Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

HOMEWORK 1 – SOLUTIONS

A quick refresher on complex numbers. Complex numbers are used in much of engineering. They are an near ideal shorthand in signal representation and they simplify expressions.

Homework 1-1

(a) Prove the Euler identity:

$$e^{j\theta} = \cos\theta + j\sin\theta.$$

Solution: Write out the Taylor series about $x = 0$ for e^x, $\cos x$ and $\sin x$:

$$\begin{aligned} e^x &= 1 + x + x^2/2! + x^3/3! + x^4/4! + \cdots \\ \cos x &= 1 - x^2/2! + x^4/4! + \cdots \\ \sin x &= x - x^3/3! + x^5/5! + \cdots \end{aligned}$$

Then, with $x = j\theta$,

$$\begin{aligned} e^{j\theta} &= 1 + j\theta + (j\theta)^2/2 + (j\theta)^3/3! + (j\theta)^4/4! + \cdots \\ &= (1 - \theta^2/2! + \theta^4/4! + \cdots) + j(\theta - \theta^3/3! + \theta^5/5! \cdots) \\ &= \cos\theta + j\sin\theta \end{aligned}$$

□

(b) Find expressions for $\cos\theta$ and $\sin\theta$ in terms of $e^{j\theta}$ and its conjugate $e^{-j\theta}$.

Solution: Combine $e^{j\theta}$ with its conjugate $e^{-j\theta}$ gives $2\cos\theta$:

$$\begin{aligned} e^{j\theta} + e^{-j\theta} &= \cos\theta + j\sin\theta + \cos\theta - j\sin\theta = 2\cos\theta \\ e^{j\theta} - e^{-j\theta} &= \cos\theta + j\sin\theta - \cos\theta + j\sin\theta = 2j\sin\theta \end{aligned}$$

then divide by 2 and $2j$, respectively. □

Homework 1-2

What is the difference between taking the conjugate of an expression and replacing every occurrence of j with $-j$?

Solution: Nothing. □

Homework 1-3

Write each of the following in polar form, that is, in $re^{j\theta}$ find r (such that $r \geq 0$) and θ.

(a) $1 + j\sqrt{3}$

Solution: $2(1/2 + j\sqrt{3}/2) = 2e^{j\pi/3}$. □

(b) $(\sqrt{3} + j^3)(1 - j)$

Solution: $2e^{-j\pi/6}\sqrt{2}e^{-j\pi/4} = 2\sqrt{2}e^{-j5\pi/12}$. □

(c) $\dfrac{e^{j\pi/3}-1}{1+j\sqrt{3}}$

Solution: $\dfrac{1/2+j\sqrt{3}/2-1}{1+j\sqrt{3}} = \dfrac{1}{2}e^{j\pi/3}$. □

(d) $j^{1,222,444,667,099,987,676,222,091,345,222,822,822,282,228}$

Solution: Magnitude $r = 1$ and $1,222,\ldots,282,228 \equiv 228 \equiv 0 \pmod 4$ so phase $\theta = 0$. □

Homework 1-4

With $j = \sqrt{-1}$ what is j^j (j to the power j)?

Solution: $1/\sqrt{e^\pi}$ (which is real) since $j = e^{j\pi/2}$ then $j^j = e^{j^2\pi/2} = e^{-\pi/2} = 1/\sqrt{e^\pi}$. □