Dynamic Monopoly Pricing With Multiple Varieties: Trading Up*

Stefan Buehler & Nicolas Eschenbaum†

This draft: June 2021

Preliminary.

Abstract

This paper studies a broad class of dynamic monopoly pricing problems, including various unexplored settings (e.g., multiple rental varieties). We show that the driving force behind pricing dynamics is the seller's incentive to trade up consumers to higher-valued consumption options. In Coasian settings, consumers can be traded up from the static optimum, and pricing dynamics arise until all trading-up opportunities are exhausted. In Non-Coasian settings, consumers cannot be traded up from the static optimum, and no pricing dynamics arise. Hence, dynamic monopoly pricing can be characterized by checking for trading-up opportunities in the static optimum.

*We are grateful to Alia Gizatulina, Paul Heidhues, Igor Letina, Marc Moeller, Georg Noeldeke, Armin Schmutzler, Philipp Zahn, and seminar audiences at DICE, the University of Southampton, the University of St. Gallen, and several conferences, for helpful discussions and comments. We gratefully acknowledge financial support from the Swiss National Science Foundation through grant No. 100018-178836.

†Stefan Buehler: University of St. Gallen, Institute of Economics, Varnbüelstrasse 19, 9000 St. Gallen, Switzerland (stefan.buehler@unisg.ch); Nicolas Eschenbaum: University of St. Gallen, Institute of Economics, Varnbüelstrasse 19, 9000 St. Gallen, Switzerland (nicolas.eschenbaum@unisg.ch)

1 Introduction

The prices of many goods and services vary across time and consumers. For instance, retail chains routinely offer temporary discounts for selected items sold through their stores; airlines run customer retention programs to make special offers to selected customer groups; sellers on online platforms use algorithms to dynamically adjust their prices. Yet, there are also firms that rarely change their prices. This raises the question of how sellers with market power set dynamic prices.

The economic literature has long emphasized the importance of "Coasian dynamics": The seller of a durable good who cannot commit to future prices has an incentive to lower the prices for (negatively selected) non-buyers over time, since high-value buyers purchase early on.[1] Yet, recent work has shown that monopoly pricing is not always governed by Coasian dynamics. For instance, there is no commitment problem for the seller, and no dynamics arise, if the potential buyers of a durable good have access to a second, durable outside option with strictly positive value (Board and Pycia, 2014). Similarly, the optimal price remains constant if only high-value rather than low-value consumers remain in the market ("positive selection") for a rental good (Tirole, 2016).[2] Finally, if the seller offers two durable varieties (rather than one), then Coasian dynamics apply, but they generally do not lead to zero profits in the limit (Nava and Schiraldi, 2019).

This paper employs a unified analytical framework to study a broad class of dynamic monopoly pricing problems, including various unexplored settings (e.g., settings with multiple rental varieties).[3] This framework highlights that the driving force behind pricing dynamics—as opposed to the repeated play of static monopoly

[1]The lack of commitment constrains the monopolist's market power, and in the limit the profit converges to zero if all trade takes place in the "twinkle of an eye", as conjectured by Coase (1972) and formally established by Stokey (1981), Bulow (1982), Fudenberg et al. (1985), Gul et al. (1986), and Ausubel and Deneckere (1989).

[2]If the seller offers a rental good and both negative selection (for non-buyers) and positive selection (for loyal buyers) are at work, then Coasian dynamics lead to "behavior-based pricing" (Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Buehler and Eschenbaum, 2020).

[3]Nava and Schiraldi (2019) study an extension of their setting in which consumers may return to the market after purchase, but focus on the case in which it is not profitable for the seller to exploit this.

prices—is the seller's incentive to "trade up" consumers to higher-valued consumption options: Faced with a set of buyers who can be traded up to a higher-valued consumption option, the seller has an incentive to cut the price of this higher-valued option and benefit from the larger surplus after trading up. The intuitive notion of trading up extends the logic of Coasian dynamics, which apply to the pricing of durable goods for previous non-buyers, to the pricing of durable or non-durable goods for any consumer who can select an option more valuable to them than the previously chosen one.

Specifically, we consider a monopolist offering two varieties of a good at zero marginal cost and (history-contingent) prices. In each period, consumers either consume one of the varieties or refrain from consumption (unit-demand). Consumers start the game in the same consumption option and have fixed values of the two varieties that are private information. A fixed set of admissible transitions between the three consumption options governs which choices are available to consumers given their choice in the previous period. Hence, if one variety is inaccessible then our setting reduces to a one-variety problem. For a consumer that may not select the outside option in a given period, we impose the price of their previous consumption choice to be set to zero today to prevent expropriation. Thus, an absorbing variety can be viewed as the sale of a durable good, and a variety that can be purchased in every period as a rental good. We are interested in characterizing the pricing dynamics in Perfect Bayesian Equilibrium (PBE).

We derive three key results. We begin by studying the pricing dynamics at histories of play at which there do not exist trading-up opportunities for the seller. In our first result, we consider a history at which all consumers previously chose their most-preferred (non-absorbing) consumption option, and where the transition to the outside option today is admissible.[4] We show that the best the seller can do at such a history is to let buyers continue with purchasing their most-preferred variety at a constant price for all future periods, and that no dynamics in realized consumption choices or paid prices emerge along the equilibrium path starting at this history. The result is reminiscent of Tirole (2016)'s finding that it is optimal to offer a constant price to loyal buyers of a single rental good in a positive selection

[4] This is the only interesting case in which there are no trading-up opportunities at a given history. In the other two (trivial) cases, pricing dynamics are de facto excluded by construction.

setting where the outside option is absorbing.[5] Our analysis of histories without trading-up opportunities demonstrates the crucial role trading up plays for pricing dynamics to emerge.

