Australian National University

Mid-Semester Examination

Semester 2, 2016

SIGNAL PROCESSING

ENGN2228

Writing period: 90 Minutes duration
Study period: 10 Minutes duration
Permitted materials: One single sided A4 page of handwritten notes and Calculator

20 multiple-choice questions, for a total of 45 marks

3 problems, for a total of 30 marks

Contribution to Final Assessment: 20%

- *Write your multiple-choice answers on the answer sheet provided and place it inside the script book.*
- *Write your 3 problem answers in the script book provided.*
- *For multiple-choice questions Q1-Q14, there is NO negative marking, each correct answer scores the number of marks indicated in the question and a no answer scores 0 marks.*
- *For multiple-choice questions Q15-Q20, a correct answer scores 3 marks, an incorrect answer scores -1 (that is, minus 1) mark and a no answer scores 0 marks.*
- *At the end of the exam, hand in the exam question sheets as well as the script book and the multiple-choice answers sheet.*

Formulas

Complex Numbers and Complex Exponentials

$$j = \sqrt{-1} \qquad e^{j\pi n} = (-1)^n$$
$$j^2 = -1 \qquad e^{-j\pi n} = (-1)^n$$
$$\tfrac{1}{j} = -j \qquad e^{j2\pi n} = 1$$
$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \qquad e^{-j2\pi n} = 1$$
$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Trigonometric Identities

$$\sin(\theta) = \cos\left(\theta - \frac{\pi}{2}\right) \qquad \cos(\theta) = \sin\left(\theta + \frac{\pi}{2}\right)$$
$$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2} \qquad \cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$$
$$\sin^3(\theta) = \frac{3\sin(\theta) - \sin(3\theta)}{4} \qquad \cos^3(\theta) = \frac{3\cos(\theta) + \cos(3\theta)}{4}$$

Geometric series

If α is a complex number then the following relationships hold:

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad |\alpha| < 1 \qquad \sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1-\alpha} \quad |\alpha| < 1$$
$$\sum_{n=0}^{\infty} n\,\alpha^n = \frac{\alpha}{(1-\alpha)^2} \quad |\alpha| < 1 \qquad \sum_{n=-k}^{-\infty} \alpha^n = \alpha^{-k}\left(\frac{\alpha}{\alpha-1}\right) \quad |\alpha| > 1$$
$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N & \alpha = 1, \\ \frac{1-\alpha^N}{1-\alpha} & \alpha \neq 1 \end{cases} \qquad \sum_{n=k}^{\ell} \alpha^n = \begin{cases} \ell - k + 1 & \alpha = 1, \\ \frac{\alpha^k - \alpha^{\ell+1}}{1-\alpha} & \alpha \neq 1 \end{cases}$$

Integration

$$\int x^n = \frac{1}{n+1}\, x^{n+1} \qquad \int e^{gx} = \frac{1}{g}\, e^{gx}$$
$$\int \cos(ax) = \frac{1}{a}\sin(ax) \qquad \int \sin(ax) = -\frac{1}{a}\cos(ax)$$
$$\int \cos^2(ax) = \frac{x}{2} + \frac{1}{4a}\sin(2ax) \qquad \int \sin^2(ax) = \frac{x}{2} - \frac{1}{4a}\sin(2ax)$$
$$\int x\cos(ax) = \frac{1}{a^2}\left(\cos(ax) + ax\sin(ax)\right) \qquad \int x\sin(ax) = \frac{1}{a^2}\left(\sin(ax) - ax\cos(ax)\right)$$
$$\int e^{gx}\cos(ax) = \frac{e^{gx}}{g^2+a^2}\left(g\cos(ax) + a\sin(ax)\right) \qquad \int e^{gx}\sin(ax) = \frac{e^{gx}}{g^2+a^2}\left(g\sin(ax) - a\cos(ax)\right)$$
$$\int xe^{gx} = \frac{1}{g^2}\, e^{gx}(gx-1)$$

Notation

- CT means continuous time, and DT means discrete time,
- A system being LTI means the system is linear and time-invariant
- The binary operator $\star$ denotes convolution for both CT and DT.
- The unit sample delta signal is given by

$$\delta[n] \triangleq \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

- $\delta(t)$ represents the unit impulse and satisfies

$$x(t) \star \delta(t - t_0) = x(t - t_0)$$

- $\overline{z}$ denotes the complex conjugate of z
- $u[n]$ represents the unit step function, given by

$$u[n] \triangleq \begin{cases} 1 & n \geqslant 0 \\ 0 & else \end{cases}$$

Question 1 (1 mark)

