Chapter 16.

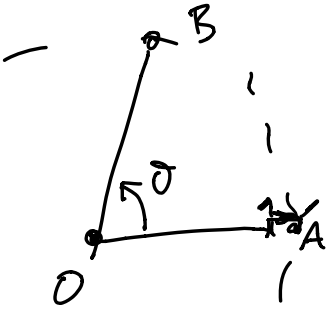
Last time: saw how to measure angles in radians and degrees, saw formula for arc length:

If θ is in degrees:

$$s = 2\pi r \frac{\theta}{360^\circ}$$

If θ is in radians:

$$s = \theta r$$



Dog walking on lease tied to pole

Want to understand how fast he's moving.

2 ways → measure angle swept in given time

→ measure distance he's traveling in given time

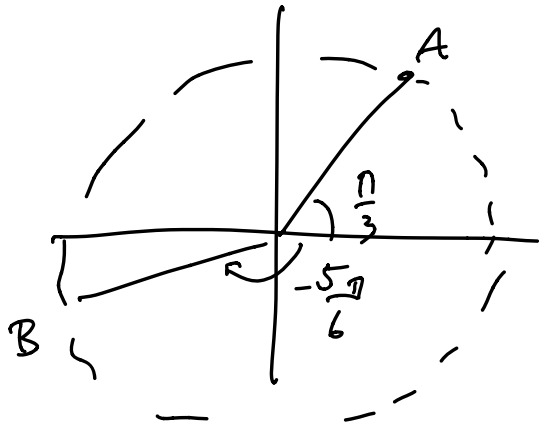
1st. $w = \dfrac{\text{measure of angle swept}}{\text{time needed to go from A to B}}$

"Angular speed"

Example: say he swept angle of $\frac{\pi}{3}$ rad counterclockwise in 3 min.

$$\omega = \frac{\frac{\pi}{3}\text{ rad}}{3\text{ min}} = \frac{\pi}{9}\ \frac{\text{rad}}{\text{min}}\ \text{counterclockwise}$$

Another Ex:



He moves from A to B clockwise in 2 s

Measure of angle $= \left|-\frac{5\pi}{6} - \frac{\pi}{3}\right| = \left|\frac{-7\pi}{6}\right|$

$= \frac{7\pi}{6}$

So $\omega = \frac{7\pi}{6}\cdot\frac{1}{2\text{sec}} = \frac{7\pi}{12}\ \frac{\text{rad}}{\text{sec}}$ clockwise

Or write $\omega = -\frac{7\pi}{12}\ \frac{\text{rad}}{\text{sec}}$ (better say clockwise - counterclockwise)

Another way to view angular speed:

$$\frac{\text{Number of revolutions}}{\text{unit of time}}$$

measured in revolutions per minute (RPM)

In last example:

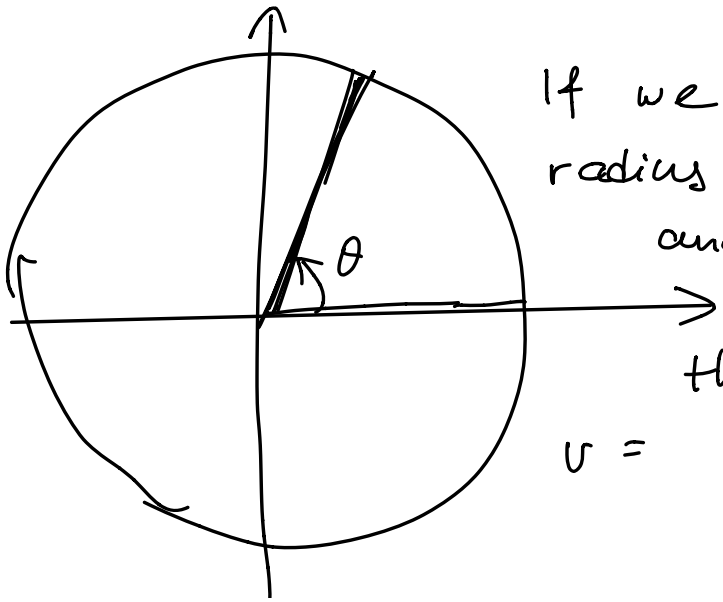
Dog completes $\frac{7\pi}{6}\text{ rad} = \frac{7\pi}{6}\text{ rad} \cdot \frac{1\text{ rev}}{2\pi\text{ rad}} = \frac{7}{12}$ of a revolution in 2 sec

So his angular speed in rpm is

$$\omega = \frac{\frac{7}{12}\text{ rev.}}{2\text{ sec}} = \frac{7}{24}\frac{\text{rev}}{\text{sec}}\,\frac{60\text{ sec}}{1\text{ min}} = \frac{420}{24}\frac{\text{rev}}{\text{min}}$$ ☒

If we'd also like to measure how far he's traveling in distance. 2 objects might have same angular speed but cover different distances in same amount of time.

