Knowledge Check: Natural Deduction for Propositional Logic

TOTAL POINTS 4

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1. What rule can be used to derive $(p \wedge q) \wedge r \vdash (p \wedge q)$? **1 / 1 point**

- (●) and-elimination (1)
- ( ) and-introduction
- ( ) implication-elimination
- ( ) and-elimination (2)

✓ **Correct**

Correct! We can derive $(p \wedge q)$ from $(p \wedge q) \wedge r$ using and-introduction. The rule is called “and-elimination (1)” because we obtain the first part of the formula in $(p \wedge q) \wedge r$.

2. Which rules may be used to derive $p \to \neg\neg q, p \vdash q \wedge p$? **1 / 1 point**

- ( ) double negation-elimination, and-introduction, and and-elimination
- ( ) double negation-elimination, implication-elimination, and and-elimination
- (●) double negation-elimination, implication-elimination, and and-introduction
- ( ) implication-elimination, implication-introduction, and and-introduction

✓ **Correct**

Correct! To derive $q \wedge p$, we need and-introduction with both p and q. p is given and q can be derived by first applying implication-elimination and then double negation-elimination.

3. **1 / 1 point**

1	$(p \wedge q) \wedge r$	*premise*
2	$p \wedge q$	$\wedge e_1\ 1$
3	r	________
4	p	$\wedge e_1\ 2$
5	q	$\wedge e_2\ 2$
6	$p \wedge r$	________
7	$(p \wedge q) \wedge r$	$\wedge i\ 4,6$

Figure 1: Derivation of *(p∧q) ∧ r ⊢ p∧(q∧ r)*

Review the figure, which gives the derivation of (p∧q) ∧ r ⊢ p∧(q∧ r). However, while the derivation itself has been written out, two of the rules needed to complete the proof are missing, specifically the ones in line 3 and line 6.

In order to make the derivation correct, which proof rules are needed in lines 3 and 6?

- $\wedge e_1 1$ in line 3, and $\wedge i 5,3$ in line 6
- $\wedge e_1 1$ in line 3, and $\wedge i 5,4$ in line 6
- $\wedge e_2 1$ in line 3, and $\wedge i 5,3$ in line 6
- $\wedge e_1 2$ in line 3, and $\wedge i 5,3$ in line 6

Correct

Correct! You can derive line 3 using line 1 with and-elimination, and you can derive line 6 using lines 3 and 5 with and-introduction.

4. **Consider this argument: "If I am guilty, I must be punished; I must not be punished. Therefore, I am not guilty." Is the argument logically correct? If so, which rules are needed to derive the conclusion?**

- Yes, the argument is logically correct. We can use negation elimination, implication introduction, and implication elimination.

- No, the argument is *not* logically correct.
- Yes, the argument is logically correct. We can use and-introduction, negation-introduction, and implication elimination.
- Yes, the argument is logically correct. We can use negation-elimination, negation-introduction, and implication elimination.

Correct

Correct! Negation-introduction allows us to make an assumption which must then allow us to derive the bottom. This can be done using negation-elimination combined with implication elimination.