Dynamic Monopoly Pricing With Multiple Varieties: Trading Up Buyers

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Abstract

This paper studies dynamic monopoly pricing in a unified analytical framework that allows for multiple durable or rental varieties, as well as other settings including positive selection. We show that the driving force behind dynamic pricing is the seller's incentive to trade up consumers to higher-valued consumption options. If there are no trading-up opportunities at static monopoly prices, then the seller can do no better than repeatedly play static monopoly prices and is guaranteed to obtain the commitment profit irrespective of commitment ability. If trading-up opportunities do exist at static monopoly prices, then the seller engages in dynamic pricing by lowering prices for any history with trading-up opportunities until reaching the static profit-maximizing prices that leave no trading-up opportunities. All trading-up opportunities will be exhausted in finite time if the seller's static profit is strictly positive at these prices, and profit in equilibrium is bounded from below by the repeated static no-trading up opportunities profit. We thus show that for a large class of settings the dynamic outcome can be characterized using the static problem.

1 Introduction

The prices of many goods and services vary across time and consumers. For instance, many retailers routinely offer temporary discounts for selected items sold through their stores. Similarly, mobile phone service providers often make introductory offers to new customers while charging higher prices to loyal ones. Airlines and retail chains, in turn, design sophisticated customer retention programs to make selected offers to specific customer groups, and an increasing number of firms makes use of repricing software to dynamically adjust the prices of their products.

The economic literature has emphasized the importance of a sellers commitment ability in explaining dynamic pricing. As is well-known, a monopolistic seller of a single, durable good will want to refrain from dynamic pricing, but faces a commitment problem: high value buyers are more likely to purchase today which results in a negative selection of remaining non-buyers and this causes the monopolist to reduce prices for these buyers if she cannot commit ('Coasian dynamics'). Forward-looking buyers exploit these falling prices resulting in a negative externality of future prices on today's profits. This limits the sellers monopoly power, and in its extreme form the sellers profit goes to zero if all trade takes place in the *twinkling of an eye*, as conjectured by Coase (1972) and proved formally by Stokey (1981), Bulow (1982), Fudenberg et al. (1985), Gul et al. (1986), and Ausubel and Deneckere (1989).

But Coasian dynamics, time inconsistency, and zero profits do not universally apply. For example, if buyers are able to exit the bargaining process at any point for a positively valued alternative, the seller no longer faces a commitment problem and no dynamic pricing arises (Board and Pycia, 2014). Similarly, if the seller offers a rental instead of a durable good and only high-value rather than low-value consumers remain in the market ('positive selection'), the profit-maximizing price is constant and pricing is time-consistent (Tirole, 2016). Yet if both high- and low-value consumers remain and thus both negative selection (for non-buyers) and positive selection (for loyal consumers) are at work, leading to "behavior-based pricing", then Coasian dynamics may arise again, but the price for positively selected buyers may no longer be constant over time (Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Acquisti and Varian, 2005). And if the seller is able to offer more than one durable variety, then time inconsistency and Coasian dynamics apply but this will no longer lead to zero profits (Nava and Schiraldi, 2019).

In this paper, we extend the existing analysis to a monopolistic seller that may offer two varieties of a good and each variety may be durable or rental. To the best of our knowledge this has not been studied before.[1] We further allow for settings with pure positive selection, and a variety of settings in which varieties are neither durable nor rental. We argue that dynamic pricing – as opposed to the repeated play of static monopoly prices – and the arising of commitment problems can be analyzed using a simple concept: the incentive to 'trade up' buyers to higher-valued consumption options. If the seller faces a set of buyers for which trading-up opportunities exist, then she has an incentive to cut the price to trade up these types. By lowering the price that these types are facing for the higher-valued consumption option, the seller can trade them up to the higher-valued consumption option and thereby tap into a larger surplus than that emerging from the currently chosen one. This is the essence of Coasian dynamics. But this logic applies not only to consumers that selected the outside option, but also to buyers that did not select their most-preferred variety and can be employed to explain pricing dynamics in a large class of settings.

Specifically, we consider a monopolist with constant marginal cost selling two varieties of a good to a continuum of buyers with unit-demand. Buyers may choose between the varieties and the outside option in each period according to a set of admissible transitions. We place almost no restrictions on this set of transitions, allowing varieties (and the outside option) to be *absorbing* (i.e. buyers must choose this option in every future period) or not, as well as numerous other combinations of transitions. Buyers' values for the varieties are private information distributed according to a measure that allows for arbitrary correlation structures, including vertical or horizontal differentiation. The seller chooses prices in each period for the two varieties, but only if the buyers facing these prices can choose the outside option in this period. That is, we require pricing to be consistent with voluntary consumption. If not, we set prices to zero and thus in our setting an absorbing variety acts like the sale of a durable variety, while a variety that can be bought or rejected in each period acts like a rental variety.

Our analysis demonstrates that the outcome of dynamic pricing problems hinges on two price profiles from the static setting: the monopoly profit-maximizing prices on the one hand and the prices that maximize the sellers profit conditional on leaving no trading-up opportunities on the other hand. By comparing these two price profiles we can understand the outcome of the repeated game. If the monopoly prices leave no trading-up opportunities, that is, the two price profiles coincide, then the seller does not face a commitment problem and the profit-maximizing solution is to play constant monopoly prices irrespective of commitment ability. If the two price profiles do not coincide, then a commitment problem can arise and the monopolist will repeatedly lower prices to trade up consumers over time. This dynamic will end in finite time, exhausting all trading-up opportunities, if the static profit from playing profit-maximizing prices that leave no trading-up opportunities is strictly positive. This logic and our main results extend previous work to, for instance, settings with two rental varieties, or "mixed" settings with one rental and one durable variety, but also to more unorthodox settings where a variety may be neither fully rental nor durable. For example, this would be the case if the seller offers only one variety to all buyers, but an additional, second variety exclusively to loyal buyers.

This paper contributes to an extensive literature on the sale of a single durable good (e.g Coase, 1972; Fudenberg et al., 1985; Gul et al., 1986; Sobel, 1991; Kahn, 1986; Bond and Samuelson, 1984; Fuchs and Skrzypacz, 2010), of multiple varieties of a durable good (e.g. Nava and Schiraldi, 2019; Board and Pycia, 2014), and of vertically differentiated durable products (Hahn, 2006; Inderst, 2008; Takeyama, 2002). Our work differs by considering a unified analytical framework that allows for both rental and durable varieties, as well as a class of other settings. In doing so we add to the analysis of settings with 'positive selection' (Tirole, 2016). In addition, we contribute to the literature on behavior-based pricing (e.g. Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Taylor, 2004; Buehler and Eschenbaum, 2020). In contrast to recent work in Rochet and Thanassoulis (2019), we do not allow varieties to be sold as a bundle. Our setup is best understood as offering two varieties of

[1] Nava and Schiraldi (2019) study an extension of their setting in which consumers may return to the market after purchase, but focus on the case in which it is not profitable for the seller to exploit this.

the same good over time.

Our work is also related to the marketing literature. Our formal notion of trading up is closely linked to what is commonly known as 'upselling' in marketing (e.g. Blattberg et al., 2008; Aydin and Ziya, 2008; Wilkie et al., 1998). However, upselling generally refers to inducing loyal customers to upgrade to a more expensive product, while trading up applies to both non-buyers and loyal buyers.

The remainder of the paper is organized as follows. Section 2 introduces the analytical framework. In subsection 2.1 we sketch some applications that our framework allows for. In subsection 2.2 we formalize our notion of trading up. Section 3 analyzes the dynamic pricing game. In subsection 3.1 we first establish a skimming result. In subsection 3.2 we provide our main results. Section 5 concludes and offers directions for future research.

2 Analytical Framework

Consider a monopolist that sells two varieties of a good, a and b, at zero marginal cost to a measure of buyers with unit-demand per period, so that each buyer consumes per period one of the varieties, or none. Buyers have per-period valuations v_a, v_b for the two varieties that are constant over time. Value profiles $v = (v_a, v_b)$ are private information of buyers and distributed according to a measure $\mathcal{F}$ on the unit square $[0,1]^2$. The associated cumulative distribution is F with density f, and V is the support. F_i denotes the marginal cumulative distribution of variety i, while f_i and V_i denote the respective density and support.

Time is discrete and indexed by $t = 0, ..., T$. After T periods, the good becomes obsolete. All players share the same discount factor $\delta \in (0,1)$. In every period t, consumers make a discrete choice $x^t \in X$, where

$$X \equiv \{(1,0), (0,1), (0,0)\},$$

with $a = (1,0)$, $b = (0,1)$, $o = (0,0)$, which are the three *states* of the game and where o indicates the outside option. Let $\bar{x} \in X$ be the initial state for all consumers. A sequence of discrete choices x^t from t onward or *consumption path* $\mathbf{x}^t = (x^t, x^{t+1}, ..., x^T)$ gives rise to a present discounted sum of *total consumption* $\chi(\mathbf{x}^t) = \sum_{\tau=t}^{T} \delta^{\tau-t} x^\tau$. A given consumption path is *admissible* if all transitions from state to state along the entire path are within the set of admissible transitions $\Gamma \subset X \times X$. We place no restrictions on Γ, except that transitions from a state to itself are always admissible, that is $(o,o), (a,a), (b,b) \in \Gamma$. A state $x \in X$ is *absorbing* if no other state $x' \in X$ is accessible from x, that is, $(x, x') \notin \Gamma$. Let $\Delta^t = \sum_{\tau=t}^{T} \delta^{\tau-t}$ denote the present discounted number of *total periods*.[2]

We allow for both varieties to be a durable or a per-period "rental" service (Hart and Tirole, 1988). The 'sale' or 'durable' model assumes that the variety is sold once for all future periods: $x^t = i$ implies $x^\tau = i$ for $\tau \in t+1, .., T$, $i \in (a, b)$. Thus, in the case of a durable variety $i \in (a, b)$, the state i is absorbing and at any time t the only path that includes consumption of variety i at t available to buyers is the path of always consuming i, with associated total consumption $\Delta^t \cdot i$. The 'rental' model instead assumes that buyers may choose to consume the variety i in each period anew so that $(i,i), (i,o), (o,i) \in \Gamma$. Note that our setup also allows for sets of admissible transitions for which a given variety is neither durable nor rental.

