# Part 2 Outline

# Complex Number Revision

Engineers use $j$ to denote the imaginary unit, mathematicians use $i$

$$j = \sqrt{-1}$$

$$j^2 = -1$$

$$\frac{1}{j} = -j$$

Three identities with complex exponentials $e^{j\theta}$ used heavily in this course:
Euler's identity:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

# Complex Number Revision

Some examples:

$$
\begin{aligned}
x[n] &= e^{j2\pi n} \\
&= (e^{j2\pi})^n \\
&= [cos(2\pi) + j\sin(2\pi)]^n \quad \leftarrow \text{Euler's} \\
&= 1^n \qquad \cos(2\pi) \to 1, \quad \sin(2\pi) \to 0 \\
&= 1
\end{aligned}
$$

Is this periodic?

$$
\begin{aligned}
x[n] &= e^{j\pi n} \\
&= (e^{j\pi})^n \\
&= [cos(\pi) + j\sin(\pi)]^n \qquad \sin(\pi) \to 0 \\
&= (-1)^n
\end{aligned}
$$

$$
\cos(\pi) \to -1 = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}
$$

What is the period? 2

# Complex Number Revision– Rectangular and Polar Form

$x$ - real
$y$ - imaginary

- Rectangular form $\boxed{x + jy}$
- Polar form $\boxed{re^{j\theta},\ -\pi < \theta \leq \pi}$ $r$ - magnitude $\theta$ - angle
- Convention in signal processing is to express angle $\theta$ in radians.
- Can use functions on your calculator to convert. Make sure it is in radians mode.
- Or use trig identities.

# Complex Number Revision– Rectangular and Polar Form

Rectangular to polar:

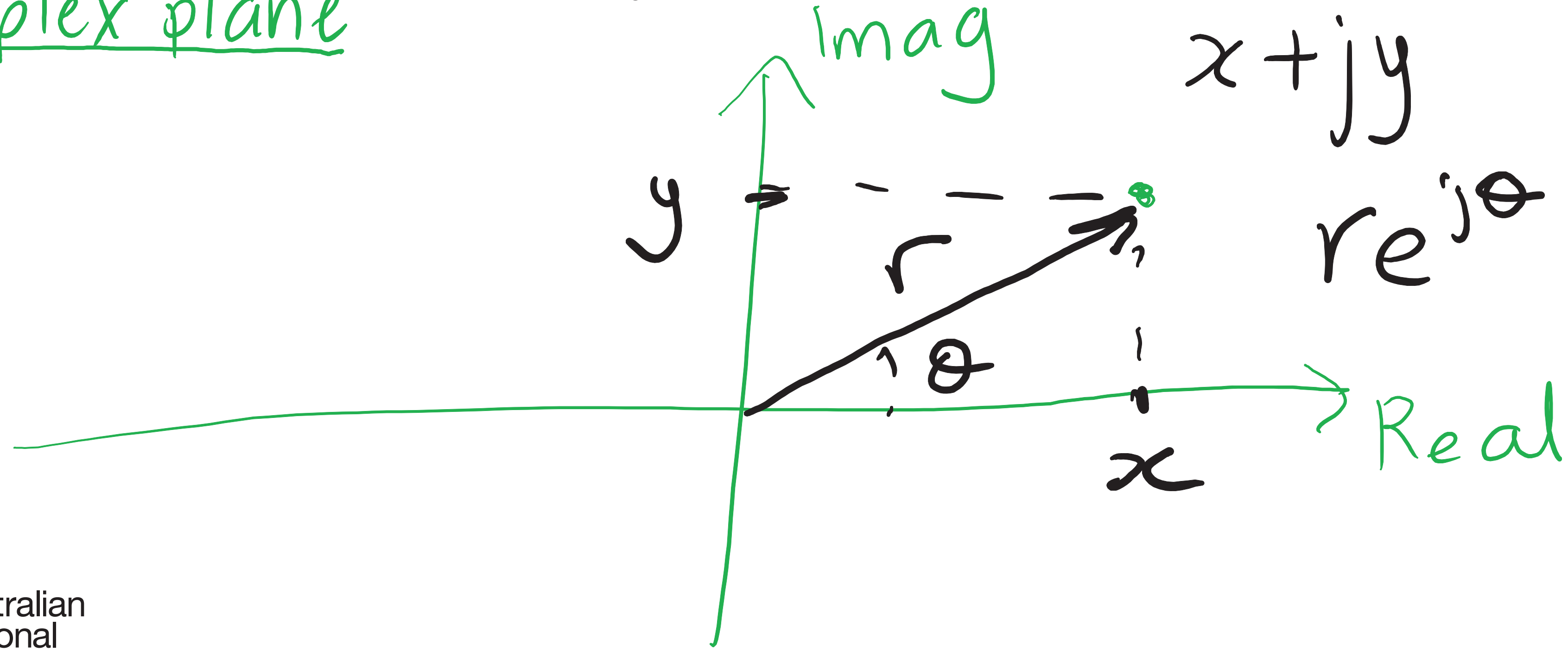
$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$-\pi < \theta \leq \pi$

Polar to rectangular:

$$x = r\cos\theta$$
$$y = r\sin\theta$$

complex plane



# Complex Number Revision– Rectangular and Polar Form

$$(\sqrt{3}+j^3)(1-j)$$

now $j^3 = j \times j^2 = -j$ so

$$\begin{aligned}(\sqrt{3}+j^3)(1-j) &= (\sqrt{3}-j)(1-j)\\ &= \sqrt{3}-j\sqrt{3}-j+j^2\\ &= (\sqrt{3}-1)-j(\sqrt{3}+1)\\ &= 2.828e^{-j1.308} = 2\sqrt{2}e^{-j\frac{5\pi}{12}}\end{aligned}$$

Either version of answer is fine.

In rectangular form:

$$z = x + jy$$

$$z^* = x - jy$$

change sign imaginary component

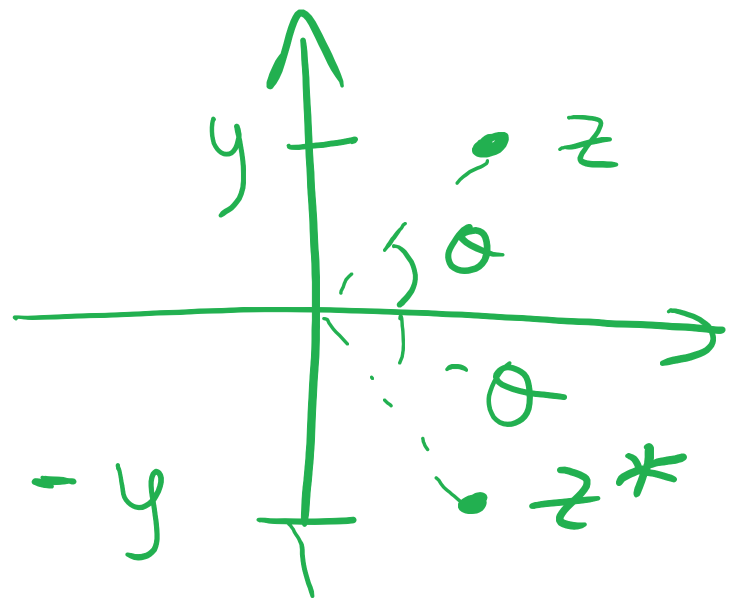
e.g. $z_1 = 4 - j3$, $z_1^* =$ $4 + j3$

In polar form:

$$re^{j\theta}$$

$$re^{-j\theta}$$

$z_1 = 5e^{j0.5\pi}$, $z_1^* =$ $5e^{-j0.5\pi}$



# Part 2 Outline

# CT Exponential Signals

- A fundamental signal class is the complex exponential signals

$$\boxed{x(t) = Ce^{\alpha t}}$$

with $C$ and $\alpha$ complex numbers ($C, \alpha \in \mathbb{C}$).

- This is a compact way to represent: pure sinusoids, real exponentials, exponentially growing or decaying sinusoids, etc. They are basic building blocks from which we can construct many signals of interest.
- With $C = |C|e^{j\theta}$ and $\alpha = r + j\omega_0$, we can write

Polar

rectangular form

$$\boxed{x(t) = |C|e^{rt}e^{j(\omega_0 t+\theta)}}$$

Parameter $|C|$ is the magnitude, $r$ is the exponential growth, $\omega_0$ is the fundamental frequency of the oscillation in rad/sec and $\theta$ is the phase offset.

- We should be comfortable with $j \triangleq \sqrt{-1}$ such that $j^2 = -1$.

## CT Exponential Signals

- Note, Euler's relation `O&W p.71`

$$e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$$

rectangula form - Euler's

which has fundamental period $\boxed{T_0 = 2\pi/|\omega_0|.}$

- Signal $x(t) = e^{j\omega_0 t}$ has finite, in fact unity, average power

$$
\begin{aligned}
P_\infty &= \lim_{T \longrightarrow \infty} \frac{1}{2T}\int_{-T}^{T} \left|x(t)\right|^2 dt \\
&= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left|e^{j\omega_0 t}\right|^2 dt \\
&= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} 1\, dt \\
&= 1
\end{aligned}
$$

and (necessarily) infinite total energy, $E_\infty = \infty$.

# CT Exponential Signals

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

- Many identities can be easily derived such as

$$A\cos(\omega_0 t + \phi) = \frac{A}{2}e^{j\phi}e^{j\omega_0 t} + \frac{A}{2}e^{-j\phi}e^{-j\omega_0 t}$$
$$= A\,\mathit{Re}\{e^{j(\omega_0 t + \phi)}\}$$

$$A\sin(\omega_0 t + \phi) = A\,\mathit{Im}\{e^{j(\omega_0 t + \phi)}\}$$

See `O&W 1.3.1 p.17`

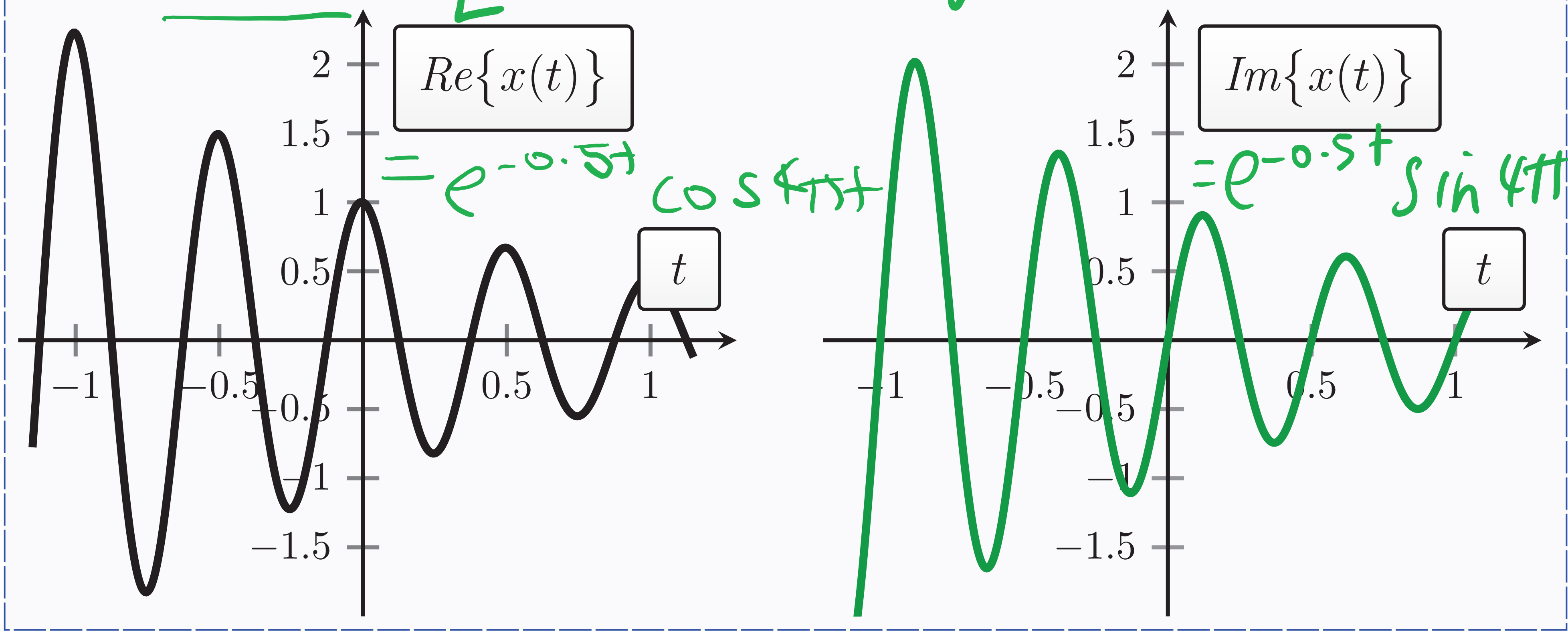
- Get comfortable and competent with such calculations.

# CT Exponential Signals – Example

With $C = 1$ and $\alpha = -0.5 + j4\pi$ in $x(t) = Ce^{\alpha t}$ we have

$$\boxed{x(t) \triangleq \exp\left(4\pi j\, t\right)\exp(-0.5\, t) \in \mathbb{C}}$$

$$= e^{-0.5t}\left[\cos 4\pi t + j \sin 4\pi t\right]$$




$= e^{-0.5t}\cos 4\pi t$

$= e^{-0.5t}\sin 4\pi t$

# CT Exponential Signals – Periodicity

$$x(t) = e^{j\frac{2\pi}{3}t}$$

- Is $x(t)$ periodic?
- Expanding $\boxed{x(t) = \cos(\frac{2\pi}{3}t) + j\sin(\frac{2\pi}{3}t).}$
- Real and imaginary parts periodic.
- Hence $x(t)$ periodic.
- Comparing with $e^{j\omega_0 t}$, $\omega_0 = \frac{2\pi}{3}$ so $T_0 = 3$.

$$T_0 = \frac{2\pi}{\omega_0} = \frac{\cancel{2\pi}}{2\pi/3} = 3$$

# Part 2 Outline

# DT Exponential Signals



- A fundamental signal class is the class of DT complex exponential signals

$$\boxed{x[n] = C\alpha^n, \quad n \in \mathbb{Z}}$$

with $C$ and $\alpha$ complex numbers ($C, \alpha \in \mathbb{C}$).

- Generally DT signals emulate the properties of CT signals or vice versa.
- Slightly weird things can emerge for DT signals though. You should rely on mathematical analysis to resolve any confusion rather than trying to memorize any details. However, you should make a mental note to be wary.
- First we look at the case where $|\alpha| = 1$ which we can write in the form

$$\boxed{x[n] = e^{j\omega_0 n}, \quad n \in \mathbb{Z},}$$

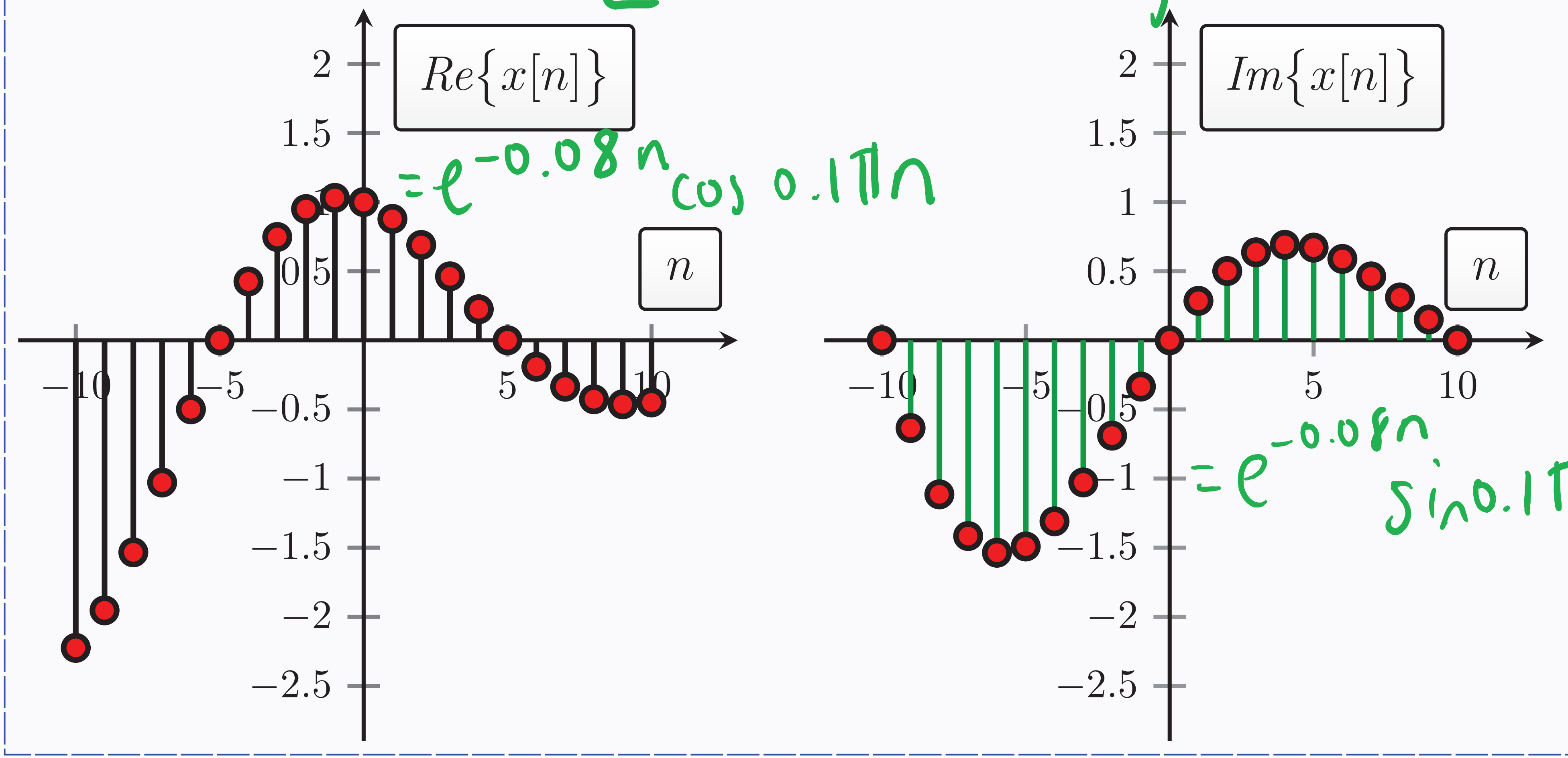
that is, $\underline{C = 1}$ and $\underline{\alpha = e^{j\omega_0}}$ for some $0 \leq \omega_0 < 2\pi$.

