Signal Processing
ENGN2228

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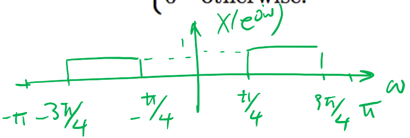
Second Semester

Lectures 31-32

Part 0 Slide 1/many Convenor: R. A. Kennedy

Examples:

$$X\left(e^{j\omega}\right) = \begin{cases} 1 & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0 & \text{otherwise.} \end{cases} \quad -\pi < \omega < \pi \quad x[n] = ?$$



$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-3\pi/4}^{-\pi/4} (1)\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\pi/4}^{3\pi/4} (1)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\frac{1}{jn}\left[e^{j\omega n}\right]_{-3\pi/4}^{-\pi/4} + \frac{1}{2\pi}\frac{1}{jn}\left[e^{j\omega n}\right]_{\pi/4}^{3\pi/4}$$

$$= \frac{1}{\pi n}\frac{1}{2j}\left[e^{-j\frac{\pi}{4}n} - e^{-j\frac{3\pi}{4}n}\right] + \frac{1}{\pi n}\frac{1}{2j}\left[e^{j\frac{3\pi}{4}n} - e^{j\frac{\pi}{4}n}\right] =$$

$$= \frac{1}{\pi n}\frac{-1}{2j}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right) + \frac{1}{\pi n}\frac{1}{2j}\left(e^{j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n}\right)$$

$$= \frac{1}{\pi n}\left[\sin\left(\frac{3\pi}{4}n\right) - \sin\left(\frac{\pi}{4}n\right)\right]$$

Examples:

$$X\left(e^{j\omega}\right) = \sum_{l=-\infty}^{\infty} \left\{2\pi\delta(\omega - 2\pi l) + \pi\delta(\omega - \frac{\pi}{2} - 2\pi l) + \pi\delta(\omega + \frac{\pi}{2} - 2\pi l)\right\} \quad -\infty < \omega < \infty \quad x[n] = ?$$

$$l=0 \quad X(e^{j\omega}) = 2\pi\delta(\omega) + \pi\delta(\omega - \pi/2) + \pi\delta(\omega + \pi/2) \quad -\pi < \omega < \pi$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} 2\pi\delta(\omega) e^{j\omega n} + \int_{-\pi}^{\pi} \pi\delta(\omega - \pi/2) e^{j\omega n} d\omega + \int_{-\pi}^{\pi} \pi\delta(\omega + \pi/2) e^{j\omega n} d\omega\right]$$

$$= \frac{1}{\cancel{2\pi}}\left[\cancel{2\pi}\, \underbrace{\cancel{e^{j(0)n}}}_{1} + \cancel{\pi}\, e^{j\frac{\pi}{2}n} + \cancel{\pi}\, e^{-j\frac{\pi}{2}n}\right] = \frac{e^{j\pi/2 n} + e^{-j\pi/2 n}}{2} + 1$$

$$= \cos\left(\frac{\pi}{2} n\right) + 1$$

Examples:

$$X\left(e^{j\omega}\right) = \frac{2}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)^2} \quad x[n] = ?$$

partial fraction expansion

let $v = e^{-j\omega}$

$$X(v) = \frac{2}{(1-\frac{1}{2}v)(1-\frac{1}{4}v)^2} = \frac{A}{(1-\frac{1}{2}v)} + \frac{B}{(1-\frac{1}{4}v)^2} + \frac{C}{(1-\frac{1}{4}v)}$$

$$A = (1-\tfrac{1}{2}v)X(v)\Big|_{v=2} = \frac{2}{(1-\frac{1}{4}v)^2}\Big|_{v=2} = \frac{2}{(\frac{1}{2})^2} = 8$$

$$B = (1-\tfrac{1}{4}v)^2 X(v)\Big|_{v=4} = \frac{2}{(1-\frac{1}{2}v)}\Big|_{v=4} = \frac{2}{1-2} = -2$$

$$C = \frac{1}{(2-1)!}(-4)^{2-1}\frac{d}{dv}\left[(1-\tfrac{1}{4}v)^2 X(v)\right]\Big|_{v=4} = (-4)\frac{d}{dv}\left(\frac{2}{1-\frac{1}{2}v}\right)\Big|_{v=4}$$

