Australian National University

*MOCK Final Examination*

*Semester 2, 2017*

**SIGNAL PROCESSING**

**ENGN2228**

*Study period:* *None*
*Writing period:* *180 Minutes duration*
*Permitted materials:* *Two A4 pages with handwritten notes on both sides*
*Calculator (memory cleared)*

*6 True-False and 14 Multiple-Choice Questions, for a total of 20 marks*
*4 Problems of 18 marks each, for a total of 72 marks*
*Total marks for exam: 92 marks*

*Contribution to Final Assessment: 50%*

- *Write your True-False and Multiple-Choice Questions answers on the answer sheet provided and place it inside the script book.*
- *Write your 4 Problem answers in the script book provided.*
- *For True-False and Multiple-Choice Questions, there is NO negative marking.*
- *At the end of the exam, hand in the exam question sheets as well as the script book and the Multiple-Choice answers sheet.*

# PART 1 — True/False Questions

## TF Question 1

The signal $x(t)$ below can be expressed as the sum of an even signal and an odd signal.

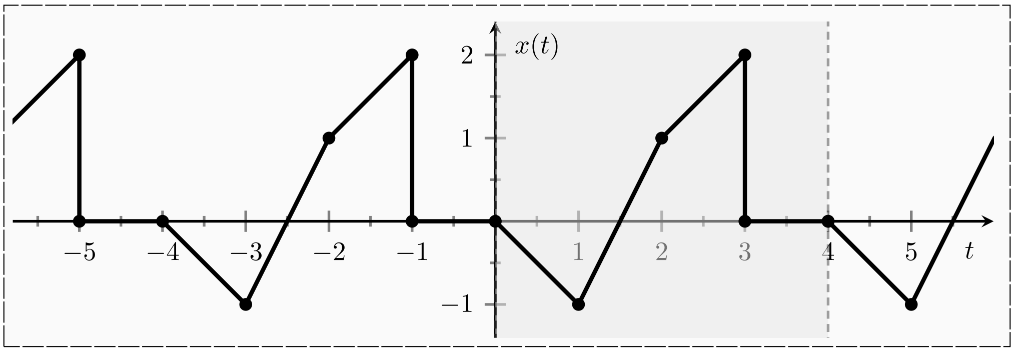


**Figure 1:** Real-valued CT Periodic Signal $x(t)$ with fundamental period $T = 4$. The shaded region indicates one period of the signal, which is periodically repeated.

a. True

b. False

## TF Question 2

The CT periodic signal $x(t) = \cos(t)$ has a Fourier series but no Fourier transform.

a. True

b. False

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## TF Question 3

For the DT signal $x[n] = \sin(n/16)$, it has only one (non-zero) term in its Fourier series.

a. True

b. False

## TF Question 4

A CT signal which is periodic with period $2\pi$ is also periodic with period $8\pi$.

a. True

b. False

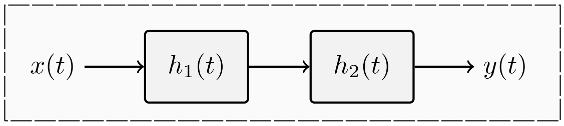



**Figure 2:** A series/cascade connection of the two CT LTI systems.

## TF Question 5

If an LTI system with impulse response $h_1(t)$ is non-causal and an LTI system with impulse response $h_2(t)$ is also non-casual then the system corresponding to the series/cascade connection of $h_1(t)$ and $h_2(t)$ (as depicted in Figure 2) is always non-causal.

a. True

b. False

## TF Question 6

Consider the series/cascade connection of the two CT LTI systems with impulse responses $h_1(t)$ and $h_2(t)$, input $x(t)$ and output $y(t)$, as shown in Figure 2:

Suppose $h_1(t) = \delta(t - 2.3)$ and $h_2(t) = \delta(t - 1.5)$ then $y(t) = x(t - 3.8)$.

a. True

b. False

# PART 2 — Multiple Choice Questions

## MC Question 1

What is the DT convolution, $y[n] = x[n] \star h[n]$, of the two signals

$$x[n] = \delta[n] + \delta[n-2] \quad \text{and} \quad h[n] = 2\,\delta[n-3]$$

a. $y[n] = \delta[n] + \delta[n-2] + 2\delta[n-3]$
b. $y[n] = 2\,\delta[n-3] + 2\,\delta[n-5]$
c. $y[n] = 2\,\delta[n+3] + 2\,\delta[n+1]$
d. $y[n] = 2\,\delta[n] + 2\,\delta[n-2]$
e. None of the above.

