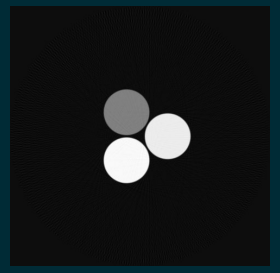
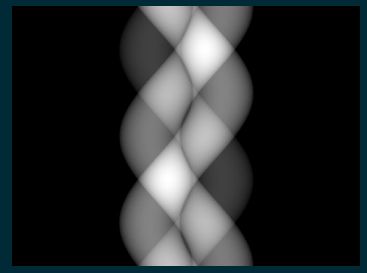
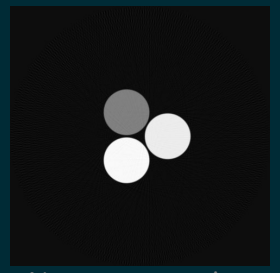
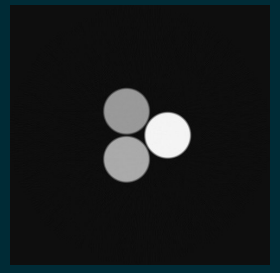
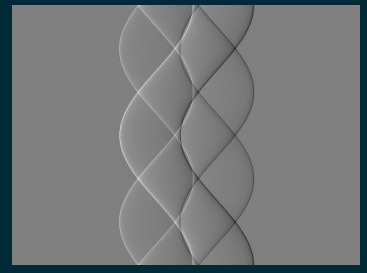
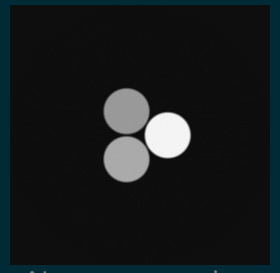
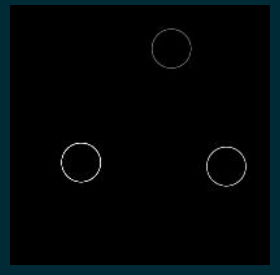
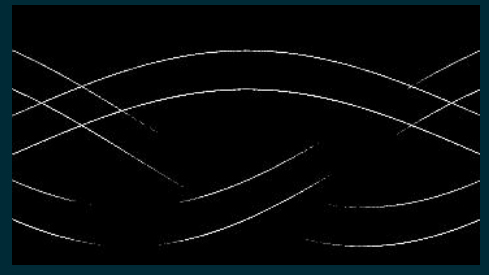
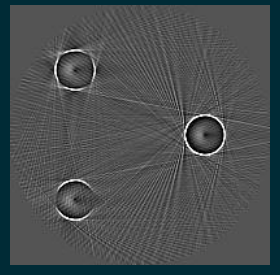
MA Kick-off

