Random Walk Algorithms and Application: A Survey

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***Abstract*—The abstract goes here.**

1. Introduction

Random walk and quantum walk have been used broadly in computer science community during the past years. They play an important role in computer vision, recommender system, social network etc.

Random walk has a very long history, which was first introduced by Pearson in 1905 [1]. A well-known application of random walk is page rank algorithm [2]. And page rank become an import kind of random walk proximity measures. There are other researchers present other proximity measure based on random walk. [3] [4] [Reversible Markov Chains]. Based on these proximity measures, a lot of widely used algorithms came up. In the era of collaborative filtering, researchers introduced some algorithm based on the proximity measures. [5] [6]. proximity measures also play an important role in recommending systems. [7] [8] [9]. Random walk also used in computer vision [**?**] [**?**] [10] [11] and semi-supervised learning [12] [13] [14] [15].

During the past few years, there many application of random walk on the analysis of social network. Many researchers have done excellent job on random walk. There are barely no complete reviews about random walk in computer science community. We would like to summarize their methods and results. Moreover, we can find out the challenges of random walk method.

The rest of this paper are organized as follows. In section 2, we will introduction some basic concepts of random walk. In section 3, we will introduce some important proximity measures in random walk. In section 4, we will introduce the application of random walk in computer science, including collaborative filtering, recommending system. In section 5, we will introduce the random walk from a quantum view the quantum walks. In section 6, we will discuss about the challenge of random walk in computer science.

2. Brief Introduction of Random Walk

Random walk is an important part of stochastic process. Stochastic process can be denoted as . Xt is a random variable. Single step transition probability can be denoted as $\{\xi_t, t = 0, 1, 2, ...\}$. T steps transition probability are defined as follow.

A graph is denoted by $G = (V, E)$, where V denotes the vertex set and E denoted the edge set. The adjacency matrix is denoted by A, where the Aij means the weigh on edge i,j. The transition probability (single step) between node I and node j on the graph can be defined as follow. $pij = Aij/$. We employ the diagonal matrix D to record for each node. In that case we can define the transition matrix of the graph. $Pij = Aij/Dii$ P denotes the transition matrix of the graph. The Laplacian of G can be defined as follow. $L = D - A$

3. Proximity Measures

3.1. PageRank

The most famous Proximity measure is the page rank. It was first proposed by Lary Page. [The PageRank Citation Ranking: Bringing Order to the Web] The purpose of this measure is to rank the webpage in World Wide Web. The network of webpage is considered as a graph where random walk happens. The graph is made up by vertexes and edges. The webpages are considered as the vertexes. if there is a webpage containing a hyperlink which pointing to anther webpage, then there should be a directed edge between these two vertexes. The direction of the edge is same as the web redirection. The most simple page rank can be describe by the mathematic equation.

$$R(u) = c \sum_{v \in B_u} \frac{R(v)}{N_v} \tag{1}$$

R(u) is the rank of web u. B(u) is the set of vertexes point to page u. out(u) is the set of pages u points to. N(v) is the number of vertexes in set out(u). The intuition behind this equation is that a page is important when it

has more backlinks and more important these backlinks are, higher rank this page gets. But this simple page rank cant be implemented because the practical situation is much more complicated. A more reliable mathematic description of page rank is as follows.

$$V = (1-\alpha)P^T V + \frac{\alpha}{n}\mathbf{1} \tag{2}$$

Alpha is the probability that the rand walk restart in given steps. Alpha is crucial for this proximity measure. It make sure the random walk procedure is aperiodic and irreducible. In that case, the random walk in the web network can converge to a certain distribution. However, the calculation of page rank uses a power method. To improve the converge speed of page rank, Quadratic Extrapolation [Extrapolation Methods for Accelerating PageRank Computations] present a novel algorithm for Page rank computation. Quadratic Extrapolation accelerate the convergence of the power method. The main strategy in this algorithm is periodically reducing estimates of the non-principal eigenvectors.

