Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

HOMEWORK 8 – SOLUTIONS

Homework 8-1

For each of the following pulse responses shown in the figures:

(i) give an expression for the pulse response $h[n]$,

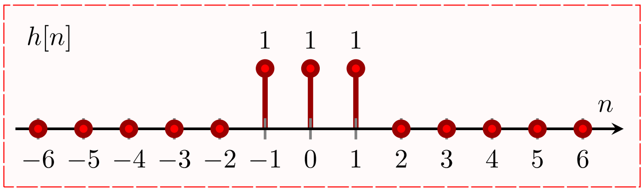
(ii) the frequency response

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-jn\omega},$$

(iii) say whether $H(e^{j\omega})$ as a function of ω is even or odd

(iv) discuss the phase of $H(e^{j\omega})$ and by looking at the slope say what the (group) delay is

(v) and sketch/plot the magnitude $\left|H(e^{j\omega})\right|$

(a) 

Real and even.

Solution:

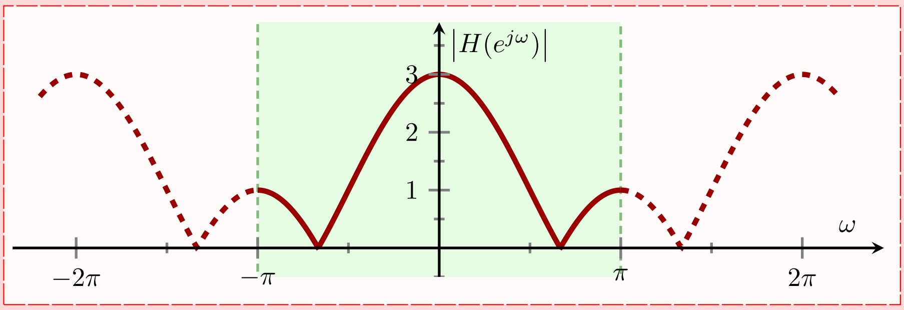
(i) $h[n] = h[-1]\delta[n+1] + h[0]\delta[n] + h[1]\delta[n-1] = \delta[n+1] + \delta[n] + \delta[n-1]$

(ii) $H(e^{j\omega}) = h[-1]e^{j\omega} + h[0] + h[1]e^{-j\omega} = e^{j\omega} + 1 + e^{-j\omega} = 1 + 2\cos\omega$

(iii) $H(e^{j\omega})$ is even

(iv) phase is flat and equals 0, the delay being the slope is 0

(v) $\left|H(e^{j\omega})\right| = |1 + 2\cos\omega|$ and is plotted below:



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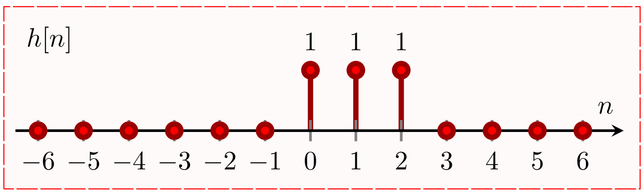
(b) 

Real and causal.

Solution:

(i) $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

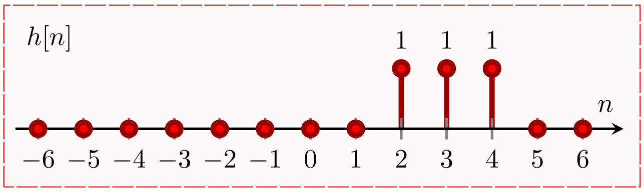
(ii) $H(e^{j\omega}) = 1 + e^{-j\omega} + e^{-j2\omega} = e^{-j\omega}(1 + 2\cos\omega)$

(iii) $H(e^{j\omega})$ is complex-valued but $\left|H(e^{j\omega})\right|$ is even

(iv) phase is linear and equals $-\omega$, the delay being the negative slope is 1

(v) $\left|H(e^{j\omega})\right| = |1 + 2\cos\omega|$ as previously plotted $\square$

(c)



Real with larger delay.

