Part 1 Outline

Signal Energy and Power (background)

Definition (Power vs Energy)

Power (Watts) is energy (Joules) transferred per unit of time (seconds).

- Power is the rate at which energy is delivered.
- Understand the difference between energy and power.

1 Watt = 1 Joule/second

Signal Energy and Power (physical motivation)

For a resistor R continuous time **instantaneous power** in a circuit is the product of voltage and current

$v(t) = Ri(t)$

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t) = R\,i^2(t)$$

The **total energy** dissipated from time t_1 to time t_2 is

$$\boxed{\int_{t_1}^{t_2} p(t)\,dt = \int_{t_1}^{t_2} \frac{1}{R}v^2(t)\,dt}$$

and the **average power** over this time interval is

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)\,dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \frac{1}{R}v^2(t)\,dt$$

such that the energy delivered over the time interval $t_2 - t_1$ at the average power equals the total energy. It is the mean rate that energy is delivered.

Signal Energy and Power (signal abstraction)

Definition (Total energy of a continuous time signal $x(t)$)

Total energy of a continuous time signal $x(t)$ between real times instants t_1 and t_2 is

$$\boxed{\int_{t_1}^{t_2} |x(t)|^2 \, dt}$$

V^2

Definition (Total energy of a discrete time signal $x[n]$)

Total energy of a discrete time signal $x[n]$ between integer time instants n_1 and n_2 is

$$\boxed{\sum_{n=n_1}^{n_2} |x[n]|^2}$$

- Signals can be real or complex valued; $|\cdot|$ means "absolute" value for real and "magnitude" for complex
- Don't worry about the physical interpretation; this is just a definition

Signal Energy and Power (Infinite Time Interval Energy)

Definition (Infinite time interval total energy of a continuous time signal $x(t)$)

$$E_\infty = \lim_{T \longrightarrow \infty} \int_{-T}^{T} \left|x(t)\right|^2 dt$$

Definition (Infinite time interval total energy of a discrete time signal $x[n]$)

$$E_\infty = \lim_{N \longrightarrow \infty} \sum_{n=-N}^{N} \left|x[n]\right|^2$$

- Total energy may be finite or infinite.

Signal Energy and Power (Infinite Time Interval Time-Average Power)

Definition (Infinite time interval time-average power of a continuous time signal $x(t)$)

$$P_\infty = \lim_{T \longrightarrow \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

Definition (Infinite time interval time-average power of a continuous time periodic signal)

$$P_\infty = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 \, dt$$

Signal Energy and Power (Infinite Time Interval Time-Average Power)

Definition (Infinite time interval time-average power of a discrete time signal $x[n]$)

$$P_\infty = \lim_{N \longrightarrow \infty} \boxed{\frac{1}{2N+1}} \sum_{n=-N}^{N} \left|x[n]\right|^2$$

Definition (Infinite time interval time-average power of a discrete time periodic signal)

$$P_\infty = \frac{1}{N} \sum_{n=0}^{N-1} \left|x[n]\right|^2$$

Signal Energy and Power (Examples)

- $$x[n] = \begin{cases} \cos(\pi n) & -4 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$$

  Using $E_\infty = \lim_{N \longrightarrow \infty} \sum_{n=-N}^{N} |x[n]|^2$: $= \sum_{n=-4}^{4} |\cos(\pi n)|^2$

  $= \cos(-4\pi)^2 + \cos(-3\pi)^2 + \ldots + \cos(4\pi)^2 = (1)^2 + (-1)^2 + \ldots (1)^2$

  $= 1 \times 9 = 9$ Joules

  $E_\infty = 9$

- $T_0 = \frac{2\pi}{\omega_0}$ $\quad T_0 = 2 \quad T_0 = \frac{2}{5}$

  $$x(t) = 5\cos(\pi t) + \sin(5\pi t)$$

  $\underline{T_0 = 2}$

  Using $P_\infty = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2\, dt$ get $P_\infty = 13$

  $P_\infty = \frac{1}{2}\int_{-1}^{1} (5\cos(\pi t) + \sin(5\pi t))^2\, dt$

Signal Energy and Power (various cases)

- P_∞ may be finite or infinite. Natural signals are expected to be finite power.
- $E_\infty < \infty$ implies $P_\infty = 0$. Finite energy signals have zero average power over the infinite interval.
- $P_\infty > 0$ implies $E_\infty = \infty$. Finite average power signals end up delivering infinite energy over the infinite interval.
- Both $P_\infty = \infty$ and $E_\infty = \infty$ also mathematically possible, but not of much engineering interest.

