FRM Term Project

Comparison of VaR estimates with backtesting

Group members	
Ankit Pandey	17MF3FP18
Ashish Gokarnkar	18IM30027
Abdul Ahadh	17AG3FP02
Nesara S R	18IM30014
Manan Khaneja	16PH20023

Assumption :
Value invested in each Index = **INR 1**

Current price of **S&P BSE 500** as on 2021-04-09 is **INR 19944.06**
Current price of **NIFTY** as on 2021-04-09 is **INR 14834.85**

Data Description

We have taken up the data from two indices: NIFTY 50 and S&P BSE 500. For the ARIMA model, volatility index data (India VIX) is used. The India VIX data is available for the last 11 years, hence to maintain uniformity and to enable cross comparisons across different models we have used only 11 year data for S&P BSE 500 and Nifty as well. Current date is assumed to be 2021-04-09 - for which VaR predictions are made using several models.

Sources of data:

1. NIFTY 50
2. S&P BSE 500
3. India VIX

S&P BSE 500 Statistics

1. Sample Standard Deviation : **0.01157**
2. Minimum Sample Standard Deviation (from standard error): **0.01127**

3. Maximum Sample Standard Deviation (from standard error): **0.01186**
4. Sample Mean : **0.000616**
5. Minimum Mean (from standard error): **0.000202**
6. Maximum Mean (from standard error): **0.001030**
7. Min Parametric Relative Loss the asset value will face is **INR 0.02624**
8. Max Parametric Relative Loss the asset value will face is **INR 0.0276**
9. Min Parametric Absolute Loss the asset value will face is **INR 0.0252**
10. Max Parametric Absolute Loss the asset value will face is **INR 0.0274**

NIFTY Statistics

1. Sample Standard Deviation : **0.012022**
2. Minimum Sample Standard Deviation (from standard error): **0.011718**
3. Maximum Sample Standard Deviation (from standard error): **0.012326**
4. Sample Mean : **0.000571**
5. Minimum Mean (from standard error): **0.000141**
6. Maximum Mean (from standard error): **0.0010012**
7. Min Parametric Relative Loss the asset value will face is **INR 0.02726**
8. Max Parametric Relative Loss the asset value will face is **INR 0.02867**
9. Min Parametric Absolute Loss the asset value will face is **INR 0.02626**
10. Max Parametric Absolute Loss the asset value will face is **INR 0.02853**

VaR Estimates : Without forecasting

VaR Terminology	S&P BSE 500	NIFTY	Relative (Nifty/ S&P)	Remarks
Empirical Absolute VaR (in rupees)	0.031	0.030	0.990	S&P is riskier
Empirical Relative VaR (in rupees)	0.030	0.030	0.993	S&P is riskier
Parametric Absolute VaR (in rupees)	0.026	0.027	1.054	NIFTY is riskier
Parametric Relative VaR (in rupees)	0.027	0.028	1.041	NIFTY is riskier
7 day loss-absolute (Efficient markets) (in rupees)	0.067	0.070	1.046	NIFTY is riskier
7 day loss-relative (Efficient markets) (in rupees)	0.071	0.074	1.039	NIFTY is riskier
7 day loss-absolute (Inefficient markets) (in rupees)	0.070	0.073	1.046	NIFTY is riskier
7 day loss-relative (Inefficient markets) (in rupees)	0.074	0.077	1.039	NIFTY is riskier
CVaR : Worst loss (Absolute Loss) given VaR (in rupees)	0.031	0.032	1.039	NIFTY is riskier
CVaR : Worst loss (Relative Loss) given VaR (in rupees)	0.031	0.033	1.038	NIFTY is riskier
Hence we can conclude that NIFTY is riskier.				

VaR Estimates : With forecasting

GARCH(1,1)

Assuming errors follow Student's T distribution, we forecasted standard deviation for both NIFTY and S&P using a GARCH(1,1) model. The model was implemented using the python package "arch" and observed for both expanding and rolling window approaches.
Considering a rolling window length of about 5 years, we obtained predictions for the time frame starting on 4th March 2014 and ending on 9th April 2021.