What is the polar form representation of the complex number $(1 - j\sqrt{3})^3$?

a. $8e^{-\pi}$

b. $2e^{j\pi}$

c. $8e^{j\frac{\pi}{3}}$

d. $2e^{-j\frac{\pi}{3}}$

e. $8e^{-j\pi}$

Solution: $(1 - j\sqrt{3})^3 = (2e^{-j\pi/3})^3 = 8e^{-j\pi}$. **(28 students got it wrong)** □

Question 2 (2 marks)

What is the rectangular form representation of the sum $\sum_{n=0}^{9} e^{\frac{j\pi n}{2}}$? (Hint: select and use the appropriate identity from the list of formulas provided)

a. $1 + j$

b. $1 - j$

c. $-(1 + j)$

d. 10

e. j.

Solution: $\sum_{n=0}^{9} e^{\frac{j\pi n}{2}} = \frac{1-e^{\frac{j\pi 10}{2}}}{1-e^{\frac{j\pi}{2}}} = 1 + j$. **(35 students got it wrong)** □

Question 3 (1 mark)

What is the fundamental period of DT signal $x[n] = \sin(n/16)$?

a. 16

b. 16π

c. 32

d. 32π

e. It is not periodic and has no fundamental period.

Solution: $N = \frac{2\pi m}{\frac{1}{16}} = 32\pi m$. The intrinsic period of sin is irrational, which implies n would need to be irrational. It needs to be an integer. **(23 students got it wrong)** □

Question 4 (3 marks)

For CT signals, which of the following statements is false:

a. A CT signal which is periodic with period 2π is also periodic with period 4π .

b. The sum of two periodic CT signals of different periods is always periodic.

c. A CT signal that is not periodic is referred to as an aperiodic signal.

d. The sum of two non-periodic CT signals is never periodic.

e. The sum of a periodic CT signal and a non-periodic CT signal is never periodic.

Solution: Counter-example for b = If one periodic CT signal has period 1 and another CT signal has period $\sqrt{2}$ then their sum is not periodic because the periods are not rationally related.

Counter-example for d (posted by students on wattle) = For the sum of two non-periodic CT signals: their non-periodic parts may cancel under summation leaving a periodic signal. For example, consider $e^{-x} + (\sin(x) - e^{-x})$. Both terms are non periodic but their sum is $\sin(x)$ which is periodic.

Counter-example for e (posted by students on wattle) = If we consider $x(t) = \sin(t) + \sin(\pi t)$ (not periodic) and $y(t) = -\sin(\pi t)$ (periodic). Their sum is $x(t) + y(t) = \sin(t)$ which is periodic.

(20 students got it wrong) □

Question 5 (2 marks)

What is the fundamental period, N, of the DT signal $x[n] = (-j)^n + \cos(\pi\, n/3) + \cos(2\pi\, n/15)$?

a. 15

b. 30

c. 45

d. 60

e. 360

Solution: $(-j)^n = e^{-\frac{j\pi n}{2}}$ has period 4 (samples 1, $-j$, -14, j repeat), $\cos(\pi\, n/3)$ has period $6 = 2 \times 3$, and $\cos(2\pi\, n/15)$ has period $15 = 3 \times 5$. The lowest common multiple is $4 \times 3 \times 5 = 60$, so $x[n + 60] = x[n]$. **(50 students got it wrong)** □