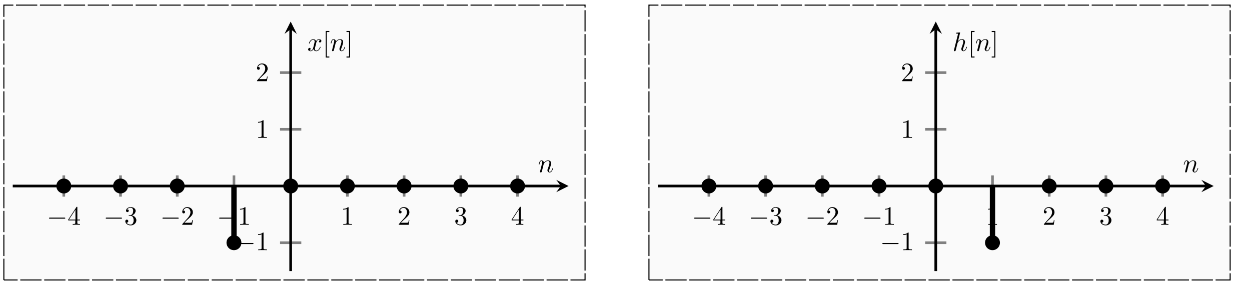
## MC Question 2

A DT LTI system with impulse response $h[n]$ is stable if

$$\sum_{n=-\infty}^{\infty} \big|h[n]\big| < \infty.$$

One of the following causal DT LTI systems is not stable. Which one is it?

a. $h[n] = u[n] - u[n-10]$
b. $h[n] = \dfrac{1}{n^2} u[n]$
c. $h[n] = \dfrac{1}{n} u[n]$
d. $h[n] = e^{-2n} u[n]$
e. $h[n] = \delta[n]$




**(a)** Input signal $x[n]$.

**(b)** System response $h[n]$.

**Figure 3:** Signal $x[n]$ and system response $h[n]$, output signal is $y[n] = h[n] \star x[n]$

## MC Question 3

For $x[n]$ shown in Figure 3(a), which of the following is correct?

a. $x[n] = -\delta[n+1]$

b. $x[n] = -\delta[n-1]$

c. $x[n] = -1$

d. $x[n] = +1$

e. $x[n+1] = \delta[n]$

## MC Question 4

For $x[n]$ and $h[n]$, shown in Figure 3(a) and Figure 3(b), what is $y[n] = h[n] \star x[n]$?

a. $y[n] = \delta[n]$

b. $y[n] = -\delta[n-1] - \delta[n+1]$

c. $y[n] = -2\delta[n]$

d. $y[n] = -\delta[n-2]$

e. $y[n] = \delta[n-2]$

## MC Question 5

[Tricky] Consider the system $y(t) = x(t)\,u(-t)$, where $u(t)$ is the unit step signal. Which of the following statements is true?

