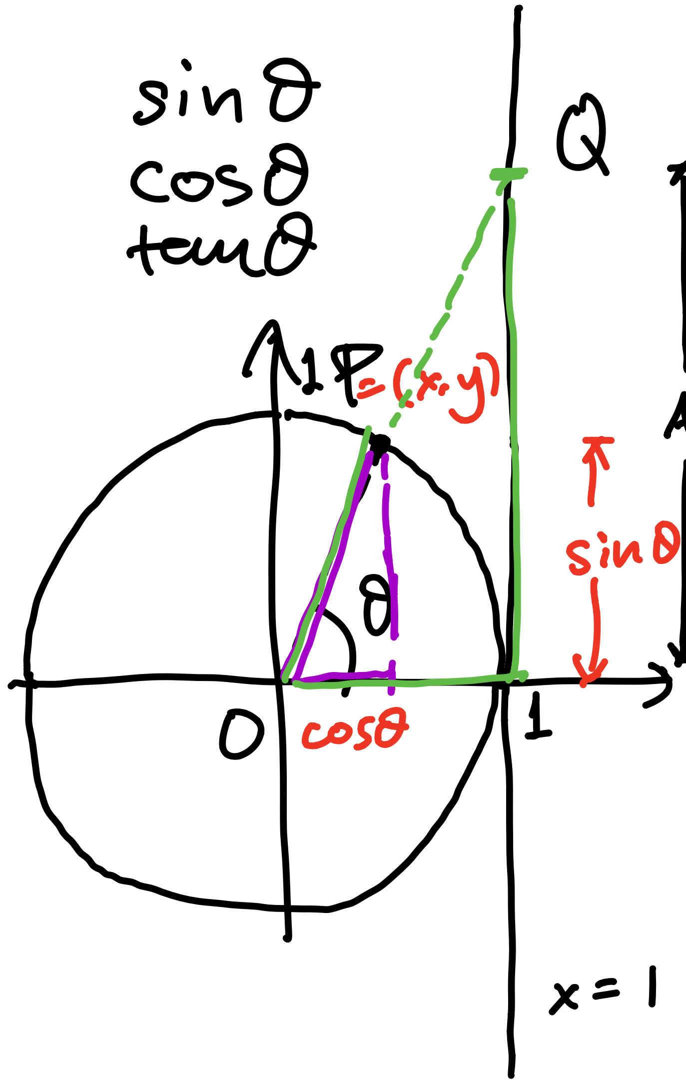
$\sin\theta$
$\cos\theta$
$\tan\theta$



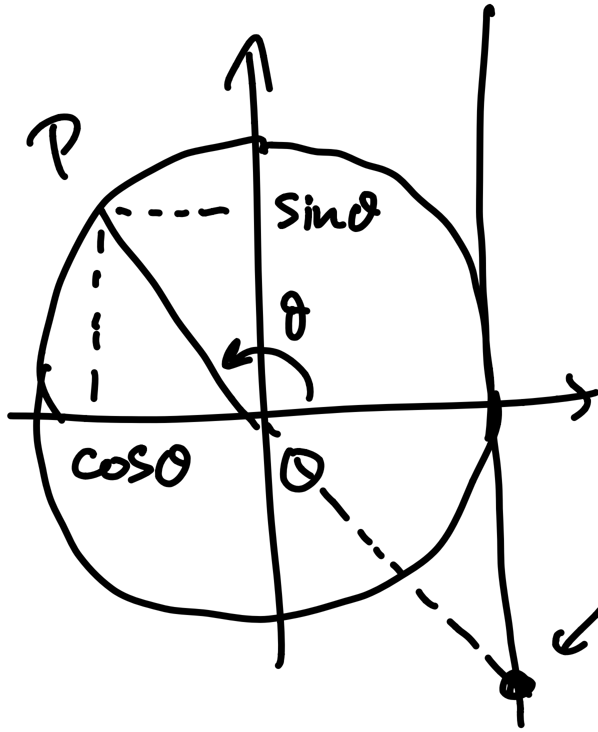
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

similar triangles

$$\frac{A}{1} = \frac{\sin\theta}{\cos\theta}$$

$$A = \tan\theta$$

So: we can find $\tan\theta$ by looking at y coordinate of the intersection of $x=1$ and line through OP.

