Signal Processing
ENGN2228

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Second Semester

Part 0 Slide 1/many Convenor: R. A. Kennedy

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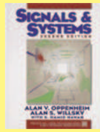
Part 1 Outline

1. **What are Signals?**
   - Examples of Signals
2. **Independent Variables**
3. **Continuous Time Signals**
4. **Discrete Time Signals**
5. **Periodic Signals**
6. **Signal Energy and Power**
7. **Odd and Even Signals**

What are Signals?



Signals & Systems
section 1.1
pages 1-5

Definition (Signals)

Signals carry information in their variations. Mathematically signals are functions of one or more independent variables.

Examples of signals are:

- speech and audio signals, output from a microphone
- voltages, currents
- images, stock market prices, currency exchange rates
- temperature, weather data
- biological signals
- ...

Signals are generally connected with physical quantities that vary with time or space or both.

Part 1 Outline

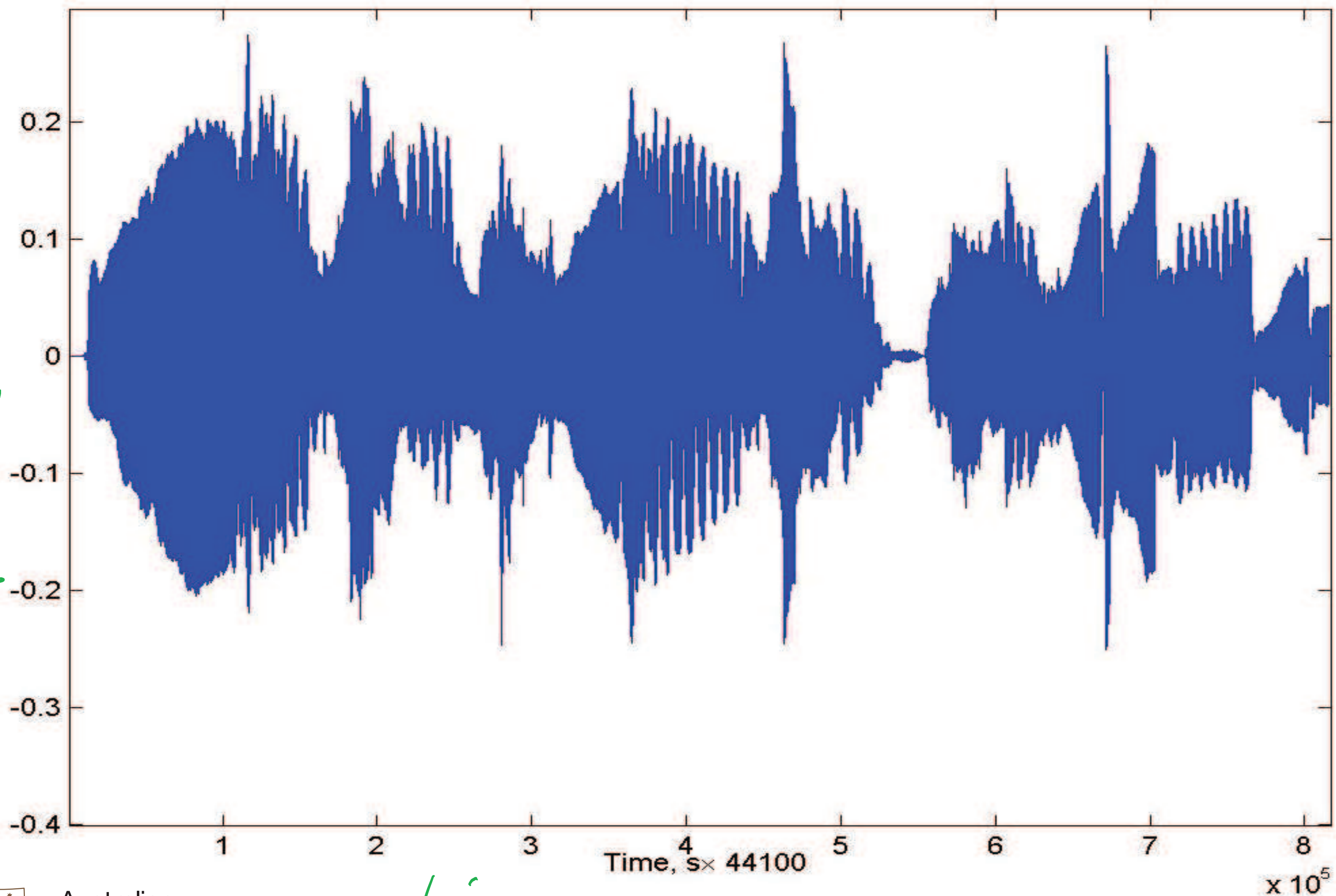
Examples of Signals – Currency (vs time)

