Introduction to single-layer perceptron

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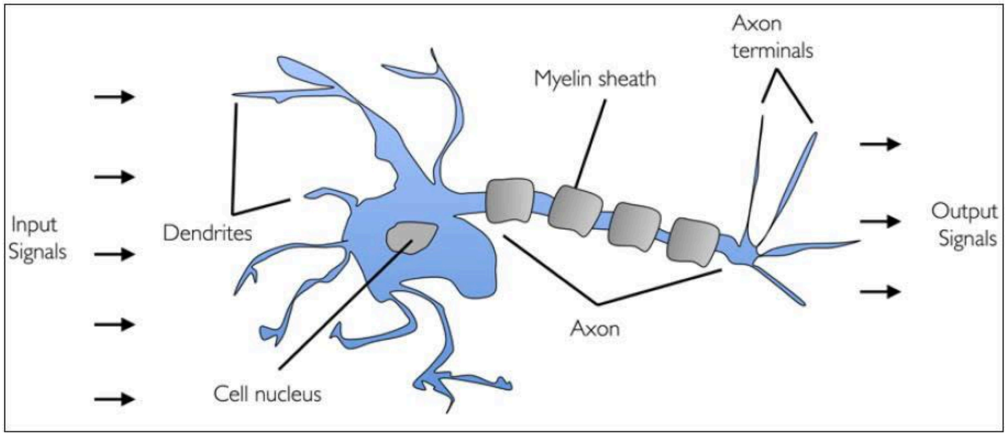
What are neural networks?

- Artificial Neural Networks (ANN) - computational predictive model inspired by neuronal structure of the mammalian cerebral cortex (brain)
- "...a computing system made up of a number of simple, highly interconnected processing elements, which process information by their dynamic state response to external inputs" - Dr. Robert Hecht-Nielsen
- In data science we are not concerned about recreating the biological structure of the brain, ANNs are simply inspired by the brain neurons
  - Typical mammalian brain has billions of neurons while a big neural network model might have hundreds thousands units

Motivation for neural networks

- Biological neurons



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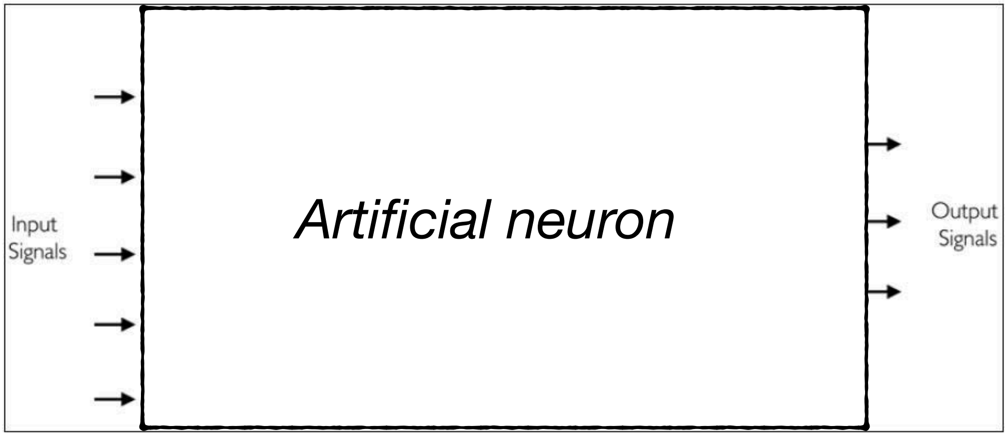
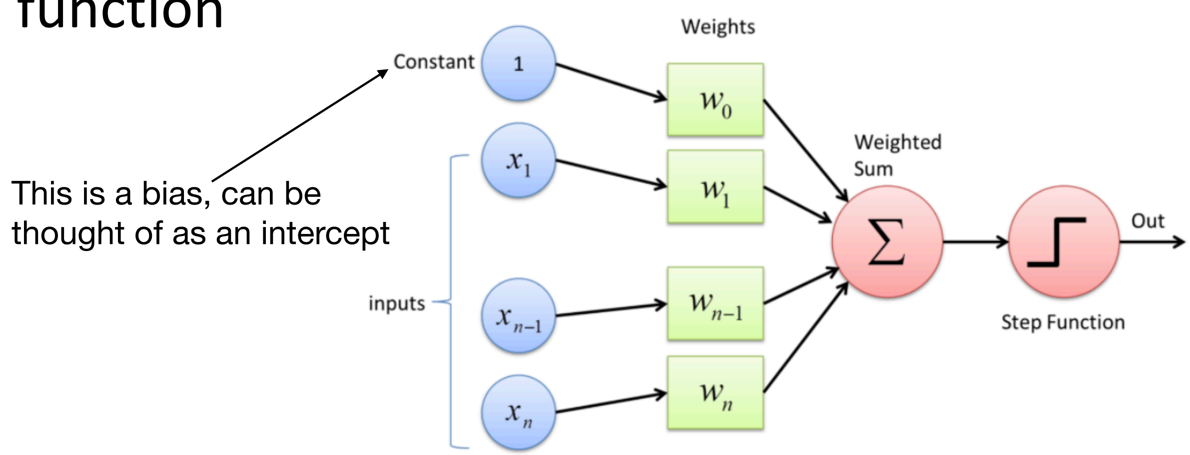
Can we imitate biological neurons?



