Australian National University

MOCK Final Examination
Semester 2, 2017

SIGNAL PROCESSING

ENGN2228

Study period: None
Writing period: 180 Minutes duration
Permitted materials: Two A4 pages with handwritten notes on both sides
Calculator (memory cleared)

6 True-False and 14 Multiple-Choice Questions, for a total of 20 marks
4 Problems of 18 marks each, for a total of 72 marks
Total marks for exam: 92 marks

Contribution to Final Assessment: 50%

- *Write your True-False and Multiple-Choice Questions answers on the answer sheet provided and place it inside the script book.*
- *Write your 4 Problem answers in the script book provided.*
- *For True-False and Multiple-Choice Questions, there is NO negative marking.*
- *At the end of the exam, hand in the exam question sheets as well as the script book and the Multiple-Choice answers sheet.*

PART 1 — True/False Questions

TF Question 1

The signal $x(t)$ below can be expressed as the sum of an even signal and an odd signal.

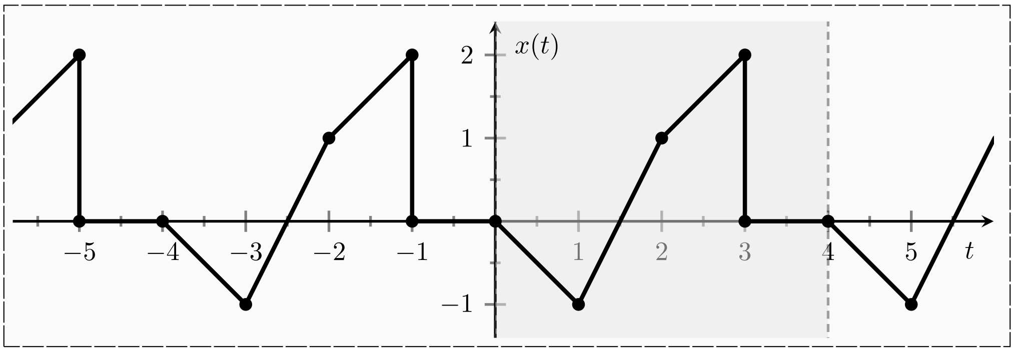


Figure 1: Real-valued CT Periodic Signal $x(t)$ with fundamental period $T = 4$. The shaded region indicates one period of the signal, which is periodically repeated.

a. True

b. False

Solution: Every signal can be expressed as the sum of an even signal and an odd signal, not just $x(t)$. □

TF Question 2

The CT periodic signal $x(t) = \cos(t)$ has a Fourier series but no Fourier transform.

a. True

b. False

□

TF Question 3

For the DT signal $x[n] = \sin(n/16)$, it has only one (non-zero) term in its Fourier series.

a. True

b. False

Solution: The intrinsic period of sin is irrational, which implies n would need to be irrational. It needs to be an integer for the signal to be DT periodic. Because the signal is not periodic then it does not have a Fourier series. □

TF Question 4

A CT signal which is periodic with period 2π is also periodic with period 8π.

a. True

b. False

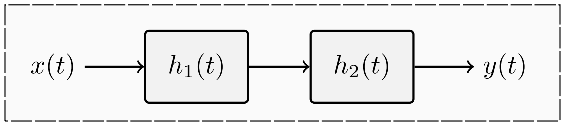


Figure 2: A series/cascade connection of the two CT LTI systems.

TF Question 5

If an LTI system with impulse response $h_1(t)$ is non-causal and an LTI system with impulse response $h_2(t)$ is also non-casual then the system corresponding to the series/cascade connection of $h_1(t)$ and $h_2(t)$ (as depicted in Figure 2) is always non-causal.

- a. True
- b. False

Solution: Intuitively the case and can be shown. □

TF Question 6

Consider the series/cascade connection of the two CT LTI systems with impulse responses $h_1(t)$ and $h_2(t)$, input $x(t)$ and output $y(t)$, as shown in Figure 2:

Suppose $h_1(t) = \delta(t - 2.3)$ and $h_2(t) = \delta(t - 1.5)$ then $y(t) = x(t - 3.8)$.

- a. True
- b. False

Solution: Here the delays add to a overall delay of -3.8. □

PART 2 — Multiple Choice Questions

MC Question 1

What is the DT convolution, $y[n] = x[n] \star h[n]$, of the two signals

$$x[n] = \delta[n] + \delta[n-2] \quad \text{and} \quad h[n] = 2\,\delta[n-3]$$

a. $y[n] = \delta[n] + \delta[n-2] + 2\delta[n-3]$

b. $y[n] = 2\,\delta[n-3] + 2\,\delta[n-5]$

c. $y[n] = 2\,\delta[n+3] + 2\,\delta[n+1]$

d. $y[n] = 2\,\delta[n] + 2\,\delta[n-2]$

e. None of the above.

