Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

HLab–3: "Frequency Response of Discrete Time Systems"

Lab Week: Week 11

Total Marks: 10

Contribution to Final Assessment: 3%

Submission: No due dates. Lab is marked based on satisfactory completion of tasks during the lab. Hence, attendance is compulsory.

Relevant Textbook Sections: 2.4, 3.9, 3.10, 3.11.

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1 Learning Outcomes

After completing this lab, students should be able to:

- understand the frequency response of discrete-time LTI systems.
- understand the operation of notch filter.
- apply notch filter to remove unwanted signals.

2 Modules and Devices Needed

Audio Oscillator, Sequence Generator, Tunable LPF, Oscilloscope, Adder, Multiplier, and VCO.

2.1 Background

In Labs 1 and Lab 2, we have been dealing with continuous time signals and observed the frequency characteristics of continuous time systems.

Here, we explore the frequency response of the discrete time systems. The discrete time system under consideration is shown in Figure 1 (inside the dashed box), with the following input/output relation

$$y[n] \;=\; B_1x[n] + B_2x[n-1] + B_3x[n-2]. \tag{1}$$

The frequency response $H(e^{j\omega})$ of this LTI system is given by following equation

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \;=\; B_1 + B_2e^{-j\omega} + B_3e^{-2j\omega}, \quad \omega = [0, 2\pi). \tag{2}$$

We will see that different sets of values of B_1, B_2 and B_3 correspond to different frequency response of a system.

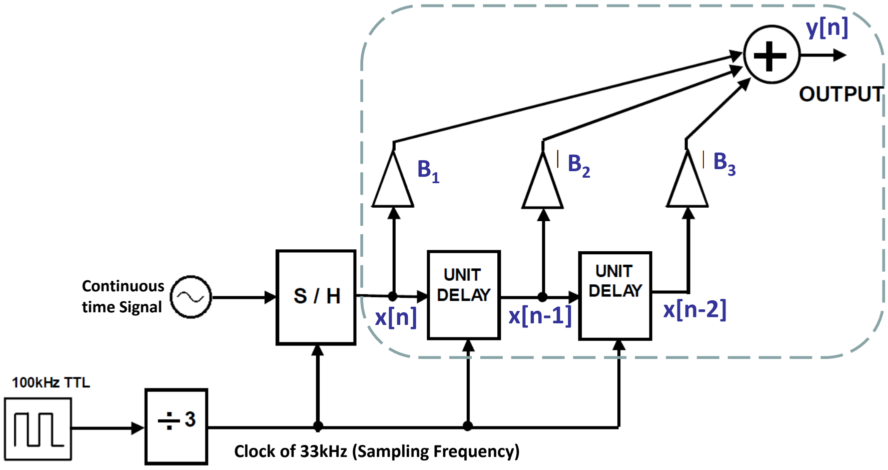

Figure 1: Block diagram of discrete time system

In order to produce a delay in the signal by one unit of time, we use Z-Transform module as indicated in Figure 1. You do not need to know the details of Z-Transform. You can only think of it as a signal processing device that delays the signal by one unit of time.

2.2 Relation between Continuous-time Frequency and Discrete-time frequency

In fact, we are processing a continuous time signal by first converting the continuous time signal to a discrete time signal and then processing a discrete time signal using a discrete time system. We carry out the conversion of continuous time (also called analog) signal to a discrete time (also called digital) signal outside the dashed box in Figure 1. Continuous time is passed through sample and hold (S/H) module to obtain a discrete time signal $x[n]$. The sample and hold module provides the simplest form of continuous to discrete time conversion, where the sample at each sampling instance is hold at the output until the next sampling instance. Note that the clock of 33 kHz is also provided to S/H module, which indicates that the continuous time is sampled with a sampling frequency of 33 kHz.