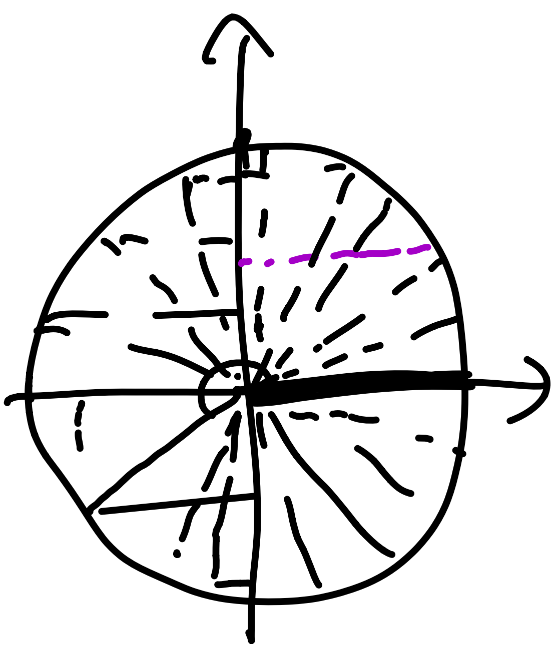


Here $\sin\theta > 0$

$\cos\theta < 0$

$\tan\theta < 0$

y coord. is $\tan\theta$



θ	$z = \sin\theta$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

Range of $z = \sin\theta$

$-1 \le z \le 1$

when $0 \le \theta \le 2\pi$

Range of $z = \cos\theta$ $\quad -1 \le z \le 1$
for $\quad 0 \le \theta \le 2\pi$

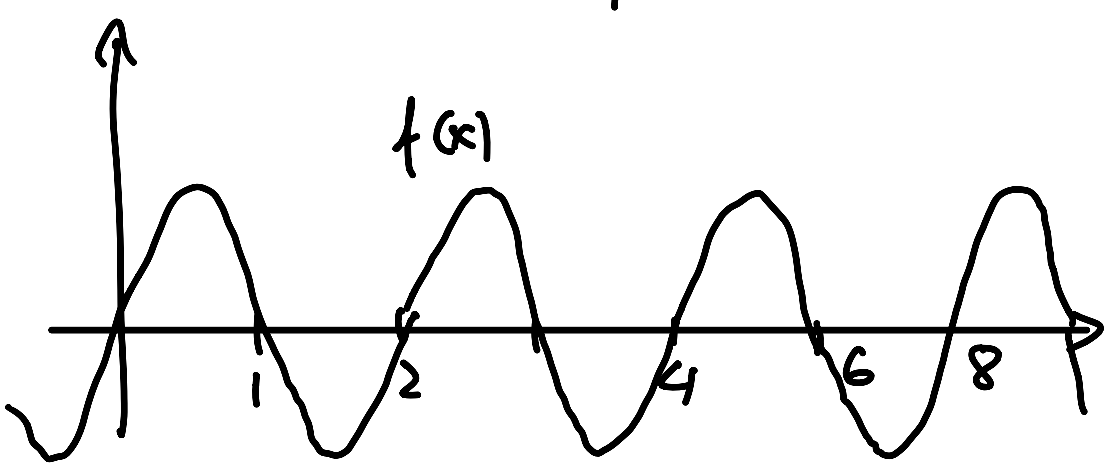
$\sin\theta$ and $\cos\theta$ are <u>periodic</u> functions.

<u>Def'n</u>. A function $f(x)$ is called c-periodic if

i) $$f(x+c) = f(x)$$
for all x, $c > 0$

ii) There isn't any other number $0 < c' < c$ such that $f(x+c') = f(x)$ for all x.