Next, we analyze settings in which there are no trading-up opportunities in the monopoly outcome of the *static* game. In our second result, we show that in this case, the seller cannot do better than obtain the static monopoly profit $\pi(p^m)$ in every period. In the essentially unique PBE of the game, the seller's present discounted profit is the repeated monopoly profit and thus the seller does not face a commitment problem.[6] The result establishes that the seller can never benefit from dynamic pricing if she can exhaust all trading-up opportunities by implementing the static monopoly outcome right at the start of the game. This is the case, for instance, if the potential buyers of a durable good have access to an additional durable outside option with strictly positive value (Board and Pycia, 2014), or in a setting with positive selection (Tirole, 2016).

We complete our analysis by studying histories at which there exist trading-up opportunities for the seller and establish how trading up drives dynamic pricing. In our third result, we show that for any history with trading up opportunities, the seller trades up buyers by lowering prices along the equilibrium path following this history. Hence, as long as trading-up opportunities remain, dynamic pricing continues as the game progresses. However, trading up does not lead to prices below $\bar{p}$, which are the prices that implement the optimal outcome in the static game for the seller conditional on leaving no trading-up opportunities, and the seller's present discounted profit is bounded from below by the repeated static profit that prices $\bar{p}$ yield, implying that in many settings the seller is guaranteed to earn a positive profit. We further show that whether or not the pricing dynamics are played out in finite time depends on the setting under study.

For example, in the setting with two durable varieties and a non-absorbing outside option, the seller is lowering prices over time in line with Coasion dynamics, but may still be able to earn a strictly positive profit at prices that exhaust all

[5]In the positive selection setting consumers all begin the game in the consumption choice of the single variety and once choosing the outside option can never return to consumption, so that all histories at which the seller can choose a price do not have trading-up opportunities.

[6]The equilibrium is essentially unique in the sense that the seller obtains the repeated monopoly profit in any PBE of the game.

trading-up opportunities in the static game (Nava and Schiraldi, 2019). These authors show that intra-temporal price discrimination can partially make up for the loss of market power due to inter-temporal price discrimination and prevent zero profits. We find that this result extends to all settings in which the seller can exhaust all trading-up opportunities at once while achieving positive profits, such as a "mixed" setting with one durable and one rental variety, provided the durable variety is less-preferred by all consumers.

Our results imply that the pricing dynamics in a broad class of dynamic monopoly pricing problems depend on whether or not the monopoly outcome in the static game leaves trading-up opportunities to the seller and we classify settings accordingly into "Coasian" and "Non-Coasian". If the monopoly outcome leaves no trading-up opportunities (Non-Coasian settings), then the seller does not face a commitment problem and the profit-maximizing solution is to implement the repeated static monopoly outcome irrespective of commitment ability. Instead, if there are trading-up opportunities at the static monopoly outcome (Coasian settings), then the monopolist will lower prices to trade up consumers over time. Our analysis highlights the key role of Coase's original insight: Pricing dynamics emerge only in settings that are Coasian in nature. This finding can be given a simple intuitive interpretation, since trading up can be understood as exploiting (additional) gains from trade. Thus, if the seller can exhaust all gains from trade at once in the first period, she has no incentive to change prices over the course of the game.

This paper contributes to an extensive literature on the pricing of a single durable good (e.g Coase, 1972; Fudenberg et al., 1985; Gul et al., 1986; Sobel, 1991; Kahn, 1986; Bond and Samuelson, 1984; Fuchs and Skrzypacz, 2010), of multiple varieties of a durable good (e.g. Nava and Schiraldi, 2019; Board and Pycia, 2014), and of vertically differentiated durable products (Hahn, 2006; Inderst, 2008; Takeyama, 2002). Our work differs by proposing a unified analytical framework that covers a broad class of dynamic monopoly pricing problems, including settings that have not been studied before (e.g., the setting with two rentals and a non-absorbing outside option). In doing so we also add to the analysis of settings with positive selection (Tirole, 2016), as we allow for an absorbing outside option. Our framework shows how the single-variety positive selection setting can be extended to cover multiple varieties while leaving the key insight that the seller faces no commitment problem and optimally chooses constant monopoly prices unaffected. In addition, we contribute to the literature on behavior-based pricing (e.g. Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Taylor, 2004; Buehler and Eschenbaum, 2020). In contrast to recent work by Rochet and Thanassoulis (2019), we focus on settings with unit-demand and do not allow for varieties to be sold as a bundle. The paper is also related to the marketing literature. Our notion of trading up is closely linked to what is commonly known as "upselling" in marketing (e.g., Blattberg et al., 2008; Aydin and Ziya, 2008; Wilkie et al., 1998). The key difference is that upselling generally refers to the upgrading of loyal buyers to a more expensive product, whereas trading up applies to buyers and non-buyers alike.

The remainder of the paper is organized as follows. Section 2 introduces the analytical framework, formalizes the notion of trading-up opportunities, and sketches various applications. Section 3 provides a skimming property for the unified analytical framework and explains how it translates into the skimming results known from the literature for specific settings. Section 4 analyzes dynamic monopoly pricing in the absence of trading-up opportunities. Section 5 characterizes dynamic monopoly pricing in the presence of trading-up opportunities. Section 6 concludes and offers directions for future research.

2 Analytical Framework

Consider a monopolist that sells two varieties of a good, a and b, at zero marginal cost to a measure of consumers with unit-demand per period. Consumers either purchase one of the varieties or select the outside option in every period. Their value profiles $v = (v_a, v_b)$ are fixed, private information, and distributed according to a measure $\mathcal{F}$ on the unit square $[0, 1]^2$. The associated cumulative distribution is F, with density f, and V is the support. F_i denotes the marginal cumulative distribution of variety i, while f_i and V_i denote the respective density and support.