a. The system is non-linear and casual

b. The system is linear and causal

c. The system is non-linear and non-casual

d. The system is linear and non-causal

e. None of the above

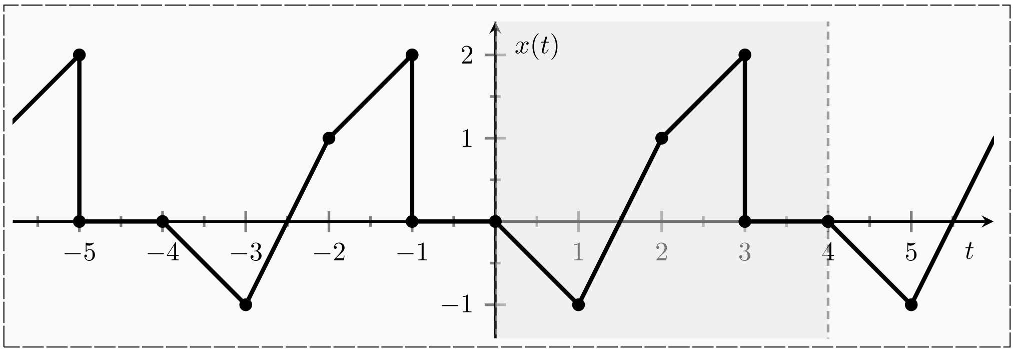



**Figure 4:** Real-valued CT Periodic Signal $x(t)$ with fundamental period $T = 4$. The shaded region indicates one period of the signal, which is periodically repeated.

## MC Question 6

Figure 4 shows a portion of a real-valued periodic CT signal whose fundamental period is $T = 4$. What is the fundamental frequency, $\omega_0$, associated with this fundamental period?

a. $\omega_0 = \pi/4$

b. $\omega_0 = \pi/2$

c. $\omega_0 = \pi$

d. $\omega_0 = 2\pi$

e. $\omega_0 = 1/4$

## MC Question 7

For the periodic CT signal, $x(t)$ in Figure 4, what is the DC value of the signal or the $k = 0$ Fourier coefficient? (The DC value is also the average value per period.)

a. $-1/4$

b. $0$

c. $1/4$

d. $2$

e. $8$

## MC Question 8

Now consider the periodic CT signal $|x(t)|$, which is the absolute value of the real-valued CT signal in Figure 4. (For example it goes through the point $(1, 1)$ instead of $(1, -1)$ and still passes through $(1.5, 0)$). Which of the following is true?

a. $|x(t)|$ has the same power per period as $x(t)$

b. $|x(t)|$ has more power per period than $x(t)$

c. $|x(t)|$ has less power per period than $x(t)$

d. $|x(t)|$ has the same DC value as $x(t)$

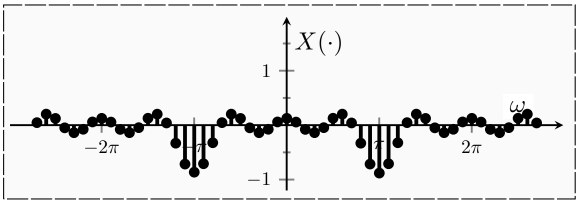
e. $|x(t)|$ has the same Fourier series coefficients as $x(t)$

For Questions 15 to 18 we have, depending on the context, either

$$x(t) \xleftrightarrow{\mathcal{F}} X(\omega) \quad \text{or} \quad x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\omega}).$$

Further, the Fourier transform $X(\cdot)$ is purely real (zero imaginary part).

## MC Question 9




Based on the structure of the frequency domain representation $X(\cdot)$ (period $2\pi$) shown to the right which of the following is the best description of the time-domain signal?

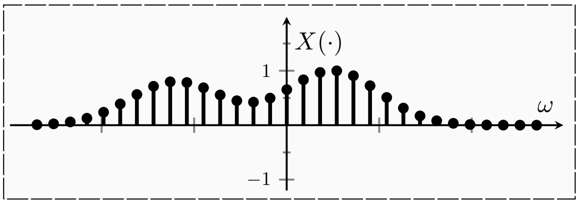
a. The time-domain signal is DT and periodic in time.

b. The time-domain signal is CT and periodic in time.

c. The time-domain signal is DT and not periodic in time.

d. The time-domain signal is CT and not periodic in time.

## MC Question 10




Based on the structure of the frequency domain representation $X(\cdot)$ shown to the right which of the following is the best description of the time-domain signal?

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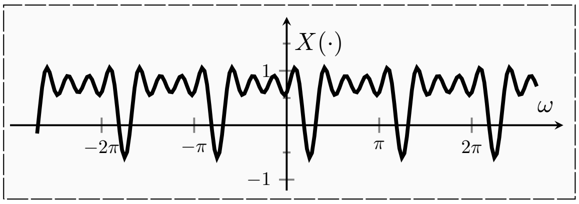
b. The time-domain signal is CT and periodic in time.

c. The time-domain signal is DT and not periodic in time.

d. The time-domain signal is CT and not periodic in time.