# DT Exponential Signals – Example

With $C = 1$ and $\alpha = -0.08 + j0.1\pi$ in $x[n] = Ce^{\alpha n}$ we have

$$x[n] \triangleq \exp\left(0.1\pi j\, n\right) \exp(-0.08\, n) \in \mathbb{C}$$

$$x[n] = e^{-0.08n}\left[\cos 0.1\pi n + j \sin 0.1\pi n\right]$$




$$= e^{-0.08n} \cos 0.1\pi n$$

$$= e^{-0.08n} \sin 0.1\pi n$$

# Part 2 Outline

# Periodicity of DT Exponential Signals

- Note that CT signal $x(t) = e^{j\omega_0 t}$ is **periodic** (with fundamental period $T_0 = 2\pi/|\omega_0|$) for all values of $\omega_0 \neq 0$). This is not quite true of the analogous DT exponential signal.
- For
$$x[n] = e^{j\omega_0 n}, \quad n \in \mathbb{Z}$$
to be periodic we need $x[n] = x[n+N]$ for some period $N$ (which may or may not be the fundamental period $N_0$).
- The issue is that $N$ may not be, and in general is not likely to be, synchronized with the period implicit in $\omega_0$ which is $2\pi/|\omega_0|$ and is generally not an integer.
- Generally $\boxed{x[n] = e^{j\omega_0 n}}$ is not periodic.
- Only when $\omega_0$ is a rational multiple of $2\pi$.

# Periodicity of DT Exponential Signals

$$\boxed{x[n] = e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{4}n}}$$

- $\omega_{01} = \frac{2\pi}{3}$
- Therefore $\boxed{N_1 = m\frac{2\pi}{\omega_{01}}} = m\frac{2\pi}{2\pi/3} = 3m, \quad N_0 = 3 \ (m=1)$
- $\omega_{02} = 3\pi/4$
- Therefore $N_2 = m\frac{2\pi}{\omega_{02}} = m\frac{\cancel{2\pi}}{3\cancel{\pi}/4} = \frac{8m}{3}, \quad N_0 = 8 \ (m=3)$
- Fundamental period, $N_0 = 24$ samples

# Part 2 Outline

# Signal Transformations–dependent variable

$x$

Already familiar with transformations of the signal such as:

| Operation | CT | DT |
|---|---|---|
| Amplitude scaling (amplifier) | $y(t) = Cx(t)$ | $y[n] = Cx[n]$ |
| Addition (summing amplifier) | $y(t) = x_1(t) + x_2(t)$ | $y[n] = x_1[n] + x_2[n]$ |
| Multiplication | $y(t) = x_1(t)x_2(t)$ | $y[n] = x_1[n]x_2[n]$ |

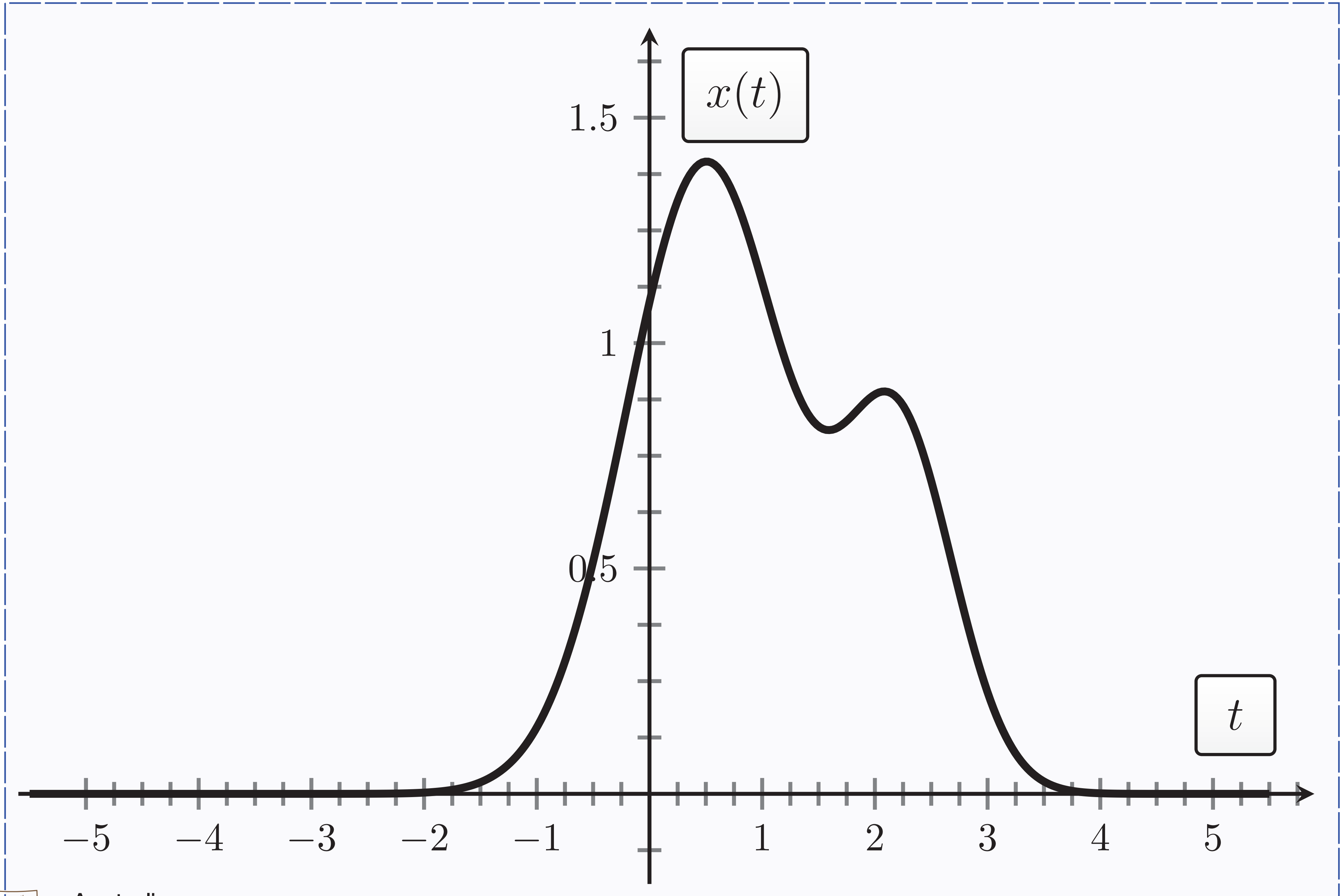
# Signal Transformations (independent var.)



$t$, $n$

- A signal $x(t)$ at some point transferred to another point (e.g., via communication) would generally suffer a time delay, $x(t - \Delta)$. This and other simple scenarios defines a simple but important class of **signal transformations** which are easily characterized.
- These signal transformations can be understood in terms of "affine" transformations on the independent variable
- For time as the independent variable, this just means scaling (compressing or expanding), reversing and shifting (delaying or advancing) time
- Other signal transformations not of this form are possible and treated later.

# Signal Transformations (CT reference signal)



## Time Shift

Independent variable change

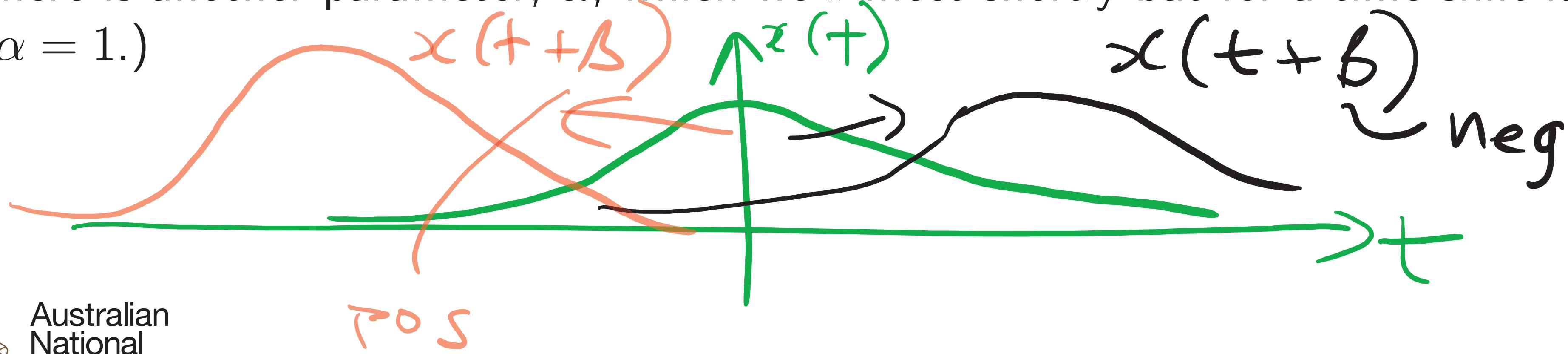
$$t \longrightarrow t + \beta$$

induces the signal change

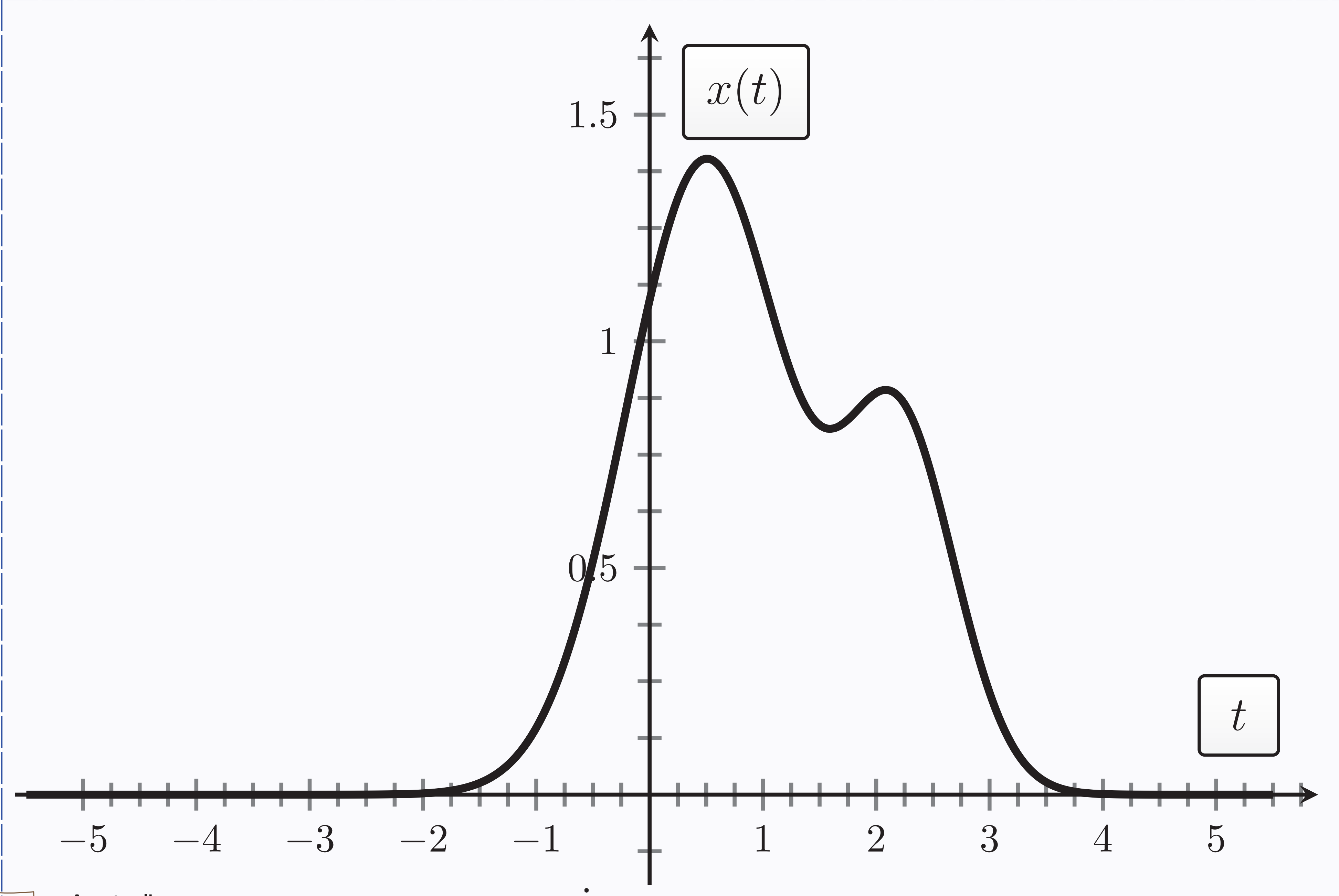
$$x(t) \longrightarrow y(t) \triangleq \underline{x(t+\beta)},$$

where $\beta \in \mathbb{R}$. If $\beta < 0$ then signal is delayed (shifts to the right), if $\beta = 0$ then no change, and if $\beta > 0$ then signal is advanced (shifts to the left).

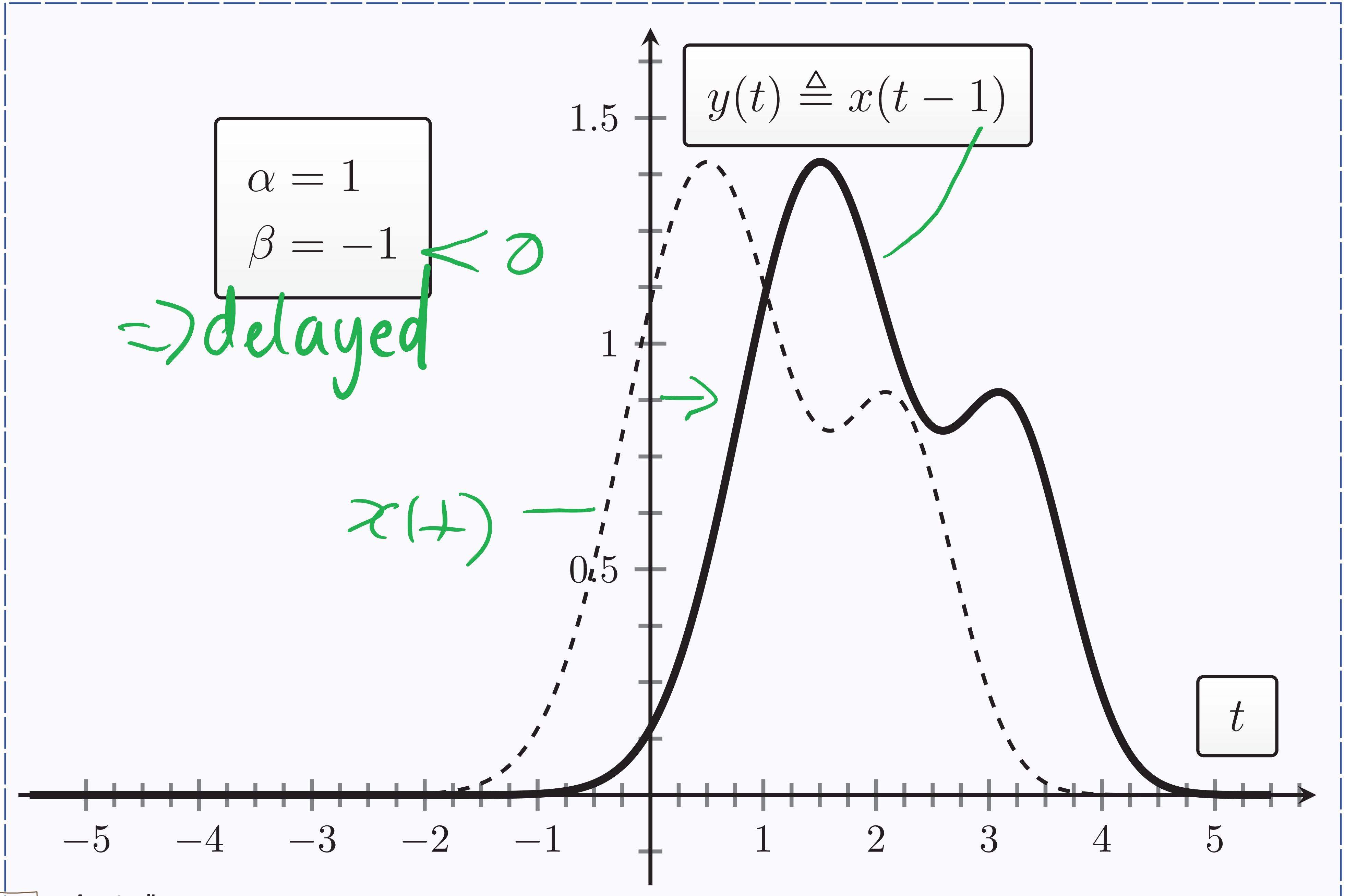
(There is another parameter, $\alpha$, which we'll meet shortly but for a time shift it is $\alpha = 1$.)