3.2. HITS

Page Rank has nothing to do with user-supplied query. Therefore, Jon M. Kleinberg [Authoritative sources in a hyperlinked environment] came up with Hyperlink-Induce topic search which can filter the search result for a broad topic. The author propose that there are two kinds of useful web-pages for topic search: authorities and hubs. He also proposed a link-base model for the conferral of authority. There are millions pages relevant to a broad topic. Authorities are the most central pages for the broad topic, which provide good information for the broad topic. Whereas the hubs are those pages contains hyper-links redirecting to the authorities. Now, we can discuss that main procedure of how the author provide a broad topic search result for the user. Given a query, the author construct a focused subgraph of the www relevant to the broad topic. The subgraph contains a set of relevant pages rich in candidate authorities. Then the author present a algorithm to discover authorities over the subgraph. The author would like to construct a subgraph denoted by S which satisfy the following requirements:

- S is relatively small
- S is rich in relevant pages
- S contains most the strongest authorities

How to construct subgraph S is the first difficult in HITS. The author rendered a solution: 1. collect the t highest ranked pages for the query as the root set R 2. expanding R along the links that enter and leave it Now that we get a qualified subgraph, we can apply the algorithm over it. The author would like to extract these authorizes from the subgraph. Hence the author use two scores to describe a vertex in the subgraph: authority score and hub score. The intuition of this idea is that A node is good hub if it points to many authorities; a node is a good authority if it is pointed to by many good hubs. In order to break this circulation, the author use a iterative method, which can be mathematically describe as follows.

I(i) denotes that the set of pages point to page i. O(i) denotes that the set of pages pointed to by page i. a(i) is the authority score of page I h(i) is the hub score of page i During the iteration, authority score and hub score are normalized so theire squares sum to 1. The two equation could be rewritten by using the matrix. A denote the unweighted adjacency matrix of the subgraph, vector a is the authority scores and vector h is the hub scores.

From the above equations, we can know that the hub scores converges to the principal eigenvector of AAt, meanwhile the authority scores converge to the principal eigenvector of AtA.

3.3. Personalized PageRank

Pagerank is a very democratic [The PageRank Citation Ranking: Bringing Order to the Web] since the walker can jump to every vertex with the Probability of alpha. On the contrary, personalized PageRank concentrate on one vertex. The intuitive idea of personalized page rank is the random walker can jump to a certain vertex with the probability of alpha. The mathematical description is the following equation. [RANDOM WALKS IN SOCIAL NETWORKS AND THEIR APPLICATIONS: A SURVEY]

$$\mathbf{v} = (1-\alpha)P^T\mathbf{v} + \alpha\mathbf{r} \tag{3}$$

There are many link prediction problem using personalized page rank as a proximity measure.[The Link-Prediction Problem for Social Networks] We will discuss these applications of personalized pagerank in the following sections.

3.4. Hitting Time and Commute Time

Hitting timehitting time [Reversible Markov Chains] can be considered as a weighted path length from I to j. The mathematic definition of hitting time is as follows:

Hij denotes the hitting time from node i to node j. P_{ik} denotes the transition probability from node I to node k. As we have mentioned before, on the undirected graph, transition probability matrix is symmetric. However, the hitting time matrix is not symmetric even on the undirected graph. Another important fact about hitting time was proved by LOVASZ [Random walk on graphs: a survey]: hitting time follows the triangle inequality. The commute time from node I to node j is defined as :

$$C_{ij} = h_{ij} + h_{ji} \tag{4}$$

In order to research the commute time on undirected graphs, Ashok K. Chandra et al gave an electrical network view. They compare commute time between two nodes on graph to resistance on electrical network. They gave us some intuition about commute time on undirected graphs: 1. The smaller resistance can make the current go through easier on electrical networks, the smaller commute time can make random walker diffuse easier on undirected graphs. 2.

commute time should be robust to small perturbation since removing or adding a few resistances do not change much on an electrical network.

3.5.

