Quiz 3

Name: _____

Student ID Number: _____

Let E be the solid of the picture below, bounded below by the paraboloid $z = 4x^2 + 4y^2$ and bounded above by the cone $z = 8 - 4\sqrt{x^2 + y^2}$. Compute the volume of E.

Use cylindrical coords

① $z = 4r^2$

② $z = 8 - 4r$

Find projection by eliminating z:

$4r^2 = 8 - 4r \Rightarrow \qquad 4r^2 + 4r - 8 = 0$

$\Rightarrow \quad r^2 + r - 2 = 0$

$\Rightarrow \quad r = \dfrac{-1 \pm \sqrt{1 + 8}}{2}$

$\Rightarrow \quad r = \dfrac{-1 \pm 3}{2} \Rightarrow \quad r = -2 \text{ or } \boxed{r = 1}$

So

$\text{Volume} = \displaystyle\int_0^1 \int_0^{2\pi} \int_{4r^2}^{8-4r} 1 \cdot r \, dz \, d\theta \, dr$

$= \displaystyle\int_0^1 \int_0^{2\pi} (8 - 4r - 4r^2) r \, d\theta \, dr$

$= 2\pi \displaystyle\int_0^1 8r - 4r^2 - 4r^3 \, dr$

$= 2\pi \left[4r^2 - \dfrac{4}{3}r^3 - r^4 \right]_0^1$

$= 2\pi \left(4 - \dfrac{4}{3} - 1 \right)$

$= 2\pi \dfrac{5}{3}$