Propositional Logic

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Objectives

- Know how to turn sentences into logic formulas
- Know the syntax of propositional logic
- Know how to construct a parse tree

Declarative vs. Non-Declarative

- **Declarative sentences**
  - The sum of the numbers 3 and 5 equals 8
  - A dog has four legs
- **Non-Declarative sentences**
  - Could you please pass me the cup?
  - Wake up!

Logic Formulas

- **Declarative sentences**
  - The sum of the numbers 3 and 5 equals 8
  - A dog has four legs
- **Symbols**
  - p
  - q
- **Connectives**
  - $\wedge$ (and)
  - $\vee$ (or)
  - $\rightarrow$ (imply)
  - $\neg$ (not)
- **Formula**
  - $p \wedge q \rightarrow p \vee q$

Example (1)

- If the train arrives late and there are no taxis at the station then John is late for his meeting. John is not late for his meeting. The train did arrive late.

p	The train is late
q	There are taxis at the station
r	John is late for his meeting

- True or False: there was a taxi at the station

- We know about:
  - $p \land \neg q \to r$
  - p
  - $\neg r$

Example (2)

- If it is raining and Jane does not have her umbrella with her, then she will get wet.

  Jane is not wet

  It is raining

p	It is raining
q	Jane has her umbrella with her
r	Jane gets wet

- True or false: Jane has

  her umbrella with her

- We know about:
  - $p \wedge \neg q \rightarrow r$
  - p
  - $\neg r$

Observation

Two examples have the same structure

	Example 1	**Example 2**
p	The train is late	It is raining
q	There are taxis at the station	Jane has her umbrella with her
r	John is late for his meeting	Jane gets wet

- If p and not q, then r $(p \land \neg q \rightarrow r)$
- not r $(\neg r)$
- p

Therefore q

Syntax of Propositional Logic

Alphabet of Propositional Logic

- Propositional signature: a set of symbols, i.e., atoms.
  - E.g., p, q, r
- Propositional connectives:
  - binary: $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication) and $\leftrightarrow$ (equivalence)
  - unary: $\neg$ (negation)
  - $\bot$ (false) and $\top$ (true)
- parentheses

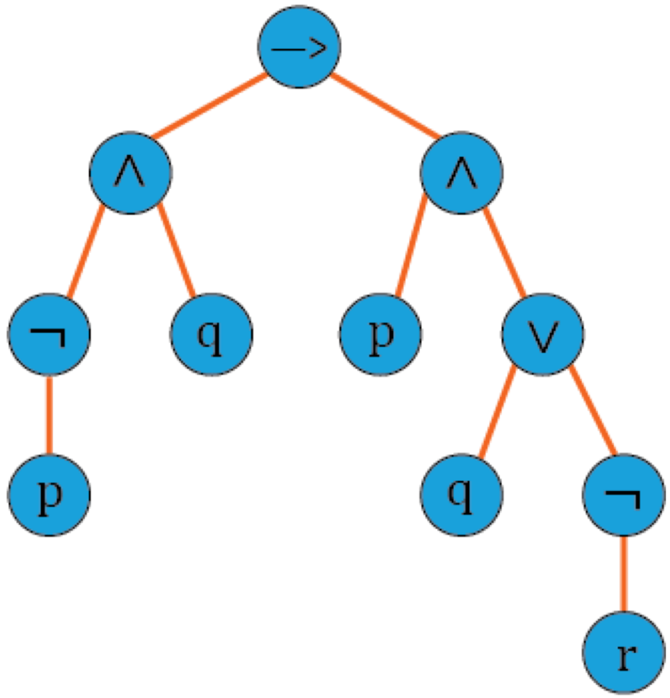
Definition of Propositional Formulas

- **A propositional formula of a signature is defined recursively as follows:**
  - every atom is a formula
  - both $\bot$ (false) and $\top$ (true) are formulas
  - if F is a formula then $(\neg F)$ is a formula
  - for any binary connective $\odot$, if F and G are formulas then $(F \odot G)$ is a formula

These are well-formed propositional formulas

Parse Tree

- $(((\neg p) \land q) \longrightarrow (p \land (q \lor (\neg r))))$



- **Subformulas** are the formulas corresponding to the subtrees of the parse tree
- Priority of connectives: $\neg, \land, \lor, \longrightarrow$

$$\neg p \land q \longrightarrow p \land (q \lor \neg r)$$

Summary

- **To turn sentences into logic formulas:**
  - Declarative vs. non-declarative sentences
  - Using connectives to connect logic symbols
- **Syntax of Propositional Logic**
  - Alphabet
    - Atoms
    - Connectives
    - Parentheses
  - Recursive definition of propositional formulas
- **Parse tree reveals the structure; unique for well-formed formulas**