

# MATH OLYMPICS

**Format:** In-Person

## **Registration Guidelines:**

1. A limit of two (2) students from each team may register for this competition.
2. Each student competes individually. As such collaboration is not permitted.

## **Procedure:**

1. No knowledge of Calculus is required. An understanding of only Geometry, Trigonometry, Algebra I & II, and Statistics & Probability are sufficient for a student.
2. Basic scientific calculators are allowed. Calculators with graphing ability, such as the TI-83 or higher, are NOT allowed.
3. Students will get a set of problems in different rounds. The student with the highest number of points moves on to the next round.
4. A ballot is not provided for this competition, as scoring will be based solely on test performance.
5. Electronic device usage is not permitted during the competition.

## **Layout:**

1. The competition will be split up into three portions.
  - a. The first portion will be the MIST Math Olympics Pretest.
  - b. The second portion will be the Ciphers Round I.
  - c. The last portion will be the Ciphers Round II.
  - d. If two or more competitors tie by the end of Ciphers Round II, then the competitors will go into the Ciphers Bonus Round.
2. The Pretest will consist of 30 different questions ranging from Geometry and Algebra and will be one hour long.
3. The two Ciphers Rounds will both contain 10 questions each of which will be given two minutes.
  - a. Anyone that answers the question correctly in the given time will be awarded a point.
  - b. Students will only be allowed to submit one answer
  - c. Additionally the first finisher with the correct answer of each question will receive 4 additional points for a total of 5, second will receive 3, third 2, and fourth will receive 1.
4. The Ciphers Round questions are not only designed to be difficult but also put the competitor under time pressure by giving him/her only a couple minutes to answer each question.
5. The top 10 competitors from the pretest make it to Ciphers Round I. The top 5 competitors from Ciphers Round I make it to Ciphers Round II.

## **Notes:**

1. The problems will be rated from (0,1,2,3) with 0 being easy, 1 being fair, 2 being difficult, and 3 being challenging.
  - a. Type zero (0) questions will only be found on the pretest.
  - b. Type one (1) may be found on the pretest or Ciphers I.
  - c. Type two (2) may be found on the last couple questions of the pretest, or Ciphers Rounds I and II.
  - d. Type three (3) questions may be found in Ciphers Round II or the Ciphers Bonus Round.
2. The topics listed below are not a comprehensive list of the topics that may be included in the exam. However, there will be no calculus or beyond in difficulty.

# MATH OLYMPICS STUDY GUIDE

- 1. Geometric Formulas (The competitor will be asked to recognize or apply these formulas)**
  - a. Area and Perimeter of two dimensional shapes
  - b. Number of Diagonals of two dimensional shapes
  - c. Interior and exterior angles of two dimensional shapes
  - d. Volume and Surface Area of three dimensional shapes
  - e. Arc length formula
  - f. Slope of Line
  - g. Midpoint and Distance Formula
  - h. Equation of a circle
- 2. Geometry (The competitor needs to have a strong understanding of these topics)**
  - a. Find the point of intersection between two lines
  - b. Find a line parallel/perpendicular to a given line
- 3. Graphing (The competitor will be asked to apply these methods)**
  - a. Graphing a function
  - b. From a given graph determine if the equation is a function
  - c. Identify an odd or even function
- 4. Algebraic Formulas and Methods (The competitor will be asked to apply these methods)**
  - a. Quadratic Formula
  - b. Pascal's Triangle
  - c. Finding a quadratic equation from given points
  - d. Factoring
  - e. Simplifying polynomial expressions
  - f. Simplifying expression with radicals
  - g. Finding max and min
  - h. Describing the nature of the roots
  - i. Given  $g(x)$  and  $f(x)$ , then find  $f(g(x))$ .
- 5. Algebra (The competitor needs to have a strong understanding of these topics)**
  - a. Mean, Median, and Mode
  - b. Logarithmic functions
  - c. Inverse functions
  - d. Range and Domain
  - e. Systems of equations
  - f. Basic rate functions
  - g. Factorials
  - h. Inequalities
  - i. Arithmetic and geometric sequences and series
- 6. Probability and Statistics**
  - a. Combination and Permutation
  - b. Binomial Probability Formula
  - c. Mean, median, mode and standard deviation
  - d. Conditional probability
  - e. Expected value
  - f. Set Theory

# MATH OLYMPICS STUDY GUIDE CONT.

## 7. Logic

- a. Puzzles
- b. Patterns
- c. Problem-solving
- d. Model Theory

## 8. Number Theory

- a. Divisibility rules
- b. Prime factorization, prime and composite numbers
- c. Number of and sum of divisors
- d. Greatest Common Divisor and Least Common Multiple
- e. Number bases

## 9. Trigonometry

- a. Triangles
- b. Similarity
- c. Special right triangles
- d. Sines, cosines, and tangents
- e. Pythagorean Theorem and Identities
- f. Law of Sines, Law of Cosines

# MATH OLYMPICS QUESTION GUIDE

## **MIST Math Olympics Question Guide:**

- The following problems are examples that may show up on either the Pretest or Ciphering Rounds.
- You will notice the ciphering round questions often involve a combination of two or more topics listed in the study guide.
- Below are a total of 35 questions. Their answers are also posted.