Using a transition diagram (Rubinstein 1986), Figure 1 illustrates the least restrictive setting with two rental varieties and a non-absorbing outside option. In this setting, all possible transitions are admissible so that $\Gamma = ((a,a), (b,b), (o,o), (a,b), (b,a), (o,a), (o,b), (a,o), (b,o))$. In the figure, the vertices indicate the states a, b, o, with initial state $\bar{x} = o$, while the arcs and brackets (x, x'), with $(x, x') \in \Gamma$, represent the admissible transitions.

[2]Throughout, we will consistently omit the exponent t for all our expressions when $t = 0$.

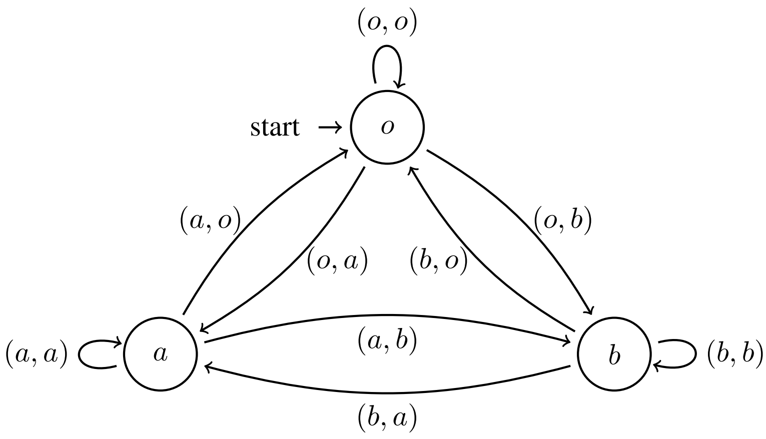


Figure 1: States and transitions in the two rentals setting with $\bar{x} = o$

2.1 Applications

Our analytical framework can be used to study a range of specific applications. We sketch some of the possible applications below, including settings that have been analyzed in the literature previously such as the canonical one durable good setting. We omit states that are inaccessible for simplicity.

One durable good: There is a single variety, say a, which is an absorbing state, while the outside option o is non-absorbing, as in e.g. Stokey (1979); Coase (1972); Gul et al. (1986); Sobel (1991). Hence, there is no transition possible from a to o. The initial state is the outside option o.

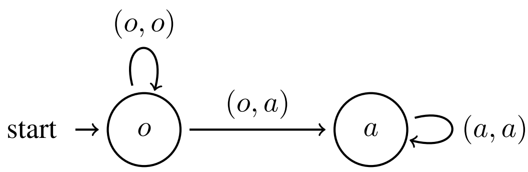


Figure 2: One absorbing variety with $\bar{x} = o$

Positive selection: There is a single non-absorbing variety, say a, while the outside option o is absorbing, as in Tirole (2016). Consumers may only transition from consumption to the outside option, but not reverse. The initial state is the variety a.

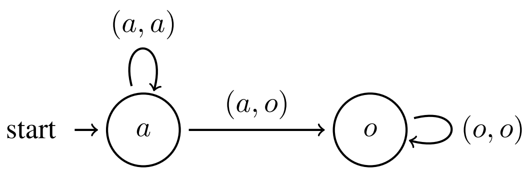


Figure 3: One variety and an absorbing outside option with $\bar{x} = a$

Behavior-based pricing: There is a single variety, say a, and the outside option o which are both non-absorbing states, as in e.g. Acquisti and Varian (2005); Armstrong (2006); Conitzer et al. (2012); Fudenberg and Villas-Boas (2007); Buehler and Eschenbaum (2020). Transitions are possible from the variety to the outside option and reverse. The initial state is the outside option o.

Two durable varieties: There are two varieties which are both absorbing states, while the outside option o is non-absorbing, as in Nava and Schiraldi (2019); Board and Pycia (2014). Transitions are only possible from the outside option to one of the two varieties but not reverse. The initial state is the outside option o.

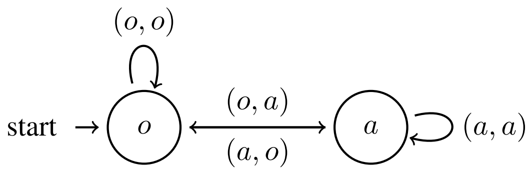


Figure 4: One non-absorbing variety with $\bar{x} = o$

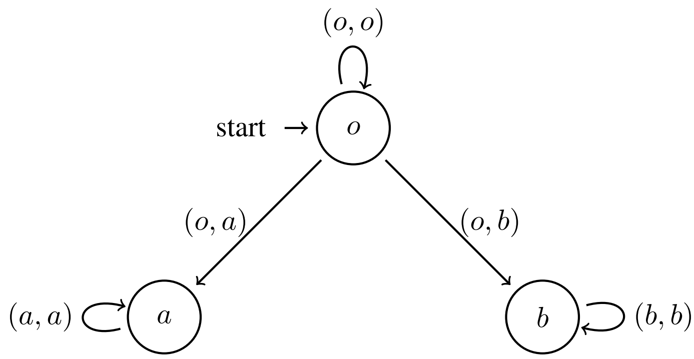


Figure 5: Two absorbing varieties with $\bar{x} = o$

One durable and one rental variety: There are two varieties one of which is an absorbing state, say b, while the outside option o is non-absorbing. Transitions are possible from the outside option to either variety and from a back to o or a to b. The initial state is the outside option o.

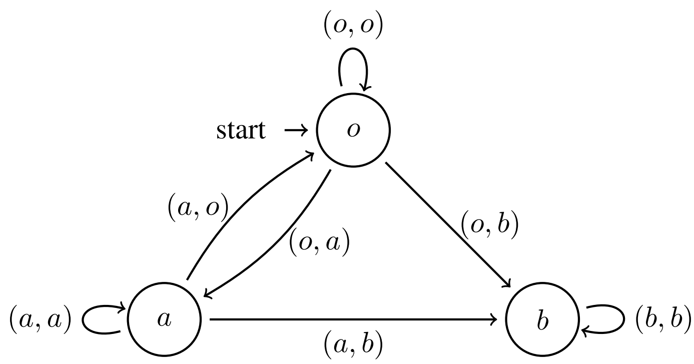


Figure 6: One absorbing and one non-absorbing variety with $\bar{x} = o$

'Loyalty upgrade': There are two varieties one of which is rental and one durable, but the durable one can only be purchased after the rental one. Transitions are possible from the outside option to one of the two varieties, say a, from a to b and to o, and from b back to a. The initial state is the outside option o.
'Consumption cycle': There are two varieties that are neither rental nor durable goods. Transitions are possible from the outside option to one of the two varieties, say b, from b to a, and from a back to o. The initial state is a.

2.2 Prices, Payoffs, and Trading Up

All players are risk-neutral. The seller chooses a price profile $p^t = (p_a^t, p_b^t)$ in each period t for every sequence of past consumption decisions $\mathbf{x}^{-t} \equiv (x^0, ..., x^{t-1})$ that satisfies $(x^{t-1}, o) \in X$. That is, we require pricing to be consistent with voluntary consumption. Whenever $(x^{t-1}, o) \notin X$, the price $p_{x^{t-1}}^t$ is

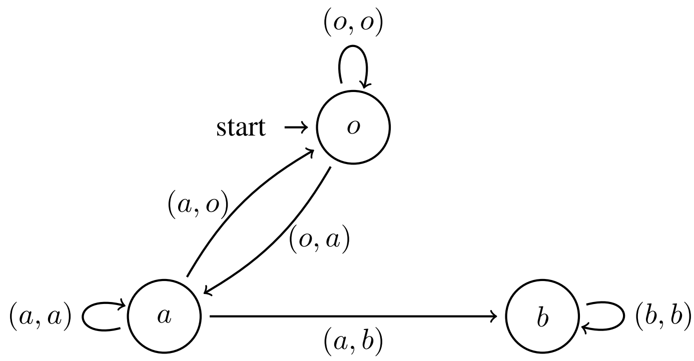


Figure 7: 'Loyalty upgrade' with $\bar{x} = o$

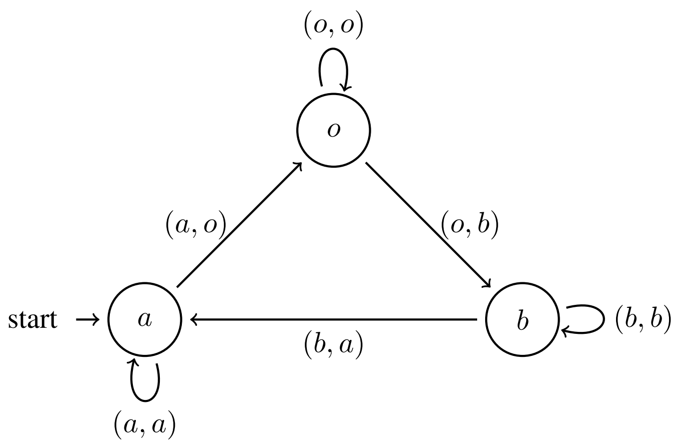


Figure 8: 'Consumption cycle' with $\bar{x} = a$

set to zero. Let $\rho(\mathbf{x})$ be the present discounted sum of *total payment* made by a buyer to the seller along a consumption path $\mathbf{x}$, or $\rho(\mathbf{x}) = \sum_{t=0}^{T} \delta^t p^t \cdot x^t$. Similarly, let $\nu(v, \mathbf{x})$ be the present discounted sum of *total value* a buyer obtains from a consumption path $\mathbf{x}$, or $\nu(v, \mathbf{x}) = v \cdot \chi(\mathbf{x})$. Then we can formulate a buyers utility U compactly as

$$U(v, \mathbf{x}) = \nu(v, \mathbf{x}) - \rho(\mathbf{x}),$$

and the sellers profit Π as

$$\Pi = \sum_{\mathbf{x}_k \in \mathbf{X}} \rho(\mathbf{x}_k) \mathcal{F}\left(v \in V;\ \mathbf{x}_k\right),$$

where $\mathcal{F}(v \in V;\ \mathbf{x}_k)$ denotes the measure of types on path $\mathbf{x}_k$ and $\mathbf{X}$ denotes the set of admissible consumption paths that is defined by the set of admissible transitions Γ, or

$$\mathbf{X}^t \equiv \{\mathbf{x}^t : (x^\tau, x^{\tau+1}) \in \Gamma\ \forall\ \tau \geq t-1 \text{ with } x^{-1} \equiv \bar{x}\}.$$

The seller's profit in the static game in turn is given by

$$\pi(p) = \sum_{(\bar{x}, x) \in \Gamma} (x \cdot p) \mathcal{F}\left(v \in V | (\bar{x}, x) = \arg \max_{(\bar{x}, x) \in \Gamma} \{x \cdot (v - p)\}\right),$$

and we denote by $p^m = (p_a^m, p_b^m)$ the prices that maximize the sellers profit in the static game and satisfy $p^m \in \arg\max_p \pi(p)$.