4. Applications of Random Walk

4.1. Computer Vision

Many researchers solve computer vision problems by using random walk. One of the common techniques is characterizing shape of image by using random walk. Gorelick et al. [Shape Representation and Classification Using the Poisson Equation] compute many useful properties of a silhouette based on the notion of random walk. For every internal pixel in the contour, they compute a value reflecting the mean time required for a random walker beginning at the pixel to reach the boundary. Based on the computed values, they can extract many properties of the silhouette such as part structure, rough skeleton, local orientation, convex part and concave part. Random walk is also utilized in image segmentation. Meila et al. [A Random Walks View of Spectral Segmentation] present an approach of image clustering and segmentation based on the view of random walk proximity measures. They also find that the spectral view of clustering and segmentation have a probabilistic foundation. They exploit the eigenvalue and eigenvector of walkers transition matrix to cluster and segment image. Grady et al. [Random Walks for Image Segmentation] propose a new algorithm for performing multilabel, interactive image segmentation. The interactive image segmentation means that the user has to label some pixels in the image manually. Given these labeled pixels, the algorithm can quickly determine the probability that a random walker starting from an unlabeled pixel will first reach the predefined pixels. Therefore, a good segmentation of that image arises from the labels of all the pixels. The predefined labels indicate that the regions of the image belong to several objects. The authors treat the image as a graph including nodes and edges. Nodes represents the pixels of the image, and edges represents the connection of two nodes where the value means the likelihood of a random walker going through that edge. The authors believe that this view of image segmentation has following advantage: there is no discretization errors or ambiguities. Because the authors use purely combinational operators that require no discretization. The segmentation algorithm only requires solution to a sparse, symmetric, positive definite system of equation, hence the efficiency of this algorithm is guaranteed. Qiu et al. [Image Segmentation using Commute times] exploits the properties of the commute time to develop image segmentation method. They compute the commute time from the spectrum. By using the discrete Greens function of graphs, they can analyze the cuts of the image from commute time. Qiu also use commute time to motion track [Robust Multi-body Motion Track using Commute Time Clustering]. The main purpose of using commute time as proximity measure is to alleviate the effect of noise on the shape interaction matrix. The noise on the shape interaction matrix result in the loss of block-diagonal structure and the difficulty of the assignment of elements to objects. Commute time is a more robust measure than row proximity matrix when facing the noise on the shape interaction matrix. The authors compute the commute time by using the Laplacian matrix. They also show us that how the ensemble the commute time, kernel of PCA (Principle Component Analysis), the Laplacian eigenmap and the diffusion map. To demonstrate the result of the robust method, they compare it with some other motion tracking algorithms on both synthetic and real world data. The function of commute time is to provide a proximity measure for clustering in the literature of Qin[Robust Multi-body Motion Track using Commute Time Clustering][Image Segmentation using Commute times].

4.2. Recommender System

Some scholars use random walk to solve their own problem during doing research. They find that it is hard for them to find out useful literature recently published in their field. A researcher is supposed to be well aware of recent development of the field he is working on. So a paper recommendation system can help them find out potential helpful papers. This is meaningful and time saving. Because publications increase exponentially, selecting useful papers really a pain in the neck for the most researchers. A very simplified algorithm to solve this problem was presented by Woodruff et al [Enhancing a Digital Book with a Reading Recommender]. The author employ spreading activation and citation data to generate recommendations. The author uses documents read by the reader as input. Then recommend the most related literature to the reader in a digital book. This method only recommends a chapter or an article in a digital book. A more applicable method for paper recommendation can be found in [On the Recommending of Citations for Research Papers]. The author exploit collaborative filtering for recommendation. The use the citation web as the graph to create ratings. The author investigated six algorithms by do experiments on the subset of ResearchIndex. The best algorithms can either provide relevant recommendations or novel recommendations, but none of them can do the both. And The use of citation web can affect the recommendations greatly. Also based on the citation graph, Gori et al exploit the idea of page rank algorithm to solve the paper recommendation problem by presenting the PaperRank algorithm [Research Paper Recommender Systems: A Random-Walk Based Approach]. The authors view is that utilizing the model expressed by the citation graph can help us find out valuable papers to suggest to a user. The author considered that the PaperRank algorithm must have two properties: propagation and attenuation. With propagation, if a paper is relevant to good papers in bibliography of researchers work, then we can find out that this paper maybe a good suggestion for him. With attenuation, the positive influence of good papers decreases if we move futher and further away

from good papers on the citation graph. PageRank algorithm has both properties. The author borrows its idea to solve paper recommendation problem. The essential of PaperRank algorithm is a random walk based score algorithm. Xia et al [Scientific Article Recommendation: Exploiting Common Author Relations and Historical Preferences] incorporates author relations and historical preferences for scientific article recommendation. The authors build a graph based on the information on common authors relation, and they employ the random walk with restart to generate a recommendation list. Compared with some baseline algorithms, the algorithm presented in the literature called CARE performs better in precision, recall and F1 score. Most studies of paper recommendation have the same algorithms for all the researchers no matter what the researchers situation is. But CARE method takes researchers own features into consideration. Hence the CARE method is more accurate than the baseline algorithms.

