Dynamic Monopoly Pricing With Multiple Varieties: Trading Up Buyers*

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Abstract

This paper studies dynamic monopoly pricing in a unified analytical framework that allows for multiple durable or rental varieties, as well as other settings including positive selection. We show that the driving force behind dynamic pricing is the seller's incentive to trade up consumers to higher-valued consumption options. We derive two key results. First, if there are no trading-up opportunities at static monopoly prices, the seller can do no better than set static monopoly prices and obtain the commitment profit irrespective of commitment ability. Second, if trading-up opportunities exist for any history, the seller engages in dynamic pricing by lowering prices until reaching prices that leave no trading-up opportunities in the static game and will exhaust all trading-up opportunities in finite time if the seller's static profit is strictly positive at profit-maximizing prices that exhaust all trading-up opportunities.

JEL-Classification: D82, D42, L12

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1 Introduction

The prices of many goods and services vary across time and consumers. For instance, many retailers routinely offer temporary discounts for selected items sold through their stores. Similarly, mobile phone service providers often make introductory offers to new customers while charging higher prices to loyal ones. Airlines and retail chains, in turn, design sophisticated customer retention programs to make selected offers to specific customer groups, and an increasing number of firms makes use of repricing software to dynamically adjust the prices of their products.

The economic literature has emphasized the importance of a sellers commitment ability in explaining dynamic pricing. As is well-known, a monopolistic seller of a single, durable good will want to refrain from dynamic pricing, but faces a commitment problem: high value buyers are more likely to purchase today which results in a negative selection of remaining non-buyers and this causes the monopolist to reduce prices for these buyers if she cannot commit ('Coasian dynamics'). Forward-looking buyers exploit these falling prices resulting in a negative externality of future prices on today's profits. This limits the sellers monopoly power, and in its extreme form the sellers profit goes to zero if all trade takes place in the *twinkling of an eye*, as conjectured by Coase (1972) and proved formally by Stokey (1981), Bulow (1982), Fudenberg et al. (1985), Gul et al. (1986), and Ausubel and Deneckere (1989).

But Coasian dynamics, time inconsistency, and zero profits do not universally apply. For example, if buyers are able to exit the bargaining process at any point for a positively valued alternative, the seller no longer faces a commitment problem and no dynamic pricing arises (Board and Pycia, 2014). Similarly, if the seller offers a rental instead of a durable good and only high-value rather than low-value consumers remain in the market ('positive selection'), the profit-maximizing price is constant and pricing is time-consistent (Tirole, 2016). Yet if both high- and low-value consumers remain and thus both negative selection (for non-buyers) and positive selection (for loyal consumers) are at work, leading to "behavior-based pricing", then Coasian dynamics may arise again, but the price for positively selected buyers may no longer be constant over time (Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Acquisti and Varian, 2005). And if the seller is able to offer more than one durable variety, then time inconsistency and Coasian dynamics apply but this will no longer lead to zero profits (Nava and Schiraldi, 2019).

In this paper, we extend the existing analysis to a monopolistic seller that may offer two varieties of a good and each variety may be durable or rental. To the best of our knowledge this has not been studied before.[1] We further allow for settings with pure positive selection, and a variety of settings in which varieties are neither durable nor rental. We argue that dynamic pricing – as opposed to the repeated play of static monopoly prices – and the arising of commitment problems can be analyzed using a simple concept: the incentive to 'trade up' buyers to higher-valued consumption options. If the seller faces a set of buyers for which trading-up opportunities exist, then she has an incentive to cut the price to trade up these types. By lowering the price that these types are facing for the higher-valued consumption option, the seller can trade them up to the higher-valued consumption option and thereby tap into a larger surplus than that emerging from the currently chosen one. This is the essence of Coasian dynamics. But this logic applies not only to consumers that selected the outside option, but also to buyers that did not select their most-preferred variety and can be employed to explain pricing dynamics in a large class of settings.

Specifically, we consider a monopolist with constant marginal cost selling two varieties of a good to a continuum of buyers with unit-demand. Buyers may choose between the varieties and the outside option in each period according to a set of admissible transitions. We place almost no restrictions on this set of transitions, allowing varieties (and the outside option) to be *absorbing* (i.e. buyers must choose this option in every future period) or not, as well as numerous other combinations of transitions. Buyers' values for the varieties are private information distributed according to a measure that allows for arbitrary correlation structures, including vertical or horizontal differentiation. The seller chooses prices in each period for the two varieties, but only if the buyers facing these prices can choose the outside option in this period. That is, we require pricing to be consistent with voluntary consumption. If not, we set prices to zero and thus in our setting an absorbing variety acts like the sale of a durable variety, while a variety that can be bought or rejected in each period acts like a rental variety.

We first show that if static monopoly prices leave no trading-up opportunities, then neither dynamic pricing nor a commitment problem will arise. For example, this is the case when buyers can exit to a strictly positively valued, absorbing second 'variety' as in Board and Pycia (2014) or in settings with pure positive selection with one variety as in Tirole (2016) or with two varieties. Second, we show that if static monopoly prices do leave trading-up opportunities for the seller, then in the absence of commitment ability the seller will be forced to lower prices over time whenever trading-up opportunities exist

[1] Nava and Schiraldi (2019) study an extension of their setting in which consumers may return to the market after purchase, but focus on the case in which it is not profitable for the seller to exploit this.

until finally reaching prices that leave no trading-up opportunities in the static game. Depending on the admissible transitions and distribution of value profiles, these prices can still result in a positive profit in the static game and we prove that if that is the case, then the dynamics are played out in finite time and all trading-up opportunities are exhausted. For example, this is the case with two absorbing (i.e. durable) varieties (Nava and Schiraldi, 2019). These results show that the insight of Nava and Schiraldi (2019) that intra-temporal price discrimination can partially make up for the loss of market power due to inter-temporal price discrimination and prevent zero profits in the case of two durables extends to a setting with a 'mix' of varieties, if the durable variety is the less-preferred one by all buyers.

Our analysis demonstrates that the outcome of dynamic pricing problems hinges on two price profiles from the static setting: the monopoly profit-maximizing prices on the one hand and the prices that maximize the sellers profit conditional on leaving no trading-up opportunities on the other hand. By comparing these two price profiles we can understand the outcome of the repeated game. If the monopoly prices leave no trading-up opportunities, that is, the two price profiles coincide, then the seller does not face a commitment problem and the profit-maximizing solution is to play constant monopoly prices irrespective of commitment ability. If the two price profiles do not coincide, then a commitment problem can arise and the monopolist will repeatedly lower prices to trade up consumers over time. This dynamic will end in finite time, exhausting all trading-up opportunities, if the static profit from playing profit-maximizing prices that leave no trading-up opportunities is strictly positive. This logic and our main results extend previous work to, for instance, settings with two rental varieties, or "mixed" settings with one rental and one durable variety, but also to more unorthodox settings where a variety may be neither fully rental nor durable. For example, this would be the case if the seller offers only one variety to all buyers, but an additional, second variety exclusively to loyal buyers.

This paper contributes to an extensive literature on the sale of a single durable good (e.g Coase, 1972; Fudenberg et al., 1985; Gul et al., 1986; Sobel, 1991; Kahn, 1986; Bond and Samuelson, 1984; Fuchs and Skrzypacz, 2010), of multiple varieties of a durable good (e.g. Nava and Schiraldi, 2019; Board and Pycia, 2014), and of vertically differentiated durable products (Hahn, 2006; Inderst, 2008; Takeyama, 2002). Our work differs by considering a unified analytical framework that allows for both rental and durable varieties, as well as a class of other settings. In doing so we add to the analysis of settings with 'positive selection' (Tirole, 2016). In addition, we contribute to the literature on behavior-based pricing (e.g. Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Taylor, 2004; Buehler and Eschenbaum, 2020). In contrast to recent work in Rochet and Thanassoulis (2019), we do not allow varieties to be sold as a bundle. Our setup is best understood as offering two varieties of the same good over time.

Our work is also related to the marketing literature. Our notion of trading up is closely linked to what is commonly known as 'upselling' in marketing (e.g. Blattberg et al., 2008; Aydin and Ziya, 2008; Wilkie et al., 1998). However, upselling generally refers to inducing loyal customers to upgrade to a more expensive product, while trading up applies to both non-buyers and loyal buyers.

The remainder of the paper is organized as follows. Section 2 introduces the analytical framework. In subsection 2.1 we sketch some applications that our framework allows for. In subsection 2.2 we formalize our notion of trading up. Section 3 analyzes the dynamic pricing game. In subsection 3.1 we first establish a skimming result. In subsection 3.2 we provide our main results. Section 5 concludes and offers directions for future research.

2 Analytical Framework

Consider a monopolist that sells two varieties of a good, a and b, at zero marginal cost to a measure of buyers with unit-demand per period, so that each buyer consumes per period one of the varieties, or none. Buyers have per-period valuations v_a, v_b for the two varieties that are constant over time. Value profiles $v = (v_a, v_b)$ are private information of buyers and distributed according to a measure $\mathcal{F}$ on the unit square $[0, 1]^2$. The associated cumulative distribution is F with density f, and V is the support. F_i denotes the marginal cumulative distribution of variety i, while f_i and V_i denote the respective density and support.