## MC Question 11




Based on the structure of the frequency domain representation $X(\cdot)$ (period $2\pi$) shown to the right which of the following is the best description of the time-domain signal?

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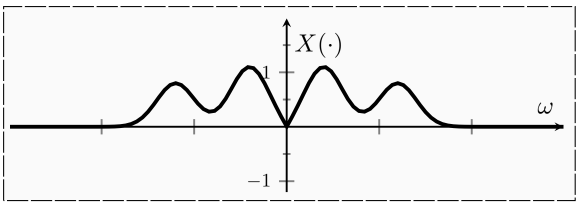
b. The time-domain signal is CT and periodic in time.

c. The time-domain signal is DT and not periodic in time.

d. The time-domain signal is CT and not periodic in time.

## MC Question 12




Based on the structure of the frequency domain representation $X(\cdot)$ shown to the right which of the following is the best description of the time-domain signal?

a. The time-domain signal is DT and periodic in time.

b. The time-domain signal is CT and periodic in time.

c. The time-domain signal is DT and not periodic in time.

d. The time-domain signal is CT and not periodic in time.

## MC Question 13

What is the frequency of the positive 3rd harmonic ($k = 3$) of the DT periodic signal

$$x[n] = (-1)^{3n} \cos(\pi n/3)?$$

a. $\pi/2$ rad/sec
b. $\pi$ rad/sec
c. $2\pi$ rad/sec
d. 6 rad/sec
e. 18 rad/sec

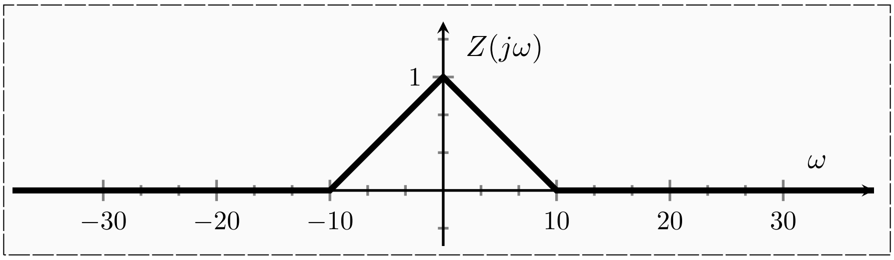



**Figure 5:** A frequency domain spectrum, $Z(j\omega)$, which is purely real. The units of $\omega$ are radians/sec

## MC Question 14

For the CT signal $z(t)$ with $Z(j\omega)$ shown in Figure 5, what is the minimum sampling frequency, $\omega_s$ so that $z(t)$ can be recovered from its samples using an ideal low-pass filter?

a. $\omega_s = 10$ radians/sec
b. $\omega_s = 20$ radians/sec
c. $\omega_s = 40$ radians/sec
d. $\omega_s = 60$ radians/sec
e. $\omega_s = 80$ radians/sec

(end of multiple choice questions)

## PART 3 — Problems

**Instructions:** Attempt all 4 problems. Each problem is worth 18 marks making a total of 72 marks (72%) available for this Part of the exam. You should target 30 minutes for each problem.

### Problem 1

(a) **[2 marks]** The analysis and synthesis equations for a CT signal $x(t)$ and its Fourier transform $X(j\omega)$ are given below:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega, \quad t \in \mathbb{R} \qquad (Synthesis)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt, \quad \omega \in \mathbb{R} \qquad (Analysis)$$

Express in a few sentences your understanding of these two equations.

(b) **[8 marks]** Show, using any method, that the FT of the signal

$$x(t) = e^{-t/10}\cos(10t)u(t)$$

can be expressed in the form

$$X(j\omega) = \frac{j\omega + \frac{1}{10}}{(j\omega + \frac{1}{10})^2 + 10^2}$$

Note: You must show all steps and name any properties used to arrive at the correct answer.