Examples of Signals – Share Price (vs time)

time

Examples of Signals – Flute Sound Waveform (vs time)

amplitude



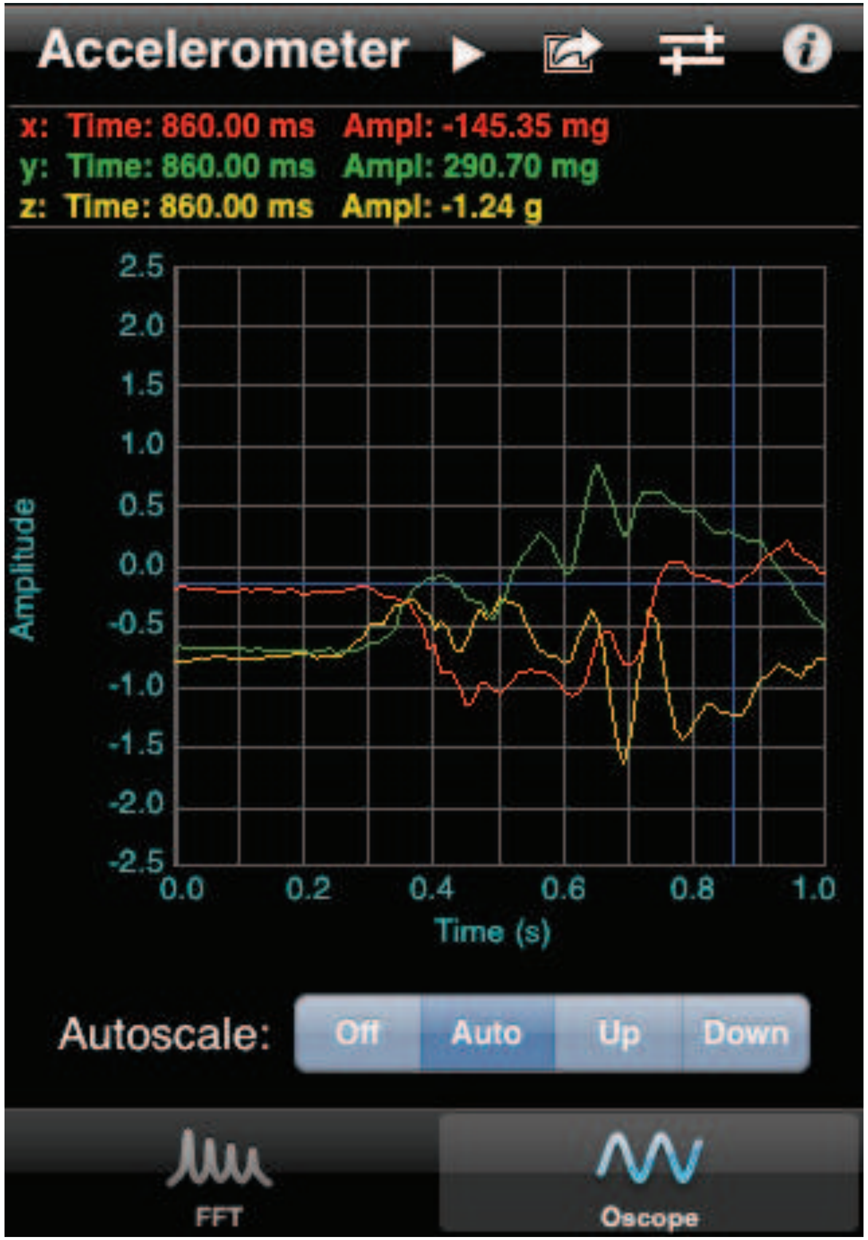
time

Examples of Signals – An Image (vs pixel space)

y

x

Examples of Signals – iPhone Accelerometers (vs time)