Perceptron model

- Perceptron - single layer neural network
- Perceptron model is the simplest form of ANN
- Binary classifier
- Perceptrons consist of: input values, weights + bias, net sum, activation function



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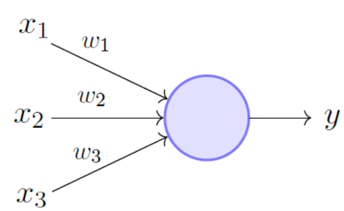
Terms perceptron vs. neural network

- “Perceptron” - single-layer ANN
- “Neural network” - multi-layer ANN
  - Also referred to as multi-layer perceptron
  - As well as feedforward neural network

Brief history of perceptron algorithm

- Invented by Frank Rosenblatt at the Cornell Aeronautical Laboratory in 1958
  - Perceptron was initially a machine, not a program
  - Was intended to perform image recognition task
  - Its popularity diminished because of its limitations, until the rise of multi-layer perceptrons
- Only capable of separating linearly separable patterns
  - Linear classifier
- If you understand how a perceptron works, you will understand how neural networks work



Perceptron Model (Minsky-Papert in 1969)

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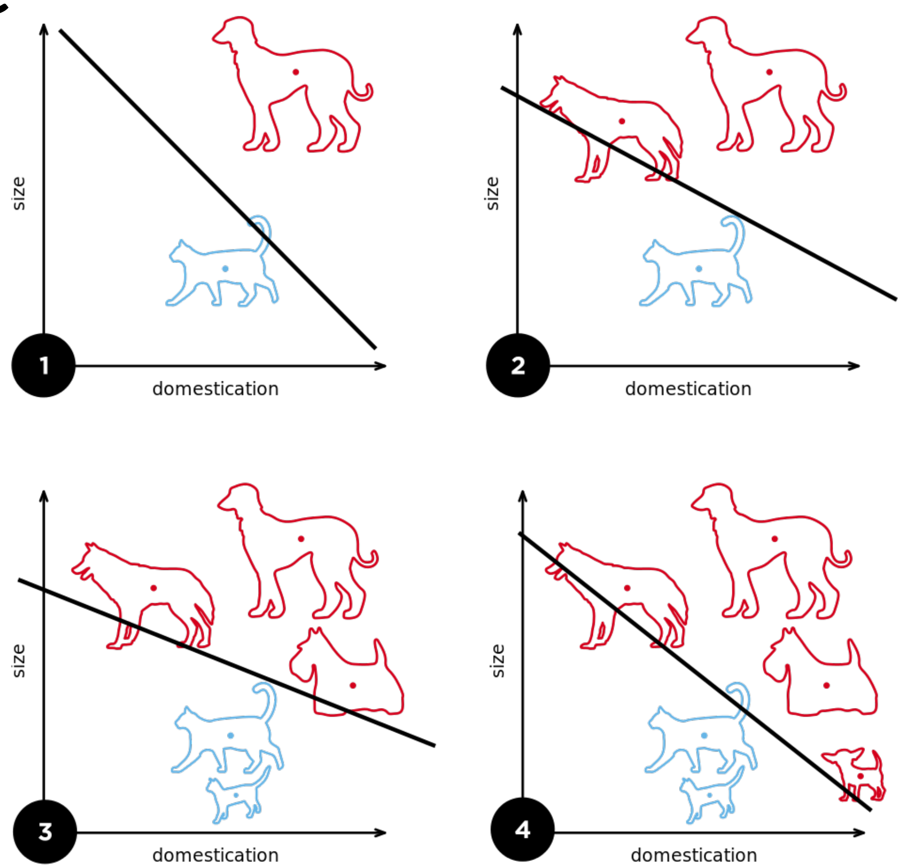
We use perceptron when our data is linearly separable