Time is discrete and indexed by $t = 0, ..., T$. After T periods, the good becomes obsolete. All players share the same discount factor $\delta \in (0, 1)$. In every period t, consumers make a discrete choice $x^t \in X$, where

$$X \equiv \{(1, 0), (0, 1), (0, 0)\},$$

with $a = (1, 0)$, $b = (0, 1)$, $o = (0, 0)$, which are the three *states* of the game and where o indicates the outside option. Let $\bar{x} \in X$ be the initial state for all consumers. A sequence of discrete choices x^t from t onward or *consumption path* $\mathbf{x}^t = (x^t, x^{t+1}, ..., x^T)$

gives rise to a present discounted sum of *total consumption* $\chi(\mathbf{x}^t) = \sum_{\tau=t}^{T} \delta^{\tau-t} x^t$. A given consumption path is *admissible* if all transitions from state to state along the entire path are within the set of admissible transitions $\Gamma \subset X \times X$. We place no restrictions on Γ, except that transitions from a state to itself are always admissible, that is $(o, o), (a, a), (b, b) \in \Gamma$. A state $x \in X$ is *absorbing* if no other state $x' \in X$ is accessible from x, that is, $(x, x') \notin \Gamma$. Let $\Delta^t = \sum_{\tau=t}^{T} \delta^{\tau-t}$ denote the present discounted number of *total periods*.[2]

We allow for both varieties to be a durable or a per-period "rental" service (Hart and Tirole, 1988). The 'sale' or 'durable' model assumes that the variety is sold once for all future periods: $x^t = i$ implies $x^\tau = i$ for $\tau \in t+1, .., T$, $i \in (a, b)$. Thus, in the case of a durable variety $i \in (a, b)$, the state i is absorbing and at any time t the only path that includes consumption of variety i at t available to buyers is the path of always consuming i, with associated total consumption $\Delta^t i$. The 'rental' model instead assumes that buyers may choose to consume the variety i in each period anew so that $(i, i), (i, o), (o, i) \in \Gamma$. Note that our setup also allows for sets of admissible transitions for which a given variety is neither durable nor rental.

Using a transition diagram (Rubinstein 1986), Figure 1 illustrates the least restrictive setting with two rental varieties and a non-absorbing outside option. In this setting, all possible transitions are admissible so that $\Gamma = ((a, a), (b, b), (o, o), (a, b), (b, a), (o, a), (o, b), (a, o), (b, o))$. In the figure, the vertices indicate the states a, b, o, with initial state $\bar{x} = o$, while the arcs and brackets (x, x'), with $(x, x') \in \Gamma$, represent the admissible transitions.

2.1 Applications

Our analytical framework can be used to study a range of specific applications. We sketch some of the possible applications below, including settings that have been analyzed in the literature previously such as the canonical one durable good setting. We omit states that are inaccessible for simplicity.

One durable good: There is a single variety, say a, which is an absorbing state, while the outside option o is non-absorbing, as in e.g. Stokey (1979); Coase (1972); Gul et al. (1986); Sobel (1991). Hence, there is no transition possible from a to o. The initial state is the outside option o.

[2]Throughout, we will consistently omit the exponent t for all our expressions when $t = 0$.

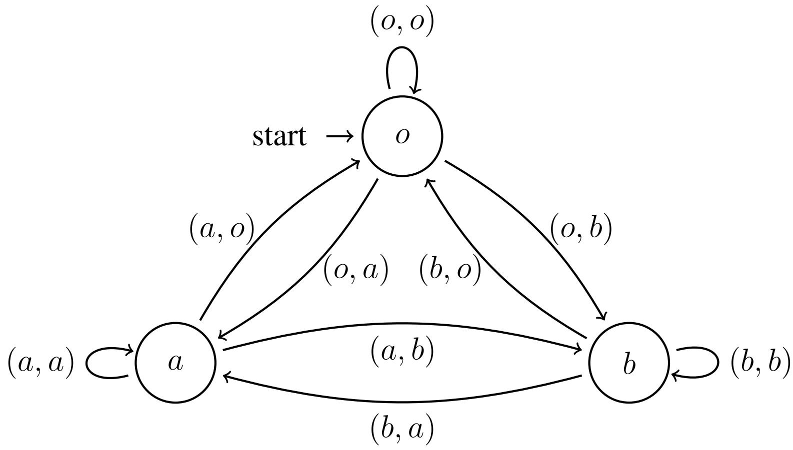


Figure 1: States and transitions in the two rentals setting with $\bar{x} = o$

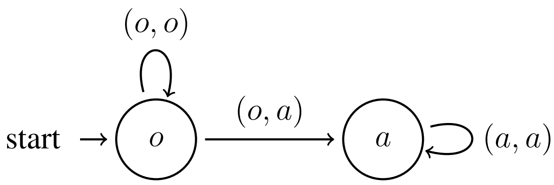


Figure 2: One absorbing variety with $\bar{x} = o$

Positive selection: There is a single non-absorbing variety, say a, while the outside option o is absorbing, as in Tirole (2016). Consumers may only transition from consumption to the outside option, but not reverse. The initial state is the variety a.

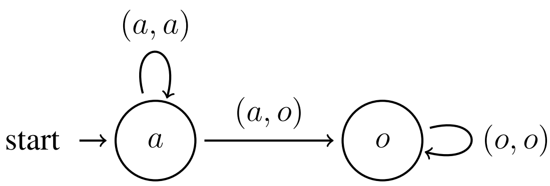


Figure 3: One variety and an absorbing outside option with $\bar{x} = a$

Behavior-based pricing: There is a single variety, say a, and the outside option o which are both non-absorbing states, as in e.g. Acquisti and Varian (2005); Armstrong (2006); Conitzer et al. (2012); Fudenberg and Villas-Boas (2007); Buehler and Eschenbaum (2020). Transitions are possible from the variety to the outside option and reverse. The initial state is the outside option o.

Two durable varieties: There are two varieties which are both absorbing states, while

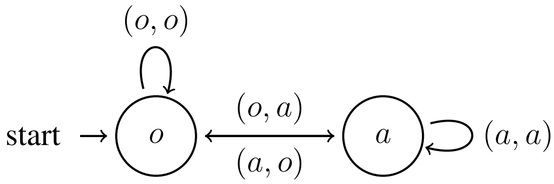


Figure 4: One non-absorbing variety with $\bar{x} = o$

the outside option o is non-absorbing, as in Nava and Schiraldi (2019); Board and Pycia (2014). Transitions are only possible from the outside option to one of the two varieties but not reverse. The initial state is the outside option o.

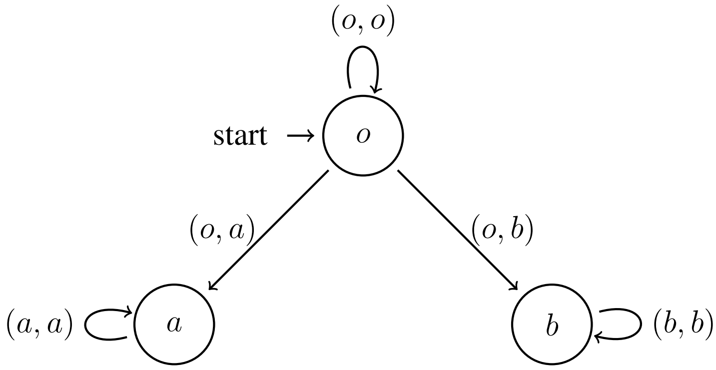


Figure 5: Two absorbing varieties with $\bar{x} = o$

One durable and one rental variety: There are two varieties one of which is an absorbing state, say b, while the outside option o is non-absorbing. Transitions are possible from the outside option to either variety and from a back to o or a to b. The initial state is the outside option o.

'Loyalty upgrade': There are two varieties one of which is rental and one durable, but the durable one can only be purchased after the rental one. Transitions are possible from the outside option to one of the two varieties, say a, from a to b and to o, and from b back to a. The initial state is the outside option o.

'Consumption cycle': There are two varieties that are neither rental nor durable goods. Transitions are possible from the outside option to one of the two varieties, say b, from b to a, and from a back to o. The initial state is a.

