Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

PROBLEM SET 1

Complex Number Refresher

A quick refresher on complex numbers. Complex numbers are used in much of engineering. They are an near ideal shorthand in signal representation and they simplify expressions.

Please read textbook Page 71: Mathematical Review of Complex numbers, before attempting this homework.

Problem Set 1-1

(a) Prove the Euler identity:

$$e^{j\theta} = \cos\theta + j\sin\theta.$$

(b) Find expressions for $\cos\theta$ and $\sin\theta$ in terms of $e^{j\theta}$ and its conjugate $e^{-j\theta}$.

Problem Set 1-2

What is the difference between taking the conjugate of an expression and replacing every occurrence of j with $-j$?

Problem Set 1-3

Write each of the following in polar form, that is, in $re^{j\theta}$ find r (such that $r \geq 0$) and θ (such that $-\pi < \theta \leq \pi$).

(a) $1 + j\sqrt{3}$

(b) $(\sqrt{3} + j^3)(1 - j)$

(c) $\dfrac{e^{j\pi/3} - 1}{1 + j\sqrt{3}}$

(d) $j^{1,222,444,667,099,987,676,222,091,345,222,822,822,282,228}$

Problem Set 1-4

With $j = \sqrt{-1}$ what is j^j (j to the power j)?

Problem Set 1-5

Solve Textbook Problems 1.1, 1.2.

Problem Set 1-6

Solve Textbook Problem 1.49.

Classification and Properties of Signals

Problem Set 1-7

Find the even and odd components of each of the following signals:

(a)

$$x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$$

(b)

$$x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$$

(c)

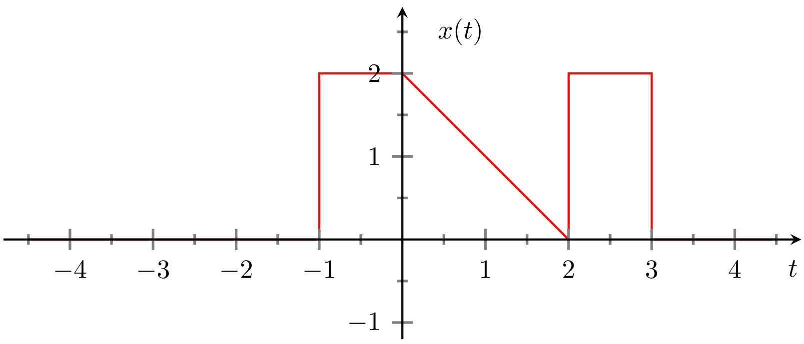
$$x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$$

(d)

$$x(t) = (1 + t^3)\cos^3(10t)$$

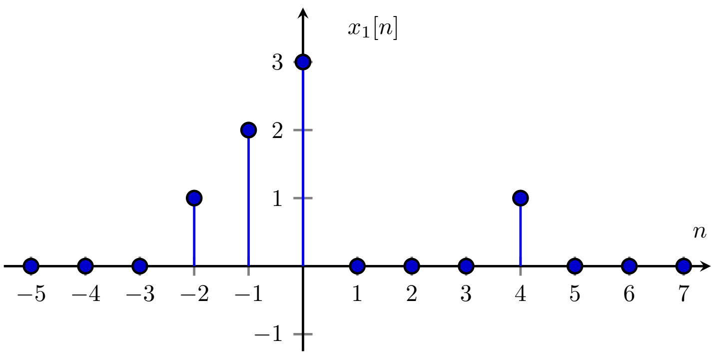
Problem Set 1-8

Determine and sketch the even and odd part of the following Continuous Time signal. Label your sketch carefully.



Problem Set 1-9

Determine and sketch the even and odd parts of the DT signal depicted below.



Problem Set 1-10

Review phasors, see phasors on wikipedia and looks at phasor diagrams for one sinusoid and phasor diagram for sum of two sinusoids, where this only works because the sinusoids are the same frequency.

Problem Set 1-11

(Hard) Movies like two sinusoids show what happens when there is the sum of two sinusoids at the same frequency. The real part of the rotating phasor maps out a sinusoid. Using the phasor ideas, suppose there is the sum of two sinusoids at different frequencies, for example at ω and 3ω such as

$$\cos(\omega t) + \frac{1}{9}\cos(3\omega t)$$

What modification happens to the phasor picture?

