Worksheet 3

December 7, 2017

1. Decide which of the three plots below corresponds to the parametrization

$$\vec{r}(u,v) = \langle (2+\cos(v))\cos(u), (2+\cos(v))\sin(u), \sin(v)\rangle, \text{ for } (u,v) \in [0,2\pi]\times[0,2\pi]$$

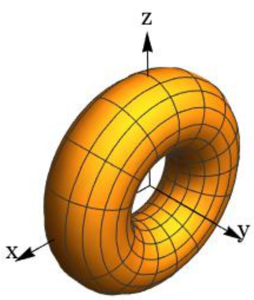

Figure 1: Plot 1

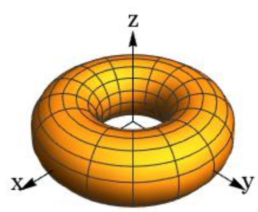

Figure 2: Plot 2

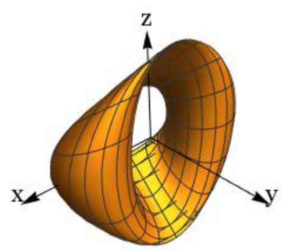

Figure 3: Plot 3

2. Prove the first step towards showing Green's First Identity: Show that for any twice differentiable functions $u(x,y,z)$ and $v(x,y,z)$ we have

$$\operatorname{div}(u\nabla v) = \nabla u \cdot \nabla v + u\Delta v \tag{1}$$

3. Compute the line integral $\int_c \vec{F}\cdot d\vec{r}$, where $\vec{F} = \langle -y, x, 2\rangle$ and c is the path that consists of the following line segments, as in Figure 5:

   - A line segment from $(0,0,1)$ to $(-1,1,0)$.
   - A line segment from $(-1,1,0)$ to $(1,1,0)$.
   - A line segment from $(1,1,0)$ back to $(0,0,1)$.

4. Let S be the surface that consists of the part of the cylinder $x^2 + y^2 = 1$ lying between the planes $z = 0$ and $z = -1$, together with the part of the sphere

$$x^2 + y^2 + (z + 1 + \sqrt{3})^2 = 4$$

that lies below the plane $z = -1$, and let S have orientation pointing away from the origin, as in picture 4.

(a) Compute $\int_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle y + x, x + z, -z + y^2 \rangle$.
Hint: Modify the surface accordingly so you can use divergence theorem.

(b) *Find the surface area of S.

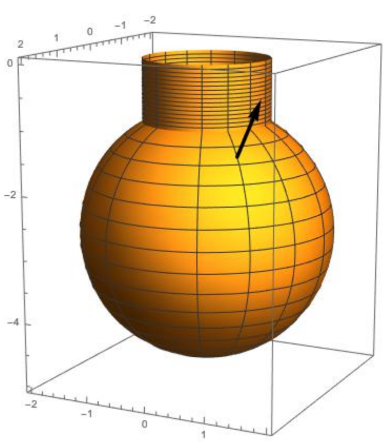

Figure 4: Problem 2

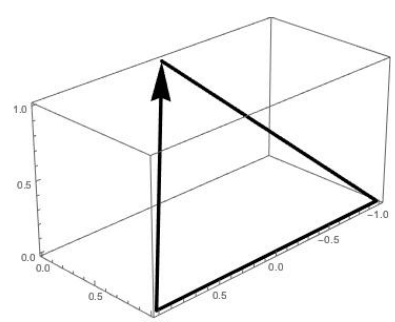

Figure 5: Problem 3