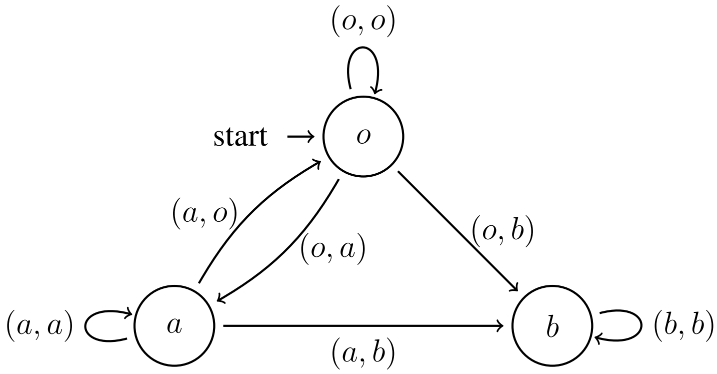


Figure 6: One absorbing and one non-absorbing variety with $\bar{x} = o$

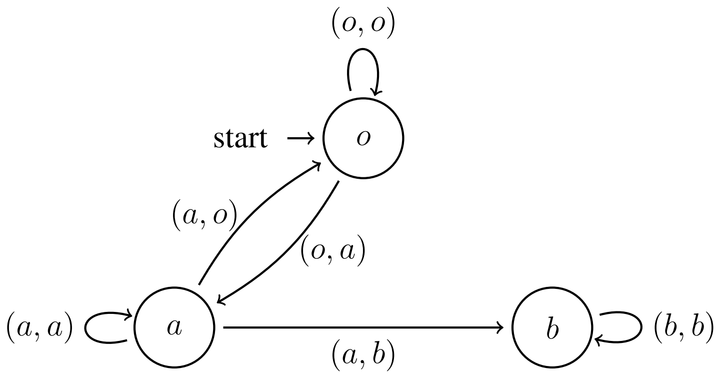


Figure 7: 'Loyalty upgrade' with $\bar{x} = o$

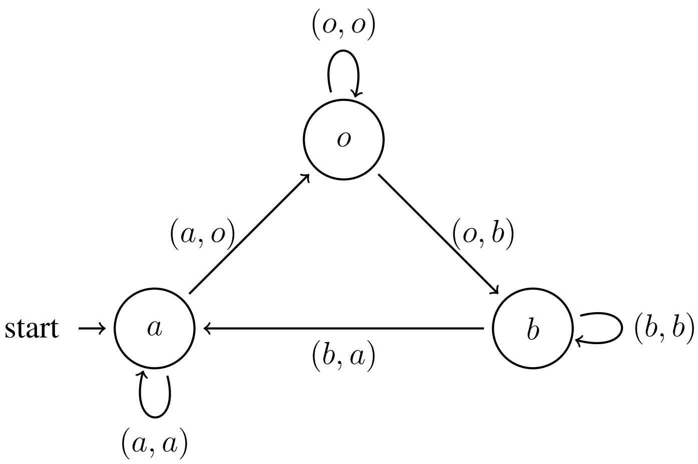


Figure 8: 'Consumption cycle' with $\bar{x} = a$

2.2 Prices, Payoffs, and Trading Up

All players are risk-neutral. The seller chooses a price profile $p^t = (p_a^t, p_b^t)$ in each period t for every sequence of past consumption decisions $\mathbf{x}^{-t} \equiv (x^0, ..., x^{t-1})$ that satisfies $(x^{t-1}, o) \in X$. That is, we require pricing to be consistent with voluntary consumption. Whenever $(x^{t-1}, o) \notin X$, the price $p_{x^{t-1}}^t$ is set to zero. Let $\rho(\mathbf{x})$ be the present discounted sum of *total payment* made by a buyer to the seller along a consumption path $\mathbf{x}$, or $\rho(\mathbf{x}) = \sum_{t=0}^{T} \delta^t p^t x^t$. Similarly, let $\nu(v, \mathbf{x})$ be the present discounted sum of *total value* a buyer obtains from a consumption path $\mathbf{x}$, or $\nu(v, \mathbf{x}) = v\chi(\mathbf{x})$. Then we can formulate a buyers utility U compactly as

$$U(v, \mathbf{x}) = \nu(v, \mathbf{x}) - \rho(\mathbf{x}),$$

and the sellers profit Π as

$$\Pi = \sum_{\mathbf{x}_k \in \mathbf{X}} \rho(\mathbf{x}_k) \mathcal{F}\left(v \in V;\ \mathbf{x}_k\right),$$

where $\mathcal{F}(v \in V;\ \mathbf{x}_k)$ denotes the measure of types on path $\mathbf{x}_k$ and $\mathbf{X}$ denotes the set of admissible consumption paths that is defined by the set of admissible transitions Γ, or

$$\mathbf{X}^t \equiv \{\mathbf{x}^t : (x^\tau, x^{\tau+1}) \in \Gamma\ \forall\, \tau \geq t-1 \text{ with } x^{-1} \equiv \bar{x}\}.$$

The seller's profit in the static game in turn is given by

$$\pi(p) = \sum_{(\bar{x}, x) \in \Gamma} xp\mathcal{F}\left(v \in V | (\bar{x}, x) = \arg \max_{(\bar{x}, x) \in \Gamma} \{x(v-p)\}\right),$$

and we denote by $p^m = (p_a^m, p_b^m)$ the prices that maximize the sellers profit in the static game and satisfy $p^m \in \arg\max_p \pi(p)$.

We will say that there exists a *trading-up opportunity* for the seller if there are consumers who can transition to a strictly higher-valued state:

Definition 1 (trading-up) *There is a trading-up opportunity for the seller if there exist consumers for whom transitions to a strictly higher-valued state are admissible,*

$$\exists v \textit{ in state } x \textit{ with } f(v) > 0 \textit{ and } x, x' \in X \textit{ s.t. } (x, x') \in \Gamma \textit{ and } vx' > vx.$$

For instance, in the setting with two absorbing varieties (durables) and initial state $\bar{x} = o$, there is a trading-up opportunity for the seller if there are non-buyers with strictly positive value for one of the varieties. The same is true in a setting with two non-absorbing varieties (rentals), but here there are also trading-up opportunities when there are buyers that buy their less-preferred variety.

Let Ω denote the set of price profiles $p = (p_a, p_b)$ that leave no trading-up opportunities for the seller in the static game,

$$\Omega = \{p \text{ s.t. } x = \arg\max_{x \in X}\{(v - p)x\} \implies vx > vx' \text{ or } (x, x') \notin \Gamma \; \forall v \in V\},$$

where

$$(\bar{x}, x), (\bar{x}, x') \in \Gamma,$$

and $\bar{p}$ a price profile that maximizes static profit while leaving no trading-up opportunities,

$$\bar{p} \in \arg\max \pi(p) \text{ s.t. } \bar{p} \in \Omega.$$

A period-t seller history, h^t, records all prices offered and consumption choices made along past consumption choices $\mathbf{x}^{-t}$, with $h^0 = \emptyset$. A period-t buyer history, $\hat{h}^t$, consists of the seller history h^t and the period-t price profile offered to consumers with seller history h^t. The set of period-t seller histories is denoted by H^t, and the set of seller histories by $H = \cup_{t=0}^{T} H^T$. Similarly, the set of period-t buyer histories is denoted by $\hat{H}^t$, and the set of buyer histories by $H = \cup_{t=0}^{T} \hat{H}^T$. Let $V(h^t) \subset V$ denote the subset of consumers with the same seller history h^t. The period-t price profile shown to a consumer with history h^t is denoted by $p^t(h^t) = (p^t_a(h^t), p^t_b(h^t))$.

A *Perfect Bayesian Equilibrium* (PBE) is a history-contingent sequence of the seller's chosen price profiles p^t for each history h^t, consumption choices x^t made by consumers, and updated beliefs about the buyers' values along the various consumption paths, such that actions are optimal given beliefs, and beliefs are derived from actions from Bayes' rule whenever possible.

3 Dynamic Pricing as Trading Up Buyers

3.1 Preliminaries

We first show that in equilibrium the seller's beliefs about the value profiles of buyers satisfy a form of top-down skimming.

Lemma 1 (skimming) *Consider buyers with common history $h^t \in H^t$.*

(i) If a buyer with value profile v prefers path $\mathbf{x}_k^t$ to path $\mathbf{x}_l^t$, $\chi(\mathbf{x}_k^t) \neq \chi(\mathbf{x}_l^t)$, then so does a buyer with value profile $\tilde{v} \neq v$ such that

$$(\tilde{v} - v) \cdot (\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t)) \geq 0. \tag{1}$$

(ii) In any PBE, if a buyer with value profile v prefers consumption choice $x^t = x$ to $x^t = x'$, $x' \neq x$, then so does a buyer with value profile $\tilde{v} \geq v$ such that

$$\begin{aligned}(\tilde{v} - v)\,(x - x') &+ \delta \left[\min_{\mathbf{x}^{t+1} \in \mathbf{X}^{t+1} | x^t = x} \left\{(\tilde{v} - v) \cdot \chi(\mathbf{x}^{t+1})\right\}\right] \\ &- \delta \left[\max_{\mathbf{x}^{t+1} \in \mathbf{X}^{t+1} | x^t = x'} \left\{(\tilde{v} - v) \cdot \chi(\mathbf{x}^{t+1})\right\}\right] \geq 0. \end{aligned} \tag{2}$$

where $\mathbf{X}^{t+1} | x^t$ denotes the sets of admissible consumption paths after consumption choice x^t.

Proof. Since type v prefers path $\mathbf{x}_k^t$ to path $\mathbf{x}_l^t$ by assumption, we must have

$$\nu(v, \mathbf{x}_k^t) - \rho(\mathbf{x}_k^t) \geq \nu(v, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t).$$

Now, consider some type $\tilde{v} \neq v$. Then, we have

$$\begin{aligned}\nu(\tilde{v}, \mathbf{x}_k^t) - \rho(\mathbf{x}_k^t) &= \nu(v, \mathbf{x}_k^t) - \rho(\mathbf{x}_k^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t) \\ &\geq \nu(v, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t),\end{aligned}$$

since type v prefers path $\mathbf{x}_k^t$ to path $\mathbf{x}_l^t$ by assumption. For type $\tilde{v}$ to prefer path $\mathbf{x}_k^t$, we must thus have

$$\nu(v, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t) \geq \nu(\tilde{v}, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t),$$

which can be rearranged to yield the result in (1). To show (2), we follow a similar line of argument. Denoting the continuation valuation of a type v following choice x by $U(v, x)$ (suppressing h^t for brevity) and considering the choices x and x', respectively, we can write

$$\begin{aligned}(\tilde{v} - p)x + \delta U(\tilde{v}, x) &= (v - p)x + \delta U(v, x) + (\tilde{v} - v)x + \delta[U(\tilde{v}, x) - U(v, x)] \\ &\geq (v - p)x' + \delta U(v, x') + (\tilde{v} - v)x + \delta[U(\tilde{v}, x) - U(v, x)] \\ &\geq (\tilde{v} - p)x' + \delta U(\tilde{v}, x')\end{aligned}$$

or

$$(\tilde{v}-v)(x-x') + \delta[U(\tilde{v},x) - U(v,x)] - \delta[U(\tilde{v},x') - U(v,x')] \geq 0. \tag{3}$$

Noting that each type can always mimic the actions of the other type by making the same consumption choices in every future period, we must have

$$\min_{\mathbf{x}^{t+1}\in\mathbf{X}^{t+1}|x^t} \left\{\chi(\mathbf{x}^{t+1})(\tilde{v}-v)\right\} \leq U(\tilde{v},x^t) - U(v,x^t) \leq \max_{\mathbf{x}^{t+1}\in\mathbf{X}^{t+1}|x^t} \left\{\chi(\mathbf{x}^{t+1})(\tilde{v}-v)\right\}.$$

Substituting and reorganizing yields (2). ■

Part (i) shows that for a buyer with value profile $\tilde{v}$ to prefer the same consumption path as a buyer with value profile v, the relative values of the two buyers and the relative total consumption along the two paths must be aligned. For example, if choosing path $\mathbf{x}_k^t$ instead of $\mathbf{x}_l^t$ implies obtaining relatively less consumption of a and relatively more of b and type v is willing to make this trade, then only types $\tilde{v}$ who do not prefer a relatively more than b compared to type v will make the same choice. This is illustrated in Figure 9, where the solid square shows the possible values of $(\tilde{v}-v)$ and the dashed lines indicate the possible values of $(\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t))$. For the difference in total consumption $(\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t))$ depicted in the figure, only consumers with value profiles in the shaded area satisfy the skimming property in (1). Part (ii) of Lemma 1 states the skimming result in terms of the consumption values resulting from current choices and the admissible consumption paths following these choices.