Periodic signals are finite average power signals

Part 1 Outline

Definition (Even Continuous Time Signals)

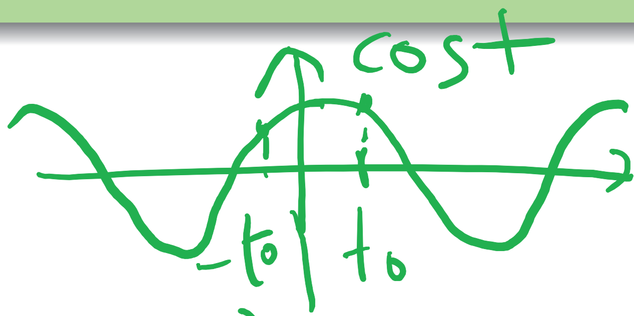
CT Signal $x(t)$ is **even** if

$$\boxed{x(-t) = x(t), \qquad \text{for all } t}$$

i.e. it is symmetric about the y-axis.

We find the even component of a signal $x(t)$ as:

$$\boxed{x_e(t) = \frac{1}{2}\big(x(t) + x(-t)\big)}$$
$$= \frac{1}{2}\big(x(-t) + x(t)\big)$$
$$= x_e(-t)$$



$x(-t) = \cos(-t)$
$= \cos(t) = x(t)$

Definition (Odd Continuous Time Signals)

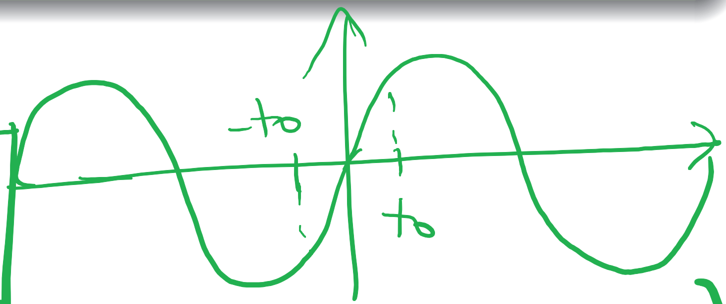
CT Signal $x(t)$ is **odd** if

$$\boxed{x(-t) = -x(t), \qquad \text{for all } t}$$

i.e. anti-symmetric about the y-axis.

We find the odd component of a signal $x(t)$ as:

$$\boxed{x_o(t) = \frac{1}{2}\big(x(t) - x(-t)\big)}$$
$$= -\frac{1}{2}\big(x(-t) - x(t)\big)$$
$$= -x_o(-t)$$



$x(-t) = \sin(-t)$
$= -\sin(t)$
$= -x(t)$

Odd and Even Signals

For any signal $x(t)$, then we can write

$$\boxed{x(t) = \underbrace{\frac{1}{2}\big(x(t) + x(-t)\big)}_{x_e(t)} + \underbrace{\frac{1}{2}\big(x(t) - x(-t)\big)}_{x_o(t)}}$$

which, in a way, seems completely daft since we introduce a time reversed signal, $x(-t)$ which is cancelled.

Other notation `(see O&W p.14)`

$$\mathit{Ev}\{x(t)\} \triangleq x_e(t) = \frac{1}{2}\big(x(t) + x(-t)\big)$$
$$\mathit{Od}\{x(t)\} \triangleq x_o(t) = \frac{1}{2}\big(x(t) - x(-t)\big)$$

