IM39003

Assignment 1 : Simulated Annealing

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Activity 2.2

This report summarizes the results of experiments conducted to study the effect of various parameters on the convergence of the Simulated Annealing algorithm.

We have the following objective function :

$$z = (1 - x1)^2 + 100\,(x2 - x1^2)^2$$
$$-5 \leq x1, x2 \leq 5$$

1. **Starting point**

Varying starting point, but keeping the following parameters constant :

```
T_init = 1000;          %Initial Temperature
max_run = 100;          %Maximum Number of runs
k = 1;                  %Boltzman constant
T_min = 0.1;            %Minimum temperature for cooling
alpha= 0.7;             %cooling factor

max_rej = 2000 ;        %Maximum number of rejections
max_accept = 80 ;       %Maximum number of acceptances in a temperature
initial_search = 500;   %Initial search period
step = 0.5;             %Local search steps-size
```

S.no	Parameter value	Total Evaluation	Best objective function value	Best Solution, x	
				x1	x2
1	(0,0)	2081	0.3443	1.284	1.700
2	(1.5,1.5)	2081	0.4478	1.408	1.929
3	(-4,-4)	2081	0.6021	1.339	1.862
4	(4,4)	2081	0.0989	1.143	1.279
5	(-1,1)	2081	0.5024	1.101	1.142

2. Starting temperature

Varying starting temperature, but keeping the following parameters constant :

```
u0 = (0,0);              %Starting point
max_run = 100;           %Maximum Number of runs
k = 1;                   %Boltzman constant
T_min = 0.1;             %Minimum temperature for cooling
alpha= 0.7;              %cooling factor

max_rej = 2000 ;         %Maximum number of rejections
max_accept = 80 ;        %Maximum number of acceptances in a temperature
initial_search = 500;    %Initial search period
step = 0.5;              %Local search steps-size
```

S.no	Parameter value	Total Evaluation	Best objective function value	Best Solution, x	
				x1	x2
1	2000	2241	0.4509	1.182	1.333
2	1000	2081	2.1593	1.148	1.463
3	500	1921	0.453	0.366	0.111
4	200	1761	0.0693	1.075	1.180
5	100	1601	0.2315	0.651	0.457

3. Maximum runs

Varying maximum runs at a particular temperature, but keeping the following parameters constant :

```
T_init = 1000            %Initial temperature
u0 = (0,0);              %Starting point
k = 1;                   %Boltzman constant
T_min = 0.1;             %Minimum temperature for cooling
alpha= 0.7;              %cooling factor

max_rej = 2000 ;         %Maximum number of rejections
max_accept = 80 ;        %Maximum number of acceptances in a temperature
initial_search = 500;    %Initial search period
step = 0.5;              %Local search steps-size
```

S.no	Parameter value	Total Evaluation	Best objective function value	Best Solution, x	
				x1	x2
1	300	2081	0.9696	1.505	2.182
2	200	2081	1.5568	1.482	2.311
3	100	2081	0.7411	1.760	3.138
4	80	2055	1.6602	2.215	4.951
5	40	781	1.8596	1.015	0.894

Note : for max_run = 40, changed max_acceptance to 30, because max_Acceptance <= max_run

4. **Cooling factor**

Varying the cooling factor, but keeping the following parameters constant :

T_init = 1000 %Initial temperature
u0 = (0,0); %Starting point
k = 1; %Boltzman constant
T_min = 0.1; %Minimum temperature for cooling
Max_run = 100 ; %Maximum runs at a particular temperature

max_rej = 2000 ; %Maximum number of rejections
max_accept = 80 ; %Maximum number of acceptances in a temperature
initial_search = 500; %Initial search period
step = 0.5; %Local search steps-size

S.no	Parameter value	Total Evaluation	Best objective function value	Best Solution, x	
				x1	x2
1	0.99	73361	0.3828	1.231	1.458
2	0.9	7041	0.0252	1.147	1.310
3	0.7	2081	0.0679	1.231	1.528
4	0.5	1121	1.1647	0.745	0.660
5	0.3	641	2.7663	1.119	1.087

5. **Maximum acceptance (in terms of fraction of max_runs)**

T_init = 1000 %Initial temperature
u0 = (0,0); %Starting point
k = 1; %Boltzman constant
T_min = 0.1; %Minimum temperature for cooling
Max_run = 200 ; %Maximum runs at a particular temperature

max_rej = 2000 ; %Maximum number of rejections
Cooling factor = 0.7 ; %cooling factor
initial_search = 500; %Initial search period
step = 0.5; %Local search steps-size

S.no	Parameter value	Total Evaluation	Best objective function value	Best Solution, x	
				x1	x2
1	0.9 (max_accept = 180)	4681	0.4676	1.197	1.368
2	0.8 (max_accept = 160)	4161	0.4321	0.773	0.537
3	0.6 (max_accept = 120)	3121	0.1571	0.740	0.517
4	0.4 (max_accept = 80)	2081	0.1413	0.659	0.419
5	0.2 (max_accept = 40)	1041	0.9598	1.146	1.216

6. Step-size for local search

```
T_init = 1000              %Initial temperature
u0 = (0,0);                %Starting point
k = 1;                     %Boltzman constant
T_min = 0.1;               %Minimum temperature for cooling
Max_run = 100 ;            %Maximum runs at a particular temperature

max_rej = 2000 ;           %Maximum number of rejections
Cooling factor = 0.7 ;     %cooling factor
initial_search = 500;      %Initial search period
max_accept = 80;           %maximum acceptance at a particular temperature
```

S.no	Parameter value	Total Evaluation	Best objective function value	Best Solution, x	
				x1	x2
1	1.5	2081	0.252	1.060	1.173
2	1	2081	0.0519	0.989	1.002
3	0.9	2081	0.8113	1.756	3.133
4	0.5	2081	1.7921	1.307	1.578
5	0.3	2081	0.3265	0.904	0.873

Inference

1. Most of the experiment runs came close the optimal solution obtained theoretically i.e z=0 and x = [1,1]
2. The effect of change in starting point seemed quite insignificant on the best solution obtained and the rate of convergence.
3. There is a direct relationship between the initial temperature and the total number evaluations to converge to an optimal. Lower is the initial temperature chosen, lower will be the number of evaluations conducted. However, the effect of initial temperature on the quality of the best solution obtained was quite randomized and so no conclusion could be obtained.
4. Cooling factor has a direct impact on the total number of evaluations taken to arrive at a near optimal solution. Increasing the cooling factor will result in slow changes in temperature parameter thereby resulting in a slow convergence rate.
5. The fraction of solutions we decide to accept at a particular temperature plays quite an important role in fastness of convergence. Lower the acceptance rate, lesser is the number of evaluations required to arrive at a near optimal solution. A possible explanation for this is that rejecting solutions and increasing exploration can result in reaching global optimum at a much faster rate.