Lemma 1 nests well-known earlier skimming results as special cases. To see this, consider the case of a single durable good and the set of types that have not purchased yet at time t. We let x be the purchase of the durable good, whereas x' is the choice of the outside option. Since the minimum difference in consumption value after x and the maximum difference in consumption value after x' is the same and given by $(\tilde{v}-v)\Delta^{t+1}$, the skimming condition (2) simplifies to the standard condition $\tilde{v} \geq v$.[3] A similar result holds in the case of a single "anti-durable" good as in Tirole (2016). Here we consider the set of types that have chosen to purchase in every previous period until time t. If x denotes the purchase of the good and x' is the outside option, the minimum difference in consumption value after x is again $(\tilde{v}-v)\Delta^{t+1}$, while the largest value following x' is 0 because the outside option is an absorbing state. The skimming condition (2) then simplifies to $\tilde{v} \geq v$.[4]

[3] We can obtain the same result from (1) by noting that purchasing today (path $\mathbf{x}_k^t$) rather than delaying (path $\mathbf{x}_l^t$) immediately implies that $\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t) \geq 0$.

[4] Again, we can obtain the result from (1) by noting that $\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t) \geq 0$ because the outside option is an absorbing state (positive selection).

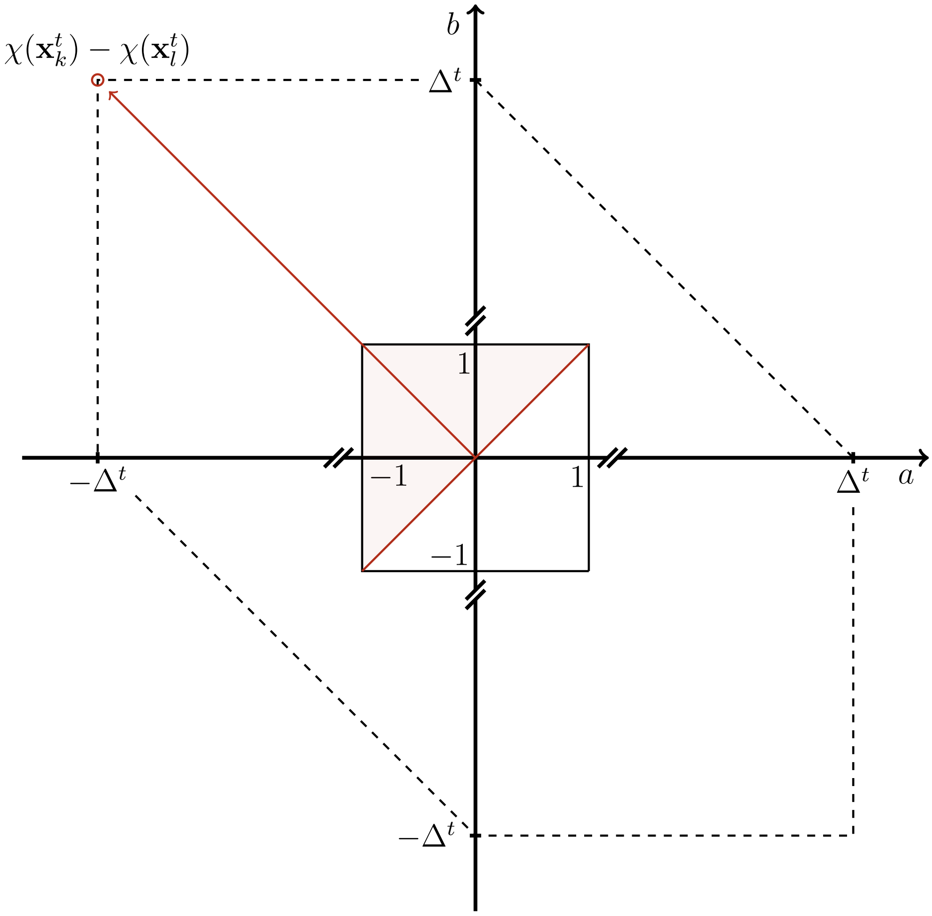


Figure 9: Illustration of the skimming condition

Finally, in the case of two durable goods (Nava and Schiraldi, 2019), the analysis above implies that skimming is satisfied for the comparison of a path with consumption of variety $i \in (a, b)$ in period t (i.e., $x^t = i$) to a path without consumption ($x^t = o$) if $\tilde{v}_i \geq v_i$. For the comparison of two paths that feature different varieties in period t, we have $(\tilde{v} - v) \cdot (\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t)) \geq 0$ iff $\tilde{v}_i - \tilde{v}_j \geq v_i - v_j$, where $i \neq j$. Therefore, skimming is satisfied if $\tilde{v}_i - v_i \geq \max\{0, \tilde{v}_j - v_j\}$. In general, as the purchase decisions may differ at every time t, for skimming to be satisfied the difference in total consumption must be considered. Therefore, the skimming conditions for any two paths in (1) and for a period-t purchase in (2) must both account for the admissible consumption paths and resulting total consumption in the future.

Figure 10 illustrates the set of admissible consumption paths $\mathbf{X}$ and the corresponding streams of discounted payoffs for the case of two rentals and two periods (i.e., $T = 1$).

For example, the lowest branch in Figure 10 depicts the "always-b-path" $\mathbf{x}_b = (b, b)$ with total consumption $\chi(\mathbf{x}_b) = (0, 1 + \delta)$, total value $\nu(\mathbf{x}_b) = (1 + \delta)v_b$ and total payment $\rho(\mathbf{x}_b) = p_b^0 + \delta p_b^1((p_a^0, p_b^0, b)$.

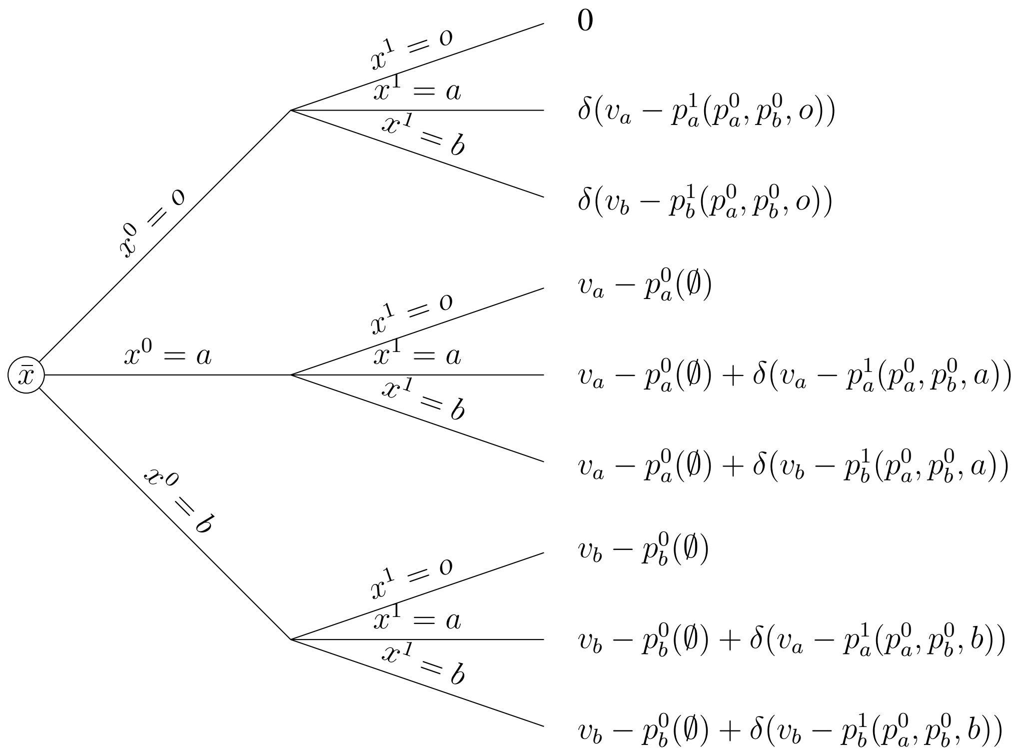


Figure 10: Consumption paths and utilities for two rentals and two periods

3.2 No Trading-Up at Static Monopoly Prices

We first consider the case without trading-up opportunities at static monopoly prices, that is, when p^m coincides with $\bar{p}$. Our first main result shows that, in this case, the seller's maximum profit attainable in the dynamic game is the repeated monopoly profit resulting from static monopoly pricing, and the seller can obtain this payoff irrespective of commitment ability. That is, the seller faces no commitment problem if there are no trading-up opportunities at static monopoly prices.