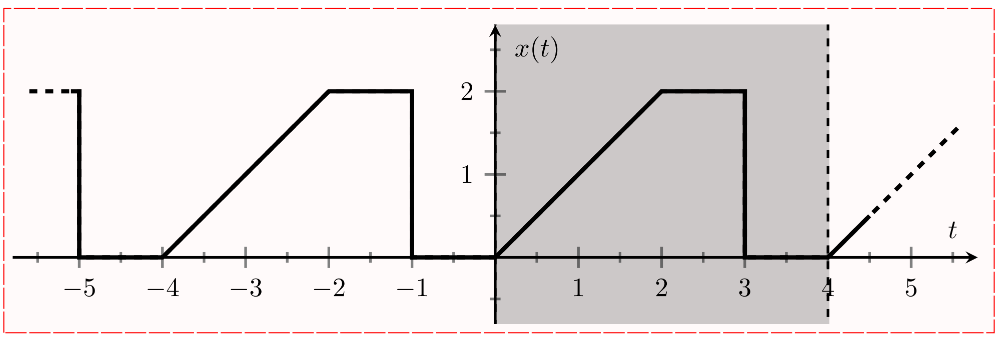
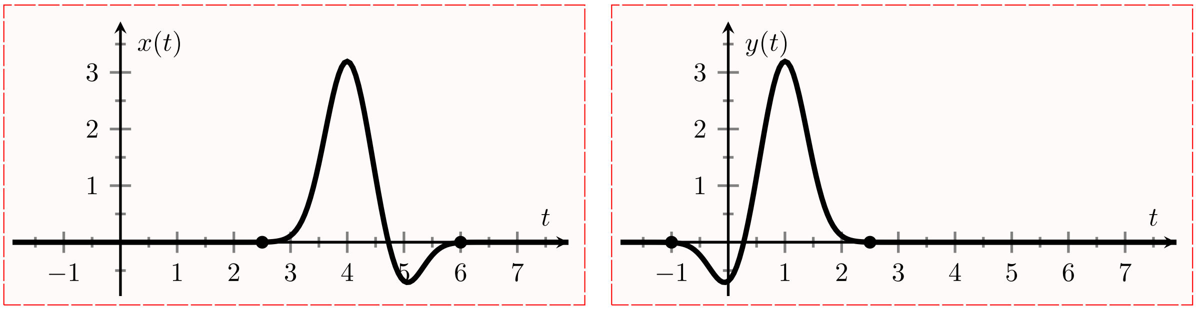


Figure 1: CT Periodic Signal $x(t)$ with Fundamental Period $T = 4$. One period has been shaded.



(a) CT signal $x(t)$ which can be taken as zero when $t < 2.5$ or $t > 6$

(b) CT signal $y(t)$ which can be taken as zero when $t < 0$ or $t > 3.5$

Figure 2: CT signals $x(t)$ and $y(t)$, which are related through an affine transformation.

Question 6 (2 marks)

The average power of a periodic signal $x(t)$ over a period T is given by

$$\frac{1}{T}\int_0^T \left(x(t)\right)^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

What is the average power per period of the signal, $x(t)$, shown in Figure 1?

a. $-1/4$

b. 0

c. 1

d. $\boxed{5/3}$

e. 4

Solution: Answer (a) is impossible because power must be non-negative. Answer (b) is also not sane given the signal is non-zero. To choose between (c), (d) and (e), do integration in the time domain

$$\left(\int_0^2 t^2\,dt + 1 \times 2^2\right)/4 = \left(2^3/3 + 4\right)/4 = 5/3$$

(55 students got it wrong) □ □

Question 7 (2 marks)

Two CT signals $x(t)$ and $y(t)$ are related through a transformation of their independent variables and are shown in Figure 2. Which of the following choices is correct?

a. $y(t) = x(t-5)$

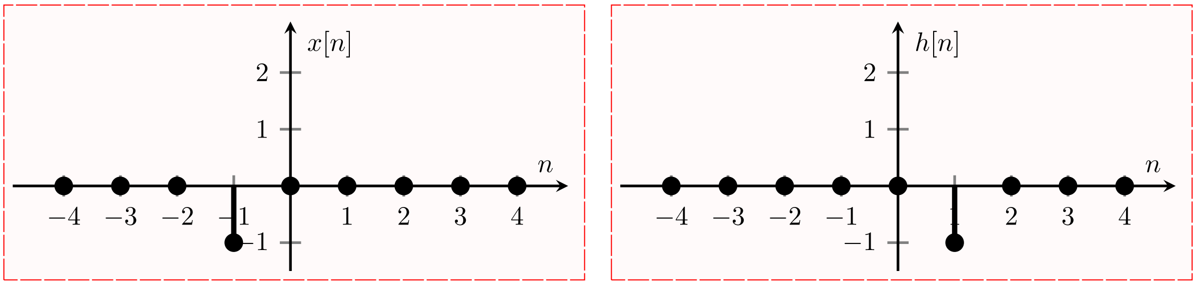
b. $\boxed{y(t) = x(5-t)}$

c. $y(t) = x(t+5)$

d. $y(t) = x(-5-t)$

e. $y(t) = x(t-3)$

Solution: The max is around $x(4) = y(1)$ and the min is around $x(5) = y(0)$ from the figures, and only the correct answer agrees with these equalities. **(13 students got it wrong)** □



(a) Input signal $x[n]$. **(b)** System response $h[n]$.

