Probability Basics

Part 2

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Independent RVs

- $X = x$ **neither** makes it **more or less** probable that $Y = y$
- X and Y are independent iff

$$P(X = x \cap Y = y) = P(X = x)P(Y = y)$$

  – No matter how many heads you get, your friend's outcome will not be affected, and vice versa

Conditional Independence

- Given Z, X and Y are conditionally independent iff once Z is **known** the value of X does not add any **additional information** about Y

$$P(X = x \cap Y = y | Z = z) = P(X = x | Z = z) P(Y = y | Z = z)$$

Example

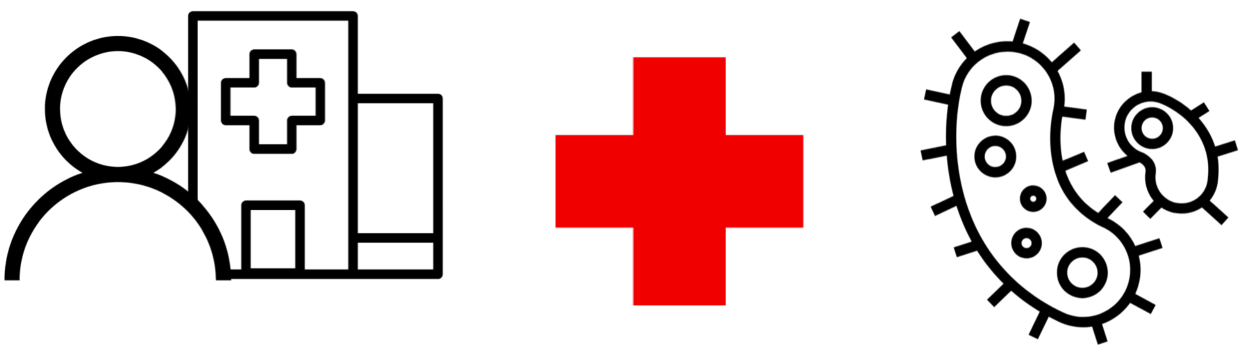
- You test for a rare disease (1 in 1000 have the disease) and the result is positive. With what probability do you actually have the disease? %99

$$P(H|E) = \frac{P(E|H) * P(H)}{P(E)} = \frac{0.99 * 0.001}{P(E)}$$

$$P(E) = P(H) * P(E|H) + P(\neg H) P(E|\neg H)$$

$$(0.99 * 0.001 + 0.01 * 0.999)$$

$$= 0.09 = 9\%$$



Example

- You go to a second clinic and the result of the test is positive for the second time. What now?

$$P(H|E) = \frac{P(E|H) * P(H)}{P(E)}$$

$$P(H|E) = \frac{0.99 * 0.09}{0.09 * 0.99 + 0.91 * .01} = 0.907 = 90.7\%$$

Continuous Random Variables

- A pdf is any function $f(x)$ that describes the probability density in terms of the input variable ***x***.

$$\forall x, f(x) \geq 0$$

$$\int_{-\infty}^{+\infty} f(x) = 1$$

- Actual probability can be obtained by taking the integral of pdf
- The probability of X being between 0 and 1 is:

$$\int_0^1 f(x)\, dx$$

Cumulative Distribution Function

$$F_x(v) = P(X \leq v)$$

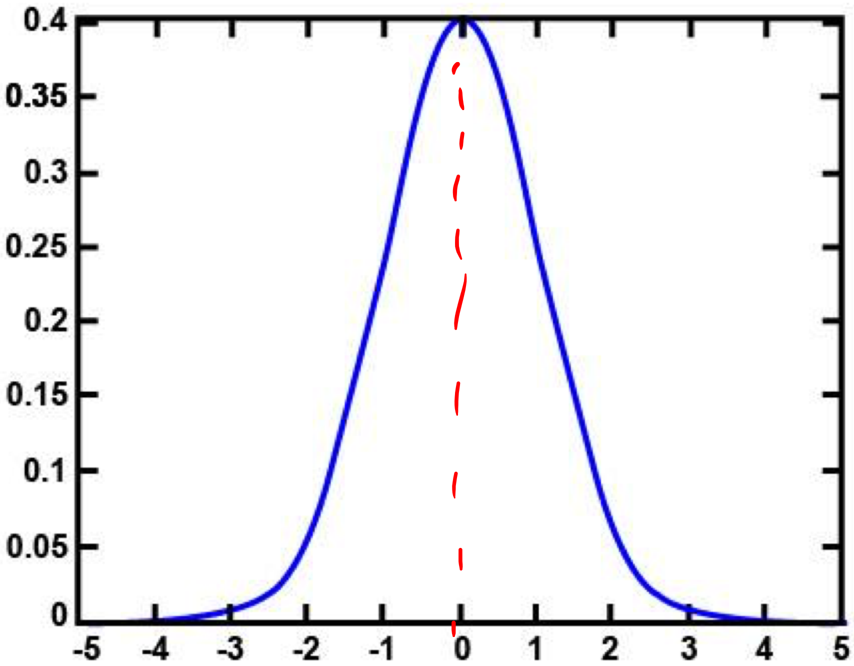
- Discrete RVs

$$F_x(v) = \sum_{v_i} P(X = v_i)$$

- Continuous
  - $F_x(v) = \int_{-\infty}^{v} f(x)d(x)$

Common Distribution

- **Normal** $X \sim N(\mu, \sigma^2)$
  - Used when the random variable distribution are not known
  - The height of the entire population



Definitions

- **Mean (Expectation):** $\mu = E(X)$
- **Discrete RVs:** $E(X) = \sum_{v_i} v_i P(X = v_i)$
- **Continuous RVs:** $E(X) = \int_{-\infty}^{+\infty} x f(x) dx$
- **Variance** $V(X) = E(X - \mu)^2$
- **Discrete RVs:** $V(X) = \sum_{v_i} (v_i - \mu)^2 P(X = v_i)$
- **Continuous RVs:** $V(X) = \int_{-\infty}^{+\infty} (x - \mu)^2 f(x) dx$

Definitions

$$E(X+Y) = E(X) + E(Y)$$

$$E(aX) = aE(X)$$

$$V(aX+b) = a^2V(X)$$

- If X and Y are independent:

$$V(X+Y) = V(X) + V(Y)$$

Summary

- Independent RVs
- Conditional Independence
- Continuous RVs
- Cumulative Distribution Function
- Mean, Variance and their properties