We will say that there exists a *trading-up opportunity* for the seller if there are buyers who can transition to a strictly higher-valued state:

Definition 1 (trading-up) *There is a trading-up opportunity for the seller if there exist buyers for whom transitions to a strictly higher-valued state are admissible,*

$$\exists v \text{ in state } x \text{ with } f(v) > 0 \text{ and } x, x' \in X \text{ s.t. } (x, x') \in \Gamma \text{ and } v \cdot x' > v \cdot x.$$

For instance, in the setting with two absorbing varieties (durables) and initial state $\bar{x} = o$, there is a trading-up opportunity for the seller if there are non-buyers with strictly positive value for one of the varieties. The same is true in a setting with two non-absorbing varieties (rentals), but here there are also trading-up opportunities when there are buyers that buy their less-preferred variety.

Let Ω denote the set of price profiles $p = (p_a, p_b)$ that leave no trading-up opportunities for the seller in the static game,

$$\Omega = \{p \text{ s.t. } x = \arg\max_{x \in X}\{(v - p) \cdot x\} \implies v \cdot x > v \cdot x' \text{ or } (x, x') \notin \Gamma \ \forall v \in V\},$$

where

$$(\bar{x}, x), (\bar{x}, x') \in \Gamma,$$

and $\bar{p}$ a price profile that maximizes static profit while leaving no trading-up opportunities,

$$\bar{p} \in \arg\max \pi(p) \text{ s.t. } \bar{p} \in \Omega.$$

A period-t seller history, h^t, records all prices offered and consumption choices made along past consumption choices $\mathbf{x}^{-t}$, with $h^0 = \emptyset$. A period-t buyer history, $\hat{h}^t$, consists of the seller history h^t and the period-t price profile offered to consumers with seller history h^t. The set of period-t seller histories is denoted by H^t, and the set of seller histories by $H = \cup_{t=0}^{T} H^T$. Similarly, the set of period-t buyer histories is denoted by $\hat{H}^t$, and the set of buyer histories by $\hat{H} = \cup_{t=0}^{T} \hat{H}^T$. Let $V(h^t) \subset V$ denote the subset of consumers with the same seller history h^t. The period-t price profile shown to a consumer with history h^t is denoted by $p^t(h^t) = (p_a^t(h^t), p_b^t(h^t))$.

A *Perfect Bayesian Equilibrium* (PBE) is a history-contingent sequence of the seller's chosen price profiles p^t for each history h^t, consumption choices x^t made by consumers, and updated beliefs about the buyers' values along the various consumption paths, such that actions are optimal given beliefs, and beliefs are derived from actions from Bayes' rule whenever possible.

3 Dynamic Pricing as Trading Up Buyers

3.1 Preliminaries

We first show that in equilibrium the seller's beliefs about the value profiles of buyers satisfy a form of top-down skimming.

Lemma 1 (skimming) *Consider buyers with common history $h^t \in H^t$.*

(i) If a buyer with value profile v prefers path $\mathbf{x}_k^t$ to path $\mathbf{x}_l^t$, $\chi(\mathbf{x}_k^t) \neq \chi(\mathbf{x}_l^t)$, then so does a buyer with value profile $\tilde{v} \neq v$ such that

$$(\tilde{v} - v) \cdot (\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t)) \geq 0. \tag{1}$$

(ii) In any PBE, if a buyer with value profile v prefers consumption choice $x^t = x$ to $x^t = x'$, $x' \neq x$, then so does a buyer with value profile $\tilde{v} \neq v$ such that

$$\begin{aligned}(\tilde{v}-v)\cdot\left(x-x'\right) &+\delta\left[\min_{\mathbf{x}^{t+1}\in\mathbf{X}^{t+1}|x^t=x}\left\{(\tilde{v}-v)\cdot\chi(\mathbf{x}^{t+1})\right\}\right]\\ &-\delta\left[\max_{\mathbf{x}^{t+1}\in\mathbf{X}^{t+1}|x^t=x'}\left\{(\tilde{v}-v)\cdot\chi(\mathbf{x}^{t+1})\right\}\right]\geq 0. \qquad (2)\end{aligned}$$

where $\mathbf{X}^{t+1}|x^t$ denotes the set of admissible consumption paths after consumption choice x^t.

Proof. Since type v prefers path $\mathbf{x}_k^t$ to path $\mathbf{x}_l^t$ by assumption, we must have

$$\nu(v,\mathbf{x}_k^t)-\rho(\mathbf{x}_k^t)\geq\nu(v,\mathbf{x}_l^t)-\rho(\mathbf{x}_l^t).$$

Now, consider some type $\tilde{v}\neq v$. Then, we have

$$\begin{aligned}\nu(\tilde{v},\mathbf{x}_k^t)-\rho(\mathbf{x}_k^t)&=\nu(v,\mathbf{x}_k^t)-\rho(\mathbf{x}_k^t)+\nu(\tilde{v},\mathbf{x}_k^t)-\nu(v,\mathbf{x}_k^t)\\&\geq\nu(v,\mathbf{x}_l^t)-\rho(\mathbf{x}_l^t)+\nu(\tilde{v},\mathbf{x}_k^t)-\nu(v,\mathbf{x}_k^t),\end{aligned}$$

since type v prefers path $\mathbf{x}_k^t$ to path $\mathbf{x}_l^t$ by assumption. For type $\tilde{v}$ to prefer path $\mathbf{x}_k^t$, we must thus have

$$\nu(v,\mathbf{x}_l^t)-\rho(\mathbf{x}_l^t)+\nu(\tilde{v},\mathbf{x}_k^t)-\nu(v,\mathbf{x}_k^t)\geq\nu(\tilde{v},\mathbf{x}_l^t)-\rho(\mathbf{x}_l^t),$$

which can be rearranged to yield the result in (1). To show (2), we follow a similar line of argument. Denoting the continuation valuation of a type v following choice x by $U(v,x)$ (suppressing h^t for brevity) and considering the choices x and x', respectively, we can write

$$\begin{aligned}(\tilde{v}-p)\cdot x+\delta U(\tilde{v},x)&=(v-p)\cdot x+\delta U(v,x)+(\tilde{v}-v)\cdot x+\delta[U(\tilde{v},x)-U(v,x)]\\&\geq(v-p)\cdot x'+\delta U(v,x')+(\tilde{v}-v)\cdot x+\delta[U(\tilde{v},x)-U(v,x)]\\&\geq(\tilde{v}-p)\cdot x'+\delta U(\tilde{v},x')\end{aligned}$$

or

$$(\tilde{v}-v)\cdot(x-x')+\delta[U(\tilde{v},x)-U(v,x)]-\delta[U(\tilde{v},x')-U(v,x')]\geq 0. \qquad (3)$$

Noting that each type can always mimic the actions of the other type by making the same consumption choices in every future period, we must have

$$\min_{\mathbf{x}^{t+1}\in\mathbf{X}^{t+1}|x^t}\left\{\chi(\mathbf{x}^{t+1})\cdot(\tilde{v}-v)\right\}\leq U(\tilde{v},x^t)-U(v,x^t)\leq\max_{\mathbf{x}^{t+1}\in\mathbf{X}^{t+1}|x^t}\left\{\chi(\mathbf{x}^{t+1})\cdot(\tilde{v}-v)\right\}.$$

Substituting and reorganizing yields (2). ■

Part (i) shows that for a buyer with value profile $\tilde{v}$ to prefer the same consumption path as a buyer with value profile v, the relative values of the two buyers and the relative total consumption along the two paths must be aligned. For example, if choosing path $\mathbf{x}_k^t$ instead of $\mathbf{x}_l^t$ implies obtaining relatively less consumption of a and relatively more of b and type v is willing to make this trade, then only types $\tilde{v}$ who do not prefer a relatively more than b compared to type v will make the same choice. This is illustrated in Figure 9, where the solid square shows the possible values of $(\tilde{v}-v)$ and the dashed lines indicate the possible values of $\left(\chi(\mathbf{x}_k^t)-\chi(\mathbf{x}_l^t)\right)$. For the difference in total consumption $\left(\chi(\mathbf{x}_k^t)-\chi(\mathbf{x}_l^t)\right)$ depicted in the figure, only consumers with value profiles in the shaded area satisfy the skimming property in (1). Part (ii) of Lemma 1 states the skimming result in terms of the consumption values resulting from current choices and the admissible consumption paths following these choices.

