Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

ASSIGNMENT 1

Due Date: Friday, 1 September 2017, 11:55 PM (Friday Week 6)

Late Submission Policy: Submit by the due date and time. Late assessment is *not accepted* for this course. That is, late submissions will get 0 marks.

This policy is to support the majority of students who complete and submit on time. This hard deadline enables the quick release of the assignment solutions.

Wattle submission: Upload your report as PDF format as a *multipage, single file* in Wattle. Use a scanner or scan-type smartphone app to create the PDF from your handwritten solutions.

Assignment format: 5 problems, for a total of 75 marks.

Value: 7% of total course assessment.

Solution: Will be posted on Wattle by Saturday, 2 September 2017. Marked assignments will be returned back in Wattle within 10–14 days.

Relationship to textbook: This assignment is related to Chapters 1–2 in the textbook, and Problem Sets 1, 2 and 3. It is intended to aid you in your preparation for the mid-semester exam.

Declaration by the Student: All assessment task submissions, regardless of mode of submission, require agreement to the following declaration by the student:

> "I declare that this work:
> - upholds the principles of academic integrity, as defined in the University Academic Misconduct Rules;
> - is original, except where collaboration (for example group work) has been authorised in writing by the course convener in the course outline and/or Wattle site;
> - is produced for the purposes of this assessment task and has not been submitted for assessment in any other context, except where authorised in writing by the course convener;
> - gives appropriate acknowledgement of the ideas, scholarship and intellectual property of others insofar as these have been used;
> - in no part involves copying, cheating, collusion, fabrication, plagiarism or recycling."

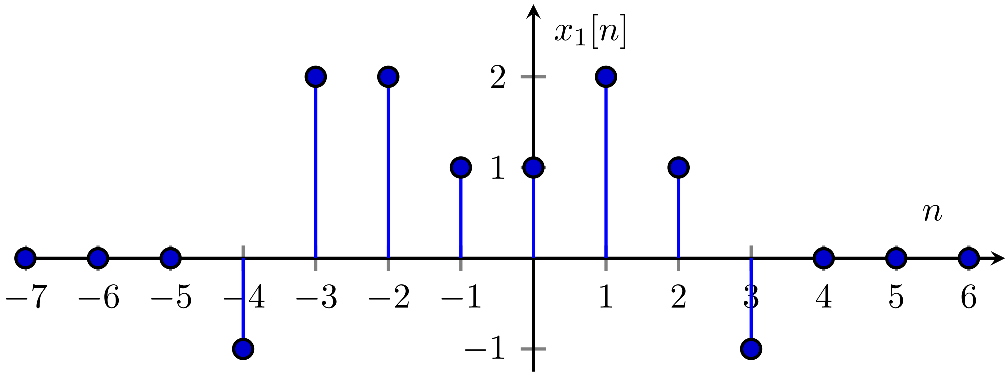
Release Date: Monday, 31 July 2017 (Monday Week 2)

In the following: $\delta[n]$ and $u[n]$ represent the Dirac and unit step functions for discrete-time (DT). Similarly $\delta(t)$ and $u(t)$ for continuous-time (CT). Convolution of signals is written $x[n] \star h[n]$ or $x(t) \star h(t)$. Please indicate any identities or formulas used in the simplification of the results.

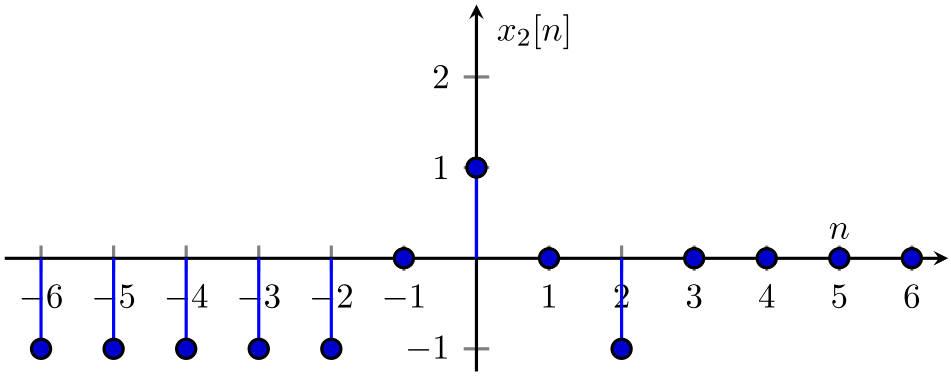
Problem 1

Determine and sketch the even and odd parts of the DT signals depicted below:

(a) **[2 marks]** For $x_1[n]$ shown below:



(b) **[2 marks]** For $x_2[n]$ shown below:



Problem 2

Determine whether or not each of the signals is periodic. If a signal is periodic, determine the fundamental period.

