Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

ASSIGNMENT 2

Due Date: Friday, 27 October 2017, 11:55 PM (Week 12)

Late Submission Policy: Submit by the due date and time. Late assessment is *not accepted* for this course. That is, late submissions will get 0 marks.

This policy is to support the majority of students who complete and submit on time. This hard deadline enables the quick release of the assignment solutions.

Wattle submission: Upload your report as PDF format as a *multipage, single file* in Wattle. Use a scanner or scan-type smartphone app to create the PDF from your handwritten solutions.

Assignment format: 7 problems, for a total of 100 marks.

Value: 8% of total course assessment.

Solution: Will be posted on Wattle by Saturday, 28 October 2017. Marked assignments will be returned back in Wattle within 10–14 days.

Relationship to textbook: This assignment is related to Chapters 3–5 in the textbook, and Problem Sets 4 and 5. It is intended to aid you in your preparation for the final exam.

Declaration by the Student: All assessment task submissions, regardless of mode of submission, require agreement to the following declaration by the student:
"I declare that this work:

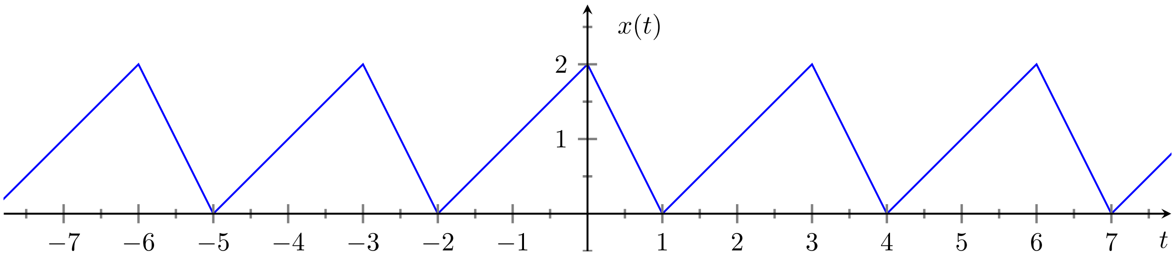
- upholds the principles of academic integrity, as defined in the University Academic Misconduct Rules;
- is original, except where collaboration (for example group work) has been authorised in writing by the course convener in the course outline and/or Wattle site;
- is produced for the purposes of this assessment task and has not been submitted for assessment in any other context, except where authorised in writing by the course convener;
- gives appropriate acknowledgement of the ideas, scholarship and intellectual property of others insofar as these have been used;
- in no part involves copying, cheating, collusion, fabrication, plagiarism or recycling."

Release Date: Monday, 31 July 2017 (Monday Week 2)

In the following: $\delta[n]$ and $u[n]$ represent the Dirac and unit step functions for discrete-time (DT). Similarly $\delta(t)$ and $u(t)$ for continuous-time (CT). Convolution of signals is written $x[n] \star h[n]$ or $x(t) \star h(t)$. Please indicate any identities or formulas used in the simplification of the results. You must show all steps taken to arrive at your answers.

Problem 1   7 marks

Determine the Fourier Series representation of the periodic CT signal in the following figure:



Problem 2   5 marks

The Fourier Series coefficients of a continuous time signal with period 4 is specified as

$$a_k = \begin{cases} j\,k, & |k| < 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine the signal $x(t)$:

Problem 3

For each of the following pulse responses shown in the figures:

(i) give an expression for the pulse response $h[n]$,

(ii) the frequency response

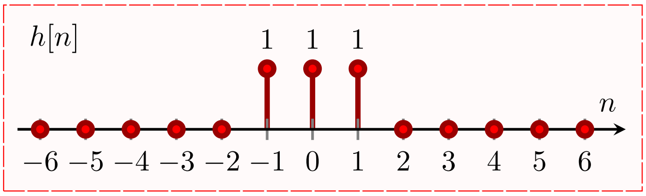
$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jn\omega},$$

(iii) say whether $H(e^{j\omega})$ as a function of ω is even or odd

(iv) discuss the phase of $H(e^{j\omega})$ and by looking at the slope say what the (group) delay is

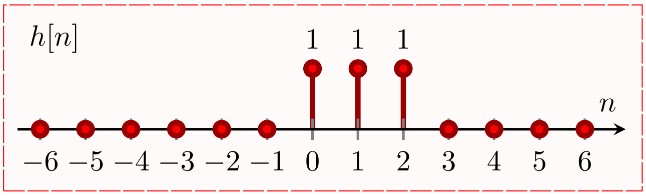
(v) and sketch/plot the magnitude $\left|H(e^{j\omega})\right|$

(a) **[2 marks]**



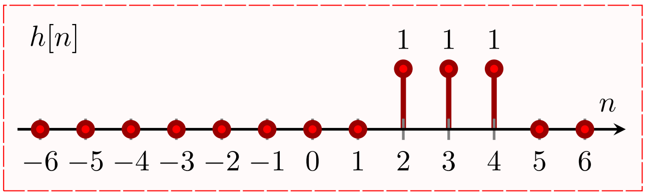
Real and even.