Problem Set 1-12

Lecture slides 55 and 56 show that adding two periodic signals of different periods can give you an output signal that is periodic:

(a) When does this occur in general?

(b) Assuming all periods under consideration are integer-valued (like 7 or 11 and not 4/5 nor $\sqrt{2}$), answer the following. If the output is periodic what is the relationship of the output period to the two input periods?

(c) Repeat the previous part in the case when the periods are not integer-valued. (For example, suppose $T_1 = 4098\sqrt{2}$ and $T_2 = 18879\sqrt{2}$.)

Problem Set 1-13

Review the type of calculation shown on lecture slide 70, repeated here

$$\begin{aligned} A\cos(\omega_0 t + \phi) &= \frac{A}{2}e^{j\phi}e^{j\omega_0 t} + \frac{A}{2}e^{-j\phi}e^{-j\omega_0 t} \\ &= A\,\Re\{e^{j(\omega_0 t+\phi)}\} \end{aligned}$$

and

$$A\sin(\omega_0 t + \phi) = A\,\Im\{e^{j(\omega_0 t+\phi)}\},$$

which needs the Euler identity (here A is real-valued).

Problem Set 1-14

Roger Federer and Leonhard Euler were both born in Basel Switzerland but they never meet. Why?

Problem Set 1-15

CT signal $x(t) = e^{j\omega t}$ is periodic for any choice of ω.

(a) True or false?

(b) What is its fundamental period when $\omega = 4$?

(c) What is its fundamental period when $\omega = 19\pi$?

Problem Set 1-16

DT signal $x[n] = e^{j\omega n}$ is not periodic for every choice of ω.

(a) True or false?

(b) Is it periodic when $\omega = 4$?

(c) Is it periodic when $\omega = 19\pi$?

(d) How does the DT signal $x[n] = e^{j19\pi n}$ differ from $x[n] = e^{j\pi n}$?

Problem Set 1-17

Solve Textbook Problem 1.9.

Problem Set 1-18

Determine whether or not each of the signals is periodic. If a signal is periodic, determine the fundamental period.

(a)

$$x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$$

(b)

$$x(t) = j\ e^{j10t}$$

(c)

$$x(t) = 2e^{j\left(t+\frac{\pi}{4}\right)}u(t)$$

(d)

$$x(t) = e^{(-1+j)t}$$

(e)

$$x(t) = \sqrt{\cos(2t)}$$

(f)

$$x(t) = \big(\cos(2t - 3\pi/4)\big)^2$$

(g)

$$x(t) = \mathrm{Ev}\big\{\cos(4\pi t)\,u(t)\big\}$$

where $\mathrm{Ev}\{\cdot\}$ denotes the even part.

Problem Set 1-19

Determine whether or not each of the following signals is periodic. If the signal is periodic determine its fundamental period.

(a) $x(t) = \sin^2(4t) \equiv \big(\sin(4t)\big)^2$

(b) $x[n] = \cos(4n + \pi/4)$

(c) $x[n] = (-1)^n \cos(2\pi n/7)$

Problem Set 1-20

If $x(t)$ and $x[n]$ are even signals, then show that

$$\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$$

and

$$\sum_{n=-k}^{k} x[n] = x[0] + 2\sum_{n=1}^{k} x[n]$$

Problem Set 1-21

Categorise each of the following signals as an energy signal or power signal, and find the energy or the time-averaged power of the signal:

(a)

$$x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2 - t & 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases},$$

(b)

$$x[n] = \begin{cases} n & 0 \leq n < 5 \\ 10 - n & 5 \leq n \leq 10 \\ 0 & \text{otherwise} \end{cases},$$

(c)

$$x(t) = 5\cos(\pi t) + \sin(5\pi t)$$

(d)

$$x(t) = \begin{cases} 5\cos(\pi t) & -1 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases},$$

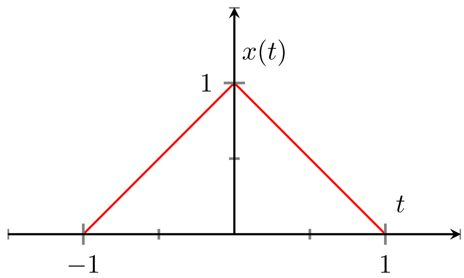
(e)

$$x[n] = \begin{cases} \cos(\pi n) & -4 \leq n \leq 4 \\ 0 & \text{otherwise} \end{cases},$$