Solution:

(i) $h[n] = \delta[n-2] + \delta[n-3] + \delta[n-4]$

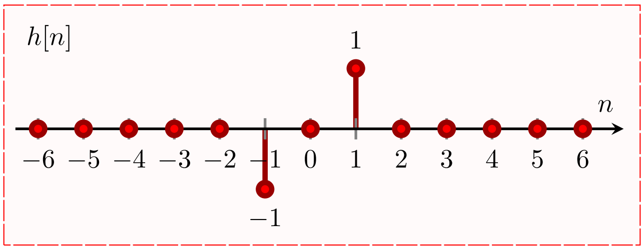
(ii) $H(e^{j\omega}) = e^{-3j\omega}(1 + 2\cos\omega)$

(iii) $H(e^{j\omega})$ is complex-valued but $\left|H(e^{j\omega})\right|$ is even

(iv) phase is linear and equals -3ω, the delay being the negative slope is 3

(v) $\left|H(e^{j\omega})\right| = |1 + 2\cos\omega|$ as previously plotted $\square$

(d)



Real and odd.

Solution:

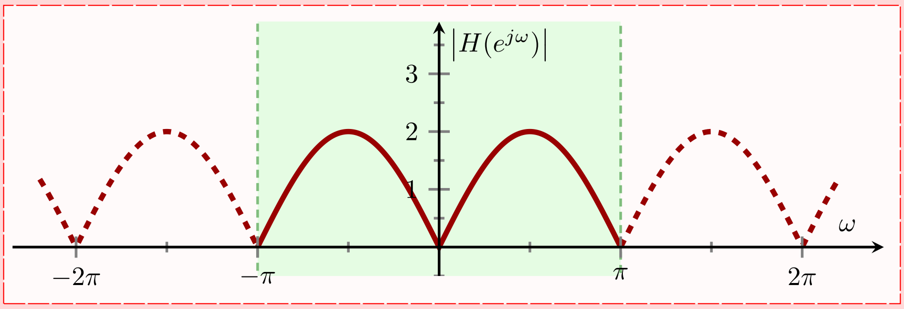
(i) $h[n] = h[-1]\delta[n+1] + h[1]\delta[n-1] = -\delta[n+1] + \delta[n-1]$

(ii) $H(e^{j\omega}) = h[-1]e^{j\omega} + h[1]e^{-j\omega} = -e^{j\omega} + e^{-j\omega} = -2j\left(\dfrac{e^{j\omega} - e^{-j\omega}}{2j}\right) = 2\,e^{j3\pi/2}\sin\omega$

(iii) $H(e^{j\omega})$ is odd since $\sin\omega$ is odd, and $\left|H(e^{j\omega})\right|$ is even

(iv) phase is constant at $3\pi/2$ and the delay (slope with respect to ω) is zero

(v) $\left|H(e^{j\omega})\right| = |2\sin\omega|$ and is plotted below:



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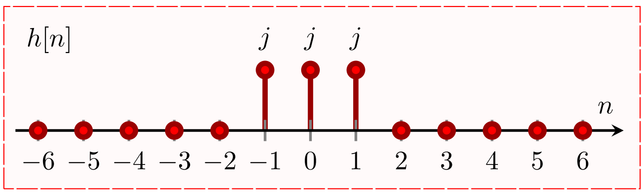
(e)



Imaginary and even.

Solution:

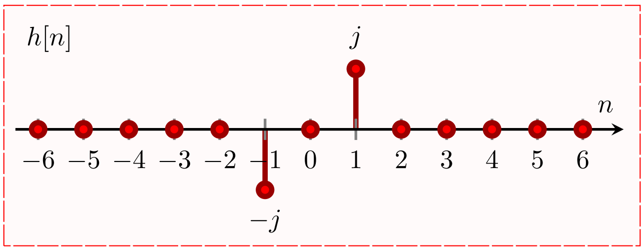
(i) $h[n] = h[-1]\delta[n+1] + h[0]\delta[n] + h[1]\delta[n-1] = j\delta[n+1] + j\delta[n] + j\delta[n-1]$

(ii) $H(e^{j\omega}) = h[-1]e^{j\omega} + h[0] + h[1]e^{-j\omega} = j\big(e^{j\omega} + 1 + e^{-j\omega}\big) = e^{j\pi/2}(1 + 2\cos\omega)$

(iii) $H(e^{j\omega})$ is even

(iv) phase is flat and equals $\pi/2$, the delay being the slope is 0

(v) $\big|H(e^{j\omega})\big| = |1 + 2\cos\omega|$ as previously plotted $\square$

(f) 

Imaginary and odd.