Ex:



Note: $f(x+c)$ is graph of $f(x)$ shifted by c

In our example, we can shift by 2 to the left and the new graph agrees with old.

$$f(x+c)=f(x)$$

$c=2$ is a good candidate for a period! Property (i) is satisfied.

$c'=1$ wouldn't work! shifting by 1 to left gives same graph.

Period = 2

$f(x+c)=f(x)$

Side note: if c is a period for $f(x)$ then $c \cdot n$, n integer satisfies $f(x+cn)=f(x)$

$$f(x+2c)=f(\underbrace{(x+c)}+c)$$

$$=f(\underbrace{(x+c)})$$

$$=f(x)$$

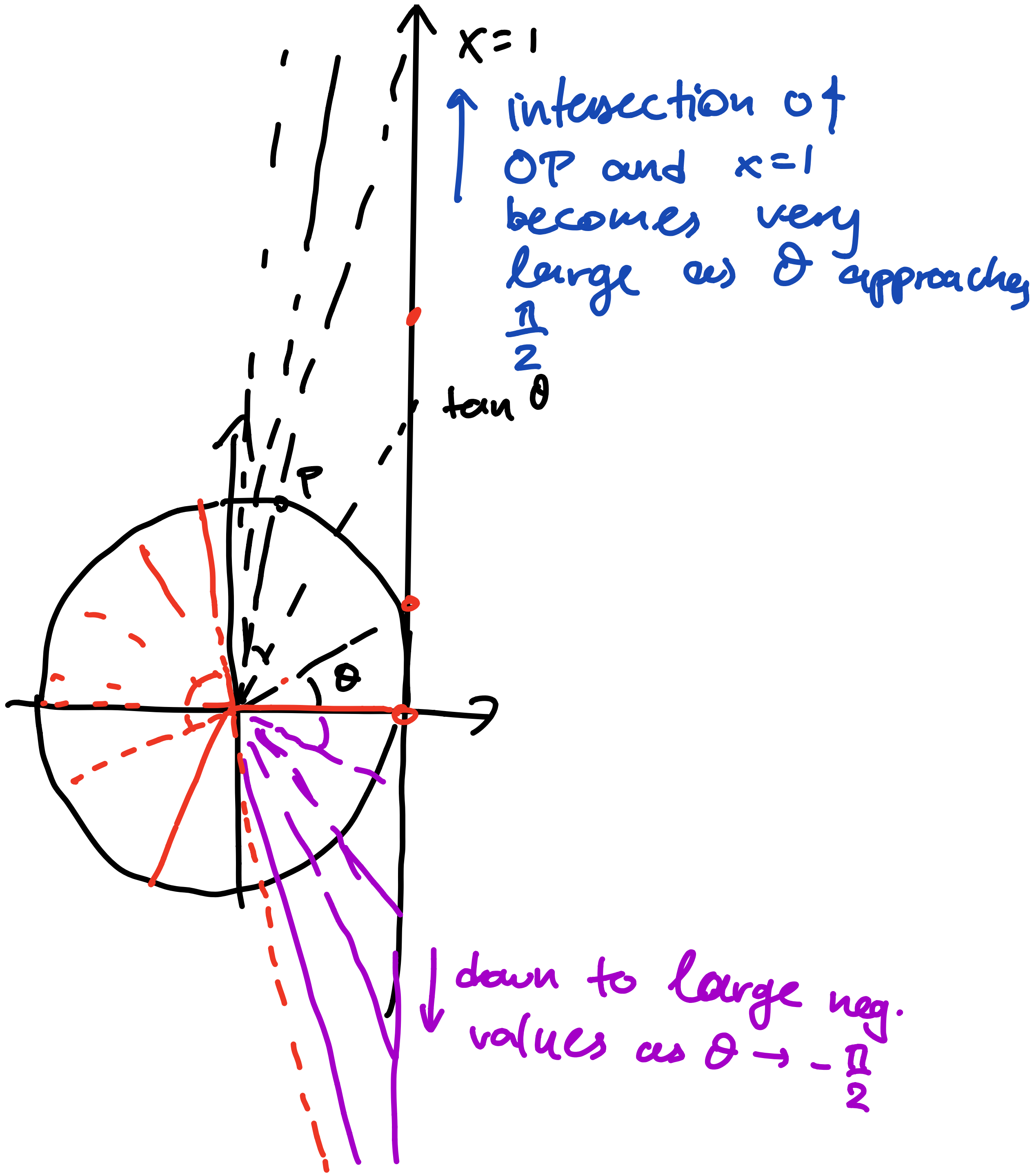
If a function repeats itself after c, it will repeat itself after 2c