# Signal Transformations (CT time shift)



# Signal Transformations (CT time shift)



## Time Scaling

Independent variable change

$$t \longrightarrow \alpha t$$

induces the signal change

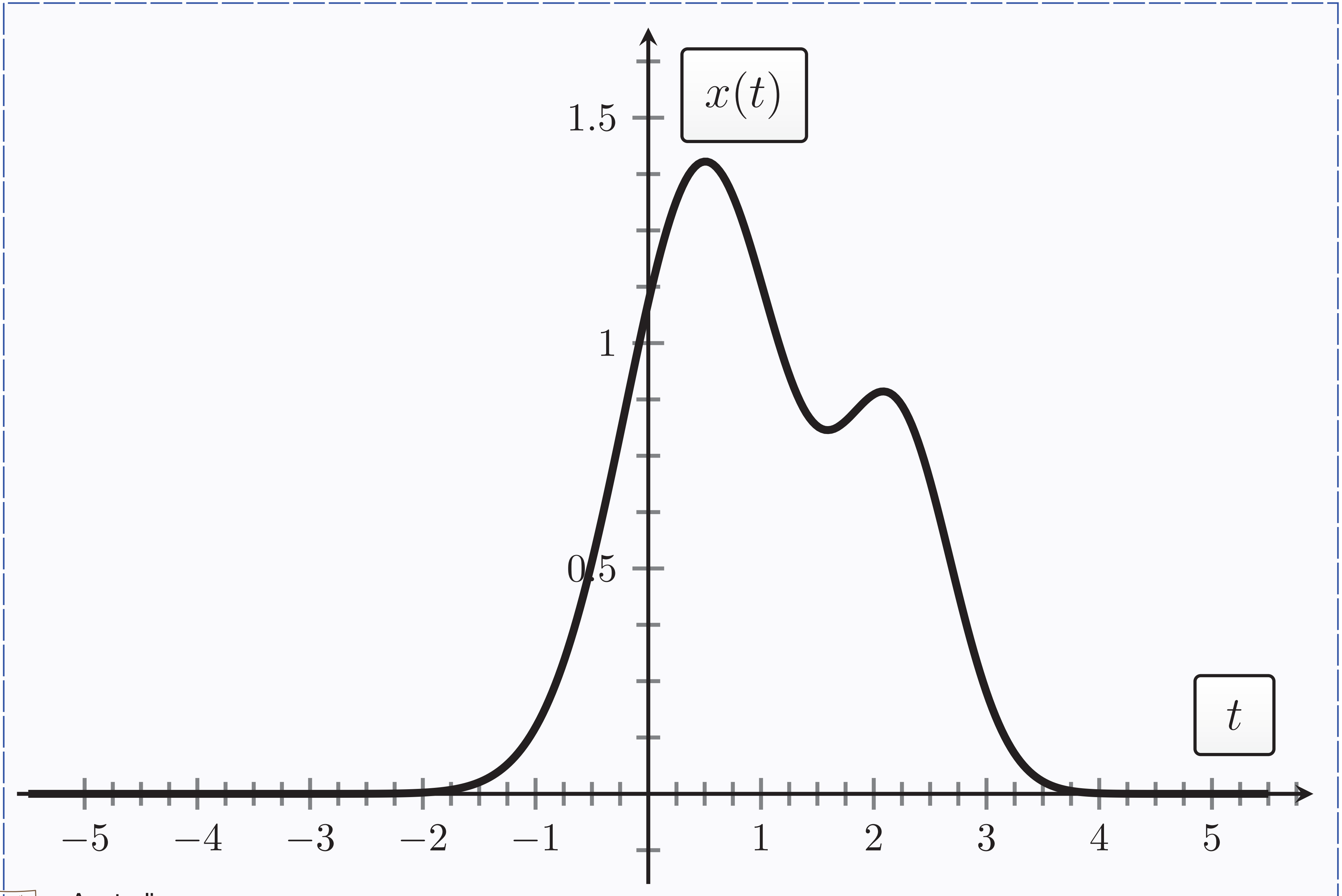
$$x(t) \longrightarrow y(t) \triangleq x(\alpha t),$$

where $\alpha \in \mathbb{R}$ and $\alpha \neq 0$.
If $\alpha > 1$ then the signal is compressed, if $\alpha < 1$ then the signal is expanded.

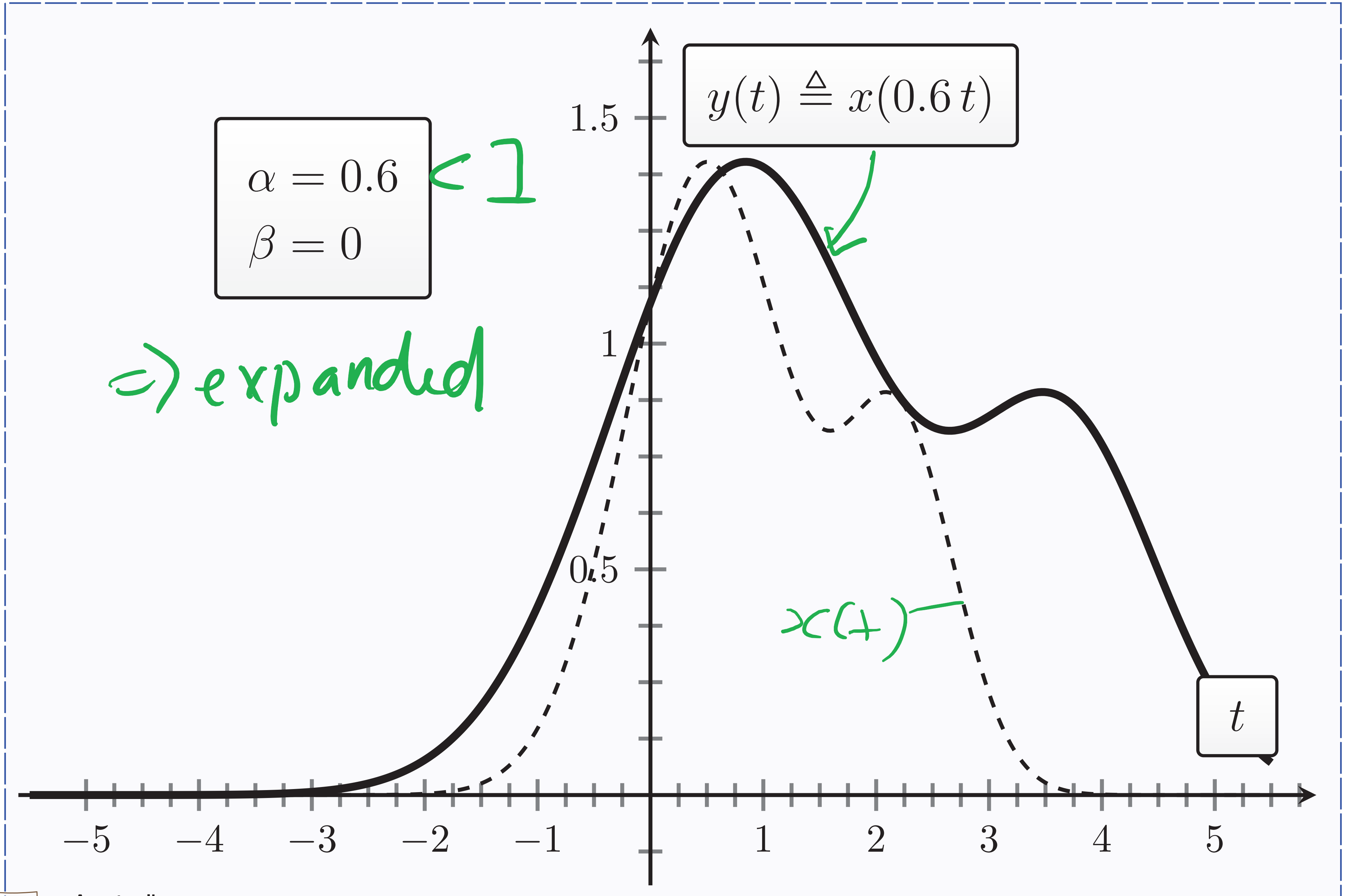
thinner

wider

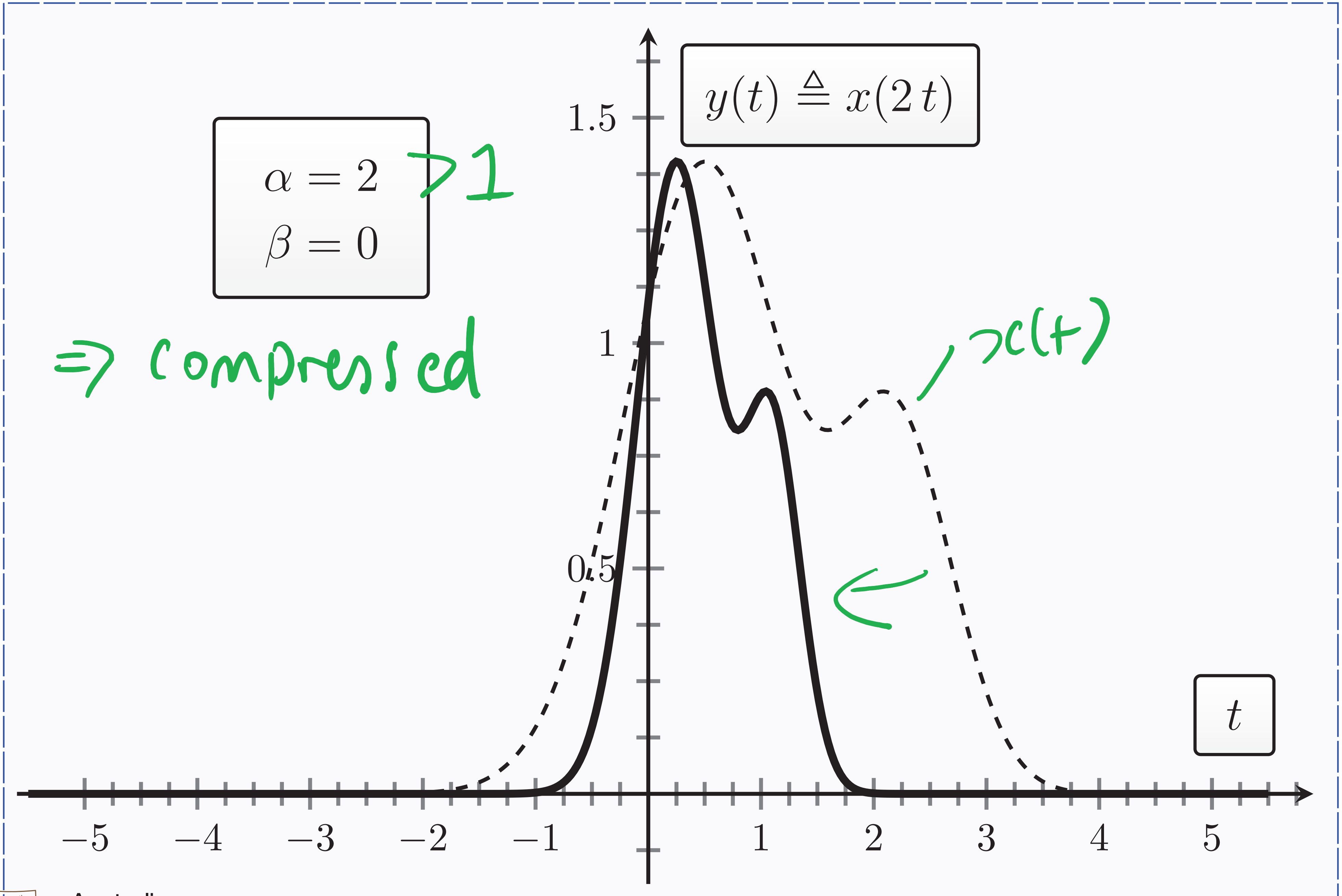
# Signal Transformations (CT time scaling)



# Signal Transformations (CT time scaling - expand)



# Signal Transformations (CT time scaling - compress)



# Signal Transformations (CT time reversal)

**Time Reverse**

Independent variable change

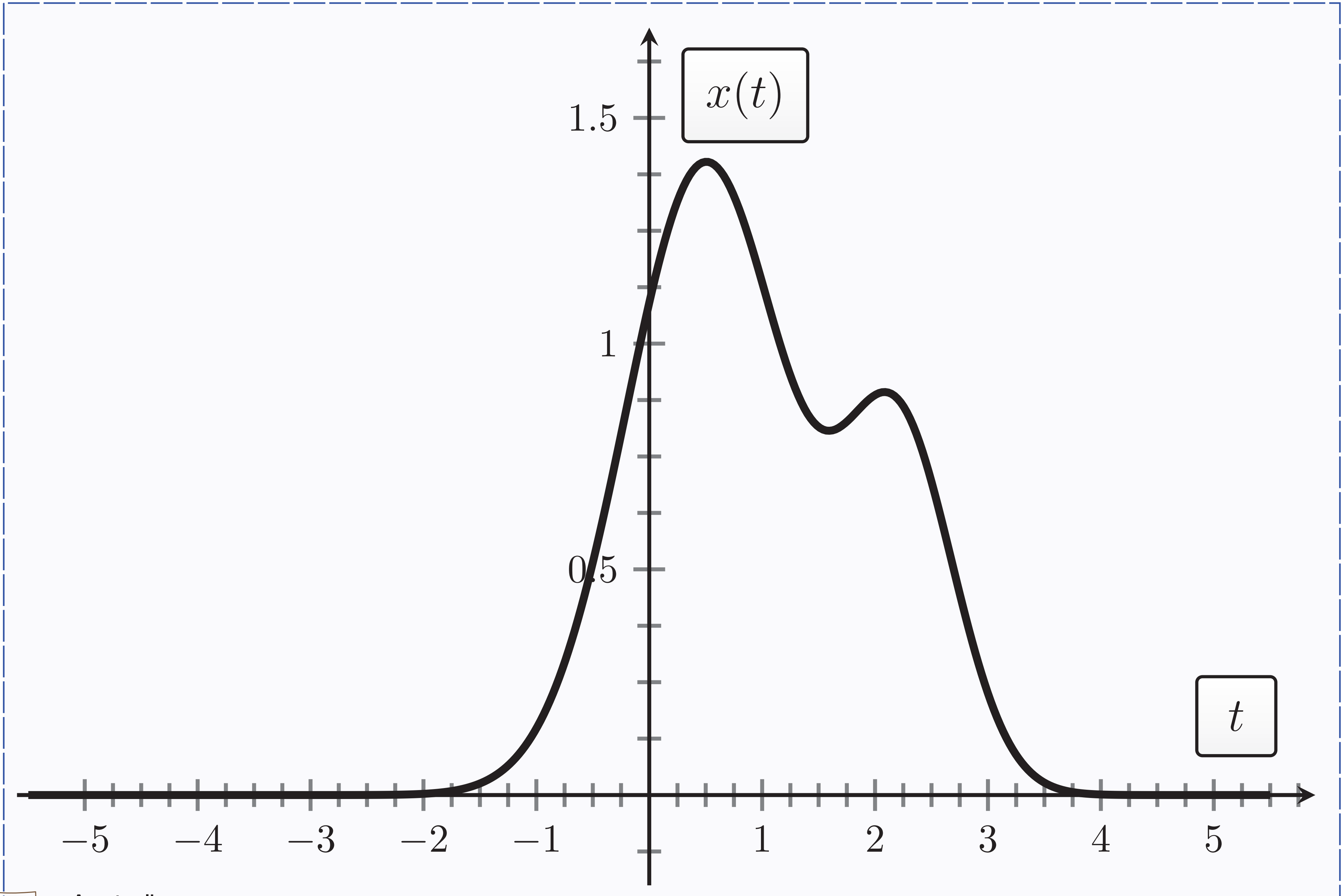
$$t \longrightarrow -t$$

induces the signal change

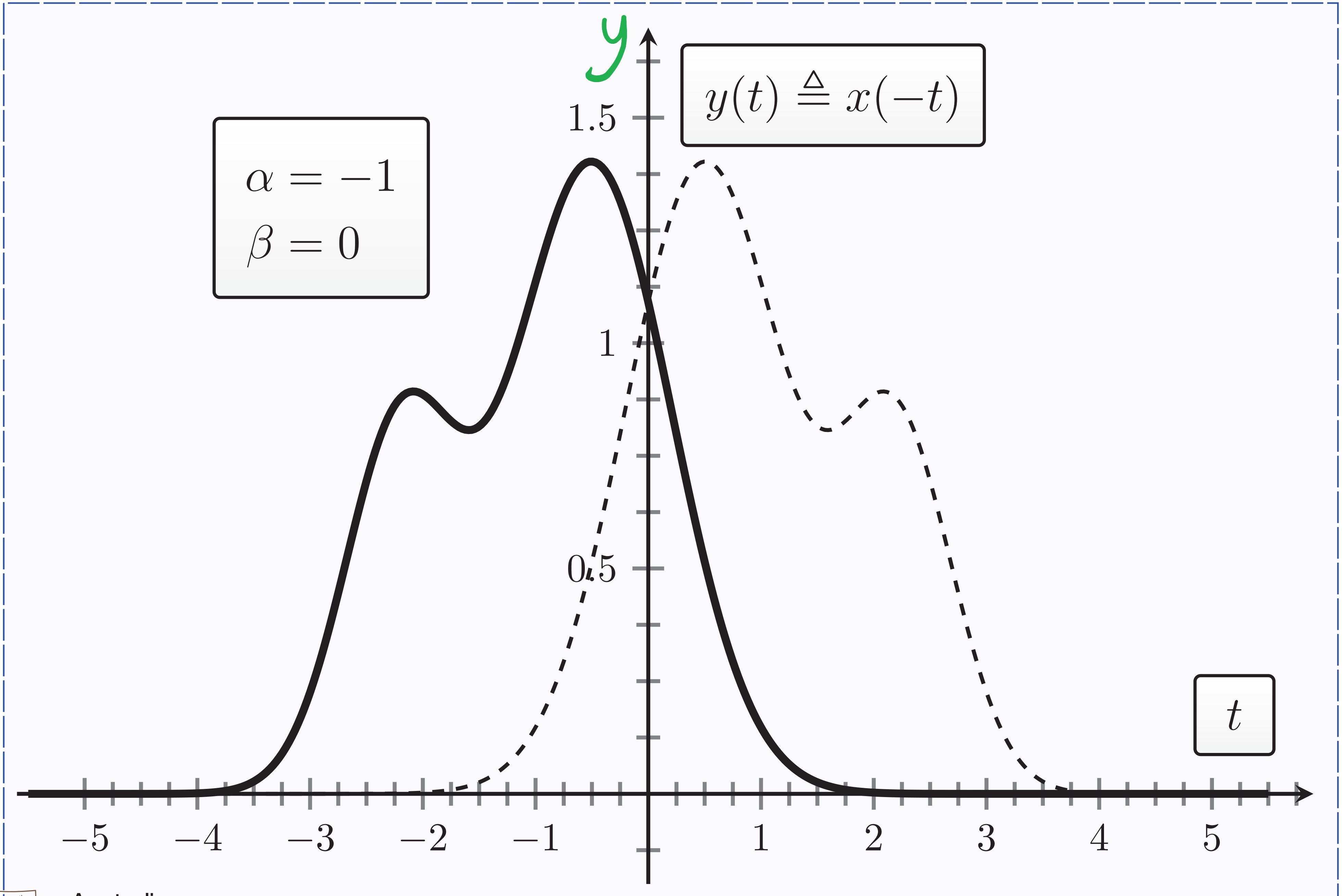
$$x(t) \longrightarrow y(t) \triangleq x(-t)$$

Where $\alpha = -1$.

