Knowledge Check: Natural Deduction for Propositional Logic

TOTAL POINTS 4

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1. **What rule can be used to derive $(p \wedge q) \vdash (p \wedge q) \wedge r$?** **1 / 1 point**

- ○ and-introduction
- ○ implication-elimination
- ○ and-elimination
- ◉ The conclusion cannot be derived by any rule

✓ **Correct**

Correct! We cannot deduce the conclusion from the premises using any rules. The proof is not a valid proof.

2. **Which rules may be used to derive $p \rightarrow \neg\neg q, p \vdash q \wedge p$?** **1 / 1 point**

- ○ implication-elimination, implication-introduction, and and-introduction
- ○ double negation-elimination, and-introduction, and and-elimination
- ◉ double negation-elimination, implication-elimination, and and-introduction
- ○ double negation-elimination, implication-elimination, and and-elimination

✓ **Correct**

Correct! To derive $q \wedge p$, we need and-introduction with both p and q. p is given and q can be derived by first applying implication-elimination and then double negation-elimination.

3. **1 / 1 point**

1	$(p \wedge q) \wedge r$	*premise*
2	$p \wedge q$	$\wedge e_1$ 1
3	r	________
4	p	$\wedge e_1$ 2
5	q	$\wedge e_2$ 2
6	$p \wedge r$	________
7	$(p \wedge q) \wedge r$	$\wedge i$ 4, 6

Figure 1: Derivation of ***(p∧q) ∧ r ⊢ p∧(q∧ r)***

Review the figure, which gives the derivation of (p∧q) ∧ r ⊢ p∧(q∧ r). However, while the derivation itself has been written out, two of the rules needed to complete the proof are missing, specifically the ones in line 3 and line 6.

In order to make the derivation correct, which proof rules are needed in lines 3 and 6?

- ( ) $\wedge e_1 2$ in line 3, and $\wedge i 5, 3$ in line 6
- ( ) $\wedge e_1 1$ in line 3, and $\wedge i 5, 4$ in line 6
- ( ) $\wedge e_1 1$ in line 3, and $\wedge i 5, 3$ in line 6
- (•) $\wedge e_2 1$ in line 3, and $\wedge i 5, 3$ in line 6

Correct

Correct! You can derive line 3 using line 1 with and-elimination, and you can derive line 6 using lines 3 and 5 with and-introduction.

4. **Consider this argument: "If I am guilty, I must be punished; I must not be punished. Therefore, I am not guilty." Is the argument logically correct? If so, which rules are needed to derive the conclusion?**

- (•) Yes, the argument is logically correct. We can use negation-elimination, negation-introduction, and implication elimination.

- Yes, the argument is logically correct. We can use negation elimination, implication introduction, and implication elimination.
- Yes, the argument is logically correct. We can use and-introduction, negation-introduction, and implication elimination.
- No, the argument is *not* logically correct.

Correct

Correct! Negation-introduction allows us to make an assumption which must then allow us to derive the bottom. This can be done using negation-elimination combined with implication