Unsupervised Learning

COMP9417, 23T2

Section 1

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The content this week is light, so I'll go straight to the lab to explain it.

Section 2

The Missing Learning Theory Tut

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But unironically

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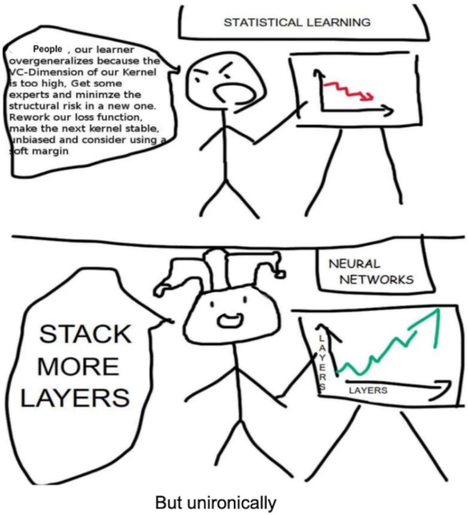

I'll focus on PAC learning and VC dimension, but we also introduce the WINNOW algorithms and the *No Free Lunch theorem* in lectures.

Section 3

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- A *concept* c generates the data i.e we have data $D = \{(x_i, c(x_i))\}_{i=1}^{m}$
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In this setting, we define the true error of a hypothesis as,

$$\text{Err}_{\mathcal{D}}(h) = \Pr_{x \in \mathcal{D}}(c(x) \neq h(x))$$

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How many training examples do we need to ϵ-exhaust the version space for a problem?

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Therefore,

$$\Pr(\mathrm{Err}_{\mathcal{D}}(h) \geq \epsilon) < |H|e^{-\epsilon m} \text{ for all } h \in H$$

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We now have a bound of the number of examples needed to assure that $(\forall h \in VS_{H,D})\text{Pr}(\text{Err}_{\mathcal{D}}(h) \leq \epsilon) \geq 1 - \delta$.

We say a concept class C is PAC-learnable by a learner L using a hypothesis class H for all $c \in C$ and distributions $\mathcal{D}$ if for all $0 < \epsilon < 1/2$ and $0 < \delta < 1/2$ the learner outputs a hypothesis $h \in H$ such that,

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with probability $1 - \delta$. In time that is polynomial in $1/\epsilon$, $1/\delta$, m and $|C|$.

Section 4

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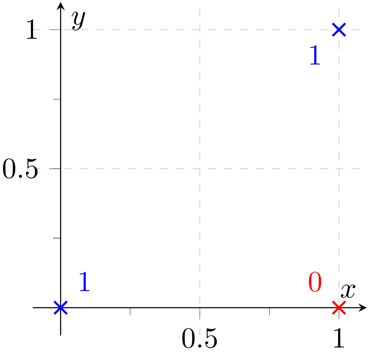
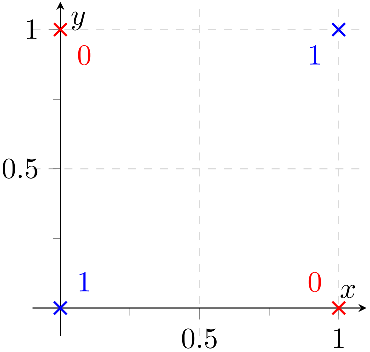
First, we define a **dichotomy** of a set as a partitioning of that set into two disjoint subsets.

We also say a set is **shattered** by a hypothesis space if for every dichotomy there is a hypothesis from that space which is consistent with that dichotomy.

Lots of big words, what does it mean?

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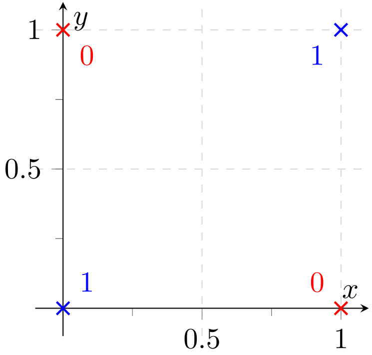

We can't shatter this dataset with the space of linear classifiers!

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We can also generalise the bound of m from PAC-learning to include possibly non-finite hypothesis classes,

$$m \geq \frac{1}{\epsilon}(4\log_2(2/\delta) + 8VC(H)\log_2(13/\epsilon))$$

That's it for the term, good luck in the exam period! 😄

Do myExperience, study hard etc. etc.