Solution: Immediately we see $y[n] = 2\,x[n-3]$ and the result follows. $\square$

MC Question 2

A DT LTI system with impulse response $h[n]$ is stable if

$$\sum_{n=-\infty}^{\infty} \big|h[n]\big| < \infty.$$

One of the following causal DT LTI systems is not stable. Which one is it?

a. $h[n] = u[n] - u[n-10]$

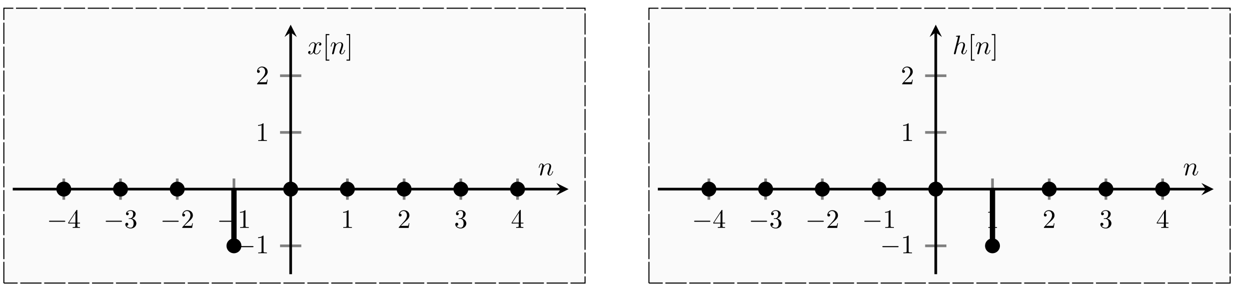
b. $h[n] = \dfrac{1}{n^2} u[n]$

c. $h[n] = \dfrac{1}{n} u[n]$

d. $h[n] = e^{-2n} u[n]$

e. $h[n] = \delta[n]$

Solution: The harmonic series $\sum_{n=0}^{\infty} 1/n$ diverges. All the others are finite sums or decay more rapidly than the harmonic series. $\square$



(a) Input signal $x[n]$. **(b)** System response $h[n]$.

Figure 3: Signal $x[n]$ and system response $h[n]$, output signal is $y[n] = h[n] \star x[n]$

MC Question 3

For $x[n]$ shown in Figure 3(a), which of the following is correct?

a. $x[n] = -\delta[n+1]$

b. $x[n] = -\delta[n-1]$

c. $x[n] = -1$

d. $x[n] = +1$

e. $x[n+1] = \delta[n]$

Solution: For $n = -1$ then the value should be 1. For $n = 0$ then value should be 0. □

MC Question 4

For $x[n]$ and $h[n]$, shown in Figure 3(a) and Figure 3(b), what is $y[n] = h[n] \star x[n]$?

a. $y[n] = \delta[n]$

b. $y[n] = -\delta[n-1] - \delta[n+1]$

c. $y[n] = -2\delta[n]$

d. $y[n] = -\delta[n-2]$

e. $y[n] = \delta[n-2]$

Solution: $\delta[n] \to -\delta[n-1]$. So $-\delta[n] \to \delta[n-1]$ and $-\delta[n+1] \to \delta[n-1+1] \equiv \delta[n]$. □

MC Question 5

[Tricky] Consider the system $y(t) = x(t)\,u(-t)$, where $u(t)$ is the unit step signal. Which of the following statements is true?

a. The system is non-linear and casual

b. The system is linear and causal

c. The system is non-linear and non-casual

d. The system is linear and non-causal

e. None of the above

Solution: The $u(-t)$ makes the system time-varying, it switches from acting like the trivial system for negative t to the zero system for positive t. However it is still linear. It is also causal; $u(-t)$ looks non-causal but it has nothing to do with the input. □

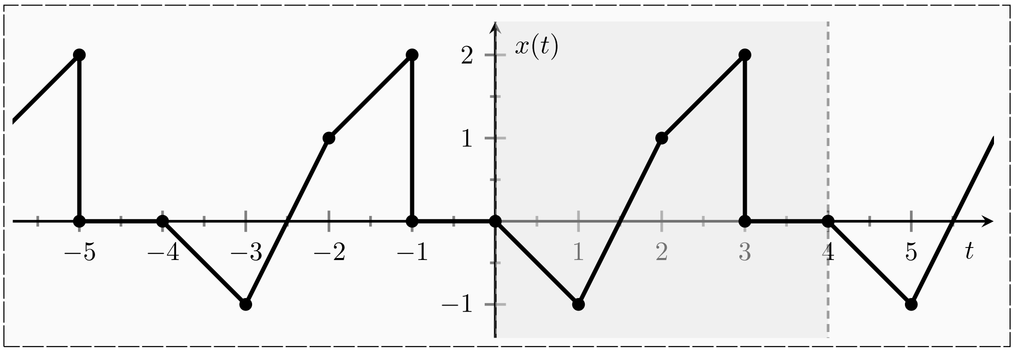


Figure 4: Real-valued CT Periodic Signal $x(t)$ with fundamental period $T = 4$. The shaded region indicates one period of the signal, which is periodically repeated.