$$\boxed{\cos\theta \text{ and } \sin\theta \text{ are periodic with period } 2\pi}$$

e.g. $\cos(\theta + 22\pi) =$

$= \cos(\theta + 11 \cdot 2\pi)$

$= \cos(\theta)$



Behavior of tan θ "repeats itself" every π, it has a smaller period than sin θ &

$\cos\theta$.

approaching ∞

Remark: $\tan\theta$ becomes "big positive" or "big negative" when $\pm\frac{\pi}{2}$

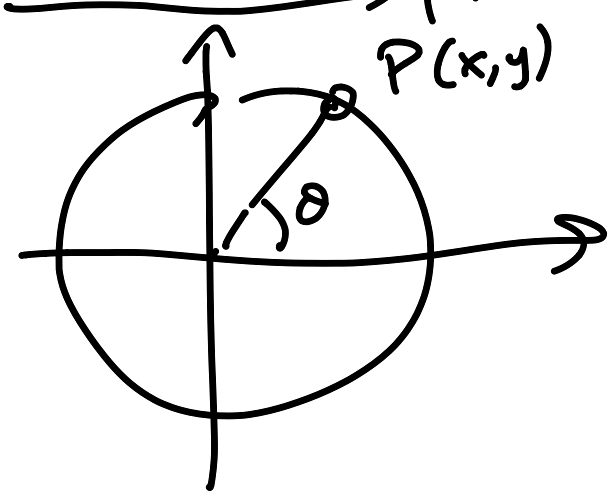
$\pi \pm \frac{\pi}{2}$

$2\pi \pm \frac{\pi}{2}$

this is when $\cos(\theta) = 0$!

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

Identities



$x = \cos\theta$
$y = \sin\theta$

P is on unit circle. So
$x^2 + y^2 = 1$

Identity 1: $\Rightarrow \boxed{\cos^2(\theta) + \sin^2(\theta) = 1}$

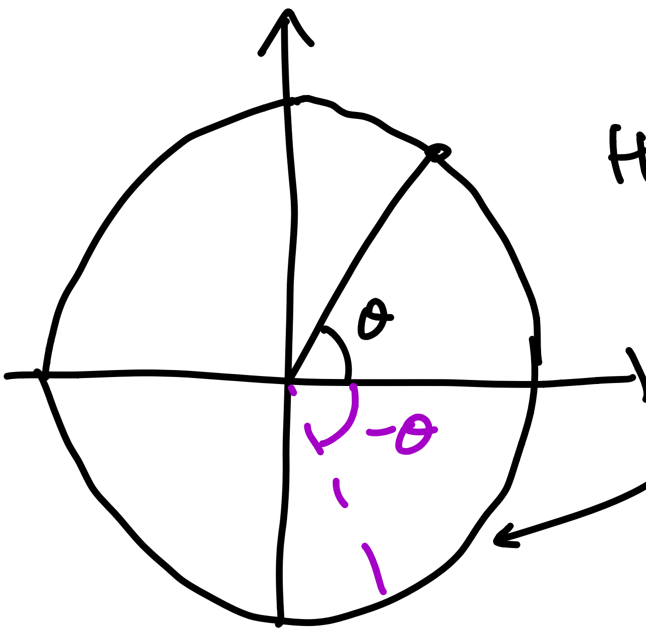
$(\cos(\theta))^2$

Even - odd

$\sin(-\theta) = -\sin(\theta)$ "$\sin(\theta)$ is an odd function"

and

$\cos(-\theta) = \cos(\theta)$ "$\cos(\theta)$ is an even function"



How to remember: Pick angle in 1st quadrant.