# Signal Transformations (CT time reversal)



# Signal Transformations (CT time reversal)



### Affine Transformation[2]

Independent variable change

$$t \longrightarrow \boxed{\alpha\, t + \beta}$$

induces the signal change

$$x(t) \;\longrightarrow\; y(t) \triangleq x(\alpha\, t + \beta)$$

where

$\alpha \in \mathbb{R}$ **time scales** expands whenever $|\alpha| < 1$ and
compresses whenever $|\alpha| > 1$ and/or
**time-reverses** whenever $\alpha < 0$

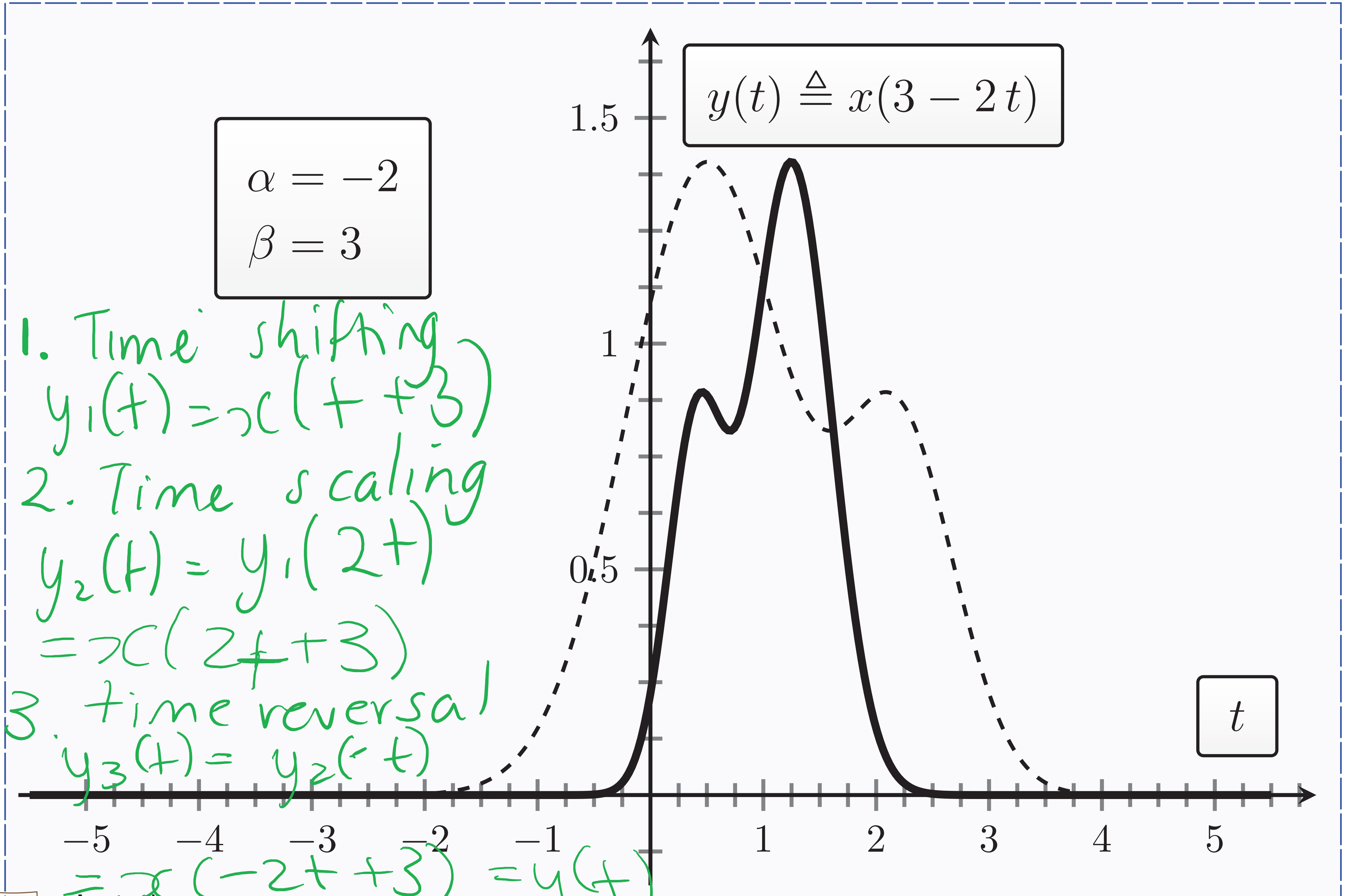
$\beta \in \mathbb{R}$ **time shifts** forward in time whenever $\beta < 0$ and
backward in time whenever $\beta > 0$

[2] affine $\equiv$ linear $+$ constant

# Signal Transformations (CT affine transformation)

- Single framework captures three transformations in one.
- Apply transformations in this order:
  1 - Apply time shift.
  2 - Then time scaling.
  3 - Finally time reversal.
- Check answer by substituting in a range of values of the independent variable.

# Signal Transformations (CT affine time change)




1. Time shifting

$y_1(t) = x(t+3)$

2. Time scaling

$y_2(t) = y_1(2t)$

$= x(2t+3)$

3. time reversal

$y_3(t) = y_2(-t)$

$= x(-2t+3) = y(t)$

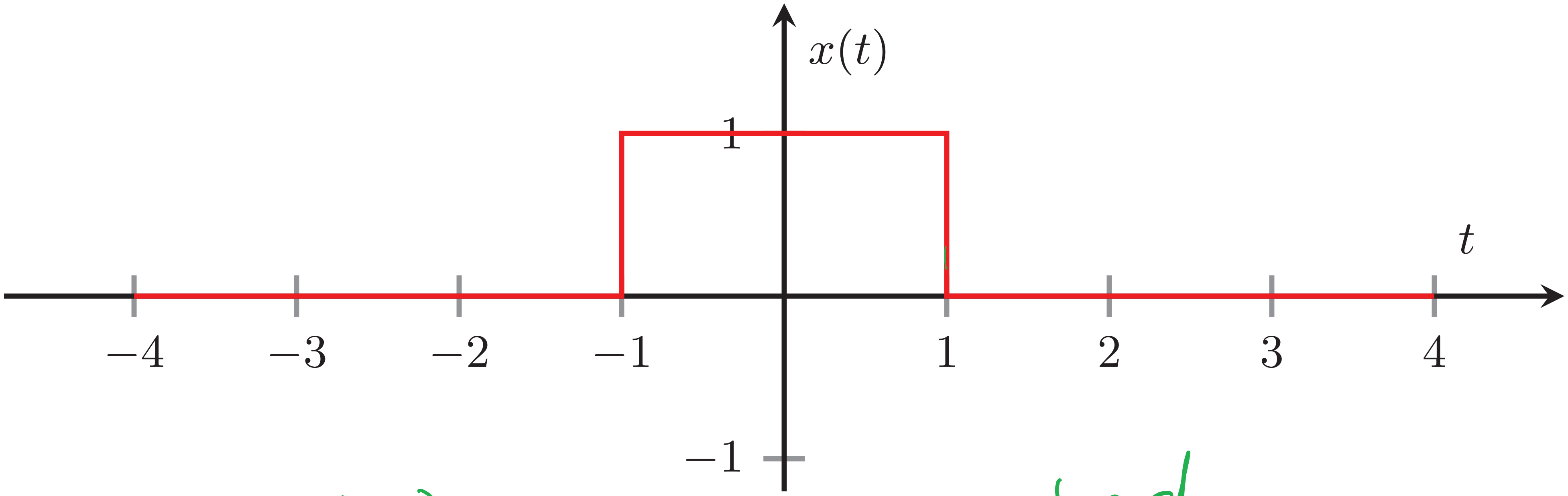
# Signal Transformations (independent variable)

- To this point we have only considered transformations

$$x(t) \longrightarrow y(t) \triangleq x(\alpha\, t + \beta)$$

  which affinely transform the independent variable.
- But this **excludes** other simple signal transformations such as

$$x(t) \longrightarrow y(t) \triangleq 3\, x(t)$$
$$x(t) \longrightarrow y(t) \triangleq x(t) + 2.7$$
$$x(t) \longrightarrow y(t) \triangleq x(t^2)$$
$$x(t) \longrightarrow y(t) \triangleq \big(x(t)\big)^5$$
$$x(t) \longrightarrow y(t) \triangleq 23$$

