Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

PROBLEM SET 2

In the following: $\delta[n]$ and $u[n]$ represent the Dirac and unit step functions for discrete-time (DT). Similarly $\delta(t)$ and $u(t)$ for continuous-time (CT). Convolution of signals is written $x[n] \star h[n]$ or $x(t) \star h(t)$. Please indicate any identities or formulas used in the simplification of the results.

Unit Impulse and Unit Step Functions

Problem Set 2-1

Draw the following signals

(a) $x(t) = 2\,\delta(t+1)$

(b) $-1.5\,\delta(t-2)$

(c) $x(t) = \displaystyle\sum_{k=3}^{7} 2^{k-5}\delta(t-2k)$

(d) $x(t) = \displaystyle\int_{-\infty}^{t} \delta(\tau-2)\,d\tau$

(e) $x(t) = \displaystyle\int_{-\infty}^{t} \delta(t-2)\,d\tau$

(f) $\displaystyle\int_{-t}^{t} \delta(t-2)\,d\tau$

Problem Set 2-2

Draw the following signals

(a) $x[n] = -2\,\delta[n+2]$

(b) $x[n] = u[n] - u[n-1]$

(c) $x[n] = -u[-n] + u[-n-1]$

(d) $x[n] = 2u[-n] + u[n-3]$

(e) $x[n] = \displaystyle\sum_{k=-\infty}^{-1} \delta[k] + u[n]$

(f) $x[n] = \displaystyle\sum_{k=-\infty}^{-1} \delta[n-k] + u[n]$

Discrete-time Convolution

Problem Set 2-3

Use the graphical flip/shift method, showing intermediate working to perform the following DT convolutions. Note: solutions can be checked in Matlab.

(a) $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$ and $h[n] = 2\delta[n-4]$

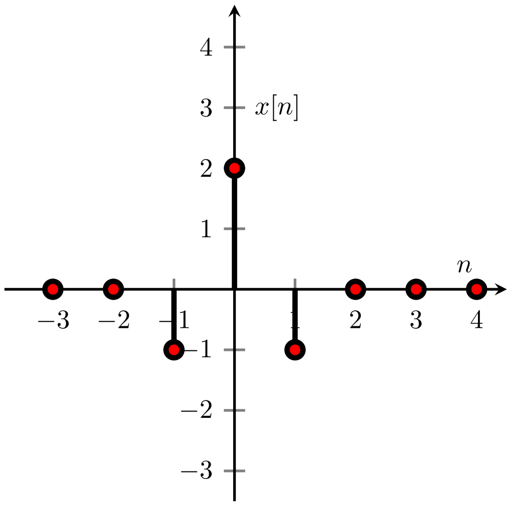
(b) $x[n] = 2\delta[n] + 4\delta[n-1] - 2\delta[n-2]$ and $h[n] = \delta[n] + 0.5\delta[n-1]$

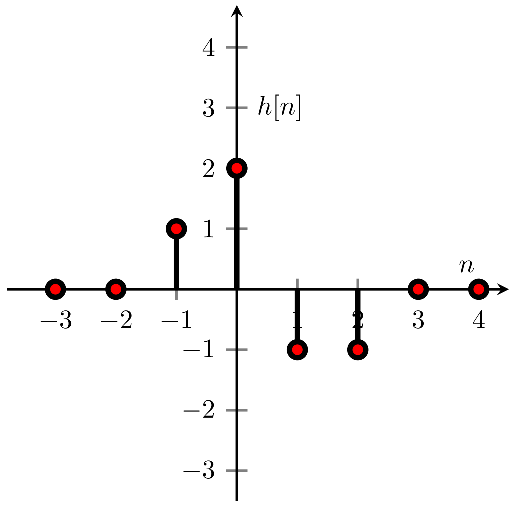
(c) $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$ and $h[n] = 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$

(d) $x[n] = \delta[n+1] + 2\delta[n] + 3\delta[n-1] + 4\delta[n-2]$ and $h[n] = -\delta[n+2] + 5\delta[n+1] + 3\delta[n]$

Problem Set 2-4

Compute the DT convolution of $x[n]$ and $h[n]$ as shown below





Problem Set 2-5

For a DT LTI system with impulse response

$$h[n] = u[n-1].$$

Find the output $y[n] = x[n] \star h[n]$ for input

$$x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1].$$

Problem Set 2-6

Compute the convolution $y[n] = x[n] * h[n]$ when $x[n] = 5^n\, u[-n-1]$ and $h[n] = u[n-1]$.