Borrowing terminology from Board and Pycia (2014), we say that the seller and buyers adopt *monopoly strategies* if, in every period t,

(i) the seller plays p^m for every history h^t, and

(ii) all consumers behave as if they were maximizing per-period utility.

Instead, if the seller varies prices over time or histories (rather than keeping them fixed), we say that she engages in *dynamic pricing*.

Proposition 1 *Suppose there are no trading-up opportunities at static monopoly prices, that is, $p^m = \bar{p}$. Then,*

(i) the seller can do no better than obtain the repeated monopoly profit $\pi(p^m)$ in every period $t = 0, ..., T$ and refrain from dynamic pricing, $\Pi \leq \pi(p^m) \cdot \Delta$.

(ii) there exists a PBE in which the seller and buyers adopt monopoly strategies in every period $t = 0, ..., T$ for any history h^t.

(iii) the seller can obtain the commitment profit irrespective of commitment ability.

Proof. We prove the three statements in turn.

(i) We first prove an intermediate Lemma that allows us to conveniently rewrite the seller's profit. Denote the set of consumers at $t = 0$ who are indifferent between two distinct consumption paths $\mathbf{x}_k$ and $\mathbf{x}_l$ by

$$V_{k,l} \equiv \{v : U(v, \mathbf{x}_k) = U(v, \mathbf{x}_l)\},$$

and the difference in the sums of discounted values obtained by indifferent consumers with value profile $v \in V_{k,l}$ along consumption paths $\mathbf{x}_k$ and $\mathbf{x}_l$, respectively, by

$$\Delta\nu_{k,l} \equiv \nu(v, \mathbf{x}_k) - \nu(v, \mathbf{x}_l) = \rho(\mathbf{x}_k) - \rho(\mathbf{x}_l t).$$

Then, the following result holds.

Lemma 2 *The present discounted profit of the seller can be written as*

$$\Pi = \rho(\mathbf{x}_0)\mathcal{F}\big[v \in V\big] + \sum_{k=1}^{K} \Delta\nu_{k,k-1}\mathcal{F}\left[v \in V; \cup_{j \geq k}\mathbf{x}_j\right],$$

where $\mathbf{X} = \{\mathbf{x}_0, ..., \mathbf{x}_K\}$ is ordered by the respective total payments such that $\rho(\mathbf{x}_0) \leq \rho(\mathbf{x}_1) \leq ... \leq \rho(\mathbf{x}_K)$.

Proof. Let $V_{0,1}$ denote the set of value profiles of consumers indifferent between $\mathbf{x}_0$ and $\mathbf{x}_1$. Then for any $v \in V_{1,0}$ we have

$$\nu(v, \mathbf{x}_1) - \rho(\mathbf{x}_1) = \nu(v, \mathbf{x}_0) - \rho(\mathbf{x}_0)$$

or

$$\rho(\mathbf{x}_1) = \rho(\mathbf{x}_0) + \Delta\nu_{1,0}$$

by construction. Next, let $V_{2,1}$ denote the set of value profiles of consumers who are indifferent between $\mathbf{x}_2$ and $\mathbf{x}_1$. Then for all $v \in V_{2,1}$ we have $\rho(\mathbf{x}_2) = \rho(\mathbf{x}_1) + \Delta\nu_{2,1}$, and thus

$$\rho(\mathbf{x}_2) = \rho(\mathbf{x}_0) + \Delta\nu_{1,0} + \Delta\nu_{2,1}.$$

Iterating this procedure for all consumption paths up to $\mathbf{x}_K$ yields

$$\rho(\mathbf{x}_k) = \rho(\mathbf{x}_0) + \sum_{k=1}^{K} \Delta\nu_{k,k-1}.$$

The present discounted profit is thus given by

$$\Pi = \rho_0(\mathbf{x}_0)\mathcal{F}\big[v \in V\big] + \sum_{k=1}^{K} \Delta\nu_{k,k-1}\mathcal{F}\left[v \in V; \cup_{j\geq k}\mathbf{x}_k\right].$$

■

Next, suppose the seller sets constant prices $p^m = (p_a^m, p_b^m)$. Denote the consumption paths that consist of the repeated consumption of a, b, and o, respectively, in every period by $\mathbf{x}_a$, $\mathbf{x}_b$, and $\mathbf{x}_o$. Then, for any consumption path $\mathbf{x}_k \in \hat{\mathbf{X}}$, where $\hat{\mathbf{X}} = \mathbf{X} \setminus \{\mathbf{x}_a, \mathbf{x}_b, \mathbf{x}_o\}$, and any value profile $v \in V$ we have that

$$\begin{aligned}
U(v, \mathbf{x}_k) &= v_a \sum_{t, x_k^t = a}^{T} \delta^t + v_b \sum_{t, x_k^t = b}^{T} \delta^t - \left(\sum_{t, x_k^t = a}^{T} \delta^t p_a^m + \sum_{t, x_k^t = b}^{T} \delta^t p_b^m \right) \\
&= (v_a - p_a^m) \sum_{t, x_k^t = a}^{T} \delta^t + (v_b - p_b^m) \sum_{t, x_k^t = b}^{T} \delta^t \\
&= U(v, \mathbf{x}_a) \frac{\sum_{t=0, x_k^t = a}^{T} \delta^t}{\sum_{t=0}^{T} \delta^t} + U(v, \mathbf{x}_b) \frac{\sum_{t=0, x_k^t = b}^{T} \delta^t}{\sum_{t=0}^{T} \delta^t}.
\end{aligned}$$

Therefore,

$$U(v, \mathbf{x}_k) \leq \max(U(v, \mathbf{x}_a), U(v, \mathbf{x}_b)) \; \forall v \in V,$$

which shows that no consumer will strictly prefer consumption path $\mathbf{x}_k$ to paths $\mathbf{x}_a$ or $\mathbf{x}_b$. Hence, the profit of the seller at time $t = 0$ is

$$\begin{aligned}\Pi =&\rho(\mathbf{x}_a)\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_a\big] \\ &+ \rho(\mathbf{x}_b)\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_b\big].\end{aligned}$$

Now suppose the seller deviates from setting constant monopoly prices p^m such that an additional path $\mathbf{x}_k$ is chosen by a positive mass of consumers. Then the profit becomes

$$\begin{aligned}\Pi =&\rho(\mathbf{x}_a)\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_a\big] \\ &+ \rho(\mathbf{x}_b)\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_b\big] \\ &+ \rho(\mathbf{x}_k)\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_k\big].\end{aligned}$$

Letting $\Delta\nu_{k,l}$ denote the difference in the sums of total values obtained by consumers who are indifferent between paths $\mathbf{x}_k$ and $\mathbf{x}_l$, $k \neq l$, and assuming that $\rho(\mathbf{x}_a) \leq \rho(\mathbf{x}_b) \leq \rho(\mathbf{x}_k)$ without loss of generality, we can rewrite this profit function by applying Lemma 2 as

$$\begin{aligned}\Pi =&\rho(\mathbf{x}_a)\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_a \vee \mathbf{x}_b \vee \mathbf{x}_k\big] \\ &+ \Delta\nu_{b,a}\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_b \vee \mathbf{x}_k\big] \\ &+ \Delta\nu_{k,b}\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_k\big].\end{aligned}$$

Note that the first two terms of the profit function are maximized by setting $p^m = (p^m_a, p^m_b)$ in every period $t = 0, ..., T$, that is,

$$\begin{aligned}&\max_{p_a,p_b}\Big\{\rho(\mathbf{x}_a)\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_a \vee \mathbf{x}_b \vee \mathbf{x}_k\big] \\ &\qquad + \Delta\nu_{b,a}\mathcal{F}\big[v \in V \big| \arg\max_{\mathbf{x}\in\mathbf{X}}\{\nu(v,\mathbf{x}) - \rho(\mathbf{x})\} = \mathbf{x}_b \vee \mathbf{x}_k\big]\Big\} \\ &= \sum_{t=0}^{T}\delta^t\pi(p^m).\end{aligned}$$

Therefore, for a deviation from repeatedly playing p^m to be profit-increasing for the seller, it is necessary that $\Delta\nu_{k,b} > 0$. However, this inequality requires that $\exists v \in V$ and $t \in T$ such that $x^t_k v > x^t_b v$ or $x^t_k v > x^t_a v$, which is excluded by assumption of no trading-up opportunities at static monopoly prices.

(ii) Consider the incentives to deviate from monopoly strategies on the equilibrium path. Part (i) of the proof shows that it is optimal for consumers to make quasi-myopic consumption decisions when faced with constant monopoly prices p^m. But then, cutoffs are equal to the static optimal cutoffs which, by statement (i), yield the maximum profit the seller can achieve. Thus the seller obtains her highest possible payoff when playing prices p^m starting at any time t and any history h^t. Therefore, the seller cannot benefit from deviating from the monopoly strategy.

(iii) Follows directly from (i) and (ii). ■

Proposition 1 shows that the emergence of dynamic pricing crucially depends on the existence of trading-up opportunities in the static optimum. A profit-maximizing seller engages in dynamic pricing only if doing so allows her to trade up buyers to more valuable consumption options. Therefore, if profit-maximizing prices in the static game leave no trading-up opportunities, $p^m = \bar{p}$, the seller can simply repeat the static monopoly prices and obtain the commitment profit irrespective of commitment ability, since the monopoly strategies of the seller and buyers form a PBE.