(c) **[8 marks]** Find, using any method, the time domain signal corresponding to the following Fourier representation

$$X(j\omega) = \frac{1}{2\pi}\Big(\frac{2\sin(\omega-2)}{(\omega-2)} \star \frac{e^{-j2\omega}2\sin(2\omega)}{\omega}\Big)$$

where $\star$ denotes the convolution. Note: You must show all steps and name any properties used in your working.

## Problem 2

**(a)** The analysis and synthesis equations for a periodic CT signal $x(t)$ and its Fourier series coefficients $a_k$ are given below:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \quad t \in \mathbb{R} \qquad \text{(Synthesis Equation)}$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt, \quad k \in \mathbb{Z} \qquad \text{(Analysis Equation)}$$

where $T$ is the fundamental period and $\omega_0 = 2\pi/T$ is the fundamental frequency in rad/sec.

i) **[3 marks]** Can we find the Fourier transform of $x(t)$? If so how?

**(b)** Consider the periodic CT signal $x(t)$, shown below:

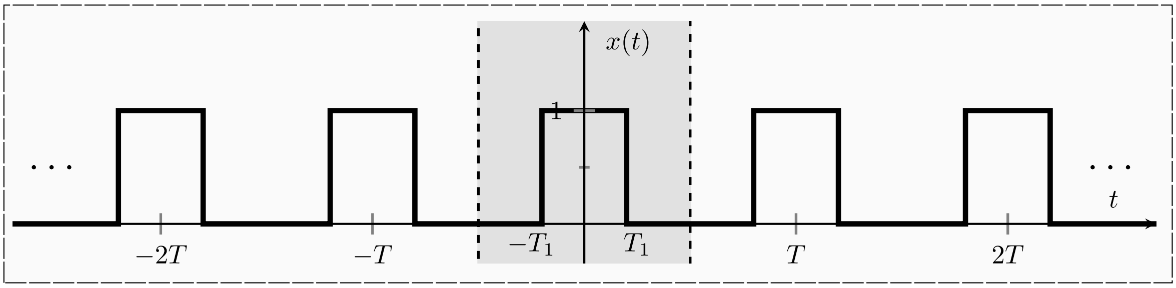



**Figure 1:** Periodic signal $x(t)$.

i) **[2 marks]** Determine the Fourier series coefficient

$$a_0 = \frac{1}{T}\int_T x(t)\, dt$$

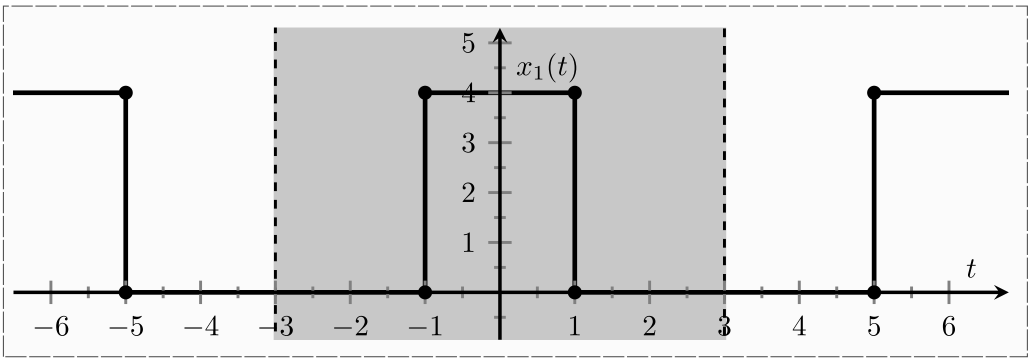
of $x(t)$.

ii) **[3 marks]** Determine the Fourier series of $x(t)$ using the analysis equation.

iii) **[2 marks]** Find the Fourier series coefficients of the signal $a(t) = x(t-1)$.