Figure 3: Signal $x[n]$ and system response $h[n]$, output signal is $y[n] = h[n] \star x[n]$

Question 8 (1 mark)

For $x[n]$ shown in Figure 3(a), which of the following is correct?

a. $x[n] = -\delta[n+1]$

b. $x[n] = -\delta[n-1]$

c. $x[n] = -1$

d. $x[n] = +1$

e. $x[n+1] = \delta[n]$

Solution: For $n = -1$, the value should be 1. For $n = 0$ the value should be 0. **(3 students got it wrong)** □

Question 9 (3 marks)

For $x[n]$ and $h[n]$, shown in Figure 3(a) and Figure 3(b), what is $y[n] = h[n] \star x[n]$?

a. $y[n] = \delta[n]$

b. $y[n] = -\delta[n-1] - \delta[n+1]$

c. $y[n] = -2\delta[n]$

d. $y[n] = -\delta[n-2]$

e. $y[n] = \delta[n-2]$

Solution: $y[n] = h[n] \star x[n] = (-\delta[n-1]) \star (-\delta[n+1]) = \delta[(n+1)-1] = \delta[n]$. **(20 students got it wrong)** □

Question 10 (2 marks)

What is the DT convolution, $y[n] = x[n] \star h[n]$, of the two signals $x[n] = \delta[n] + \delta[n-2]$ and $h[n] = 2\,\delta[n-3]$?

a. $y[n] = \delta[n] + \delta[n-2] + 2\delta[n-3]$

b. $y[n] = 2\,\delta[n-3] + 2\,\delta[n-5]$

c. $y[n] = 2\,\delta[n+3] + 2\,\delta[n+1]$

d. $y[n] = 2\,\delta[n] + 2\,\delta[n-2]$

e. None of the above.

Solution: $h[n]$ as a system delays $x[n]$ by 3 and amplifies by 2. (L10-L12: Convolving a signal with an impulse shifts the signal to the location of the impulse). **(14 students got it wrong)** □

Question 11 (2 marks)

The equation for a LTI system with input $x[n]$ and output $y[n]$ is given by

$$y[n] = 0.5\,x[n] - 0.3\,x[n-1] + 0.1\,x[n-2].$$

What is the impulse response $h[n]$ such that $y[n] = x[n] \star h[n]$?

a. $h[n] = 0.5\,\delta[n] - 0.3\,\delta[n-1] + 0.1\,\delta[n-2]$.
b. $h[n] = 0.1\,\delta[n] - 0.3\,\delta[n-1] + 0.5\,\delta[n-2]$.
c. $h[n] = 0.5\,\delta[n] - 0.3\,\delta[n+1] + 0.1\,\delta[n+2]$.
d. $h[n] = 0.1\,\delta[n] - 0.3\,\delta[n+1] + 0.5\,\delta[n+2]$.
e. $h[n] = 0.5\,\delta[n-2] - 0.3\,\delta[n-1] + 0.5\,\delta[n]$.

Solution: Just read it off the difference equation coefficients when $x[n] = \delta[n]$. **(6 students got it wrong)** □

Question 12 (2 marks)

What is the even part of $\delta(t)$?

a. 1
b. 0.5
c. $\delta(t)$
d. $\delta(t/2)$
e. $0.5\,\delta(t)$

Solution: An odd signal is zero at 0 and so $\delta(t)$ is even. Alternatively, if you start with rectangular waveform, which is even, and take the limit, then in the limit $\delta(t)$ is even. Alternatively, $\delta(-t) = \delta(t)$ and so $0.5\delta(t) + 0.5\delta(-t) = \delta(t)$.**(35 students got it wrong)** □

Question 13 (3 marks)

Let $\delta(t)$ be the CT unit impulse function. Which of the following is false?