Differentiable projection operations for X-ray computed tomography

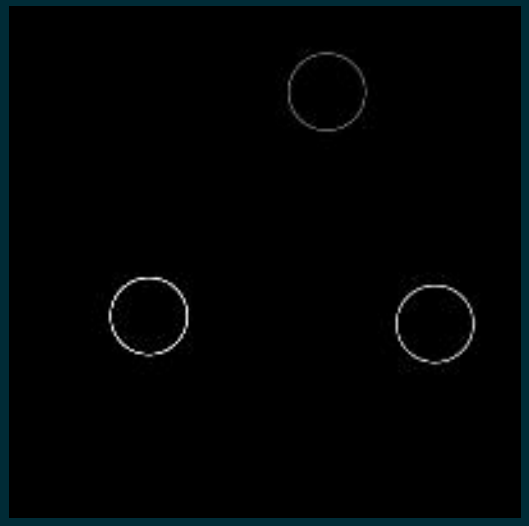
David Frank, on the 8th February, 2022

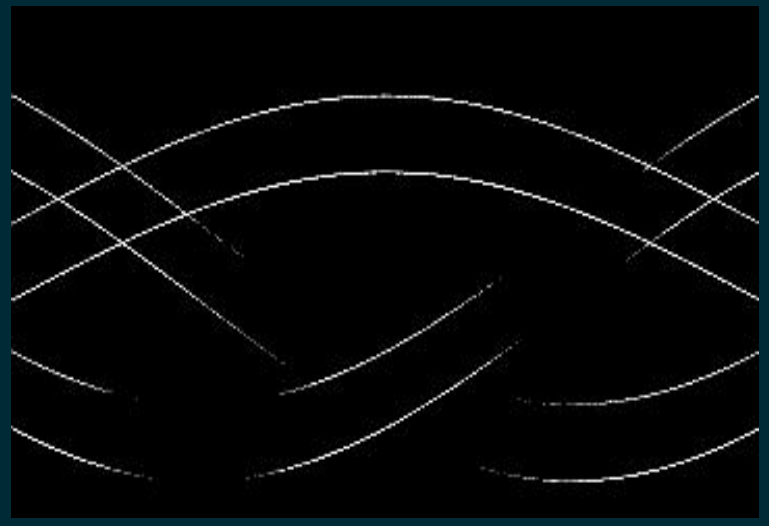
Tomographic Reconstruction

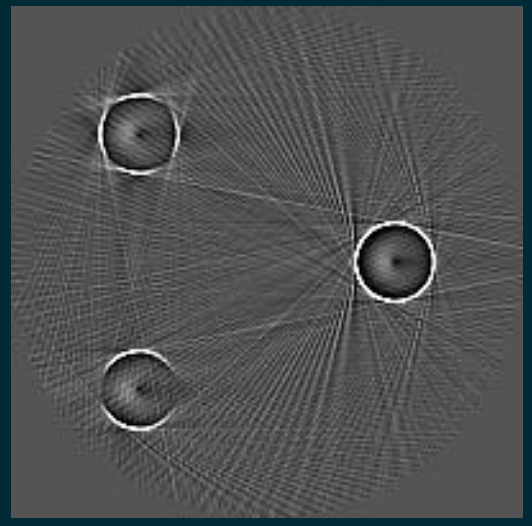
	Object	Measurement	Reconstruction
Attenuation CT			 No reconstruction
Phase Contrast CT			 No reconstruction
Dark-field CT			

- Images of the first and second row taken form: P. Modregger et al., "Artifacts in X-Ray Dark-Field Tomography" (2011)
- Images of the third row taken form: L. Felsner et al., "Phase-Sensitive Region-of-Interest Computed Tomography" (2018)

Model







$$f : \Omega \to \mathbb{R}$$

$$\Omega \subset \mathbb{R}^n$$

$$m_j = \mathcal{M}_j(f)$$

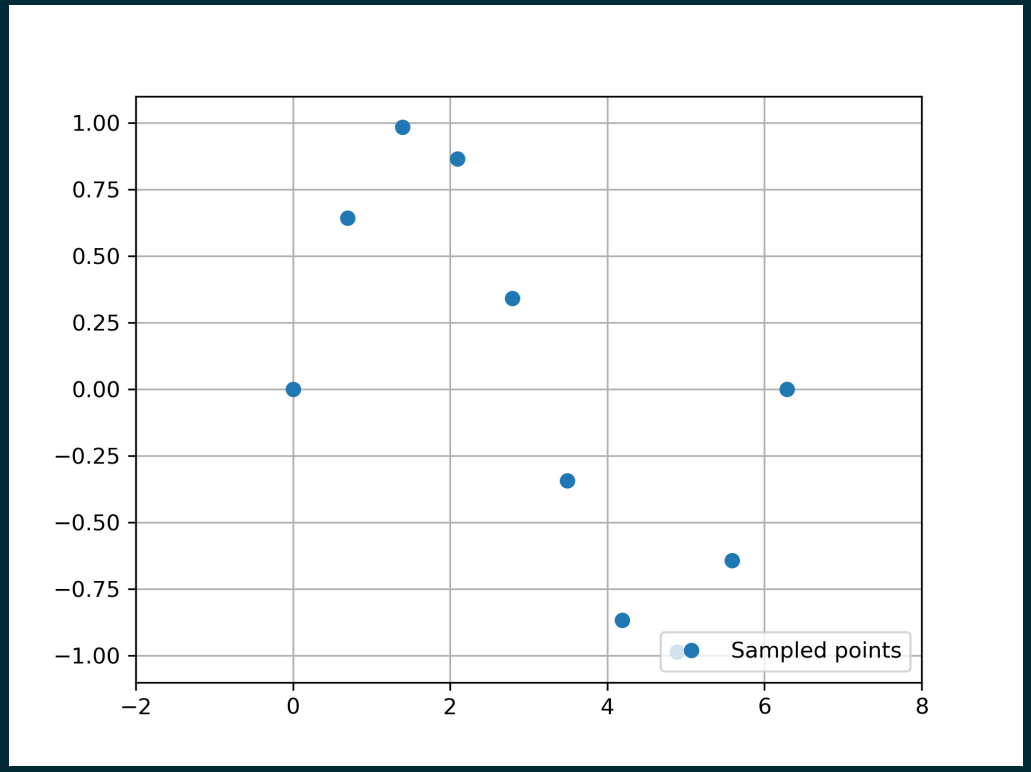
$$\mathcal{M}_j : (\Omega \to \mathbb{R}) \to \mathbb{R}$$