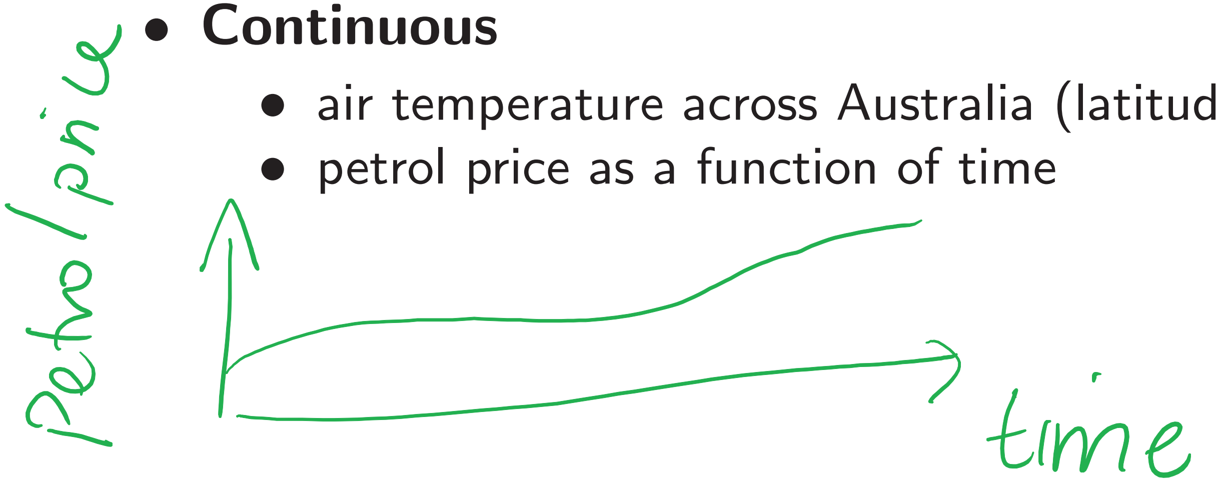


Part 1 Outline

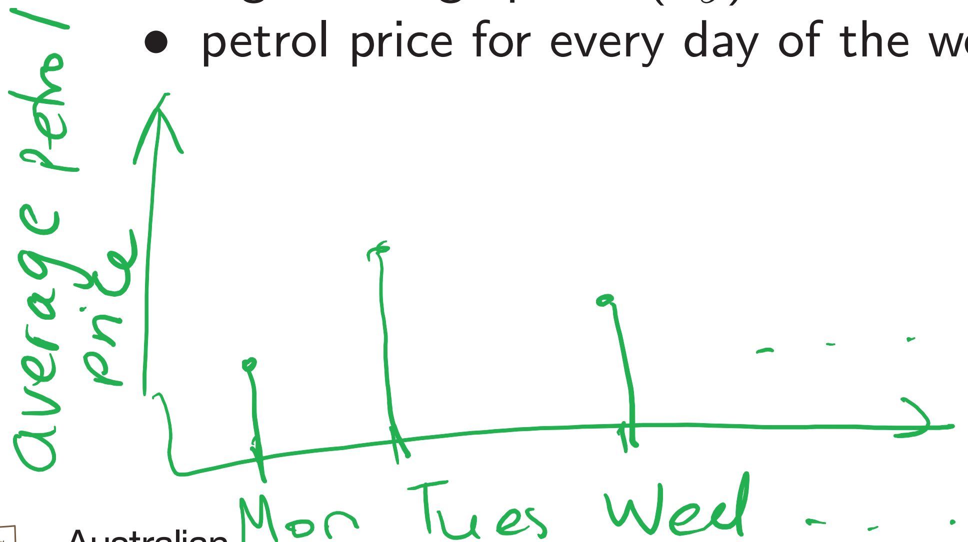
Independent Variables (types)

Signals vary with respect to the “independent variables”

- **Continuous**
  - air temperature across Australia (latitude and longitude)
  - petrol price as a function of time



- **Discrete**
  - digital image pixels (xy), 3D medical image voxels (xyz)
  - petrol price for every day of the week



Independent Variables – (dimension)

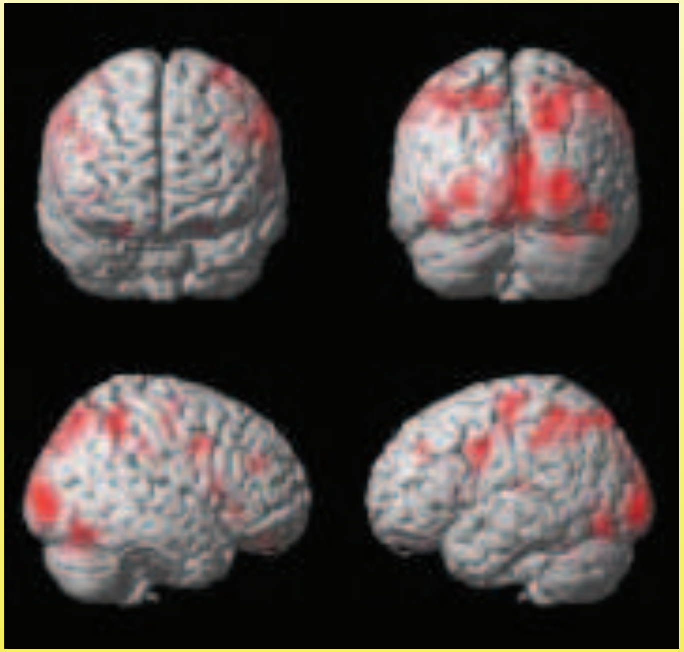
The independent variables can be one dimensional 1D, 2D, 3D, etc.

- A signal may vary with time (1D)
- An image varies with cartesian coordinates x and y in space (2D)
- The temperature can vary with position in a room horizontal x and y, and vertical z (3D)
- A movie is a 2D image that varies with time (3D)
- What dimension is a 3D movie? 4D

Independent Variables – (dimension)

- fMRI – functional Magnetic Resonance Imaging – 3D volume of patient's brain is imaged every one or two seconds (4D, i.e., 3 space and 1 time dimensions)

