Australian National University

Research School of Engineering
College of Engineering and Computer Science

# ENGN2228 Signal Processing

# PROBLEM SET 3

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## Continuous-time Convolution

### Problem Set 3-1

Use the graphical flip/shift method, showing intermediate working to perform the following CT convolutions to find $y(t)$. NOTE: solution can be checked using Matlab.

(a) $x(t) = (t-1)\{u(t-1) - u(t-3)\}$ and $h(t) = u(t+1) - 2u(t-2)$

(b) $x(t) = u(t) - u(t-2)$ and $h(t) = e^{-t}u(t)$

(c) $x(t) = u(t) - 2u(t-2) + u(t-5)$ and $h(t) = e^{2t}u(1-t)$. Hint: convolution is commutative

### Problem Set 3-2

Compute the convolution $y(t) = x(t) * h(t)$ when $x(t) = u(t-1) - u(t-3)$ and $h(t) = u(t) - u(t-2)$.

### Problem Set 3-3

Consider the following convolution

$$y(t) = x(t) \star h(t)$$

where

$$x(t) = e^{-|t|} \quad \text{and} \quad h(t) = 2\,u(t-3) - 2\,u(t-5).$$

(a) Draw $x(t)$ and $h(t)$.

(b) Evaluate the integral

$$y(t) = x(t) \star h(t) \triangleq \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

(c) Draw $y(t)$.

### Problem Set 3-4

In 3-3, we note that the centre of $h(t)$ is at $t = 4$. Review and understand the following:

$$\begin{aligned} h(t) &= 2\big(u(t-3) - u(t-5)\big) \\ &= 2\big(u(t+1) - u(t-1)\big) \star \delta(t-4) \end{aligned}$$

Therefore

$$\begin{aligned} y(t) &= x(t) \star h(t) \\ &= \Big(x(t) \star 2\big(u(t+1) - u(t-1)\big)\Big) \star \delta(t-4). \end{aligned}$$

So we can adjust for the delay of 4 at the end of a convolution that uses the even function

$$h_1(t) = 2\big(u(t+1) - u(t-1)\big)$$

instead of the original $h(t)$.

(a) Draw

$$y_1(t) = x(t) \star h_1(t) = x(t) \star 2\big(u(t+1) - u(t-1)\big).$$

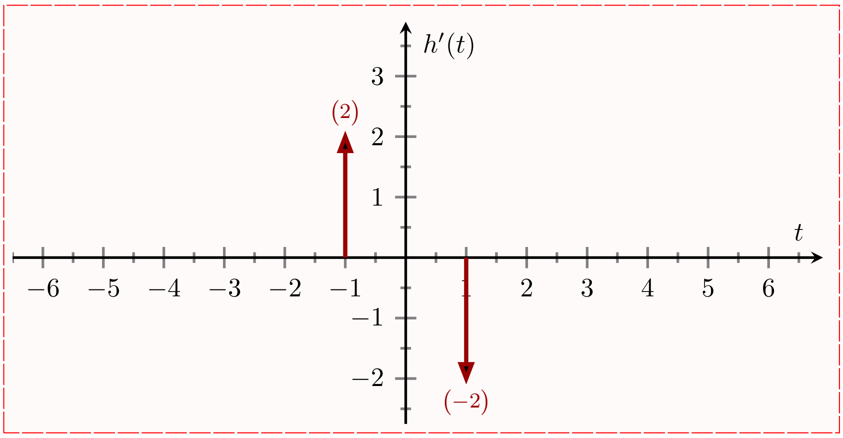
(b) Confirm $y_1(t)$'s relationship with $y(t)$.

## Problem Set 3-5

(Hard) The derivative of $h(t)$ is given by

$$h'(t) = 2\,\delta(t+1) - 2\,\delta(t-1)$$

and is drawn below




In the lecture notes it was shown how to compute the convolution using the $u_n(t)$ functions (where $\delta(t) = u_0(t)$, $u(t) = u_{-1}(t)$, etc.).

(a) Confirm $x(t) \star h(t) = x(t) \star u_1(t) \star u_{-1}(t) \star h(t)$.

(b) Use this expression to evaluate the convolution.

## Problem Set 3-6

Let $h(t)$ be the triangular pulse shown in Fig. 1 and $x(t)$ be the impulse train depicted in Fig. 2. That is,

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT).$$

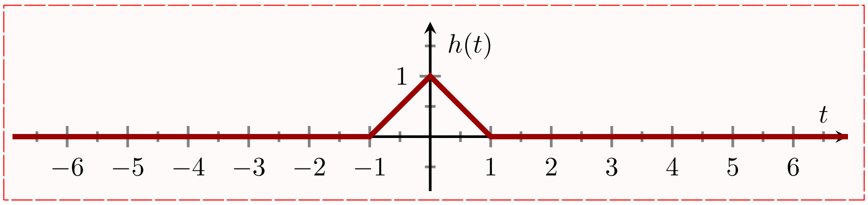



**Figure 1:** Triangular pulse $h(t)$.

Determine and sketch $y(t) = x(t) \star h(t)$ for the following values of $T$:

(a) $T = 2$

(b) $T = 1.5$

## Problem Set 3-7

A continuous-time LTI system has impulse response given by the shifted unit step

$$h(t) = u(t - t_0) \equiv \begin{cases} 1 & t \geq t_0 \\ 0 & \text{otherwise,} \end{cases}$$

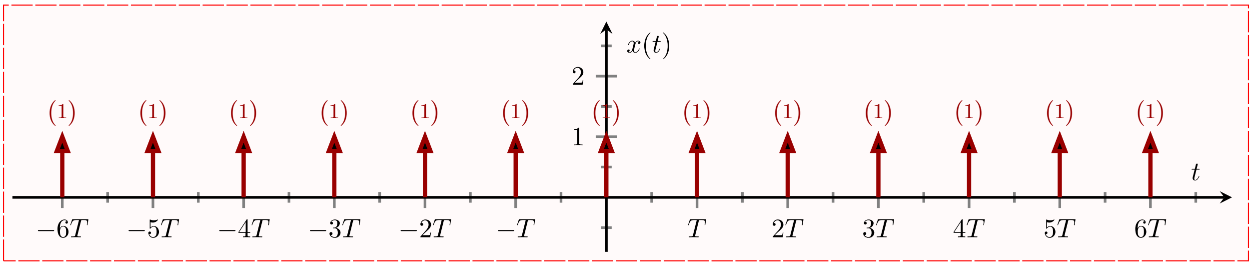



**Figure 2:** Impulse train $x(t)$ with spacing $T$.

where $t_0$ is a fixed time shift. If $y(t) = x(t) \star h(t)$, show that

$$y(t) = \int_{-\infty}^{t} x(\tau - t_0)\, d\tau.$$

$\big($*Hint:* Note the similarity to $x(t) \star h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau.\big)$

# Continuous-time Impulse Response

## Problem Set 3-8

Consider a continuous-time LTI system with input signal $x(t)$ and output signal $y(t)$ given by

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau.$$

Find the impulse response of the system, $h(t)$ where $y(t) = x(t) \star h(t)$. You should express your answer in terms of the unit step function nothing that it can be written

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau.$$

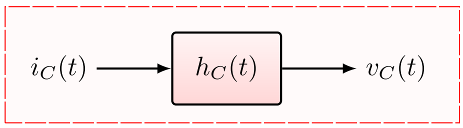
$\big($*Hint:* Remember that $h(t)$ is given by $y(t)$ when $x(t) = \delta(t)$.$\big)$

## Problem Set 3-9

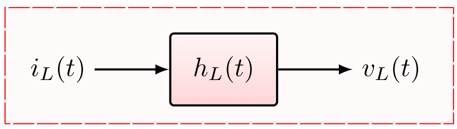
(a) The voltage and current for a capacitor $C$ are related through

$$i_C(t) = C \frac{dv_C(t)}{dt}.$$

Consider the LTI system with input $i_C(t)$ and output $v_C(t)$. What is the impulse response $h_C(t)$ of the system? (You can express the result using the $u_k(t)$ functions defined in Part 7 of the lecture notes, or Section 2.5 of the text.)




(b) The inductor $L$ can be thought of as the dual of the capacitor $C$ where the transformation can be achieved by $L \leftrightarrow C$, $v_L(t) \leftrightarrow i_C(t)$ and $i_L(t) \leftrightarrow v_C(t)$. What is the impulse response $h_L(t)$ for the LTI system with input $i_L(t)$ and output $v_L(t)$



# Continuous-time System Properties

## Problem Set 3-10

If a system is memoryless is it causal?

## Problem Set 3-11

If a system is non-causal can it be memoryless?

## Problem Set 3-12

Determine whether each of the following systems, where $x(t)$ or $x[n]$ is the input signal and $y(t)$ or $y[n]$ is the output signal, are: i) linear, ii) time-invariant, and iii) causal and iv) Memoryless.

| System | Linear | Time-Invariant | Causal | Memoryless |
|---|---|---|---|---|
| $y(t) = x(t-1)$ | | | | |
| $y[n] = x[1-n]$ | | | | |
| $y(t) = 2x(t) + 3$ | | | | |
| $y(t) = x(5t)$ | | | | |
| $y(t) = x(t/5)$ | | | | |
| $y(t) = \text{Real}\{x(t)\}$ | | | | |
| $y[n] = \sum_{k=0}^{\infty} x[k]$ | | | | |
| $y[n] = \sum_{k=-10}^{n-3} x[k]$ | | | | |
| $y(t) = \sin\big(2\pi x(t/5)\big)$ | | | | |
| $y[n] = \cos(2\pi n)x[n]$ | | | | |
| $y[n] = \cos(\pi n)x[n]$ | | | | |
| $y[n] = \sum_{k=-10}^{5} x[k]$ | | | | |