4.3. collaborative filtering

collaborative filtering is a method of making automatic predictions about the interests of a user by collecting preferences or taste information from many

users. The assumption of collaborative filtering is that the two people who have the same taste on one issue will have the same interest on the other issue.

Much literature has recorded methods of collaborative filtering, with successful demonstrations of Bayesian, nonparametric, linear methods etc.[Gediminas Adomavicius and Alexander Tuzhilin. Recommendation technologies: Survey of current methods and possible extensions. MISRC working paper 03-29,http://misrc.umn.edu/workingpapers/abstracts/0329.aspx,May 2003.] All these methods are essentially the same. They are all match the individual to others based on there choices, and use combination of their experiences to predict future choices. But Brand et al [A random walks perspective on maximizing satisfaction and profit] introduced a random walk view to collaborative filtering. The goal of Brand is to find out what products a customer wants to buy next, what product categories are preferred by specific demographic groups. They derived a weighted association graph from a relational database. These weighted association graph include consumers and their web browsing behavior, shopping behavior and entertainment choices etc. **a fig of collaborative filtering** The author look at the expected behavior of random walk on the association graph. Based on the hitting time and commute time, the authors employ a novel measure of similarityłthe cosine correlation between states. Compared with other methods of collaborative filtering, one of the biggest advantages of random walks view is that it can incorporate large amounts of contextual information. By using cross-validation, the author proved that the random walk view collaborative filtering is more predictive and robust to perturbations of edges on the association graph than other methods. The flow of random walk view is the heavy computing price. In that case the author employ approximation strategies to alleviate time complexity. Fouss et al [A novel way of computing similarities between nodes of a graph, with application to collaborative recommendation] also use random walk to movie collaborative recommendation. The author also considers relational database as a collection of element sets linked by their connection. The author exploits the graph structure of the relational database to compute dissimilarity measure between elements in sets. The dissimilarity is of course based on the hitting time and commute time. It was also the first time that hitting time and commute time measures was used in collaborative recommendation. For better understanding, the author gives us a specific example of the collaborative movie recommendation. If we get three elements, people, movie and movie category, and two relationships between people and movie and between movie and movie category, we have to do following things for movie recommendation. (1) compute dissimilarity measure between people based on the movies that they have watched

(2) compute dissimilarity measure between people and movies for recommendation

(3) compute dissimilarity measure between people and categories to give a prefer category for each person.

As a conclusion, Fouss et al introduced a general procedure for computing similarity between elements of a relational database. These elements are not necessarily directly connected. The authors use movie recommendation as an example to show us that their method has better performance than shortest path method on recommendation. However, there are two shortcomings of the authors method. For large databases, this method is time consuming and does not scale well. The other short coming is that the method is valid on a weighted, undirected graph. Although the random walk view of collaborative filtering is useful and has good performance, it also faces several challenges. One of the biggest problem is the start-up problem presented by Resnick [An Open Architecture for Collaborative Filtering of Netnews]. All collaborative filtering systems are based on an existed database. If there isnt an existed database, the system cant be built up.

4.4. Semi-supervised Learning

Semi-supervised learning uses both labeled data and unlabeled data for training. The goal is to classify the unlabeled data when the labeled data is just a small fraction of the dataset.