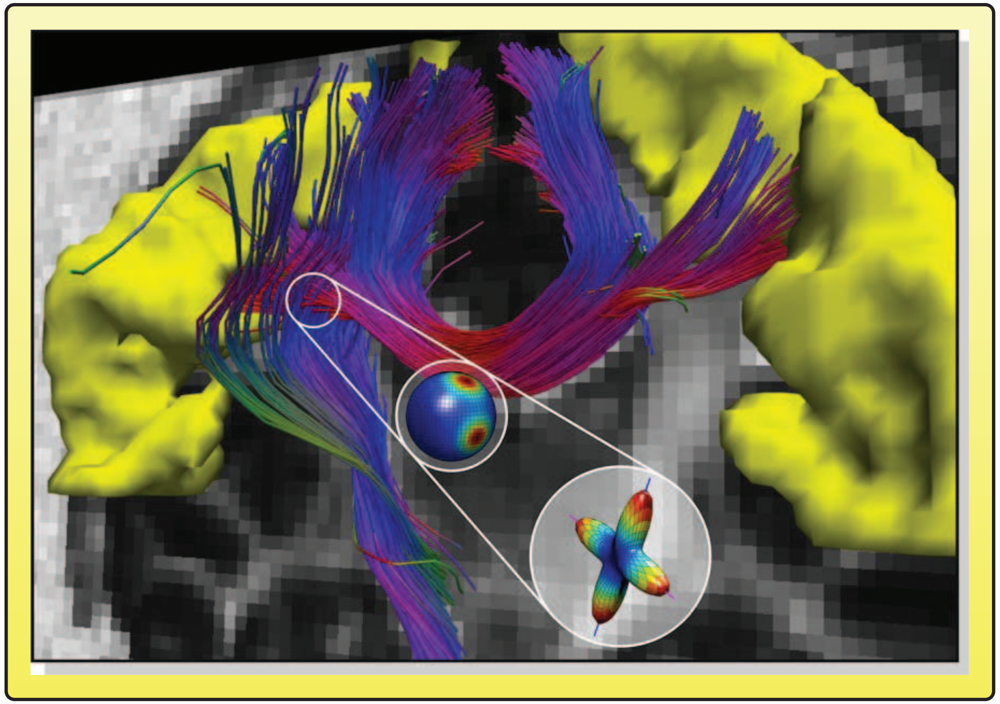
Oversized pencil sharpener



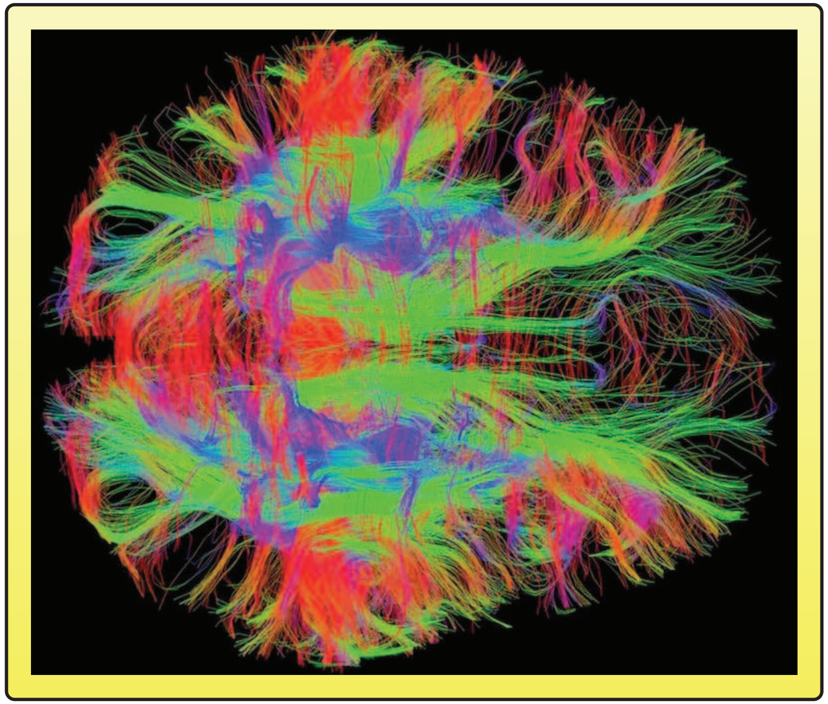
fMRI

Independent Variables – (dimension)

- dMRI – diffusion Magnetic Resonance Imaging – 3D volume of patient's brain is imaged and the diffusion of water molecules is measured in 3D within every voxel - 6D image.



6D dMRI image



Brain wiring

Independent Variables – (course focus on 1D)

- For this course we will focus on 1D
- That is, a single independent variable
- Most cases this is "time"

Part 1 Outline

Continuous Time Signals

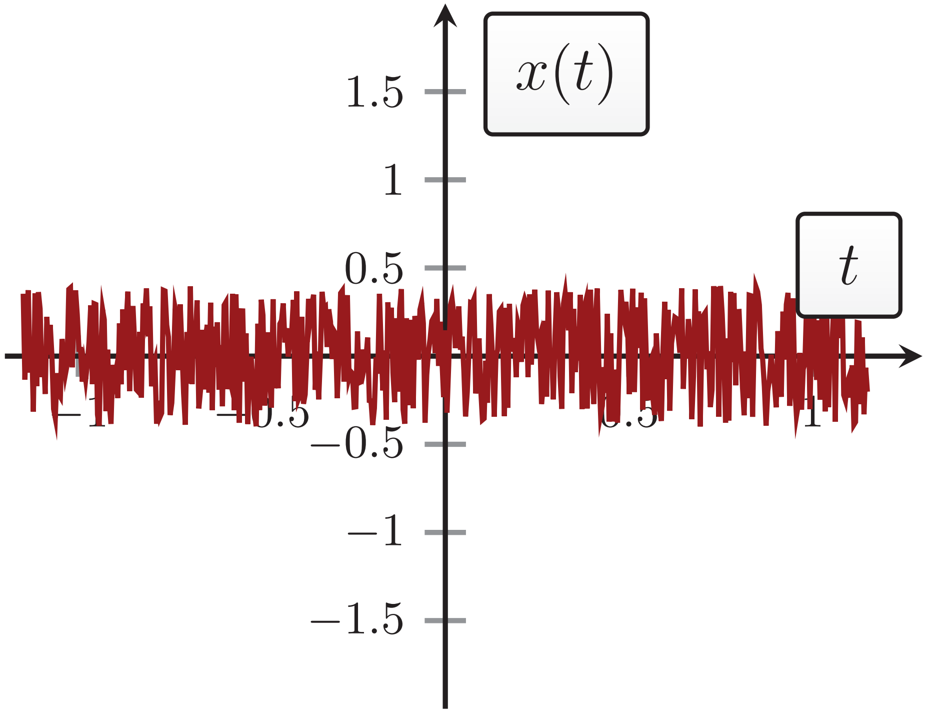
Definition (Continuous-Time, CT, Signals)