Lemma 1 nests well-known earlier skimming results as special cases. To see this, consider the case of a single durable good and the set of types that have not purchased yet at time t. We let x be the

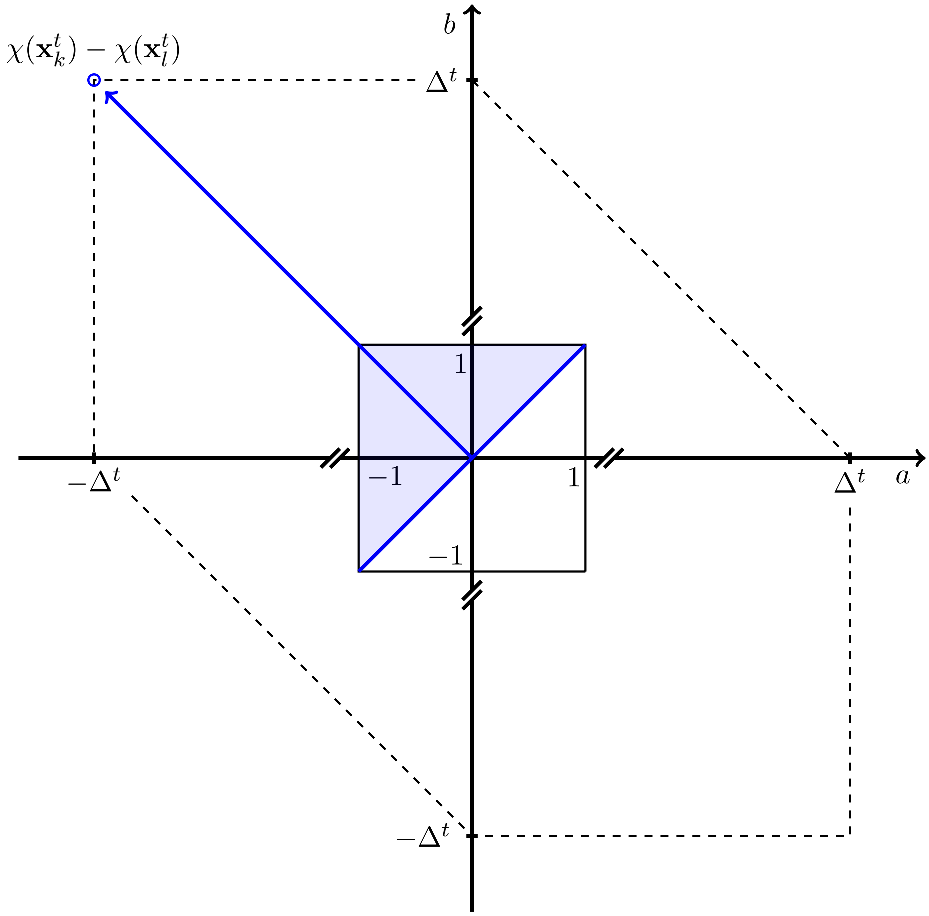


Figure 9: Illustration of the skimming condition

purchase of the durable good, say a, whereas x' is the choice of the outside option. Since the minimum difference in total value after x and the maximum difference in total value after x' is the same and given by $(\tilde{v}_a - v_a)\Delta^{t+1}$, the skimming condition (2) simplifies to the standard condition $\tilde{v}_a \geq v_a$.[3] A similar result holds in the case of a single "anti-durable" good as in Tirole (2016). Here we consider the set of types that have chosen to purchase in every previous period until time t. If x denotes the purchase of the good a and x' is the outside option, the minimum difference in total value after x is again $(\tilde{v}_a - v_a)\Delta^{t+1}$, while the largest value following x' is 0 because the outside option is an absorbing state. The skimming condition (2) then simplifies to $\tilde{v}_a \geq v_a$.[4]

Finally, in the case of two durable varieties (Nava and Schiraldi, 2019) we again consider the types that have not yet purchased and let x be the purchase of one of the two varieties, say a. Then x' is either b or o. The analysis above for the case of one durable variety shows that when $x' = o$, skimming is satisfied whenever $\tilde{v}_a \geq v_a$ if the maximum difference in total value is $\Delta^{t+1}(\tilde{v}_a - v_a)$. If instead it is $\Delta^{t+1}(\tilde{v}_b - v_b)$, we find skimming is satisfied whenever $(\tilde{v}_a - v_a)\Delta^t \geq \delta\Delta^{t+1}(\tilde{v}_b - v_b)$. When $x' = b$ in turn, the minimum difference in total value after x is $(\tilde{v}_a - v_a)\Delta^{t+1}$ while the maximum difference in total value after x' is $(\tilde{v}_b - v_b)\Delta^{t+1}$, since both varieties are absorbing states, and so (2) becomes $\tilde{v}_a - v_a \geq \tilde{v}_b - v_b$. In conjunction, we obtain that skimming is satisfied if $\tilde{v}_a - v_a \geq \max\{0, \tilde{v}_b - v_b\}$.[5] In

[3]We can obtain the same result from (1) by noting that purchasing today (path $\mathbf{x}_k^t$) rather than delaying (path $\mathbf{x}_l^t$) immediately implies that $\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t) \geq 0$.

[4]Again, we can obtain the result from (1) by noting that $\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t) \geq 0$ because the outside option is an absorbing state (positive selection).

[5]We can obtain the same result from (1) by noting that for the comparison of two paths that feature different varieties in

general, as the purchase decisions may differ at every time t, for skimming to be satisfied the difference in total consumption must be considered. Therefore, the skimming conditions for any two paths in (1) and for a period-t purchase in (2) must both account for the admissible consumption paths and resulting total consumption in the future.

Figure 10 illustrates the set of admissible consumption paths $\mathbf{X}$ and the corresponding streams of discounted payoffs for the case of two rentals and two periods (i.e., $T = 1$). For example, the lowest branch in Figure 10 depicts the "always-b-path" $\mathbf{x}_b = (b, b)$ with total consumption $\chi(\mathbf{x}_b) = (0, 1 + \delta)$, total value $\nu(\mathbf{x}_b) = (1 + \delta)v_b$ and total payment $\rho(\mathbf{x}_b) = p_b^0 + \delta p_b^1((p_a^0, p_b^0, b)$.

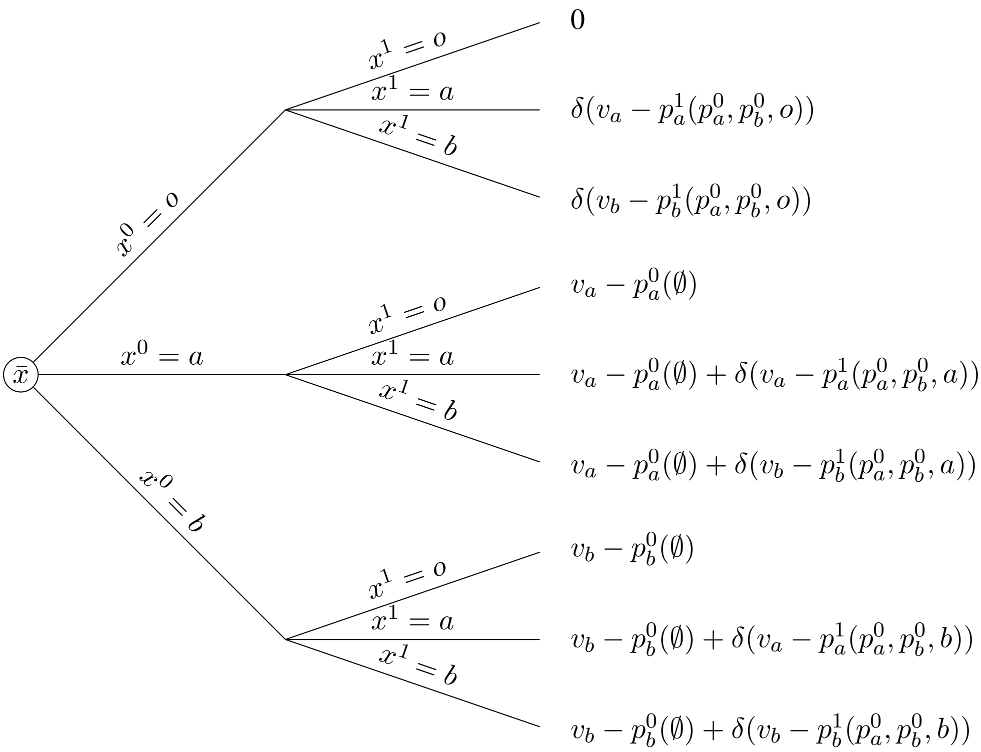


Figure 10: Consumption paths and utilities for two rentals and two periods

3.2 Optimal Pricing Without Trading-Up

We first study the implications of an absence of trading-up opportunities on the dynamics in equilibrium. If the seller varies prices over time or histories (rather than keeping them fixed), we say that she engages in *dynamic pricing*. In our first main result, we show that for histories with no trading-up opportunities for the seller, in any PBE the seller will not engage in dynamic pricing but instead repeat the static optimal price of this history in all future periods, if the static optimal price is equal to the lower bound of the set.

Proposition 1 *Consider a history $h^t \in H^t$ with associated state $x^{t-1} = i$, where $i \in \{a, b\}$, at which there exist no trading-up opportunities, so that $v_i \geq v_j \ \forall \ v \in V(h^t)$. Then in any PBE, if $p_i^*(h^t) = \min\{v_i\} \in V(h^t)$ where $p^*(h^t) \in \arg\max_p \pi(h^t)$, the seller optimally plays in every period $\tau \in t, t+1, ..., T$,*

(i) $p_i^\tau(h^\tau) = p_i^*(h^t)$, and

period t, we have $(\tilde{v} - v) \cdot (\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t)) \geq 0$ iff $\tilde{v}_a - \tilde{v}_b \geq v_a - v_b$.

(ii) $p_j^\tau(h^\tau) \geq p_j^*(h^t)$ *if* $(x^{t-1}, j) \in \Gamma$,

all buyers behave as if they were myopic, and no dynamics occur on the equilibrium path starting at history h^t.

Proof. We proceed in two steps. We first prove that in the absence of trading-up opportunities the highest profit the seller can obtain is the repeated static optimal profit. We then show that if the static optimal price is not above the lowest value in the set, then all types always accept at this price and thus the seller obtains the maximum profit by repeatedly playing the static optimal prices. Finally we also show that any other prices result in a strictly smaller profit for the seller, thus this is the only PBE.