Zhu et al. [Semi-supervised learning using Gaussian Fields and harmonic functions] present a new approach of semi-supervised learning based on the random walk. They do classification task in continuous state space rather than in the discrete label set. The intuition of the approach is that the data points should be labeled the same as their neighbors. And their neighbors are given by the random walk on graph. The authors strategy is to employ a harmonic function f: V-¿R on graph G. The harmonic function has a constrain on the labeled data i: $f(i) \equiv fl(i)y(i)$. The harmonic func-

tion, which provides a consistent probabilistic semantics, is the basis of this semi-supervised classification approach. Since the author do classification in the continuous state space, they have to turn the continuous state space into discrete label set. Instead of employing a simple threshold in terms of the interpretation of random walk, the authors incorporate the prior knowledge by using CMN (class mass normalization) procedure. The promising result has shown that the approach can improve the accuracy of classification by exploiting the structure of unlabeled data.

Szummer et al [Partially labeled classification with Markov random walks] the partially labeled data may be in the submanifold space, hence a measure of global similarity is needed for semi-supervised learning. In the meanwhile, the authors also hope the measure can incorporate the structure of manifold. Based on these consideration, they present a Markov random walk model to classify the data. The research of [Data clustering by Markovian relaxation and the information Botteneck Method], which shows how to turn the distance matrix into a Markov process, helps a lot with the construction of graph. In that case, the representation of dataset arises naturally. Given a partially labeled data set in which L is much smaller than N, the author represent the data set as a graph where node k represents the data (xk,yk) or xk. For node k, P0—t(i—k) denotes the probability of the random walker from node I to node k after t steps. They classify node k with the label c when c maximizes the following formula. y=c. P(y—i) is an unknown parameter, which can be estimated by two techniques: maximum likelihood with EM, and maximum margin subject to constraints. They discuss the two techniques in the paper and empirically show that the margin estimation has better performance. In a word, the authors provide a novel approach for semi-supervised learning task when the data sets with significant manifold structure. The parameter t in this approach is also important. T , denoting the number of transitions, determines the smoothness of random walk. However, the choice of t can be tricky and subjective. To overcome this little problem, Azran [the rendezvous algorithm: multiclass semi-supervised learning with markov random walks] presents the rendezvous algorithm. Just the same as the work of Szummer [Partially labeled classification with Markov random walks], the author represents the data points as nodes of a graph and employ the random walk view to do classification. The intuition of [Partially labeled classification with Markov random walks] is the labels propagation over the graph. But the rendezvous algorithm is different. The labeled points dont propagate, but absorb the states of the random walk. The probability of each unlabeled data to be absorbed by different labeled points can be used to derive a distribution as the transition steps increase to infinity. Hence the rendezvous algorithm doesnt bother to choose a good value of the parameter t. The author draws a conclusion that the location of labeled point in the data set is important as the size of labeled data set in terms of the experiments results.

5. Quantum view of Random Walk

Kempe et al[quantum random walks - an introductory overvew] presented us two kinds of quantum walks. They are discrete time quantum walk and continus time random walk.

5.1. Discrete Time Quantum Walk

The discrete time model first appeared in the work of Feynman[Quantum mechanics and

path integrals.] in 1966. In the field of quantum computation, Meyer rediscovered the discrete time model of quantum walk in [From quantum cellular automata to quantum

lattice gases.][On the absence of homogeneous scalar unitary cellular automata]. we define a space $H = H_p \bigotimes H_c$ for one dimensional quantum walk, . H_p denotes Hilbert space. For one dimensional Hilbert space, it can be represented as follows.

$$H_p = \{|i\rangle : i \in Z\} \tag{5}$$

H_c is spanned by two basic states $\{|\uparrow\rangle, |\downarrow\rangle\}$. Operation S defines the translation on space H.

$$S = |\uparrow\rangle\langle\uparrow| \otimes \sum_i |i+1\rangle\langle i| + |\downarrow\rangle\langle\downarrow| \otimes \sum_i |i-1\rangle\langle i| \tag{6}$$

S can transform the basic state $|\uparrow\rangle \otimes |i\rangle$ to $|\uparrow\rangle \otimes |i+1\rangle$ and $|\downarrow\rangle \otimes |i\rangle to |\downarrow\rangle \otimes |i-1\rangle$.