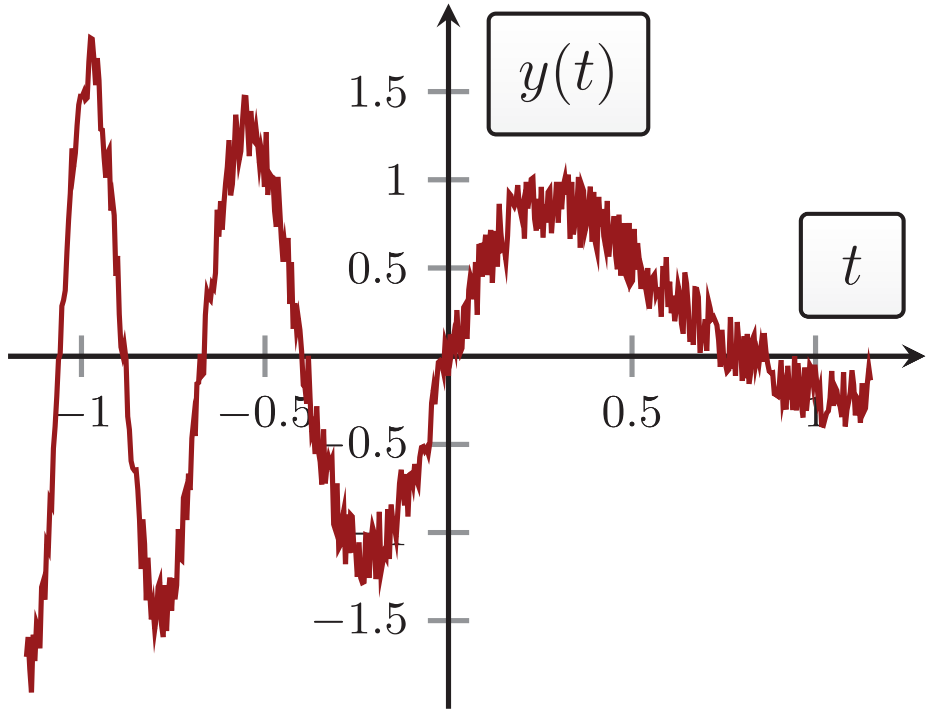
Continuous-Time Signals are signals whose independent variable is **continuous** and taken as time. That is, $x(t)$ with continuum t.

$$\boxed{x(t), \quad t \in \mathbb{R} \text{ (real numbers/time)}}$$

- Signals from the real physical world are generally CT, such as voltage, pressure, velocity, etc, as functions of time t
- Examples of real-valued CT signals (functions) are shown next: one noisy and the other some dying signal with additive noise.
- Followed by an example of a complex-valued CT signal.

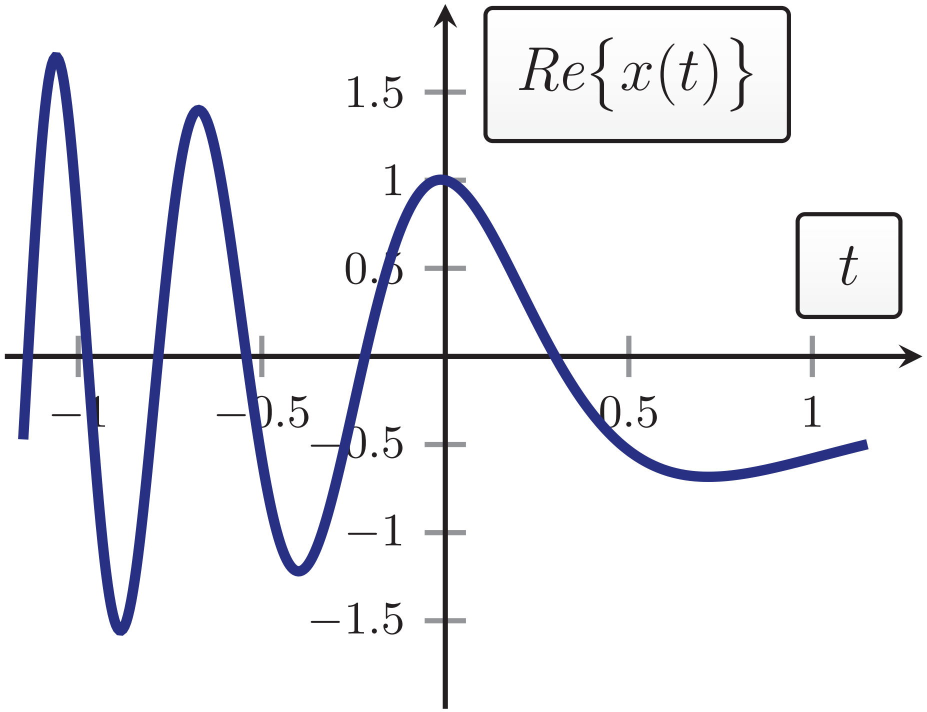
Continuous Time Signals – Examples (note (t))