Solution:

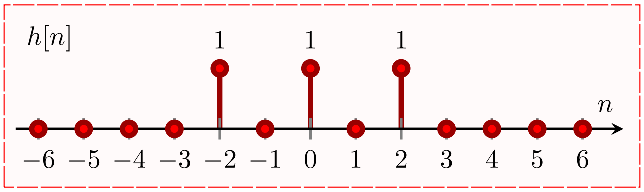
(i) $h[n] = h[-1]\delta[n+1] + h[1]\delta[n-1] = -j\delta[n+1] + j\delta[n-1]$

(ii) $H(e^{j\omega}) = h[-1]e^{j\omega} + h[1]e^{-j\omega} = -je^{j\omega} + je^{-j\omega} = 2\Big(\dfrac{e^{j\omega} - e^{-j\omega}}{2j}\Big) = 2\sin\omega$ is purely real-valued. Notice that $h[-1] = \overline{h[1]}$.

(iii) $H(e^{j\omega})$ is odd since $\sin\omega$ is odd, and $\big|H(e^{j\omega})\big|$ is even

(iv) phase is constant at 0 and the delay is zero

(v) $\big|H(e^{j\omega})\big| = |2\sin\omega|$ as previously plotted $\square$

(g) 

Real, even and spaced out.

Solution:

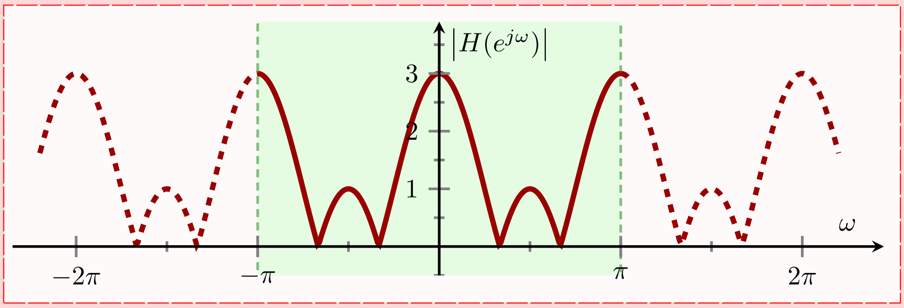
(i) $h[n] = h[-2]\delta[n+2] + h[0]\delta[n] + h[2]\delta[n-2] = \delta[n+2] + \delta[n] + \delta[n-2]$

(ii) $H(e^{j\omega}) = h[-2]e^{j\omega} + h[0] + h[2]e^{-j\omega} = e^{j2\omega} + 1 + e^{-j2\omega} = 1 + 2\cos 2\omega$

(iii) $H(e^{j\omega})$ is even and purely real-valued

(iv) phase is flat and equals 0, the delay being the slope is 0

(v) $\big|H(e^{j\omega})\big| = |1 + 2\cos 2\omega|$ and is plotted below:



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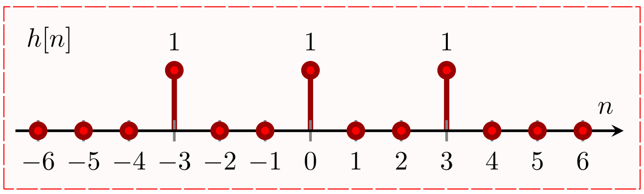
(h) 

Real, even and spaced out.

Solution:

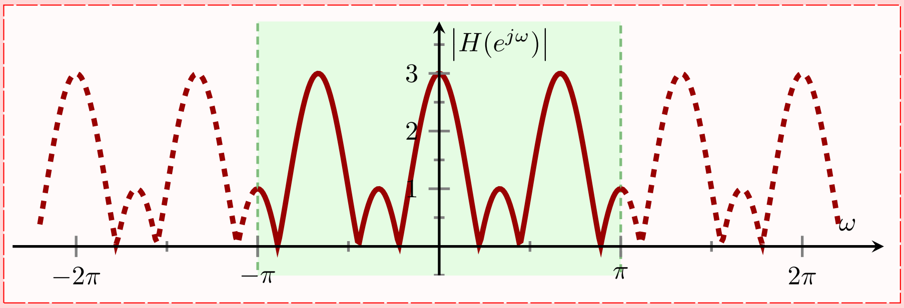
(i) $h[n] = h[-3]\delta[n+3] + h[0]\delta[n] + h[3]\delta[n-3] = \delta[n+3] + \delta[n] + \delta[n-3]$

(ii) $H(e^{j\omega}) = h[-3]e^{j\omega} + h[0] + h[3]e^{-j\omega} = e^{j3\omega} + 1 + e^{-j3\omega} = 1 + 2\cos 3\omega$