Discrete-time Impulse Response

Problem Set 2-7

Find the impulse response of the system, with following input and output relation where $x[n]$ denotes the input and $y[n]$ denotes the output,

$$y[n] + \frac{1}{3}y[n-2] = x[n].$$

Assume initial condition of rest, i.e., $x[n] = 0$ and $y[n] = 0$ for all $n < 0$.

Problem Set 2-8

Consider the LTI system initially at rest and described by the difference equation

$$y[n] + 2y[n-1] = x[n] + 2\,x[n-2].$$

(a) Find the response of this system to the unit pulse input $\delta[n]$ by solving the difference equation recursively or otherwise.

(b) Find the response of this system to the input depicted in Fig. 1 by solving the difference equation recursively or otherwise.

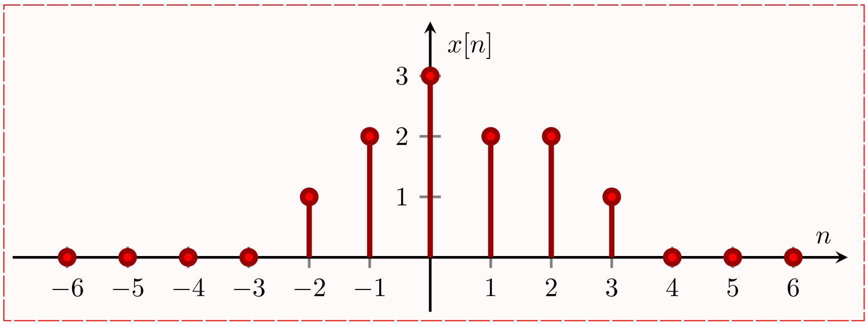


Figure 1: Input signal $x[n]$.

Discrete-time System Properties

Problem Set 2-9

The equation

$$y[n] - a\,y[n-1] = x[n]$$

describes a DT system, with input $x[n]$ and output $y[n]$, assumed to be initially at rest, that is,

$$x[n] = 0 \text{ and } y[n] = 0 \quad \text{for all } n < 0.$$

(a) Show that the impulse response $h[n]$ for this system is

$$h[n] = a^n u[n],$$

where $y[n] = x[n] \star h[n]$.

(b) Is this system (provide reasoning for each of your answer)

i) linear?

ii) time-invariant?

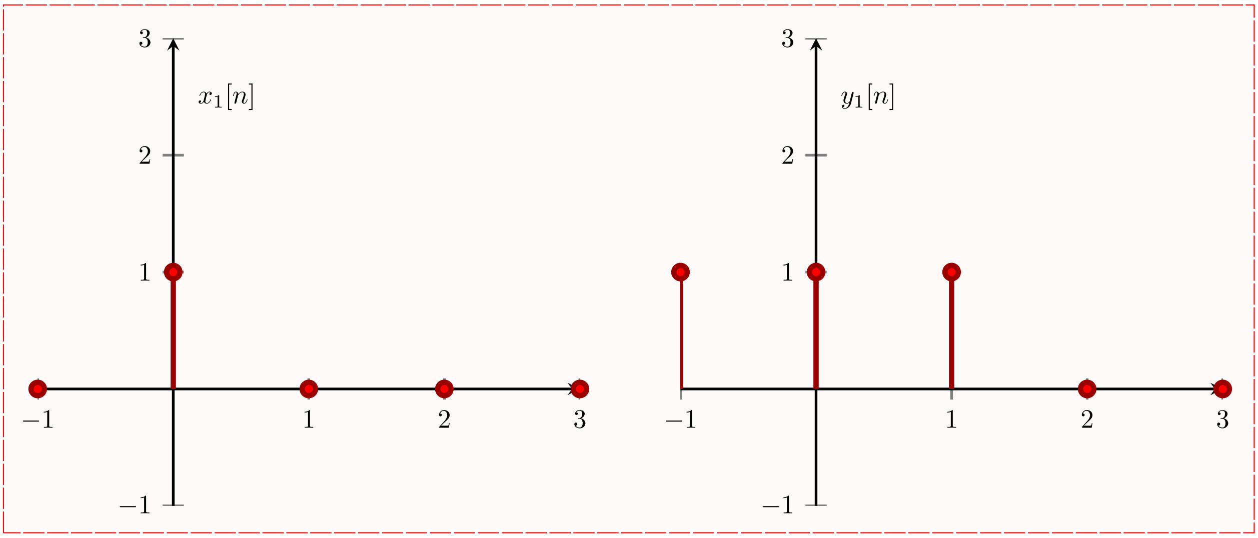
iii) memoryless?