## **Geometric Formula Questions:**

1. The circumference of a circle is  $8\pi$ , find the area of the circle. (0)
2. Calculate the volume + surface area of a box with the dimensions: l = 30cm, w = 5cm, h = 10cm. (1)
3. A right triangle has a hypotenuse which measures 10cm. If one of the sides measures 8cm, then find the length of the other side. (0)
4. There exists an arc with the length  $6\pi$ cm which is formed by the angle of  $270^\circ$ . Find the radius. (1)
5. A line passes through the point (4,5) and (-2,8). Find the equation of the line. (0)
6. There exists a line segment with its endpoints at (-1,-2) and (4,8). Find the midpoint of the line segment. (0)
7. A circle has a radius of 8 with its center at (2,4). Find the equation of the circle. (0)
8. Find the distance between the points (1,2) and (6,7). Leave the answer in simplest radical form. (1)
9. Given the points A (1,-3) and B (-5,11), find the equation of the line perpendicular to the line segment AB which crosses at the midpoint of AB. (2)
10. A regular polygon has  $n$  interior angles that sum up to  $1980^\circ$ . What is the value of  $n$ ? (3)

## **Geometry Questions:**

1. Find the distance between the lines  $y = 3x + 4$  and  $y = 3x - 5$  and give the equation of the line perpendicular to  $y = 3x + 4$  which crosses the point (2,1). (2)
2. Given a 45-45-90 triangle with the height of 5cm, find the hypotenuse of the triangle. (0)
3. If  $f(x) = 2x + 8$  and  $g(x) = (x/3) - 4$ , then find the point of intersection of the two lines algebraically. (1)
4. Given a 30-60-90 triangle, if the length of its smallest side is 4 cm, then find the length of the height and the hypotenuse of the triangle. (1)
5. Find the area of a triangle with side lengths 13, 14, and 15. (Hint: What is the altitude perpendicular to the side with length 14?) (3)

## **Graphing Questions:**

1. Solve and graph the function  $3 \geq |x + y|$  using complete solution curves. (2)
2. What is the minimum number of distinct points needed to uniquely define the graph of a cubic polynomial? (0)
3. A quadratic has roots 1 and 4 and passes through (0, -8). Find its equation. (2)

## **Algebraic Formulas and Method Questions:**

1. Simplify\* the following expression:  $(1/(2+\sqrt{3}))$  (0)
  - a. \*Remember simplifying an expression means to annihilate any radicals in the denominator and leave any radicals in the numerator in simplest radical form.
2. The polynomial  $x^3 + 7x^2 - 305x + 1449$  has 3 distinct roots  $a$ ,  $b$ , and  $c$ . Find  $ab + bc + ca$ . (3)
3. Simplify the following expression:  $(x^3+5x^2+6x)/(x^2+2x)$ . (1)
4. Find the roots of the function:  $y = x^2+10x+5$ . (1)
5. Find the minimum value of  $x^2 - 6x + 20$ . (2)
6. Hamza scored a grade of 79 and 92 on his first two tests. What is the minimum score Hamza needs on his next exam to achieve a final grade of 90? Assume all exam grades are weighted equally. (1)
7. Find the coefficient of  $x^3$  in the expansion  $(2x+3)^6$ . (2)
8. Simplify  $\sqrt{50} - \sqrt{8}$ . (1)

## **Algebra Questions:**

1. If  $f(x) = 3x + 8$  and  $g(x) = x^2-2x$ , then find  $f^{-1}(g(3))$ . (2)
2. If  $\log_{264} = n$  then find  $n$ . (0)
3. If  $x + 1/x = 3$ , find the value of  $x^2 + 1/x^2$ . (3)
4. Find the range and domain of  $y = \sqrt[3]{(x+2)}$ . (0)

- An automated tennis ball dispenser dispenses 45 ball per minute. If an automated baseball dispenser dispenses baseballs 20% faster than the tennis ball dispenser, then how fast does the baseball dispenser dispense every hour? (1)
- A car was purchased at \$45,000. After three years the car is now worth \$32,000. Find the rate of depreciation per year. Leave the answer to the nearest percent. (2)
- Simplify the following expression:  $((x+2)!) / (x!)$ . (1)