MC Question 6

Figure 4 shows a portion of a real-valued periodic CT signal whose fundamental period is $T = 4$. What is the fundamental frequency, ω_0, associated with this fundamental period?

a. $\omega_0 = \pi/4$

b. $\omega_0 = \pi/2$

c. $\omega_0 = \pi$

d. $\omega_0 = 2\pi$

e. $\omega_0 = 1/4$

Solution: With $T = 4$ then the $\omega_0 = 2\pi/T = \pi/2$. (That is, in one period, being $T = 4$, the signal traverses 2π radians; so in one time unit it covers $\pi/2$ radians.) □

MC Question 7

For the periodic CT signal, $x(t)$ in Figure 4, what is the DC value of the signal or the $k = 0$ Fourier coefficient? (The DC value is also the average value per period.)

a. $-1/4$

b. 0

c. $1/4$

d. 2

e. 8

Solution: Looking at $x(t)$ in Figure 4, then its DC value is clearly greater than zero. Further the DC value of 2, being the maximum of $x(t)$, is impossible. □

MC Question 8

Now consider the periodic CT signal $|x(t)|$, which is the absolute value of the real-valued CT signal in Figure 4. (For example it goes through the point $(1, 1)$ instead of $(1, -1)$ and still passes through $(1.5, 0)$). Which of the following is true?

a. $|x(t)|$ has the same power per period as $x(t)$

b. $|x(t)|$ has more power per period than $x(t)$

c. $|x(t)|$ has less power per period than $x(t)$

d. $|x(t)|$ has the same DC value as $x(t)$

e. $|x(t)|$ has the same Fourier series coefficients as $x(t)$

Solution: It has the same (total) power per period. It has greater power at DC and therefore must have less total power in the frequencies excluding DC. □

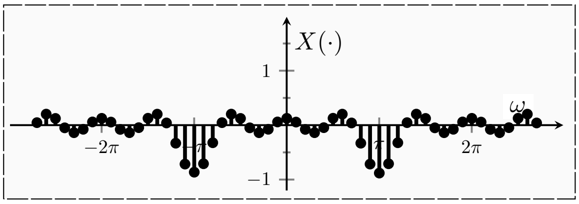
For Questions 15 to 18 we have, depending on the context, either

$$x(t) \xleftrightarrow{\mathcal{F}} X(\omega) \quad \text{or} \quad x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\omega}).$$

Further, the Fourier transform $X(\cdot)$ is purely real (zero imaginary part).

MC Question 9

Based on the structure of the frequency domain representation $X(\cdot)$ (period 2π) shown to the right which of the following is the best description of the time-domain signal?



a. The time-domain signal is DT and periodic in time.

b. The time-domain signal is CT and periodic in time.

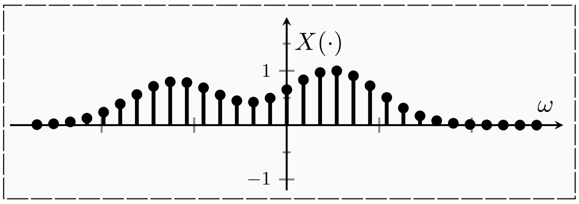
c. The time-domain signal is DT and not periodic in time.

d. The time-domain signal is CT and not periodic in time.

Solution: $X(\cdot)$ periodic iff DT and $X(\cdot)$ uniform discrete iff periodic in time. □

MC Question 10

Based on the structure of the frequency domain representation $X(\cdot)$ shown to the right which of the following is the best description of the time-domain signal?



a. The time-domain signal is DT and periodic in time.

b. The time-domain signal is CT and periodic in time.

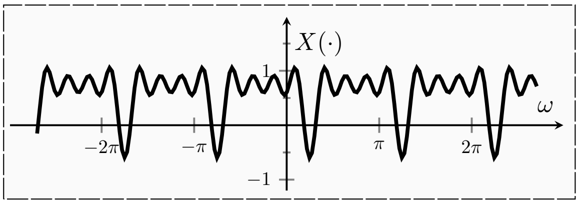
c. The time-domain signal is DT and not periodic in time.

d. The time-domain signal is CT and not periodic in time.

Solution: $X(\cdot)$ aperiodic iff CT and $X(\cdot)$ uniform discrete iff periodic in time. □

MC Question 11

Based on the structure of the frequency domain representation $X(\cdot)$ (period 2π) shown to the right which of the following is the best description of the time-domain signal?



a. The time-domain signal is DT and periodic in time.

b. The time-domain signal is CT and periodic in time.

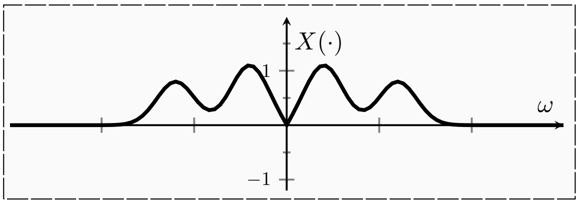
c. The time-domain signal is DT and not periodic in time.

d. The time-domain signal is CT and not periodic in time.

Solution: $X(\cdot)$ periodic iff DT and $X(\cdot)$ continuous iff aperiodic in time. □

MC Question 12

Based on the structure of the frequency domain representation $X(\cdot)$ shown to the right which of the following is the best description of the time-domain signal?



a. The time-domain signal is DT and periodic in time.

b. The time-domain signal is CT and periodic in time.

c. The time-domain signal is DT and not periodic in time.

d. The time-domain signal is CT and not periodic in time.