Let "linear speed" = "distance traveled in unit time"



If we have circle of radius r and sweep angle θ in time t (in radians) then

$$v = \frac{r\theta}{t}$$ ★

Ex. before: if the leash is 5 ft and dog sweeps angle of $\frac{7\pi}{6}$ in 2 min then his linear speed is

$$v = \frac{(5 \cdot \frac{7\pi}{6})\text{ ft}}{2 \text{ sec}} = \frac{35\pi}{12} \text{ ft/sec}$$

Wrap up:

- angular speed $\omega = \frac{\text{revolutions}}{\text{time}} = \frac{\text{degrees}}{\text{time}}$

$$= \frac{\text{radian}}{\text{time}}$$

- linear speed: $v = \frac{\text{distance traveled}}{\text{time}}$

Relate them through following formulas:

If θ is in RADIANS:

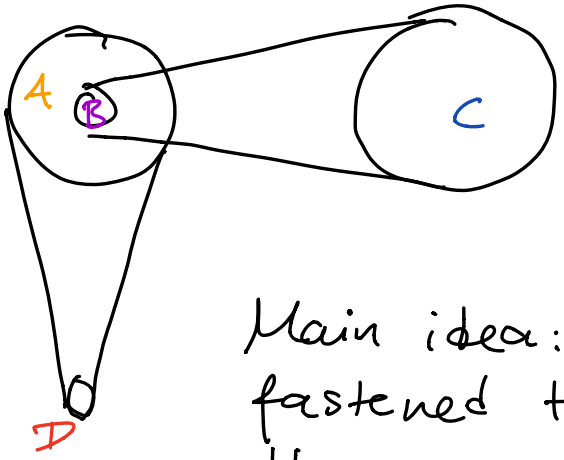
- ω in radians per unit time.

$s = r\theta$

$\theta = \omega t$ } see ⊗

$v = r\omega$

Belt and wheel problems



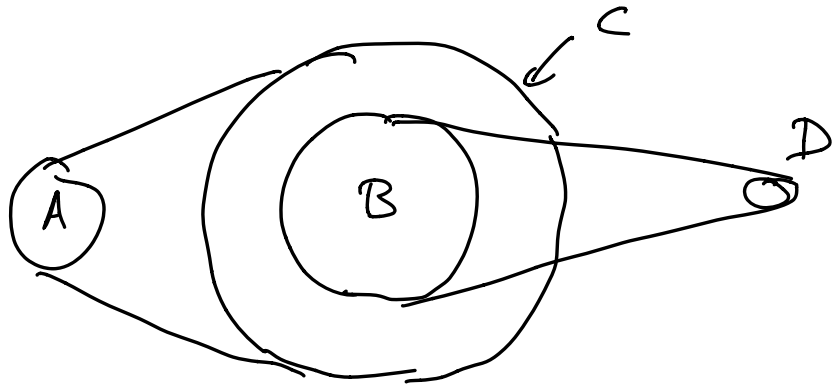
Main idea: 2 wheels fastened together have the same angular speed (not same linear speed)

A, B → same ang. speed

2 wheels connected with a belt or chain have same linear speed along ~~the~~ outer edge as the belt

Ex: B & C, A & D

Ex: Problem 1 from Courog Sp. 17



B, C rotate together

A: radius 3cm, $w_A = 13$ rpm

B: radius 4cm

D: 1 rev. in 2.1 sec, radius 1 cm

Radius of C.?

$$V_A = 13 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \cdot 3 \text{ cm}}{\text{rev}} = 78\pi \frac{\text{cm}}{\text{min}}$$

So $\boxed{V_C = 78\pi \frac{\text{cm}}{\text{min}}}$

$$w_D = \frac{1 \text{ rev}}{2.1 \text{ sec}} = \frac{1 \text{ rev}}{2.1 \text{ sec}} \frac{60 \text{ sec}}{1 \text{ min}} = \frac{60}{2.1} \text{ rpm}$$

$$v_D = \frac{60}{2.1} \, 2\pi \cdot 1 \frac{\text{cm}}{\text{min}} = \frac{120\pi}{2.1} \frac{\text{cm}}{\text{min}}$$

So $v_B = \frac{120\pi}{2.1}\ \frac{cm}{min}$

$v_B = 4\omega_B \Rightarrow \omega_B = \frac{30}{2.1}\ \frac{rad}{min}$

So $\boxed{\omega_C = \frac{30}{2.1}\ \frac{rad}{min}}$

$v_C = r\omega_C \Rightarrow \quad 78\pi = r\,\frac{30}{2.1}$

$\Rightarrow \quad r = \frac{2.1 \cdot 78}{30}\ cm$