Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

HOMEWORK 3

Homework 3-1

Consider the following convolution

$$y(t) = x(t) \star h(t)$$

where

$$x(t) = e^{-|t|} \quad \text{and} \quad h(t) = 2\,u(t-3) - 2\,u(t-5).$$

(a) Draw $x(t)$ and $h(t)$.

(b) Evaluate the integral

$$y(t) = x(t) \star h(t) \triangleq \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

(c) Draw $y(t)$.

Homework 3-2

In 3-1, we note that the centre of $h(t)$ is at $t = 4$. Review and understand the following:

$$\begin{aligned} h(t) &= 2\big(u(t-3) - u(t-5)\big) \\ &= 2\big(u(t+1) - u(t-1)\big) \star \delta(t-4) \end{aligned}$$

Therefore

$$\begin{aligned} y(t) &= x(t) \star h(t) \\ &= \Big(x(t) \star 2\big(u(t+1) - u(t-1)\big)\Big) \star \delta(t-4). \end{aligned}$$

So we can adjust for the delay of 4 at the end of a convolution that uses the even function

$$h_1(t) = 2\big(u(t+1) - u(t-1)\big)$$

instead of the original $h(t)$.

(a) Draw

$$y_1(t) = x(t) \star h_1(t) = x(t) \star 2\big(u(t+1) - u(t-1)\big).$$

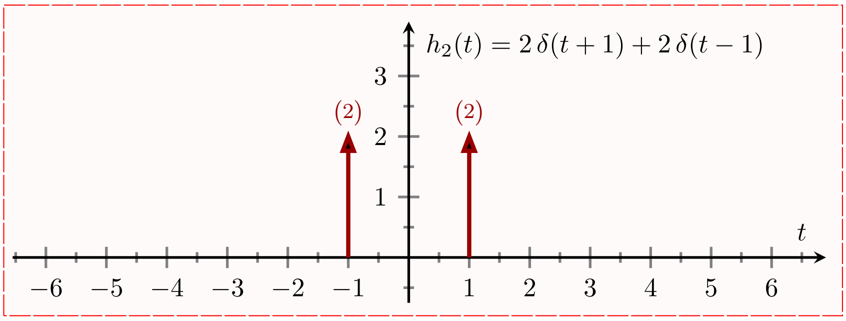
(b) Confirm $y_1(t)$'s relationship with $y(t)$.

Homework 3-3

Computing a convolution with $h_1(t)$, still requires an integral to be determined. Let's try a new "system" called $h_2(t)$ consisting of two Dirac delta functions:

$$h_2(t) = 2\,\delta(t+1) + 2\,\delta(t-1),$$

which is shown below:



The value of the delta function is presented in round brackets and that number represents the area, here (2) means the delta function has area 2 (so $h_2(t)$ has total area 4).

(a) Compute the following convolution

$$y_2(t) = x(t) \star h_2(t),$$

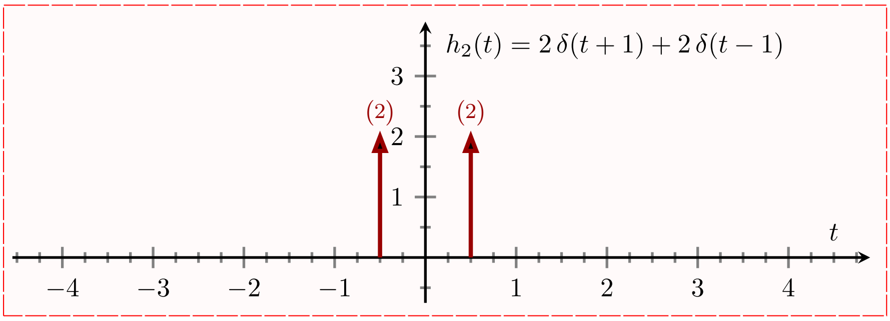
where $x(t) = e^{-|t|}$ as before.

(b) Plot $y_2(t) = x(t) \star h_2(t)$ and compare with $y(t) = x(t) \star h(t)$.

(c) It is clear that $h_2(t)$ is a rough approximation to $h(t)$ and $y_2(t)$ is a (less) rough approximation to $y(t)$. Show and argue why

$$h_3(t) = 2\,\delta(t + 1/2) + 2\,\delta(t - 1/2),$$

which is shown below, is a better approximation to $h(t)$ than $h_2(t)$.



Compute and plot

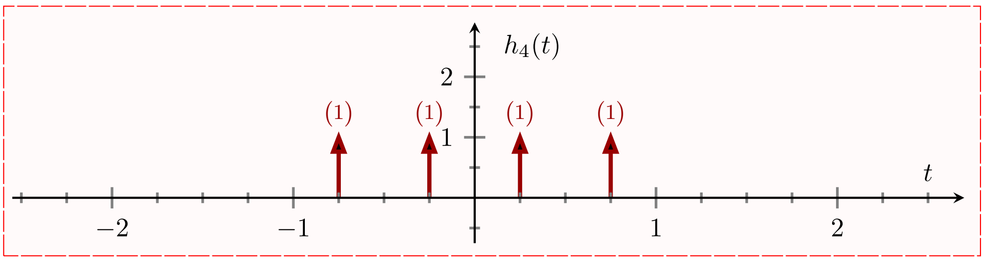
$$y_3(t) = x(t) \star h_3(t),$$

where $x(t) = e^{-|t|}$ as before. Compare $y_3(t)$ with $y_2(t)$ and $y(t)$.

(d) Repeat (c) again for

$$h_4(t) = \delta(t + 3/4) + \delta(t + 1/4) + \delta(t - 1/4) + \delta(t - 3/4)$$

shown here



and

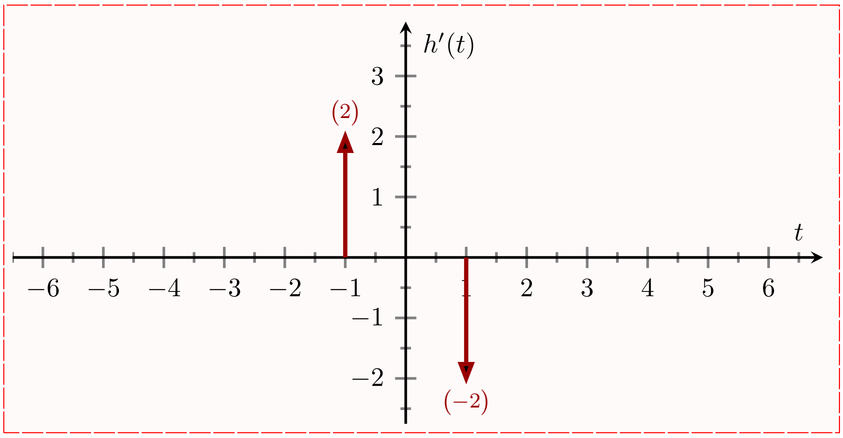
$$y_4(t) = x(t) \star h_4(t).$$

Homework 3-4

(Hard) The derivative of $h(t)$ is given by

$$h'(t) = 2\,\delta(t + 1) - 2\,\delta(t - 1)$$

and is drawn below



In the lecture notes it was shown how to compute the convolution using the $u_n(t)$ functions (where $\delta(t) = u_0(t)$, $u(t) = u_{-1}(t)$, etc.).

(a) Confirm

$$x(t) \star h(t) = x(t) \star u_1(t) \star u_{-1}(t) \star h(t).$$

(b) Use this expression to evaluate the convolution.

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