a. $x(t)\,\delta(t - t_0) = x(t_0)$
b. $\displaystyle\int_{-1}^{1} \delta(t)\,dt = 1$
c. $\displaystyle\int_{-\infty}^{\infty} \delta(t-1)\,dt = 1$
d. $\displaystyle\int_{-1}^{1} \delta(t-2)\,dt = 0$
e. $\delta(t) \star x(t) \star \delta(t) = x(t)$

Solution: The first one needs to be $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$ (the correct expression for point-wise multiplication with impulse) When $t \neq t_0$ then $x(t_0)\,\delta(t-t_0) = 0 \neq x(t_0)$. And when $t = t_0$ the LHS is ∞ and the RHS is $x(t_0)$.**(35 students got it wrong)** □

Question 14 (1 mark)

Consider the series/cascade connection of the two CT LTI systems $h_1(t)$ and $h_2(t)$ with input $x(t)$ and output $y(t)$ as shown in Figure 4. Which of the following statements is false?

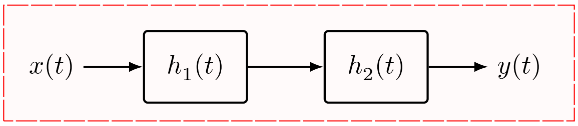


Figure 4: Series/cascade connection of two CT LTI systems.

a. $y(t) = x(t) \star h_1(t) \star h_2(t)$

b. $y(t) = x(t) \star h_2(t) \star h_1(t)$

c. $y(t) = h_1(t) \star x(t) \star h_2(t)$

d. $y(t) = h_1(t) \star \delta(t) \star x(t) \star \delta(t) \star h_2(t)$

e. $y(t) = x(t) \star \big(h_1(t)h_2(t)\big)$

Solution: From L11. $h_1(t)h_2(t)$ is wrong whereas $h_1(t) \star h_2(t)$ would be ok. Convolution with $\delta(t)$ does nothing. **(23 students got it wrong)** □

For Q15-Q20, a correct answer scores 3 marks, an incorrect answer scores -1 (that is, minus 1) mark and a no answer scores 0 marks.

Question 15 (3 marks)

Which is true about the following DT system:

$$y[n] = 3\,x[n] + 5^{-n}\,x[n+1],$$

where $x[n]$ is the input signal and $y[n]$ is the output signal?

a. Linear, time-invariant and causal
b. Non-linear, time-invariant and causal
c. Linear, time-varying and causal
d. Non-linear, time-varying and causal
e. Linear, time-invariant and non-causal
f. Non-linear, time-invariant and non-causal
g. Linear, time-varying and non-causal
h. Non-linear, time-varying and non-causal

Solution: This DT system is linear; the 5^{-n} makes it time-varying and $y[n]$ requires future $x[n+1]$, which make is non-causal.**(38 students got it wrong; 3 did not attempt)** □

Question 16 (3 marks)

Which is true about the following DT system:

$$y[n] = x[n-1]\,x[n],$$

where $x[n]$ is the input signal and $y[n]$ is the output signal:

a. Linear, time-invariant and causal
b. Non-linear, time-invariant and causal
c. Linear, time-varying and causal
d. Non-linear, time-varying and causal
e. Linear, time-invariant and non-causal
f. Non-linear, time-invariant and non-causal
g. Linear, time-varying and non-causal
h. Non-linear, time-varying and non-causal

Solution: $x[n-1]$ makes it causal. Nonlinear: $2x[n] \to 4y[n]$. It is time-invariant.**(32 students got it wrong; 5 did not attempt)** □

Question 17 (3 marks)

Consider the DT system with input signal $x[n]$ and output signal $y[n]$ given by

$$y[n] = \overline{x[n-1]} \quad \text{(complex conjugate)}$$

Which of the following sets of properties holds for this system?

(The signals in question can be complex-valued.)

a. Linear, time-invariant and causal

b. Non-linear, time-invariant and causal

c. Linear, time-varying and causal

d. Non-linear, time-varying and causal

e. Linear, time-invariant and non-causal

f. Non-linear, time-invariant and non-causal

g. Linear, time-varying and non-causal

h. Non-linear, time-varying and non-causal

Solution: It is **non-linear** because it doesn't scale properly with complex-valued scalars. See also example 1.19 in textbook.

