Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

CLab–2: "Fourier Series and their Properties"

Lab Week: Week 8

Total Marks: 10

Contribution to Final Assessment: 2%

Submission: Marked by tutors during the lab time based on completion of the lab tasks.

Relevant Textbook Sections: 3.3 and 3.5.

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1 Learning Outcomes

After completing this lab, the student should be able to:

- understand the trade-off between the number of Fourier series coefficients and the accuracy of signal representation.
- apply the concepts of linearity and time-shift to generate a new periodic signal from a given one.
- investigate the time-scaling and symmetry properties of Fourier series, with application to real-valued signals.

2 Background on Fourier Series

In Lectures, the following properties of Fourier series are studied:

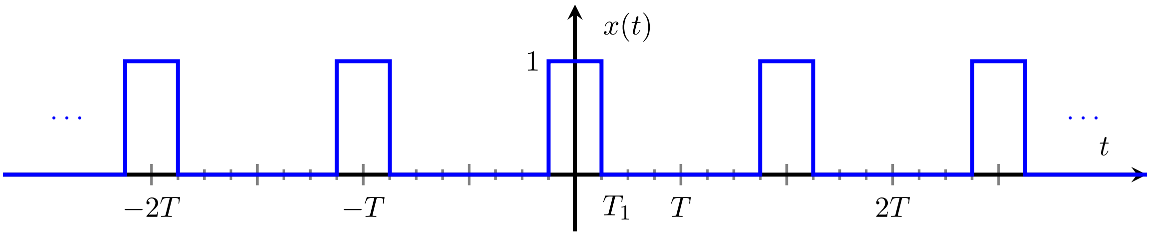
Given

$$x(t) \xleftrightarrow{FS} a_k,$$

then

Time-shift	$x(t - t_0) \xleftrightarrow{FS} a_k\, e^{-jk\,2\pi t_0/T}$
Time-scaling	$x(\alpha t) \xleftrightarrow{FS} a_k$, with period T/α. $\quad \alpha > 0$
Time-reversal	$x(-t) \xleftrightarrow{FS} a_{-k}$
Conjugation	$x^*(t) \xleftrightarrow{FS} a^*_{-k}$

3 Periodic Rectangular Wave



A Periodic Rectangular Wave with fundamental period T and half width T_1 (that is, the pulse width is $2T_1$ centered at 0) has Fourier coefficients

$$a_k = \frac{\sin(k\pi 2T_1/T)}{k\pi}, \quad k \in \mathbb{Z} \text{ (integers)} \tag{1}$$

We derived this formula in Lectures.

Task 1

1. In this task, we first understand the following MATLAB code which draws the time domain waveform and the Fourier coefficients for the above Periodic Rectangular Wave with $T = 8$ and $T_1 = 1$.

```
T=8;                                % fundamental period
omega0=2*pi/T;                      % fundamental frequency (radians/sec)
tmax = 10;                          % define time limit
t = [-tmax:0.01:tmax];              % define points along time axis
T1=1;                               % half-width of pulse
K=50; k=[-K:K];                     % max number of Fourier Series Coefficients
k(K+1)=0.0001;                      % lazy way to deal with 0/0 limit

% compute the Fourier Series Coefficients of the rectangular waveform
% matlab is a matrix/vector language. k is a vector and k*omega0*T1 is a scaled version
% of k; The denominator is a vector, doing a ./ division is an element-wise division.
a=sin(k*omega0*T1) ./ (k*pi);
k(K+1)=0;                           % cover our deception that dealt with 0/0 limit
subplot( 2, 1, 1 );
stem( k, real(a));                  % "stem" is "plot" without joining points
axis([-K,K,-0.1,0.6]);
waveform = a*exp( 1i*omega0*k'*t );
subplot( 2, 1, 2 );
plot(t,real(waveform));
axis([-tmax, tmax, -0.3, 1.2]);
```

2. In the given code, the maximum number of Fourier series coefficients $K = 50$ have been considered. Use $K = 5$, 10 and 20 and observe the difference in the plot of the signal in time domain. Record your observation and explain this difference to the tutor at the end of this lab.