This result implies that in settings with $p^m = \bar{p}$ it is sufficient to know the solution of the static game to determine the outcome of the repeated game. The following corollary to Proposition 1 provides an explicit characterization of such settings. Intuitively, there are two classes of settings in which $p^m = \bar{p}$: first, settings that exclude trading-up opportunities for arbitrary price profiles (including static monopoly prices); second, settings where the distribution of consumption values is such that profit-maximizing static prices happen to leave no trading-up opportunities.

Lemma 3 *The dynamic setting is characterized by the tuple* $(\mathcal{F}, \bar{x}, \Gamma)$. *There are no trading-up opportunities for arbitrary price profiles* p *in the static game if*

(i) the initial state $\bar{x}$ *is absorbing.*

(ii) the initial state $\bar{x}$ *is the (weakly) most-preferred state for all buyers, and all accessible states are absorbing (positive selection).*

(iii) the initial state $\bar{x}$ *is the (weakly) least-preferred of the accessible states for all buyers, all accessible states are absorbing, and the lowest-value consumer obtains a strictly positive utility in at least one of the accessible states (Board and Pycia (2014)) .*

(iv) all buyers have the same preference ranking over all accessible states and only transitions from a preferred to a (weakly) less-preferred state are admissible (trading down).

Otherwise, there exist trading-up opportunities at static monopoly prices, except if $\mathcal{F}$ is such that $p^m = \bar{p}$.

Proof. Follows directly from substituting the conditions of the respective case into the static profit function and Ω. ■

Lemma 3 identifies dynamic settings in which Proposition 1 applies. Case (i) is a trivial setting in which no transition out of the initial state is admissible. Case (ii) describes a setting of positive selection where the initial state is the most-preferred state for all consumers who can transition to less-preferred absorbing states only. (Tirole, 2016) provides an in-depth analysis of such a setting with a single non-absorbing variety as the initial state and the outside option as an absorbing state. Lemma 3 shows that we can extend this setting to allow for a second variety while ensuring that $p^m = \bar{p}$ continues to apply, by requiring that the second variety is less-preferred and absorbing. This arguably is the essence of positive selection: all buyers start in the most-preferred state and can only transition to less-preferred ones, that is, can only trade down but never up. With a single variety this is ensured if the outside option is absorbing. Case (iii) describes a setting in which the initial state is the least-preferred state for all consumers who can transition to more-preferred absorbing states. Board and Pycia (2014) provide a detailed analysis of such a setting where the initial state is the non-absorbing outside option, and the seller offers a single absorbing variety, but consumers can also choose a second absorbing outside option with a strictly positive utility for all buyers. Case (iv) describes settings in which the initial state is allowed to be any of the three states.

In any other setting, it is not the case that arbitrary price profiles in the static game leave no trading-up opportunities. For example, consider a setting with a 'mix' of varieties, where one variety is absorbing and one is non-absorbing. We will assume that the outside option is non-absorbing and the initial state. We can see that for a price profile in the static setting to not leave any trading-up opportunities, we must have that the market clears and that all types allocating themselves to the non-absorbing variety prefer it to the absorbing one. Thus, we need to check if prices p^m happen to satisfy these conditions, which will depend on the measure $\mathcal{F}$, and we cannot rule out the existence of trading-up opportunities a priori.

3.3 Trading Up and Dynamic Pricing

In most settings that our framework allows for, the price profile p^m will leave trading-up opportunities. In our second main result, we analyze the pricing dynamics when $p^m \neq \bar{p}$ and the seller cannot commit to prices ex-ante. We show that in the absence of seller commitment the existence of trading-up opportunities is the driving force behind dynamic pricing. A profit-maximizing seller will engage in dynamic pricing for each consumer segment in which trading-up opportunities exist by continually lowering prices until all trading-up opportunities are exhausted. This dynamic ends at prices $\bar{p}$ and is played out in finite time, if the static profit from prices $\bar{p}$ is strictly positive.

Proposition 2 *In any PBE,*

(i) for any history h^t at which there exist trading-up opportunities, the seller trades up a strictly positive measure of types.

(ii) the seller will never set a price for a variety i below $\bar{p}_i$ at any history h^t at which the transition to state i is admissible.

(iii) the sellers present discounted profit satisfies $\Pi \geq \pi(\bar{p}) \cdot \Delta$.

(iv) if $\pi(\bar{p}) > 0$, all trading-up opportunities are exhausted in finite time.

Proof. We prove the four statements in turn.

(i) Fix a PBE. Consider a history h^t on the equilibrium path and denote the state consumers are in by x^{t-1}. Suppose that there exist trading-up opportunities, so that there exists a consumption option $x \in X$ for which some types $v \in V(h^t)$ satisfy $vx > vx^{t-1}$ and $(x^{t-1}, x) \in \Gamma$. Denote the set of types that satisfy these conditions by $V^{TU}(h^t) \subseteq V(h^t)$. By definition, we must have $x \equiv i \in \{a, b\}$. Let the highest value for i for types $v \in V(h^t)$ be $\bar{v}_i$, the lowest value $\underline{v}_i$, and analogously for types $v \in V^{TU}(h^t)$ we have $\bar{v}_i^{TU}$ and $\underline{v}_i^{TU}$. We similarly define $\bar{v}_j, \bar{v}_j^{TU}, \underline{v}_j, \underline{v}_j^{TU}$ for $j \in \{a, b\}, j \neq i$.

Further denote the measure of types traded up, if the seller decides to trade up some types by $\mathcal{F}(v \in V(h^t)|TU)$ and the remaining measure of types not traded up by $\mathcal{F}(v \in V(h^t)|NTU)$. By definition, $\mathcal{F}(v \in V(h^t)) = \mathcal{F}(v \in V(h^t)|TU) + \mathcal{F}(v \in V(h^t)|NTU)$. There are four cases to distinguish.

Consider the trivial case first, in which $x^{t-1} = o$ or $x^{t-1} = j$ and $(x^{t-1}, o) \notin \Gamma$. If $(x^{t-1}, o) \notin \Gamma$, $p_j^t(h^t)$ is set to zero by assumption and the definition of trading up opportunities immediately implies that the seller can offer a strictly positive price $p_i^t(h^t)$ at which (some) types $v \in V^{TU}(h^t)$ purchase variety i, resulting in a positive profit. Similarly, if $x^{t-1} = o$, then inducing any types $v \in V^{TU}(h^t)$ to choose $x^t = i$ constitutes trading up and $(o, i) \in \Gamma$ by assumption at history h^t. It is straightforward to show that as the seller earns no profit from types in the outside option, inducing a positive mass to choose i (or j, or both) must be profit-increasing, as the definition of trading-up opportunities guarantees that (some) types are willing to pay a strictly positive price for i.

In the remaining three cases we have $x^{t-1} = j$ and $(x^{t-1}, o) \in \Gamma$. In the first one, we assume that $\bar{v}_i^{TU} > \bar{v}_j$. Then the equilibrium profit for the seller if she decides not to trade up any buyers, $\hat{\Pi}(h^t)$, satisfies

$$\hat{\Pi}(h^t) < \bar{v}_j \Delta^t \mathcal{F}(v \in V(h^t)) \tag{4}$$

as the seller cannot extract the full surplus of types with a linear price. However, if the seller trades up (some) types $v \in V^{TU}(h^t)$, then the equilibrium profit obtained from trading up, $\Pi^*(h^t)$, satisfies

$$\Pi^*(h^t) \geq v_i^* \Delta^t \mathcal{F}(v \in V(h^t)|TU) \tag{5}$$

where v_i^* denotes the lowest value for v_i for the cutoff types indifferent to trading up to i, as the seller can always obtain at least the value of the lowest type in the set, while the equilibrium profit obtained from types not traded up, $\Pi^\circ(h^t)$, satisfies

$$\Pi^\circ(h^t) < \bar{v}_j \Delta^t \mathcal{F}(v \in V(h^t)|NTU), \tag{6}$$

because as before the seller cannot extract the full surplus using a linear price. As $\bar{v}_i^{TU} > \bar{v}_j$ by assumption, there exists a v_i^* that satisfies $\bar{v}_i^{TU} > v_i^* > \bar{v}_j$. This immediately implies by (4), (5), (6), and $\mathcal{F}(v \in V(h^t)) = \mathcal{F}(v \in V(h^t)|TU) + \mathcal{F}(v \in V(h^t)|NTU)$ that

$$\Pi^*(h^t) + \Pi^\circ(h^t) > \hat{\Pi}(h^t).$$

In the remaining two cases, we assume that $\bar{v}_i^{TU} < \bar{v}_j$. Note first that since at any h^t the seller will leave no rent to the lowest type, no types $v_i < p_i^t(h^t)$ will ever choose $x^t = i$ to continue purchasing i as the loss cannot be recouped in future periods, and thus no cutoff types below $(p_i^t(h^t), p_j^t(h^t))$ can ever be induced. For case three, suppose that when not trading up any types, the equilibrium profit of the seller includes (some) types

$v \in V^{TU}(h^t)$. Then it follows that the equilibrium profit for the seller if she decides not to trade up any types satisfies

$$\hat{\Pi}(h^t) < (\bar{v}_j^{TU} + \min\{v_i\}\Delta^{t+1})\mathcal{F}(h^t). \tag{7}$$

The equilibrium profit from types traded up if the seller chooses to do so in turn satisfies

$$\Pi^*(h^t) \geq v_i^* \Delta^t \mathcal{F}(v \in V^{TU}(h^t)|TU). \tag{8}$$

As $\bar{v}_i^{TU} > \bar{v}_j^{TU}$ by definition, there exists a v_i^* such that $\bar{v}_i^{TU} > v_i^* > \bar{v}_j^{TU}$, implying that trading up (some) types $v \in V^{TU}(h^t)$ is strictly profit-increasing.