It is **time-invariant** and requires old inputs, so is **causal**.**(55 students got it wrong; 29 did not attempt)** □

Question 18 (3 marks)

Consider the CT system with input signal $x(t)$ and output signal $y(t)$ given by

$$y(t) = x(t^2)$$

Which of the following sets of properties holds for this system?

a. Linear, time-invariant and causal

b. Non-linear, time-invariant and causal

c. Linear, time-varying and causal

d. Non-linear, time-varying and causal

e. Linear, time-invariant and non-causal

f. Non-linear, time-invariant and non-causal

g. Linear, time-varying and non-causal

h. Non-linear, time-varying and non-causal

Solution: The example for $y(4) = x(16)$ shows it is **non-causal**.

Similarly, if it were time-invariant then we should have $y(0+4) = x(0+4)$ given $y(0) = x(0)$. But that is not the case, so it is **time-varying**.

It is actually **linear** (apply the definition): Let $y_1(t) = x_1(t^2)$ and $y_2(t) = x_2(t^2)$, and form

$$x_3(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t),$$

then

$$\begin{aligned} y_3(t) &= x_3(t^2) \\ &= \alpha_1 x_1(t^2) + \alpha_2 x_2(t^2) \\ &= \alpha_1 y_1(t) + \alpha_2 y_2(t) \end{aligned}$$

and Bob's your uncle. **(52 students got it wrong; 3 did not attempt)** □ □

Question 19 (3 marks)

Consider the DT system with input signal $x[n]$ and output signal $y[n]$ given by

$$y[n] = \sum_{k=1}^{9} x[k]$$

Which of the following sets of properties holds for this system?

(Note that there is no typo in this system equation. There is no n on the right-hand side.)

a. Linear, time-invariant and causal

b. Non-linear, time-invariant and causal

c. Linear, time-varying and causal

d. Non-linear, time-varying and causal

e. Linear, time-invariant and non-causal

f. Non-linear, time-invariant and non-causal

g. Linear, time-varying and non-causal

h. Non-linear, time-varying and non-causal

Solution: It is **linear**.

It is **time-varying**, because for $n > 10$ it acts in a non-causal way and for $n \leq 9$ it acts in a casual way.

Considering $y[0]$ (that is, for $n = 0$) then the use of future inputs, $x[1]$ through to $x[9]$, makes it **non-causal**.**(65 students got it wrong; 7 did not attempt)** ☐

Question 20 (3 marks)

Consider the CT system with input signal $x(t)$ and output signal $y(t)$ given by

$$y(t) = \int_{-\infty}^{t/3} x(\tau)\, d\tau$$

Which of the following sets of properties holds for this system?

a. Linear, time-invariant and causal

b. Non-linear, time-invariant and causal

c. Linear, time-varying and causal

d. Non-linear, time-varying and causal

e. Linear, time-invariant and non-causal

f. Non-linear, time-invariant and non-causal

g. Linear, time-varying and non-causal

h. Non-linear, time-varying and non-causal

Solution: $y(-30)$ needs $x(t)$ up to $t = -10$ and so is **non-causal**.

It is **linear** because integration is linear.

It is **time-varying**, because for negative t it acts in a non-causal way and for positive t it acts in a casual way.**(58 students got it wrong; 12 did not attempt)** ☐

(end of multiple choice questions)

(start of problem questions)

Problem 1

Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k],$$

where n_0 is a finite positive integer.

(a) [4 marks] Use a mathematical-checking procedure, showing all steps, to determine whether the system is linear or nonlinear.

Solution: Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.

$$x_1[n] \to y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

$$x_2[n] \to y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is

$$x_3[n] = ax_1[n] + bx_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output $y_3[n]$ is

$$\begin{aligned} y_3[n] &= \sum_{k=n-n_0}^{n+n_0} x_3[k] \\ &= \sum_{k=n-n_0}^{n+n_0} (ax_1[k] + bx_2[k]) \\ &= a \sum_{k=n-n_0}^{n+n_0} x_1[k] + b \sum_{k=n-n_0}^{n+n_0} x_2[k] \\ &= ay_1[n] + by_2[n] \end{aligned}$$

Therefore, the system is **linear**. □

(b) [6 marks] Use a mathematical-checking procedure, showing all steps, to determine whether the system is time-invariant or time-varying.