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Steps of perceptron classification

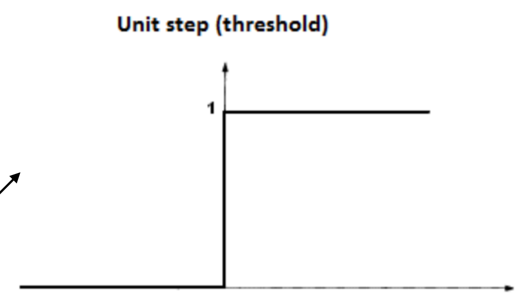
- Perceptron algorithm:
  - All input values are multiplied by the corresponding weights
  - Add up all the results from the multiplication step (we call it "weighted sum")
  - Use the weighted sum as the input into the activation function
    - Remember activation functions from the regression lecture?
      - Activation function transforms input into a different space

Activation function for a perceptron

- For linear perceptron we use an activation function called "unit step activation"

$$f(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{w} \cdot \mathbf{x} + b > 0, \\ 0 & \text{otherwise} \end{cases}$$

https://en.wikipedia.org



Maps values to 0 or 1

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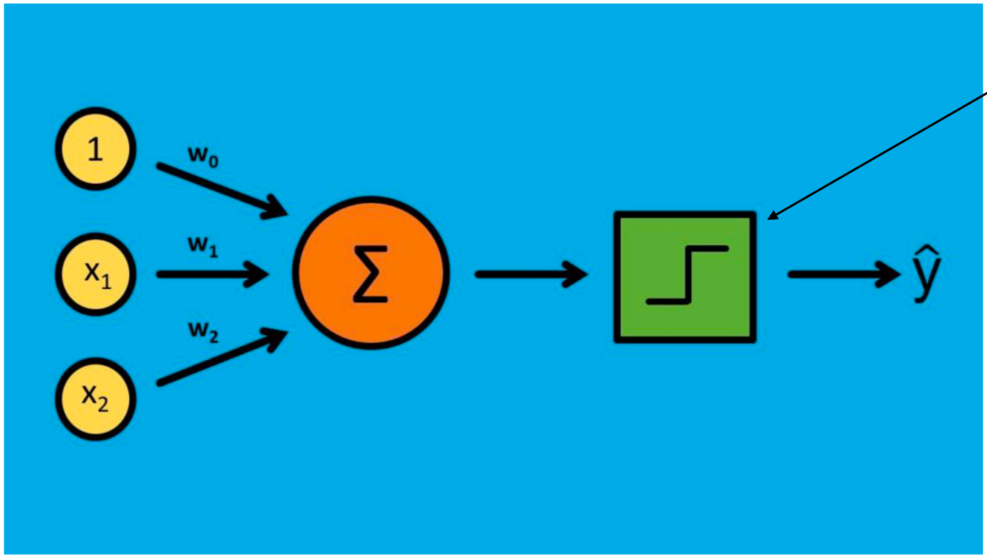
Activation function for a perceptron (cont'd)