In the final case, we have both $x^{t-1} = j$ and assume that the equilibrium profit when not trading up any types excludes all types $v \in V^{TU}(h^t)$. That is, prices are such that all types $v \in V^{TU}(h^t)$ play $x^t = o$. If the seller can induce a v_i^* such that only (some) types $v \in V^{TU}(h^t)$ choose i and thus profit from types not traded up remains constant, by case 3 this must be strictly profit increasing. This is impossible if there exists a $\tilde{v}$ such that $\tilde{v}_j - v_j \geq \tilde{v}_i - v_i$ and $\tilde{v}_j > v_j$, $\tilde{v}_j > \tilde{v}_i$, where we consider types $v = (\bar{v}_i^{TU}, v_j) \in V^{TU}(h^t)$. We now show that types $\tilde{v}$ cannot prefer $x^t = i$ on the equilibrium path.

First, note that Lemma 2 can be applied to any history h^t, not only the empty history, and define $p^* \in \max_p \pi(h^t)$, where $\pi(h^t)$ denotes the period-t profit at history h^t. Further denote the histories following history h^t for types that accept at a price $p_j^{\tau-1}(h^{\tau-1})$ as h^τ a fortiori for all future periods, so that $\tau \in (t+1, ...T)$. Then applying Lemma 2 implies that the equilibrium profit if no trading up opportunities exist at all histories h^τ satisfies $\Pi(h^t) \leq \Delta^t \pi(p^*|h^t)$. As buyers behave as if they were myopic when facing constant prices, the seller can achieve the maximum profit by continually playing p^* from t onward. By definition of case four, this applies to the equilibrium profit for the types that play $x^t = j$. Second, applying Lemma 1 we have that if types v prefer $x^t = i$ instead of $x^t = j$, then for types $\tilde{v}$ to have the same preferences requires

$$\tilde{v}_i - v_i \geq \Delta^t \tilde{v}_j - v_j, \quad \text{or} \quad \Delta^t(\tilde{v}_i - v_i) \geq \Delta^t \tilde{v}_j - v_j, \tag{9}$$

depending on the accessibility of the outside option following choice $x^t = i$ and imposing that following $x^t = j$ types $\tilde{v}$ will repeatedly play j along the equilibrium path. As (9) contradicts our assumption that $\tilde{v}_j - v_j \geq \tilde{v}_i - v_i$ we conclude that we can always find an appropriate v_i^*.

Then in conjunction statement (i) follows.

(ii) Denote by Λ the set of price profiles p that leave no trading-up opportunities for any history h^t in the dynamic game. We will show that $\Omega \setminus \Lambda = \emptyset$ and $\bar{p} \in \Lambda$. Consider prices $\bar{p} = (\bar{p}_a, \bar{p}_b)$. Note first, that if $\bar{p} \in \Omega$, then by the definition of Ω it follows that for some price profile $\tilde{p}$ we have

$$\tilde{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\}) - (\eta, \eta) \implies \tilde{p} \in \Omega, \tag{10}$$

for any $\eta \geq 0$, and

$$\tilde{p} = \begin{cases} (\bar{p}_a, \bar{p}_b) - (0, \eta), & \text{if } \bar{p}_b > \bar{p}_a \\ (\bar{p}_a, \bar{p}_b) - (\eta, 0), & \text{if } \bar{p}_b < \bar{p}_a \end{cases} \implies \tilde{p} \in \Omega, \tag{11}$$

for any $\eta \in [0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$. Second, observe that the price profile $p^\circ = (-\Delta^{t+1}, -\Delta^{t+1})$ is contained in Λ. To see this, recall from the proof of Lemma 1 that since all types can always mimic each others behavior (i.e. make the same choices from t onward), we have that

$$U(\tilde{v}, h^t) - U(v, h^t) \leq \Delta^{t+1} \max_{i \in \{a,b\}} \{\tilde{v}_i - v_i\}, \;\; v \neq \tilde{v},$$

where U denotes the continuation valuation. Therefore we know that $p^\circ \in \Lambda$ and by (10) we also have that $p^\circ \in \Omega$.

Now pick a price profile $\hat{p}$ that satisfies $\hat{p} = p^\circ + (\varepsilon, \varepsilon)$ for some $\Delta^{t+1} + \min\{\bar{p}_a, \bar{p}_b\} \geq \varepsilon > 0$. By (10) we know $\hat{p} \in \Omega$. Denote by x° the choice buyers make in the static game when facing prices p°. By (10) we therefore have

$$x^\circ(v - p^\circ - \varepsilon) \geq x'(v - p^\circ - \varepsilon), \;\; x', x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V, \tag{12}$$

and since $p^\circ \in \Lambda$ we also have that

$$\begin{aligned} x^\circ(v - p^\circ) &+ \delta U^\circ(v, h^t) \\ &\geq x'(v - p^\circ) + \delta U'(v, h^t), \;\; x', x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V, \end{aligned} \tag{13}$$

where U° and U' denote the continuation valuations associated with choice x° and x' respectively, given history h^t. By (12), (13), and the definition of p° it then follows that

$$\delta(U^\circ - U') \leq \delta\Delta^{t+1} \;\; \forall\, v \in V$$

as otherwise (13) would not hold. But then, for any ε small enough, we must have that

$$\begin{aligned} x^\circ(v - p^\circ - \varepsilon) &+ \delta U^\circ(v, h^t) \\ &\geq x'(v - p^\circ - \varepsilon) + \delta U'(v, h^t), \;\; x', x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V, \end{aligned} \tag{14}$$

since $\delta < 1$. Thus $\hat{p} \in \Lambda$. By the same argument as before, (14) then implies that

$$\delta(U^\circ - U') \leq \delta\Delta^{t+1} - \varepsilon,$$

allowing us to construct the same contradiction again. Hence, $\tilde{p} \in \Lambda$ for any $\tilde{p}$ that satisfies (10) with $\eta \geq 0$.

Now fix the price profile $\hat{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\})$. By (11) we have $\hat{p} \in \Omega$ and as shown above we also have $\hat{p} \in \Lambda$. Consider a price profile $p' = \hat{p} + (0, \varepsilon)$ if $\bar{p}_b > \bar{p}_a$ and $p' = \hat{p} + (\varepsilon, 0)$ if $\bar{p}_b < \bar{p}_a$ where $\varepsilon \in (0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$. By (11) we have $p' \in \Omega$ and therefore for an ε small enough we can construct the same argument as before for any $\delta < 1$. Thus we find that $p' \in \Lambda$ and hence $\bar{p} \in \Lambda$. Finally note that the proof applies not only to the price profile $\bar{p}$ but to any $p \in \Omega$ and therefore we have that $\Omega \setminus \Lambda = \emptyset$. Then statement (ii) follows by optimality.

(iii) Follows directly from (ii) by optimality.

(iv) Fix a candidate PBE. Consider a history h^t with associated support $V(h^t)$ and state x at which there exist trading-up opportunities for some types $v \in V(h^t)$, or $\exists v \in V(h^t)$ with $vx' > vx$ where $x' \neq x$ and $(x, x') \in \Gamma$. Suppose that there exists a history h^{t+1} with support $V(h^{t+1})$ and state x, such that $V(h^{t+1}) \subset V(h^t)$ and $\exists v \in V(h^{t+1})$ with $vx' > vx$ where $x' \neq x$ and $(x, x') \in \Gamma$. That is, along equilibrium play the seller trades up some of the types in the support $V(h^t)$ for whom there exist trading-up opportunities. Denote the measure of types with history h^t that can be traded up by ω, or

$$\mathcal{F}\left(v \in V(h^t) \text{ s.t. } vx' > vx \text{ and } (x, x') \in \Gamma\right) = \omega > 0.$$

Let $\Delta\nu$ denote the difference in the sums of discounted values of the indifferent types the seller induced by trading up at time t, $\Delta\underline{\nu}$ the lowest difference in the sums of discounted values in the support $V(h^t)$ for the types that satisfy the trading-up conditions, and by $\Delta\bar{\nu}$ the corresponding highest difference, so that $\Delta\bar{\nu} > \Delta\nu > \Delta\underline{\nu}$.

Now consider some $\varepsilon > 0$ that satisfies the following condition at the history h^t

$$\varepsilon \geq \Delta\bar{\nu} - \Delta\underline{\nu} > 0.$$

As shown in the proof of (i), whenever there exist trading-up opportunities for any history h^t, the seller trades up a strictly positive measure of types. By Lemma 1 we have that $\Delta\bar{\nu} - \Delta\underline{\nu}$ becomes smaller over time. Therefore, for any history with a support of types that contains types with trading-up opportunities, as the game continues the possible values for ε decrease. Thus the longer T, the smaller the values ε can take and satisfy the condition. We now show that for ε small enough, the seller strictly prefers to trade up all types if $\Delta\underline{\nu} > 0$.