To begin, we prove an intermediate Lemma that allows us to conveniently rewrite the seller's profit. Denote the set of consumers at history h^t who are indifferent between two distinct consumption paths $\mathbf{x}_k^t$ and $\mathbf{x}_l^t$ by

$$V_{k,l}(h^t) \equiv \{v : U(v, \mathbf{x}_k^t, h^t) = U(v, \mathbf{x}_l^t, h^t)\},$$

and the difference in the sums of discounted values obtained by indifferent consumers with value profile $v \in V_{k,l}(h^t)$ along consumption paths $\mathbf{x}_k^t$ and $\mathbf{x}_l^t$, respectively, by

$$\Delta\nu_{k,l}^t \equiv \nu(v, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_l^t) = \rho(\mathbf{x}_k^t, h^t) - \rho(\mathbf{x}_l^t, h^t).$$

Then, the following result holds.

Lemma 2 *Consider a history $h^t \in H^t$ with associated support $V(h^t) \subseteq V$. The seller's present discounted profit at history h^t can be written as*

$$\Pi^t(h^t) = \rho(\mathbf{x}_0^t, h^t)\mathcal{F}\big[v \in V(h^t)\big] + \sum_{k=1}^{K} \Delta\nu_{k,k-1}^t \mathcal{F}\left[v \in V(h^t); \cup_{j \geq k} \mathbf{x}_j^t\right],$$

where $\mathbf{X}^t = \{\mathbf{x}_0^t, ..., \mathbf{x}_K^t\}$ is ordered by the respective payments such that $\rho(\mathbf{x}_0^t, h^t) \leq \rho(\mathbf{x}_1^t, h^t) \leq ... \leq \rho(\mathbf{x}_K^t, h^t)$.

Proof. Let $V_{1,0}(h^t)$ denote the set of value profiles of consumers with common history h^t who are indifferent between $\mathbf{x}_1^t$ and $\mathbf{x}_0^t$. Then for any $v \in V_{1,0}(h^t)$

$$\nu(v, \mathbf{x}_1^t) - \rho(\mathbf{x}_1^t, h^t) = \nu(v, \mathbf{x}_0^t) - \rho(\mathbf{x}_0^t, h^t)$$
or
$$\rho(\mathbf{x}_1^t, h^t) = \rho(\mathbf{x}_0^t, h^t) + \Delta\nu_{1,0}^t$$

by construction. Next, let $V_{2,1}(h^t)$ denote the set of value profiles of consumers with common history h^t who are indifferent between $\mathbf{x}_2^t$ and $\mathbf{x}_1^t$. Then for all $v \in V_{2,1}(h^t)$ we have $\rho(\mathbf{x}_2^t, h^t) = \rho(\mathbf{x}_1^t, h^t) + \Delta\nu_{2,1}^t$, and thus

$$\rho(\mathbf{x}_2^t, h^t) = \rho(\mathbf{x}_0^t, h^t) + \Delta\nu_{1,0}^t + \Delta\nu_{2,1}^t.$$

Iterating this procedure for all consumption paths up to $\mathbf{x}_K^t$ yields

$$\rho(\mathbf{x}_k^t, h^t) = \rho(\mathbf{x}_0^t, h^t) + \sum_{k=1}^{K} \Delta\nu_{k,k-1}^t.$$

The present discounted profit at history h^t is thus given by

$$\Pi^t(h^t) = \rho(\mathbf{x}_0^t, h^t)\mathcal{F}\big[v \in V(h^t)\big] + \sum_{k=1}^{K} \Delta\nu_{k,k-1}^t \mathcal{F}\left[v \in V(h^t); \cup_{j \geq k} \mathbf{x}_k^t\right].$$

■

Now consider a history h^t that satisfies the conditions in the stated result. Then applying Lemma 2, we immediately find that the profit of the seller at every history h^τ is maximized at the optimal $\rho(\mathbf{x}_0^\tau, h^t)$, as all $v \in V(h^\tau)$ satisfy $v_i \geq v_j$ and $(x^{\tau-1}, i) \in \Gamma$ by assumption and thus we cannot find any $\Delta\nu_{k,k-1}^\tau > 0$. Hence, if $\min\{v_i\} \in V(h^\tau) = p_i^*(h^t)$, the optimal $\rho(\mathbf{x}_0^\tau, h^\tau)$ satisfies $\rho(\mathbf{x}_0^\tau, h^\tau) = \Delta^\tau p_i^*(h^t)$ for every $\tau \geq t$ and hence the profit of the seller is maximized at the repeated static optimal profit,

$$\max\{\Pi^\tau(h^\tau)\} = \Delta^\tau \pi(p_i^*(h^t), p_j^*(h^t)).$$

Now consider buyers' incentives to deviate from playing $x^t = i$. Suppose a subset of types deviates by playing j, or o, and denote the resulting history by $\hat{h}^\tau$. As the seller will never leave rent to the lowest type, we immediately find that the most any type $v \in V(\hat{h}^\tau)$ can obtain from τ onward is $\Delta^\tau(v_i - \min\{v_i\} \in V(\hat{h}^\tau))$, where by definition we have $\min\{v_i\} \in V(\hat{h}^\tau) \geq \min\{v_i\} \in V(h^t)$. But by not deviating they can obtain $\Delta^t(v_i - p_i^*(h^t))$, which since $\delta < 1$ must be greater for all types $v \in V(h^t)$. Thus, all types will accept at $p_i^*(h^t)$ at history h^t. In addition, it is easy to see that the same result obtains if the seller plays some $p_j^\tau(h^\tau) > p_j^*(h^t)$ if $(x^{t-1}, j) \in \Gamma$, and with any $p_j^\tau(h^\tau) \in \mathbb{Q}$ if $(x^{t-1}, j) \notin \Gamma$.

Finally, to prove that this is the only PBE, consider that since the profit maximum is the repeated static optimal profit, any sequence of prices that results in all types accepting at the price for variety i in every period such that the sum of prices is equal to $\Delta^t p_i^*(h^t)$ is a PBE. If the seller plays a price $p_i^\tau > p_i^*(h^t)$, then some types $v_i < p_i^\tau$ will make an instantaneous loss if they accept. As the seller will leave no rent to the lowest type in any future period at any history, this loss can never be recouped. Thus, some types will not play $x^t = i$ which must result in a lower profit by Lemma 2. Similarly, playing any price $p_i^\tau < p_i^*(h^t)$ must be strictly sub-optimal, as the seller will obtain a smaller profit at time τ that can only be recouped by playing a $p_i^\tau > p_i^*(h^t)$ in a future period, at which point some types will not accept. Thus, playing constant prices at $p_i^*(h^t)$ is the only PBE. ■

Proposition 1 shows that no dynamic pricing will occur at any history in the game at which there are no trading-up opportunities and the static optimal price for the preferred variety is at the lower bound of the set of types. In such a case, the seller optimally plays the same static optimal price every period until the end of the game and no dynamic pricing takes place along the equilibrium path starting at history h^t. In fact, the sole PBE is the one in which the seller repeatedly plays the static optimal prices, but there may be multiple price profiles that maximize static profit.

This result demonstrates the importance of the existence of trading-up opportunities for any dynamic pricing to occur in equilibrium in our setting. It leads us directly to our second main result in which we consider the subset of possible settings in which no trading-up opportunities exist at static monopoly prices, that is, the case when p^m coincides with $\bar{p}$. We show that, in this case, the seller's maximum profit attainable in the dynamic game is the repeated monopoly profit resulting from static monopoly pricing, and the seller will obtain this payoff irrespective of commitment ability. That is, the seller faces no commitment problem if there are no trading-up opportunities at static monopoly prices and this equilibrium is *outcome-unique*, i.e. the seller will obtain this profit in any PBE.

Borrowing terminology from Board and Pycia (2014), we say that the seller and buyers adopt *monopoly strategies* if, in every period t,

(i) the seller plays p^m for every history h^t, and

(ii) all consumers behave as if they were maximizing per-period utility.

Then we can state the following result.

Proposition 2 *Suppose there are no trading-up opportunities at static monopoly prices, that is, $p^m = \bar{p}$. Then,* *(i) the seller can do no better than obtain the repeated monopoly profit $\pi(p^m)$ in every period $t = 0, ..., T$ and refrain from dynamic pricing, $\Pi \leq \pi(p^m) \cdot \Delta$.*

(ii) there exists a PBE in which the seller and buyers adopt monopoly strategies in every period $t = 0, ..., T$ for any history h^t.

(iii) this equilibrium is outcome-unique and thus the seller will always obtain the commitment profit irrespective of commitment ability.

Proof. We prove the result by applying Proposition 1. If there are no trading-up opportunities at static optimal prices, then if the seller plays p^m at time $t = 0$ for any resulting history h^t at time $t = 1$ we have that the conditions in Proposition 1 are satisfied and we immediately find that in any PBE the seller will continually play the static optimal prices for all histories, since this results in cutoffs at p^m in the first period. As the optimal profit for the seller at time $t = 0$ is the repeated static monopoly profit by Lemma 2, the seller obtains her maximum profit and thus this is a PBE. It is then easy to show by the same argument as in the proof of Proposition 1 that playing any other prices at time $t = 0$ will cause (some) types to not play their static optimal choice or result in the present discounted sum of prices to not be equal to $\Delta \cdot p^m$, either of which by Lemma 2 is strictly profit-decreasing for the seller. Hence, this is the only equilibrium. ■

Proposition 2 shows that the emergence of dynamic pricing crucially depends on the existence of trading-up opportunities in the static optimum. A profit-maximizing seller engages in dynamic pricing only if doing so allows her to trade up buyers to more valuable consumption options. Therefore, if profit-maximizing prices in the static game leave no trading-up opportunities, $p^m = \bar{p}$, the seller can simply repeat the static monopoly prices and obtain the commitment profit irrespective of commitment ability, since the monopoly strategies of the seller and buyers form a PBE. Moreover, the seller is guaranteed to obtain the maximum profit despite being unable to commit as the described PBE is outcome-unique.