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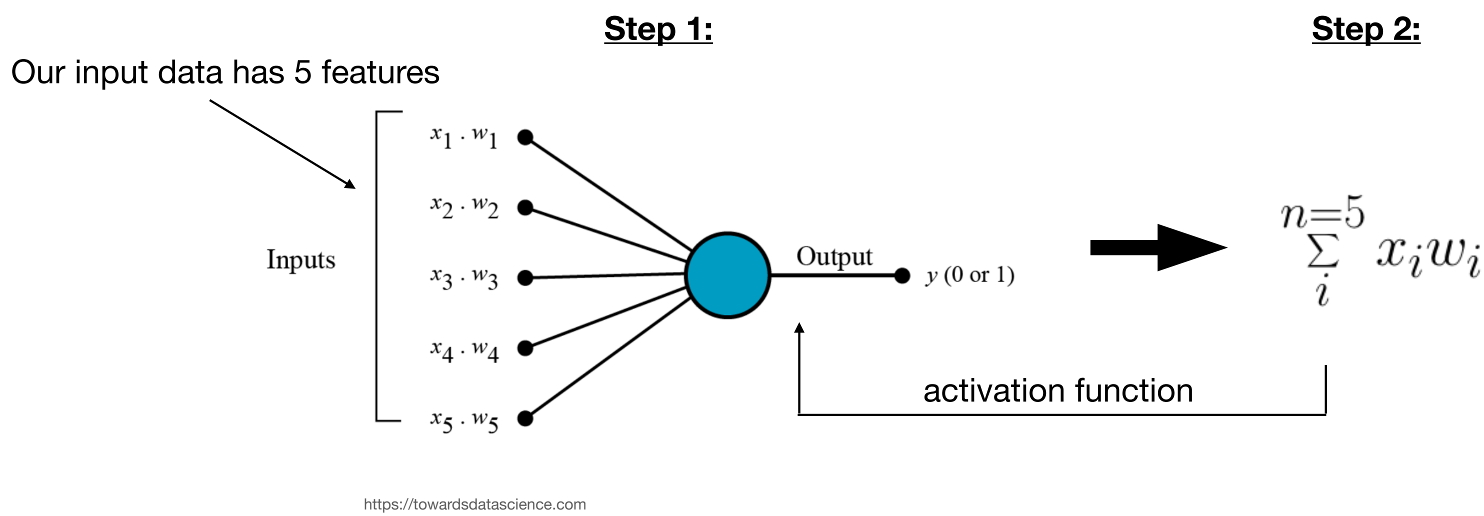
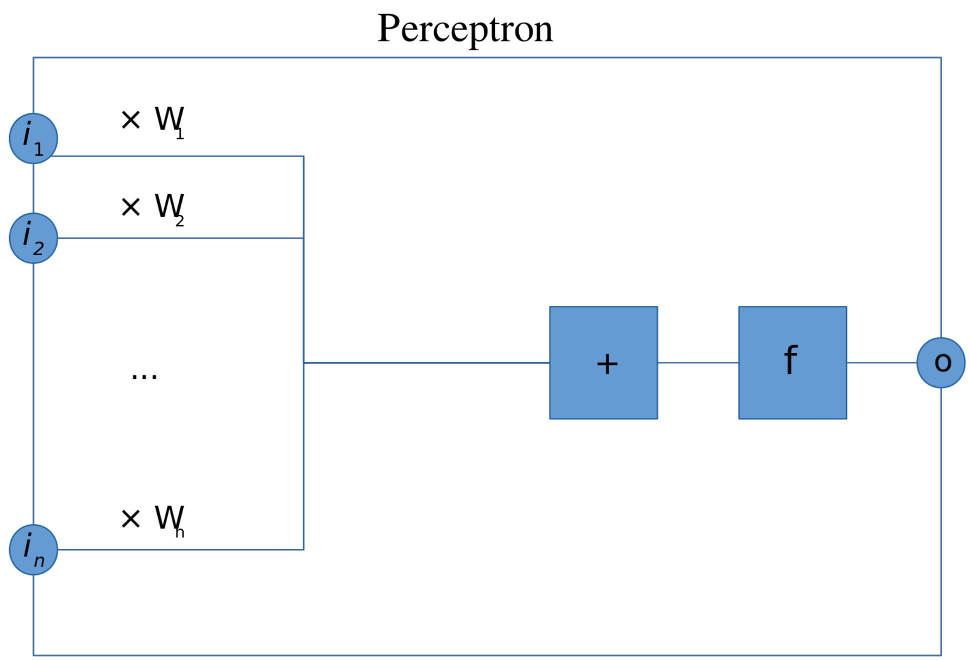
$$f(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{w} \cdot \mathbf{x} + b > 0, \\ 0 & \text{otherwise} \end{cases}$$

