Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

HLab–2: “Fourier Series Analyzer”

Lab Week: Week 9

Total Marks: 10

Contribution to Final Assessment: 3%

Submission: No due dates. Lab is marked based on satisfactory completion of tasks during the lab. Hence, attendance is compulsory.

Relevant Textbook Sections: 3.3, 3.8, 3.9.

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1 Learning Outcomes

After completing this lab, students should be able to:

- understand the operation of a low-pass filter.
- build a Fourier series analyzer.
- analyze square waves and fundamental components of sinusoids using the Fourier series analyzer.

2 Modules and Devices Needed

Audio Oscillator, Sequence Generator, Tunable LPF, Oscilloscope, Adder, Multiplier, and VCO.

3 Background

3.1 Motivation

In HLab-1, we discovered that sine-waves are special in the context of linear systems (time invariance assumed). Unlike other waveforms, a sine-wave input emerges at the output as a sine-wave. It is possible to completely characterize the behaviour of a system in this class by simply measuring the output/input amplitude ratio and the phase shift of the sine-wave as a function of frequency. Now, this is fine if we only need to process sine-waves – is it feasible to make use of this when dealing with other kinds of inputs? For example, when the input is a waveform like square wave. One possibility is to see whether a nonsinusoidal waveform could be expressed as a sum of sine-waves, over a suitable frequency range, either exactly, or even approximately. If this could be done, the system output can then be obtained by exploiting the additivity property of linear systems, i.e., first obtain the output corresponding to each sinusoidal component of the input signal, then take the sum of the outputs.

In this Lab we explore this idea by means of an ancient technique based on the generation of beat frequencies – somewhat like when a musician uses a tuning fork. We will look for sine-waves where there is no obvious indication that any should be present.

3.2 Summary of Tasks

The lab consists of three parts. In Part 1, we study the operation of low-pass filter and the effect of low-pass filtering on the square wave. In part 2, we build the wave analyzer to identify the different frequency components of a signal. In part 3, we use the wave analyzer to determine the frequency components of a square wave.

3.3 Operation of Wave Analyzer

We first see the operation of wave analyzer. Figure 1 shows the block diagram of wave analyzer. It is just a multiplier that multiplies the reference known signal to the waveform under consideration. We use an oscillator (VCO) to provide a reference sine-wave. As shown in Figure 3, an analog MULTIPLIER generates the product of this signal and the waveform under consideration.

We know the trigonometrical formula for converting a product to a sum, i.e.,

$$2\cos(\omega_1 t)\cos(\omega_2 t) = \cos((\omega_1 + \omega_2)t) + \cos((\omega_1 - \omega_2)t) \tag{1}$$

A low frequency output is generated when the inputs are close in frequency. The detection process consists of separating this low frequency product from the unwanted sine-waves at higher frequencies. All that is required is to progressively sweep through the frequency range and watch for the appearance of a low frequency. The exact frequency of the hidden sine-wave is readily deduced from frequency readings of the reference and output. Instruments based on this technique are known as wave analyzers.

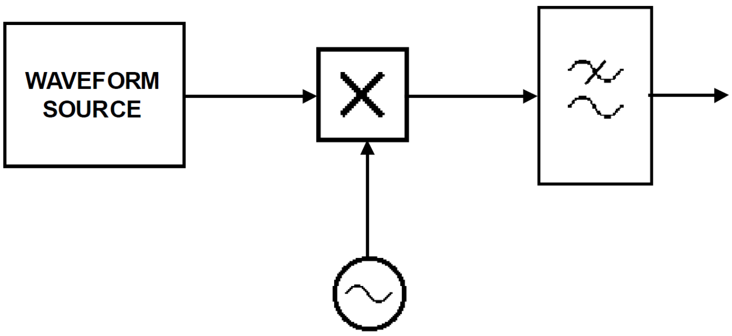

Figure 1: Block Diagram of Wave Analyzer

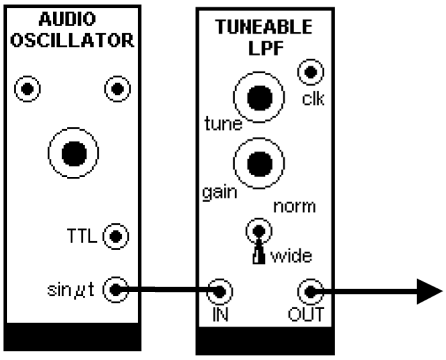

Figure 2: Task 1 - Response of Low Pass Filter

4 Part 1: Low-Pass Filter

We study the frequency response and phase response of low-pass filter (LPF) for different ranges of the low-pass filter module. Then we use the low-pass filter to smooth the square wave. We examine what happens to a square wave when the rate at which the voltage is permitted to vary versus time is progressively constrained.

Task 1: Response of Low-Pass Filter

1. Connect the circuit according to Figure 2. Reduce the frequency to the minimum setting of the AUDIO OSCILLATOR (around 250 Hz). Set the range switch to Normal position in order to allow tuning up to 5 kHz (Wide position allows up to 12 kHz). Vary the cut-off frequency by setting the TUNE knob of LPF module 9 o'clock position. Also, vary the filter gain by setting the GAIN knob to around 12 o'clock position.