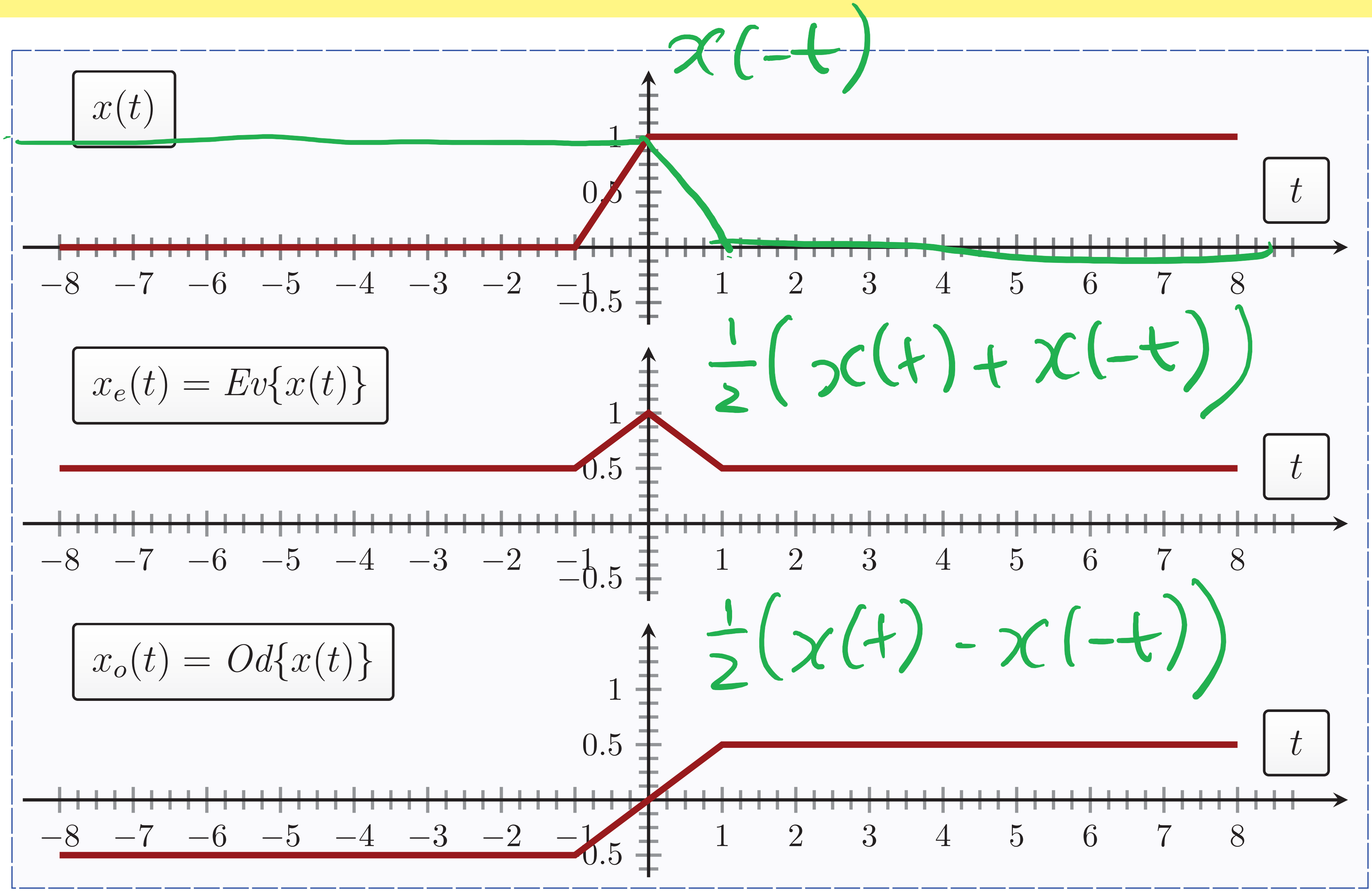
Odd and Even Signals (CT Signal Decomposition)

$$x(t) = \underbrace{\frac{1}{2}\big(x(t) + x(-t)\big)}_{x_e(t)} + \underbrace{\frac{1}{2}\big(x(t) - x(-t)\big)}_{x_o(t)}$$