Let’s visualize the steps for the perceptron algorithm



Putting it all together



$$o = f\left(\sum_{k=1}^{n} i_k \cdot W_k\right)$$

Sometimes you will see this equation written with a bias and sometimes without a bias, but bias is a part of the perceptron model

https://en.wikipedia.org

Perceptron classification in matrix form

- Weighted sum is just a dot product of input values and weights

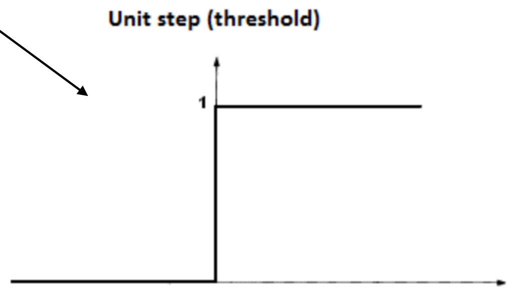
$$w^T x = w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_n x_n$$

$$\Downarrow$$

$$f(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{w} \cdot \mathbf{x} + b > 0, \\ 0 & \text{otherwise} \end{cases}$$

Interpretation of weights and bias in ANN

- Weights show the strength and direction of each node's contribution
- Bias gives a shift to the activation function (intercept)
  - Where the function cuts the x-axis



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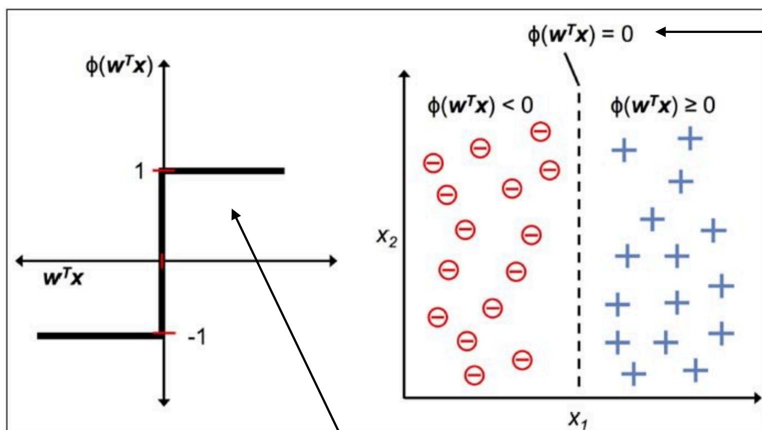
Why do we need an activation function?

- Activation function maps the output to [0,1] space
- Remember that the input values and weights are unbounded numeric values, potentially weighted sum value can belong to (-inf, +inf) interval
- Activation function allows us to have a reliable output space

How do we learn perceptron weights?