Solution: $X(\cdot)$ aperiodic iff CT and $X(\cdot)$ continuous iff aperiodic in time. □

MC Question 13

What is the frequency of the positive 3rd harmonic ($k = 3$) of the DT periodic signal

$$x[n] = (-1)^{3n} \cos(\pi n/3)?$$

a. $\pi/2$ rad/sec

b. π rad/sec

c. 2π rad/sec

d. 6 rad/sec

e. 18 rad/sec

Solution: The periodic signal has fundamental period $T = 6$, and so the fundamental frequency is $\omega_0 = 2\pi/T = \pi/3$. The positive 3rd harmonic is at frequency $3\,\omega_0 = \pi$. □

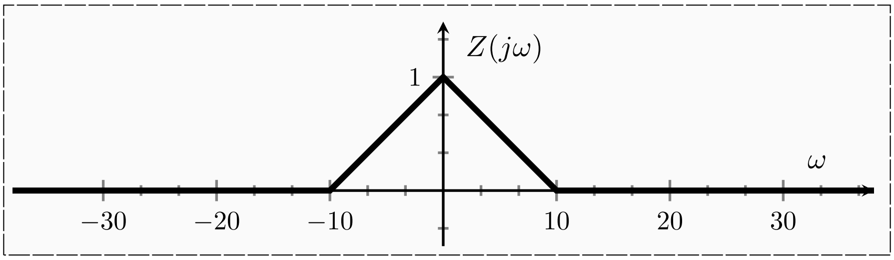


Figure 5: A frequency domain spectrum, $Z(j\omega)$, which is purely real. The units of ω are radians/sec

MC Question 14

For the CT signal $z(t)$ with $Z(j\omega)$ shown in Figure 5, what is the minimum sampling frequency, ω_s so that $z(t)$ can be recovered from its samples using an ideal low-pass filter?

a. $\omega_s = 10$ radians/sec

b. $\omega_s = 20$ radians/sec

c. $\omega_s = 40$ radians/sec

d. $\omega_s = 60$ radians/sec

e. $\omega_s = 80$ radians/sec

Solution: Twice the maximum frequency. □

(end of multiple choice questions)

PART 3 — Problems

Instructions: Attempt all 4 problems. Each problem is worth 18 marks making a total of 72 marks (72%) available for this Part of the exam. You should target 30 minutes for each problem.

Problem 1

(a) **[2 marks]** The analysis and synthesis equations for a CT signal $x(t)$ and its Fourier transform $X(j\omega)$ are given below:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\, d\omega, \quad t \in \mathbb{R} \qquad (Synthesis)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt, \quad \omega \in \mathbb{R} \qquad (Analysis)$$

Express in a few sentences your understanding of these two equations.

Solution: In the time-domain it shows the signal $x(t)$ can be built-up or synthesized by a superposition of complex exponentials.
In the frequency-domain it shows the frequency components, through analysis, that make-up the signal. □

(b) **[8 marks]** Show, using any method, that the FT of the signal

$$x(t) = e^{-t/10}\cos(10t)u(t)$$

can be expressed in the form

$$X(j\omega) = \frac{j\omega + \frac{1}{10}}{(j\omega + \frac{1}{10})^2 + 10^2}$$

Note: You must show all steps and name any properties used to arrive at the correct answer.

Solution:

$$x(t) = e^{-t/10}\cos(10t)u(t) = (e^{-t/10}u(t))(\cos(10t))$$

$e^{-t/10}u(t) \xleftrightarrow{\mathcal{F}} \frac{1}{j\omega + \frac{1}{10}}$

$\cos(10t) \xleftrightarrow{\mathcal{F}} \pi\delta(\omega+10) + \pi\delta(\omega-10)$

Using the multiplication property

$$X(j\omega) = \frac{1}{2\pi}\left(\frac{1}{j\omega + \frac{1}{10}}\right) \star (\pi\delta(\omega+10) + \pi\delta(\omega-10)) = \frac{1}{2}\left(\frac{1}{j(\omega+10) + \frac{1}{10}} + \frac{1}{j(\omega-10) + \frac{1}{10}}\right)$$

If simplify get the answer you were asked to prove.

(c) **[8 marks]** Find, using any method, the time domain signal corresponding to the following Fourier representation

$$X(j\omega) = \frac{1}{2\pi}\left(\frac{2\sin(\omega-2)}{(\omega-2)} \star \frac{e^{-j2\omega}2\sin(2\omega)}{\omega}\right)$$

where $\star$ denotes the convolution. Note: You must show all steps and name any properties used in your working.

Solution:

$$x(t) = e^{j2t}(u(t) - u(t-1))$$

Problem 2

(a) The analysis and synthesis equations for a periodic CT signal $x(t)$ and its Fourier series coefficients a_k are given below:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \quad t \in \mathbb{R} \qquad \text{(Synthesis Equation)}$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt, \quad k \in \mathbb{Z} \qquad \text{(Analysis Equation)}$$

where T is the fundamental period and $\omega_0 = 2\pi/T$ is the fundamental frequency in rad/sec.

i) **[3 marks]** Can we find the Fourier transform of $x(t)$? If so how?

Solution: Yes Fourier transform (FT) is defined for any CT signal. Can use the Fourier transform analysis equation but if have Fourier series coefficient easiest to use the FT pair for periodic signals.