2. Using this setup, observe the output and input of the LPF module on oscilloscope. Now change the frequency of input using Audio oscillator frequency control. Observe the change in amplitude and change in phase relative to the input. Measure the phase difference using the oscilloscope 'Measure' function. Identify the cut-off frequency, that is, the frequency at which the output signal peak-to-peak has 0.707 times the peak-to-peak output voltage at the minimum frequency. Using your observations, draw graphs of amplitude against frequency and phase against frequency. Recall that a rule of thumb is to go two decades above and below the cut-off frequency when plotting the filter amplitude and phase responses.

3. Repeat the above step for the TUNE knob at 12 o'clock position.

4. Show the four graphs (two for each position of TUNE knob) to the tutor. [***Marks:*** *2*]

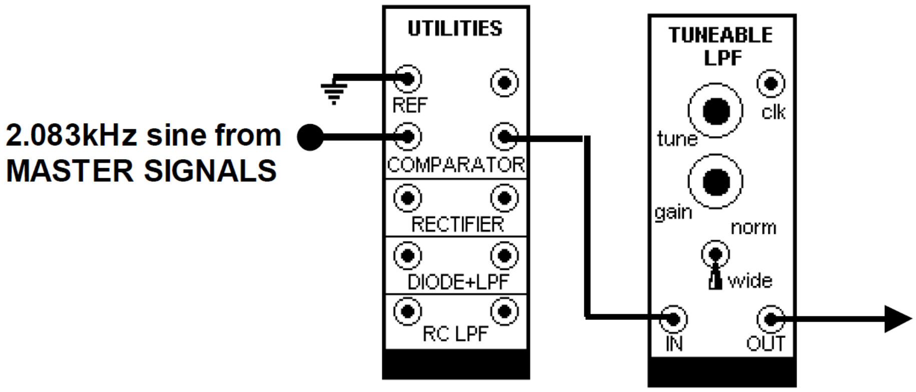

Figure 3: Task 2 - Smoothing of a square wave

Task 2: Smoothing Square Wave using LPF

1. Patch the TIMS model in Figure 3. Set the TUNEABLE LPF switch to WIDE to allow tuning up to 12 kHz. Set the TUNE knob fully clockwise and the GAIN knob near 12 o'clock. Display the filter input and output on scope.

2. Observe the output waveform as the TUNE knob is gradually rotated to maximum counter-clockwise. Observe that at some point all the kinks and twists in the output waveform disappear and a smooth waveform remains.

3. Return to the maximum clockwise position and change switch to NORMAL. Repeat the rotation of the TUNE control as above. With the rotation of the control, note that the amplitude of the waveform becomes progressively smaller.

4. Record your findings, your conclusions, and whether this outcome is a surprise or a confirmation of your expectations. If the latter, indicate the reason. Hint: What are the first three fundamental frequencies of the applied square wave? [***Marks:*** *1*]

5 Part 2: Testing the Operation of the Wave Analyzer with Two Sine-waves

We want to find the sinusoidal components that may be concealed in a square wave (Task 4). Before proceeding with the search, we will validate the method. This is done here in Task 3 by means of a test signal consisting of the sum of two known sine-waves.

Task 3: Working of Wave Analyzer

1. BEFORE patching up the diagram in Figure 4, set the AUDIO OSCILLATOR output frequency to 5 kHz. Connect its output to socket A of the ADDER, and vary the corresponding knob 'G' to obtain a signal of 1.5 Vp-p. Keep this knob unchanged. Now, disconnect socket A, and connect a signal of 2.083kHz from MASTER SIGNALS to socket B of the ADDER. Vary the knob 'g' to obtain a signal of 1 Vp-p. Do not change these two knob until you finish this part. Now reconnect socket A, from the AUDIO OSCILLATOR. Briefly display the signal at the output of ADDER. [***Marks:*** *1*]