Solution: Consider an arbitrary input $x_1[n]$. Let

$$y_1[n] = \sum_{k-n-n_0}^{n-n_0} x_1[k]$$

be the corresponding output. Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n - n_1]$$

The corresponding output to this input is

$$y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k] = \sum_{k=n-n_0}^{n+n_0} x_1[k - n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k]$$

Also note that

$$y_1[n - n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k]$$

Therefore,

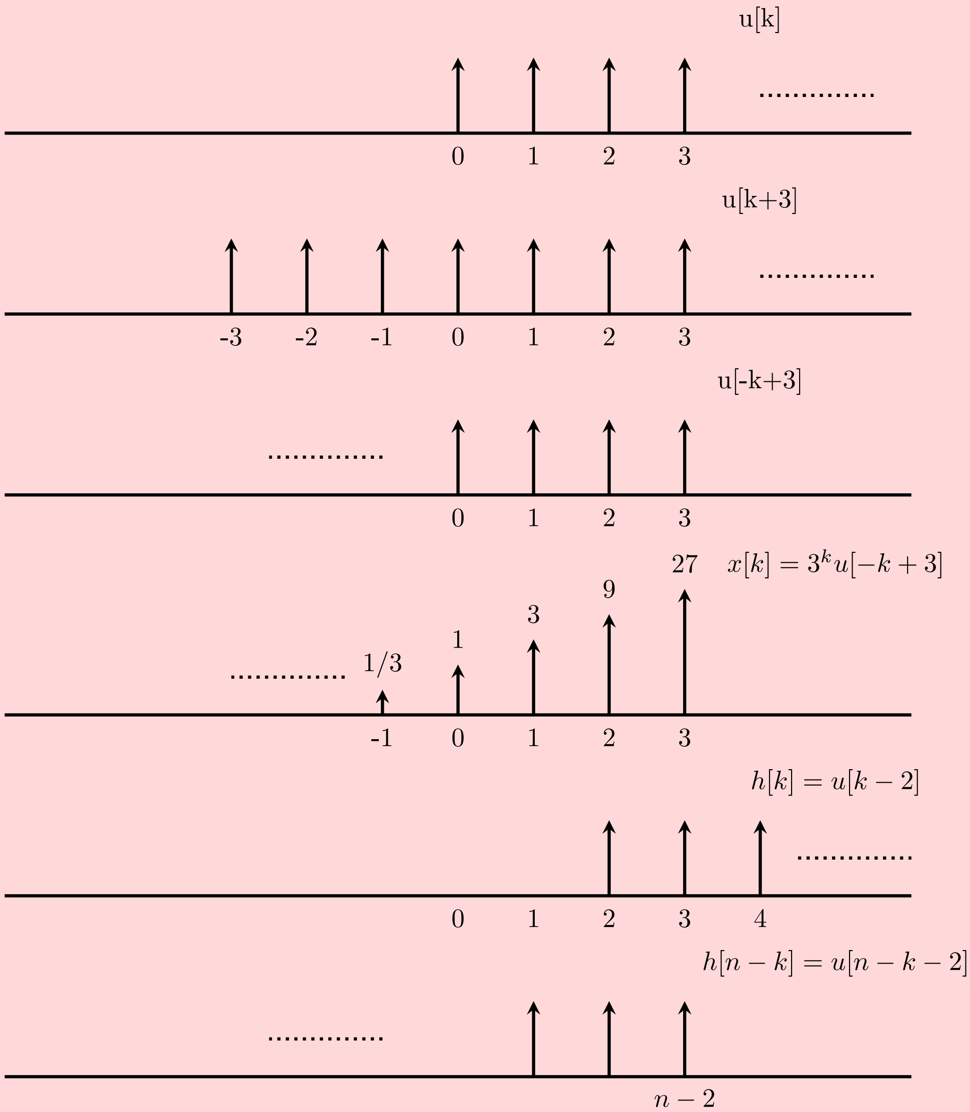
$$y_2[n] = y_1[n - n_1]$$

This implies that the system is **time-invariant**. **Average mark was 6.5/10** □

Problem 2

(a) [10 marks] Compute the convolution $y[n] = x[n] * h[n]$ when $x[n] = 3^n\ u[3-n]$ and $h[n] = u[n-2]$.