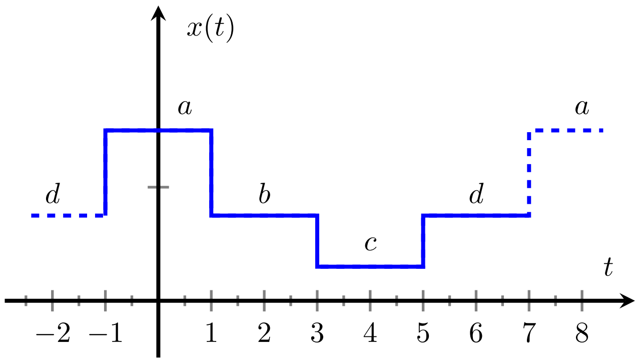
4 Linearity and Time Shift Property of Fourier Series

In this part, we examine the linearity and time-shift properties of Fourier series.

Task 2

1. For a signal $x(t)$ in Task 1, use time shift property to plot the signal $x(t-2)$ and $x(t-5)$ by *modifying the Fourier series coefficients* in the given code. Show the plot to the tutor.

2. Now look at a specific periodic stepped waveform shown in the figure below with fundamental period $T = 8$ which generalizes the Periodic Rectangular Wave. It has the same periodicity as the waveform in Task 1 and can be thought of as a *linear sum* of suitably scaled and shifted versions of the waveform in Task 1. This waveform is parameterized by four parameters: a, b c and d. Using the linearity and time shift property of the Fourier series, determine the Fourier coefficients of the periodic stepped wave in terms of a_k given in (1). Write your result on the paper and show to the tutor at the end of the lab. **Use the 4 rightmost figures of your UniID as the parameters a, b c and d.**



3. Modify the Fourier coefficients in the MATLAB code provided in Task 1 and plot the Periodic Stepped Wave for your parameter values. Show the plot to the tutor.

   Note: In the code you need to adjust the `axis` settings for the plots to appear properly.

5 Scaling in Time Domain

We now see the impact of scaling of signal in time domain on its Fourier series coefficients.

Task 3

1. Run the provided m-file with following values of T and T_1:

   - $T = 4$, $T_1 = 1$
   - $T = 6$, $T_1 = 1.5$
   - $T = 8$, $T_1 = 2$

2. Note down the difference in plots both in time and Fourier(frequency) domain. In above exercises, duty cycle $(2T_1/T)$ has been kept constant. Now we see the effect of changing duty cycle on Fourier series. Repeat the MATLAB exercise for $T = 8$ and $T_1 = 1, 1.5$ and 2. Note the difference between different plots both in time and frequency domain.

3. Now show your working, observations and plots to the tutor.

6 Symmetry Property of Fourier Series

In this part, we examine the symmetry prosperities of Fourier series. Consider the signal

$$x_1(t) = \cos(\omega_0 t) + \sin(2\omega_0 t), \tag{2}$$

where $\omega_0 = 2\pi$. To evaluate this signal in MATLAB, use the time vector

```
>>t=linspace(-1,1,1000);
```

which creates a vector of 1000 time samples over the region $-1 \leq t \leq 1$.

Task 4

1. What is the smallest period T, for which $x_1(t) = x_1(t+T)$? Analytically find the Fourier coefficients of $x_1(t)$.
2. Using the time-reversal and linearity properties find the coefficients of $y(t) = x_1(t) + x_1(-t)$.
3. Plot $y(t)$ over $-1 \leq t \leq 1$. What type of symmetry do you observe? Explain the reason.
4. Using the time-reversal, linearity and conjugation properties find the coefficients of $z(t) = x_1(t) + x_1^*(-t)$.
5. Plot $z(t)$ over $-1 \leq t \leq 1$. What type of symmetry do you observe? Explain the reason.

OPTIONAL: Task 5

Repeat steps 1–5 in Task 4 for $x_1(t) = \cos(\omega_0 t) + j\sin(2\omega_0 t)$. Note that $x_1(t)$ is now complex. So, when plotting make sure to plot the real part and the imaginary parts separately, and study the symmetry you observe in each.