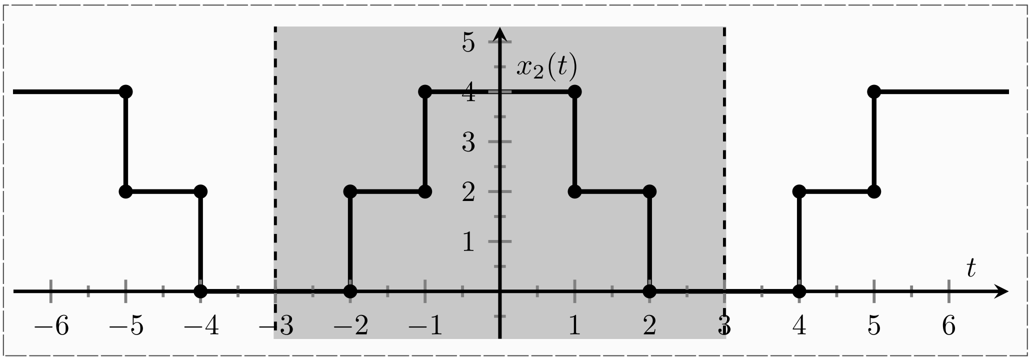
iv) **[2 marks]** Find the Fourier series coefficients of the signal $b(t) = x(t) \star a(t) = \int_T x(\tau)\, a(t-\tau)\, d\tau$.

v) **[2 marks]** Using your answer from part (b) ii), determine the Fourier Series coefficients for the periodic signal $x_1(t)$:




One period of width 6 has been shaded.

vi) **[4 marks]** Determine the Fourier Series coefficients for the periodic signal $x_2(t)$:




One period of width 6 has been shaded.

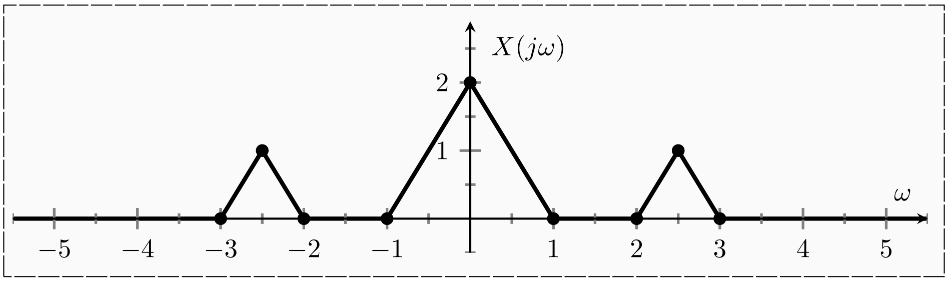



**Figure 2:** The Fourier transform $X(j\omega)$ of some CT signal $x(t)$.

## Problem 3

(a) [**5 marks**] Consider a sampled version of the CT signal $x(t)$, with Fourier transform $X(j\omega)$, shown in Figure 2, given by the CT signal

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT) \xleftrightarrow{\mathcal{F}} X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\big(j(\omega - k\,\omega_s)\big)$$

where $T$ is the sampling interval in time and the sampling rate is given by $\omega_s = 2\pi/T$.

Draw $X_p(j\omega)$, the Fourier transform of $x_p(t)$, in the range $-10 \le \omega \le 10$ for the following values of the sampling rate $\omega_s = 2\pi/T$ (for each case copy and use the template in Figure 3 into your exam script book):

i) $\omega_s = 4$ rad/sec
ii) $\omega_s = 5$ rad/sec
iii) $\omega_s = 6$ rad/sec
iv) $\omega_s = 7$ rad/sec

corresponding to $T = \pi/2,\ 2\pi/5,\ \pi/3,\ 2\pi/7$ sec.

