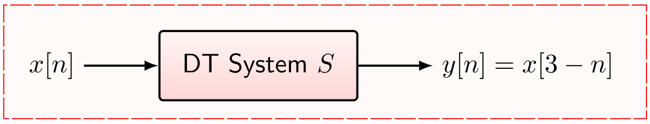
Australian National University

Research School of Engineering
College of Engineering and Computer Science

ENGN2228 Signal Processing

HOMEWORK 5 – SOLUTIONS

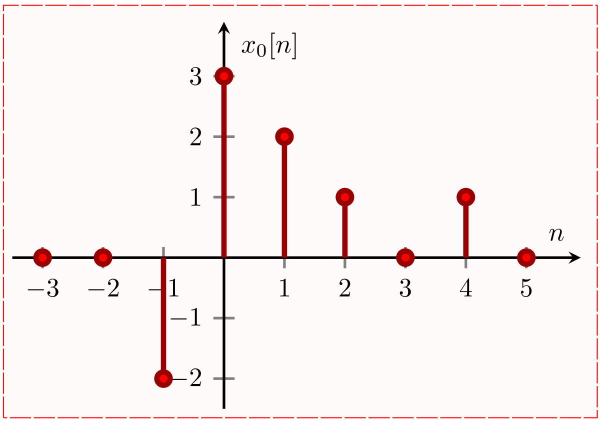
Homework 5-1



Consider the DT system, S, with input signal $x[n]$ and output signal given by

$$\boxed{S: \quad y[n] = x[3-n].} \tag{1}$$

(a) Write signal $x_0[n]$, shown below, in terms of linear combinations of shifted $\delta[n]$.



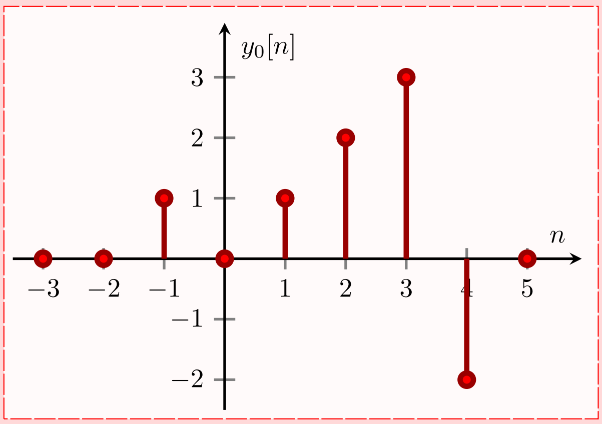
Solution:

$$x_0[n] = -2\delta[n+1] + 3\delta[n] + 2\delta[n-1] + \delta[n-2] + \delta[n-4]$$

□

(b) Draw the output $y_0[n]$ when the input is given by $x_0[n]$ shown above.

Solution:



□

(c) Write this signal $y_0[n]$ in terms of linear combinations of shifted $\delta[n]$.

Solution:
$$y_0[n] = \delta[n+1] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3] + -2\delta[n-4]$$
$\square$

(d) Shown that the system is linear.

Solution: Suppose $x_1[n] \longrightarrow y_1[n]$ and $x_2[n] \longrightarrow y_2[n]$. Then, by (1), $y_1[n] = x_1[3-n]$ and $y_2[n] = x_2[3-n]$. Create a third input signal from the linear combination
$$x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n].$$
Then, by (1), $x_3[n] \longrightarrow y_3[n] = x_3[3-n]$. But
$$\begin{aligned} x_3[3-n] &= \alpha_1 x_1[3-n] + \alpha_2 x_2[3-n] \\ &= \alpha_1 y_1[n] + \alpha_2 y_2[n]. \end{aligned}$$
In summary
$$\boxed{\alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n],}$$
and this holds for every α_1, $x_1[n]$, α_2, $x_2[n]$, meaning it is linear. $\square$

(e) Shown that the system (1) is non-causal.

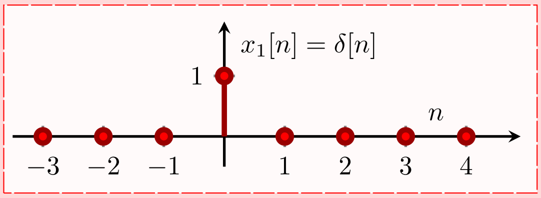
Solution: See the following solution. $\square$

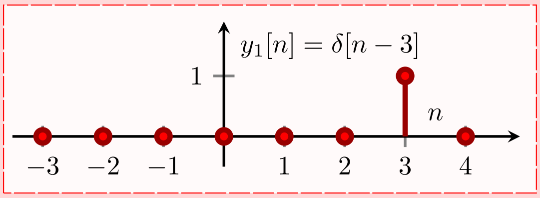
(f) Shown that the system (1) is time-varying.

Solution: We could memorise that a time reversal destroys causality and is time-varying. But it is better to provide a sound demonstration.