(a) **[2 marks]**

$$x(t) = Ev\{\sin(4\pi t)\, u(t)\}$$

(b) **[2 marks]**

$$x(t) = \sum_{n=-\infty}^{\infty} e^{(2t-n)}$$

(c) **[2 marks]**

$$x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$$

(d) **[2 marks]**

$$x[n] = e^{j10n}\cos(\pi n/22)$$

(e) **[2 marks]**

$$x[n] = 3\, e^{j0.6(n+0.5)}$$

(f) **[2 marks]**

$$x(t) = \sin(4t) + \sin(5t)$$

Problem 3

Determine whether the following systems are: i) time-invariant, ii) linear and iii) casual:

(a) **[3 marks]** The CT system:

$$y(t) = x(t+3) - x(1-t),$$

with input $x(t)$ and output $y(t)$.

(b) **[3 marks]** The DT system:

$$y[n] = \begin{cases} (-1)^n x[n] & \text{if } x[n] \geq 0 \\ 2\,x[n] & \text{if } x[n] < 0 \end{cases},$$

with input $x[n]$ and output $y[n]$.

(c) **[3 marks]** The DT system :

$$y[n] = \sum_{k=n}^{\infty} x[k],$$

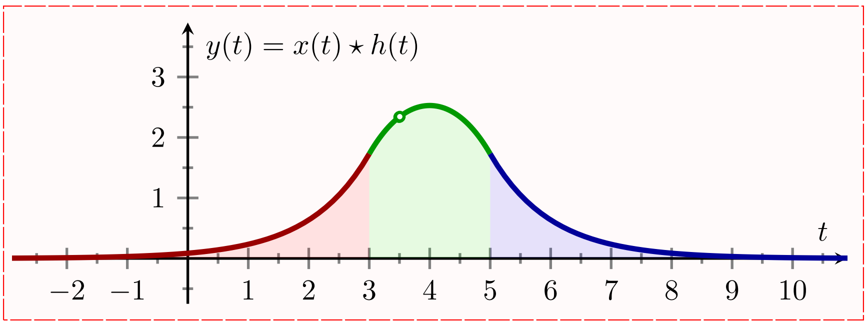
with input $x[n]$ and output $y[n]$.

Problem 4

The impulse response of a system is given by $h(t) = 2\,u(t-3) - 2\,u(t-5)$. For an input of $x(t) = e^{-|t|}$, the system produces the output

$$y(t) = \begin{cases} 2(e^{t-3} - e^{t-5}) & t < 3 \\ 4 - 2\,e^{t-5} - 2\,e^{3-t} & 3 \leq t \leq 5 \\ -2(e^{3-t} - e^{5-t}) & t > 5 \end{cases}$$

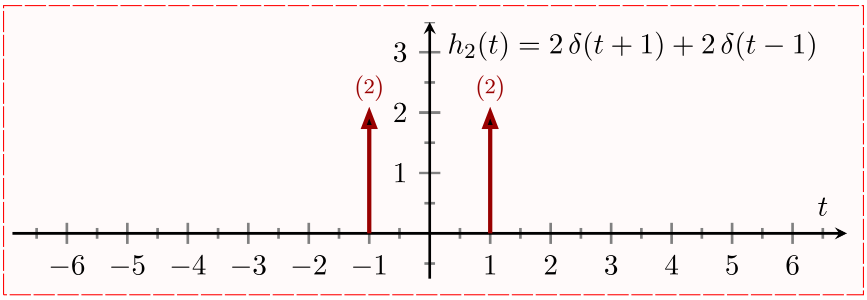
shown in the figure below:



We want to approximate the system with that of a simpler system that gives approximately $y(t)$ for the input of interest $x(t) = e^{-|t|}$. Let's try a new "system" called $h_2(t)$ consisting of two Dirac delta functions:

$$h_2(t) = 2\,\delta(t+1) + 2\,\delta(t-1),$$

which is shown below:



The value of the delta function is presented in round brackets and that number represents the area, here (2) means the delta function has area 2 (so $h_2(t)$ has total area 4).

(a) [**7 marks**] Compute the following convolution

$$y_2(t) = x(t) \star h_2(t),$$

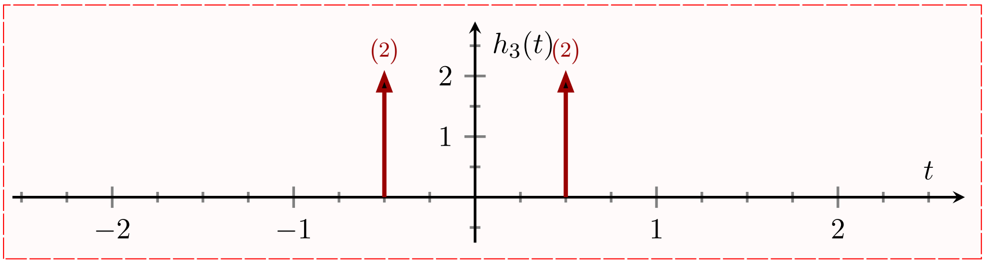
where $x(t) = e^{-|t|}$ as before.

