Convex Optimization

ZTS

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0 Recommended Books

Boyd. Vandenberghe. 《 Convex Optimization 》

Nocedal. Wright. 《 Numerical Optimization >

Nesterow. 《 Introductory Lectures On Convex Optimization 》

0 Convex Set

1 凸集与凸函数

$C \subseteq \mathbb{R}^n$,C is Convex

$$\alpha x + (1-\alpha)y \in C, \quad \forall x, y \in C, \quad \forall \alpha \in [0,1]$$

Convex Hull

$$S \subset \mathbb{R}^n, \quad \text{conv}(S) = \{\alpha x + (1-\alpha)y : x \in S, y \in S, 0 < \alpha < 1\}$$

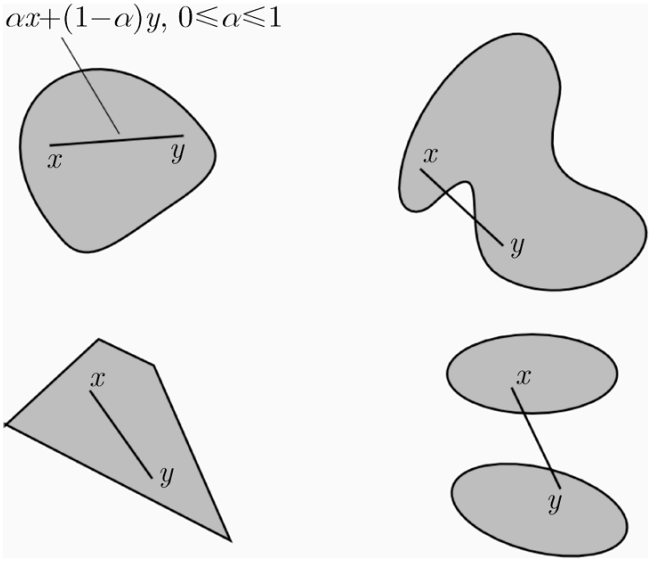

Figure 1: 凸集的定义.:凸集中任意两点的连线线段都包含在集合内部，因此左图中的集合是凸集，而右图中的不是.

11 prove

1. $\{x : Ax = b\}$

$$Ax = b, Ay = b \rightarrow A(\alpha x + (1-\alpha)y) = (\alpha + (1-\alpha))b = b$$

2. $\{x : Ax \subseteq b\}$

   $A(\alpha x + (1-\alpha)y) = \alpha Ax + (1-\alpha)Ay \subseteq b$

3. $\left\{x : \|x\|_2^2 \leqslant 1\right\}$

   $\|\alpha x + (1-\alpha)y\|_2^2 \leqslant \alpha^2\|x\|^2 + (1-\alpha)^2\|y\|^2 + 2\alpha(1-\alpha)X^T y$

   $\leqslant \alpha^2\|x\|^2 + (1-\alpha)^2\|y\|^2 + 2\alpha(1-\alpha)\|x\|\|y\|$

   $= (\alpha\|x\| + (1-\alpha)\|y\|)^2$

   $\leqslant (\alpha + (1-\alpha))^2 = 1$