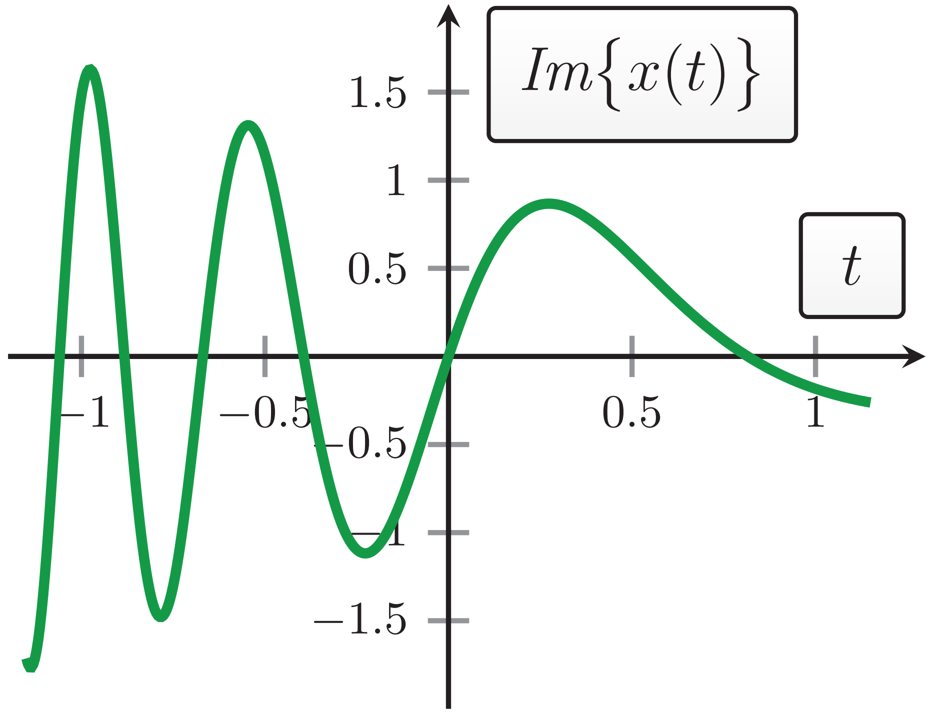




Continuous Time Signals – Complex Example

$$x(t) \triangleq \exp\big(2\pi j\, t\, \exp(-0.6\, t)\big)\, \exp(-0.5\, t) \in \mathbb{C}$$





Part 1 Outline

Discrete Time Signals

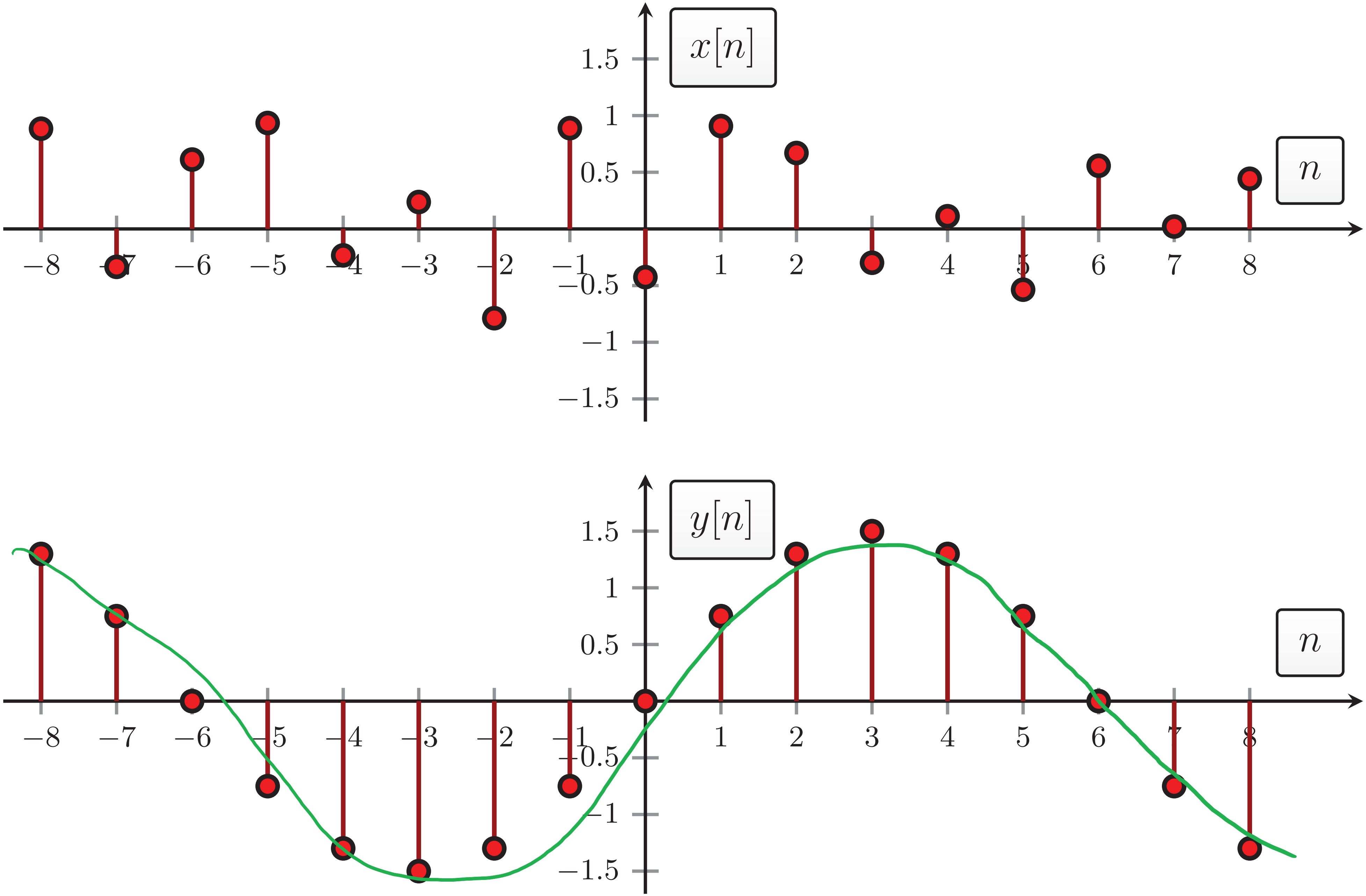
Definition (Discrete-Time (DT) Signals)

