iiint_E f(x, y, z)dV$ in **cylindrical coordinates**, where E is the solid that lies inside the sphere $x^2 + y^2 + z^2 = 9$, under the cone $z = -\sqrt{x^2 + y^2}$ and satisfies $y \leq 0$.

3. (18 pts) [You should be able to answer each part regardless of whether you have answered the other one] Let $f(x, y, z) = z$.

(a) Set up **but do not evaluate** $\displaystyle\iiint_E f(x, y, z)dV$ **in the order** $dxdydz$, where E is the solid **in the first octant** bounded by the coordinate planes, the circular cylinder $x^2 + y^2 = 4$ and circular cylinder $x^2 + z^2 = 4$. (make sure to involve the given function in your formula!)

(b) **Evaluate** $\iiint_E f(x, y, z)dV$ using cylindrical coordinates.

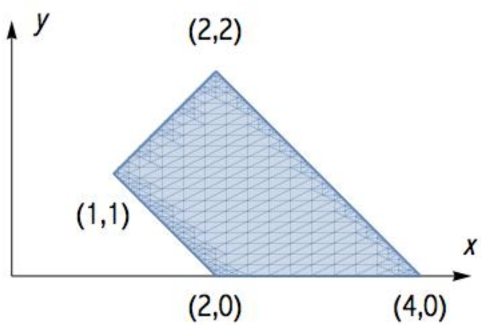
4. (12 pts) Let R be the trapezoid in the xy plane defined by the points (1,1), (2,2), (2,0) and (4,0), as in the picture, and you are given the transformation $x = u + v$ and $y = u - v$.

   (a) Compute the Jacobian determinant $\frac{\partial(x,y)}{\partial(u,v)}$.

   (b) Find the inverse transformation T^{-1} (that is, $u = u(x, y)$ and $v = v(x, y)$).

   (c) Find the image S of R under T^{-1} in the uv plane (that is, the set $S = T^{-1}(R)$) and **draw a picture of it.**

   (d) Use your work in the parts (a)-(c) to calculate $\iint_R e^{\frac{x-y}{x+y}}\, dA$ (you can use the back of the page if you run out of space).



5. (8 pts) The temperature at a point (x, y) of the plane is given in degrees Celcius by

$$T(x, y) = x^2 y^3 + 2\cos(3x\pi + y\pi),$$

where x and y are in meters. You are standing at the point (1,2) and you want to move towards the direction in which the temperature **drops** most rapidly (that is, the direction in which you have the minimum net rate of change of temperature).

(a) **Find a vector** that gives this direction.

(b) **Find the directional derivative of** T in the direction determined by this vector. **Make sure to include units in your answer.**

6. (9 pts) Let $z = z(x, y)$ be a twice differentiable function with continuous second partial derivatives and $x = x(t)$, $y = y(t)$ be differentiable with continuous first partial derivatives. You are given the following table with data. Use the chain rule to evaluate $\dfrac{d^2z}{dt^2}(0)$.

$x(0) = 1$	$y(0) = -1$	$z(1, -1) = -1$
$\dfrac{\partial z}{\partial x}(1, -1) = -2$	$\dfrac{\partial z}{\partial y}(1, -1) = 3$	$\dfrac{dx}{dt}(0) = 1$
$\dfrac{dy}{dt}(0) = 0$	$\dfrac{d^2x}{dt^2}(0) = 0$	$\dfrac{d^2y}{dt^2}(0) = 2$
$\dfrac{\partial^2 z}{\partial x^2}(1, -1) = -2$	$\dfrac{\partial^2 z}{\partial y^2}(1, -1) = -6$	$\dfrac{\partial^2 z}{\partial x \partial y}(1, -1) = 6$

Hint: Note that some of the partial derivatives given are 0; this may simplify your calculation.