$\sin(-\theta)$ is here so it must be negative.

$\sin(\theta)$ is > 0 bec. θ is in 1st quadrant.

So $\sin(-\theta) = -\sin(\theta)$

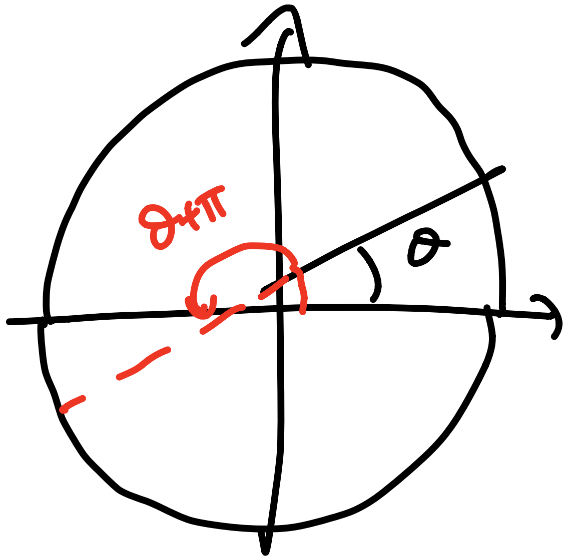
In exactly same way:

$\cos(-\theta) > 0$, $\cos(\theta) > 0$

so $\cos(-\theta) = \cos(\theta)$

3. <u>Plus π</u>

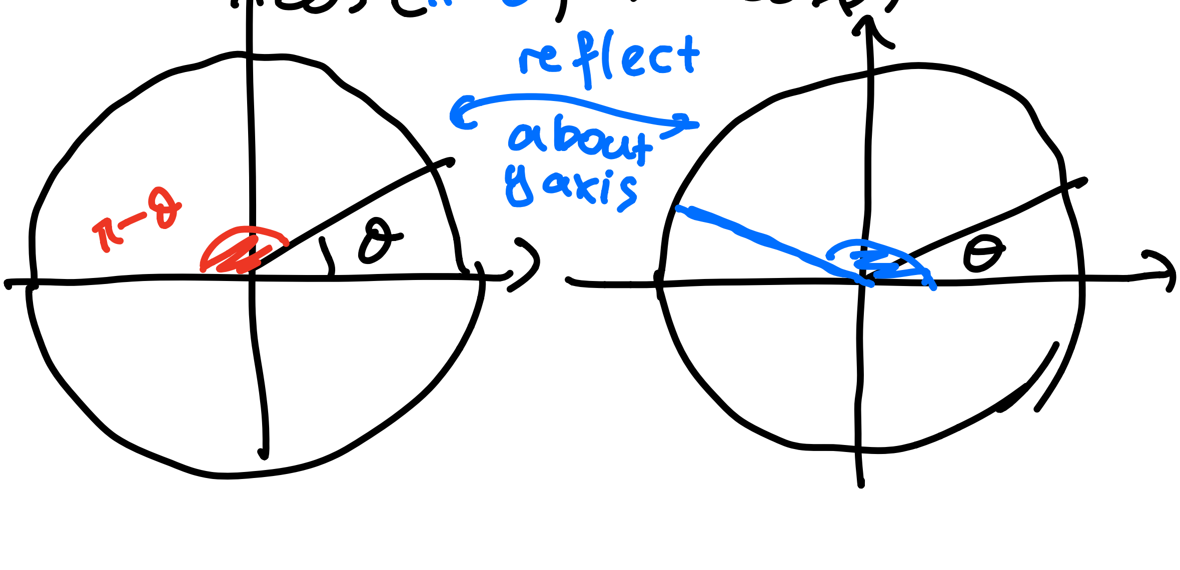
$$\sin(\theta+\pi) = -\sin(\theta)$$
$$\cos(\theta+\pi) = -\cos(\theta)$$



How to remember: $+\pi$ throws us from 1st to 3rd quadrant so $\cos(\theta+\pi)$, $\sin(\theta+\pi)$ are negative if θ in 1st quadrant.