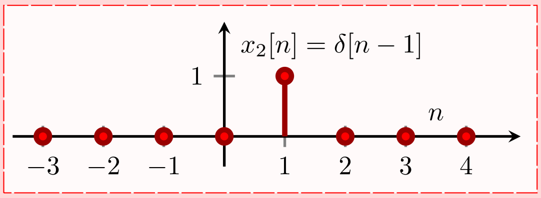
Consider the behavior of the system to three different input and corresponding output signals.

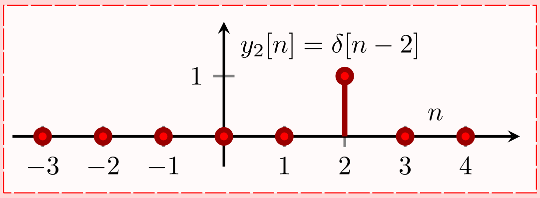
1) Input signal $x_1[n] = \delta[n]$ which yields output signal $y_1[n] = \delta[n-3]$.



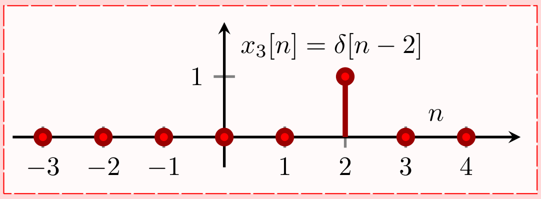


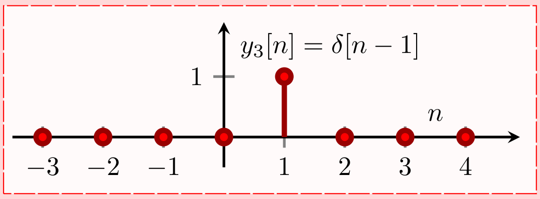
2) Input signal $x_2[n] = \delta[n-1]$ which yields output signal $y_2[n] = \delta[n-2]$.





3) Input signal $x_3[n] = \delta[n-2]$ which yields output signal $y_3[n] = \delta[n-1]$.





Non-causal: Firstly, the case $x_3[n] \longrightarrow y_3[n]$ shows it is non-causal because the output appears before the input.

Time-varying: Given $x_1[n] \longrightarrow y_1[n]$ then time shifting the input $x_1[n]$ to the right yields $x_2[n]$ (which also equals $x_1[n-1]$ or $\delta[n-1]$). However, time shifting the output $y_1[n]$ to the right, which is $y_1[n-1]$ or $\delta[n-4]$, does not equal $y_2[n]$. $\square$

(g) Suppose we have the same system but we don't know its defining relationship (1).
Let $h[n]$ be the output when $\delta[n]$ is applied. We observe $h[n] = \delta[n-3]$.
Can the system be fully characterised by this $h[n]$, that is, if we only know $h[n]$ can we determine the output for any input signal $x[n]$ for such an unknown system?

Solution: No. Firstly it is not LTI so we cannot rely only on the unit pulse response $h[n]$ to tell us everything. If it were a LTI system with unit pulse response $h[n] = \delta[n-3]$ then it must be a delay of 3 system. This is quite different from our system. $\square$

Homework 5-2

(a) The voltage and current for a capacitor C are related through

$$i_C(t) = C\frac{dv_C(t)}{dt}.$$

Consider the LTI system with input $i_C(t)$ and output $v_C(t)$. What is the impulse response $h_C(t)$ of the system? (You can express the result using the $u_k(t)$ functions defined in Part 7 of the lecture notes, or Section 2.5 of the text.)

$$i_C(t) \longrightarrow \boxed{h_C(t)} \longrightarrow v_C(t)$$

Solution: We write the output $v_C(t)$ in terms of input $i_C(t)$ as follows:

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)\,dt$$

Then the impulse response is given by

$$\boxed{h_C(t) = \frac{1}{C}u_{-1}(t) \equiv \frac{1}{C}u(t),}$$

$$i_C(t) \longrightarrow \boxed{\frac{1}{C}u_{-1}(t)} \longrightarrow v_C(t)$$

which is a scaled unit step. $\square$

(b) The inductor L can be thought of as the dual of the capacitor C where the transformation can be achieved by $L \leftrightarrow C$, $v_L(t) \leftrightarrow i_C(t)$ and $i_L(t) \leftrightarrow v_C(t)$. What is the impulse response $h_L(t)$ for the LTI system with input $i_L(t)$ and output $v_L(t)$

$$i_L(t) \longrightarrow \boxed{h_L(t)} \longrightarrow v_L(t)$$

Solution: We write the output $v_L(t)$ in terms of input $i_L(t)$ as follows:

$$v_L(t) = L\frac{i_L(t)}{dt}.$$

Then the impulse response is given by

$$\boxed{h_L(t) = Lu_1(t),}$$

$$i_L(t) \longrightarrow \boxed{Lu_1(t)} \longrightarrow v_L(t)$$

which is a scaled unit doublet (derivative of $\delta(t)$). $\square$