Affine Transformations

Problem Set 1-22

Sketch each of the following signals derived from the triangular pulse $x(t)$ shown below



(a)

$$x(3t)$$

(b)

$$x(3t+2)$$

(c)

$$x(-2t-1)$$

(d)

$$x(2(t+2))$$

(e)

$$x(2(t-2))$$

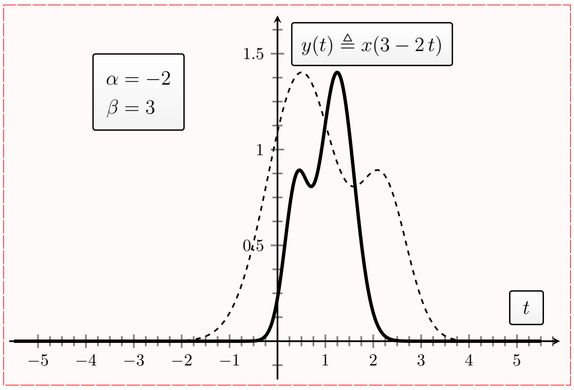
(f)

$$x(3t)+x(3t+2)$$

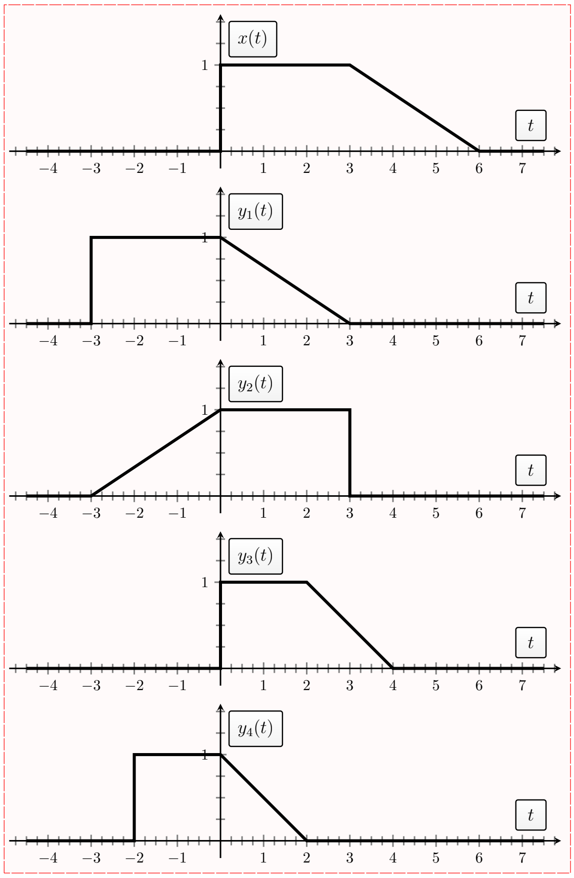
Problem Set 1-23

Review the independent variable affine transformation examples on lecture slides 50 and 52, shown below.

(a) Confirm the solid blue curve in the figure (blow) is indeed $y(t) = x(3-2t)$ where $x(t)$ is the dashed curve.

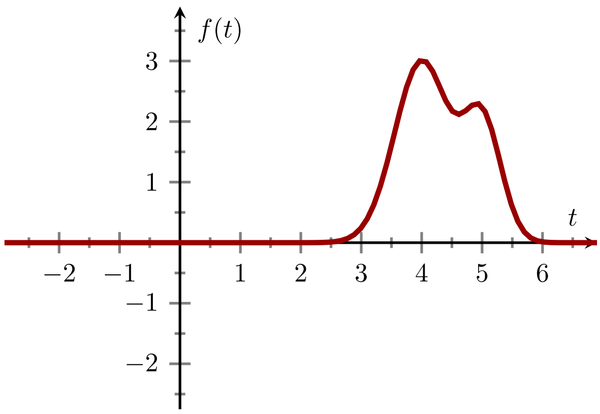


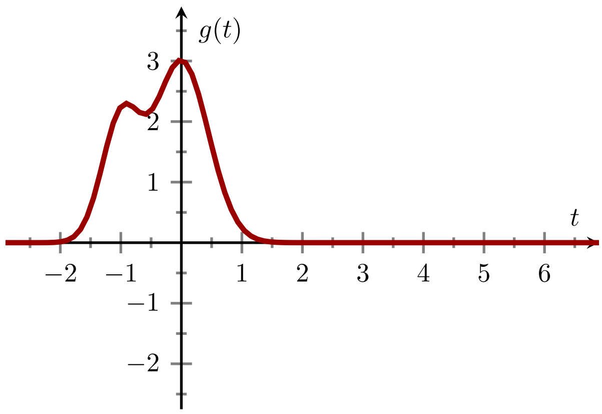
(b) Express each of $y_1(t)$, $y_2(t)$, $y_3(t)$, $y_4(t)$ in terms of $x(t)$ (shown to the right).