**Probability and Statistics Questions:**

- A committee of 6 people is to be selected from a group of 10 women and 4 men. What is the probability that the committee consists of 3 women and 3 men? Express your answer as a decimal to the nearest hundredth. (2)
- Ali is a professional basketball player who has determined that he makes nine 3pt shots per every ten attempts. What is the probability that out of 25 shots he misses 4? Express your answer as a decimal to the nearest hundredth. (1)
- A coin has probability  $p$  of landing on heads. You flip until the first time you get 2 consecutive heads. Find the probability that the process ends exactly on flip 4. Express your answer in terms of  $p$ . (3)

**Number Theory Questions:**

- State whether 135 is prime or composite (0).
- Find the prime factorization of 420 (1).
- Compute the sum of the positive divisors of 360 (2).
- The greatest common divisor of 84 and a positive integer  $N$  is 12. Their lowest common multiple is 420. Find  $N$ . (3)

**Trigonometry Questions:**

- A right triangle has angle  $\theta$  such that  $\sin(\theta) = 0.96$ . Find the value of  $\tan(\theta)$  to the nearest hundredth. (1)
- If  $\sin(\theta) = -0.57$ , find the value of  $\csc(\theta)$  to the nearest thousandth. (0)
- Is it possible to form a triangle with side lengths 5, 12, and 18? (0)
- Evaluate the sum:  $\cos(10^\circ) + \cos(20^\circ) + \cos(30^\circ) + \dots + \cos(160^\circ) + \cos(170^\circ) + \cos(180^\circ)$ . (Hint:  $\cos(x) = -\cos(180^\circ - x)$ ) (3)
- Triangle ABC has side lengths  $AB = 6$ ,  $BC = 3$ , and angle  $\angle ABC = 120^\circ$ . Find the exact length of side AC. Simplify your answer as much as possible. (2)

**Logic Questions:**

- How many positive integers less than 1000 contain at least one digit equal to 7? (2)
- If the operation  $a \cdot b$  is defined by  $a \cdot b = a^b + b$ , such that  $a \cdot b = a^b + b$ , then find  $n$ , where  $n \cdot 2 = 38$ . (1)
- What is the remainder when  $13^{2026}$  is divided by 10? (1)
- Simplify the sum:  $1/(\sqrt{1} + \sqrt{3}) + 1/(\sqrt{3} + \sqrt{5}) + 1/(\sqrt{5} + \sqrt{7}) + \dots + 1/(\sqrt{2019} + \sqrt{2021}) + 1/(\sqrt{2021} + \sqrt{2023}) + 1/(\sqrt{2023} + \sqrt{2025})$ . (3)

**Answers begin on the next page**

If you have any questions regarding how to solve these problems, please contact  
[hkhattak@getmistified.com](mailto:hkhattak@getmistified.com).

Please find a clear copy of these questions and answers [here](#)

# MATH OLYMPICS GUIDE ANSWER

## Geometry Formula Answers:

1.  $16\pi$
2. Volume =  $1,500\text{cm}^3$ , Surface Area =  $1,000\text{cm}^2$ .
3. 6cm
4.  $r=4$
5.  $y = -\frac{1}{2}x + 7$
6. (5,3)
7.  $(x-2)^2 + (y-4)^2 = 64$
8.  $5\sqrt{2}$
9.  $y = \frac{3}{7}x + \frac{34}{7}$
10. 13

## Geometry Answers:

1. distance = 9 units\*,  $y = -(x/3+5/3)$  \*Since the units were not provided, distance = 9 will also be accepted. However, be sure to provide units in your answer if it is provided in your question.
2.  $5\sqrt{2}$  cm
3. (- $36/5$ , - $32/5$ )
4. height =  $4\sqrt{3}$  cm, hypotenuse = 8 cm
5. 84

## Graphing Answers:

1.  $3 - x \geq y \geq -3 - x$ . The graph of the inequality will look like the shaded region between the lines  $y = 3 - x$  and  $y = -3 - x$ .
2. 4
3.  $y = 2x^2 - 10x + 8$

## Algebraic Formulas and Method Answers:

1.  $2 - \sqrt{3}$
2. -305
3.  $X+3$
4.  $x = -5 \pm 2\sqrt{5}$
5. 11
6. 99%
7. 4320
8.  $3\sqrt{2}$

## Algebra Answers:

1.  $-5/3$
2.  $n=6$
3. 7
4.  $D: [-2, \infty), R: [0, \infty)$
5. 3240 baseballs/hour
6. rate of depression = 11%
7.  $x^2 + 3x + 2$

## Probability and Statistics Answers:

1. 0.16
2. 0.14
3.  $p^2(1-p)$

## Number Theory Answers:

1. Composite
2.  $(2^2)(3)(5)(7)$
3. 1170
4. 60

## Trigonometry Answers:

1. 3.43
2. -1.754
3. No
4. -1
5.  $3\sqrt{7}$

## Logic Answers

1. 271
2. 6
3. 9
4. 22