Australian National University

Research School of Engineering
College of Engineering and Computer Science

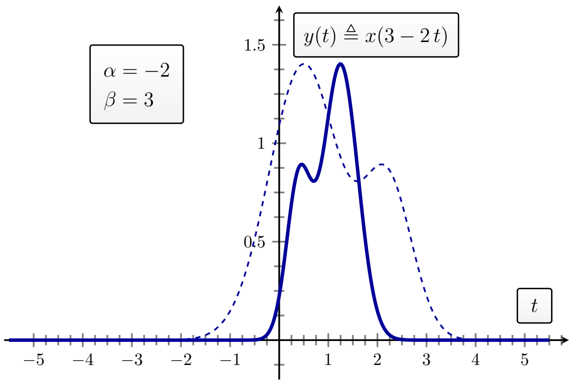
ENGN2228 Signal Processing

HOMEWORK 2 – SOLUTIONS

Homework 2-1

Review the independent variable affine transformation examples on lecture slides 50 and 52, shown below.

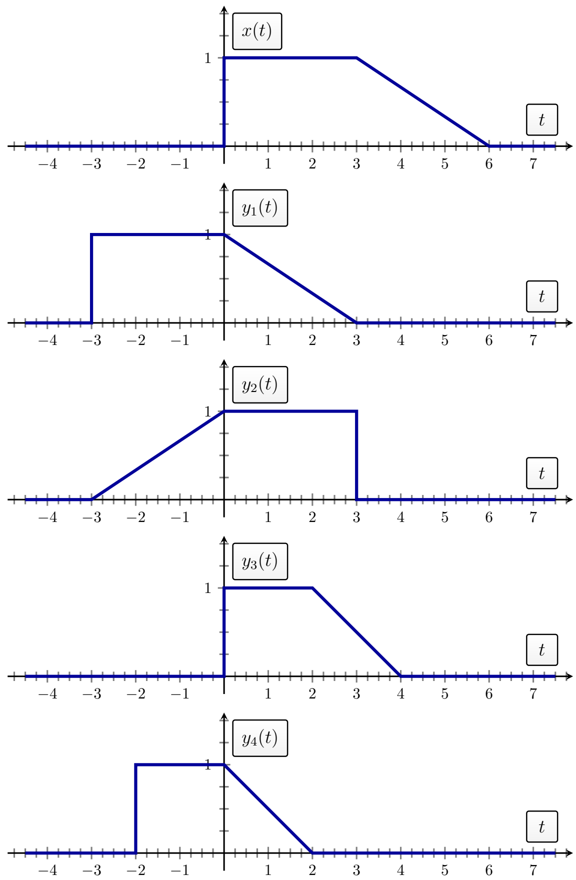
(a) Confirm the solid blue curve in the figure (blow) is indeed $y(t) = x(3 - 2t)$ where $x(t)$ is the dashed curve.



Solution: Test with different values of t. For example, for $t = 0.5$ the portion around $y(0.5)$ comes from the reversed dashed portion around $x(3 - 2 \times 0.5) = x(2)$. □

(b) Express each of $y_1(t)$, $y_2(t)$, $y_3(t)$, $y_4(t)$ in terms of $x(t)$ (shown to the right).

Solution: $y_1(t) = x(t + 3)$, $y_2(t) = x(-t + 3)$, $y_3(t) = x(1.5\,t)$, and $y_4(t) = x(1.5\,t + 3)$. □



Homework 2-2

Review phasors, see phasors on wikipedia and looks at phasor diagrams for one sinusoid and phasor diagram for sum of two sinusoids, where this only works because the sinusoids are the same frequency.

Homework 2-3

(Hard) Movies like two sinusoids show what happens when there is the sum of two sinusoids at the same frequency. The real part of the rotating phasor maps out a sinusoid. Using the phasor ideas, suppose there is the sum of two sinusoids at different frequencies, for example at ω and 3ω such as

$$\cos(\omega t) + \frac{1}{9}\cos(3\omega t)$$

What modification happens to the phasor picture?

Solution: Instead of a sinusoid a non-sinusoidal but periodic signal coming out. For $\cos(\omega t) + \frac{1}{9}\cos(3\omega t)$ you get something more triangular:

plot cos(t)+cos(3t)/9

and with more terms even more so triangular:

plot cos(t)+cos(3t)/9+cos(5t)/25

Plots can also be generated using WolframAlpha. □

Homework 2-4

Lecture slides 55 and 56 show that adding two periodic signals of different periods can give you an output signal that is periodic:

(a) When does this occur in general?

Solution: If T_1 is the period of the first signal and T_2 is the period of the second signal then T_1 and T_2 need to be rationally related, for example,

$$\frac{T_1}{T_2} = \frac{4099}{298}$$

means T_1 and T_2 are rationally related. □

(b) Assuming all periods under consideration are integer-valued (like 7 or 11 and not 4/5 nor $\sqrt{2}$), answer the following. If the output is periodic what is the relationship of the output period to the two input periods?

Solution: The output period is the least or lowest common multiple of T_1 and T_2, for example, if $T_1 = 4098$ and $T_2 = 18879$ then the LCM is $T_3 = 25788714$ (here T_1 and T_2 have a greatest common divisor of 3 and so the LCM is $T_1 \times T_2/3$.) □

(c) Repeat the previous part in the case when the periods are not integer-valued. (For example, suppose $T_1 = 4098\sqrt{2}$ and $T_2 = 18879\sqrt{2}$.)

Solution: Much the same as previously, T_1 and T_2 need to have a common not rational part that can be factored out. □

Homework 2-5

Review the type of calculation shown on lecture slide 70, repeated here

$$\begin{aligned} A\cos(\omega_0 t + \phi) &= \frac{A}{2}e^{j\phi}e^{j\omega_0 t} + \frac{A}{2}e^{-j\phi}e^{-j\omega_0 t} \\ &= A\,\Re\{e^{j(\omega_0 t+\phi)}\} \end{aligned}$$

and

$$A\sin(\omega_0 t + \phi) = A\,\Im\{e^{j(\omega_0 t+\phi)}\},$$

which needs the Euler identity (here A is real-valued).

Homework 2-6

Roger Federer and Leonhard Euler were both born in Basel Switzerland but they never meet. Why?

Solution: It would be bad PR for Roger to exhume Euler's body. □

Homework 2-7

CT signal $x(t) = e^{j\omega t}$ is periodic for any choice of ω.

(a) True or false?

Solution: True □

(b) What is its fundamental period when $\omega = 4$?

Solution: $T_0 = \pi/2$, that is, $x(t + \pi/2) = x(t)$ for all t. □

(c) What is its fundamental period when $\omega = 19\pi$?

Solution: $T_0 = 2/19$ □

Homework 2-8

DT signal $x[n] = e^{j\omega n}$ is not periodic for every choice of ω.

(a) True or false?

Solution: True □

(b) Is it periodic when $\omega = 4$?

Solution: No, because otherwise we would need $4n$ to be a multiple of 2π which would imply π is rational , which is false ($\pi \neq 2n/k$ for integer k). □

(c) Is it periodic when $\omega = 19\pi$?

Solution: Yes, because it has fundamental period $N_0 = 2$. $x[0] = 1$, $x[1] = -1$, $x[2] = 1$, etc. □

(d) How does the DT signal $x[n] = e^{j19\pi n}$ differ from $x[n] = e^{j\pi n}$?

Solution: It doesn't differ. □