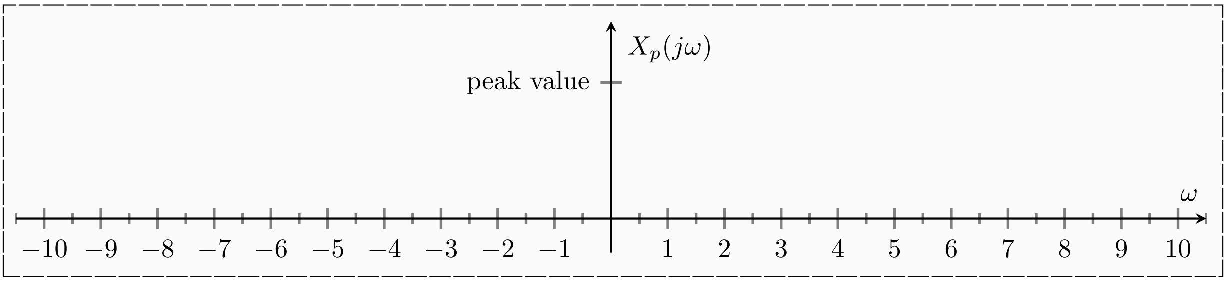



**Figure 3:** Template to draw $X_p(j\omega)$.

(b) [**7 marks**] Consider recovery of $x(t)$ from $x_p(t)$ using only an ideal low-pass filter whose cutoff or bandwidth is given by $\omega_c$ rad/sec. For each of the 4 values of the sampling rate from part (a) state whether recovery using only an ideal pass filter is possible (yes/no) and, if it is possible, what is the least bandwidth $\omega_c$ (in rad/sec), and greatest bandwidth $\omega_c$ (in rad/sec), that can be used to perfectly recover $x(t)$ from $x_p(t)$.

You can answer this question by copying and filling in the missing entries of the table below.

| Sampling Rate $\omega_s$ | Recovery Possible? | Least $\omega_c$ | Greatest $\omega_c$ |
|---|---|---|---|
| 4 rad/sec | | | |
| 5 rad/sec | | | |
| 6 rad/sec | | | |
| 7 rad/sec | | | |

In the case that recovery is not possible, then there are no values for least and greatest $\omega_c$ and you should indicate "n/a" for not applicable.

(c) **[2 marks]** Given recovery of $x(t)$ from $x_p(t)$ using only an ideal pass filter, what is the Nyquist sampling rate in rad/sec?

(d) **[4 marks]** This part considers recovery of $x(t)$ from $x_p(t)$ but not restricted to using an ideal pass filter. In any of the four cases where recovery of $x(t)$ from $x_p(t)$ using only an ideal pass filter fails, indicate whether the signal can still be recovered and how (what combination of LTI filters could be used). Justify your answer.

## Problem 4

(a) **[2 marks]** The analysis and synthesis equations for a DT signal $x[n]$ and its Fourier transform $X(e^{j\omega})$ are given below:

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega, \quad n \in \mathbb{Z} \qquad \text{(Synthesis Equation)}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}, \quad \omega \in \mathbb{R} \qquad \text{(Analysis Equation)}$$

and can be represented as $x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega})$. Express in a few sentences your understanding of these two equations.

(b) **[3 marks]** Using the identities in part (a), or otherwise, show:

$$x[n - n_0] \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega n_0} X(e^{j\omega}) \tag{1}$$

(c) **[2 marks]** Describe in words the physical meaning of the property (1), that is, what it means in the time-domain and what it means in the frequency-domain.

(d) **[4 marks]** Consider the causal LTI system characterized by the difference equation:

$$y[n] - \frac{3}{4}\, y[n-1] + \frac{1}{8}\, y[n-2] = 3\, x[n] \tag{2}$$

relating its input $x[n]$ to its output $y[n]$.

Using the property (1), or otherwise, determine an expression for the frequency response:

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}.$$

where $x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega})$ and $y[n] \overset{\mathcal{F}}{\longleftrightarrow} Y(e^{j\omega})$.

(e) **[1 mark]** What is the DC gain ($\omega = 0$) of the system in (2)?

(f) **[2 marks]** What is the gain at the maximum frequency ($\omega = \pi$) of the system in (2)?

(g) **[4 marks]** Use a partial-fraction expansion to show the (unit) impulse response is

$$h[n] = 6\left(\frac{1}{2}\right)^n u[n] - 3\left(\frac{1}{4}\right)^n u[n]$$

The following DT Fourier transform pair should be useful

$$\alpha^n\, u[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{1 - \alpha e^{-j\omega}}, \quad |\alpha| < 1.$$

(end of exam paper)