This result implies that in settings with $p^m = \bar{p}$ it is sufficient to know the solution of the static game to determine the outcome of the repeated game. The following lemma provides an explicit characterization of such settings. Intuitively, there are two classes of settings in which $p^m = \bar{p}$: first, settings that exclude trading-up opportunities for arbitrary price profiles (including static monopoly prices); second, settings where the distribution of consumption values is such that profit-maximizing static prices happen to leave no trading-up opportunities.

Lemma 3 *The dynamic setting is characterized by the tuple $(\mathcal{F}, \bar{x}, \Gamma)$. There are no trading-up opportunities for arbitrary price profiles p in the static game if*

(i) the initial state $\bar{x}$ is absorbing.

(ii) the initial state $\bar{x}$ is the (weakly) most-preferred state for all buyers, and all other accessible states are absorbing (positive selection).

(iii) the initial state $\bar{x}$ is the (weakly) least-preferred of the accessible states for all buyers, all accessible states are absorbing, and the lowest-value consumer obtains a strictly positive utility in at least one of the accessible states (Board and Pycia (2014)) .

(iv) all buyers have the same preference ranking over all accessible states and only transitions from a preferred to a (weakly) less-preferred state are admissible (trading down).

Otherwise, there exist trading-up opportunities at static monopoly prices, except if $\mathcal{F}$ is such that $p^m = \bar{p}$.

Proof. Follows directly from substituting the conditions of the respective case into the static profit function and Ω. ■

Lemma 3 identifies dynamic settings in which Proposition 2 applies. Case (i) is a trivial setting in which no transition out of the initial state is admissible. Case (ii) describes a setting of positive selection where the initial state is the most-preferred state for all consumers who can transition to less-preferred absorbing states only. (Tirole, 2016) provides an in-depth analysis of such a setting with a single non-absorbing variety as the initial state and the outside option as an absorbing state. Lemma 3 shows that we can extend this setting to allow for a second variety while ensuring that $p^m = \bar{p}$ continues to apply, by requiring that the second variety is less-preferred and absorbing. This arguably is the essence of positive selection: all buyers start in the most-preferred state and can only transition to less-preferred ones, that is, can only trade down but never up. With a single variety this is ensured if the outside option is absorbing. Case (iii) describes a setting in which the initial state is the least-preferred state for all consumers who can transition to more-preferred absorbing states. Board and Pycia (2014) provide a detailed analysis of such a setting where the initial state is the non-absorbing outside option, and the seller offers a single absorbing variety, but consumers can also choose a second absorbing outside option with a strictly positive utility for all buyers. Case (iv) describes settings in which the initial state is allowed to be any of the three states.

In any other setting, it is not the case that arbitrary price profiles in the static game leave no trading-up opportunities. For example, consider a setting with a 'mix' of varieties, where one variety is absorbing and one is non-absorbing. We will assume that the outside option is non-absorbing and the initial state. We can see that for a price profile in the static setting to not leave any trading-up opportunities, we must have that the market clears and that all types allocating themselves to the non-absorbing variety prefer it to the absorbing one, i.e. that the allocation is efficient. Thus, we need to check if prices p^m happen to satisfy these conditions, which will depend on the measure $\mathcal{F}$, and we cannot rule out the existence of trading-up opportunities a priori.

3.3 Trading Up and Dynamic Pricing

In most settings that our framework allows for, the price profile p^m will leave trading-up opportunities. In our next result, we analyze the pricing dynamics when $p^m \neq \bar{p}$ and the seller cannot commit to prices ex-ante. We show that in the absence of seller commitment the existence of trading-up opportunities is the driving force behind dynamic pricing. A profit-maximizing seller will engage in dynamic pricing for each consumer segment in which trading-up opportunities exist by continually lowering prices until all trading-up opportunities are exhausted. This dynamic ends at prices $\bar{p}$ and is played out in finite time, if the static profit from prices $\bar{p}$ is strictly positive.

Proposition 3 *In any PBE,*

(i) for any history h^t at which there exist trading-up opportunities, the seller trades up a strictly positive measure of types along the equilibrium path.

(ii) the seller will never set a price for a variety i below $\bar{p}_i$ at any history h^t at which the transition to state i is admissible.

(iii) the sellers present discounted profit satisfies $\Pi \geq \pi(\bar{p}) \cdot \Delta$.

(iv) if $\pi(\bar{p}) > 0$, all trading-up opportunities are exhausted in finite time.

Proof. We prove the four statements in turn.

(i) Fix a PBE. Consider a history h^t on the equilibrium path and denote the state consumers are in by x^{t-1}. Suppose that there exist trading-up opportunities, so that there exists a consumption option $x \in X$ for which some types $v \in V(h^t)$ satisfy $vx > vx^{t-1}$ and $(x^{t-1}, x) \in \Gamma$. Denote the set of types that satisfy these conditions by $V^{TU}(h^t) \subseteq V(h^t)$. By definition, we must have $x \equiv i \in \{a, b\}$. Let the highest value for i for types $v \in V(h^t)$ be $\bar{v}_i$, the lowest value $\underline{v}_i$, and analogously for types $v \in V^{TU}(h^t)$ we have $\bar{v}_i^{TU}$ and $\underline{v}_i^{TU}$. We similarly define $\bar{v}_j, \bar{v}_j^{TU}, \underline{v}_j, \underline{v}_j^{TU}$ for $j \in \{a, b\}, j \neq i$.

Further denote the measure of types traded up, if the seller decides to trade up some types by $\mathcal{F}(v \in V(h^t)|TU)$ and the remaining measure of types not traded up by $\mathcal{F}(v \in V(h^t)|NTU)$. By definition, $\mathcal{F}(v \in V(h^t)) = \mathcal{F}(v \in V(h^t)|TU) + \mathcal{F}(v \in V(h^t)|NTU)$. There are four cases to distinguish.

Consider the trivial case first, in which $x^{t-1} = o$ or $x^{t-1} = j$ and $(x^{t-1}, o) \notin \Gamma$. If $(x^{t-1}, o) \notin \Gamma$, $p_j^t(h^t)$ is set to zero by assumption and the definition of trading up opportunities immediately implies that the seller can offer a strictly positive price $p_i^t(h^t)$ at which (some) types $v \in V^{TU}(h^t)$ purchase variety i, resulting in a positive profit. Similarly, if $x^{t-1} = o$, then inducing any types $v \in V^{TU}(h^t)$ to choose $x^t = i$ constitutes trading up and $(o, i) \in \Gamma$ by assumption at history h^t. It is straightforward to show that as the seller earns no profit from types in the outside option, inducing a positive mass to choose i (or j, or both) must be profit-increasing, as the definition of trading-up opportunities guarantees that (some) types are willing to pay a strictly positive price for i.

In the remaining three cases we have $x^{t-1} = j$ and $(x^{t-1}, o) \in \Gamma$. In the first one, we assume that $\bar{v}_i^{TU} > \bar{v}_j$. Then the equilibrium profit for the seller if she decides not to trade up any buyers, $\hat{\Pi}(h^t)$, satisfies

$$\hat{\Pi}(h^t) < \bar{v}_j \Delta^t \mathcal{F}(v \in V(h^t)) \tag{4}$$

as the seller cannot extract the full surplus of types with a linear price. However, if the seller trades up (some) types $v \in V^{TU}(h^t)$, then the equilibrium profit obtained from trading up, $\Pi^*(h^t)$, satisfies

$$\Pi^*(h^t) \geq v_i^* \Delta^t \mathcal{F}(v \in V(h^t)|TU) \tag{5}$$

where v_i^* denotes the lowest value for v_i for the cutoff types indifferent to trading up to i, as the seller can always obtain at least the value of the lowest type in the set, while the equilibrium profit obtained from types not traded up, $\Pi^\circ(h^t)$, satisfies

$$\Pi^\circ(h^t) < \bar{v}_j \Delta^t \mathcal{F}(v \in V(h^t)|NTU), \tag{6}$$

because as before the seller cannot extract the full surplus using a linear price. As $\bar{v}_i^{TU} > \bar{v}_j$ by assumption, there exists a v_i^* that satisfies $\bar{v}_i^{TU} > v_i^* > \bar{v}_j$. This immediately implies by (4), (5), (6), and $\mathcal{F}(v \in V(h^t)) = \mathcal{F}(v \in V(h^t)|TU) + \mathcal{F}(v \in V(h^t)|NTU)$ that

$$\Pi^*(h^t) + \Pi^\circ(h^t) > \hat{\Pi}(h^t).$$

In the remaining two cases, we assume that $\bar{v}_i^{TU} < \bar{v}_j$. In the third case, we assume that the outside option is non-absorbing. Suppose the seller would not trade up any types along the equilibrium path. For any types that play $x^t = o$ we have that at time $t+1$ case 1 applies and thus trading up occurs. Moreover, suppose types $v \in V^{TU}(h^t)$ play $x^t = o$. Since all types will only ever accept at a price at which they earn a (weakly) positive utility over the course of the game, we know that if the seller never trades up any types $v \in V^{TU}(h^t)$ to i along equilibrium play after $x^t = o$, then since case 1 applies at any history at which the state is the outside option we must have $p_j^\tau(h^\tau) \leq \bar{v}_j^{TU}$ for some $\tau > t$. But then we can find a $p_i^\tau(h^\tau) > p_j^\tau(h^\tau)$ such that $v_i^*(h^\tau) > \min\{v_j\} \in V(h^\tau)$ such that inducing some types $v \in V^{TU}(h^t)$ to play $x^\tau = i$ is strictly profit-increasing, since the equilibrium profit of the seller when trading up satisfies

$$\hat{\Pi}(h^\tau) \geq v_i^* \Delta^t \mathcal{F}(v \in V(h^\tau)|TU).$$

Now suppose instead (some) types $v \in V^{TU}(h^t)$ play $x^t = j$. Then again, if the seller does not induce any types to play $x^\tau = o$ for $\tau \in t, ..., T$, we must have $p_j^\tau(h^\tau) \leq v_j^{TU}(h^\tau)$ for (some) types $v \in V^{TU}(h^t)$ to be willing to play $x^\tau = j$, whereas if he does then the argument above applies again. Thus we find as before that trading up must be strictly profit-increasing.