(b) Consider the periodic CT signal $x(t)$, shown below:

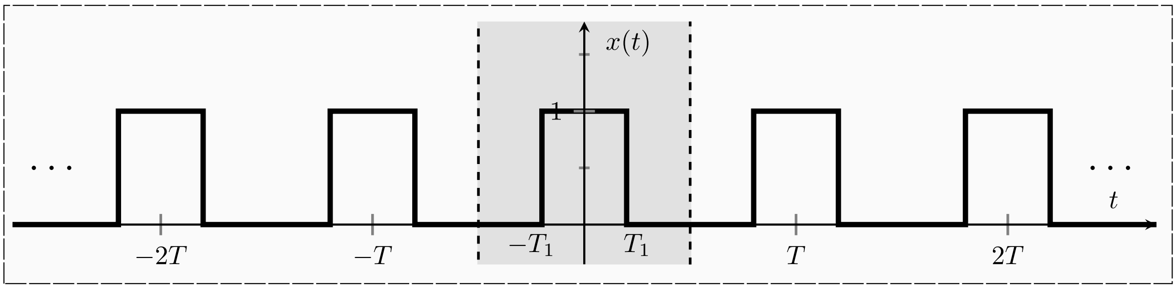


Figure 1: Periodic signal $x(t)$.

i) **[2 marks]** Determine the Fourier series coefficient

$$a_0 = \frac{1}{T}\int_T x(t)\, dt$$

of $x(t)$.

Solution:

$$a_0 = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} x(t)\, dt$$
$$= \frac{1}{T}\int_{-T_1}^{T_1} 1\, dt = \frac{2T_1}{T}.$$

□

ii) **[3 marks]** Determine the Fourier series of $x(t)$ using the analysis equation.

Solution:

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$$
$$= \frac{1}{T}\int_T e^{-jk\omega_0 t}\, dt$$
$$= \frac{\sin(k\omega_0 T_1)}{k\pi}$$

□

iii) **[2 marks]** Find the Fourier series coefficients of the signal $a(t) = x(t-1)$.

Solution: Let b_k denote the Fourier series coefficients of $a(t)$. Then using the time delay property of the Fourier transform:

$$b_k = a_k e^{-jk\omega_0 t_0} = \frac{\sin(k\omega_0 T_1)}{k\pi} e^{-jk\omega_0}$$

□

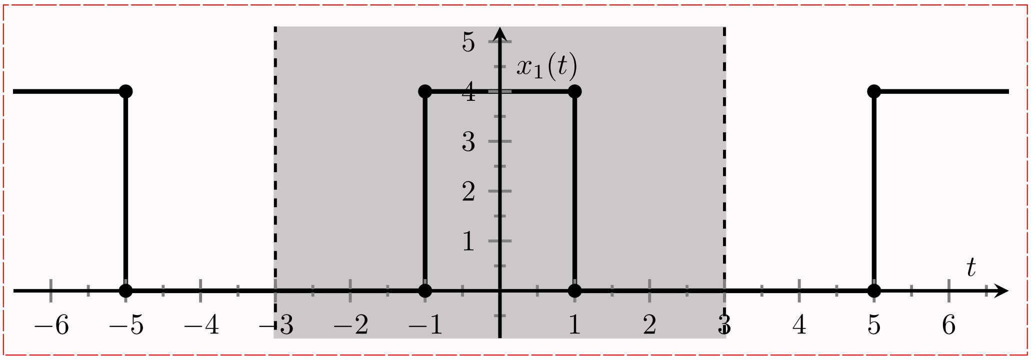
iv) **[2 marks]** Find the Fourier series coefficients of the signal $b(t) = x(t)\star a(t) = \int_T x(\tau)\, a(t-\tau)\, d\tau$.

Solution: Let c_k denote the Fourier series coefficients of $b(t)$. Using the convolution property of the Fourier transform:

$$c_k = T a_k b_k = T\left(\frac{\sin(k\omega_0 T_1)}{k\pi}\right)^2 e^{-jk\omega_0}$$

□

v) **[2 marks]** Using your answer from part (b) ii), determine the Fourier Series coefficients for the periodic signal $x_1(t)$:



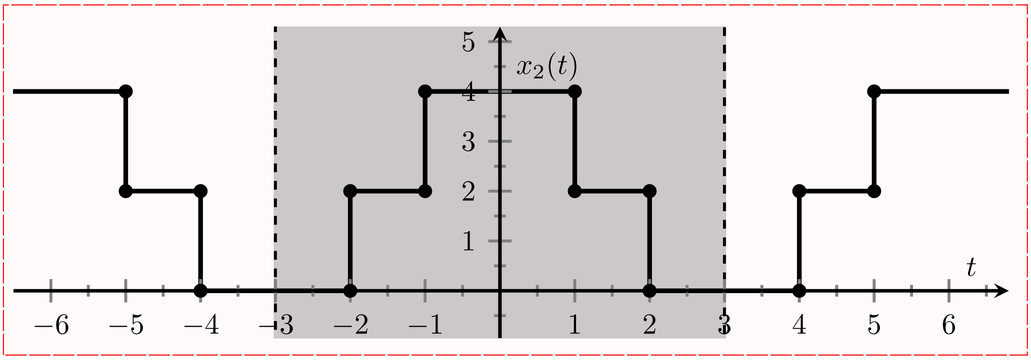
One period of width 6 has been shaded.