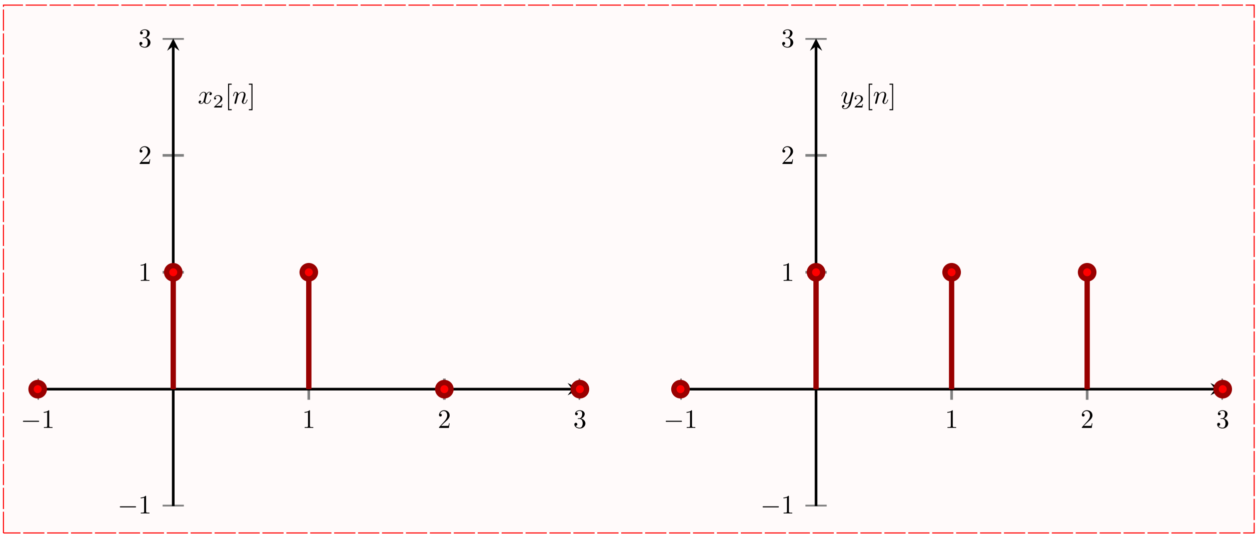
iv) causal?

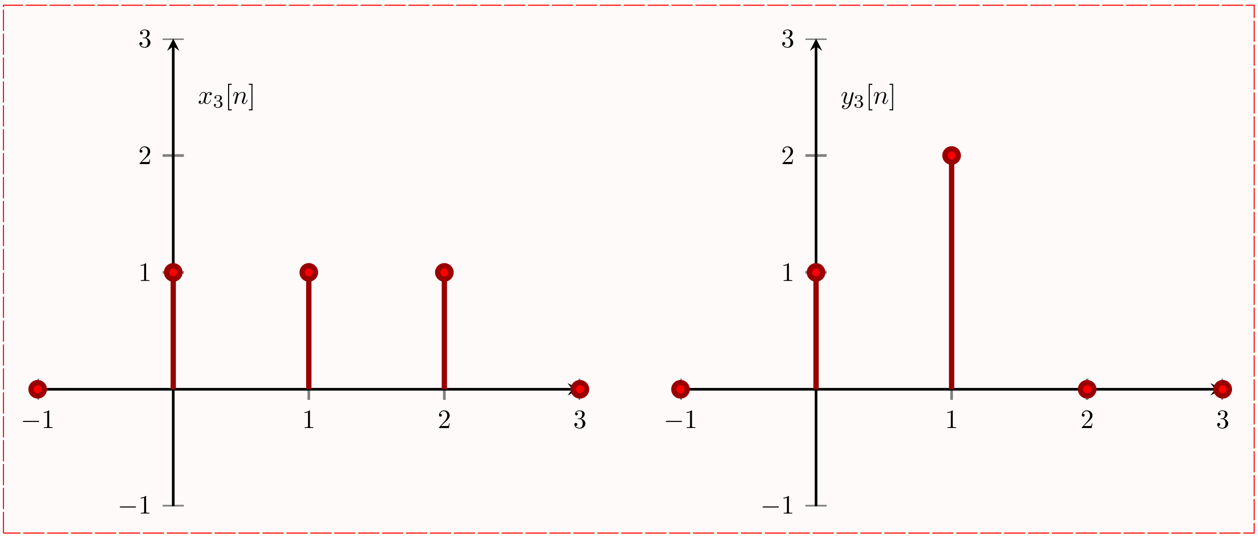
v) stable?