Discrete-Time Signals are signals whose independent variable takes on only a **discrete set of values** and are generally taken to be integer values. That is, $x[n]$ with discrete/integer n

$$\boxed{x[n], \quad n \in \mathbb{Z} \text{ (integers)}}$$

- Signals from the real physical world are generally not naturally DT, but most man-made and "sampled" signals are DT
- Examples of real-valued DT signals (discrete functions) are shown next slide:
  - $x[n]$ is somewhat random
  - $y[n]$ looks like a sampled sinusoid

Discrete Time Signals – Real-Valued Examples (note $[n]$)



Discrete Time Signals

Reminder:

Continuous-Time (CT) signals: $x(t)$ with t – continuous values[1]

Discrete-Time (DT) signals: $x[n]$ with n – integer values[1]

[1]Round brackets $\equiv$ continuous. Square brackets $\equiv$ discrete.

Discrete Time Signals

- Natural DT signals? Less common, e.g., DNA base sequence
- Most DT signals are man-made
- Images, digital music, stock market data, etc

DT signals are increasingly important because they are in a form that permit calculations, that is, “processing”, via computers (CPUs), graphics processing units (GPU’s) and digital signal processors (DSPs)

i) Intel CPU, ii) ATI RADEON GPU and iii) TI TMS320 DSP

Part 1 Outline

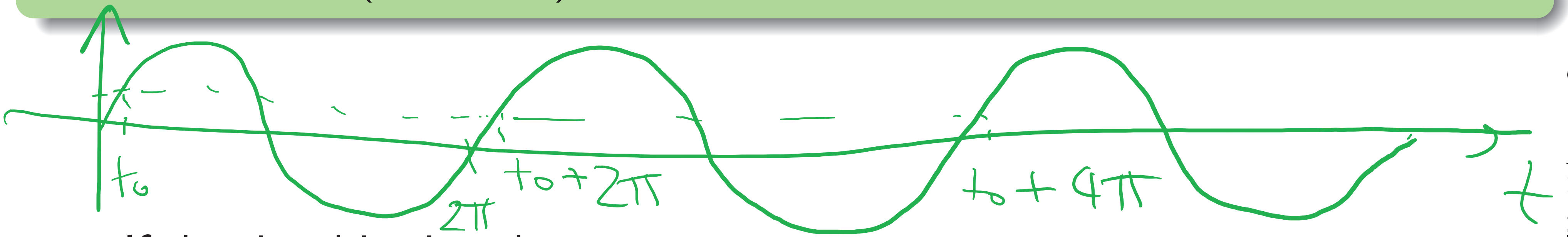
Periodic Signals (CT definition)

Definition (Periodic Continuous Time Signals)

CT Signal $x(t)$ is **periodic with period** $T > 0$ if

$$x(t) = x(t + T)$$

for all real $t \in \mathbb{R}$ (or $\forall t \in \mathbb{R}$).



- If the signal is given by

$$x(t) = \sin(t),$$

  then $x(t) = x(t + 2\pi)$, $\forall t$, and $x(t) = x(t + 4\pi)$, $\forall t$, etc.
- So, $x(t) = \sin(t)$ is periodic with period $T = 4\pi$. But, of course, $T = 4\pi$ is not the smallest period. This motivates the following definition:

Periodic Signals (CT definition)

Definition (Fundamental Period T_0)

The fundamental period, $T_0 > 0$, is the **smallest positive period** T for which $x(t)$ is periodic.