Solution: With $T_1 = 1$ and $T = 6$ we just substitute, and scale by 4:

$$a_k = 4\frac{\sin(k\pi/3)}{k\pi}.$$

□

vi) **[4 marks]** Determine the Fourier Series coefficients for the periodic signal $x_2(t)$:



One period of width 6 has been shaded.

Solution: By linearity we can add: $T_1 = 1$ rectangle of height 2 and $T_1 = 2$ rectangle of height 2. This is the simplest combination and requires no time shift.
We get

$$a_k = 2\frac{\sin(k\pi/3)}{k\pi} + 2\frac{\sin(2k\pi/3)}{k\pi}$$

or one of many other possible equivalent answers. □

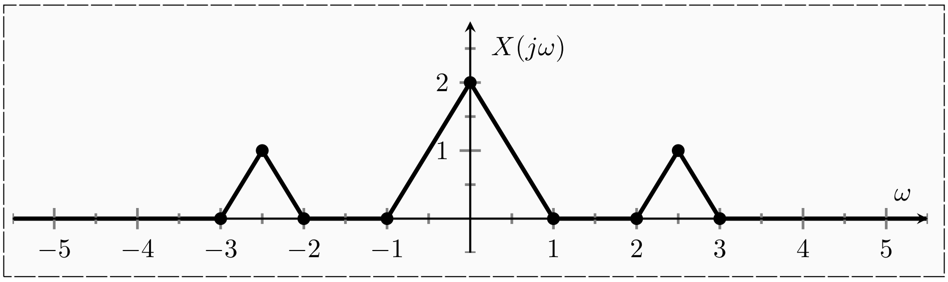


Figure 2: The Fourier transform $X(j\omega)$ of some CT signal $x(t)$.

Problem 3

(a) **[5 marks]** Consider a sampled version of the CT signal $x(t)$, with Fourier transform $X(j\omega)$, shown in Figure 2, given by the CT signal

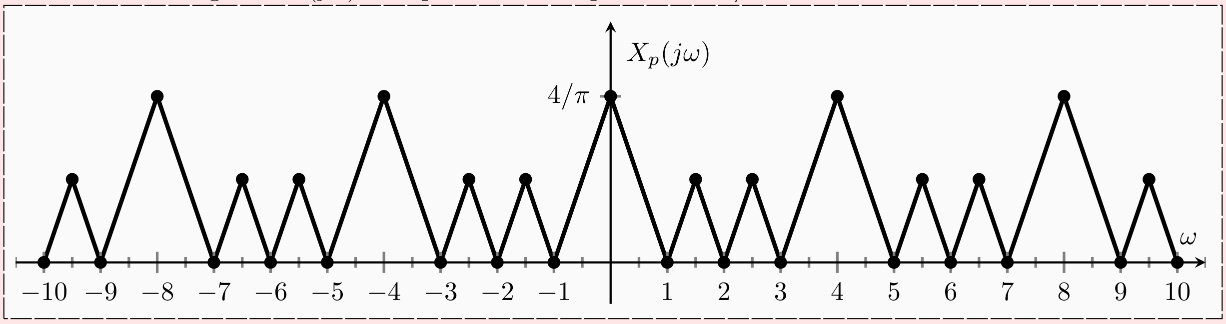
$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t - nT) \xleftrightarrow{\mathcal{F}} X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\big(j(\omega - k\,\omega_s)\big)$$

where T is the sampling interval in time and the sampling rate is given by $\omega_s = 2\pi/T$.

Draw $X_p(j\omega)$, the Fourier transform of $x_p(t)$, in the range $-10 \leq \omega \leq 10$ for the following values of the sampling rate $\omega_s = 2\pi/T$ (for each case copy and use the template in Figure 3 into your exam script book):

i) $\omega_s = 4$ rad/sec

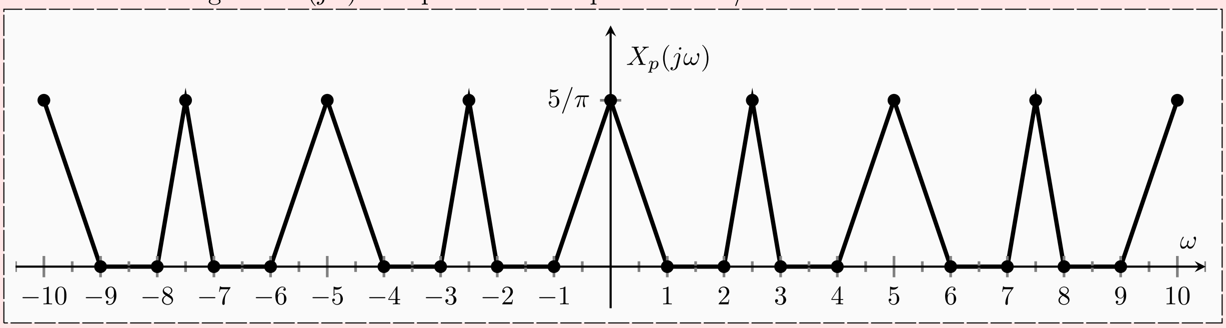
Solution: The images of $X(j\omega)$ are spaced at multiples of 4 rad/sec.