S&P BSE 500

1. Standard Deviation of S&P BSE 500 using GARCH(1,1) with expanding window : **0.01072**
2. Standard Deviation of S&P BSE 500 using GARCH(1,1) with rolling window : **0.00991**
3. VaR of S&P BSE 500 using Expanding Window GARCH(1,1) : **INR 0.0249**
4. VaR of S&P BSE 500 using Rolling Window GARCH(1,1) : **INR 0.02303**

NIFTY

1. Standard Deviation of NIFTY using GARCH(1,1) with rolling window : **0.01080**
2. Standard Deviation of NIFTY using GARCH(1,1) with expanding window : **0.01158**
3. VaR of NIFTY using Expanding Window GARCH(1,1) : **INR 0.02699**
4. VaR of NIFTY using Rolling Window GARCH(1,1) : **INR 0.025125**

NIFTY is riskier by both methods (full dataset & rolling window)

RiskMetrics (EWMA)

In the following section, we have applied the Exponentially Weighted Moving Average method to forecast the volatility, and hence the VaR in S&P BSE and NIFTY. Volatility has been calculated using the following two methods:

1. Rolling window with a time period of 5-years
2. Expanding window (with the complete dataset)

To forecast the volatility using the rolling window, we assigned exponential weights to the squared values of the returns for the last five years using the decay factor to be 0.94.

The expanding window model is implemented straightforwardly in excel using recursion whereas the rolling window model is implemented in python with manually defined exponential weights with a decay factor of 0.94.

S&P BSE 500

1. Standard Deviation of S&P BSE 500 using RiskMetrics Expanding Window : **0.0118**
2. Standard Deviation of S&P BSE 500 using RiskMetrics Rolling Window : **0.0115**

3. VaR of S&P BSE 500 using RiskMetrics Expanding Window : **INR 0.02745**
4. VaR of S&P BSE 500 using RiskMetrics Rolling Window : **INR 0.02675**

NIFTY

1. Standard Deviation of NIFTY using RiskMetrics Expanding Window : **0.0126**
2. Standard Deviation of NIFTY using RiskMetrics Rolling Window : **0.0122**
3. VaR of NIFTY using RiskMetrics Expanding Window : **INR 0.029312**
4. VaR of NIFTY using RiskMetrics Rolling Window : **INR 0.028381**

ARIMA

We forecasted standard deviation of NIFTY by fitting ARIMA model on India VIX. The India VIX data obtained was transformed as shown below to obtained daily data of NIFTY volatility :

Assuming T=365,

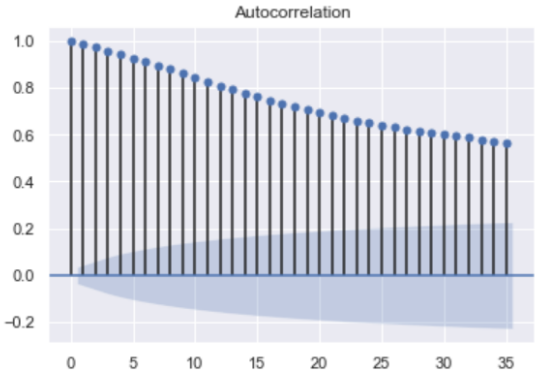
$$\sigma_{NIFTY}[i] = india\_vix[\text{'Close'}][i] / (100 * \sqrt{T})$$

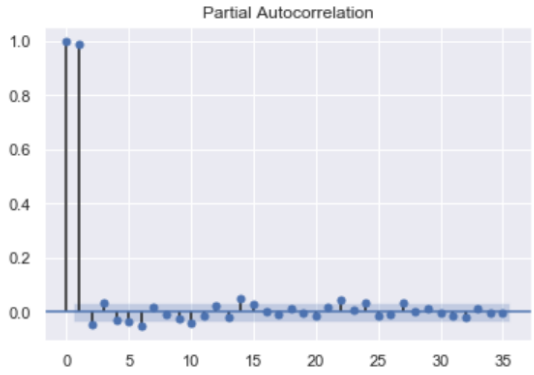
Volatility recorded at the end of each day is divided by 100 to convert from percentage and then divided by sqrt(T) to account for the time aggregation since it is known that India VIX depicts the annualized volatility of NIFTY.