Problem Set 1-24

Two signals $f(t)$ and $g(t)$ are related through an affine transformation of their independent variable





a. $g(t) = f(t-4)$

b. $g(t) = f(4-t)$

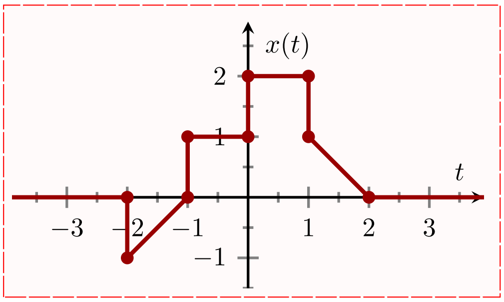
c. $g(t) = f(t+4)$

d. $g(t) = f(t+4)$

e. None of the above

Problem Set 1-25

For the CT signal $x(t)$ in the Text Figure P1.21 on page 60, shown below,



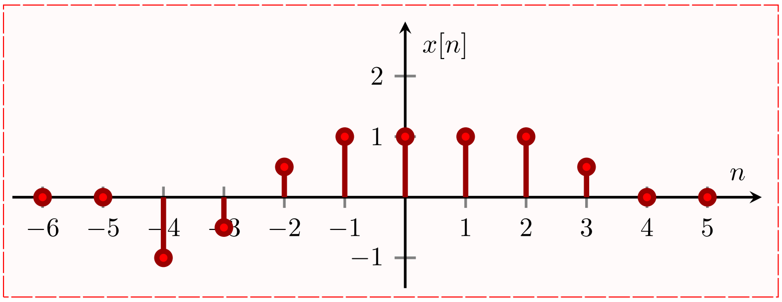
sketch and carefully label:

(a) $x(1-t/3)$

(b) $x(t-2)\big(\delta(t-1/2)+u(3-t)\big)$ where $\delta(t)$ is the delta function and $u(t)$ is the step function.

Problem Set 1-26

For the DT signal $x[n]$ from the Text Figure P1.22 on page 60, shown below,



sketch and carefully label:

(a) $x[2-n]$

(b) $x[2n+1]$

(c) $x[n/3]$

Classification and Properties of Systems

Problem Set 1-27

Classify each of the following systems, where $x(t)$ or $x[n]$ is the input signal and $y(t)$ or $y[n]$ is the output signal, as:

i) linear or non-linear,

ii) time-invariant or time-varying,

iii) causal or non-causal.

For each choice, provide either a brief (typically one sentence) justification or a mathematical-checking procedure.

(a)

$$y[n] = 2\,x[n-1]$$

(b)

$$y[n] = x[-n]$$

(c)

$$y[n] = 0$$

(d)

$$y[n] = -7$$

(e)

$$y(t) = 2.571489635789999999$$

(f)

$$y[n] = (\sin n\pi)\,x[n]$$

(g)

$$y(t) = \int_{-\infty}^{4t} x(\tau)\,d\tau$$

(h)

$$y(t) = \int_{-\infty}^{t+4} x(\tau)\,d\tau$$

(i)

$$y[n] = \left(\frac{n+0.5}{n-0.5}\right)^2 x[n]$$

(j)

$$y(t) = \frac{dx(t)}{dt} + x(t-2)$$

(k)

$$y(t) = x^2(t) + 2x(t+1)$$

(l)

$$y[n] = \sum_{k=-\infty}^{n} x[k+3]$$

Problem Set 1-28

Consider the following four systems:

$$\text{System A:} \qquad y[n] = \cos\big(2\pi\, x[n+1]\big) + x[n]$$

$$\text{System B:} \qquad y[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-2k]$$

$$\text{System C:} \qquad y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$\text{System D:} \qquad y[n] = \Big(-\frac{1}{5}\Big)^{2n} \big(x[n] + 1.5\big)$$