Time is discrete and indexed by $t = 0, ..., T$, where T is finite or infinite. All players share the same discount factor $\delta \in (0, 1)$. In every period t, buyers make a discrete choice $x^t \in X$, where

$$X \equiv \{(1, 0), (0, 1), (0, 0)\},$$

is the set of states with elements $a = (1, 0)$, $b = (0, 1)$, and $o = (0, 0)$, where o is the outside option. Let $\bar{x} \in X$ be the initial state for all buyers. A sequence of choices x^t from period t onward is a *consumption path* $\mathbf{x}^t = (x^t, x^{t+1}, ..., x^T)$ that gives rise to (present discounted) *total consumption* $\chi(\mathbf{x}^t) = \sum_{\tau=t}^{T} \delta^{\tau-t} x^\tau$. A consumption path is *admissible* if all transitions from state to state along the entire path are within the set of admissible transitions $\Gamma \subset X \times X$. We place no restrictions on Γ, except that transitions from a state to itself are always admissible, that is, $(o, o), (a, a), (b, b) \in \Gamma$. A state $x \in X$ is *absorbing* if no other state $x' \in X$ is accessible from x, that is, $(x, x') \notin \Gamma$. Let $\Delta^t = \sum_{\tau=t}^{T} \delta^{\tau-t}$ denote the (present discounted) *number of periods* from t on.[7]

Each variety can be sold as a durable good or a per-period "rental" service (Hart and Tirole, 1988). A durable variety is sold once and for all future periods. For a buyer who purchases the durable variety $i \in (a, b)$ in period t, we therefore have $x^t = i$ and $x^\tau = i$ for all $\tau \geq t$ by definition. A rental variety i, in turn, is sold in every period anew, and allows for transitions to and from i in every period. To simplify exposition, we will henceforth refer to the setting with two absorbing varieties and a non-absorbing initial state $\bar{x} = o$ as the "two durables" setting. Similarly, we will refer to the least restrictive setting with all transitions being admissible and initial state $\bar{x} = o$ as the "two rentals" setting.

Figure 1 illustrates the two rentals setting. All possible transitions are admissible, that is, $\Gamma = \{(a, a), (b, b), (o, o), (a, b), (b, a), (o, a), (o, b), (a, o), (b, o)\}$. Note that the vertices indicate the states $X = \{a, b, o\}$, with initial state $\bar{x} = o$, while the arcs and brackets $(x, x') \in \Gamma$ represent the admissible transitions.

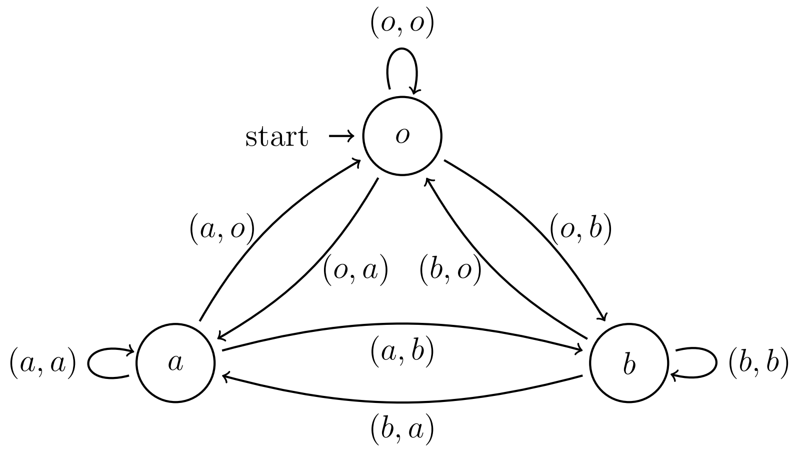
2.1 Prices, Histories, and Solution Concept

All players are risk-neutral. In each period t, the monopolist selects a price profile $p^t = (p_a^t, p_b^t) \in [\psi, 1]^2$, with $\psi < 0$,[8] for every history of play. Buyers then choose to purchase one of the varieties or forego consumption. If buyers cannot transition from their current state $i \in (a, b)$ to the outside option ($(i, o) \notin \Gamma$), then the period-t price for variety i, p_i^t, is set to zero by assumption, and the seller only chooses

[7] Throughout, we will consistently omit the exponent for all expressions if $t = 0$.

[8] The assumption on the set of prices ensures the monopolist's action set is compact.