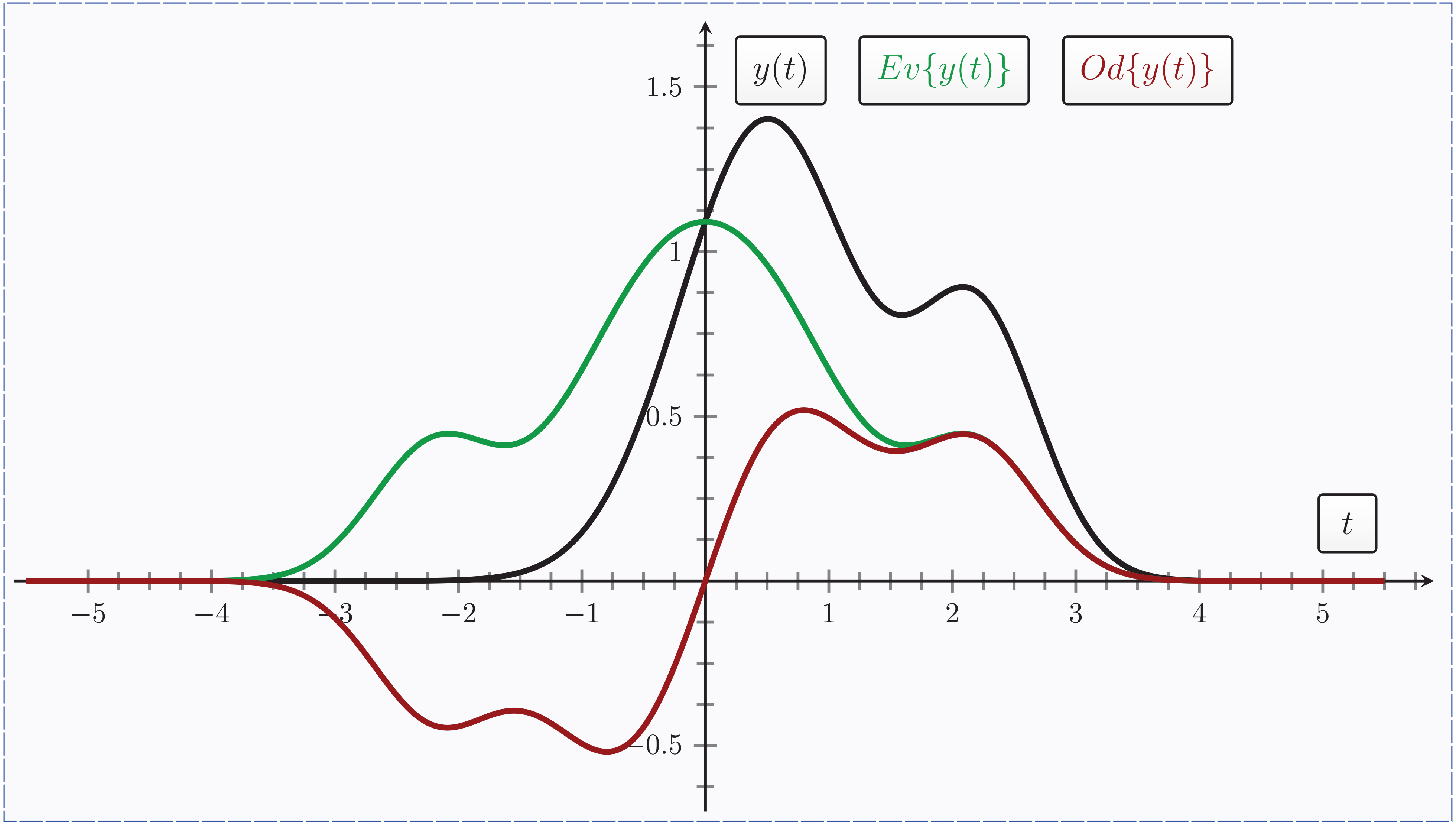
means we can always decompose any CT signal into the sum of an odd CT signal and an even CT signal.

Even signals are **symmetric** and odd signal are **anti-symmetric**. Both properties play an important role in simplifying signal processing.

Odd and Even Signals (CT Example)



Odd and Even Signals (CT 2nd Example)



Odd and Even Signals (CT 3rd Example)

$$x(-t) = 3(-t)^3 - 2(-t)^2 + 5(-t) - 7$$

$$= -3t^3 - 2t^2 - 5t - 7$$

$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right) = \frac{1}{2}\left(\cancel{3t^3} - 2t^2 + \cancel{5t} - 7 + \left(\cancel{-3t^3} - 2t^2 - \cancel{5t} - 7\right)\right) = \frac{1}{2}\left(-4t^2 - 14\right)$$

$$x(t) = 3t^3 - 2t^2 + 5t - 7$$

$$= -2t^2 - 7$$

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$

$$\vdots$$

$$= 3t^3 + 5t$$

Odd and Even Signals (DT Signal Decomposition)

