Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

CLab–3: "DTFT and Frequency Response of Discrete Time Systems"

Lab Week: Week 10

Total Marks: 10

Contribution to Final Assessment: 2%

Submission: Marked by tutors during the lab time based on completion of the lab tasks.

Relevant Textbook Sections: 5.3, 5.4, 5.5, 5.6, and 5.8.

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1 Learning Outcomes

After completing this lab, the student should be able to:

- implement a moving average filter, as an example of an LTI systems characterized by a difference equation.
- demonstrate an understanding of the frequency-domain systems analysis using magnitude/phase plots and zero-pole plots.

2 Discrete-Time Fourier Transform (DTFT)

The discrete-time Fourier transform (DTFT) of a discrete-time signal $x[n]$ is defined as

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}. \tag{1}$$

Task 1

1. For $x[n] = (0.5)^n u[n]$, use MATLAB to evaluate $X(e^{j\omega})$ at 501 equispaced points between $[0, \pi]$ and plot its: magnitude, phase, real part and imaginary part.
2. Repeat step 1 for $x[n] = \{1, \underset{\uparrow}{2}, 3, 4, 5\}$. By this notation, the arrow indicates $x[0]$, that is to say,

$$x[-1] = 1, \qquad x[0] = 2, \qquad x[1] = 3, \qquad x[2] = 4, \qquad x[3] = 5.$$

3. Let $x[n] = 0.9^n e^{j\pi n/3}, 0 \leq n \leq 10$. Determine $X(e^{j\omega})$ and investigate its periodicity. Use 401 equispaced points between $[-2\pi, 2\pi]$. What can you say about its symmetry? Is it conjugate symmetric?
4. Repeat step 3 for $x[n] = 0.9^n, -10 \leq n \leq 10$.

3 LTI System Characterized by Difference Equation

The class of LTI discrete-time systems with which we shall be mostly concerned in this exercise is characterized by a linear constant-coefficient difference equation of the form

$$\sum_{k=0}^{N} d_k y[n-k] = \sum_{k=0}^{M} p_k x[n-k], \tag{2}$$

where $x[n]$ and $y[n]$ are, respectively, the input and the output of the system, and $\{d_k\}$ and $\{p_k\}$ are constants. The order of the discrete-time system is $\max(N, M)$, which is the order of the difference equation characterizing the system. If we assume the system to be causal, then we can rewrite Eq. (2) to express $y[n]$ explicitly as a function of $x[n]$:

$$y[n] = -\sum_{k=1}^{N} \frac{d_k}{d_0} y[n-k] + \sum_{k=0}^{M} \frac{p_k}{d_0} x[n-k], \tag{3}$$

provided $d_0 \neq 0$. The output $y[n]$ can be computed using Eq. (**??**) for all $n \geq n_0$ knowing $x[n]$ and the initial conditions $y[n_0-1], y[n_0-2], ..., y[n_0-N]$.

To implmenet (2) in Matlab, we use `filter` as follows:

```
num = [p0 p1 . . . pM];
den = [d0 d1 . . . dN];
y=filter(num,den,x,ic);
```

where `ic` is an array containing the initial conditions $y[-1], ..., y[-N]$.

Task 2

We want to implement an M-point moving average filter defined by

$$y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x[n-k]. \tag{4}$$

```
% Task 1
% Simulation of an M-point Moving Average Filter
% Generate the input signal
n = 0:100;
s1 = cos(2*pi*0.05*n); % A low frequency sinusoid
s2 = cos(2*pi*0.47*n); % A high frequency sinusoid
x = s1+s2;
% Implementation of the moving average filter
M = input('Desired length of the filter = ');
num = ones(1,M);
y = filter(num,1,x)/M;
% Display the input and output signals
clf;
subplot(2,2,1);
plot(n,s1);
axis([0, 100, -2, 2]);
xlabel('Time index n'); ylabel('Amplitude');
title('Signal # 1');
subplot(2,2,2);
plot(n,s2);
axis([0, 100, -2, 2]);
```

```
xlabel('Time index n'); ylabel('Amplitude');
title('Signal # 2');
subplot(2,2,3);
plot(n,x);
axis([0, 100, -2, 2]);
xlabel('Time index n'); ylabel('Amplitude');
title('Input Signal');
subplot(2,2,4);
plot(n,y);
axis([0, 100, -2, 2]);
xlabel('Time index n'); ylabel('Amplitude');
title('Output Signal');
axis;
```

1. Without running the program, try to understand the code above, and write down in your own words what the code is doing.
2. Run the above program for `M = 2` to generate the output signal with `x[n] = s1[n]+ s2[n]` as the input. Which component of the input `x[n]` is suppressed by the discrete-time system simulated by this program?
3. If the LTI system is changed from `y[n]=0.5(x[n]+x[n-1])` to `y[n] =0.5(x[n]-x[n-1])`, what would be its effect on the input `x[n]=s1[n]+s2[n]`?
4. Run the program for other values of filter length `M`, and various values of the frequencies of the sinusoidal signals `s1[n]` and `s2[n]`. Comment on your results.

4 Transfer Function and Frequency Response of LTI systems

The transfer function of the LTI system characterized by the difference equation given in (2) is given by

$$H(e^{j\omega}) \triangleq \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{k=0}^{M} p_k e^{-jk\omega}}{\sum_{k=0}^{M} d_k e^{-jk\omega}} \tag{5}$$

We can use this transfer function to plot the frequency response of the system by setting discrete values of ω using the command `freqz`.

Task 3

1. Modify the code in **Task 1** to compute and plot the magnitude and phase spectra of a moving average filter of (**??**) for three different values of length M and for $0 \leq \omega \leq 2\pi$. Justify the type of symmetries exhibited by the magnitude and phase spectra. What type of filter does it represent? Can you now explain the results of Question 2 in Task 1?
2. Using the modified Program, compute and plot the frequency response of a causal LTI discrete-time system with a transfer function given by
$$H_1(e^{j\omega}) = \frac{0.15\left(1 - e^{-j2\omega}\right)}{1 - 0.5e^{-j\omega} + 0.7e^{-j2\omega}}.$$
where $0 \leq \omega \leq \pi$. What type of filter is this?
3. Repeat step 2 for
$$H_2(e^{j\omega}) = \frac{0.15\left(1 - e^{-j2\omega}\right)}{0.7 - 0.5e^{-j\omega} + e^{-j2\omega}}.$$
What is the difference between the two filters in $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$? Would you prefer to use one of them over the other? Why?