Signal Processing
ENGN2228

Lecturer: Dr. Amin Movahed

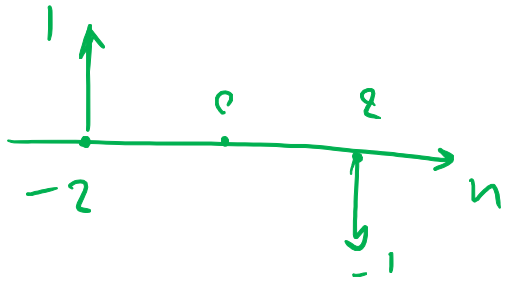
Research School of Engineering, CECS
The Australian National University
Canberra ACT 2601 Australia

Second Semester

Lecture 30

Examples:

$$x[n] = \delta[n+2] - \delta[n-2] \qquad X\left(e^{j\omega}\right) = ?$$



$\delta[n] \longleftrightarrow 1$ (11)

$\delta[n-n_0] \longleftrightarrow e^{-j\omega n_0}$ (13)

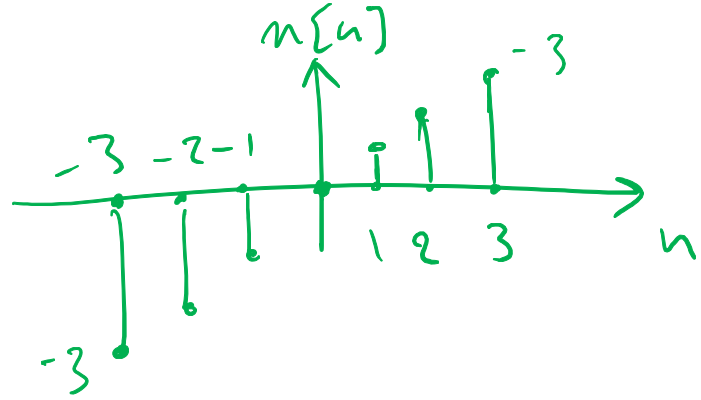
$\delta[n-2] \longleftrightarrow e^{-j2\omega}$

$\delta[n+2] = \delta[n-(-2)] \longleftrightarrow e^{+j2\omega}$

$$X(e^{j\omega}) = \frac{e^{j2\omega} - e^{-j2\omega}}{2j} \times 2j = 2j\sin(2\omega)$$

Examples:

$$x[n] = \begin{cases} n & -3 \le n \le 3 \\ 0 & \text{otherwise} \end{cases} \qquad X\left(e^{j\omega}\right) = ?$$



$$n[n] = -3\delta[n+3] - 2\delta[n+2] - \delta[n+1] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$$

$$\delta[n-n_0] \longleftrightarrow e^{-j\omega n_0}$$

$$X(e^{j\omega}) = -3e^{j3\omega} - 2e^{j2\omega} - e^{j\omega} + e^{-j\omega} + 2e^{-j2\omega} + 3e^{-j3\omega}$$

$$= -3(e^{j3\omega} - e^{-3j\omega}) - 2(e^{j2\omega} - e^{-j2\omega}) - (e^{j\omega} - e^{-j\omega})$$

$$= -6j\sin(3\omega) - 4j\sin(2\omega) - 2j\sin(\omega)$$

Examples:

$$x[n] = \begin{cases} 2^n & 0 \le n \le 9 \\ 0 & \text{otherwise} \end{cases} \qquad X\left(e^{j\omega}\right) = ?$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=0}^{9} 2^n e^{-j\omega n} = \sum_{n=0}^{9} (2e^{-j\omega})^n$$

$$* \sum_{n=0}^{M-1} \alpha^n = \frac{1-\alpha^M}{1-\alpha} \qquad \alpha \neq 1$$

$$= \frac{1-(2e^{-j\omega})^{10}}{1-(2e^{-j\omega})}$$

Examples:

$$x[n] = \left(-\frac{1}{5}\right)^n u[n] - 6\left(-\frac{1}{5}\right)^{n-2} u[n-2] \qquad X\left(e^{j\omega}\right) = ?$$

$$a^n u[n] \longleftrightarrow \frac{1}{1 - a e^{-j\omega}}$$

$$(-1/5)^n u[n] \longleftrightarrow \frac{1}{1 + 1/5\, e^{-j\omega}}$$

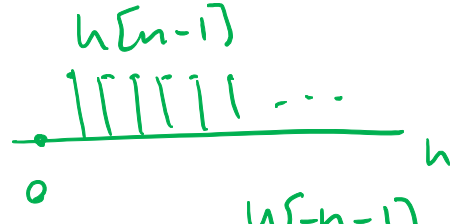
$$n[n-n_0] \longleftrightarrow e^{-j\omega n_0} X(e^{j\omega}) \qquad (-1/5)^{n-2} u[n-2] \longleftrightarrow e^{-j2\omega} \frac{1}{1 + 1/5\, e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{1}{1 + 1/5\, e^{-j\omega}} - \frac{6 e^{-j2\omega}}{1 + 1/5\, e^{-j\omega}}$$