For DT signals

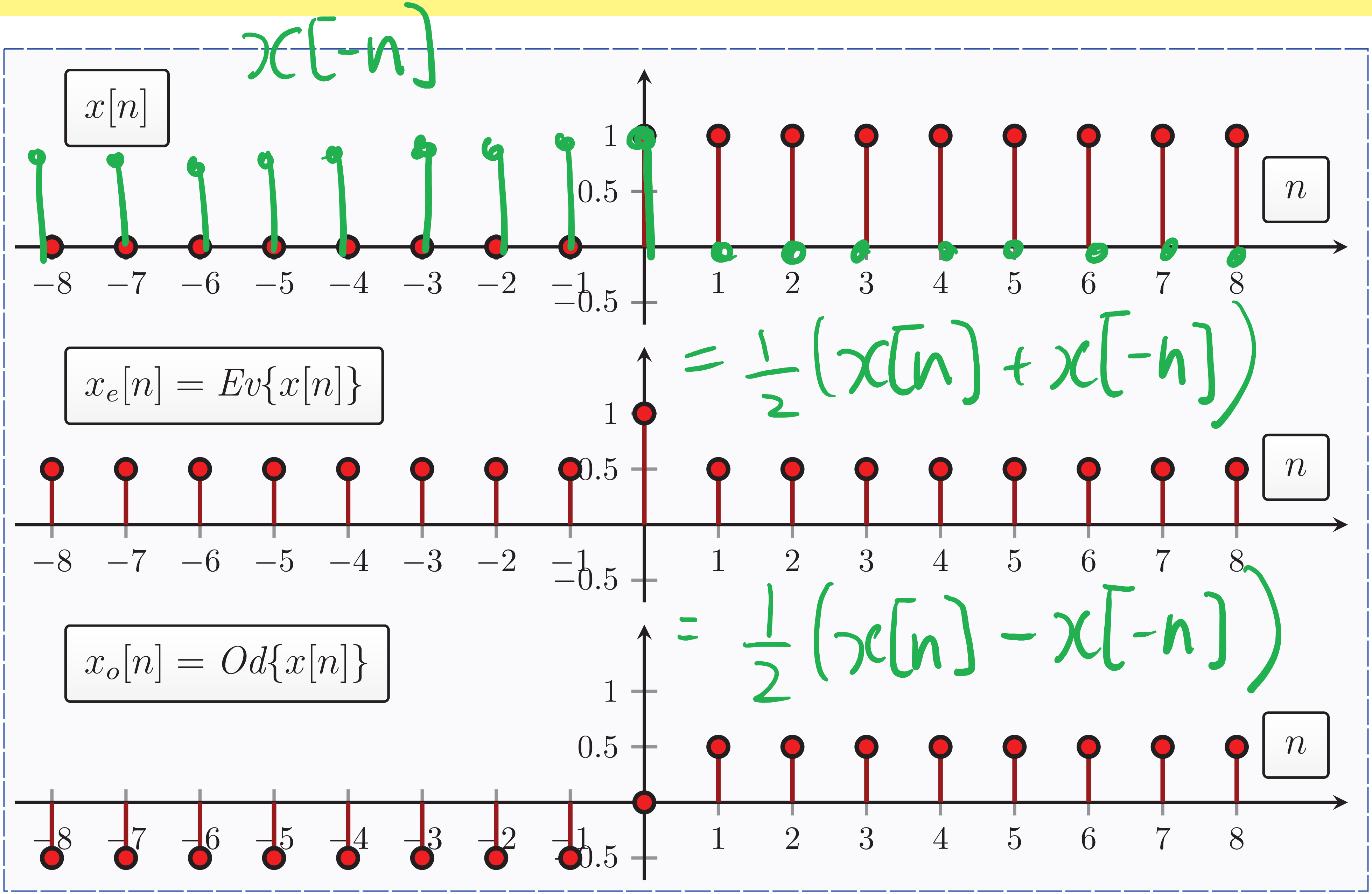
$$x[n] = \underbrace{\frac{1}{2}\left(x[n] + x[-n]\right)}_{x_e[n]} + \underbrace{\frac{1}{2}\left(x[n] - x[-n]\right)}_{x_o[n]}$$

means we can always decompose any DT signal into the sum of an odd DT signal and an even DT signal.

Same formulas:

$$x_e[n] = \frac{1}{2}\left(x[n] + x[-n]\right), \quad x_o[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$$

Odd and Even Signals (DT Example)



Odd and Even Signals

Useful property, e.g.:

- Diffusion magnetic resonance imaging (dMRI) signal is antipodally symmetric (even in 3D)
- This property can be used to reduce the number of samples required
- Important - number of measurements governs scan time

Classifications of signals

1). Continuous-time (CT) or discrete-time (DT)

$x(t)$ $\qquad$ $x[n]$, n integer/discrete

2). Periodic

$x(t+T) = x(t)$ $\qquad$ $x[n+N] = x[n]$

$T_0 = 2\pi/\omega_0$ $\qquad$ $N_0 = \frac{2\pi}{\omega_0} m$ (Sinusoidal signals)

3) Energy/power

$0 < P_\infty < \infty \Rightarrow$ power signal ($E_\infty = \infty$) e.g. periodic signal

$0 < E_\infty < \infty \Rightarrow$ Energy signal ($P_\infty = 0$)

4) odd/even $\rightarrow x(-t) = x(t)$ $\qquad$ $x_e(t) = \frac{1}{2}(x(t) + x(-t))$

$x(-t) = -x(t)$ $\qquad$ $x_o(t) = \frac{1}{2}(x(t) - x(-t))$