Knowledge Check: Natural Deduction for Propositional Logic

TOTAL POINTS 4

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1. What rule can be used to derive $(p \land q) \land r \vdash (p \land q)$? **1 / 1 point**

- ○ implication-elimination
- ● and-elimination (1)
- ○ and-introduction
- ○ and-elimination (2)

✓ **Correct**

Correct! We can derive $(p \land q)$ from $(p \land q) \land r$ using and-introduction. The rule is called “and-elimination (1)” because we obtain the first part of the formula in $(p \land q) \land r$.

2. Which rules may be used to derive $p \to \neg\neg q, p \vdash q \land p$? **1 / 1 point**

- ○ double negation-elimination, and-introduction, and and-elimination
- ○ implication-elimination, implication-introduction, and and-introduction
- ● double negation-elimination, implication-elimination, and and-introduction
- ○ double negation-elimination, implication-elimination, and and-elimination

✓ **Correct**

Correct! To derive $q \land p$, we need and-introduction with both p and q. p is given and q can be derived by first applying implication-elimination and then double negation-elimination.

3. **1 / 1 point**

1	________	*assumption*
2	________	$\wedge e_1$ 1
3	$(p \wedge q) \rightarrow p$	$\rightarrow i$ 1-2

Figure 1: Derivation of $\vdash (p \wedge q) \rightarrow p$

Review the figure, which gives a partial derivation of ⊢ (p∧q) → p. While the rules for deriving the conclusion have been written out, two steps of the proof are missing, specifically the ones in line 1 and line 2.

Given the rules required to derive the conclusion, which formulas are the missing steps for the proof?

- p in line 1, and (p∧q) in line 2
- p in line 1, and q → p in line 2
- (p∧q) in line 1, and p → q in line 2
- (p∧q) in line 1, and p in line 2

Correct

Correct! To derive the conclusion, we need to apply implication-introduction, which requires us to make an assumption on (p∧q) and derive p.\

4. **Consider this argument: "If I am guilty, I must be punished; I must not be punished. Therefore, I am not guilty." Is the argument logically correct? If so, which rules are needed to derive the conclusion?**

- Yes, the argument is logically correct. We can use and-introduction, negation-introduction, and implication elimination.
- Yes, the argument is logically correct. We can use negation-elimination, negation-introduction, and implication elimination.
- No, the argument is *not* logically correct.

- Yes, the argument is logically correct. We can use negation elimination, implication introduction, and implication elimination.

Correct

Correct! Negation-introduction allows us to make an assumption which must then allow us to derive the bottom. This can be done using negation-elimination combined with implication elimination.