In the final case, we assume that the outside option is absorbing. Suppose that (some) types $v \in V^{TU}(h^t)$ play $x^t = j$. If the seller never trades up any types to i, then by the same argument as above we know that $p_j^t(h^t) \leq \underline{v}_j^{TU}$ and thus we can find a $p_i^t(h^t) > p_j^t(h^t)$ again that ensures trading up types is strictly profitable. Suppose instead now that all types $v \in V^{TU}(h^t)$ play $x^t = o$. We know that at time $t-1$ we must have had $p_j^{t-1}(h^{t-1}) < p_i^{t-1}(h^{t-1})$ by incentive compatibility, since the indifference condition at $t-1$ is

$$v_i - p_i^{t-1}(h^{t-1}) + \delta U(v, h^{t-1}, x^{t-1} = i) = v_j - p_j^{t-1}(h^{t-1}) + 0, \tag{7}$$

for types $v \in V^{TU}(h^t)$. As all types $v \in V^{TU}(h^t)$ play $x^t = o$ we know that $p_j^t(h^t) \geq \bar{v}_j^{TU}(h^t)$ and that $p_j^t(h^t) > p_j^{t-1}(h^{t-1})$. But then, the seller could have played $p_j^t(h^t) = p_i^t(h^t)$ at time $t-1$ instead, which would satisfy $p_j^t(h^t) \geq \bar{v}_j^{TU}(h^t)$, and which must be profit-increasing given that we assume that $p_j^t(h^t) \geq \bar{v}_j^{TU}(h^t)$ is profit-maximizing. Additionally, the equilibrium profit from types playing $x^{t-1} = i$ at h^{t-1} must be increasing as well, as $p_i^{t-1}(h^{t-1})$ remains unchanged, but the measure of types playing $x^{t-1} = i$ increases. As the seller can always induce these additional types to play $x^t = o$ at time t, the continuation profit is unaffected. Thus, a history h^t satisfying the conditions stated at the beginning and resulting in the seller optimally not trading up any types cannot arise in equilibrium.

Then in conjunction statement (i) follows.

(ii) Denote by Λ the set of price profiles p that leave no trading-up opportunities for any history h^t in the dynamic game. We will show that $\Omega \setminus \Lambda = \emptyset$ and $\bar{p} \in \Lambda$. Consider prices $\bar{p} = (\bar{p}_a, \bar{p}_b)$. Note first, that if $\bar{p} \in \Omega$, then by the definition of Ω it follows that for some price profile $\tilde{p}$ we have

$$\tilde{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\}) - (\eta, \eta) \implies \tilde{p} \in \Omega, \tag{8}$$

for any $\eta \geq 0$ as all types willing to purchase at prices $\tilde{p}$ choose their most-preferred variety and all types choosing the outside option must do so when facing prices $\bar{p}$ too, and

$$\tilde{p} = \begin{cases} (\bar{p}_a, \bar{p}_b) - (0, \eta), & \text{if } \bar{p}_b > \bar{p}_a \\ (\bar{p}_a, \bar{p}_b) - (\eta, 0), & \text{if } \bar{p}_b < \bar{p}_a \end{cases} \implies \tilde{p} \in \Omega, \tag{9}$$

for any $\eta \in [0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$ as all types optimally now purchasing a different variety than when facing prices $\bar{p}$ must now choose their most-preferred variety and all types switching from the outside option when facing prices $\bar{p}$ to consumption must equally now choose their most-preferred variety. Second, observe that the price profile $p^\circ = (-\Delta^{t+1}, -\Delta^{t+1})$ is contained in Λ. To see this, recall from the proof of Lemma 1 that since all types can always mimic each others behavior (i.e. make the same choices from t onward), we have that

$$U(\tilde{v}, h^t, x^t) - U(v, h^t, x^t) \leq \Delta^{t+1} \max_{i \in \{a,b\}} \{\tilde{v}_i - v_i\}, \;\; v \neq \tilde{v},$$

where U denotes the continuation valuation following choice x^t. Therefore we know that $p^\circ \in \Lambda$ since the largest value difference in the support V is 1 and when both prices are equally low, then given that all types accept we have that all types choose their most-preferred variety. In addition, by (8) we also have that $p^\circ \in \Omega$.

Now pick a price profile $\hat{p}$ that satisfies $\hat{p} = p^\circ + (\varepsilon, \varepsilon)$ for some $\Delta^{t+1} + \min\{\bar{p}_a, \bar{p}_b\} \geq \varepsilon > 0$. By (8) we know $\hat{p} \in \Omega$. Denote by x° the choice buyers make in the static game when facing prices p°. By (8) we therefore have

$$x^\circ \cdot (v - p^\circ) \geq x' \cdot (v - p^\circ), \;\; x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V, \tag{10}$$

where we know $x^\circ \in \{a, b\}$ as $p_a^\circ = p_b^\circ < 0$ and since $p^\circ \in \Lambda$ we also have that

$$\begin{aligned} x^\circ \cdot (v - p^\circ) &+ \delta U^\circ(v, h^t) \\ &\geq x' \cdot (v - p^\circ) + \delta U'(v, h^t), \;\; x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V, \end{aligned} \tag{11}$$

where U° and U' denote the continuation valuations associated with choice x° and x' respectively, given history h^t. By the definition of p° and (11) it then follows that for $x' \in \{a, b\}$ we have

$$\delta(U' - U^\circ) \leq (x^\circ - x') \cdot v \;\; \forall\, v \in V, \tag{12}$$

and for $x' = o$ we have

$$\delta(U' - U^\circ) \leq v \cdot x^\circ \;\; \forall\, v \in V. \tag{13}$$

Then we find that for any ε, given (12), that

$$\begin{aligned} x^\circ \cdot (v - p^\circ - \varepsilon) &+ \delta U^\circ(v, h^t) \\ &\geq x' \cdot (v - p^\circ - \varepsilon) + \delta U'(v, h^t), \;\; x', x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V, \end{aligned} \tag{14}$$

and for any $\varepsilon <= \Delta^{t+1} + x^\circ \cdot v$, given (13), that

$$\begin{aligned} x^\circ \cdot (v - p^\circ - \varepsilon) &+ \delta U^\circ(v, h^t) \\ &\geq x' \cdot (v - p^\circ - \varepsilon) + \delta U'(v, h^t), \;\; x^\circ \in \{a, b\}, x' = o, \;\; \forall\, v \in V. \end{aligned} \tag{15}$$

Thus, for any $\varepsilon <= \Delta^{t+1} + \min\{\bar{p}_a, \bar{p}_b\}$ we find that only types $v < \min\{\bar{p}_a, \bar{p}_b\}$ now prefer o to x°, which continues to leave no trading-up opportunities as $\bar{p} \in \Omega$, and thus $\hat{p} \in \Lambda$, and hence for any $\tilde{p}$ that satisfies (8) we have $\tilde{p} \in \Lambda$.

Now fix the price profile $\hat{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\})$. By (9) we have $\hat{p} \in \Omega$ and as shown above we also have $\hat{p} \in \Lambda$. Consider a price profile $p' = \hat{p} + (0, \varepsilon)$ if $\bar{p}_b > \bar{p}_a$ and $p' = \hat{p} + (\varepsilon, 0)$ if $\bar{p}_b < \bar{p}_a$ where $\varepsilon \in (0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$. Then by the same logic as above, for any $\max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\} \geq \varepsilon > 0$, we find that the only types that now prefer the outside option to consumption also prefer the outside option at prices $\bar{p}$ and the only types now preferring the other variety also do so at prices $\bar{p}$. Thus, we find $p' \in \Lambda$ or equally that any $\tilde{p}$ that satisfies (9) satisfies $\tilde{p} \in \Lambda$ and therefore $\bar{p} \in \Lambda$.

Finally note that we can construct (8) and (9) for any price profile $p \in \Omega$ and thus we find that $\Omega \setminus \Lambda = \emptyset$. Then statement (ii) follows by optimality.

(iii) Follows directly from (ii) by optimality.

(iv) Fix a candidate PBE. Consider a history h^t with associated support $V(h^t)$ and state x at which there exist trading-up opportunities for some types $v \in V(h^t)$, or $\exists v \in V(h^t)$ with $v \cdot x' > v \cdot x$ where $x' \neq x$ and $(x, x') \in \Gamma$. Suppose that there exists a history h^{t+1} with support $V(h^{t+1})$ and state x, such that $V(h^{t+1}) \subset V(h^t)$ and $\exists v \in V(h^{t+1})$ with $v \cdot x' > v \cdot x$ where $x' \neq x$ and $(x, x') \in \Gamma$. That is, along equilibrium play the seller trades up some of the types in the support $V(h^t)$ for whom there exist trading-up opportunities. Denote the measure of types with history h^t that can be traded up by ω, or

$$\mathcal{F}\left(v \in V(h^t) \text{ s.t. } v \cdot x' > v \cdot x \text{ and } (x, x') \in \Gamma\right) = \omega > 0.$$

Denote the highest value for $x' = i \in \{a, b\}$ among these types as $\bar{v}_i^{TU}$, and the lowest value by $\underline{v}_i^{TU}$.

Now consider some $\varepsilon > 0$ that satisfies the following condition at the history h^t

$$\varepsilon \geq \bar{v}_i^{TU} - \underline{v}_i^{TU} > 0.$$

As shown in the proof of (i), whenever there exist trading-up opportunities for any history h^t, the seller trades up a strictly positive measure of types. By Lemma 1 we have that $\bar{v}_i^{TU} - \underline{v}_i^{TU}$ becomes smaller over time. Therefore, for any history with a support of types that contains types with trading-up opportunities, as the game continues the possible values for ε decrease. Thus the longer T, the smaller the values ε can take and satisfy the condition. We now show that for ε small enough, the seller strictly prefers to trade up all types if $\underline{v}_i^{TU} > 0$.