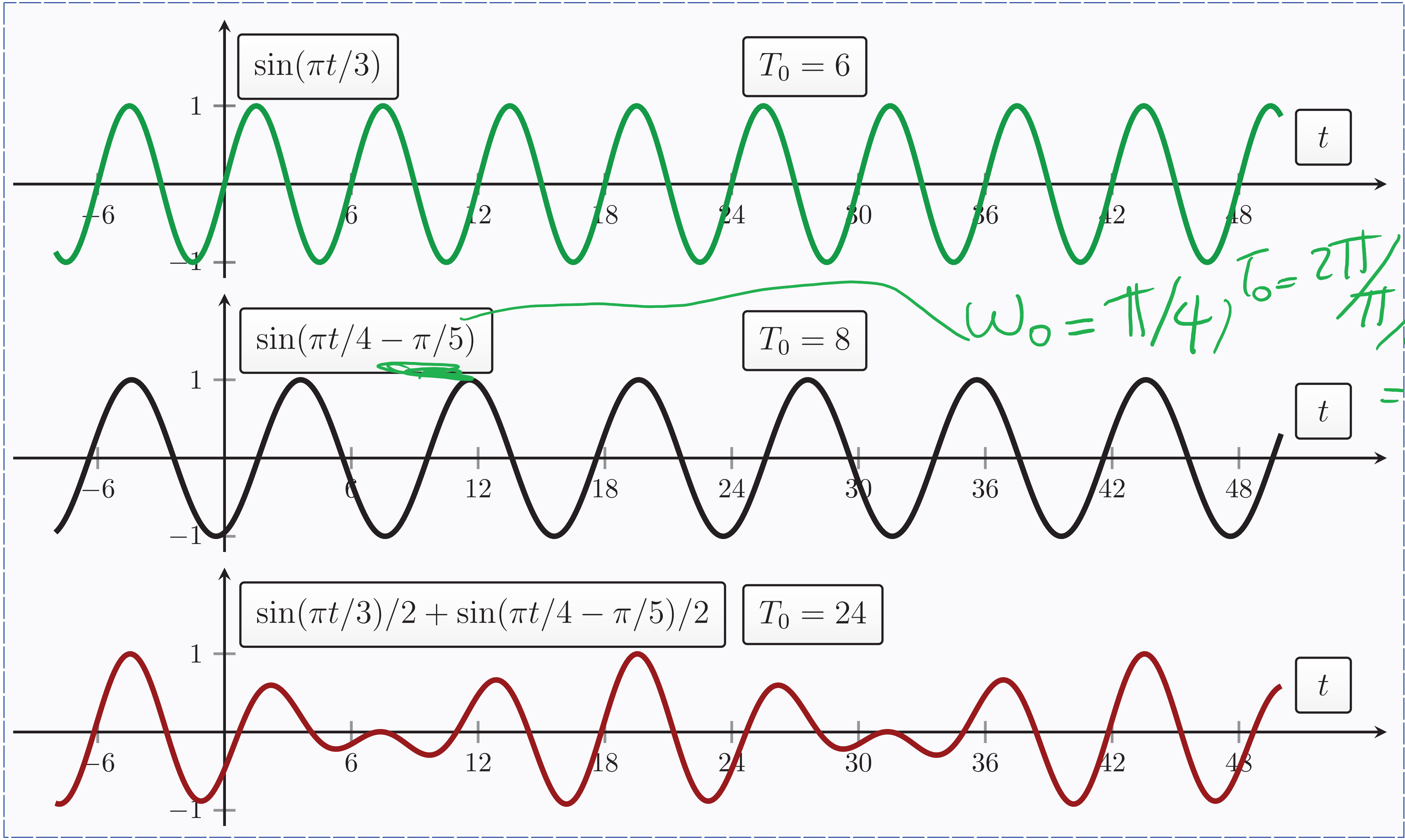
- If $x(t) = \sin(t)$, then $T_0 = 2\pi$.
- If $x(t) = A\sin(\omega_0 t + \theta)$ or $x(t) = A\cos(\omega_0 t + \theta)$, then

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

- Also $x(t) = \exp(jt)$ has fundamental period $T_0 = 2\pi$. So complex signals can be periodic too. Use same formula calculate T_0
- $x(t) = \sin(4t)$ has fundamental period $T_0 = \pi/2$.
- $x(t) = \sin(3t)$ has fundamental period $T_0 = 2\pi/3$.
- $x(t) = \sin(t) + \sin(4t)$ has fundamental period $T_0 = 2\pi$.
- $x(t) = \sin(3t) + \sin(4t)$ has fundamental period $T_0 = 2\pi$.
- $x(t) = \sin(3t) + \sin(4t + \pi/7)$ has fundamental period $T_0 = 2\pi$.
- $x(t) = \sin(\pi\, t/3)/2 + \sin(\pi\, t/4 - \pi/5)/2$ has fundamental period $T_0 = 24$.

$2\pi / \pi/3 = 6$ $\quad$ $2\pi / \pi/4 = 8$

Periodic Signals – Example



$\omega_0 = \pi/4$, $T_0 = 2\pi/(\pi/4) = 8$

Periodic Signals (DT definition)

Definition (Periodic Discrete Time Signals)

DT Signal $x[n]$ is **periodic with integer period** $N > 0$ if

$$x[n] = x[n+N]$$

for all integer $n \in \mathbb{Z}$.

Definition (Fundamental Period N_0)

The fundamental period, $N_0 > 0$, is the **smallest positive integer** N for which $x[n]$ is periodic.

- For DT sinusoidal and exponential signals:

$$\boxed{N = \frac{2\pi}{\omega_0} m}$$

where $m = 1, 2, \ldots$ such that N is an integer. $\frac{2\pi}{\omega_0}$ $= m$

- $x[n] = \cos[2\pi n]$, $\omega_0 = 2\pi$ therefore $N = \frac{2\pi}{2\pi} m$. For $m = 1$, $N_0 = 1$.
- $x[n] = 5\sin[\frac{6\pi n}{35}]$, $\omega_0 = \frac{6\pi}{35}$ therefore $N = \frac{35}{3} m$. For $m = 3$, $N_0 = 35$.
- $x[n] = \cos\left[\frac{\pi}{2} n\right] + \cos\left[\frac{\pi}{4} n\right]$, $N_0 = 8$ as lowest common multiple.

$\omega_0 = \pi/2$

$N = \frac{2\pi}{\pi/2} m = 4m$

$N_0 = 4 \ (m=1)$

$\omega_0 = \pi/4$

$N = \frac{2\pi}{\pi/4} m = 8m$

$N_0 = 8 \ (m=1)$

Periodic Signals[(DT definition)]1.2.2page 12

- Are CT sinusoids always periodic? YES
- Are DT sinusoids always periodic? NO - depends on the sampling rate
  - ω_0 must be a rational multiple of 2π.
- $x[n] = \cos[\frac{n}{6}]$, $\omega_0 = \frac{1}{6}$, $N = \frac{2\pi}{\frac{1}{6}} \overset{m}{=} 12m\pi$ so non-periodic.

Part 1 Outline