Digital Image Processing

Assignment 1 – Dry part

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Question 1:

Assuming $g_k[n] = (f * h_k)(n)$ where $h_k(x)$ is the effective point spread function corresponding to the k-th frame, and $f(x)$ is the image that would be formed on the image plane if the camera were absolutely still. The actually formed digital image of the k-th frame can be described as $g_k[n]$ as written above. We can write $h_k(x)$ as:

Let $o(t)$ be the function representing the position on the camera in the point of time t.

$$h_k = \int_k^{k+T} box(x)\tau_{o(t)}dt$$

The box function represents the exposure time of the sensor.

Question 2:

$$\mathcal{F}[h_k(x)](w) = \mathcal{F}\left[\int_k^{k+T} box(x)\tau_{o(t)}dt\right](w) = \int_k^{k+T} \mathcal{F}\left[box(x)\tau_{o(t)}dt\right](w)$$
$$= \int_k^{k+T} sinc(w)e^{-2\pi i w^T o(t)}dt$$

- The first transition is based on the linearity of the Fourier transform.
- The 3rd is based on $\mathcal{F}[box(x)](w) = sinc(w)$.

Question 3:

We already know that the action of the camera is: $g_k[n] = (f * h_k)(n)$

To express the camera action in the frequency domain, we must calculate $\mathcal{F}\left[g_k[n]\right](w)$:

$\mathcal{F}\left[g_k[n]\right](w) = \mathcal{F}[(f * h_k)(n)] = \mathcal{F}[f](w)\mathcal{F}[h_k](w) =$
$F(w)\int_{kT}^{k+T} sinc(w)e^{-2\pi i w^T o(t)}dt = sinc(w)F(w)P_k(w)$

- Third transition is based on the linearity of Fourier transforms.
- Last transition is based on that $sinc(w)$ is independent of t.

From the calculations above we can denote that:

$$P_k(w) = \int_k^{k+T} e^{-2\pi i w^T o(t)}dt$$

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Question 4:

$$|P_k(w)| = \left|\int_k^{k+T} e^{-2\pi i w^T o(t)} dt\right| \le \int_k^{k+T} \left|e^{-2\pi i w^T o(t)}\right| dt$$
$$= \int_{kT}^{k+T} \left|e^{i\pi^{-2w^T o(t)}}\right| dt$$
$$= \int_k^{k+T} \left|(-1)^{-2w^T o(t)}\right| dt = \int_k^{k+T} \left|1^{w^T o(t)}\right| dt = \int_k^{k+T} |1| dt = k + T - k$$
$$= T$$

- 2^{nd} transition is justified by the integral triangle inequality.
- 4^{th} transition is based on $e^{i\,\pi} + 1 = 0$.

From the calculations above we can denote that:

$$|P_k(w)| \le T$$

Question 5:

Assuming that $o(t) = (t - k - 0.5)ve_1$ and $e_1 = (1,0,0)^T$ we can calculate $P_k(w)$ as follows:

$$P_k(w) = \int_k^{k+T} e^{-2\pi i w^T o(t)} dt = \int_k^{k+T} e^{-2\pi i w^T (t-k-0.5)v} dt$$
$$= e^{2\pi i w^T (k+0.5)v} \int_k^{k+T} e^{-2\pi i w^T t} dt = e^{2\pi i w^T (k+0.5)v} \left(\frac{e^{-2\pi i w^T T}}{-2\pi i w^T}\right)$$
$$= \left(\frac{e^{2\pi i w^T (k+0.5-T)}}{-2\pi i w^T}\right)$$

Therefore:

$$|P_k(w)| = \left|\left(\frac{e^{2\pi i w^T (k+0.5-T)}}{-2\pi i w^T}\right)\right| = \frac{1}{2\pi w^T}$$

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Question 6:

Now we are given a more general case; $o(t) = q + tv, where\ t \in [k, k + T)$.

$$P_k(w) = \int_k^{k+T} e^{-2\pi i w^T(q+tv)} dt = e^{-2\pi i w^T q} \int_k^{k+T} e^{-2\pi i w^T tv} dt = \left(\frac{e^{-2\pi i w^T T v}}{-2\pi i w^T v}\right) e^{-2\pi i w^T q}$$
$$= \left(\frac{e^{-2\pi i w^T (Tv+q)}}{-2\pi i w^T v}\right)$$

Therefore:

$$|P_k(w)| = \left|\left(\frac{e^{-2\pi i w^T (Tv+q)}}{-2\pi i w^T v}\right)\right| = \frac{1}{2\pi w^T v}$$

Question 7:

As we conclude from calculations above, we know that for each frame the camera is moving in a certain velocity\direction that is independent of the other frames in the burst. Therefore, the Fourier frequencies will be weakened depending on the direction/velocity of the camera during the taking of the frame, so in order to reconstruct the original image we can try to combine the Fourier frequencies with the largest magnitudes, which can be done iteratively as we get a new frame from the burst, we can compare the frequency magnitude of the new frame to the last reconstructed image (for the first frame we can take the reconstructed image as the frame itself) then we chose the frequency with the highest magnitude. This method tries to minimize the weakening done by the blurring.