(b) **[2 marks]**



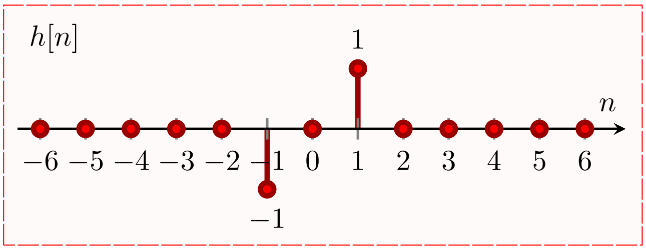
Real and causal.

(c) **[2 marks]**



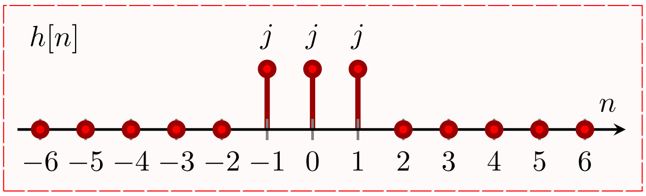
Real with larger delay.

(d) **[2 marks]**



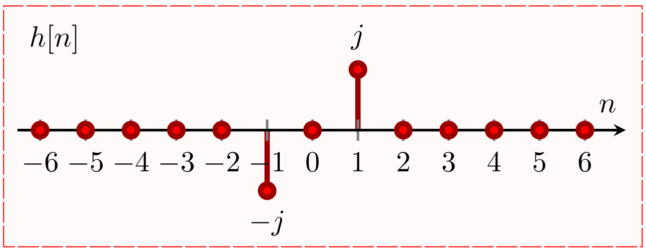
Real and odd.

(e) **[2 marks]**



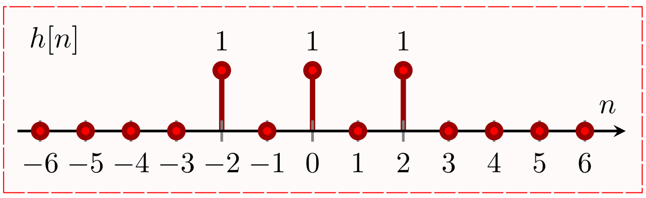
Imaginary and even.

(f) **[2 marks]**



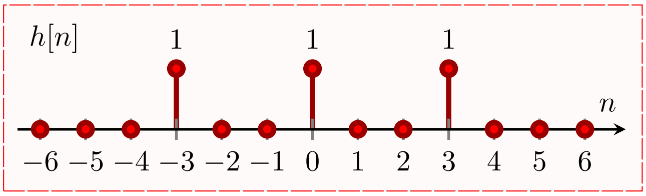
Imaginary and odd.

(g) **[2 marks]**



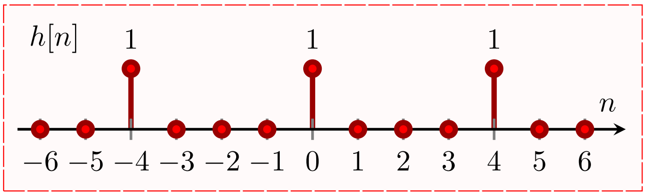
Real, even & spaced out.

(h) **[2 marks]**



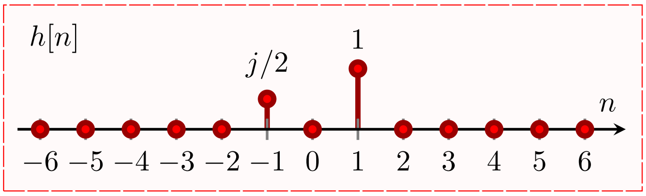
Real, even & spaced out.

(i) **[2 marks]**



Real, even & spaced out.