Denote the profit along the equilibrium path from the measure of types with history h^t that can be traded up by $\Pi^*(h^t)$ and the profit from trading up all buyers at time t instead by $\bar{\Pi}(h^t)$. Since the seller cannot fully extract the surplus of types with trading-up opportunities at once by inducing $\Delta\nu$, a $\lambda \in (0, 1)$ exists such that

$$\Pi^*(h^t) < \lambda\Delta\bar{\nu}\omega + \delta(1-\lambda)\Delta\bar{\nu}\omega.$$

Then it follows that

$$\begin{aligned}\Pi^*(h^t) - \bar{\Pi}(h^t) &< \lambda\Delta\bar{\nu}\omega + \delta(1-\lambda)\Delta\bar{\nu}\omega - \omega\Delta\underline{\nu} \\ &= ((\lambda + \delta - \lambda\delta)\Delta\bar{\nu} - \Delta\underline{\nu})\omega \\ &\leq ((\lambda + \delta - \lambda\delta)(\varepsilon + \Delta\underline{\nu}) - \Delta\underline{\nu})\omega \\ &= ((\lambda + \delta - \lambda\delta)\varepsilon - (1 - (\lambda + \delta - \lambda\delta))\Delta\underline{\nu})\omega.\end{aligned}$$

Therefore, $\bar{\Pi}(h^t) > \Pi^*(h^t)$ whenever

$$\varepsilon \leq \Delta\underline{\nu}(1 - (\lambda + \delta - \lambda\delta))/(\lambda + \delta - \lambda\delta).$$

Thus, if $\Delta\underline{\nu} > 0$ and T is sufficiently long, there exists a period t such that all trading-up opportunities are exhausted at any $\tau \geq t$ in any PBE.

It remains to check that $\Delta\underline{\nu} > 0$. It is straightforward to see that if $\pi(\bar{p}) > 0$, then $\Pi(\bar{p}|h^t) = \sum_{\tau=t}^{T} \delta^{\tau-1}\pi(\bar{p}|h^t) > 0$ for any history h^t at which there exist trading-up opportunities for the seller and therefore $\Delta\underline{\nu} > 0$ for any such history h^t. Then statement (iv) follows. ■

Proposition 2 demonstrates that the driving force behind dynamic pricing is trading up. Whenever the seller faces a set of buyers for whom trading-up opportunities exist, it is strictly profit-maximizing to induce (some) buyers to trade up to a higher-valued option than the currently chosen one. By doing so, the seller can extract a larger surplus from these types. However, in order to induce consumers to trade up, the seller must lower the prices relative to the prices these types were facing previously. Thus, as the game progresses, the seller will continually lower prices to trade up buyers. Since prices $\bar{p}$ also leave no trading-up opportunities when played in the dynamic game, the seller will never want to set a price below $\bar{p}$ and thus this dynamic ends at prices $\bar{p}$ as long as the transitions to consumption are admissible. This is required for dynamics to end at $\bar{p}$, because whenever they are not, any price is a best-response, including prices below $\bar{p}$.

This implies that the sellers profit in the absence of commitment ability is bounded from below at $\pi(\bar{p})\Delta$. The time it takes for this dynamic to play out depends on whether the seller can obtain positive profits from playing prices $\bar{p}$. If trading all buyers up at once yields a strictly positive profit in the static game, then the seller will eventually exhaust all trading-up opportunities in finite time provided that T is sufficiently long.

The result clarifies the connection between different strands of literature. In the canonical setting with one durable good and an outside option with value zero, trading-up opportunities exist for non-buyers only, since buyers of the durable good are captured in an absorbing state. Thus, if the seller lacks commitment ability, profit-maximizing prices are falling for non-buyers, reflecting the classic notion of 'Coasian dynamics' (Fudenberg et al., 1987; Coase, 1972). Since all buyers value the good more than the outside option, trading-up opportunities exist whenever a positive measure of non-buyers remains. The same is true in a setting with two durable goods and an outside option with value zero. Once again the seller is compelled to lower the prices of the varieties until the market clears (Nava and Schiraldi, 2019). However, it is possible to clear the market with only one of the two prices at zero, allowing the seller to still obtain a strictly positive profit. In settings of behavior-based price discrimination with one rental good and an outside option with value zero, the same logic prevails. Trading-up opportunities exist only for previous non-buyers, while loyal buyers cannot be traded up. As a result, the seller has no incentive to adjust the price for these 'positively selected' types in order to trade them up and hence prices fall only for previous non-buyers (Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Tirole, 2016).

Using Proposition 2 we can study settings that have not been previously analyzed (to the best of our knowledge). For example, when applied to a setting with two rental goods and a non-absorbing outside option, Proposition 2 shows that in the absence of commitment ability the seller may eventually have to set both prices to zero. Consider that in the one-shot game, the profit-maximizing prices that leave no trading-up opportunities $\bar{p}$ are equal to the lowest values for the two respective varieties in the entire support V. If these are equal to zero, then $\bar{p} = (0, 0)$. Thus, the seller will trade up buyers until prices finally fall to zero. However, as $\pi(\bar{p}) = 0$ in this case, part (iii) of Proposition 2 does not apply and this dynamic may take infinite periods to play out. Proposition 2 also shows that a setting with mixed varieties can protect the seller from eventually charging prices of zero if the rental variety is considered the superior one by all buyers and thus a seller of a single good may want to introduce a second, absorbing, low-quality variety to shield herself from being forced down to selling at zero prices.

Proposition 1 and 2 jointly show that the outcome of repeated monopoly pricing problems depends on two price profiles from the static game: p^m and $\bar{p}$. By comparing these two price profiles, we can understand the outcome and dynamics of the repeated game. If the two price profiles coincide, $p^m = \bar{p}$, then the seller does not face a commitment problem and with or without commitment by the seller, the profit-maximizing solution is to play constant prices at p^m. If the two price profiles do not coincide, $p^m \neq \bar{p}$, then in the absence of commitment ability by the seller dynamic pricing emerges in the form of 'Coasian dynamics': prices fall in every period for types that can be traded up. This dynamic continues as long as trading-up opportunities remain, but will end in finite time if the profit from playing prices $\bar{p}$ is strictly positive, $\pi(\bar{p}) > 0$. At the very least, the seller is guaranteed to obtain the repeated, discounted profit that prices $\bar{p}$ yield, $\pi(\bar{p})\Delta$, as she can always play prices $\bar{p}$ to exhaust all trading-up opportunities and end the dynamics. These findings can be understood as offering a simple 'checklist' for analyzing dynamic pricing problems, allowing one to focus on the simpler task of determining prices in the static game, instead of solving the more complex dynamic game.

**Tentative

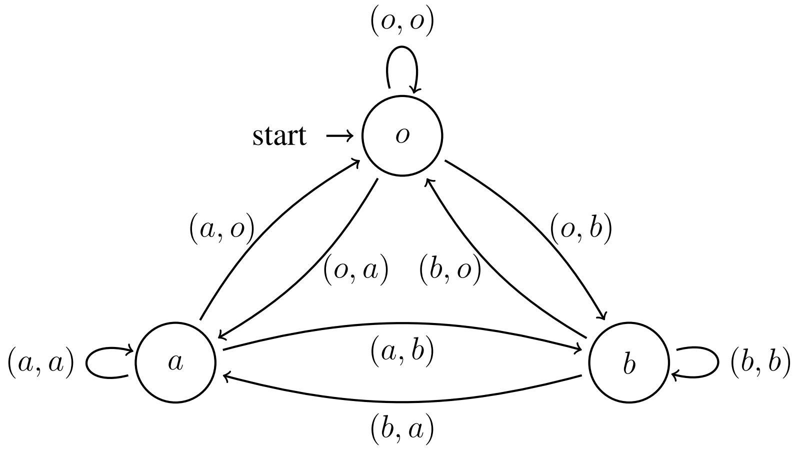
Table 1: Equilibrium characteristics without seller commitment

Case	Setting	Dynamic Pricing	Convergence in $T < \infty$	Zero profit bound
1	1 "anti-durable"	No	—	No
1	2 "anti-durables"	No	—	No
1	B+P (2014)	No	—	No
2	2 durables	Yes	Yes	No
2	Mixed varieties	Yes	Yes	No
2	1 rental	Yes	No	Yes
2	1 durable	Yes	No	Yes
2	2 rentals	No	No	Yes

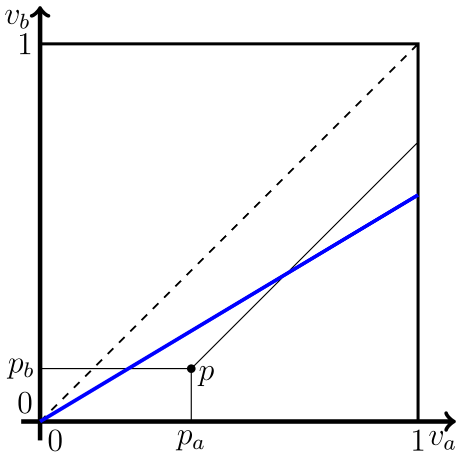
We assume that the support V contains types with value zero for both varieties.

Table 1 shows how knowledge of the price profiles p^m and $\bar{p}$ can be used to characterize the outcomes in the repeated game. We assume that the support V contains types with value zero for both varieties and no strict vertical differentiation.(Note, 2 anti durables should be wrong then?) The settings in the case where $p^m \neq \bar{p}$ are ordered by Π^{NC}/Π^{C} in descending order. Case 1 corresponds to $p^m = \bar{p}$ and case 2 corresponds to $p^m \neq \bar{p}$.

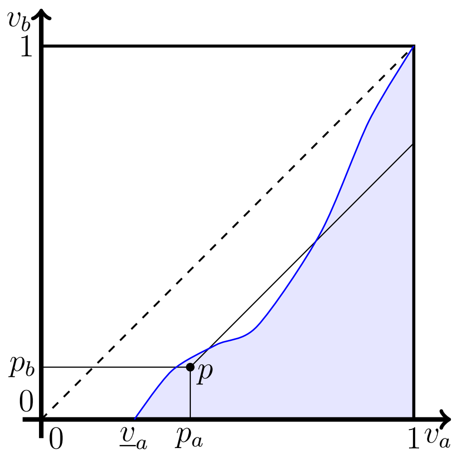
4 Example: Vertical Differentiation



(a) Transitions



(b) Linear valuation function



(c) General valuation distribution

5 Conclusion

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