The peak value is $2/T = \omega_s/\pi = 4/\pi$. □

ii) $\omega_s = 5$ rad/sec

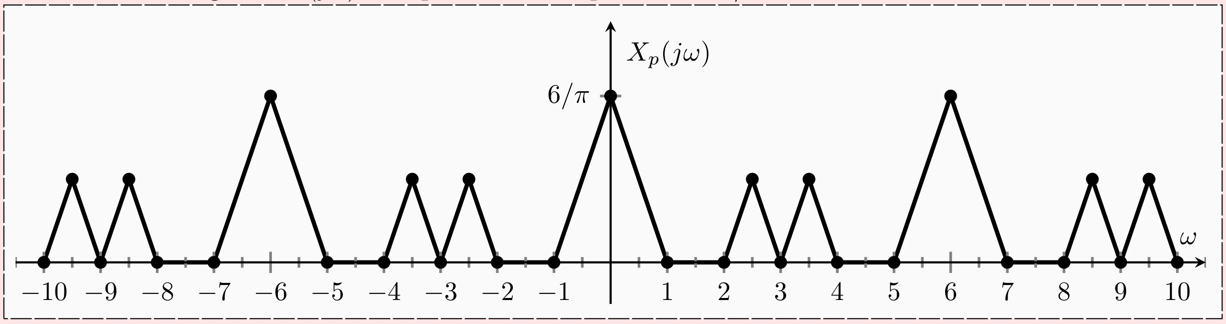
Solution: The images of $X(j\omega)$ are spaced at multiples of 5 rad/sec.



The peak value is $2/T = \omega_s/\pi = 5/\pi$. □

iii) $\omega_s = 6$ rad/sec

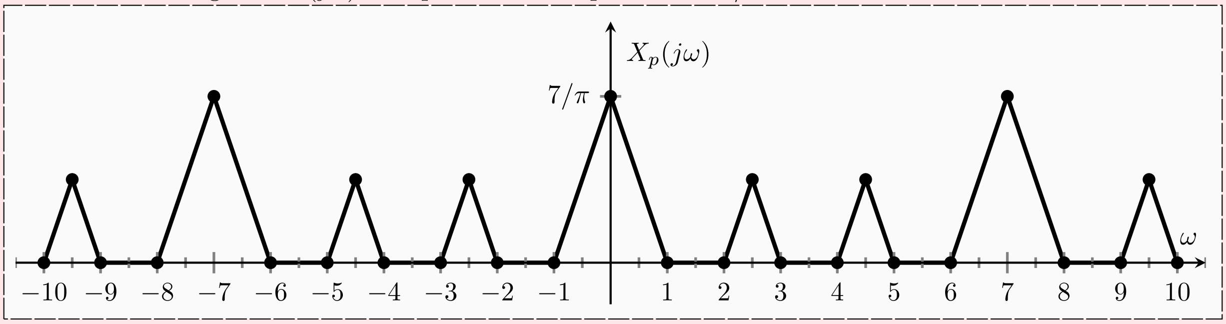
Solution: The images of $X(j\omega)$ are spaced at multiples of 6 rad/sec.



The peak value is $2/T = \omega_s/\pi = 6/\pi$. □

iv) $\omega_s = 7$ rad/sec

Solution: The images of $X(j\omega)$ are spaced at multiples of 7 rad/sec.



The peak value is $2/T = \omega_s/\pi = 7/\pi$. □

corresponding to $T = \pi/2,\ 2\pi/5,\ \pi/3,\ 2\pi/7$ sec.

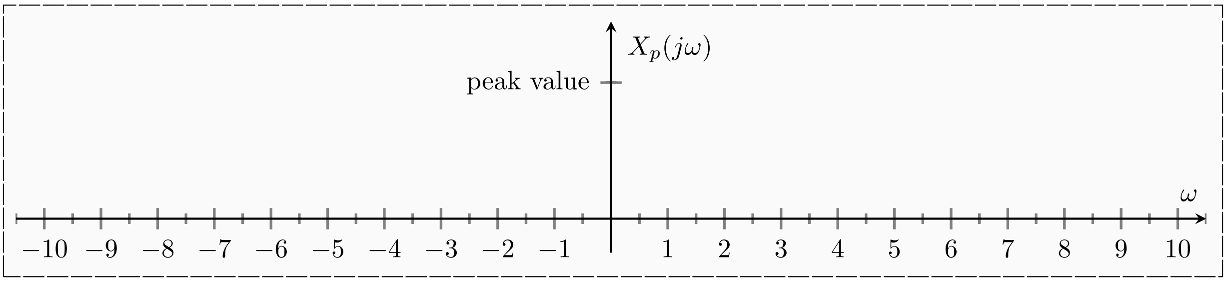


Figure 3: Template to draw $X_p(j\omega)$.

(b) [**7 marks**] Consider recovery of $x(t)$ from $x_p(t)$ using only an ideal low-pass filter whose cutoff or bandwidth is given by ω_c rad/sec. For each of the 4 values of the sampling rate from part (a) state whether recovery using only an ideal pass filter is possible (yes/no) and, if it is possible, what is the least bandwidth ω_c (in rad/sec), and greatest bandwidth ω_c (in rad/sec), that can be used to perfectly recover $x(t)$ from $x_p(t)$.

You can answer this question by copying and filling in the missing entries of the table below.