$$\frac{d(A/B)}{dx} = \frac{B\frac{dA}{dx} - A\frac{dB}{dx}}{B^2}$$

$$C = \left.\frac{-4\left(1-\frac{1}{2}r\right)\frac{d(2)}{dr} - 2\frac{d}{dr}\left(1-\frac{1}{2}r\right)}{\left(1-\frac{1}{2}r\right)^2}\right|_{r=4} = \left.\frac{-4\left(0-2\left(-\frac{1}{2}\right)\right)}{\left(1-\frac{1}{2}r\right)^2}\right|_{r=4} = -4$$

$$X(e^{j\omega}) = \frac{8}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2} - \frac{4}{\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

$$* \quad \frac{1}{1-ae^{-j\omega}} \longleftrightarrow a^n u[n]$$

$$* \quad \frac{1}{\left(1-ae^{-j\omega}\right)^2} \longleftrightarrow (n+1)a^n u[n]$$

$$\frac{8}{1-\frac{1}{2}e^{-j\omega}} \longleftrightarrow 8\left(\frac{1}{2}\right)^n u[n]$$

$$\frac{-4}{1-\frac{1}{4}e^{-j\omega}} \longleftrightarrow (-4)\left(\frac{1}{4}\right)^n u[n]$$

$$\frac{-2}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2} \longleftrightarrow (-2)(n+1)\left(\frac{1}{4}\right)^n u[n]$$

$$x[n] = 8\left(\frac{1}{2}\right)^n u[n] - 4\left(\frac{1}{4}\right)^n u[n] - 2(n+1)\left(\frac{1}{4}\right)^n u[n]$$

Examples:

$$X\left(e^{j\omega}\right) = 1 + 2e^{-j\omega} + \underbrace{\frac{\frac{1}{6}e^{-j\omega}}{1+\frac{5}{6}e^{-j\omega}+\frac{1}{6}e^{-j2\omega}}}_{X_1(e^{-j\omega})} \qquad x[n] = ?$$

let $r = e^{j\omega}$

$$X_1(r) = \frac{1/6\, r}{1+5/6\, r + 1/6\, r^2} = \frac{1/6\, r}{(1+1/2\, r)(1+1/3\, r)} = \frac{A}{(1+1/2\, r)} + \frac{B}{(1+1/3\, r)}$$

$$A = (1+1/2\, r)X_1(r)\Big|_{r=-2} = \frac{1/6\, r}{1+1/3\, r}\Big|_{r=-2} = \frac{-1/3}{1-2/3} = -1$$

$$B = (1+1/3\, r)X_1(r)\Big|_{r=-3} = \frac{1/6\, r}{1+1/2\, r}\Big|_{r=-3} = \frac{-1/2}{1-3/2} = 1$$

$$X(e^{j\omega}) = 1 + 2e^{-j\omega} - \frac{1}{1+\frac{1}{2}e^{-j\omega}} + \frac{1}{1+\frac{1}{3}e^{-j\omega}}$$

* $1 \longleftrightarrow \delta[n]$

* $e^{-j\omega n_0} \longleftrightarrow \delta[n-n_0]$

* $\dfrac{1}{1-ae^{-j\omega}} \longleftrightarrow a^n u[n]$

$$x[n] = \delta[n] + 2\,\delta[n-1] - \left(-\tfrac{1}{2}\right)^n u[n] + \left(-\tfrac{1}{3}\right)^n u[n]$$

DTFT Properties:

- **Convolution:** `O&W 5.4 pp.382-388`

$$\boxed{y[n] = h[n] \star x[n] \xleftrightarrow{\mathscr{F}} Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})}$$

Frequency Response $H(e^{j\omega})$ is the DTFT of the unit sample response.

Convolution using DTFT:

Find the convolution $y[n] = x[n] * h[n]$ where

$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$

$h[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$

$* \ \delta[n-n_0] \longleftrightarrow e^{-j\omega n_0}$

$* \ \delta[n] \longleftrightarrow 1$

$X(e^{j\omega}) = 1 + 2e^{-j\omega} - e^{-j2\omega}$

$H(e^{j\omega}) = 1/3 + 1/3\, e^{-j\omega} + 1/3\, e^{-j2\omega}$

$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$

$$Y(e^{j\omega}) = (1 + 2e^{-j\omega} - e^{-j2\omega})(1/3 + 1/3\, e^{-j\omega} + 1/3\, e^{-j2\omega})$$

$$= 1/3 + 1/3\, e^{-j\omega} + 1/3\, e^{-j2\omega} + 2/3\, e^{-j\omega} + 2/3\, e^{-j2\omega} + 2/3\, e^{-j3\omega} - 1/3\, e^{-j2\omega} - 1/3\, e^{-j3\omega} - 1/3\, e^{-j4\omega}$$

$$= 1/3 + e^{-j\omega} + 2/3\, e^{-j2\omega} + 1/3\, e^{-j3\omega} - 1/3\, e^{-j4\omega}$$

$$y[n] = 1/3\,\delta[n] + \delta[n-1] + 2/3\,\delta[n-2] + 1/3\,\delta[n-3] - 1/3\,\delta[n-4]$$

