Ira A. Fulton Schools of Engineering
Arizona State University
Engineering | School of Computing, Informatics, and Decision Systems Engineering

CSE 571: Artificial Intelligence

Derivation of Logic Proofs

Purpose

This assignment is an exercise for using natural deduction to derive logic proofs. This exercise not only tests students' ability to apply natural deduction but also familiarizes students with logic proofs that are derived in computers.

Objectives

Students will be able to:

- Apply natural deduction to derive logic proofs.

Technology Requirements

- N/A

Assignment Description

Part 1: Proving the Validity of Sequents

Assuming binding priority ($\neg$, $\wedge$, $\vee$, $\rightarrow$), how would you prove the validity of the sequents?

1. $p \rightarrow (q \vee r), \neg q, \neg r \vdash \neg p$ using MT rule
2. $\vdash (p \rightarrow q) \vee (q \rightarrow r)$ without using LEM
3. $(p \rightarrow q) \wedge (q \rightarrow r) \vdash (p \rightarrow r)$
4. $(p \rightarrow r) \wedge (q \rightarrow \neg r) \vdash (q \rightarrow \neg p)$
5. $(p \rightarrow r) \vee (q \rightarrow r) \vdash (p \wedge q) \rightarrow r$

6. ¬(p ∨ q) is equivalent to (¬p ∧ ¬q) (prove → in both directions)
7. (p → ¬q) is equivalent to ¬(p ∧ q)

Part 2: Determining the Validity of Sequents, Proofs, and Truth Tables

Assuming binding priority (¬, ∧, ∨, →), are these sequents valid or not? If they are valid, how do you prove it? If they are not valid, what would the truth table be?

1. A → B, C → D ⊢ A ∨ C → B ∧ D
2. A ∧ ¬A ⊢ ¬(B → C) ∧ (B → C)
3. (A ∧ B) → C, C → D, B ∧ ¬D ⊢ ¬A

Part 3: Drawing Parse Trees

Assuming binding priority (¬, ∧, ∨, →), what are **all** the possible parse trees for each formula? Draw the trees for each formula.

1. (p → r) ∨ (q → r) → (p ∧ q) → r
2. (p → r) ∧ (q → ¬r) ∧ (q → ¬p)

Submission Directions for Deliverables

Drafting your answers in Microsoft Word is strongly encouraged. Office 365 is Microsoft's productivity suite, and includes Word, Excel, PowerPoint, Access, OneNote and more. It is available for offline and online use. Currently enrolled students can use this software for free. Refer to the article "ASU providing Microsoft Office 365 to all Students, Faculty & Staff" (you must be logged into my.asu.edu to view the article) in the "Welcome and Start Here" section of the course.

Type your answers on the provided "**CSE 571_Unit 2_ Assignment_Derivation of Logic Proofs_Submission Template**". Submit the template as a single PDF titled "**Last Name_First Name_Derivation of Logic Proofs**".

Rubric

	No Attempt	Undeveloped	Developing	Approaching	Meets
Part 1: Proving the Validity of Sequents	Provides no proof.	Provides incorrect steps. Fully incorrect answer with no merit in approach showing how proof rules should be applied to solve a problem.	Provides some steps that are correct. Answer may demonstrate surface-level understanding of logic proof rules, but lacks the overall view to develop a proof toward the conclusion. Application of proof rules has clear imperfections.	Provides a mostly correct answer. Answer may demonstrate mid-level understanding of proof rules, with a reasonable approach to developing a proof toward the conclusion. Some inconsistency with specificity may be present, such as providing incorrect rule names. Application of proof rules may have slight imperfections.	Provides a clear, fully correct answer. Answer demonstrates high-level understanding of proof rules, with a logical approach to developing the proof. All proof names are provided correctly, and a correct proof is present. Application of proof rules has no imperfections.
Part 2: Determining the Validity of Sequents, Proofs, and Truth Tables	**For valid sequents:** Provides no proof.	**For valid sequents:** Provides incorrect proof. Fully incorrect answer with no merit in approach to show how proof rules should be applied to solve a problem.	**For valid sequents:** Provides some steps that are correct. Answer may demonstrate surface-level understanding of logic proof rules, but lacks the overall view to develop a proof toward	**For valid sequents:** Provides a mostly correct answer. Answer may demonstrate mid-level understanding of logic proof rules, with a reasonable approach to developing a proof toward the conclusion. Some	**For valid sequents:** Provides a clear, fully correct answer. Answer demonstrates high-level understanding of proof rules, with a logical approach to developing the proof. All proof names are provided correctly,

			the conclusion. Application of proof rules has clear imperfections. **For invalid sequents:** Answer demonstrates a general understanding of how to develop a truth table, but entries may be missing or inaccurate.	inconsistency with specificity may be present, such as providing incorrect rule names. Application of proof rules may have slight imperfections. **For invalid sequents:** Answer provides a fully developed truth table, though a few entries may be inaccurate.	and a correct proof is present. Application of proof rules has no imperfections. **For invalid sequents:** Answer provides a fully developed and correct truth table.
Part 3: Drawing Parse Trees	Provides no parse trees.	Provides incorrect parse trees. Fully incorrect answer with no merit in approach to show how parse trees should be developed for a given formula.	Provides some subtrees that are correct. Answer may demonstrate surface-level understanding of parse trees, but lacks the overall view to develop a tree for a given formula. Drawings of parse trees have clear imperfections.	Provides a mostly correct answer. Answer may demonstrate mid-level understanding of the parse trees, with a reasonable approach to developing the correct tree for a given formula. Some nodes may be missing, some incorrect rule names may be provided. Drawings of parse trees may have slight imperfections.	Provides a clear, fully correct answer. A high-level of understanding of the parse tree is provided. All nodes in the tree are correctly given. Drawings of parse trees have no imperfections.