Let $\Pi^*(h^t)$ denote the equilibrium profit for the seller obtained from trading up only some of the types with trading-up opportunities in the support $V(h^t)$. As the seller cannot extract the full surplus of types with a linear price and at the earliest can trade up the remaining types at history h^{t+1} at time $t+1$, a $\lambda \in (0,1)$ exists such that

$$\Pi^*(h^t) < \lambda\omega\bar{v}_i^{TU}\Delta^t + \delta(1-\lambda)\omega\bar{v}_i^{TU}\Delta^{t+1}.$$

Similarly, let $\bar{\Pi}(h^t)$ denote the equilibrium profit for the seller obtained from trading up all buyers at time t instead. As the seller can always obtain at least the lowest value of the types in each period, we have that

$$\bar{\Pi}(h^t) \geq \omega\underline{v}_i^{TU}\Delta^t.$$

Then it follows that

$$\begin{aligned}
\Pi^*(h^t) - \bar{\Pi}(h^t) &< \lambda\omega\bar{v}_i^{TU}\Delta^t + \delta(1-\lambda)\omega\bar{v}_i^{TU}\Delta^{t+1} - \underline{v}_i^{TU}\Delta^t\omega \\
&= ((\Delta^t - 1 + \lambda)\bar{v}_i^{TU} - \Delta^t\underline{v}_i^{TU})\omega \\
&\leq ((\Delta^t - 1 + \lambda)(\varepsilon + \underline{v}_i^{TU}) - \Delta^t\underline{v}_i^{TU})\omega \\
&= ((\Delta^t - 1 + \lambda)\varepsilon - (1-\lambda)\underline{v}_i^{TU})\omega.
\end{aligned}$$

Therefore, $\bar{\Pi}(h^t) > \Pi^*(h^t)$ whenever

$$\varepsilon \leq \underline{v}_i^{TU}(1-\lambda)/(\Delta^t - 1 + \lambda).$$

Thus, if $\underline{v}_i^{TU} > 0$ and T is sufficiently long, there exists a period t such that all trading-up opportunities are exhausted at any $\tau \geq t$ in any PBE.

It remains to check that $\underline{v}_i^{TU} > 0$. It is straightforward to see that if $\pi(\bar{p}) > 0$, then $\Pi(\bar{p}|h^t) = \sum_{\tau=t}^{T} \delta^{\tau-1}\pi(\bar{p}|h^t) > 0$ for any history h^t at which there exist trading-up opportunities for the seller and therefore $\underline{v}_i^{TU} > 0$ for any such history h^t. Then statement (iv) follows. ■

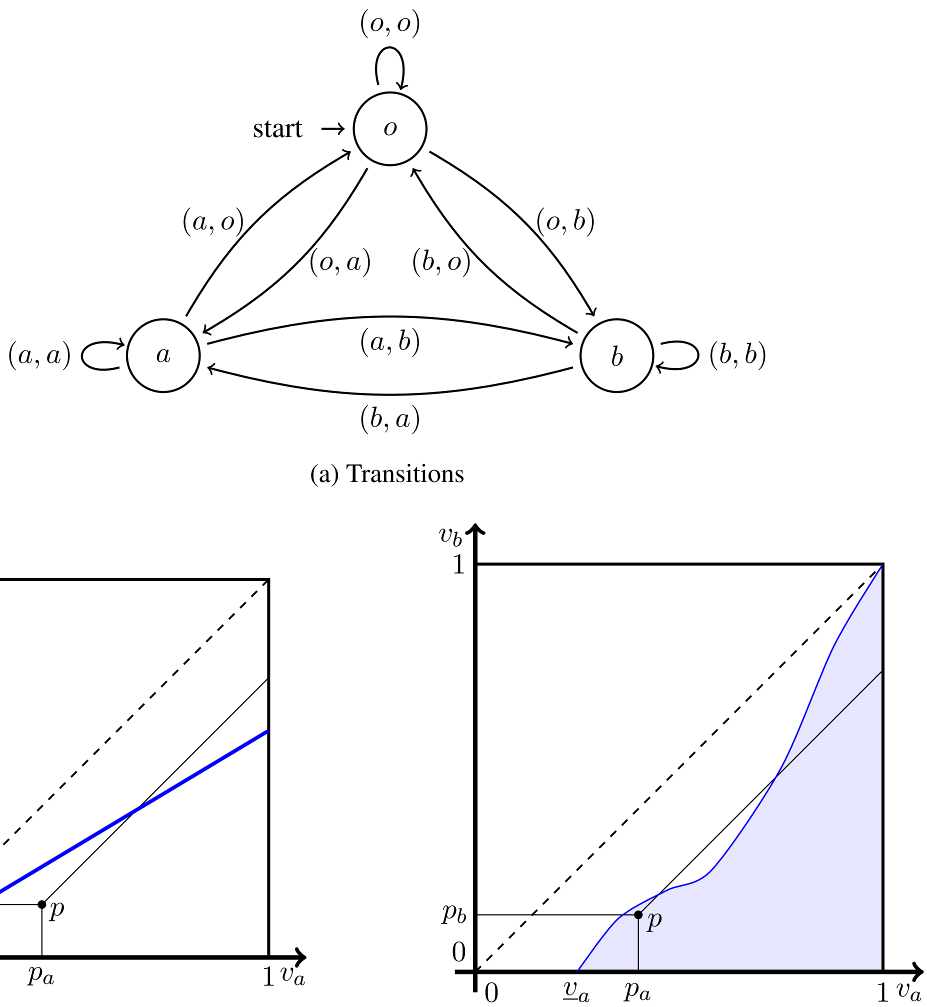
Proposition 3 demonstrates that the driving force behind dynamic pricing is trading up. Whenever the seller faces a set of buyers for whom trading-up opportunities exist, it is strictly profit-maximizing to induce (some) buyers to trade up to a higher-valued option than the currently chosen one. By doing so, the seller can extract a larger surplus from these types. However, in order to induce consumers to trade up, the seller must lower the prices relative to the prices these types were facing previously. Thus, as the game progresses, the seller will continually lower prices to trade up buyers. Since prices $\bar{p}$ also leave no trading-up opportunities when played in the dynamic game, the seller will never want to set a price below $\bar{p}$ and thus this dynamic ends at prices $\bar{p}$ as long as the transitions to consumption are admissible. This is required for dynamics to end at $\bar{p}$, because whenever they are not, any price is a best-response, including prices below $\bar{p}$. This implies that the sellers profit in the absence of commitment ability is bounded from below at $\pi(\bar{p}) \cdot \Delta$. The time it takes for this dynamic to play out depends on whether the seller can obtain positive profits from playing prices $\bar{p}$. If trading all buyers up at once yields a strictly positive profit in the static game, then the seller will eventually exhaust all trading-up opportunities in finite time provided that T is sufficiently long.

The result clarifies the connection between different strands of literature. In the canonical setting with one durable good and an outside option with value zero, trading-up opportunities exist for non-buyers only, since buyers of the durable good are captured in an absorbing state. Thus, if the seller lacks commitment ability, profit-maximizing prices are falling for non-buyers, reflecting the classic notion of 'Coasian dynamics' (Fudenberg et al., 1987; Coase, 1972). Since all buyers value the good more than the outside option, trading-up opportunities exist whenever a positive measure of non-buyers remains. The same is true in a setting with two durable goods and an outside option with value zero. Once again the seller is compelled to lower the prices of the varieties until the market clears (Nava and Schiraldi, 2019). However, it is possible to clear the market with only one of the two prices at zero, allowing the seller to still obtain a strictly positive profit. In settings of behavior-based price discrimination with one rental good and an outside option with value zero, the same logic prevails. Trading-up opportunities exist only for previous non-buyers, while loyal buyers cannot be traded up. As a result, the seller has no incentive to adjust the price for these 'positively selected' types in order to trade them up and hence prices fall only for previous non-buyers (Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Tirole, 2016).

Similarly, Proposition 3 shows that in a setting with two rental goods and a non-absorbing outside option the seller may eventually have to set both prices to zero. Consider that in the one-shot game, the profit-maximizing prices that leave no trading-up opportunities $\bar{p}$ are equal to the lowest values for the two respective varieties in the entire support V. If these are equal to zero, then $\bar{p} = (0, 0)$. Thus, the seller will trade up buyers until prices finally fall to zero. However, as $\pi(\bar{p}) = 0$ in this case, part (iii) of Proposition 3 does not apply and this dynamic may take infinite periods to play out. Proposition 3 also shows that a setting with mixed varieties can protect the seller from eventually charging prices of zero if the rental variety is considered the superior one by all buyers and thus a seller of a single good may want to introduce a second, absorbing, low-quality variety to shield herself from being forced down to selling at zero prices. Thus, Nava and Schiraldi (2019)'s insight that intra-temporal price discrimination can shield the seller from zero profits that arise from inter-temporal price discrimination extends to a setting with mixed varieties.

Proposition 2 and 3 jointly show that the outcome of repeated monopoly pricing problems depends on two price profiles from the static game: p^m and $\bar{p}$. By comparing these two price profiles, we can understand the outcome and dynamics of the repeated game. In 'non-Coasian' settings the two price profiles coincide, $p^m = \bar{p}$, and the seller does not face a commitment problem. With or without commitment ability by the seller, the profit-maximizing solution is to play constant prices at p^m. In 'Coasian' settings the two price profiles do not coincide, $p^m \neq \bar{p}$, and in the absence of commitment ability by the seller dynamic pricing emerges in the form of 'Coasian dynamics': prices keep falling for types that can be traded up. This dynamic continues as long as trading-up opportunities remain, but will end in finite time if the profit from playing prices $\bar{p}$ is strictly positive, $\pi(\bar{p}) > 0$. At the very least, the seller is guaranteed to obtain the repeated, discounted profit that prices $\bar{p}$ yield, $\pi(\bar{p})\Delta$, as she can always play prices $\bar{p}$ to exhaust all trading-up opportunities and end the dynamics. These findings can be understood as offering a simple 'checklist' for analyzing dynamic pricing problems, allowing one to focus on the simpler task of determining prices in the static game, instead of solving the more complex dynamic game.

4 Example: Vertical Differentiation



(a) Transitions

(b) Linear valuation function

(c) Generic valuation distribution

5 Conclusion

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