Example:

Determine the convolution $y[n] = x[n] * h[n]$ if

$x[n] = \left(\frac{1}{4}\right)^n u[n]$

$h[n] = \frac{1}{6}[3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n]u[n]$

$$* \quad a^n u[n] \longleftrightarrow \frac{1}{1 - a e^{-j\omega}}$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$X(e^{j\omega}) = \frac{1}{1 - (1/4) e^{-j\omega}}$$

$$h[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] - \frac{1}{3}\left(\frac{1}{3}\right)^n u[n]$$

$$H(e^{j\omega}) = \frac{1/2}{1 - \frac{1}{2} e^{-j\omega}} - \frac{1/3}{1 - \frac{1}{3} e^{-j\omega}} = \frac{\frac{1}{2}\left(1 - \frac{1}{3} e^{-j\omega}\right) - \frac{1}{3}\left(1 - \frac{1}{2} e^{-j\omega}\right)}{\left(1 - \frac{1}{2} e^{-j\omega}\right)\left(1 - \frac{1}{3} e^{-j\omega}\right)}$$

$$= \frac{1/6}{\left(1 - \frac{1}{2} e^{-j\omega}\right)\left(1 - \frac{1}{3} e^{-j\omega}\right)}$$

$$Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \frac{1/6}{(1-\frac{1}{2}e^{-jw})(1-\frac{1}{3}e^{-jw})(1-\frac{1}{4}e^{-jw})}$$

let $r = e^{-jw}$

$$\frac{1/6}{(1-\frac{1}{2}r)(1-\frac{1}{3}r)(1-\frac{1}{4}r)} = \frac{A}{(1-\frac{1}{2}r)} + \frac{B}{(1-\frac{1}{3}r)} + \frac{C}{(1-\frac{1}{4}r)}$$

$$A = \frac{1/6}{(1-\frac{1}{3}r)(1-\frac{1}{4}r)}\Bigg|_{r=2} = \frac{1/6}{(1-2/3)(1-1/2)} = \frac{1/6}{\frac{1}{3}\,\frac{1}{2}} = 1$$

$$B = \frac{1/6}{(1-\frac{1}{2}r)(1-\frac{1}{4}r)}\Bigg|_{r=3} = \frac{1/6}{(1-3/2)(1-3/4)} = \frac{1/6}{(-1/2)(1/4)} = -4/3$$

$$C = \frac{1/6}{(1-\frac{1}{2}r)(1-\frac{1}{3}r)}\Bigg|_{r=4} = \frac{1/6}{(1-2)(1-4/3)} = \frac{1/6}{(-1)(-1/3)} = 1/2$$

$$Y(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} - \frac{4/3}{1-\frac{1}{3}e^{-j\omega}} + \frac{1/2}{1-\frac{1}{4}e^{-j\omega}}$$

$$y[n] = \left(\tfrac{1}{2}\right)^n u[n] - \tfrac{4}{3}\left(\tfrac{1}{3}\right)^n u[n] + \tfrac{1}{2}\left(\tfrac{1}{4}\right)^n u[n]$$

Good luck if you want to work it out in time-domain!

DT LTI System Representation:

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$\delta[n] \longrightarrow \boxed{h[n]} \longrightarrow h[n]$$

DT LTI systems can be described via:

- Direct-form I implementation (block diagram)
- Difference equation
- Frequency response $H\left(e^{j\omega}\right)$
- Impulse response $h[n]$
- Output $y[n]$ and input $x[n]$ are given

$$y[n] = x[n] * h[n]$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

Note: (1) using $h[n]$ we can check stability and causality.
(2) using $h[n]$ we can determine if the DT system is IIR or FIR.