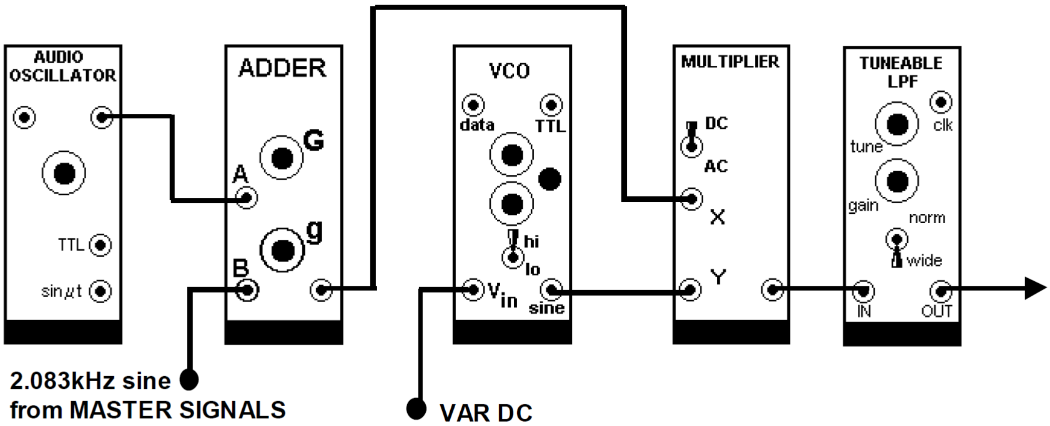

Figure 4: Task 3 - Wave Analyzer

2. Patch up the TIMS model in Figure 4. On the VCO, set the switch to the LO position. Set the GAIN control to 10 o'clock. On the VARIABLE DC module, set the control to 12 o'clock. Set the switch on TUNEABLE LPF to NORM, the TUNE control near 9 o'clock, GAIN near 12 o'clock. Connect the VCO output to oscilloscope Channel X and the LPF output to Channel Y.

3. Set the VCO output frequency near 2 kHz with the f_0 control. Now, with the external DC control, carefully vary the VCO output frequency up to 2.5 kHz and 1.66 kHz, and observe the LPF output in each case. It should be possible to observe an output or around 420 Hz. If an output is not apparent, rotate the TLPF tuning control clockwise, to 10 o'clock. Now sit back and try to understand the system you have constructed. [***Marks:*** *1*]

4. Verify that the TLPF tuning control is suitably adjusted. The fully counter clockwise position (minimum bandwidth) should provide a convenient frequency window.

5. Display the frequency of the VCO output on the FREQUENCY COUNTER and of the beat signal at the TLPF output. Verify that you are able to deduce the frequency of the sine-wave under test from these readings. Note that there is a possibility of ambiguity – you need to know whether the VCO freq is above or below the target frequency. Measure and record the maximum amplitude of the beat signal. Show to the tutor. [***Marks:*** *1*]

6. Repeat the procedure for the 5 kHz target. That is, repeat the above steps for 5.42 kHz and 4.58 kHz. Compare the amplitudes of the beat signals and the test sine-waves. Could the amplitude of the beat signal be used to deduce the *relative* amplitudes of the target sine-waves. Reflect on your observations? [***Marks:*** *1*]

6 Part 3: Use of Wave Analyzer

Task 4: Analysis of Square Wave using Wave Analyzer

1. Setup the TIMS model in Figure 5. Note that this is essentially the same as in Task 3, except for the signal under test, which is a square wave, generated with the model in Figure 2.

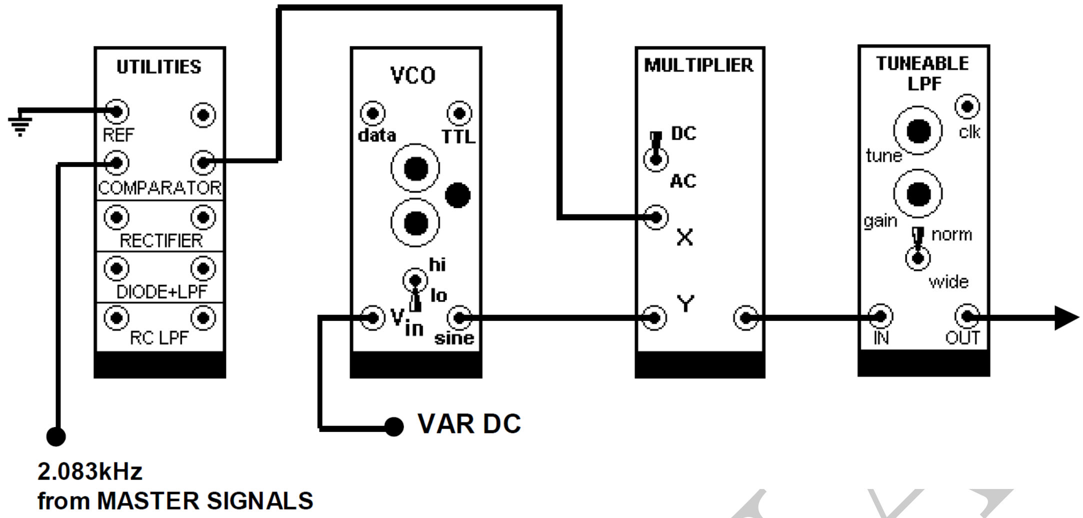

Figure 5: Task 3 - Wave Analyzer

2. Using the procedure already explained in Task 3, begin your search near 2083 Hz and confirm the presence of a target at that frequency. Record the amplitude of the beat signal. [***Marks:*** *1*]

3. Repeat the search near 6250 Hz. Proceed as before. Measure the amplitude of the beat signal and determine the frequency of the target. Remember that the WIDE position of the LPF module allows tuning up to 12 kHz. [***Marks:*** *1*]

4. Should we expect to find targets at 4167 Hz and 8333 Hz? Confirm that no components are present at these frequencies. Can you suggest the reason for this? Find one more target frequency [***Marks:*** *1*]