Figure 1: States and transitions in the two rentals setting



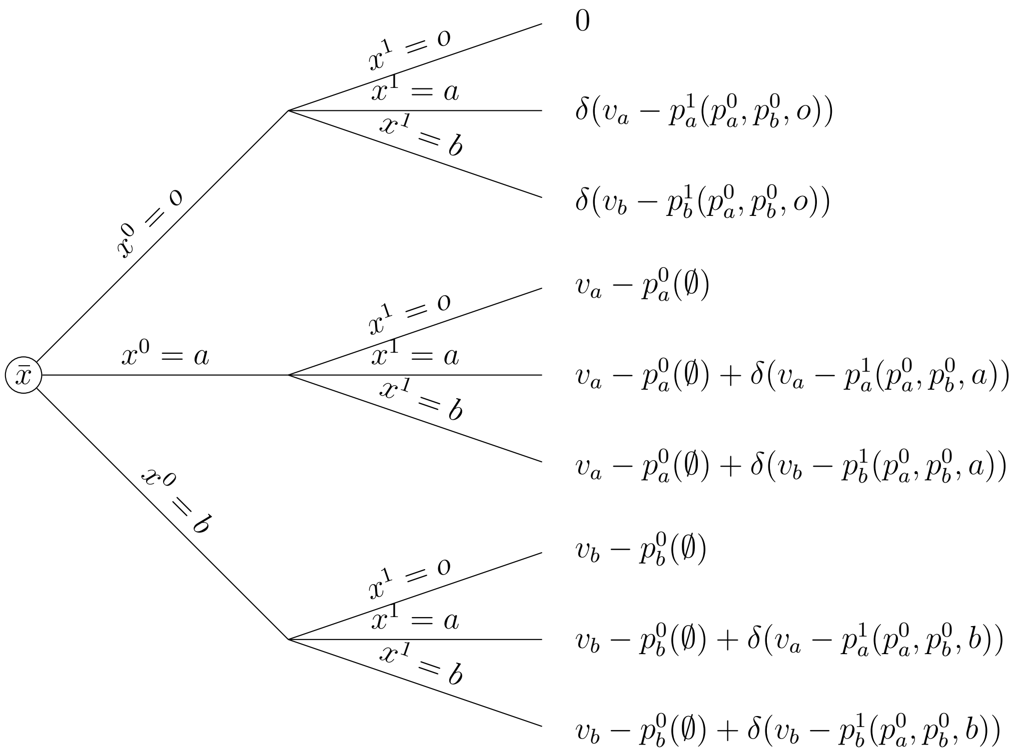
the price for the other variety $j \neq i$.[9] Let $\rho(\mathbf{x}) = \sum_{t=0}^{T} \delta^t p^t \cdot x^t$ denote the (present discounted) *total payment* made along consumption path $\mathbf{x} = (x^0, x^1, ..., x^T)$. Similarly, let $\nu(v, \mathbf{x}) = v \cdot \chi(\mathbf{x})$ be the (present discounted) *total value* obtained by a buyer with value profile v along consumption path $\mathbf{x}$. We can then write the (present discounted) total indirect utility obtained by a buyer with value profile v along consumption path $\mathbf{x}$ compactly as $\nu(v, \mathbf{x}) - \rho(\mathbf{x})$.

Figure 2 illustrates the admissible consumption paths $\mathbf{x}$ and corresponding utilities obtained by a consumer with value profile $v = (v_a, v_b)$ for a setting with two rentals and two periods ($T = 1$). For instance, the lowest branch in Figure 2 depicts the "always-b" path $\mathbf{x}_b = (b, b)$ with consumption $\chi(\mathbf{x}_b) = (0, 1 + \delta)$, value $\nu(v, \mathbf{x}_b) = (1 + \delta)v_b$, payment $\rho(\mathbf{x}_b) = p_b^0 + \delta p_b^1(p_a^0, p_b^0, b)$, and utility $\nu(v, \mathbf{x}_b) - \rho(\mathbf{x}_b) = v_b - p_b^0(\emptyset) + \delta(v_b - p_b^1(p_a^0, p_b^0, b))$.

A period-t seller history, h^t, records a sequence of previous consumption choices $(x^0, ..., x^{t-1})$ and price profiles $(p^0, ..., p^{t-1})$, with $h^0 = \emptyset$. A period-t buyer history, $\hat{h}^t$, consists of the seller history h^t and the period-t price profile $p^t(h^t) = (p_a^t(h^t), p_b^t(h^t))$ offered to consumers with seller history h^t. The set of period-t seller histories is denoted by H^t, and the set of all seller histories by $H = \cup_{t=0}^{T} H^T$. Similarly, the set of period-t buyer histories is denoted by $\hat{H}^t$, and the set of all buyer histories by $\hat{H} = \cup_{t=0}^{T} \hat{H}^T$. Let $V(h^t) \subseteq V$ denote the subset of consumers

[9]This assumption is consistent with our interpretation of an absorbing variety as the sale of a durable good and excludes the expropriation of "captured" buyers.

Figure 2: Consumption paths and utilities for two rentals and two periods



with the same seller history h^t.

We let $\Pi(h^t)$ denote the (present discounted) value of the seller's profit in the dynamic game obtained from buyers with history $h^t \in H^t$. The seller's profit in the static game, in turn, is given by

$$\pi(p) = \sum_{i \in (a,b)} p_i \mathcal{F}\left(v \in V \middle| i = \arg \max_{x \in X \wedge (\bar{x}, i) \in \Gamma} \{(v - p) \cdot x\}\right).$$

Let $\pi(p^m)$ denote the supremum of the seller's profit in the static game, with associated price profile p^m. The profit supremum with associated prices must always exist since profits are bounded from below by zero (the seller can always price out all buyers) and from above by 1, whereas price profiles p that maximize the seller's profit, $p \in \arg\max \pi(p)$, are not guaranteed to exist. For convenience, we will refer to $\pi(p^m)$ as the monopoly profit.

A behavioral strategy for buyers is denoted by $\hat{\sigma}$ and determines the probability distributions over the consumption choices $x \in X$ made by buyers at every possible history. In line with the literature, we assume that at any possible history the set of buyers making the same consumption choice is a measurable set. A behavioral strategy for the seller is denoted by σ and determines the probability distribution over the prices $p \in [\psi, 1]^2$ set by the seller as a function of the history of play. A *Perfect Bayesian Equilibrium* (PBE) is a strategy profile $\{\sigma, \hat{\sigma}\}$ and updated beliefs about the buyers' values along the various consumption paths, such that actions are optimal given beliefs, and beliefs are derived from actions from Bayes' rule whenever possible.