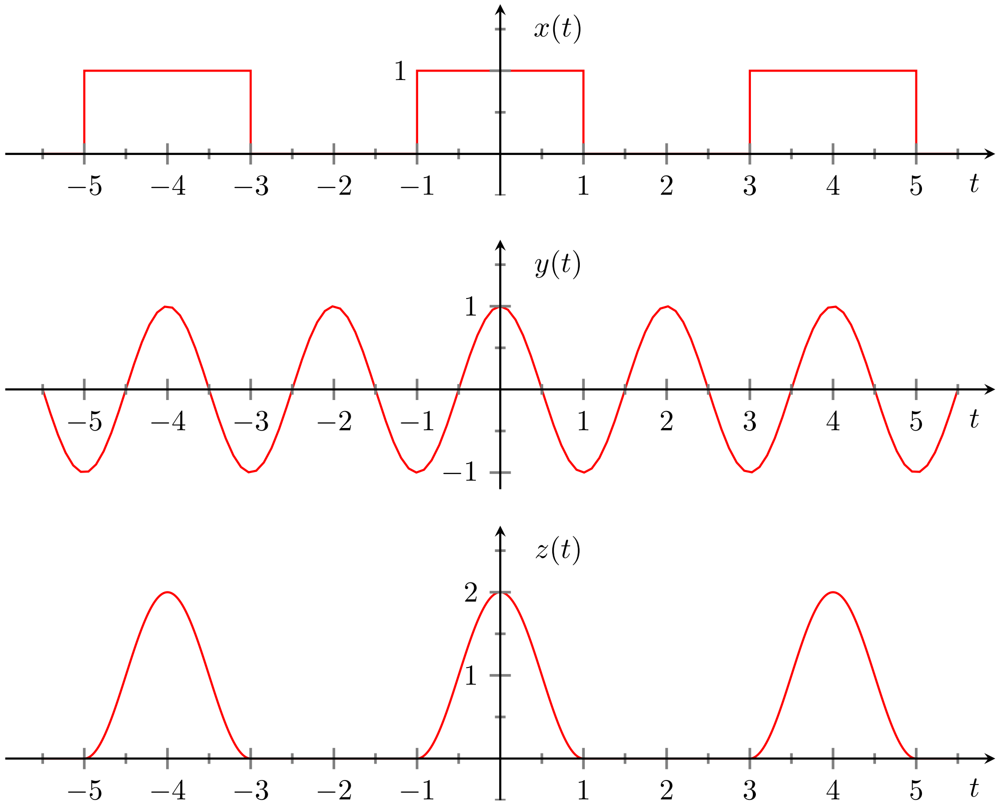
(j) **[2 marks]**



Mixed real and imaginary.

Problem 4

Consider the following CT periodic signals $x(t)$, $y(t)$ and $z(t)$



(a) **[5 marks]** Determine the fundamental frequency, ω_0, period T_0 and Fourier series coefficients, a_k, for $x(t)$ as shown (the central portion is given by $x(t) = \chi_{[-1,1]}(t)$).

(b) **[7 marks]** Determine the fundamental frequency, period and Fourier series coefficients, b_k, for $y(t) = \cos(\pi t)$.

(c) **[2 marks]** Determine the fundamental frequency and fundamental period for

$$z(t) = x(t)\big(1 + \cos(\pi t)\big).$$

(d) **[6 marks]** Using the results of parts (a) and (b) determine the Fourier coefficients c_k for $z(t)$. (It is recommended that you use, with care, the Fourier Series Multiplication Property.)

Problem 5

Determine the following convolutions using the approach specified:

(a) [**6 marks**] Use discrete time convolution sum to determine the convolution $y[n] = x[n] \star h[n]$ where $x[n] = \left(\frac{3}{4}\right)^n u[n]$ and $h[n] = \left(\frac{1}{2}\right)^n u[n]$.

You may use any of the following sums in your calculations:

$$\sum_{k=0}^{n} \alpha^k \beta^{n-k} = \frac{\alpha^{1+n} - \beta^{1+n}}{\alpha - \beta}$$

$$\sum_{k=0}^{n} \alpha^k \beta^k = \frac{\alpha^{1+n}\beta^{1+n} - 1}{\alpha\beta - 1}$$

$$\sum_{k=0}^{\infty} \alpha^k \beta^{n-k} = \frac{\beta^{n+1}}{\beta - \alpha}$$