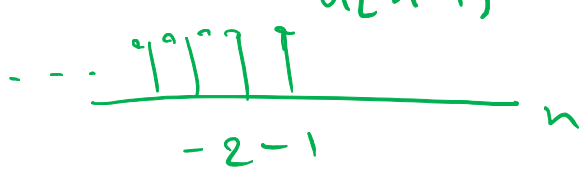
Examples:

$$x[n] = \left(\frac{1}{2}\right)^{-n} u[-n-1] \qquad X\left(e^{j\omega}\right) = ?$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} u[n]\, e^{-j\omega n}$$



$$= \sum_{n=-\infty}^{-1} (1/2)^{-n} e^{-j\omega n}$$



$$= \sum_{n=-\infty}^{-1} (1/2\, e^{j\omega})^{-n} = \sum_{n=-\infty}^{-1} (2e^{-j\omega})^{n} = \sum_{n=-1}^{-\infty} (2e^{-j\omega})^{n}$$

$$\text{If} \sum_{n=-k}^{-\infty} \alpha^{n} = \alpha^{-k}\left(\frac{\alpha}{\alpha-1}\right) \quad |\alpha|>1 \Rightarrow X(e^{j\omega}) = (2e^{-j\omega})^{-1}\left(\frac{2e^{-j\omega}}{2e^{-j\omega}-1}\right)$$

$$= \frac{e^{j\omega}}{2}\left(\frac{1}{1-1/2\, e^{j\omega}}\right)$$

Examples:

$$x[n] = \sin\left(\frac{\pi}{3}n + \frac{\pi}{4}\right) \qquad X\left(e^{j\omega}\right) = ?$$

$\omega_0 = \pi/3$ $\quad N = \frac{2\pi}{\omega_0} m$

$= \frac{2\pi}{\pi/3} m = 6m$ $\quad N_0 = 6$

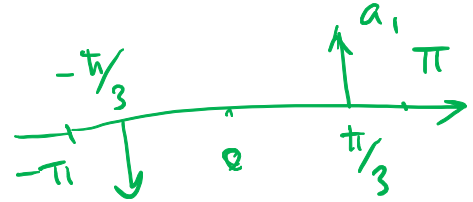
$$x[n] = \frac{e^{j(\pi/3 n + \pi/4)} - e^{-j(\pi/3 n + \pi/4)}}{2j}$$

$$= \frac{e^{j\pi/4}}{2j} e^{j\pi/3 n} - \frac{e^{-j\pi/4}}{2j} e^{-j\pi/3 n}$$

$$x[n] = \sum_{k=-2}^{3} a_k e^{jk\omega_0 n} = a_{-2} e^{-j2\pi/3 n} + a_{-1} e^{-j\pi/3 n} + a_0 + a_1 e^{j\pi/3 n} + a_2 e^{j2\pi/3 n} + a_3 e^{j\pi n}$$

$$a_1 = \frac{e^{j\pi/4}}{2j}, \quad a_{-1} = -\frac{e^{-j\pi/4}}{2j}$$

$-\pi < \omega < \pi$



$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - \omega_0) = 2\pi a_1 \delta(\omega - \omega_0) + 2\pi a_{-1} \delta(\omega + \omega_0)$$

$$\pi/j\, e^{j\pi/4} \delta(\omega - \omega_0) - \pi/j\, e^{-j\pi/4} \delta(\omega + \omega_0)$$

Examples:

$N_0 = 4$ $\quad \omega_0 = 1$

$$x[n] = \sin\left(n\frac{\pi}{2}\right) + \cos(n) \qquad X\left(e^{j\omega}\right) = ?$$

not periodic

$\sin \omega_0 n \longleftrightarrow \pi/j\,[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)] \quad -\pi < \omega < \pi$

$\sin \pi/2\, n \longleftrightarrow \pi/j\,[\delta(\omega-\pi/2) - \delta(\omega+\pi/2)]$ (A)

$\cos n = \dfrac{e^{jn} + e^{-jn}}{2} \longrightarrow e^{j\omega n} \longleftrightarrow 2\pi\,\delta(\omega-\omega')$

(B) $\frac{1}{2} e^{jn} \longleftrightarrow \pi\,\delta(\omega-1)$ $\qquad X(e^{j\omega}) = A + B + C$

(C) $\frac{1}{2} e^{-jn} \longleftrightarrow \pi\,\delta(\omega+1)$