4 π-minus

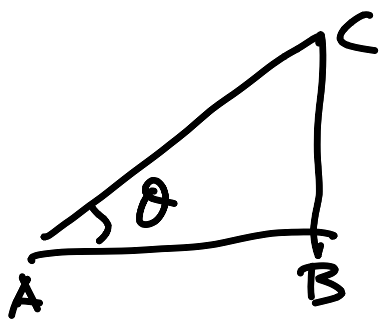
$$\sin(\pi-\theta) = \sin(\theta)$$
$$\cos(\pi-\theta) = -\cos(\theta)$$



5. $\frac{\pi}{2}$ minus

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin(\theta)$$

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos(\theta)$$



$$\cos\left(\frac{\pi}{2}-\theta\right) = \cos(\hat{C}) = \frac{|BC|}{|AC|} = \sin(\hat{A}) = \sin(\theta)$$

Idea: what's opposite for θ is adjacent for $\frac{\pi}{2}-\theta$.

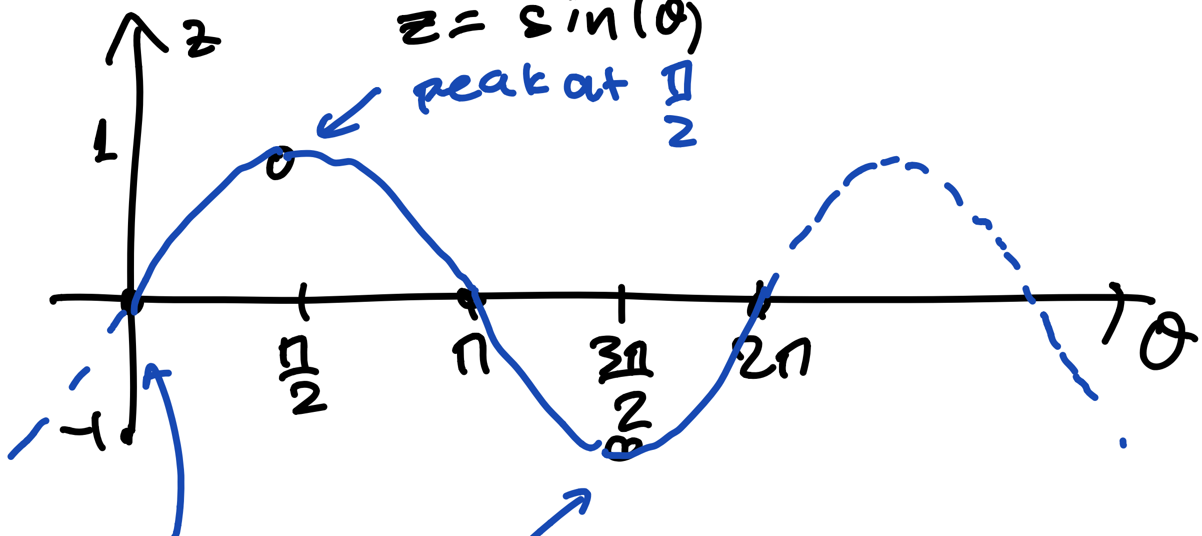
As a general rule:

Identities involving π:
keep cos as cos
sin as sin

Identities involving $\frac{\pi}{2}$:
chang cos to sin
sin to cos

Graphs of trig. functions

$z = \sin(\theta)$



at 0 starts at 0, goes up.

