



中国科学院大学

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Probability plot

ZTS

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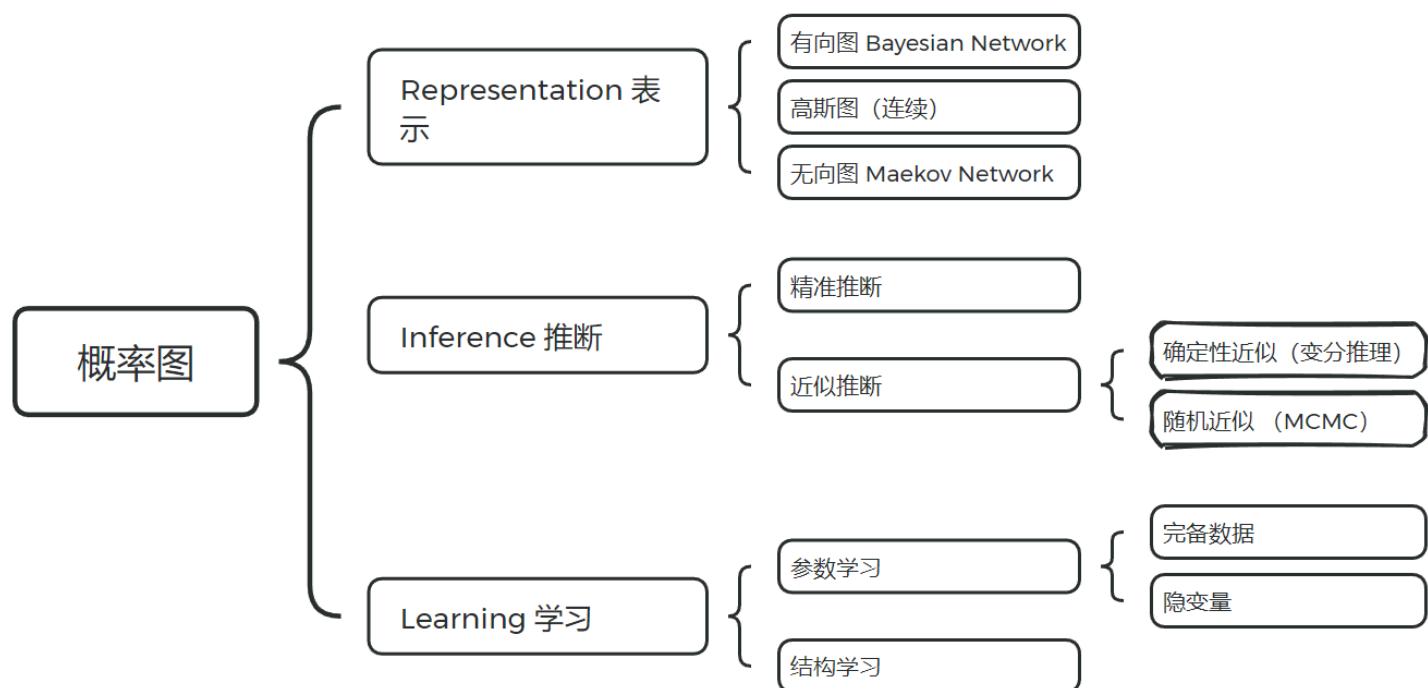


Figure 1: Introduction to Probabilistic Graphical Models.

$$\text{高维随机变量 } P(x_1, x_2 \cdots x_p) = \begin{cases} \text{边缘概率 } p(x_i) \\ \text{条件概率 } p(x_j | x_i) \end{cases}$$

✓ Sum rule: $p(x_1) = \int p(x_1, x_2) dx_2$

✓ Product Rule: $p(x_2) = p(x_1)p(x_2|x_1) = p(x_2)p(x_1|x_2)$

✓ Chain Rule: $p(x_1, x_2, \dots, x_p) = \prod_{i=1}^p p(x_i | x_1, x_3, \dots, x_{p-1})$

✓ Bayesian Rule: $p(x_2 | x_1) = \frac{p(x_1, x_2)}{p(x_1)} = \frac{p(x_1, x_2)}{\int p(x_1, x_2) dx_2} = \frac{p(x_2)p(x_1|x_2)}{\int p(x_2)p(x_1|x_2) dx_2}$

高维随机变量的困难:

✓ 维度高, 计算复杂, $p(x_1, x_2, \dots, x_p)$ 的计算量太大

0 Bayesian network

$$P(x_1, x_2, \dots, x_p) = P(x_1) \cdot \prod_{i=2}^p P(x_i \mid x_{1:i-1})$$

条件独立性来简化依赖链过长 $x_A \perp x_c \mid x_B$

$$\text{因子分解: } P(x_1, x_2, \dots, x_p) = \prod_{i=1}^p P(x_i \mid x_{p_{a(i)}})$$

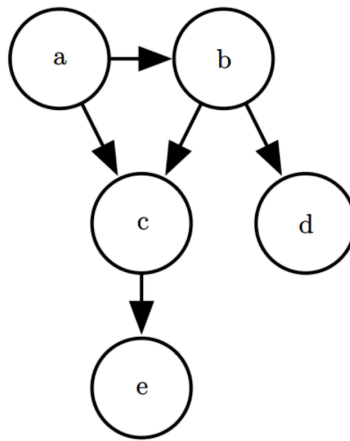


Figure 2: Directed Graph Model.

这个概率分布就可以写成:

$$p(a, b, c, d, e) = p(a)p(b|a)p(c|a, b)p(d|b)p(e|c).$$

01 怎么构建图? – 拓扑排序

只要存在 $p(x_i|x_j)$ 我们就可以称 j 是 i 的父节点, $j \rightarrow i$