Problem Set 2-10

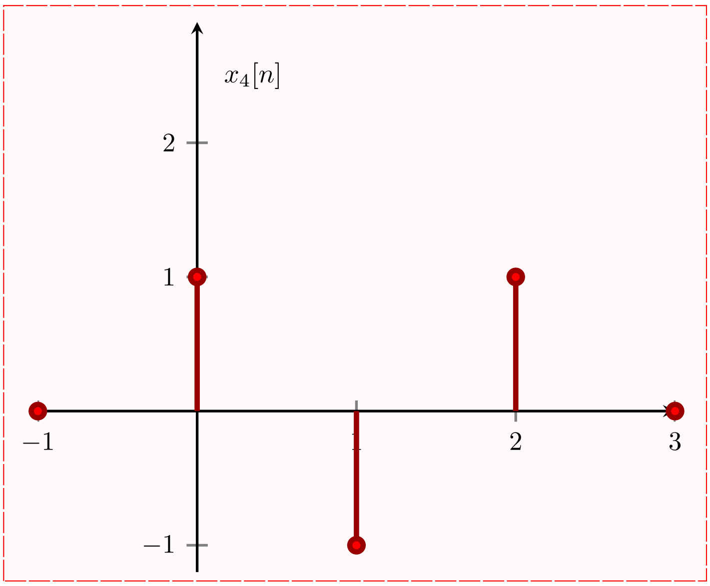
Suppose we have an unknown linear system, for which the superposition principle applies. Further, suppose we have knowledge of three outputs $y_1[n]$, $y_2[n]$, $y_3[n]$, generated by inputs $x_1[n]$, $x_2[n]$, $x_3[n]$, as shown below:





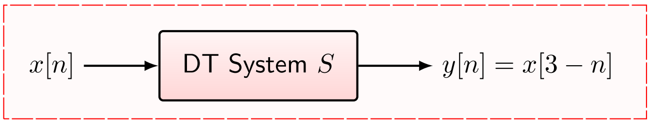


(a) Determine the response of the system $y_4[n]$ when the input is $x_4[n]$ as shown below.



(b) Do we need the system to also be time-invariant?

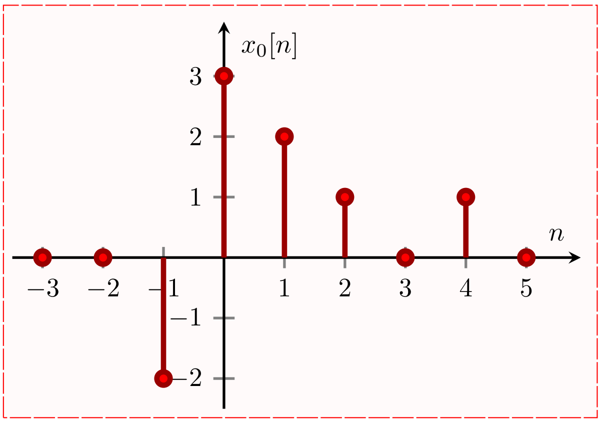
Problem Set 2-11



Consider the DT system, S, with input signal $x[n]$ and output signal given by

$$\boxed{S: \quad y[n] = x[3-n].} \tag{2}$$

(a) Write signal $x_0[n]$, shown below, in terms of linear combinations of shifted $\delta[n]$.



(b) Draw the output $y_0[n]$ when the input is given by $x_0[n]$ shown above.

(c) Write this signal $y_0[n]$ in terms of linear combinations of shifted $\delta[n]$.

(d) Shown that the system is linear.

(e) Shown that the system (2) is non-causal.

(f) Shown that the system (2) is time-varying.

(g) Suppose we have the same system but we don't know its defining relationship (2).
Let $h[n]$ be the output when $\delta[n]$ is applied. We observe $h[n] = \delta[n-3]$.
Can the system be fully characterised by this $h[n]$, that is, if we only know $h[n]$ can we determine the output for any input signal $x[n]$ for such an unknown system?