C is a unitrary transformation to rotate the spin in H_c. A frequently used unitary transformation is called Hadamard coin H. Here is an example of H.

$$|\uparrow\rangle \otimes |0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |0\rangle \tag{7}$$

The single quantum walk transformation can be defined as follows.

$$U = S \cdot (C \otimes I) \tag{8}$$

Here is a example of single step transformation.

$$|\uparrow\rangle \otimes |0\rangle \xrightarrow{U} \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |1\rangle + |\downarrow\rangle \otimes |-1\rangle) \tag{9}$$

The T steps of tansformation can be represented by U^T.

5.2. Continuous Time Quantum Walk

The original purpose of continuous time quantum walk is to speed up many a algorithm using classic random walks. The concept of continuous time quantum walk was first presented by Farhi et al. in 1997.[Quantum computation and Decision tree] The authors exploit quantum walk in the decision tree algorithm instead of classic random walk to . Different from discrete time quantum walk, continuous time quantum walk don't need a coin space Hc, taking place entirely in the Hilbert space Hp.[quantum random walks - an introductory overview] . The idea of continuous

time quantum walk is from continuous random walk. The continuous time random walk can be defined as

$$P(t) = exp(-Ht)P(0) \tag{10}$$

. Similarly, the unitary time evolution operator of continuous time quantum walk is

$$\hat{U}(t) = exp(-i\hat{H}t) \tag{11}$$

5.3. Properties of Quantum walk

5.4. Algorithms based on Quantum walk

5.4.1. quantum decision tree algorithm. Fahri [**?**] originally presented the idea of continuous time quantum walk with the example of decision tree algorithm. He chooses the approach that systematically exploring the whole tree with a probabilistic rule. The author wants that the n-level nodes can be reach in polynomial time with a considerable probability. A tree is penetrable when its node or nodes in n-level meet the requirement above. If a tree is penetrable for an specific algorithm, we believe that the problem corresponding to the decision tree is solvable with this algorithm in polynomial time. The author present the quantum walk algorithm for decision tree with following intuition. He considers decision tree nodes as quantum states in Hilbert space. Then He constructs a Hamiltonian $\hat{H}$ which determines the time evolution of the quantum system. With the basis of Hamiltonian, The author presented the unitary time evolution operator shown in 11. The author compare quantum walk decision tree algorithm with classical counterpart, and find that there is a family of trees which are both classical penetrable and quantum penetrable. However, Some decision trees is quantum penetrable but not classical penetrable. With these findings, the author draw the conclusion that quantum algorithms are more faster with respect to the classical decision tree algorithms for some decision tree problems.

5.4.2. Traversal quantum algorithm.

5.4.3. Quantum page rank algorithm . page rank algorithm is one of the most important random walk algorithms. When quantum computation is widely considered in the era of random walk, it is natural and inevitable to apply quantum computation on page rank algorithm. There are many literature of quantum networks [**?**] [**?**] [**?**] [**?**] [**?**] [**?**]. In order to study the behavior of pagerank algorithm on the quantum network, the authors present the quantum page rank algorithm [**?**]. However, the authors don't give a specific defination of quantum page rank algorithms, but give a admissible class shown as follows.

P1: The classical PageRank must be embedded into the quantum class with its undirected graph structure preserved.

P2: The sum of all quantum PageRanks must equal to 1.

P3: The quantum PageRank obey a quantized Markov Chain (MC) rules.

P4: The classical algorithm to compute the quantum PageRank is also feasible.

The author exploit the idea of discrete time quantum walk. Hence we have to define the coin space $\mathcal{H}_{\lrcorner}$and hilbert space $\mathcal{H}_{\swarrow}$ which are mentioned in the section of discrete time quantum walk. The definition of coin space is similar to the one dimension quantum walk.

$$\mathcal{H}_{\lrcorner} = span\{|L\rangle, |R\rangle\} \tag{12}$$

However the Hilbert space $\mathcal{H}_{\swarrow}$ here is a little tricky. Since the page rank algorithm is on a graph, the author define the Hilbert space as the space of oriented edges.

$$\mathcal{H}_{\swarrow} = span\{|i\rangle_1, |j\rangle_2 \quad | \quad i, j \in N\} \tag{13}$$

where N denotes the all the vertices of the graph. Since the edge is oriented, We use the subscript 1,2 to show the direction.