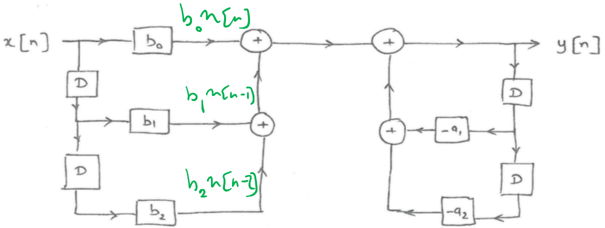
Direct-Form I implementation:

2nd order Difference Equations

$$y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

Rearranging

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] - a_1 y[n-1] - a_2 y[n-2]$$



DT System Properties – Causality Property

For DT LTI Systems the **Causality Property** can be written:

Theorem (Causal DT LTI System)

A DT LTI system is **causal** *if and only if its pulse response, $h[n]$, satisfies*

$$h[n] = 0, \quad \textit{for all } n < 0.$$

- If $h[n] \neq 0$ for at least one $n = -n_0$ $(n_0 > 0)$ then the output at time n, $y[n]$, would contain term

$$h[-n_0]\, x[n + n_0],$$

for example, if $n_0 = 1$ and $h[-1] = 2$ then $y[n] = \cdots + h[-1]\, x[n + 1] + \cdots$, and hence would not be causal.

Stability: a bounded input $x[n]$ produces a bounded output $y[n]$.

Definition (DT LTI System Stability)

A DT LTI system is **stable**, with pulse respond $h[n]$, if and only if

$$\sum_{n=-\infty}^{\infty} \left|h[n]\right| < \infty$$

is bounded if and only if the input is bounded.

Finite Impulse Response (FIR): `O&W 2.4.2 p.122`

A DT system with a finite length impulse response (total duration) is called a Finite Impulse Response (FIR) filter. Just need a finite number of parameters to describe it. Ideally suited to digital signal processing implementation.

Three examples:

$$y_1[n] = x[n]$$
$$y_2[n] = x[n+1] - 3x[n] + 7x[n-89]$$
$$y_3[n] = \sum_{k=0}^{100} 0.5^k x[n-k]$$

They have total durations 1, 91 and 101 (which are clearly finite), respectively.

Infinite Impulse Response (IIR): `O&W 2.4.2 p.123`

If a DT system is not FIR then it is Infinite Impulse Response (IIR).

For example,

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{n} a^{n-k} x[k], \quad |a| < 1, \quad n \in \mathbb{Z} \\ &= \sum_{k=0}^{\infty} a^{k} x[n-k], \quad |a| < 1, \quad n \in \mathbb{Z} \end{aligned}$$

However, IIR doesn't necessarily mean infinite complexity. In the next few slides we see why.

DT Fourier Stuff – Miscellanea

LCC Difference Equations: `O&W 5.8 pp.396-399`

Linear constant coefficient difference equation (LCCDEs)

$$\boxed{\sum_{k=0}^{N} a_k \, y[n-k] = \sum_{k=0}^{M} b_k \, x[n-k]}$$

where we can interpret this as describing a DT-LTI system

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

for some impulse response $h[n]$. (Here $x[n]$ is the input and $y[n]$ is the output.) We can infer this system response; well at least its frequency response. That is, we can find the frequency response $H(e^{j\omega})$ corresponding to the LCCDE.

DT Fourier Stuff – Miscellanea

The LCCDE can be simplified by using

$$\boxed{x[n-k] \overset{\mathscr{F}}{\longleftrightarrow} e^{-j\omega k} X(e^{j\omega}), \quad k \in \mathbb{Z}}$$

By taking the Fourier Transform of both sides of:

$$\boxed{\sum_{k=0}^{N} a_k \, y[n-k] = \sum_{k=0}^{M} b_k \, x[n-k]}$$

we get:

$$\boxed{\sum_{k=0}^{N} a_k e^{-jk\omega} \, Y(e^{j\omega}) = \sum_{k=0}^{M} b_k e^{-jk\omega} \, X(e^{j\omega})}$$

This type of transformation is directly analogous of what we did for differential equations.

DT Fourier Stuff – Miscellanea

$$Y(e^{j\omega}) = \frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{K} a_k e^{-jk\omega}} X(e^{j\omega})$$

$b = [b_0, b_1, \ldots]$

$a = [a_0, a_1, \ldots]$

So

$$\boxed{H(e^{j\omega}) = \frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{K} a_k e^{-jk\omega}}}$$

freqz(b, a) = H

is the frequency response of the linear, constant coefficient difference equation system.

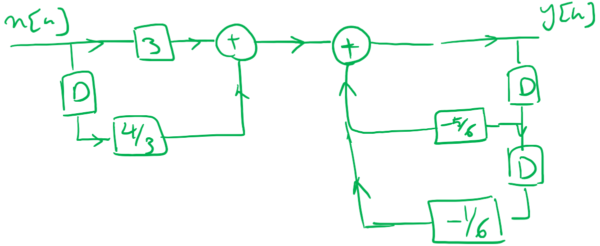
$H(e^{j\omega})$ is a rational function of $e^{-jk\omega}$ amenable to the use of partial fraction expansions.