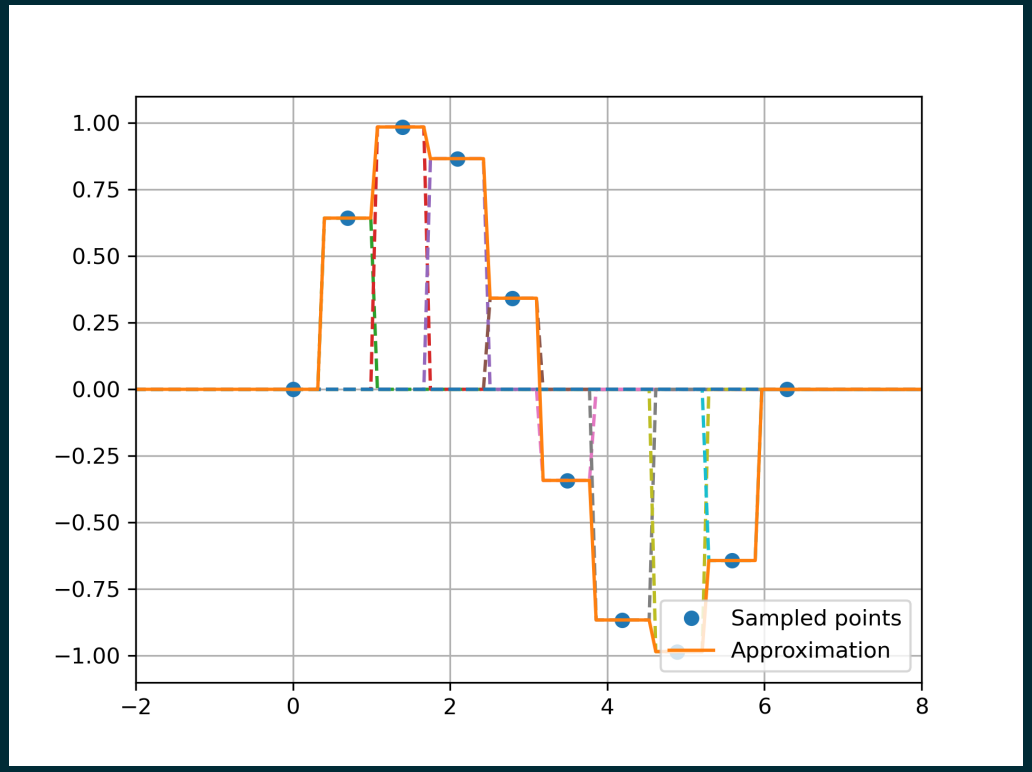
$$\hat{f} \approx f$$

$$\hat{f}(\cdot) = \sum_{i=1}^{I} c_i b_i(\cdot - x_i)$$

Images again from: P. Modregger et al., "Artifacts in X-Ray Dark-Field Tomography" (2011)