"valley" at $\frac{3\pi}{2}$

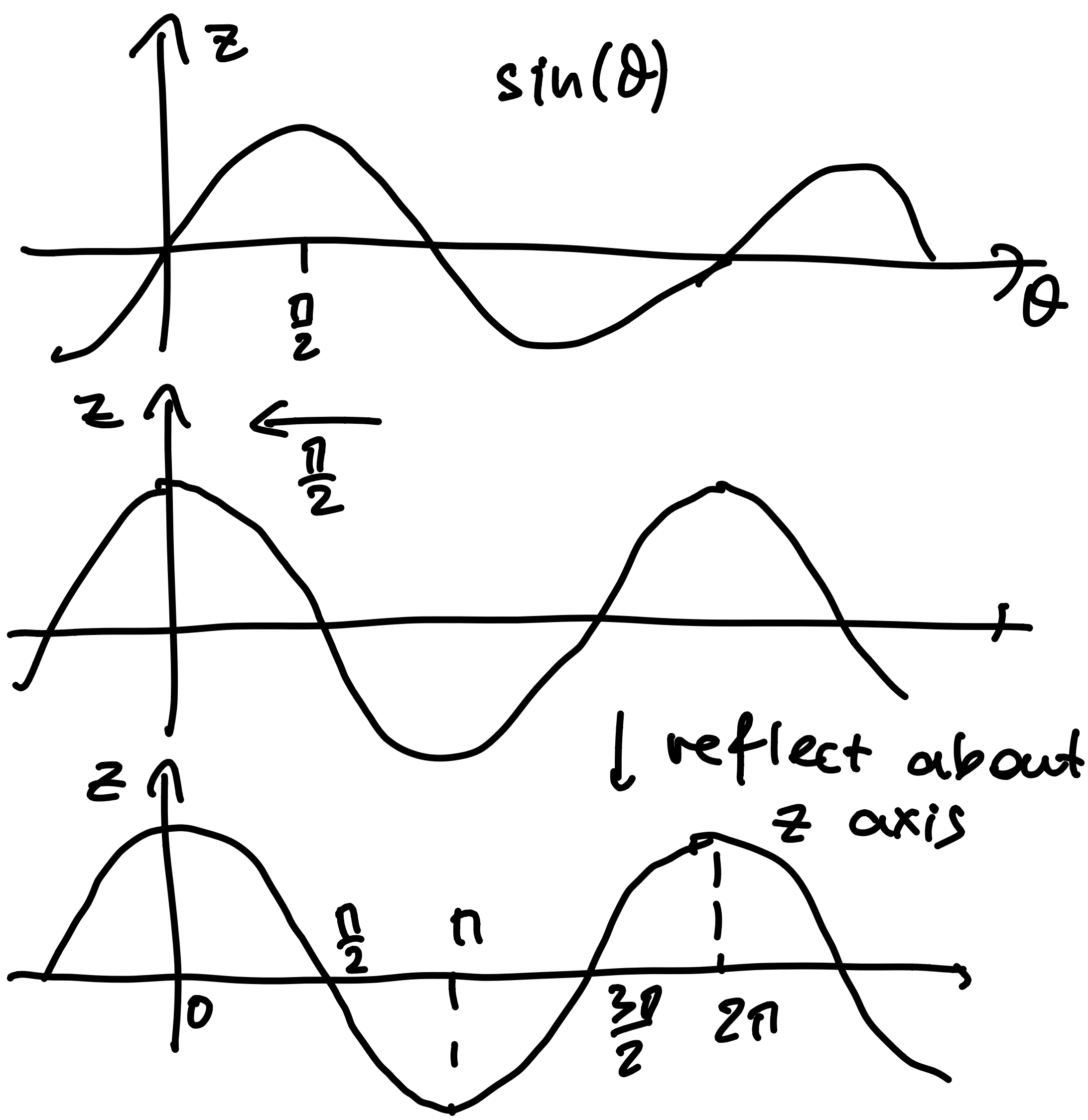
Graph $z = \cos(\theta)$

$$z = \cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$= \sin\left((-\theta) + \frac{\pi}{2}\right)$$

① shift by $\frac{\pi}{2}$ to left

② Reflect about z axis


z
sin(θ)
π/2
θ
z
π/2
reflect about
z axis
z
0
π/2
π
3π/2
2π