$$\sum_{k=0}^{\infty} \alpha^k \beta^k = \frac{1}{1 - \alpha\beta}$$

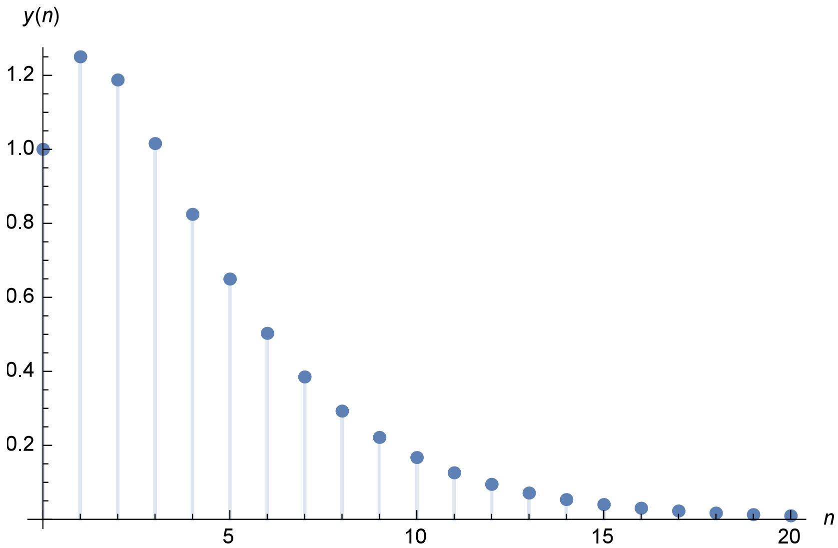


Figure 1: The convolution output $y[n]$ for parts (a) and (b).

(b) [**6 marks**] Use Fourier transforms to determine the convolution $y[n] = x[n] \star h[n]$ where $x[n] = \left(\frac{3}{4}\right)^n u[n]$ and $h[n] = \left(\frac{1}{2}\right)^n u[n]$. Show that your answer is the same as in part (a) above.

(c) [**4 marks**] Use Fourier transforms to determine the convolution $y[n] = x[n] \star h[n]$ where $x[n] = (n+1)\left(\frac{1}{4}\right)^n u[n]$ and $h[n] = \left(\frac{1}{2}\right)^n u[n]$.

(d) [**5 marks**] Use Fourier transforms to determine the convolution $y(t) = x(t) \star h(t)$ where $x(t) = te^{-2t}u(t)$ and $h(t) = e^{-4t}u(t)$.

(e) [**3 marks**] Use Fourier transforms to determine the convolution $y(t) = x(t) \star h(t)$ where $x(t) = te^{-2t}u(t)$ and $h(t) = te^{-4t}u(t)$.

Problem 6

A casual and stable LTI system has the frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$

and frequency domain description:

$$Y(j\omega) = H(j\omega)\, X(j\omega)$$

(a) [**4 marks**] Using the property:

$$\frac{d^k z(t)}{dt^k} \stackrel{\mathcal{F}}{\longleftrightarrow} (j\omega)^k\, Z(j\omega), \quad k = 0, 1, 2, \ldots$$

where

$$z(t) \stackrel{\mathcal{F}}{\longleftrightarrow} Z(j\omega),$$

find the **differential equation** relating the input $x(t)$ (and its derivatives) and output $y(t)$ (and its derivatives) of the system with frequency response $H(j\omega)$.

(b) [**4 marks**] Find the **partial fraction expansion** of $H(j\omega)$.

(c) [**4 marks**] Using the partial fraction expansion and the property

$$e^{-at}\, u(t) \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{1}{a + j\omega},$$

where $u(t)$ is the unit step function, determine the **impulse response** $h(t)$ corresponding to frequency response $H(j\omega)$.

Problem 7

(a) **[3 marks]** Determine the frequency response $H(e^{j\omega})$ and the impulse response $h[n]$ for the system whose input is $x[n]$ and output is $y[n]$:

$$y[n] = x[n] - 0.5\,x[n-1]$$

(b) **[3 marks]** Determine the frequency response $H(e^{j\omega})$ and the impulse response $h[n]$ for the system whose input is $x[n]$ and output is $y[n]$:

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]$$

(c) **[3 marks]** Determine the frequency response $H(e^{j\omega})$ and the impulse response $h[n]$ for the system whose input is $x[n]$ and output is $y[n]$:

$$y[n] - 0.3\,y[n-1] + 0.1\,y[n-2] = 3x[n] - x[n-1]$$

(d) **[3 marks]** The filter described by

$$y[n] = x[n] + \frac{1}{3}\,x[n-1]$$

is connected in parallel with another filter with impulse response $g[n]$ such that the resulting parallel interconnection has the frequency response

$$H(e^{j\omega}) = \frac{-\frac{1}{9}e^{-j2\omega}}{1 - \frac{1}{3}e^{-j\omega}}.$$

Determine the frequency response $G(e^{j\omega})$ and impulse response $g[n]$.

— End of Assignment —