(b) [**7 marks**] Plot $y_2(t) = x(t) \star h_2(t)$ and compare with $y(t) = x(t) \star h(t)$.

(c) [**5 marks**] It is clear that $h_2(t)$ is a rough approximation to $h(t)$ and $y_2(t)$ is a (less) rough approximation to $y(t)$. Show and argue why

$$h_3(t) = 2\,\delta(t + 1/2) + 2\,\delta(t - 1/2),$$

which is shown below, is a better approximation to $h(t)$ than $h_2(t)$.



Compute and plot

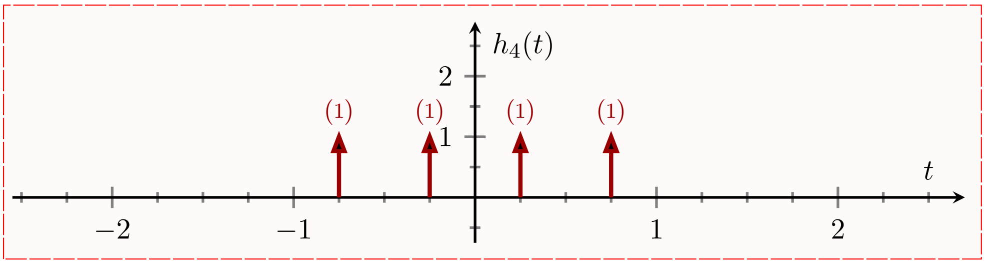
$$y_3(t) = x(t) \star h_3(t),$$

where $x(t) = e^{-|t|}$ as before. Compare $y_3(t)$ with $y_2(t)$ and $y(t)$.

(d) [**5 marks**] Repeat (c) again for $y_4(t) = x(t) \star h_4(t)$ where

$$h_4(t) = \delta(t + 3/4) + \delta(t + 1/4) + \delta(t - 1/4) + \delta(t - 3/4),$$

which is shown below:



Problem 5 5 marks

Consider an RC circuit shown in Fig. 1, which is an example of a linear time-invariant (LTI) system. The impulse response of this circuit is given by

$$h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$$

Assume that the capacitor is initially uncharged and the circuit's time constant is $RC = 1$ seconds. Use convolution to determine the voltage across the capacitor, $y(t)$, resulting from an input voltage $x(t) = u(t) - u(t - 2)$ volts.

Problem 6 5 marks

Compute the convolution $y[n] = x[n] * h[n]$ when $x[n] = \left(-\frac{1}{2}\right)^n u[n-4]$ and $h[n] = 4^n\, u[2 - n]$.

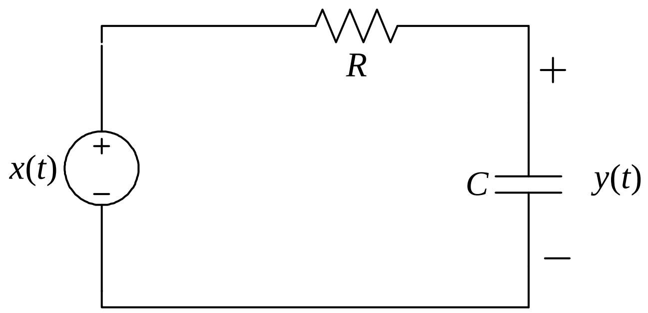


Figure 1: *RC* circuit which is an LTI system.

Problem 7

Let $h_1[n]$, $h_2[n]$ and $h_3[n]$ be the unit pulse responses of three systems S1, S2 and S3, respectively.

(a) **[5 marks]** Relate $y[n]$ and $x[n]$ in terms of h_1, h_2 and h_3 for the system configuration in Fig. 2.

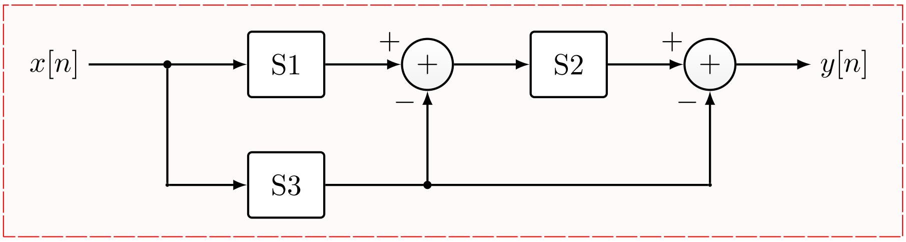


Figure 2: Interconnection of systems.

(b) **[8 marks]** Given

$$h_1[n] = \delta[n-1], \quad h_2[n] = n\,\alpha^n\,u[n], \quad \text{and} \quad h_3[n] = \beta^n\,u[n-2],$$

where $|\alpha| < 1$ and $|\beta| < 1$, determine $h[n]$ such that $y[n] = x[n] \star h[n]$.

(c) **[3 marks]** Is the overall system causal? Why?

— End of Assignment —