Solution: Using the graphical method for convolution,



Now, we have two cases. Either $n-2 \geq 3$ or $n-2 < 3$.
For $n-2 \geq 3 \Rightarrow n \geq 5$

$$y[n] = \sum_{k=-\infty}^{3} x[k]h[n-k]$$

$$= \sum_{k=-\infty}^{3} 3^k = 3^3\left(\frac{3}{3-1}\right) = \frac{81}{2}$$

For $n-2 < 3 \Rightarrow n < 5$

$$y[n] = \sum_{k=-\infty}^{n-2} 3^k$$

$$= 3^{n-2}\left(\frac{3}{3-1}\right) = \frac{3}{2}\cdot 3^{n-2} = \frac{3^{n-1}}{2}$$

Finally, we have

$$y[n] = \begin{cases} \frac{3^{n-1}}{2}\ (\text{or} \frac{3^n}{6}) & n < 5 \\ \frac{81}{2} & n \geq 5 \end{cases}$$

Average mark was 7.3/10 □

Problem 3

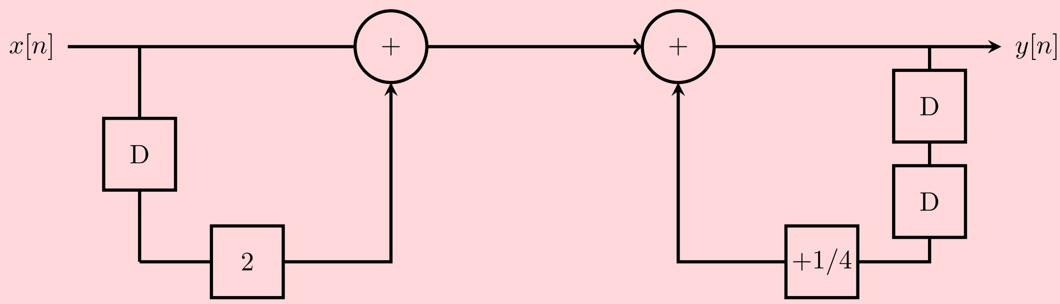
Consider the LTI system initially at rest and described by the difference equation

$$y[n] - \frac{1}{4}\,y[n-2] = x[n] + 2\,x[n-1].$$

(a) [3 marks] Draw the direct form I implementation of the given LTI system.

Solution:

$$y[n] = \frac{1}{4}\,y[n-2] + x[n] + 2\,x[n-1].$$



(b) [7 marks] Find the impulse response of this system by solving the difference equation recursively or otherwise.

Solution: Let $x[n] = \delta[n]$, then $y[n] \to h[n]$. Then from the given difference equation,

$$h[n] = \frac{1}{4}h[n-2] + \delta[n] + 2\delta[n-1]$$

Calculating $h[n]$ for different values of n,

$$\begin{aligned}
h[0] &= \frac{1}{4}h[-2] + \delta[0] + 2\delta[-1] = 1\\
h[1] &= \frac{1}{4}h[-1] + \delta[1] + 2\delta[0] = 2\\
h[2] &= \frac{1}{4}h[0] = \frac{1}{4}\\
h[3] &= \frac{1}{4}h[1] = \frac{1}{4}\cdot 2 = \frac{1}{2}\\
h[4] &= \frac{1}{4}h[2] = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}\\
h[5] &= \frac{1}{4}h[3] = \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}\\
h[6] &= \frac{1}{4}h[4] = \frac{1}{4}\cdot\frac{1}{16} = \frac{1}{64}
\end{aligned}$$

Following on the recursion for $h[n]$, we have

$$h[n] = \begin{cases} \frac{1}{2^n} \text{ or } \left(\frac{1}{2}\right)^n \text{ or } \left(\frac{1}{4}\right)^{n/2} \text{ or } 2^{-n} & n = 0, 2, 4, \ldots \\ \frac{1}{2^{n-2}} \text{ or } \left(\frac{1}{2}\right)^{n-2} \text{ or } 2\left(\frac{1}{2}\right)^{n-1} \text{ or } 2\left(\frac{1}{4}\right)^{(n-1)/2} \text{ or } 2^{-n+2} & n = 1, 3, 5, \ldots \end{cases}$$

Average mark was 6.6/10 □

(start of problem questions)

Question	Points	Score
1	1	
2	2	
3	1	
4	3	
5	2	
6	2	
7	2	
8	1	
9	3	
10	2	
11	2	
12	2	
13	3	
14	1	
15	3	
16	3	
17	3	
18	3	
19	3	
20	3	
21	10	
22	10	
23	10	
Total:	75	