Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

HOMEWORK 6 – SOLUTIONS

Homework 6-1

Determine whether each of the following systems, where $x(t)$ or $x[n]$ is the input signal and $y(t)$ or $y[n]$ is the output signal, are: i) linear, ii) time-invariant, and iii) causal and iv) Memoryless.

System	Linear	Time-Invariant	Causal	Memoryless
$y(t) = x(t-1)$				
$y[n] = x[1-n]$				
$y(t) = 2x(t) + 3$				
$y(t) = x(5t)$				
$y(t) = x(t/5)$				
$y(t) = \text{Real}\{x(t)\}$				
$y[n] = \sum_{k=0}^{\infty} x[k]$				
$y[n] = \sum_{k=-10}^{n-3} x[k]$				
$y(t) = \sin\left(2\pi x(t/5)\right)$				
$y[n] = \cos(2\pi n)x[n]$				
$y[n] = \cos(\pi n)x[n]$				
$y[n] = \sum_{k=-10}^{5} x[k]$				

Solution:

System	Linear	Time-Invariant	Causal	Memoryless
A: $y(t) = x(t-1)$	✓	✓	✓	X
B: $y[n] = x[1-n]$	✓	X	X	X
C: $y(t) = 2x(t) + 3$	X	✓	✓	✓
D: $y(t) = x(5t)$	✓	X	X	X
E: $y(t) = x(t/5)$	✓	X	X	X
F: $y(t) = \text{Real}\{x(t)\}$	X	✓	✓	✓
G: $y[n] = \sum_{k=0}^{\infty} x[k]$	✓	X	X	X
H: $y[n] = \sum_{k=-10}^{n-3} x[k]$	✓	X	X	X
I: $y(t) = \sin\big(2\pi x(t/5)\big)$	X	X	X	X
J: $y[n] = \cos(2\pi n)x[n]$	✓	✓	✓	✓
K: $y[n] = \cos(\pi n)x[n]$	✓	X	✓	✓
L: $y[n] = \sum_{k=-10}^{5} x[k]$	✓	X	X	X

For the more challenging and trickier ones.

- C: put zero function in and the constant value of 3 comes out. Then scale this input by 7, say, and the output is still 3 and not 21 (which it would be if it were linear).
- D: $y(t)$ at $t = 1$ needs $x(t)$ at (future) $t = 5$.
- E: $y(t)$ at $t = -1$ needs $x(t)$ at (future) $t = -1/5$.
- F: Very tricky. Multiply the input by a complex scalar (e.g., $j = \sqrt{-1}$) generally doesn't multiply the output by the same complex scalar.
- G: If input is $x_1[n] = \delta[n+1]$ then $y_1[n] = 0$; then shifted input $x_2[n] = x[n-1] = \delta[n]$ gives $y_2[n] = 1 \neq y_1[n] = 0$.
- H: If the summation is permitted to reverse then having $n = -20$ requires future $x[-10]$. If the summation is forward only then it is time-invariant and causal.
- J: A trick because $\cos(2\pi n) = 1$ (for integer n).
- K: $\cos(2\pi n) = (-1)^n$
- L: If input is $x_1[n] = \delta[n+11]$ then $y_1[n] = 0$; then shifted input $x_2[n] = x[n-1] = \delta[n+10]$ gives $y_2[n] = 1 \neq y_1[n] = 0$.

□

Homework 6-2

If a system is memoryless is it causal?

Solution: Yes. □

Homework 6-3

If a system is non-causal can it be memoryless?

Solution: No, by the contrapositive of the previous question. That is, saying memoryless implies causal (which is true) is the same as saying not causal implies not memoryless. If you are not familiar with the contrapositive then: rain implies clouds, therefore no clouds implies no rain. □