- Perceptron learning rule
  - Computed output is compared to the true output
  - The error (difference between predicted and true outputs) is propagated back and weights are adjusted



Perceptron decision function (defines the decision hyperplane)

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We say “the neuron fired” if the result is in this interval

Training a perceptron model

- Steps:
  - Initialize the weights and the bias
  - Compute the classification result (see slide 9 “Steps of perceptron classification”)
  - Compare the prediction results to the true labels
  - Update the weights
  - Repeat until the convergence and predefined number of iterations

Updating perceptron weights

- “Eta” is a learning rate of the update step
- We update the weights by the difference between the predicted and true labels weighted by input value and eta parameter

$$w^{n+1} = w_i^n + \eta(y_i - \hat{y}_i x_i)$$

eta - learning rate
y_i - true label for observation i
hat(y_i) - predicted label (output of perceptron) for observation i
x_i - input values for misclassified observation

Let's go step by step

weight vector

$$\begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

features

$$\begin{array}{c} x_0 \quad\; x_1 \quad\; x_2 \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\Sigma w_i \cdot x_i = (w_0 \cdot x_0) + (w_1 \cdot x_1) + (w_2 \cdot x_2)$$

$$\Sigma w_i \cdot x_i = (0 \cdot 0) + (0 \cdot 1) + (0 \cdot 0)$$

$$f = 0$$

threshold

$z = 0$

if $f > z$, $\hat{y} = 1$

otherwise, $\hat{y} = 0$

$f = 0$

$\hat{y}_0 = 0$

our estimate

$\hat{y}_0 = 0$

$y_0 = 1$

actual output

outputs

$$y_i = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \boxed{1} \\ 1 \\ 0 \end{bmatrix}$$

https://towardsdatascience.com

Let’s go step by step (cont’d)

current weights

$n=1$

$$\begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

features

$$\begin{array}{ccc} x_0 & x_1 & x_2 \\ \end{array}$$
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

estimate $\hat{y} = 0$

actual $y = 1$

learning rate $\eta = 0.1$

$$w_0^2 = 0 + 0.1(1 - 0)0 = 0$$
$$w_1^2 = 0 + 0.1(1 - 0)1 = 0.1$$
$$w_2^2 = 0 + 0.1(1 - 0)0 = 0$$

new weights

$n=2$

$$\begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.1 \\ 0 \end{bmatrix}$$

learning rate (“eta”)

$$w_i^{n+1} = w_i^n + \eta(y_i - \hat{y}_i)x_i$$

new weight current weight

$\eta = 0.1$

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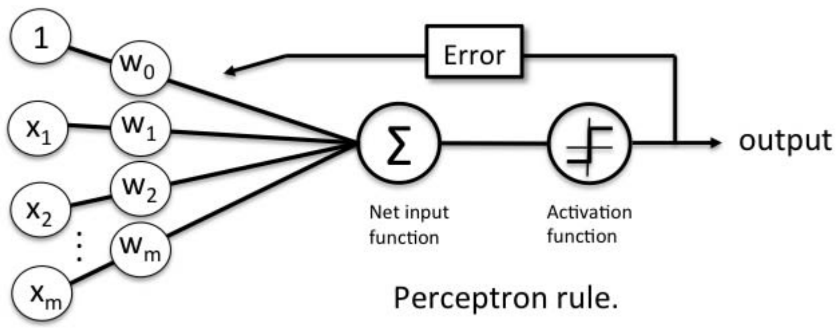
Iterate until optimal weights are learned

- We continue through every observation in the dataset
- Then, continue to iterate over the dataset until
  - Convergence to acceptable prediction accuracy
  - Predetermined number of iterations is reached
  - Other termination criteria can be used
- Epoch - complete sweep through the dataset
  - In the previous toy example it takes 3 iterations to complete 1 epoch
- Number of iterations or epochs is a hyperparameter you can tune
- Perceptron algorithm will converge for a linearly separable data

Perceptron training overview



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Let’s look at some code

- Classifying breast cancer using perceptron
  - *Perceptron.Breast.ipynb*

- Also, this is a great explanation of how to implement logic gates with perceptron (we will not be going over these in class)
  - https://towardsdatascience.com/perceptrons-logical-functions-and-the-xor-problem-37ca5025790a