If a continuous time signal of frequency f (in Hertz) is sampled at the sampling frequency f_s (in Hertz), the normalized discrete time frequency, denoted by ω, is related to the frequency of continuous time signal through the following expression:

$$\omega = 2\pi \frac{f}{f_s}. \tag{3}$$

3 Task 1: Setting Gains and Patching up of Discrete Time System

[***Total Marks:*** *2*]

Here, we use the TIMS modules to implement the LTI system in Figure 1. First, we adjust the gains B_1, B_2 and B_3 of the second Z-transform module in Fig. 3. See the onboard switches on the Z-transform module., illustrated in Figure 2

1. Position the Z-TRANSFORM module in a slot with at least 2 vacant slots to the left of it so you can access the gain controls with your fingers. Do not connect the circuit in Figure 3 at this point. You will do that in step 7.

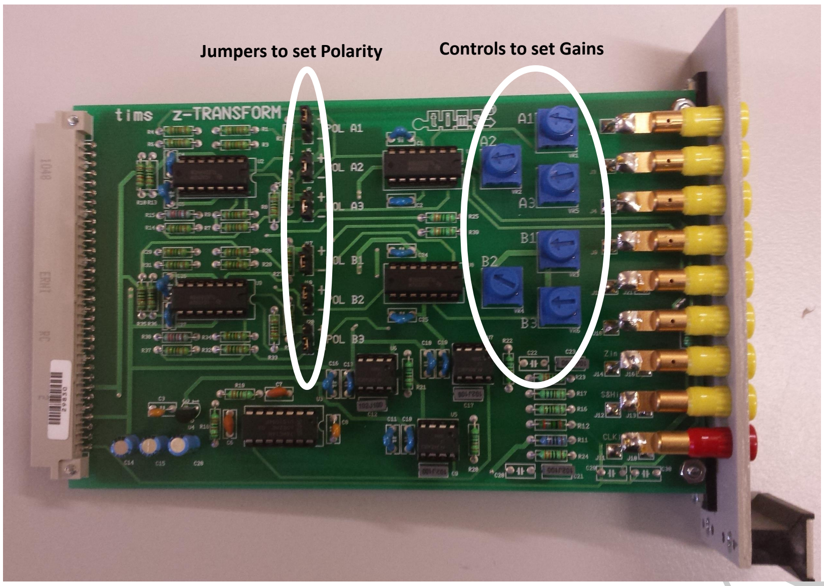

Figure 2: Z-Transform module with jumpers to set polarity and controls to set gains.

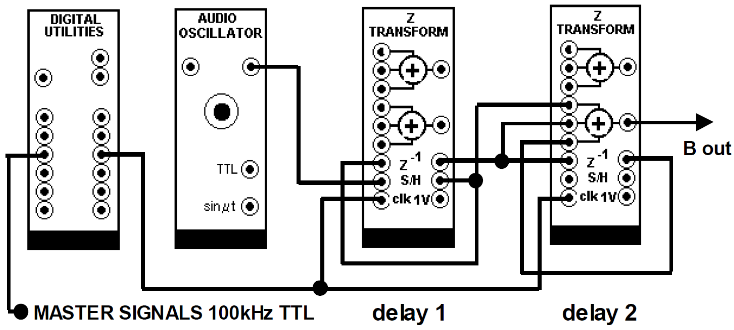

Figure 3: Task 1 - TIMS model of discrete-time system for frequency response measurement.

2. The following procedure is to have the gains B_1 and B_3 of the *second Z-transform module* closely matched by setting $B_3 - B_1$ to zeros. To begin, set the polarity jumpers to INVERTING[1] for B_1 and NON-INVERTING for B_3. Adjust both control knobs to 9 o'clock. Connect the AUDIO OSCILLATOR analog output via the BUFFER AMPLIFIER to B_1 and B_3 in parallel. Set the BUFFER AMPLIFIER gain to 12 o'clock position. Display the output of the B adder. The signal should be approximately zero. If that is not the case, carefully vary B_1 to get the signal as close to zero as possible ($< 150mV_{p-p}$).

3. Remove the B_3 input and measure the gain B_1. That is divide the adder output by its input. Recall that the gain $B_3 = B_1$.

4. Without varying the control knobs, disconnect the inputs, move the B_3 polarity jumper to the INVERTING position, and show the measured gains to the tutor.