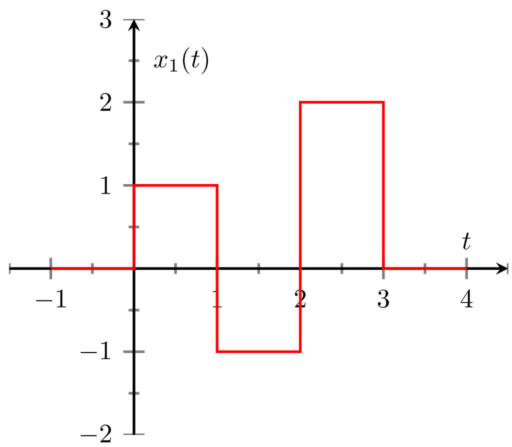
where $x(t)$ or $x[n]$ is the system input and $y(t)$ or $y[n]$ is the system output.

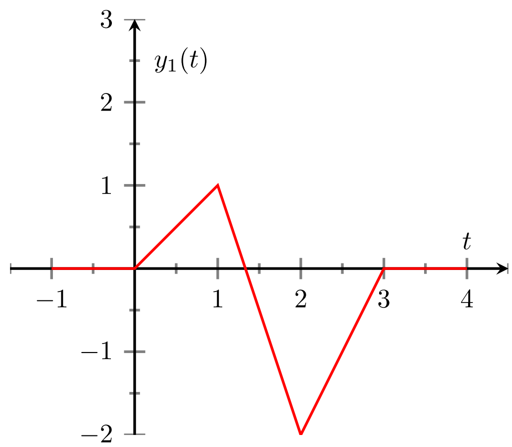
Answer yes or no for each of the following questions for each of the four systems. No explanation is required. Is the system:

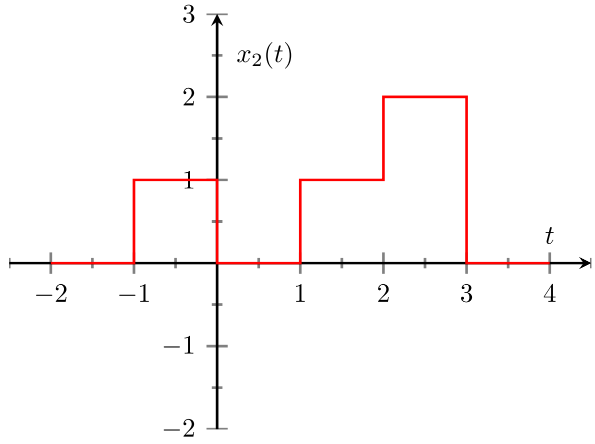
	System A	System B	System C	System D
memoryless ?	yes no	yes no	yes no	yes no
stable ?	yes no	yes no	yes no	yes no
casual ?	yes no	yes no	yes no	yes no
linear ?	yes no	yes no	yes no	yes no
time invariant ?	yes no	yes no	yes no	yes no

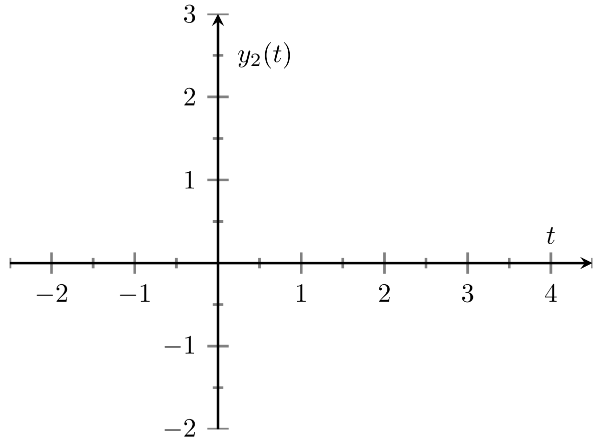
Problem Set 1-29

Consider a LTI system whose reponse to the signal $x_1(t)$ is the signal $y_1(t)$ depicted below. Determine and provide a labelled sketch of the response $y_2(t)$ of the system to the input $x_2(t)$ depicted below. Provide concise statements to explain any reasoning. [Hint: $x_2(t) = x_1(t) + x_1(t+1)$]






$x_2(t)$
t


$y_2(t)$
t