2.2 Trading-Up Opportunities

We will say that there exists a *trading-up opportunity* for the seller if there are buyers in a given state who can transition to a strictly higher-valued state:

Definition 1 (TUO) *There exists a trading-up opportunity for the seller if there are buyers for whom transitions to a strictly higher-valued state are admissible, that is,*

$$\exists v \textit{ in state } x \textit{ with } f(v) > 0 \textit{ and } x, x' \in X \textit{ s.t. } (x, x') \in \Gamma \textit{ and } v \cdot x' > v \cdot x.$$

Let Ω denote the set of price profiles $p = (p_a, p_b)$ that induce an allocation which leaves no trading-up opportunities for the seller in the static game,

$$\Omega = \left\{ p \in \mathbb{R}^2 \middle| \max_{x \in X \wedge (\bar{x}, x) \in \Gamma} (v - p) \cdot x \;\Rightarrow v \cdot x > v \cdot x' \text{ or } (x, x') \notin \Gamma \;\forall v \in V \right\}.$$

Intuitively, any price profile $p \in \Omega$ must induce an allocation at which all buyers either choose their most-preferred state among those that are accessible from the initial state, or an absorbing state. Thus, in a setting with two durables, $p \in \Omega$ is equivalent to market-clearing. In a setting with two rentals, $p \in \Omega$ is equivalent to market-clearing and efficiency.[10] Finally, we let $\bar{p} \in \Omega$ denote a price profile that is associated with the supremum of the profit obtainable in the static game conditional on leaving no trading-up opportunities, $\pi(\bar{p})$. The price profile $\bar{p}$ will play a key role in our analysis below. This profit supremum with associated prices

[10] We follow Nava and Schiraldi (2019) in referring to price profiles that ensure all buyers choose their preferred (accessible) variety as *efficient*, since they maximize total welfare when marginal costs are zero.

always exists as long as $\Omega \neq \emptyset$. This is guaranteed, unless there exists a state that is accessible over the course of the game, but not accessible from the initial state, since otherwise we always have $p = (0,0) \in \Omega$. We do not study these more unorthodox settings in detail.

Figure 3: Demand segments in the static game for given p (panel (a)), and profiles $p \in \Omega$ with full support (panel (b)) or linear support (panel (c)) for two rentals

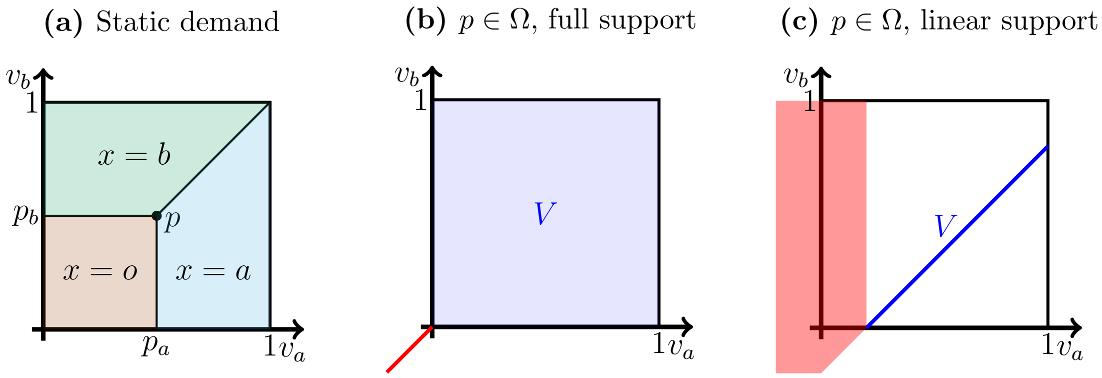


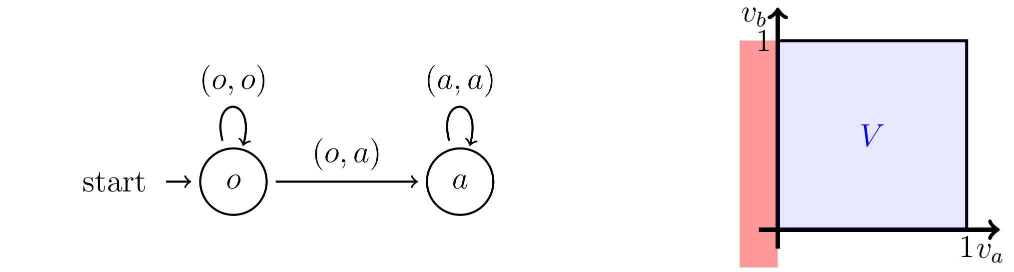
Figure 3 illustrates for two rentals how buyers self-select in the static game for a given price profile p and price profiles (in red) that therefore satisfy $p \in \Omega$ for two different supports. Specifically, panel (a) shows the static demand segments for a given price profile $p = (0.5, 0.5)$ and indicates, for instance, that all buyers with a value profile $v < p$ choose the outside option, $x = o$. Panels (b) and (c) depict the price profiles that leave no trading-up opportunities with full and linear support, respectively. With two rentals, $p \in \Omega$ requires that all buyers choose their most-preferred variety, as otherwise there are trading-up opportunities from one variety to the other, or from the initial state to each variety. Thus, with full support only non-positive price profiles on the diagonal satisfy $p \in \Omega$ (panel (b)), whereas with an increasing linear support that lies to the right of the diagonal through the type space ("vertical differentiation", panel (c)), any price profile that ensures $x = a$ for all types in the support satisfies $p \in \Omega$, since all buyers prefer a to b .

2.3 Applications

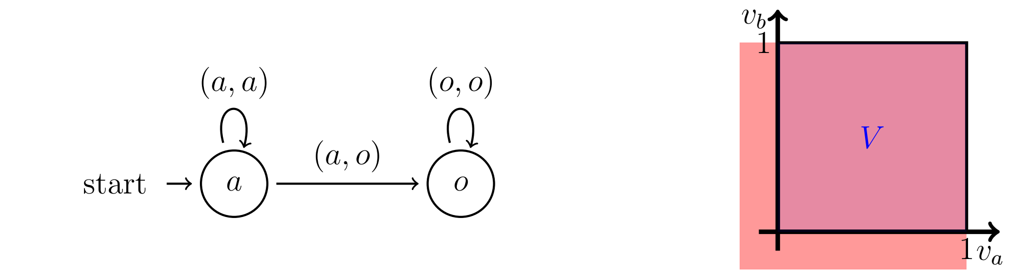
Our analytical framework covers a broad class of settings that are characterized by the tuple $(\bar{x}, \Gamma, \mathcal{F})$ and can be illustrated in two complementary graphs: one