Sampling Rate ω_s	Recovery Possible?	Least ω_c	Greatest ω_c
4 rad/sec			
5 rad/sec			
6 rad/sec			
7 rad/sec			

In the case that recovery is not possible, then there are no values for least and greatest ω_c and you should indicate "n/a" for not applicable.

Solution:

Sampling Rate ω_s	**Recovery Possible?**	**Least** ω_c	**Greatest** ω_c
4 rad/sec	No	n/a	n/a
5 rad/sec	No	n/a	n/a
6 rad/sec	Yes	3	3
7 rad/sec	Yes	3	4

□

(c) [**2 marks**] Given recovery of $x(t)$ from $x_p(t)$ using only an ideal pass filter, what is the Nyquist sampling rate in rad/sec?

Solution: Clearly 6 rad/sec. □

(d) [**4 marks**] This part considers recovery of $x(t)$ from $x_p(t)$ but not restricted to using an ideal pass filter. In any of the four cases where recovery of $x(t)$ from $x_p(t)$ using only an ideal pass filter fails, indicate whether the signal can still be recovered and how (what combination of LTI filters could be used). Justify your answer.

Solution: When $\omega_s = 5$ rad/sec there is overlap, meaning aliasing and recovery is impossible. When $\omega_s = 4$ rad/sec there is no overlap (at the edges the values are zero and not a problem). So there is no irreversible loss of information. To recover then you cannot use a low pass filter but instead a filter whose gain is one for value of ω within the support of $X(j\omega)$, and zero otherwise. □

Problem 4

(a) **[2 marks]** The analysis and synthesis equations for a DT signal $x[n]$ and its Fourier transform $X(e^{j\omega})$ are given below:

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega, \quad n \in \mathbb{Z} \qquad \text{(Synthesis Equation)}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}, \quad \omega \in \mathbb{R} \qquad \text{(Analysis Equation)}$$

and can be represented as $x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega})$. Express in a few sentences your understanding of these two equations.

Solution: In the time-domain it shows the signal $x[n]$ can be built-up or synthesized by a superposition of complex exponentials.

In the frequency-domain it shows the frequency components, through analysis, that make-up the signal. □

(b) **[3 marks]** Using the identities in part (a), or otherwise, show:

$$x[n-n_0] \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega n_0} X(e^{j\omega}) \tag{1}$$

(c) **[2 marks]** Describe in words the physical meaning of the property (1), that is, what it means in the time-domain and what it means in the frequency-domain.

Solution: The Fourier transform of a time-shifted DT signal is equal to the Fourier transform of the original DT signal multiplied by a linear phase term, where the slope of the linear phase is equal to the time-shift.

Shifting in time corresponds to adding a linear phase factor in the Fourier transform (where the slope of the linear phase is equal to the time-shift). □

(d) **[4 marks]** Consider the causal LTI system characterized by the difference equation:

$$y[n] - \frac{3}{4}\, y[n-1] + \frac{1}{8}\, y[n-2] = 3\, x[n] \tag{2}$$

relating its input $x[n]$ to its output $y[n]$.

Using the property (1), or otherwise, determine an expression for the frequency response:

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}.$$

where $x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega})$ and $y[n] \overset{\mathcal{F}}{\longleftrightarrow} Y(e^{j\omega})$.

Solution: Directly we have

$$Y(e^{j\omega}) - \frac{3}{4}\, e^{-j\omega}\, Y(e^{j\omega}) + \frac{1}{8}\, e^{-j2\omega}\, Y(e^{j\omega}) = 3\, X(e^{j\omega})$$

which can be written

$$\Big(1 - \frac{1}{2}e^{-j\omega}\Big)\Big(1 - \frac{1}{4}e^{-j\omega}\Big) Y(e^{j\omega}) = 3\, X(e^{j\omega})$$

and hence

$$\boxed{H(e^{j\omega}) = \frac{3}{\big(1 - \frac{1}{2}e^{-j\omega}\big)\big(1 - \frac{1}{4}e^{-j\omega}\big)}.}$$

□

(e) **[1 mark]** What is the DC gain ($\omega = 0$) of the system in (2)?

Solution: The DC gain is

$$\boxed{H(e^{j0}) = H(1) = 3/(0.5 \times 0.75) = 8.}$$

□

(f) [**2 marks**] What is the gain at the maximum frequency ($\omega = \pi$) of the system in (2)?

Solution: The maximum frequency gain is

$$\boxed{H(e^{j\pi}) = H(-1) = 3/(1.5 \times 1.25) = 8/5.}$$

□

(g) [**4 marks**] Use a partial-fraction expansion to show the (unit) impulse response is

$$h[n] = 6\left(\frac{1}{2}\right)^n u[n] - 3\left(\frac{1}{4}\right)^n u[n]$$

The following DT Fourier transform pair should be useful

$$\alpha^n\, u[n] \xleftrightarrow{\mathcal{F}} \frac{1}{1 - \alpha e^{-j\omega}}, \quad |\alpha| < 1.$$

(end of exam paper)

Question	Points	Score
1	1	
2	1	
3	1	
4	1	
5	1	
6	1	
7	1	
8	1	
9	1	
10	1	
11	1	
12	1	
13	1	
14	1	
15	1	
16	1	
17	1	
18	1	
19	1	
20	1	
21	18	
22	18	
23	18	
24	18	
Total:	92	