5. Now, we set the B_2 gain. The sine-wave output is again used as the test signal. Set gain B_2

[1] An inverting setting is obtained by placing the jumper to select the bottom two pins, while the non-inverting is obtained by the top two pins.

to 10 o'clock with polarity jumper to NONINVERTING. Measure and record the B_2 gain. For this experiment, B_2 should be less than $2B_1$

6. Disconnect the input. At a glance, the positions of the B_1, B_2 and B_3 gain controls are 9, 10 and 9 o'clock respectively, and polarities set to INVERTING, NON-INVERTING, INVERTING respectively.

7. Now, patch up the TIMS model in Figure 3. Make sure you use the TTL output of the 100kHz MASTER SIGNALS. Use the DIGITAL UTILITY frequency divider to obtain 33.33 kHz. Confirm that the clock frequency is set 33.33 kHz using the FREQUENCY COUNTER.
8. Set the AUDIO OSCILLATOR to approximately 2 kHz. We take this 2 kHz sine-wave and pass it through a sample-and-hold amplifier. The output is a staircase shaped sine-wave. This is our $x[n]$. We pass it through z^{-1} to get $x[n-1]$. We pass $x[n-1]$ through another z^{-1} to get $x[n-2]$. Then we use an adder whose polarity and gain have been set to get $y[n]$.Display the outputs of the SAMPLE-HOLD and of the B adder. Use the Cursors button on the oscilloscope to check that the time interval between samples is consistent with the clock frequency. Ask the tutor to check the circuit and output waveforms on oscilloscope.

4 Task 2: Amplitude Response and Time Delay

[***Total Marks:*** *5*]

So far, we have implemented the discrete LTI system. Now, we see the frequency response of discrete LTI system.

1. Measure and plot the amplitude response over the range of the AUDIO OSCILLATOR. To start with, scan over the AUDIO OSCILLATOR range to carefully find the notch frequency[2]. With the increase in frequency, the output first decreases and then increases again. Confirm that the obtained nothch frequency is in agreement with the theoretical value computed by

$$f_0 = \frac{\omega_0}{2\pi} f_s, \tag{4}$$

$$\omega_0 = \cos^{-1}\left(\frac{B_2}{2B_1}\right), \qquad \big(\text{Prove this by setting } |H(e^{j\omega})| = 0\big)\,. \tag{5}$$

2. Take several frequency steps upto 1kHz on both sides of the notch frequency, to obtain a representative graph of amplitude against frequency. Note that because the waveforms are represented by sample values, the measurement of the peak-peak amplitude on the scope will be challenging at the upper frequencies.
3. Theoretically, calculate the value of the magnitude response for different frequencies using the measured gains and equation (2). Show that the measured values are in agreement with the calculated values.

4. Keeping B_1 and B_3 to the set values, calculate the frequency of the notch when $B_2 = 0$. First calculate analytically the notch frequency and then show the similar observation using the experimental set-up to the tutor.

[2]This is the frequency where the output goes to the minimum. Consequently, the discrete-time system is called as **Notch Filter**

5 Task 3: Suppression of Unwanted Single Frequency Component

[***Total Marks:*** *3*]

We can regard the discrete-time LTI system as consisting of only three samples as Finite Impulse Response Notch filter. In Task 2, we saw that the notch frequency can be controlled by changing the gain control B_2 only. Here, we use the notch filter to remove an interference tone.

Consider this situation: we are trying to receive a message at frequency f_1 from a distant transmitter, but in our neighborhood there is another transmitter sending an unwanted signal at frequency f_2 affecting our reception of f_1.

1. First prove equation (5), the frequency of the sampled signal which varies from 0 to 2π. Show this derivation to the tutor.

2. Assume $f_1 = 2$ kHz, $f_2 = 5$ kHz. Use the AUDIO OSCILLATOR and the VCO to generate f_1 and f_2, respectively. Add these using the ADDER module and set the ADDER gains around unity for both. Connect the output of ADDER to the input of the notch filter.

3. Find the gain B_2 corresponding to the notch frequency f_2 using (4) and (5). Accordingly, carefully adjust B_2 to the computed value to suppress the f_2 component. Remember to disconnect B_1 and B_3, when setting B_2.

4. Show that the f_2 signal can be suppressed with the notch filter.