(iii) $H(e^{j\omega})$ is even and purely real-valued

(iv) phase is flat and equals 0, the delay being the slope is 0

(v) $\left|H(e^{j\omega})\right| = |1 + 2\cos 3\omega|$ and is plotted below:



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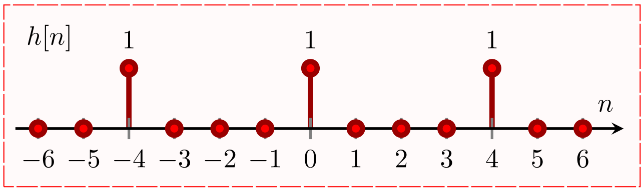
(i) 

Real, even and spaced out.

Solution:

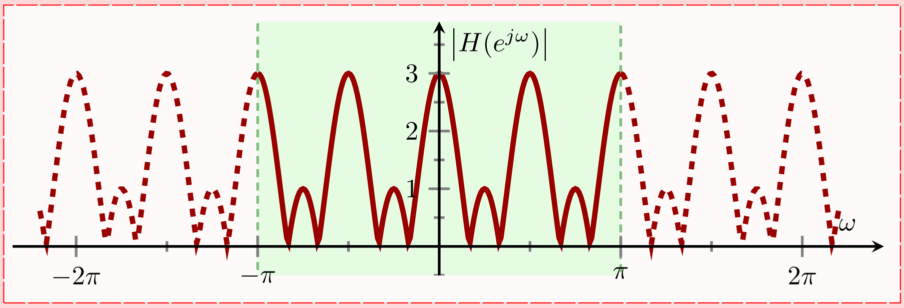
(i) $h[n] = h[-4]\delta[n+4] + h[0]\delta[n] + h[4]\delta[n-4] = \delta[n+4] + \delta[n] + \delta[n-4]$

(ii) $H(e^{j\omega}) = h[-4]e^{j\omega} + h[0] + h[4]e^{-j\omega} = e^{j4\omega} + 1 + e^{-j4\omega} = 1 + 2\cos 4\omega$

(iii) $H(e^{j\omega})$ is even and purely real-valued

(iv) phase is flat and equals 0, the delay being the slope is 0

(v) $\left|H(e^{j\omega})\right| = |1 + 2\cos 4\omega|$ and is plotted below:



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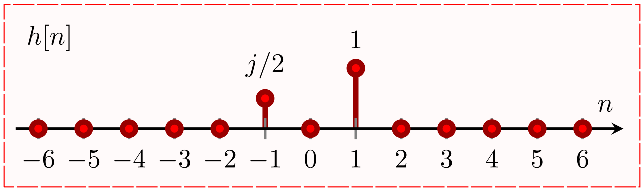
(j) 

Mixed real and imaginary.

Solution:

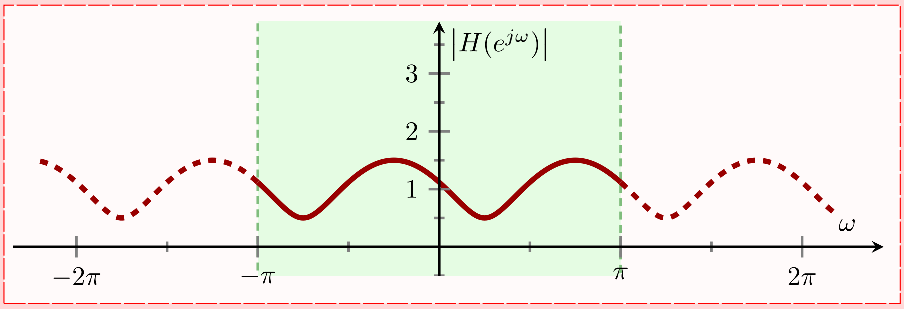
(i) $h[n] = h[-1]\delta[n+1] + h[1]\delta[n-1] = (j/2)\delta[n+1] + \delta[n-1]$

(ii) $H(e^{j\omega}) = (j/2)e^{j\omega} + e^{-j\omega}$ then use Euler

(iii) $H(e^{j\omega})$ is neither even nor odd and mixed real- and complex-valued

(iv) phase is all over the shop

(v) $\left|H(e^{j\omega})\right| = \left((\cos\omega + (1/2)\sin\omega)^2 + ((1/2)\cos\omega - \sin\omega)^2\right)^{1/2}$ and is plotted below:



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