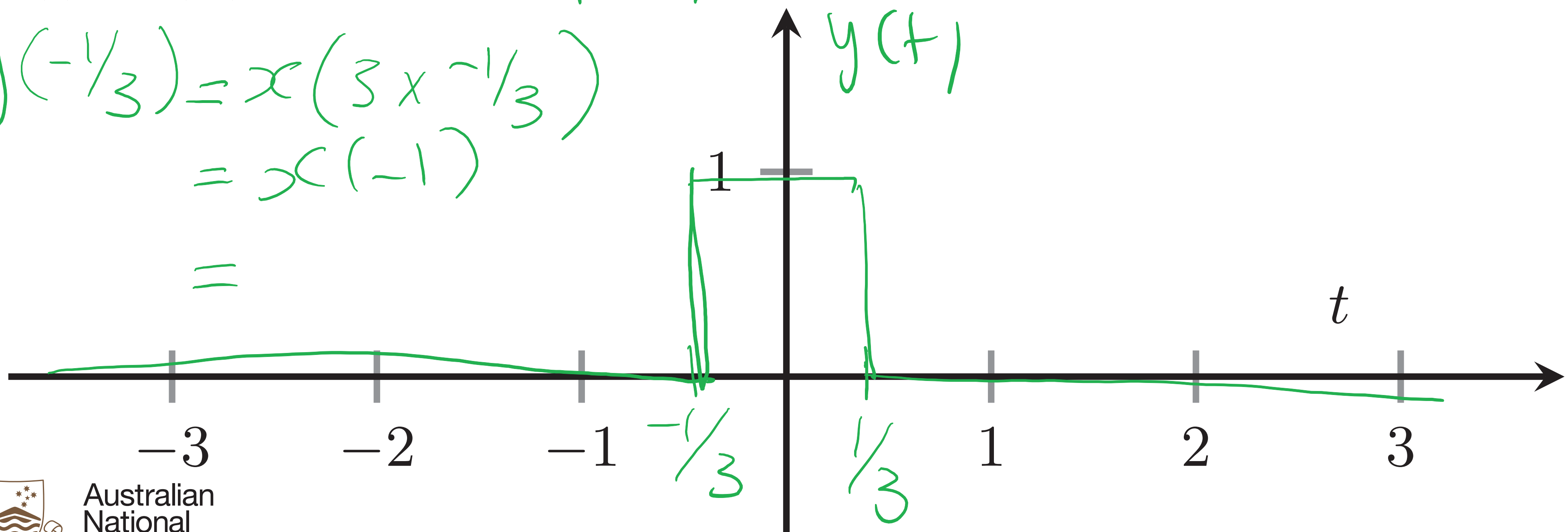



$y(t) = x(3t)$: $\alpha = 3 > 1 \Rightarrow$ compressed

$y(-1/3) = x(3 \times -1/3)$

$= x(-1)$

$=$

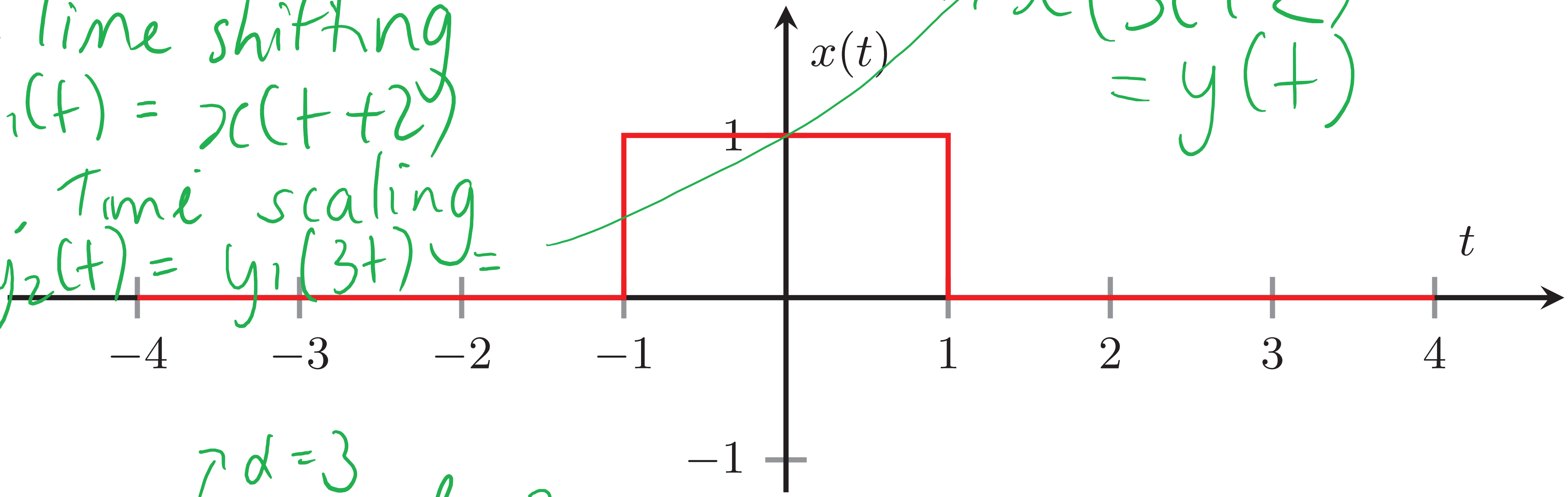


1. Time shifting

$y_1(t) = x(t+2)$

2. Time scaling

$y_2(t) = y_1(3t) =$ $x(3t+2)$ $= y(t)$

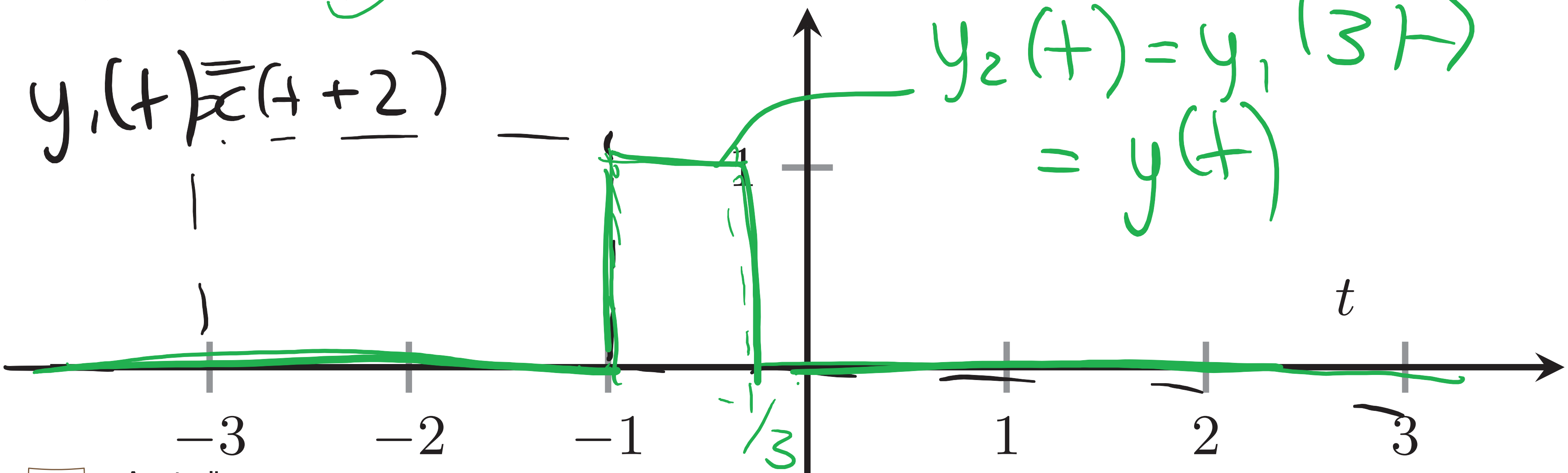



$\alpha = 3$

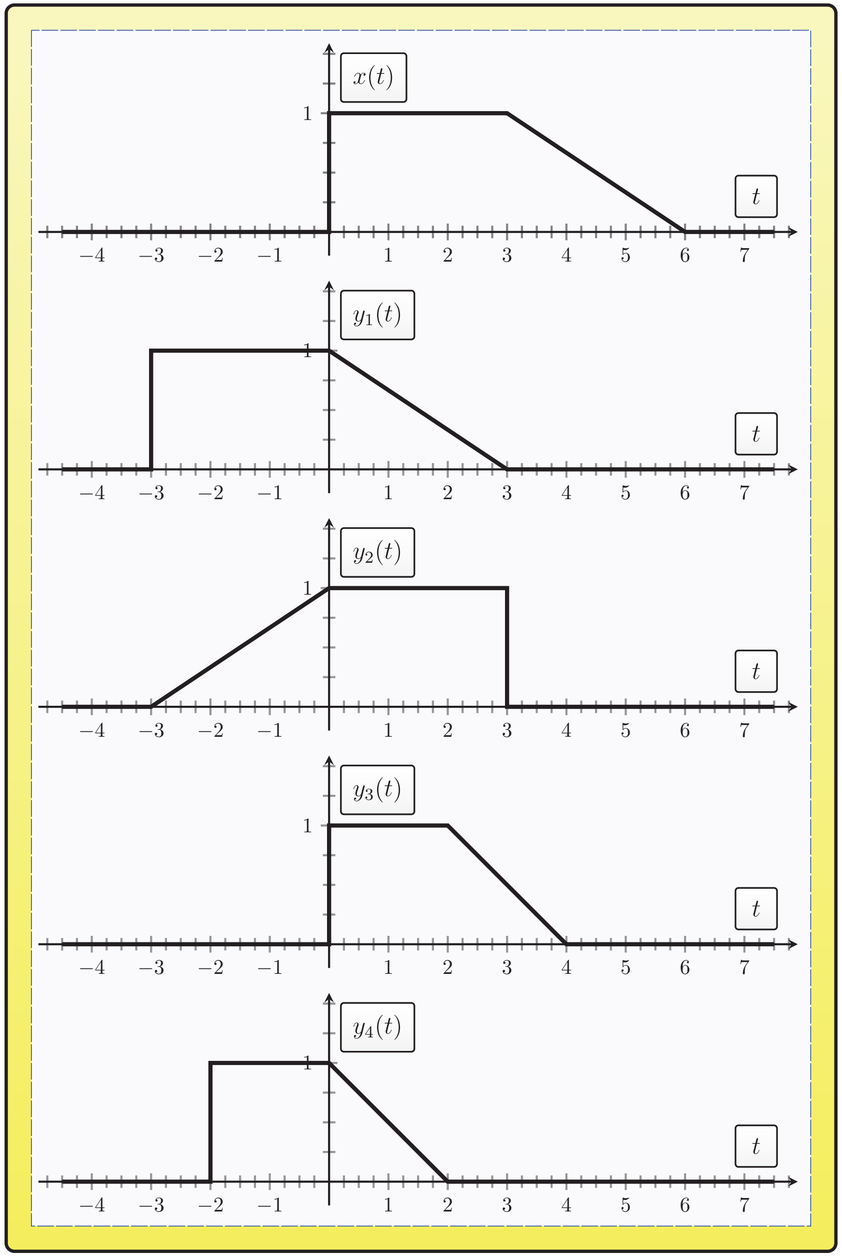
$y(t) = x(3t+2)$: $\beta = 2$

$y_1(t) = x(t+2)$

$y_2(t) = y_1(3t) = y(t)$

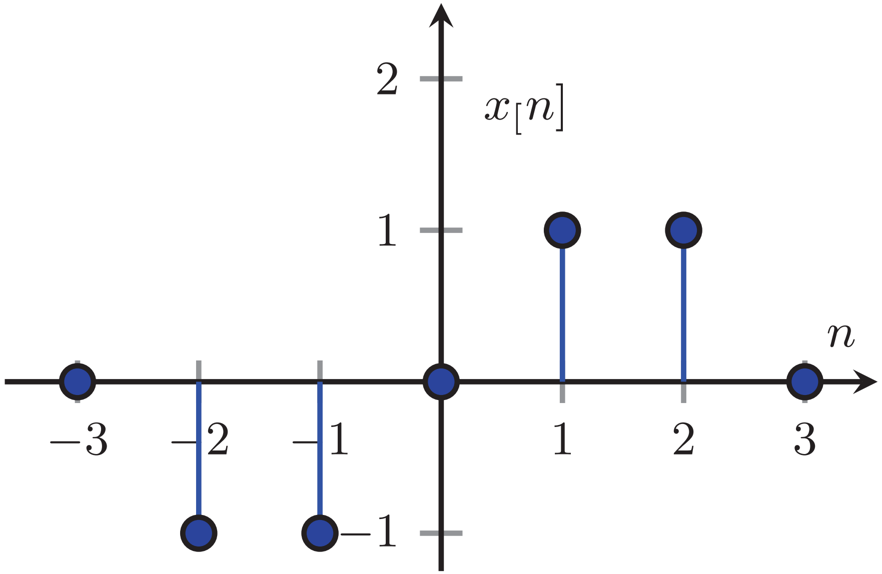


# Signal Transformations (CT examples)

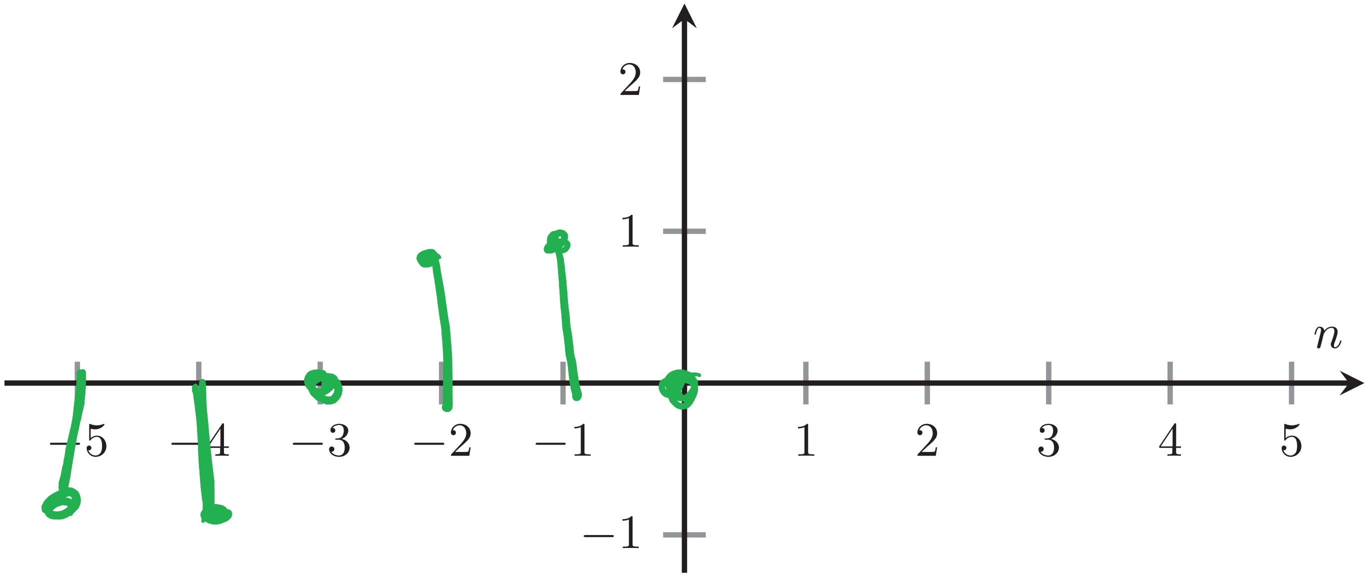



- $x(t)$
- $y_1(t) = x(t+3)$, that is, $\alpha = 1 \quad \beta = 3$
- $y_2(t) = x(-t+3)$, that is, $\alpha = -1 \quad \beta = 3$
- $y_3(t) = x(1.5\,t)$, that is, $\alpha = 1.5 \quad \beta = 0$
- $y_4(t) = x(1.5\,t+3)$, that is, $\alpha = 1.5 \quad \beta = 3$

# Signal Transformations (DT examples)



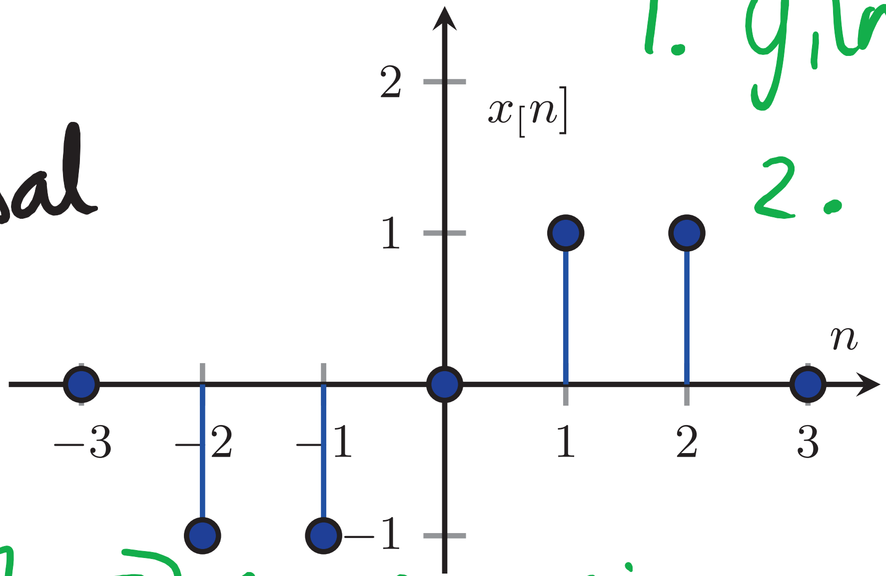

Draw $y[n] = x[n + 3]$: $\beta = 3 > 0 \Rightarrow$ advance



# Signal Transformations (DT examples)

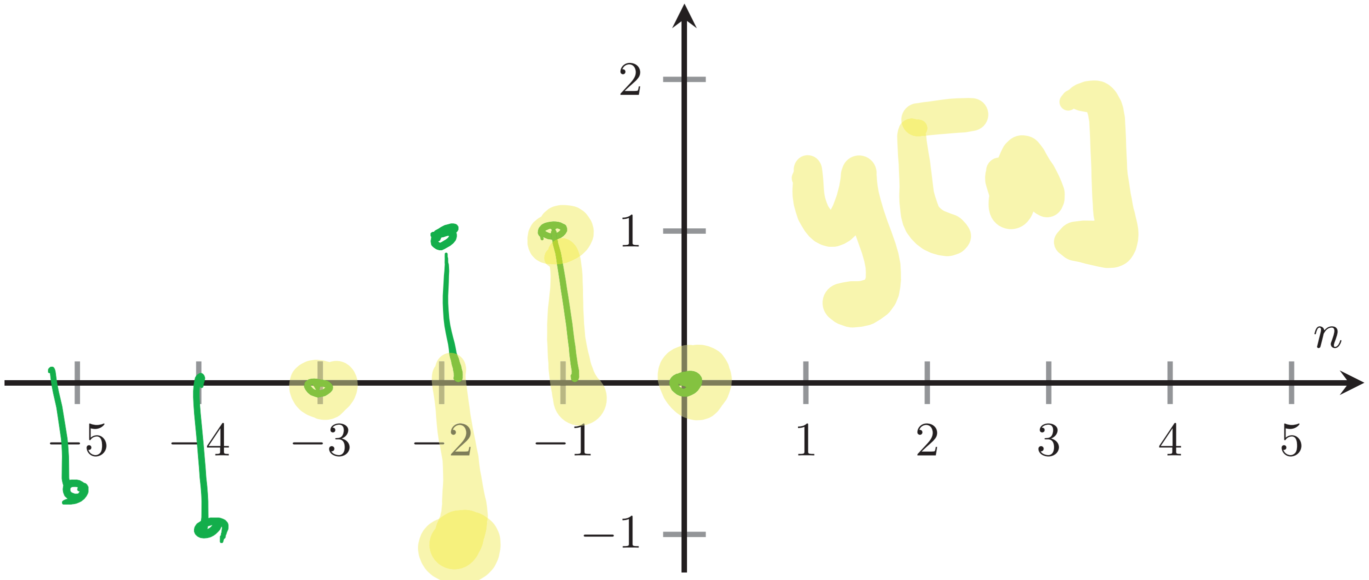
1. time shift
2. time scale
3. time reversal

1. $y_1[n] = x[n+3]$
2. $y_2[n] = y_1[2n] = x[2n+3] = y[n]$

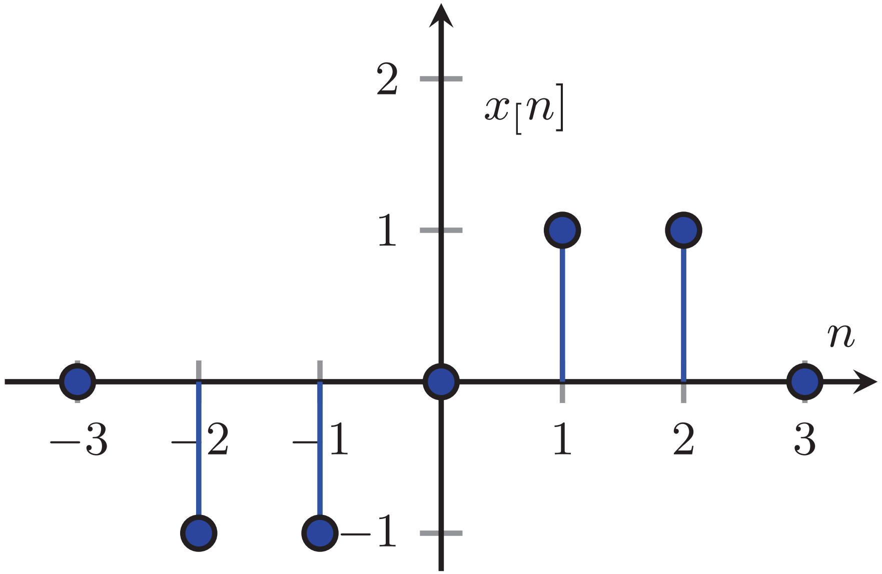



$\alpha = 2 > 1 \Rightarrow$ compressing

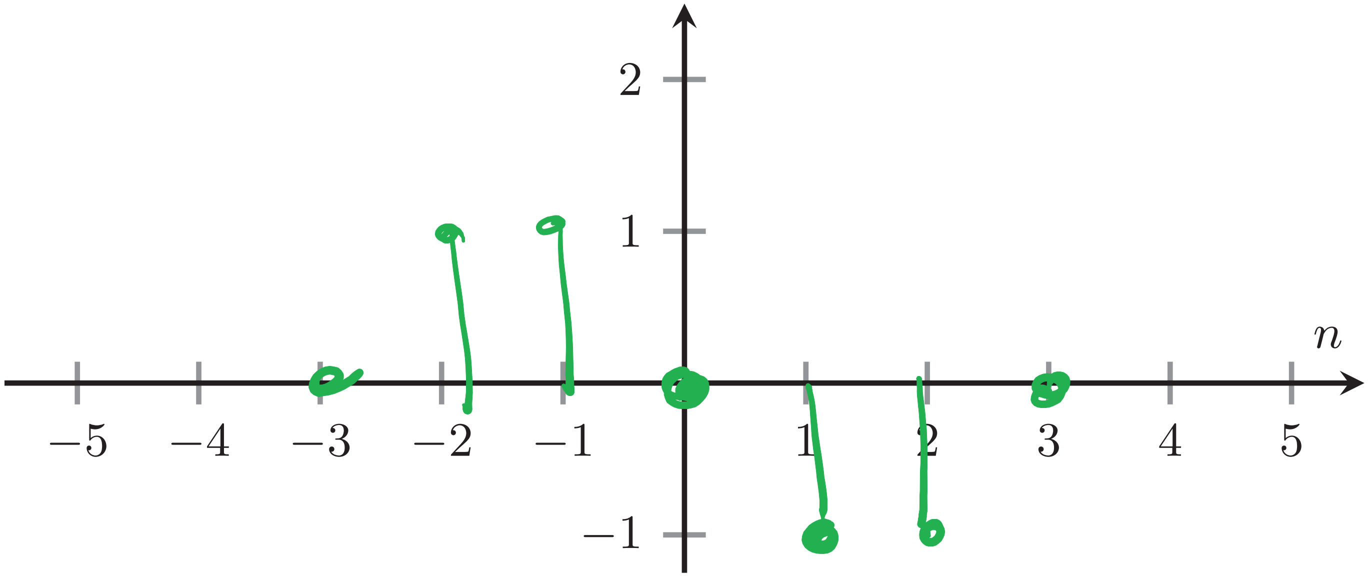
Draw $y[n] = x[2n+3]$:



# Signal Transformations (DT examples)



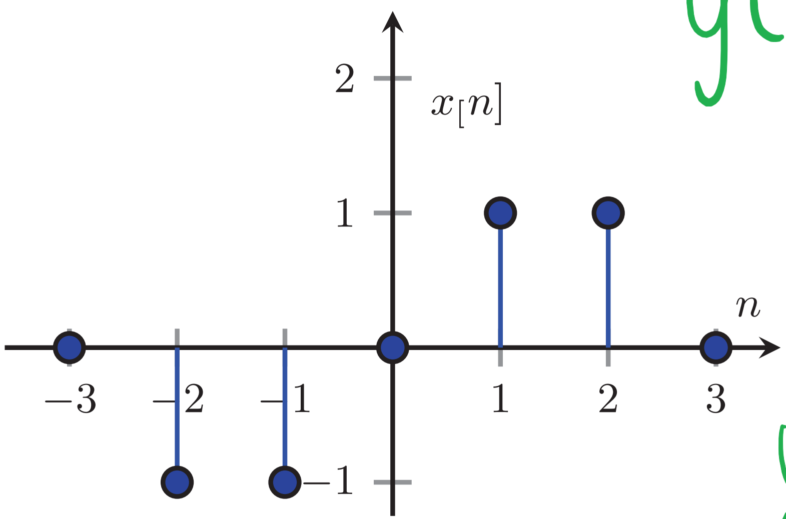

Draw $y[n] = x[-n]$:



# Signal Transformations (DT examples)

What about $x\left[\frac{n}{2}\right]$?

- Original samples remain.
- Then introduce more values by interpolation, zero-padding etc.
- Won't get questions on this as ambiguous.