Figure 4: Three examples: Accessible states and admissible transitions (left), and price profiles $p \in \Omega$ with a full support (right)

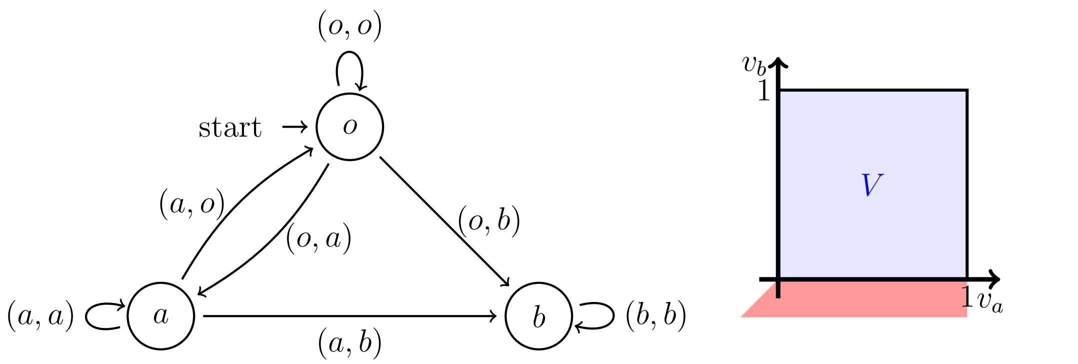
(a) Single durable variety a



(b) Positive selection, with single variety a and initial state $\bar{x} = a$



(c) Mixed setting, with rental variety a, durable variety b, and initial state $\bar{x} = o$



showing the accessible states and admissible transitions, and one showing the support V of the measure $\mathcal{F}$. Figure 4 provides three examples that are drawn for a full support V on the unit square $[0, 1]^2$. We also indicate price profiles that satisfy $p \in \Omega$ in red.[11]

In the setting with a single durable variety a (Figure 4a), $p \in \Omega$ requires that $p_a \leq 0$ (whereas the price p_b remains unrestricted) given a full support, which implies that $\pi(\bar{p}) = 0$. In the positive selection setting with one variety a and initial state $\bar{x} = a$ (Figure 4b), in turn, all price profiles satisfy $p \in \Omega$, which implies that $\pi(\bar{p}) = \pi(p^m) > 0$ with a full support. Finally, in the mixed setting with rental variety a, durable variety b, and initial state $\bar{x} = o$ (Figure 4c), $p \in \Omega$ requires that the price of the durable variety b is non-positive, whereas the price of the rental variety a can be positive and thus $\pi(\bar{p}) > 0$ with a full support.

We pay particular attention to the price profiles $p \in \Omega$ that leave no trading-up opportunities in the static game, because our analysis below will demonstrate that in order to characterize the dynamics in equilibrium, it is sufficient to determine the supremum of the profit obtained in the static game conditional on leaving no trading-up opportunities, $\pi(\bar{p})$, and examine whether it coincides with the monopoly profit $\pi(p^m)$.

3 A Unified Skimming Result

We first show that in equilibrium the seller's beliefs about the value profiles of buyers satisfy a skimming property (see the Appendix for the proof):

Lemma 1 *Consider buyers with common history $h^t \in H^t$.*

(i) If a buyer with value profile v obtains a higher (present discounted) total indirect utility along path $\mathbf{x}_k^t$ than along path $\mathbf{x}_l^t$, $\chi(\mathbf{x}_k^t) \neq \chi(\mathbf{x}_l^t)$, then so does a buyer with value profile $\tilde{v} \neq v$ such that

$$(\tilde{v} - v) \cdot (\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t)) \geq 0. \tag{1}$$

(ii) In any PBE, if a buyer with value profile v prefers consumption choice $x^t = x$ to $x^t = x'$, $x' \neq x$, then so does a buyer with value profile $\tilde{v} \neq v$ such that

$$\begin{aligned}(\tilde{v} - v) \cdot (x - x') &+ \delta \left[\min_{\mathbf{x}^{t+1} \in \{\mathbf{X}^{t+1} | x^t = x\}} \left\{ (\tilde{v} - v) \cdot \chi(\mathbf{x}^{t+1}) \right\} \right] \\ &- \delta \left[\max_{\mathbf{x}^{t+1} \in \{\mathbf{X}^{t+1} | x^t = x'\}} \left\{ (\tilde{v} - v) \cdot \chi(\mathbf{x}^{t+1}) \right\} \right] \geq 0, \end{aligned} \tag{2}$$

[11] For simplicity, inaccessible states (and transitions out of these states) are omitted.

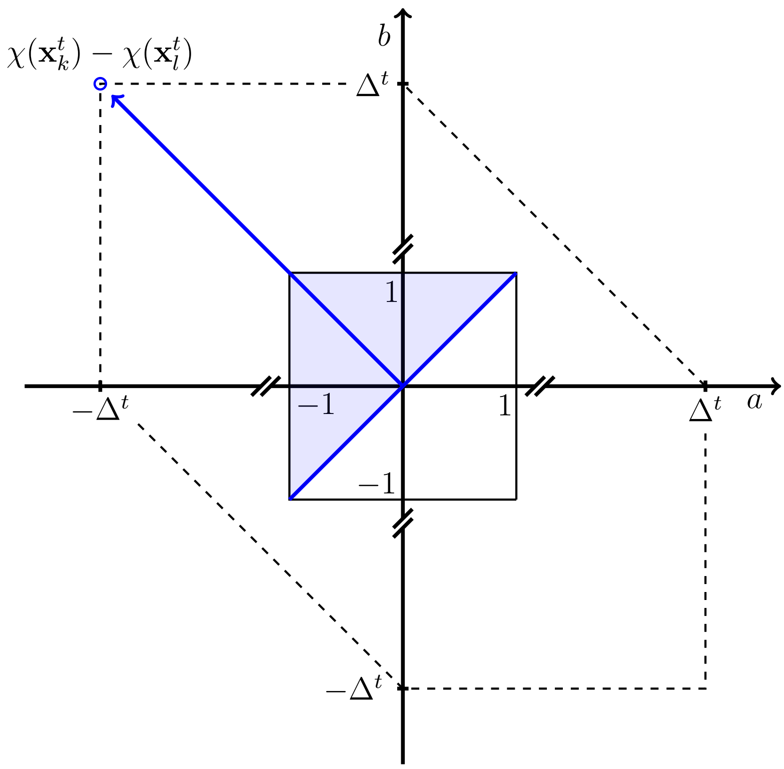
where $\{\mathbf{X}^{t+1}|x^t\}$ is the set of admissible consumption paths after consumption choice x^t.