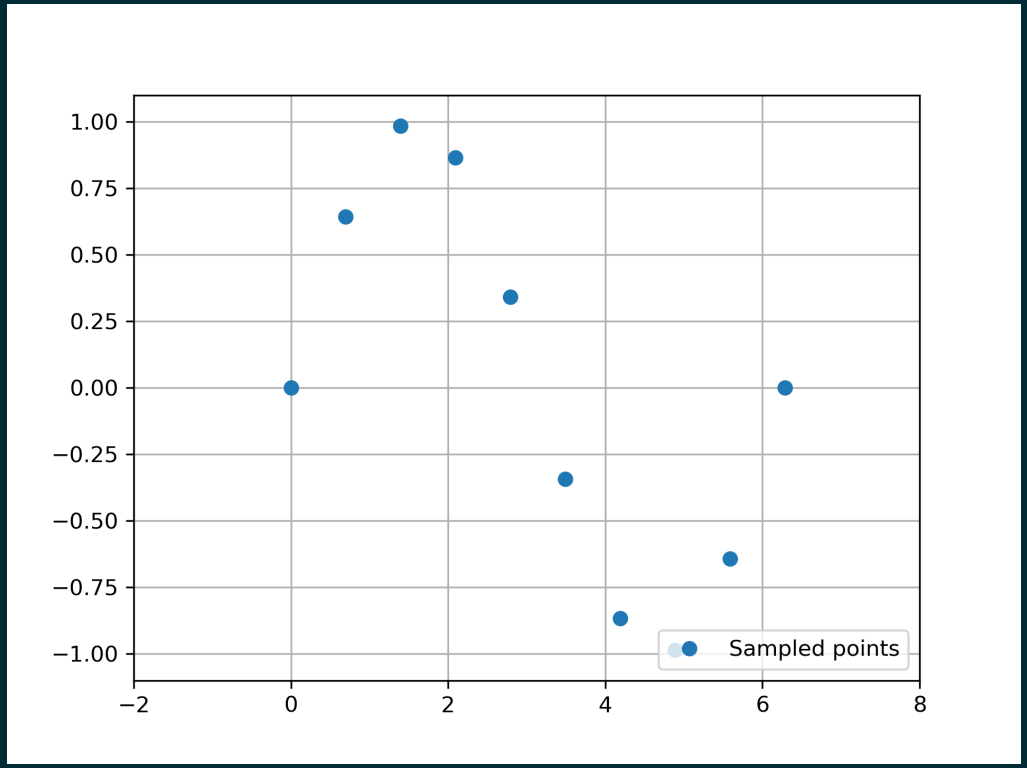
Pixel Basis Functions

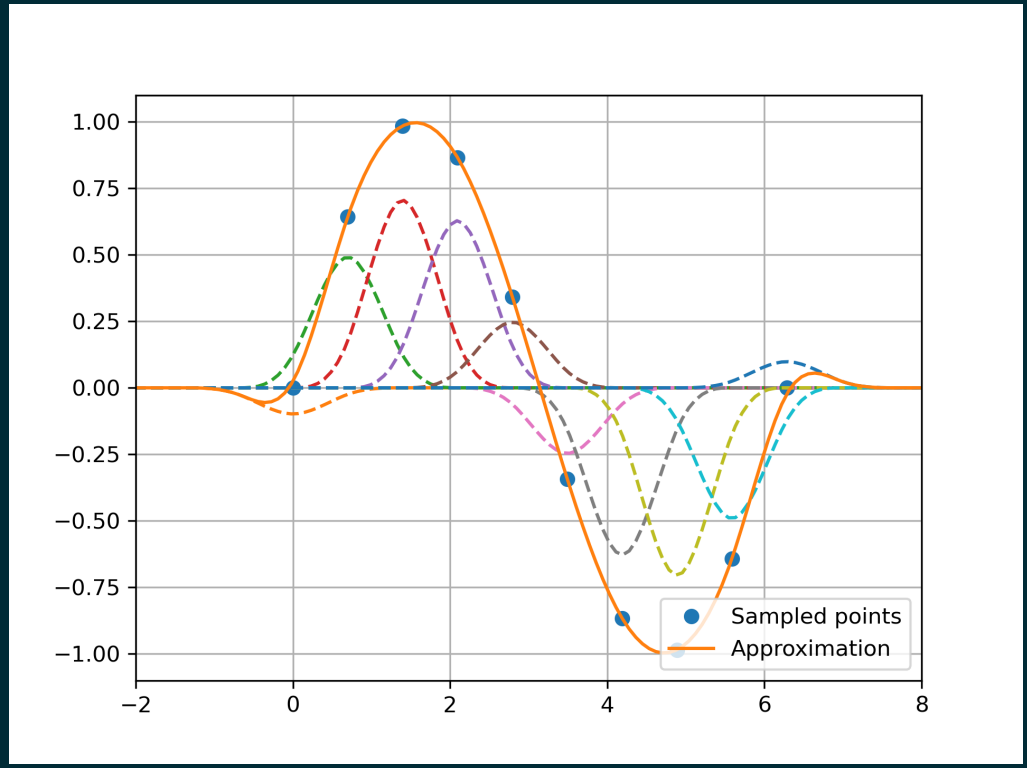




$$b_i(x) = \begin{cases} 1, & \|x\| < h/2 \\ 0, & \text{otherwise} \end{cases}$$

B-Spline Basis Functions

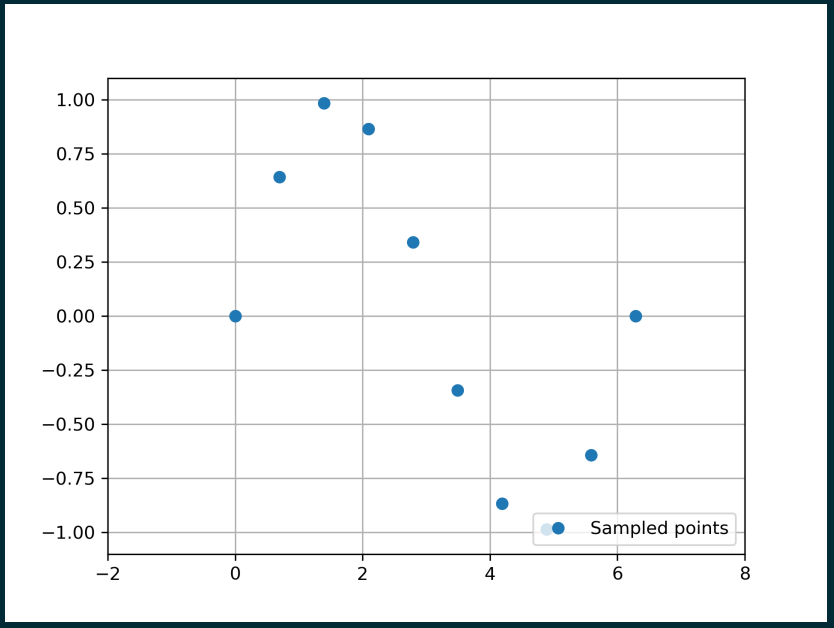




$$\beta^0(x) = \begin{cases} 1, & \|x\| < h/2 \\ 0, & \text{otherwise} \end{cases}, \qquad \beta^d(x) = \beta^0 * \cdots * \beta^0(x)$$

Literature: F. Momey et. al, "A new representation and projection model for tomography, based on separable B-splines" (2011)

Spherically Symmetric Basis Functions



$$b_i(\|x\|) = \begin{cases} \frac{\left(\sqrt{1-(\frac{\|x\|}{a})^2}\right)^m}{I_m(\alpha)} I_m\left(\alpha\sqrt{1-(\frac{\|x\|}{a})^2}\right), & 0 \leq \|x\| \leq \\ 0, & \text{otherwise} \end{cases}$$

Literature: R. M. Lewitt, "Alternatives to voxels for image representation in iterative reconstruction algorithms" (1992)

Motivation for (differentiable) basis functions

- Support for Phase Contrast CT
- Increased accuracy
- Continuous representation of the volume domain

Questions?

Thanks for listening!