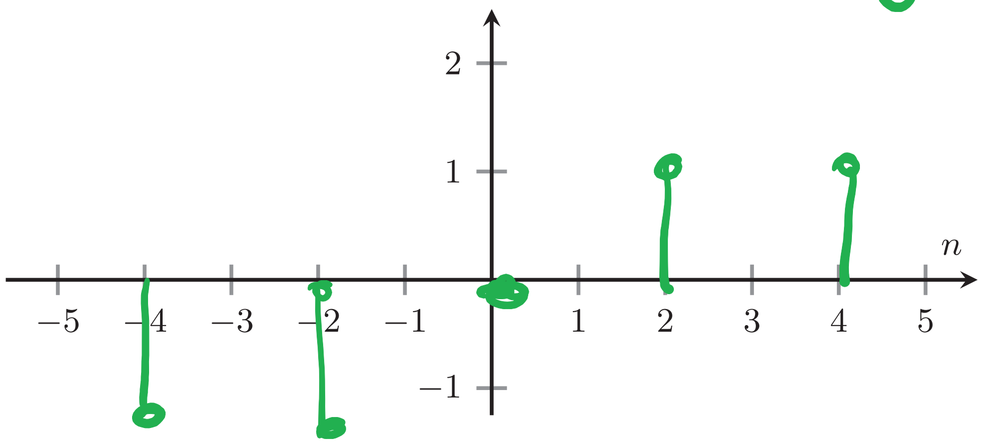

$y[0] = x\left[\frac{0}{2}\right]$

$= x[0]$

$y[1] = x\left[\frac{1}{2}\right]$

$y[2] = x\left[\frac{2}{2}\right]$

$= x[1]$



# Part 3 Outline

13 **Special Test Signals**

14 CT and DT Systems

15 Interconnections of Systems

16 System Examples
- Electrical
- Mechanical
- Thermal
- Edge Detector

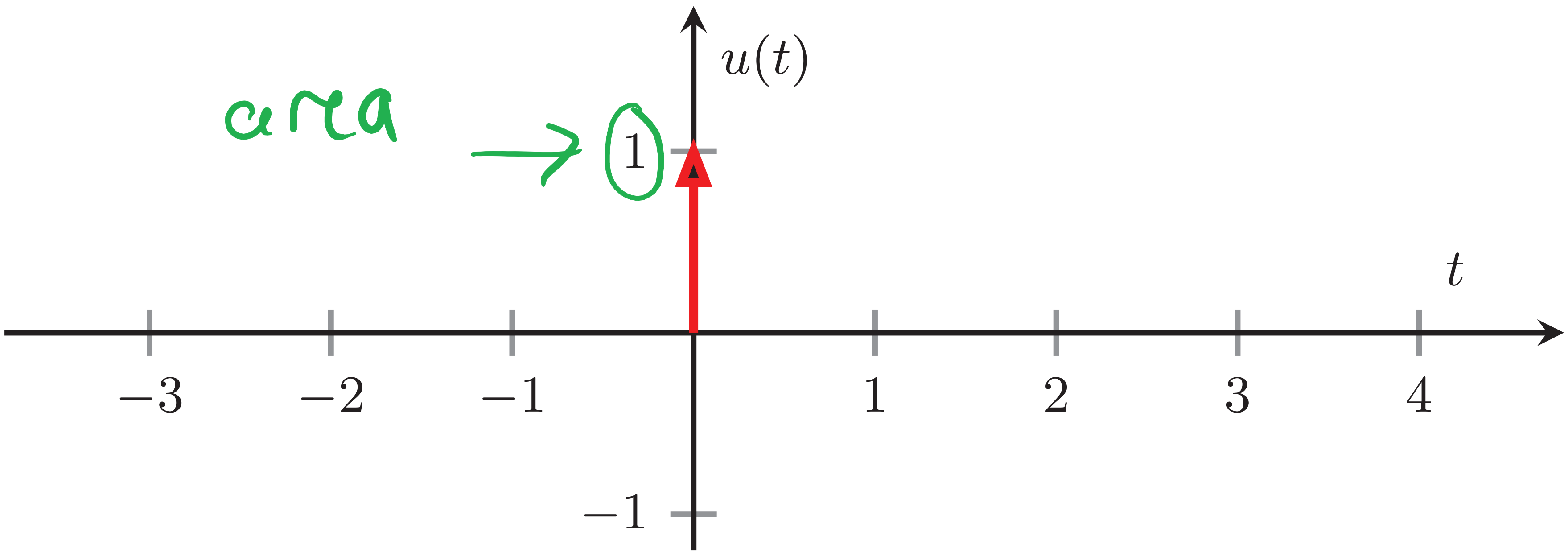
17 System Properties
- Causality
- Memory
- Time-Invariance
- Linear & Nonlinear
- Audio Example

# Special Test Signals - Impulse Function

Define the CT **impulse/delta function** (this is a signal) `O&W 1.4 pages 30-32`

$$\delta(t) \triangleq \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

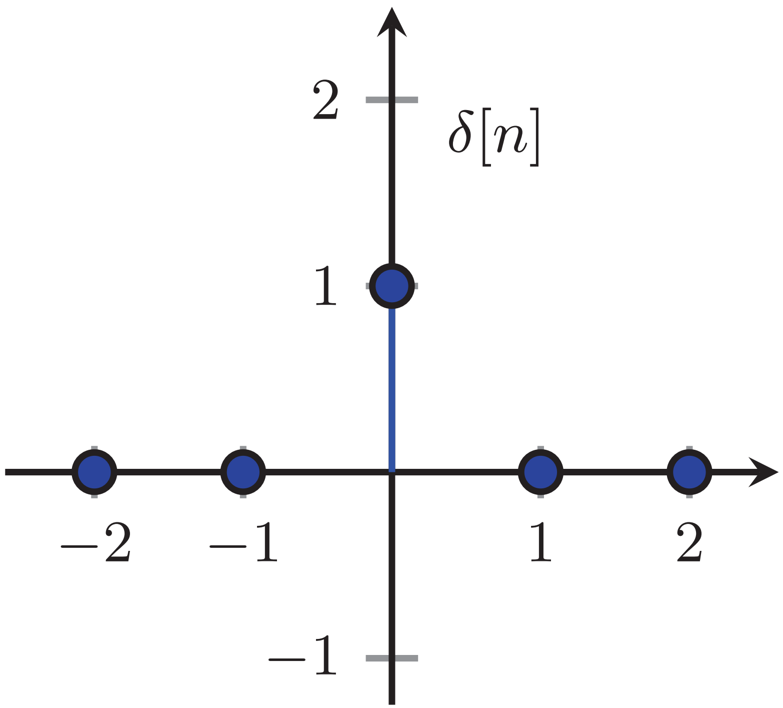
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$



# Special Test Signals - Impulse Function

Define **unit impulse/ sample** (this is a signal) `O&W 1.4 pages 30-32`

$$\delta[n] \triangleq \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

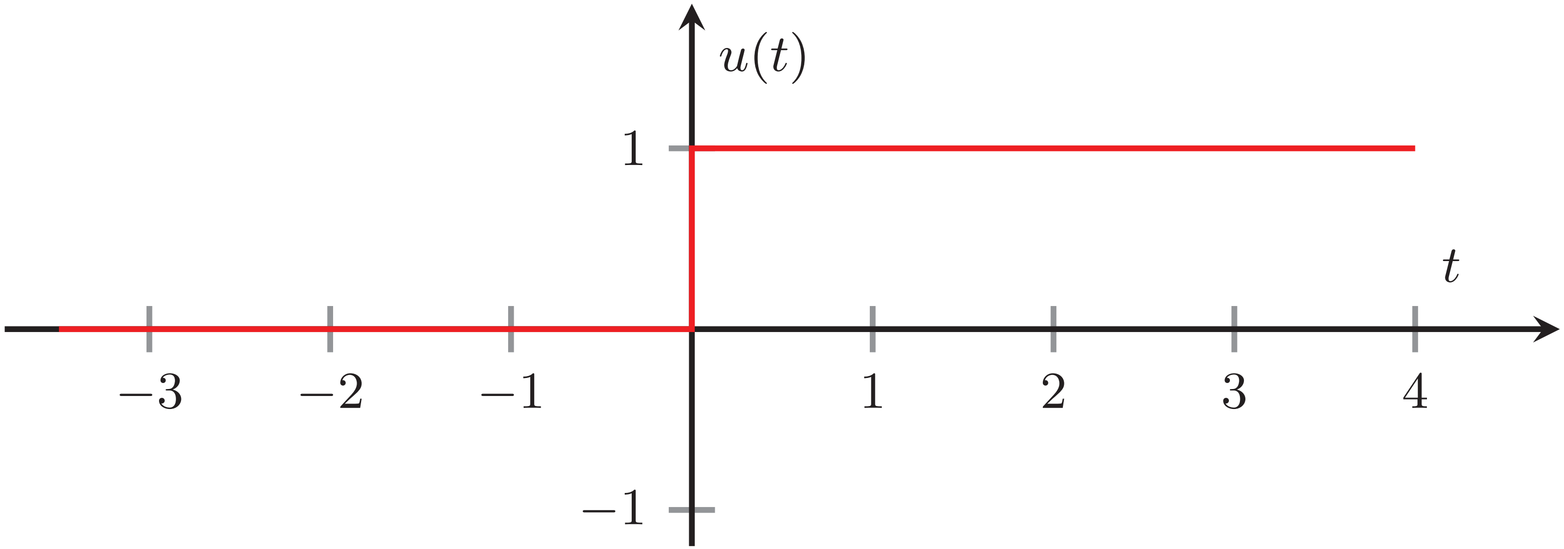


# Special Test Signals - Unit Step

- Define CT **unit step** (this is a signal) `O&W 1.4.2 pages 32-38`

$$u(t) \triangleq \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

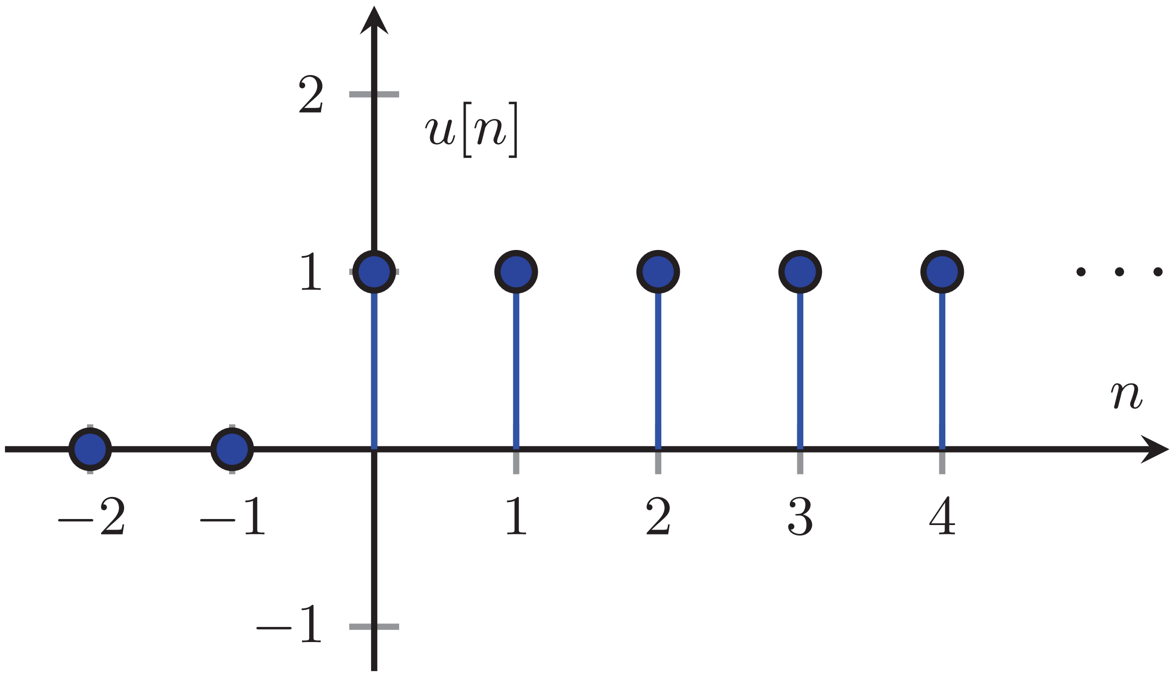
Can think of as a switch closing at $t = 0$.



# Special Test Signals - Unit Step

- Define **unit step** (this is a signal) `O&W 1.4.2 pages 32-38`

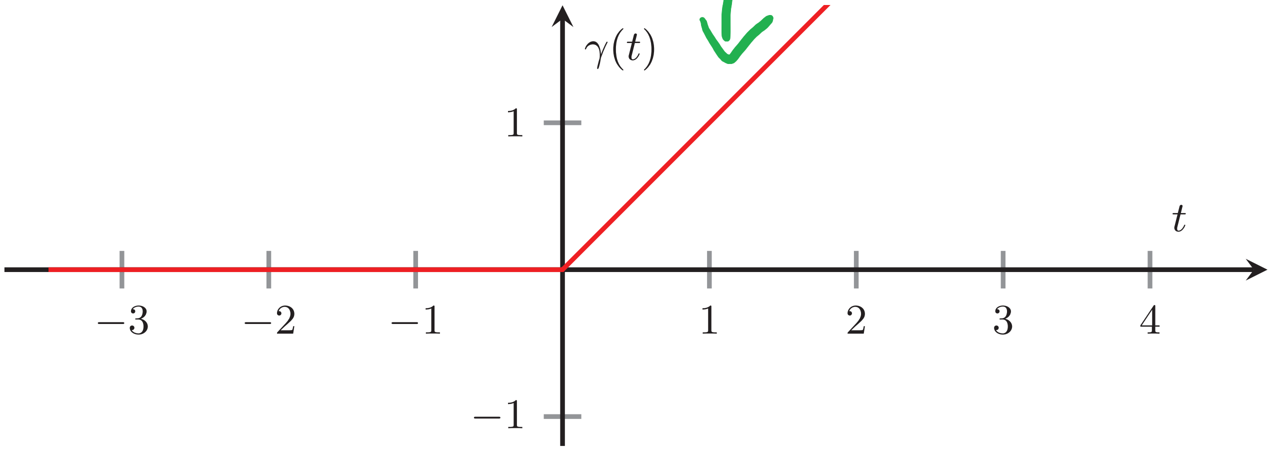
$$u[n] \triangleq \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