Part (i) shows that for two buyers with different value profiles to have the same preferences over the total indirect utilities obtained along two distinct consumption paths, the relative value profiles of the two buyers and the relative consumption along the two paths must be aligned. This implies that it is generally not sufficient for a type to have strictly higher values for both varieties to satisfy the condition; instead, the relative values $(v_a - v_b)$ must be considered. For example, if following path $\mathbf{x}_k^t$ instead of $\mathbf{x}_l^t$ implies obtaining relatively less consumption of a and relatively more consumption of b and type v is willing to follow path $\mathbf{x}_k^t$, then only types $\tilde{v}$ who do not prefer a relatively more than b compared to type v will make the same choice. However, if path $\mathbf{x}_k^t$ implies obtaining more consumption of a compared to path $\mathbf{x}_l^t$, while the consumption of b is equal along the two paths, then for type $\tilde{v}$ to have the same preference $\tilde{v}_a > v_a$ is sufficient. Hence, restricting the set of admissible consumption paths will make it easier to satisfy skimming, and restricting the setting to a one-variety problem will yield the classic skimming results.

This intuition from part (i) can be captured geometrically as illustrated in Figure 5, where the solid square shows the possible differences in value profiles $(\tilde{v} - v)$ and the dashed lines indicate the possible differences in total consumption $(\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t))$ along the two consumption paths. For the particular difference in total consumption $(\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t))$ depicted, only consumers with value profiles in the shaded area satisfy condition (i) in Lemma 1. Note that if b is an inaccessible state, then differences in total consumption will yield vectors along the a-axis, in which case any higher type $\tilde{v}_a > v_a$ will satisfy (1) if $\mathbf{x}_k^t$ implies more consumption of a than $\mathbf{x}_l^t$.

Part (ii) states the skimming property applicable in PBE in terms of the values resulting from current consumption choices and (future) admissible consumption paths following these choices. As the purchase decisions may differ at every time t, again the difference in total consumption must be considered for skimming to be satisfied. Therefore, the conditions for any two paths in (1) and for a period-t purchase in (2) must both account for the admissible consumption paths and the resulting total consumption in the future.

Figure 5: Illustration of the skimming condition



Condition (2) nests well-known earlier skimming results for settings that are covered by our analytical framework. To see this, consider the classic setting with a single durable good, say a, and focus on consumers that have not yet purchased at time t. Let x be the purchase of the durable good, whereas x' is the choice of the outside option, and distinguish the following two cases: $\tilde{v}_a \geq v_a$ and $\tilde{v}_a < v_a$. In the first case, the minimum difference in total value after x equals the maximum difference in total value after x' and is given by $(\tilde{v}_a - v_a)\Delta^{t+1}$, so that (2) simplifies to $\tilde{v}_a \geq v_a$. In the second case, the minimum difference in total value after x is unchanged (but negative), while the maximum difference in total value after x' is 0, such that (2) cannot be satisfied. Thus, we obtain the standard condition $\tilde{v}_a \geq v_a$.[12] A similar result holds in the positive selection setting introduced by Tirole (2016) with a single variety a and an absorbing outside option. Consider the set of types that have purchased a in every previous period until time t. Let x

[12]We can obtain the same result from (1) by noting that purchasing today (path $\mathbf{x}_k^t$) rather than delaying (path $\mathbf{x}_l^t$) immediately implies that $\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t) \geq 0$.

denote the purchase of the good a and x' the choice of the outside option. Then, the minimum difference in total value after x is 0 if $\tilde{v}_a \geq v_a$ and $(\tilde{v}_a - v_a)\Delta^{t+1}$ if $\tilde{v}_a < v_a$, while the maximum difference in total value following x' is 0 for $\tilde{v}_a \geq v_a$ and $\tilde{v}_a < v_a$ because the outside option is an absorbing state. The skimming condition (2) then simplifies to $\tilde{v}_a \geq v_a$.[13]

Finally, consider the two durables setting (Nava and Schiraldi, 2019). Let x be the purchase of one of the two varieties, say a. Then x' is either b or o. Our above analysis of the setting with one durable shows that when $x' = o$, skimming is satisfied whenever $\tilde{v}_a \geq v_a$ if the maximum difference in total value after x' is $(\tilde{v}_a - v_a)\Delta^{t+1}$. Instead, if the maximum difference in total value after x$'$ is $(\tilde{v}_b - v_b)\Delta^{t+1}$, then skimming is satisfied whenever $(\tilde{v}_a - v_a)\Delta^{t} \geq \delta\Delta^{t+1}(\tilde{v}_b - v_b)$. When $x' = b$ in turn, the minimum difference in total value after x is $(\tilde{v}_a - v_a)\Delta^{t+1}$ while the maximum difference in total value after x' is $(\tilde{v}_b - v_b)\Delta^{t+1}$, since both varieties are absorbing, and so (2) becomes $\tilde{v}_a - v_a \geq \tilde{v}_b - v_b$. In conjunction, we obtain that skimming is satisfied if $\tilde{v}_a - v_a \geq \max\{0, \tilde{v}_b - v_b\}$.[14]

4 Dynamic Pricing Without Trading Up

We first study optimal pricing when there are no trading-up opportunities for the seller. It is useful to distinguish three different cases in which there are no trading-up opportunities at history $h^t \in H^t$ with associated state x^{t-1}:

(i) The state x^{t-1} is absorbing.