With these defination and the method of Szegedys Quantization of Markov Chains [**?**] We can present the unitatry step operator of quantum walk as follows.

$$U = S(2\prod - \not\Vdash)S = \sum_{i,k=1}^{N} |j,k\rangle\langle k,j| \prod := \sum_{j=1}^{N} |\psi_j\rangle\langle\psi_j||\psi_j\rangle = |j\rangle_1 \otimes \sum_{k=} \tag{14}$$

Where G_{ij} means the weight of edge ij.

The authors apply quantum page rank algorithm on small generated network to have a insight of the behavior of it. They find that the quantum pagerank algorithm obtain a larger score than the classical value. In the meanwhile, the quantum algorithm break down the hierarchy of classical values. The author also look into the properties of quantum page rank algorithm in complex real-world network [**?**]. The authors find that quantum page rank algorithm can reveal the underlying topology of the network more univocally with repect to classical page rank algorithm. The ability of detecting hub for network is enhanced with respect to classical counterpart.

5.5. element distinctness

We introduce the element distinctness problem first. Element distinctness problem is to tell whether all the elements in a given sequence are distinct. More precisely, $M = x_i, i \in N$, are there $x_i \in Mandx_jinM$ and $i \neq j$ such that $x_i = x_j$? There is a simple classical algorithm to solve this problem with $Nlog(N) + O(N)$ comparisons. Buhrman et al. present an quantum algorithm to speedup [**?**]. Their algorithm give a upper bound $O(N^{\frac{3}{4}}log(N))$. Ambainis improve the quantum way to solve element distinctness with $O(N^{\frac{2}{3}})$ comparisons. The intuition of this optimal quantum algorithm is to construct a graph, and transform the element distinctness problem to finding a marked vertex in the graph. In order to search marked vertex efficiently, the author improve the Grover's quantum search algorithm.

6. Conclusion

The conclusion goes here.

7. open issues

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References

[1] K. Pearson, "The problem of the random walk," *Nature*, vol. 72, no. 1865, p. 294, 1905.

[2] L. Page, "The pagerank citation ranking : Bringing order to the web," *Stanford Digital Libraries Working Paper*, vol. 9, no. 1, pp. 1–14, 1998.

[3] J. M. Kleinberg, "Authoritative sources in a hyperlinked environment," *Journal of the Acm*, vol. 46, no. 5, pp. 604–632, 1999.

[4] T. H. Haveliwala, *Topic-sensitive PageRank: A context-sensitive ranking algorithm for Web search*. IEEE Educational Activities Department, 2003.

[5] F. Fouss, A. Pirotte, and M. Saerens, "A novel way of computing similarities between nodes of a graph, with application to collaborative recommendation," *web intelligence*, pp. 550–556, 2005.

[6] M. Brand, "A random walks perspective on maximizing satisfaction and profit." pp. 12–19, 2005.

[7] A. Woodruff, R. Gossweiler, J. E. Pitkow, E. H. Chi, and S. K. Card, "Enhancing a digital book with a reading recommender," pp. 153–160, 2000.

[8] S. M. Mcnee, I. Albert, D. Cosley, P. Gopalkrishnan, S. K. Lam, A. M. Rashid, J. A. Konstan, and J. Riedl, "On the recommending of citations for research papers," *conference on computer supported cooperative work*, pp. 116–125, 2002.

[9] M. Gori and A. Pucci, "Research paper recommender systems: A random-walk based approach," *web intelligence*, pp. 778–781, 2006.

[10] L. Grady, "Random walks for image segmentation," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 28, no. 11, pp. 1768–1783, Nov 2006.

[11] H. Qiu and E. R. Hancock, "Image segmentation using commute times." 2005.

[12] X. Zhu, Z. Ghahramani, and J. D. Lafferty, "Semi-supervised learning using gaussian fields and harmonic functions," pp. 912–919, 2003.

[13] M. Szummer and T. S. Jaakkola, "Partially labeled classification with markov random walks," vol. 14, pp. 945–952, 2002.

[14] A. Azran, "The rendezvous algorithm: multiclass semi-supervised learning with markov random walks," pp. 49–56, 2007.

[15] N. Tishby and N. Slonim, "Data clustering by markovian relaxation and the information bottleneck method," pp. 640–646, 2001.