Example 1:

$y[n-2] + 5y[n-1] + 6y[n] = 18x[n] + 8x[n-1]$, Direct form I implementation? $H(e^{j\omega})$ and $h(n)$? Stable? Causal?

$6y[n] = -y[n-2] - 5y[n-1] + 18x[n] + 8x[n-1]$

$y[n] = 3x[n] + \frac{4}{3}x[n-1] - \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2]$



$$y[n-2]+5y[n-1]+6y[n]=18x[n]+8x[n-1]$$

$$* \quad z[n-n_0] \longleftrightarrow e^{-j\omega n_0} Z(e^{j\omega})$$

Taking DTFT of both sides

$$e^{-j2\omega} Y(e^{j\omega}) + 5e^{-j\omega} Y(e^{j\omega}) + 6Y(e^{j\omega}) = 18X(e^{j\omega}) + 8e^{-j\omega} X(e^{j\omega})$$

$$(e^{-2j\omega} + 5e^{-j\omega} + 6)\, Y(e^{j\omega}) = (18 + 8e^{-j\omega})\, X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{18 + 8e^{-j\omega}}{e^{-2j\omega} + 5e^{-j\omega} + 6}$$

let $v = e^{-jw}$.

$$H(v) = \frac{18+8v}{v^2+5v+6} = \frac{18+8v}{6\left(1+\frac{5}{6}v+\frac{1}{6}v^2\right)} = \frac{3+\frac{4}{3}v}{\left(1+\frac{1}{2}v\right)\left(1+\frac{1}{3}v\right)} = \frac{A}{1+\frac{1}{2}v} + \frac{B}{1+\frac{1}{3}v}$$

$$A = \left.\frac{3+\frac{4}{3}v}{1+\frac{1}{3}v}\right|_{v=-2} = \frac{3-\frac{8}{3}}{1-\frac{2}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

$$B = \left.\frac{3+\frac{4}{3}v}{1+\frac{1}{2}v}\right|_{v=-3} = \frac{3-4}{1-\frac{3}{2}} = \frac{-1}{-\frac{1}{2}} = 2$$

$$H(e^{jw}) = \frac{1}{1+\frac{1}{2}e^{-jw}} + \frac{2}{1+\frac{1}{3}e^{-jw}}$$

Using $a^n u[n] \longleftrightarrow \dfrac{1}{1-ae^{-j\omega}}$

$h[n] = \left(-\frac{1}{2}\right)^n u[n] + 2\left(-\frac{1}{3}\right)^n u[n]$ (IIR-filter)

the system is causal $u[n]=0$ for $n<0$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left|\left(-\tfrac{1}{2}\right)^n\right| + 2\sum_{n=0}^{\infty} \left|\left(-\tfrac{1}{3}\right)^n\right|$$

$$\ast \sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad |\alpha|<1$$

$$\frac{1}{1-\left(\frac{1}{2}\right)} + \frac{2}{1-\left(\frac{1}{3}\right)} = 2+3=5 < \infty$$

DT LTI system is stable.

Example 2:

A DTI system has impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{1}{4}\right)^n u[n]$

Find the difference equation.

$$* \; a^n u[n] \longleftrightarrow \frac{1}{1 - a e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2} e^{-j\omega}} + \frac{\frac{1}{2}}{1 - \frac{1}{4} e^{-j\omega}} = \frac{\frac{3}{2} - \frac{1}{2} e^{-j\omega}}{1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-j\omega}}$$

we know $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{\frac{3}{2} - \frac{1}{2} e^{-j\omega}}{1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-j\omega}}$

$$Y(e^{j\omega}) - \frac{3}{4} e^{-j\omega} Y(e^{j\omega}) + \frac{1}{8} e^{-j2\omega} Y(e^{j\omega}) = \frac{3}{2} X(e^{j\omega}) - \frac{1}{2} e^{-j\omega} X(e^{j\omega})$$

$$* \; e^{-j\omega n_0} Y(e^{j\omega}) \longleftrightarrow y[n - n_0]$$

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = \frac{3}{2}x[n] - \frac{1}{2}x[n-1]$$

the DT LTI system is causal because of u[n]

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left|\left(\frac{1}{2}\right)^n\right| + \frac{1}{2}\sum_{n=0}^{\infty} \left|\left(\frac{1}{4}\right)^n\right| = \frac{1}{1-\frac{1}{2}} + \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{8}{3} < \infty$$

The DT LTI system is stable.