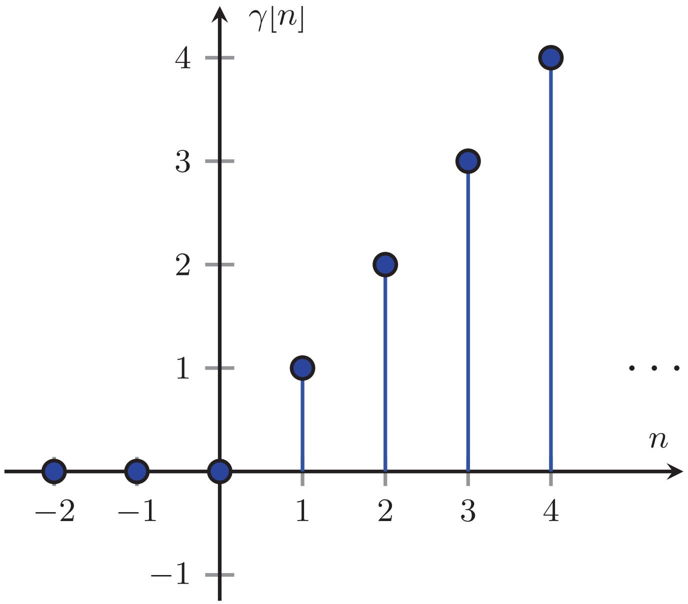


# Special Test Signals - Ramp Function

Is the integral of the step function

gradient = 1

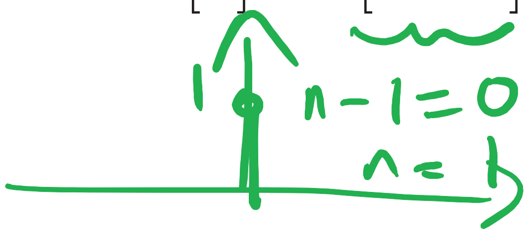


# Special Test Signals - Examples

$\beta = -1 < 0$

- $x[n] = \delta[\underbrace{n-1}] + \delta[\underbrace{n+2}]$



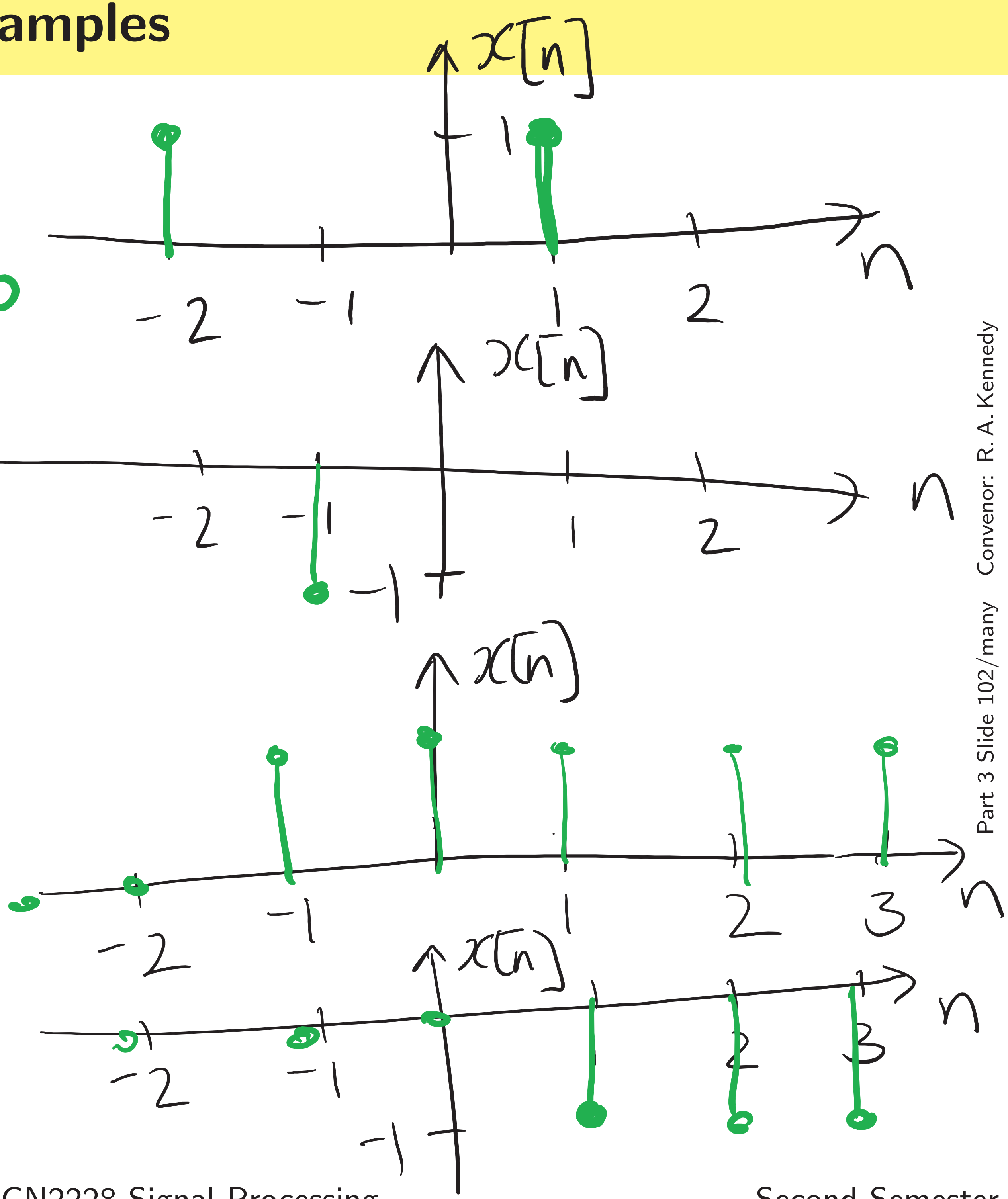

$n-1=0 \quad n+2=0$

$n=1 \quad n=-2$

- $x[n] = -\delta[\underbrace{n+1}_{n=-1}]$

- $x[n] = u[n+1]$

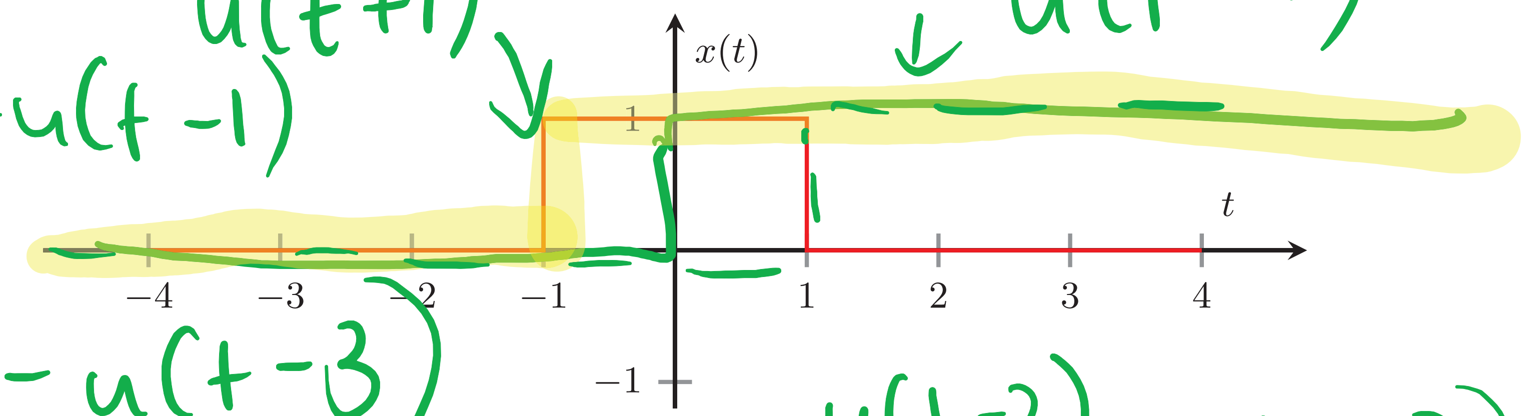
- $x[n] = -u[\underbrace{n-1}]$



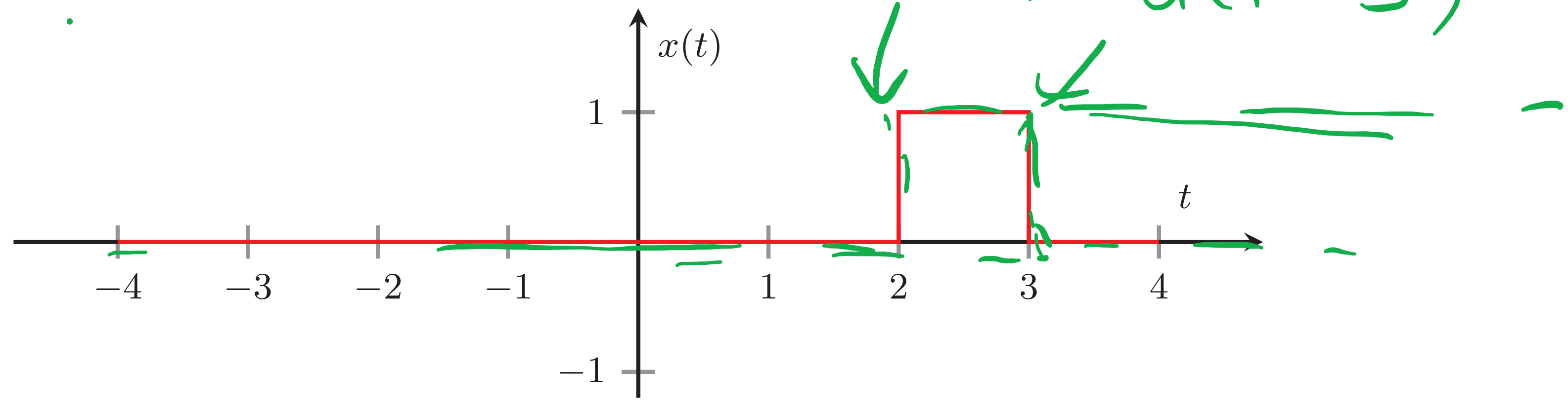
# Special Test Signals - Signal Representation

Decomposing functions into sum of shifted step functions e.g.:

$u(t+1) - u(t-1)$



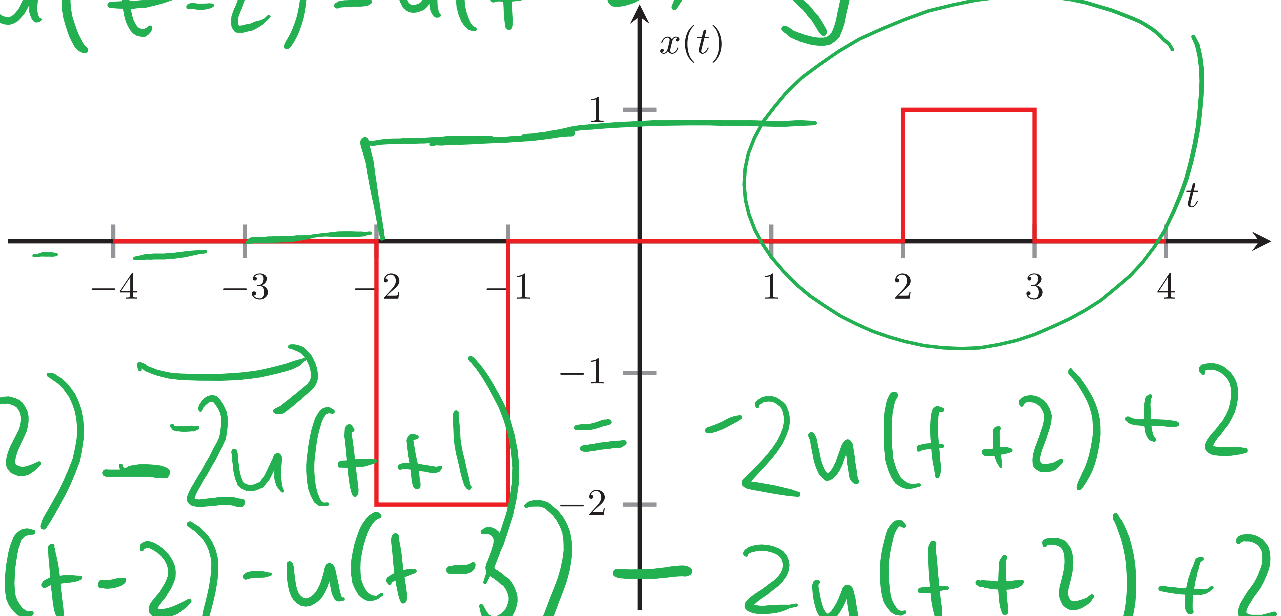

$u(t-2) - u(t-3)$



# Special Test Signals - Signal Representation

Decomposing functions into sum of scaled unit step functions e.g.:

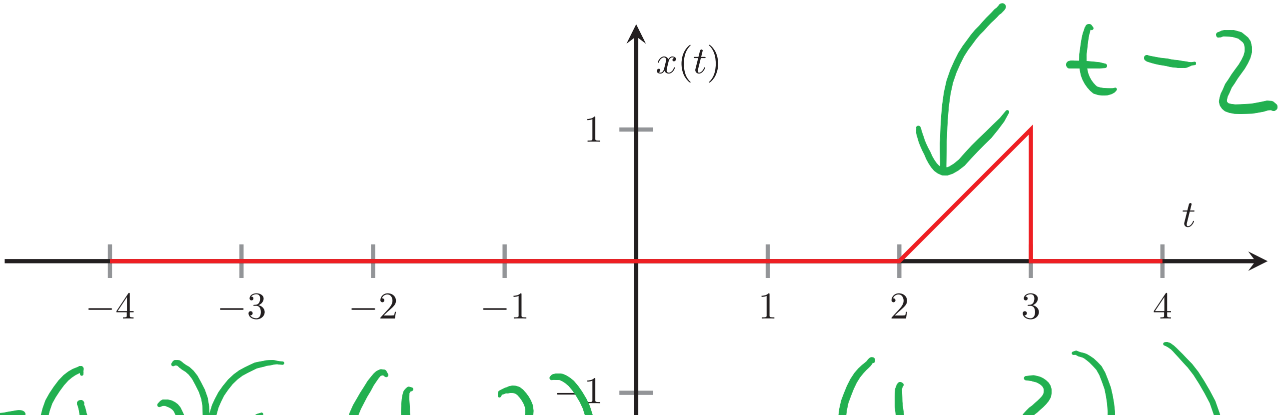
$u(t-2) - u(t-3)$




$-2u(t+2) - -2u(t+1) = -2u(t+2) + 2u(t+1)$

$x(t) = u(t-2) - u(t-3) - 2u(t+2) + 2u(t+1)$

$t-2$




$x(t) = (t-2)\big(u(t-2) - u(t-3)\big)$

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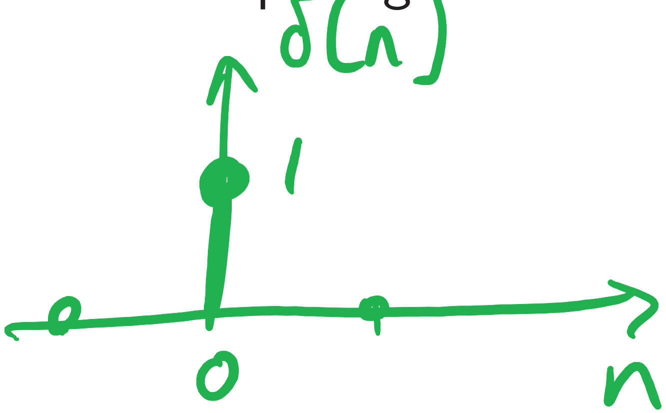
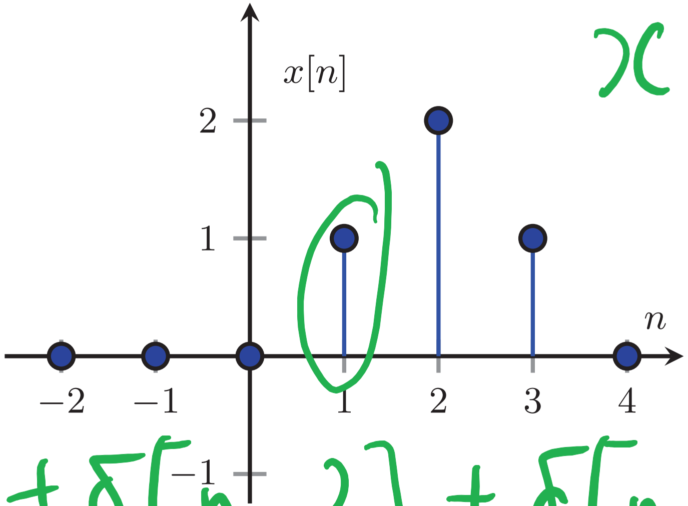
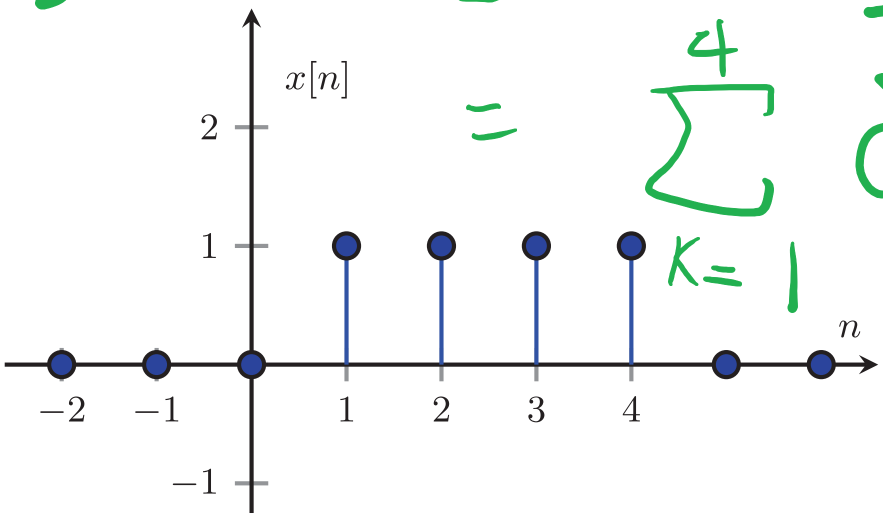
Decomposing functions into sum of shifted impulse functions e.g.:



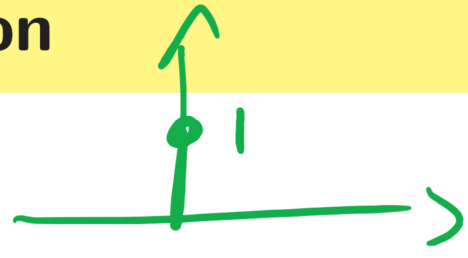

$$x[n] = \delta[n-1] + 2\delta[n-2] + \delta[n-3]$$

$$x[n] = \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$$



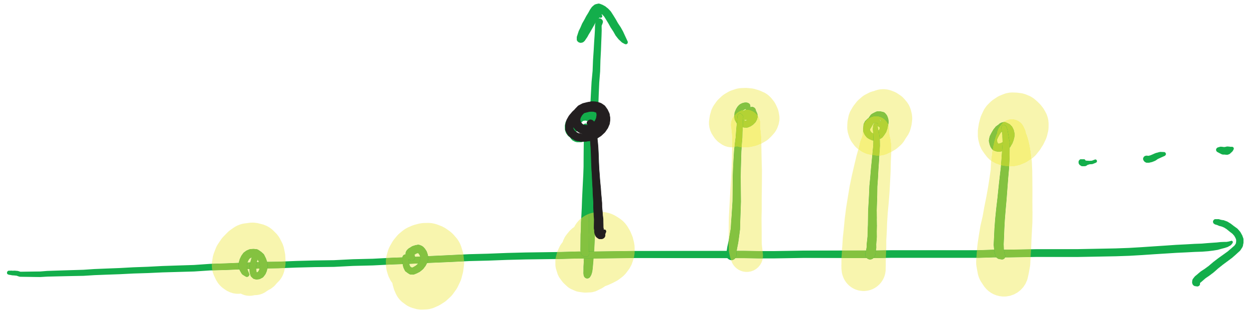

$$= \sum_{k=1}^{4} \delta[n-k]$$

# Special Test Signals - Signal Representation

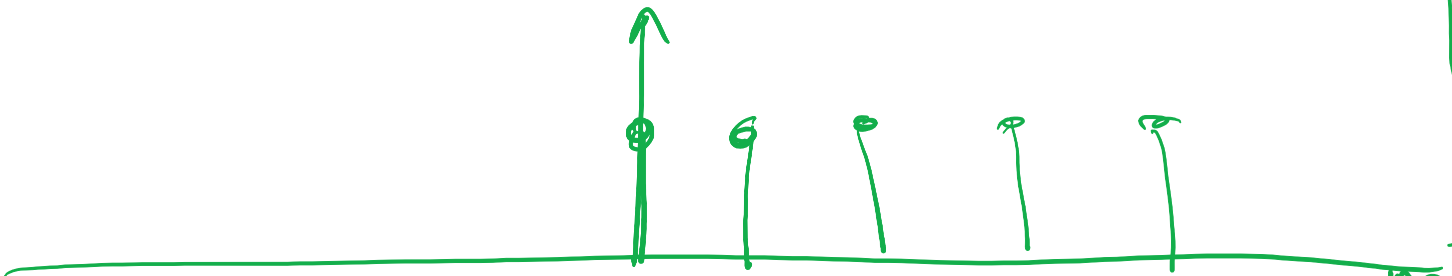


Some important relations:

- $$\delta[n] = u[n] - u[n-1]$$



- $$u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \ldots = \boxed{\sum_{k=0}^{\infty} \underbrace{\delta[n-k]}_{m}}$$



- setting $m = n - k$ (p.31 textbook):

$m = n - K$

$m = n - \infty = -\infty$

$$\boxed{u[n] = \sum_{m=-\infty}^{n} \delta[m]}$$

$$\sum_{m=-\infty}^{m=n} \delta[m] = \sum_{m=-\infty}^{n} \delta[m]$$

# Part 3 Outline

# CT and DT Systems

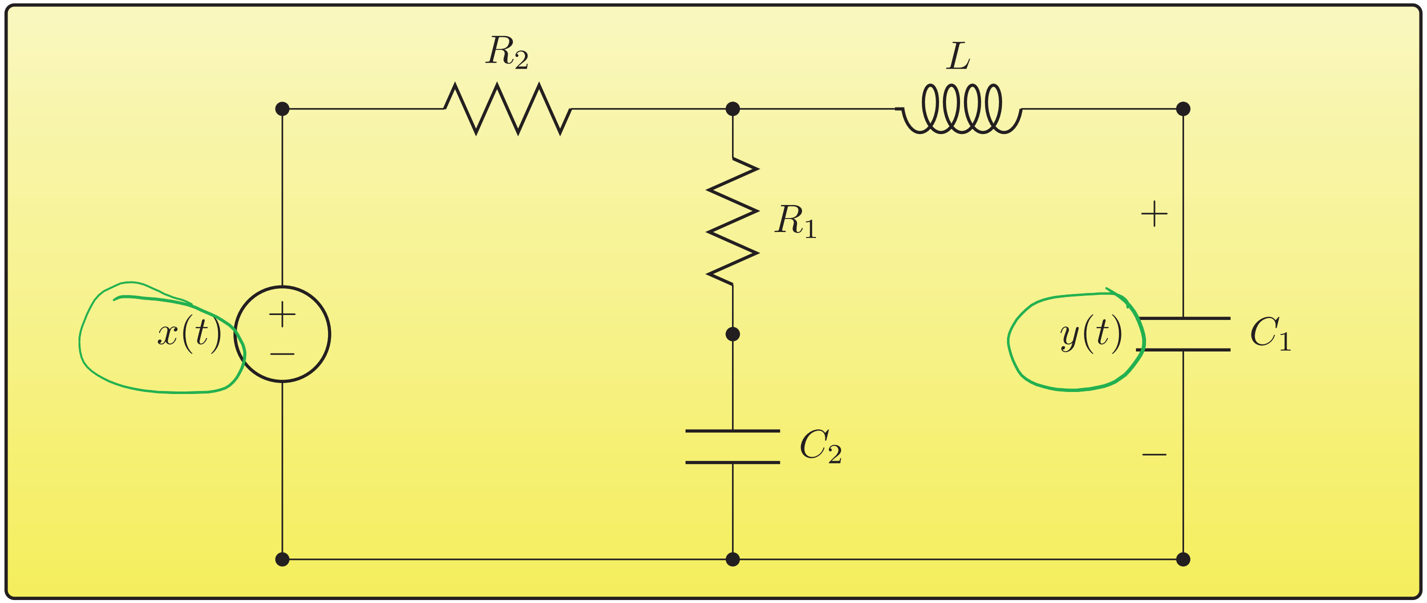
- A **system is a box** with an input signal $x(t)$ and an output signal $y(t)$


- In the discrete case with an input signal $x[n]$ and an output signal $y[n]$


- Mathematicians and physicists would say a system is an operator
- Signals are functions (containing useful information) and systems are things that transform one type of signal (function) to another signal (function)

# CT and DT Systems (examples)

- An RLC circuit can be regarded as a system




where the $x(t)$ is a voltage source and $y(t)$ is the voltage across the capacitor.

