

Quiz 5

Name: _____

UW email address: _____

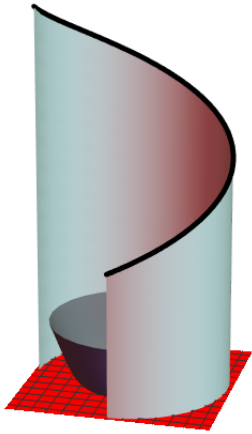
Nadine bought a new fancy curtain for her bathroom. The curtain rod (black in the picture) lies on the graph of the function

$$f(x, y) = 3 + \arctan\left(\frac{y}{x}\right),$$

where x and y are measured in feet, over the arc of the circle

$$x^2 + y^2 = 4$$

that starts at $(-2, 0)$ and goes clockwise all the way to $(0, -2)$, as in the picture. If the curtain touches the floor, which is represented as the xy plane, find the area of the curtain. **Make sure to include units.**



Need to compute

$$\int_C f(x, y) ds, \quad C \text{ is the arc of circle.}$$

Param. doesn't matter. Set

$$c(t) = \langle 2\cos t, 2\sin t \rangle, \quad t \in \left[-\frac{\pi}{2}, \pi\right]$$

$$\begin{aligned} x(t) &= 2\cos t \\ y(t) &= 2\sin t \end{aligned} \Rightarrow \begin{aligned} x'(t) &= -2\sin t \\ y'(t) &= 2\cos t \end{aligned}$$

$$\text{Area} = \int_{-\frac{\pi}{2}}^{\pi} 3 + \arctan\left(\frac{2\sin t}{2\cos t}\right) \sqrt{4\sin^2 t + 4\cos^2 t} dt$$

$$= \int_{-\frac{\pi}{2}}^{\pi} (3 + t) 2 dt = 6t + t^2 \Big|_{-\frac{\pi}{2}}^{\pi}$$

$$= 6\pi + \pi^2 + 6 \cdot \frac{\pi}{2} - \frac{\pi^2}{4}$$

$$= 9\pi + \frac{3\pi^2}{4}$$