Since ARIMA model requires the time series data to be stationary, the series σ_{NIFTY} obtained after transformation was checked for stationarity using Augmented Dickey-Fuller Test with null hypothesis H_0 = Series has a unit root or the series is non-stationary.

```
ADF Statistic: -5.089934
p-value: 0.000015
Critical Values:
        1%: -3.433
        5%: -2.863
        10%: -2.567
```

ADF statistic < Critical value indicates that we can reject the null hypothesis. We therefore conclude that the **series at hand is stationary.** We then proceeded to analyze the Autocorrelation and Partial autocorrelation plots to determine the AR parameter 'p' & MA parameter 'q.





1. Since the autocorrelation plot is monotonously decreasing without approaching zero, we take q=0
2. Since the partial autocorrelation plot peaked at lag=1 (ignoring lag=0), we take p=1
3. Since the series is already stationary, there is no necessity for differencing and therefore d=0

With above listed parameters for the ARIMA model, we forecasted standard deviation for the current date. The model was implemented using the python package "Statsmodels" and observed for both expanding and rolling window approaches.
Considering a rolling window length of about 5 years, we obtained predictions for the time frame starting on 4th March 2014 and ending on 9th April 2021.

NIFTY (using India VIX)

1. Standard deviation of NIFTY using ARIMA Expanding Window : **0.01036**
2. Standard deviation of NIFTY using ARIMA Rolling Window : **0.010617**
3. VaR of NIFTY using ARIMA Expanding Window : **INR 0.02410**
4. VaR of NIFTY using ARIMA Rolling Window : **INR 0.024698**

BACKTESTING: Conditional Coverage Model

Model has been back tested using the **last 200 business day data**. The VaR model used is Parametric Relative VaR for both Nifty and S&P BSE 500 The backtesting model used is Binomial Distribution for Black Swan event possibility.

Confidence Interval = 99% (two tailed)

S&P BSE 500
(N, T) = 2, 200
(Zcalc, Ztab) = 1.00251, 2.57583
Model is GOOD as per conditional coverage test

NIFTY
(N, T) = 2, 200

(Zcalc, Ztab) = 1.00251, 2.57583
Model is GOOD as per conditional coverage test

In cases of both NIFTY and S&P BSE 500 we find that only 2 days out of 200 days had a loss marked greater than their 99% Parametric Relative VaRs (INR 536.84 and INR 414.89 respectively). We then compare it with the output that a binomial distribution (approximated to a Normal Distribution) might have given us. For concluding the analysis, we compare the calculated Z statistic with the tabulated Z statistic from the Normal Distribution table.

In both the indices, Nifty and S&P BSE 500, the Zcalc is found to lie within the range [- Ztab, + Ztab].
Thus our VaR models lie within the ranges of error as per the Conditional Coverage Model.

Summary and Inferences

The following points were observed for all the 3 forecasting models implemented :

1. NIFTY has more VaR (within 10%) compared to S&P BSE 500 using both the expanding window and the rolling window. Hence we can conclude that NIFTY is riskier. This might be due to a variety of reasons including more diversification in S&P index, more weightage of stocks that are in less risky industries in S&P compared to NIFTY and so on.

2. Among the two methods of rolling window and expanding window, expanding window gives a greater VaR for both S&P BSE 500 and NIFTY. The reason can be that the expanding window takes the complete dataset of 11 years in forecasting the current volatility whereas the rolling window is fixed for 5 years. The probability of capturing extreme loss events is more in the former case, and hence we speculate that the model should in fact give a higher VaR for expanding window.

3. The RiskMetrics method gave the highest VaR whereas ARIMA gave the lowest VaR for NIFTY. Similarly for the S&P BSE index, RiskMetrics approach yielded the highest VaR.