- There are a plethora of systems derived from the RLC circuit. The input $x(t)$ can be any voltage or current and output $y(t)$ can be any voltage or current.

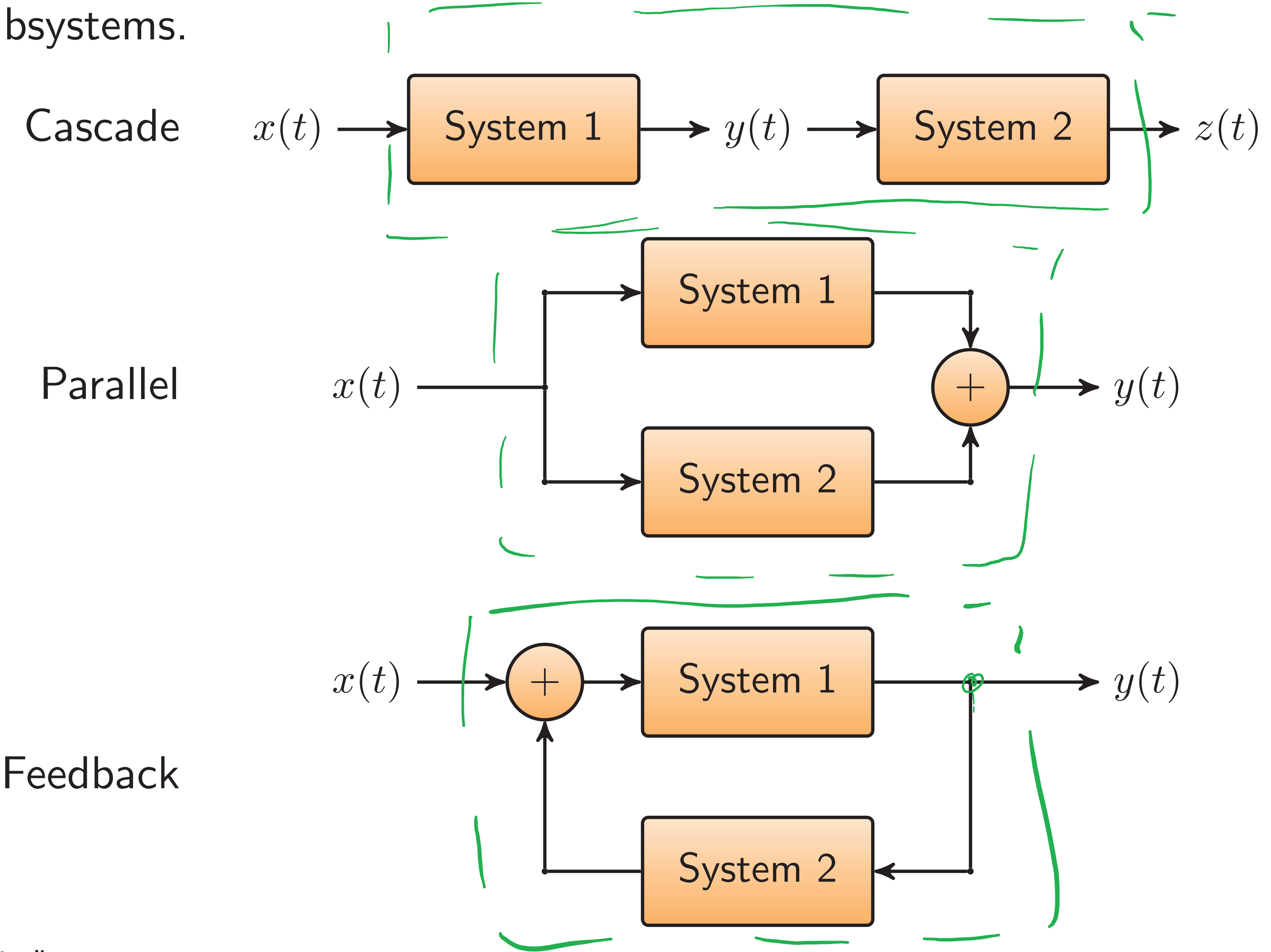
# CT and DT Systems

- Dynamics of a car in response to steering.
- An algorithm for predicting the BHP stock price.
- Medical image feature enhancement processing algorithms.
- Basically anything that has an output that responds to an input, e.g., a horse
- This course we focus on systems with one input and one output.
- This course we focus on systems that are linear (whatever that means).

# Part 3 Outline

# Interconnections of Systems

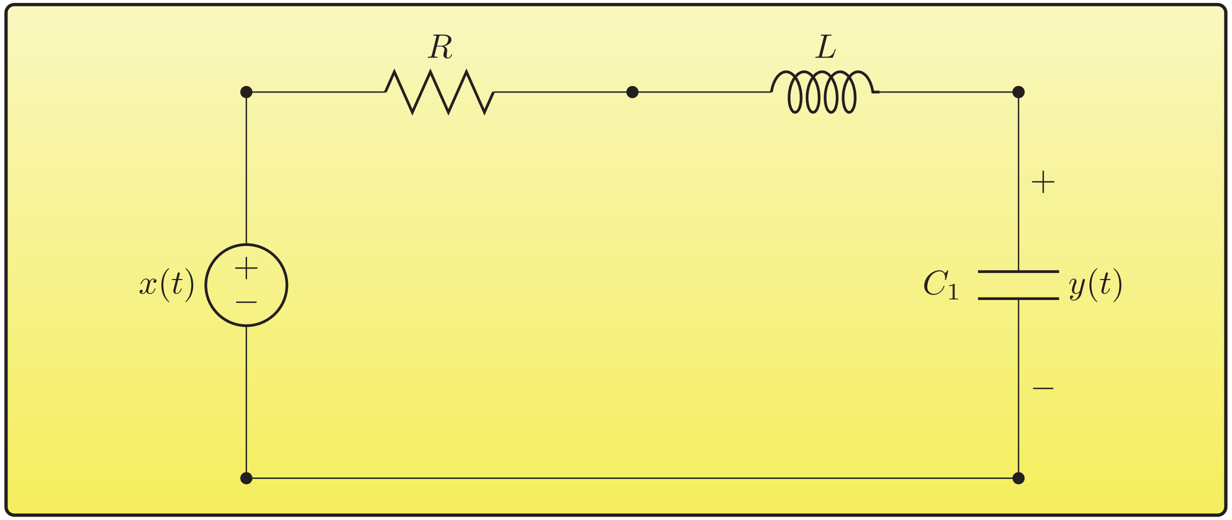
- More complex systems are the interconnection of simpler or component subsystems.



# Interconnections of Systems

- Understanding, designing and analysing complex interconnections of systems, in cascade (series), parallel and feedback is a core element of modern engineering.
- Complex control systems for a modern aircraft with high order dynamical flight models.
- Almost bewilderingly complex mobile communications systems (that work).

# Part 3 Outline




$$Ri(t) + L\frac{di(t)}{dt} + y(t) = x(t)$$

1st order DE

$$i(t) = C\frac{dy(t)}{dt}$$

1st order DE

$$\Downarrow$$

$$LC\frac{d^2y(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t) = x(t)$$

2nd order DE

# System Examples – Electrical (cont'd)


$$LC\frac{d^2y(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t) = x(t) \tag{1}$$

This is a system? (Yes)

- Signal $x(t)$ is the input. Signal $y(t)$ is the output and responds to, or depends on, this input.
- The coefficients $LC$ and $RC$ are constants and don't depend on time or $x(t)$ or $y(t)$.

CT systems can be described by differential equations.

## System Examples – Electrical (cont'd)

$$LC\frac{d^2y(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t) = x(t)$$

- [Addition] The system in (1) is a second order differential equation. It is **linear**. If $y_1(t)$ is the result of setting $x(t) = x_1(t)$ and $y_2(t)$ is the result of setting $x(t) = x_2(t)$ then $y(t) = y_1(t) + y_2(t)$ is the result of setting $x(t) = x_1(t) + x_2(t)$
- [Scaling] If $y_1(t)$ is the result of setting $x(t) = x_1(t)$ then $3y_1(t)$ is the result of setting $x(t) = 3x_1(t)$

# System Examples – Electrical (cont'd)

- In combination (that is, added and scaling together), for input

$$x(t) = \alpha_1 \, x_1(t) + \alpha_2 \, x_2(t)$$

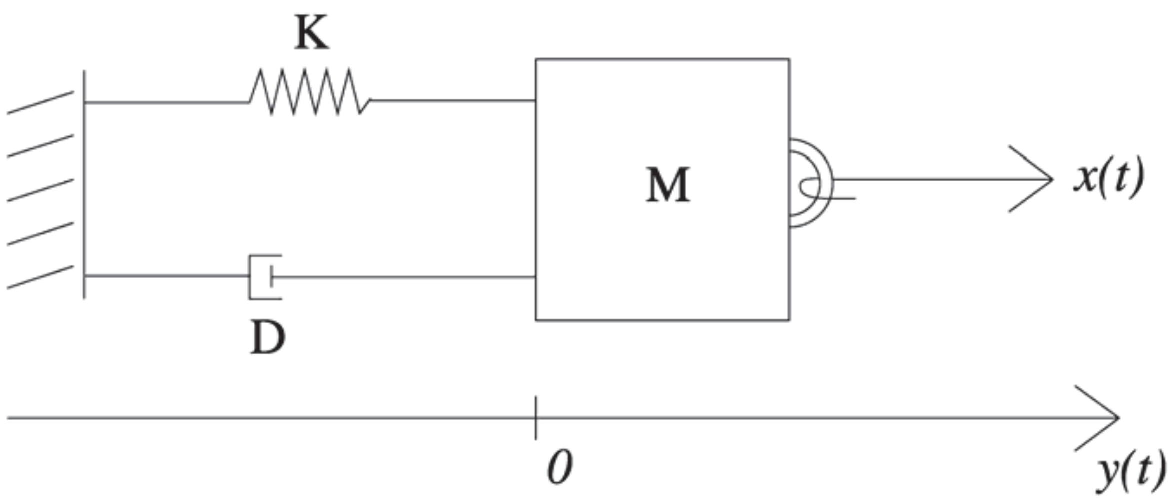
  the output is

$$y(t) = \alpha_1 \, y_1(t) + \alpha_2 \, y_2(t)$$

  where $\alpha_1$ and $\alpha_2$ are (complex) scalars.
- This is linearity / superposition.

# Part 3 Outline

# System Examples – Mechanical




- Force Balance

$$M\frac{d^2y(t)}{dt^2} + D\frac{dy(t)}{dt} + Ky(t) = x(t)$$

where $M$ is mass, $K$ is string constant, $D$ is damping and $x(t)$ is the applied force.

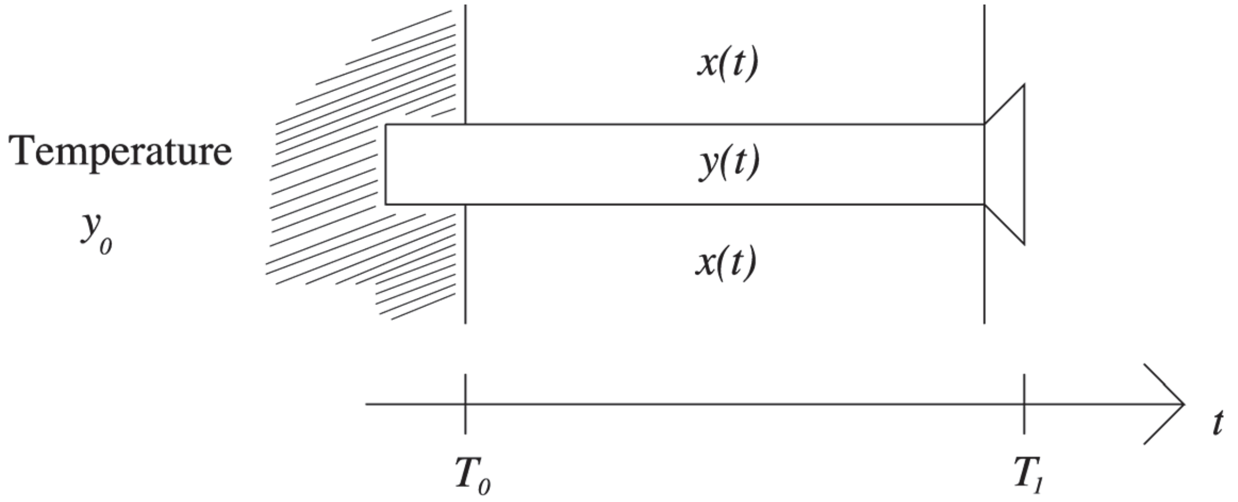
- The coefficients $M$, $D$ and $K$ are $RC$ constants and don't depend on time or $x(t)$ or $y(t)$.

**Observations:**

- this can be viewed as a mechanical analogue of the previous electrical example
- different physical systems / analogues may have identical or very similar mathematical descriptions
- generally you have a strong or familiar domain, say electrical, from which you can interpret other systems (e.g., resistance interpretation of damping)

# Part 3 Outline

# System Examples – Thermal




- $t$ – distance along the cooling fin
- $y(t)$ – fin temperature as a function of distance
- $x(t)$ – surrounding temperature along fin

# System Examples – Thermal (cont'd)

$$
\begin{aligned}
\frac{d^2 y(t)}{dt^2} &= k\big(y(t) - x(t)\big) \\
y(T_0) &= y_0 \\
\frac{dy}{dt}(T_1) &= 0
\end{aligned}
$$

- Here the independent variable, $t$, is space (not time). (OK so the notation is not the greatest.)
- Here we have boundary conditions rather than initial conditions.

# Part 3 Outline

# System Examples – Edge Detector


- A rough edge detector acting on a DT signal (sequence)

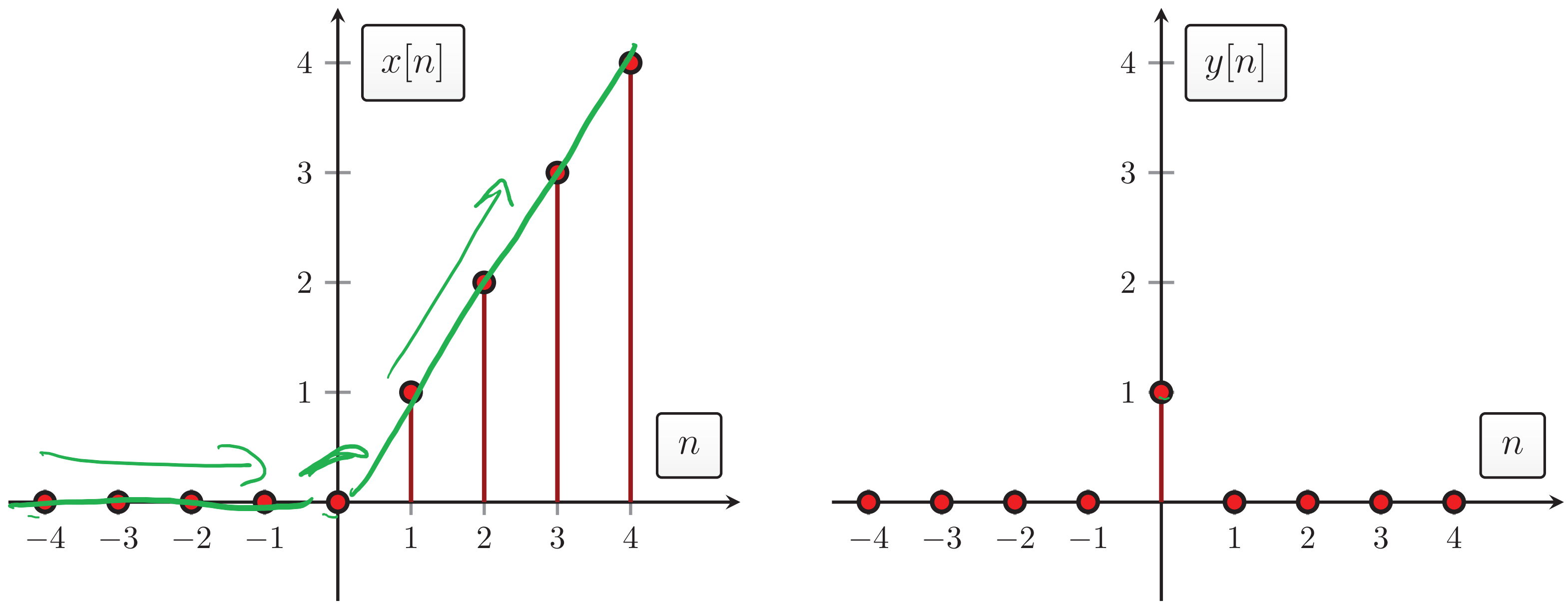
$$\begin{aligned} y[n] &= \boxed{x[n+1] - 2x[n] + x[n-1]} \\ &= \Big(x[n+1] - x[n]\Big) - \Big(x[n] - x[n-1]\Big) \end{aligned}$$

  which is a second difference. This emulates the second derivative for CT signals "change of slope".

- This is a system, $x[n]$ is the input DT signal and $y[n]$ is output detector.
- If $x[n] = n$ (a linear ramp) then $y[n] = 0$ for all $n$.

## System Examples – Edge Detector (cont'd)

- If $x[n] = n\,u[n]$ $(= n \times u[n])$, where $u[n]$ is a step then $y[n] = 0$ for all $n$ except $n = 0$ where $y[0] = 1$.




$x[n]$ $y[n]$

- Here $y[n]$ is a bit weird because how did it know at time $n = 0$ that the input was changing at time $n = 1$? This system is not "causal" which we treat shortly.

$$y[n] = x[n+1] - 2x[n] + x[n-1]$$

# System Examples – Observations

**Observations:**

- Differential equations and difference equations form an important class of systems.
- A system is not fully characterized by just the “dynamical” equations but also the initial conditions (or boundary conditions).