(ii) The state x^{t-1} is the most-preferred (accessible) state $i \in (a, b)$ for all buyers, and the transition to the outside option is *not* admissible, $(i, o) \notin \Gamma$.

(iii) The state x^{t-1} is the most-preferred (accessible) state $i \in (a, b)$ for all buyers, and the transition to the outside option is admissible, $(i, o) \in \Gamma$.

It is straightforward to see that pricing dynamics are excluded by construction in cases (i) and (ii). In case (i), buyers either cannot buy any variety $i \in (a, b)$ (if

[13]Again, we can obtain the result from (1) by noting that $\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t) \geq 0$ because the outside option is an absorbing state.

[14]We can obtain the same result from (1) by noting that for the comparison of two paths that feature different varieties in period t, we have $(\tilde{v} - v) \cdot (\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t)) \geq 0$ iff $\tilde{v}_a - \tilde{v}_b \geq v_a - v_b$.

$x^{t-1} = o$) or must make the same consumption choice i in every future period (if $x^{t-1} = i$), with the price p_i set to zero for all future periods by assumption (and the price p_j set arbitrarily). In case (ii), buyers can consume their most-preferred variety i in every future period at price zero by assumption. For case (iii), our first main result establishes that the seller cannot benefit from dynamic pricing, either.

Proposition 1 *Consider a history $h^t \in H^t$ at which all buyers purchased their most-preferred variety $i \in (a, b)$ in the previous period, $(i, o) \in \Gamma$, and assume that at the supremum of the one-shot game profit, $\pi^*(h^t)$, all buyers purchase variety i. Then, in any PBE no dynamics in realized consumption choices or paid prices emerge along the equilibrium path starting at history h^t.*

Proposition 1 shows that if, at a given history h^t, all buyers are in their most-preferred state and the seller cannot benefit from pricing out buyers in the one-shot game at this history, then the best the seller can do is to let all buyers continue with purchasing their most-preferred variety at a constant price. Thus, no dynamics in paid prices or realized consumption choices emerge along the equilibrium path following this history. Note that this result only pins down the price of the most-preferred variety i. For the price of variety j, the only requirement is that it must be sufficiently high to ensure that all buyers continue with purchasing variety i. Thus, dynamics in the price p_j^τ may still emerge, and numerous strategies for the seller may constitute a PBE. However, the seller cannot *benefit* from dynamic pricing.

The above analysis of histories at which there do not exist trading-up opportunities suggests that, if the profit-maximizing solution to the static game does not leave any trading-up opportunities to the seller, then no dynamics will emerge in equilibrium since the seller exhausts all gains from trade at once if she plays static-optimal prices in the first period. Our next result shows that this is indeed the case.

Specifically, we consider settings in which there are no trading-up opportunities in the static optimum, so that the monopoly profit $\pi(p^m)$ coincides with the supremum of the profit the seller can obtain in the static game conditional on leaving no trading-up opportunities, $\pi(\bar{p})$. We show that, in this case, the seller's maximum profit attainable in the dynamic game is the repeated monopoly profit, and the seller will be able to obtain this payoff irrespective of commitment ability. That is, the seller faces no commitment problem if there are no trading-up opportunities in the static monopoly optimum, and the equilibrium is *essentially unique* in the sense that the seller will obtain this profit in any PBE.

Borrowing terminology from Board and Pycia (2014), we say that the seller and buyers adopt *monopoly strategies* if, in every period t, (i) the seller plays p_i^m for every non-absorbing variety and $p_i^m \Delta^t$ for every absorbing variety for every history h^t at which a positive price for i is allowed, and (ii) all consumers behave as if they were maximizing per-period utility. We can then state the following result.

Proposition 2 *Suppose there are no trading-up opportunities in the static monopoly outcome, that is,* $\pi(p^m) = \pi(\bar{p})$*. Then,*

(i) the seller can do no better than obtain the repeated monopoly profit $\pi(p^m)$ *in every period* $t = 0, ..., T$*, that is,* $\Pi \leq \pi(p^m)\Delta$*.*

(ii) there exists a PBE in which the seller and buyers adopt monopoly strategies in every period $t = 0, ..., T$ *for any history* h^t*.*

(iii) the PBE is essentially unique, and the seller will thus always obtain the commitment profit irrespective of commitment ability.

Proposition 2 shows that the emergence of dynamic pricing crucially depends on the existence of trading-up opportunities in the static monopoly outcome. A profit-maximizing seller engages in dynamic pricing only if doing so allows her to trade up buyers to more valuable consumption options over the course of the game. Therefore, if the profit-maximizing solution in the static game leaves no trading-up opportunities, $\pi(p^m) = \pi(\bar{p})$, then the seller can simply repeat the static monopoly solution and obtain the commitment profit irrespective of commitment ability, since the monopoly strategies of the seller and buyers form a PBE. Moreover, the seller is guaranteed to obtain the maximum profit despite being unable to commit as the described PBE is essentially unique.

Therefore, it is sufficient to know the profit-maximizing solution of the static game to determine the outcome of the repeated game in settings with $\pi(p^m) = \pi(\bar{p})$. The following lemma provides an explicit characterization of such settings. Intuitively, there are two classes of settings in which $\pi(p^m) = \pi(\bar{p})$: first, settings that exclude trading-up opportunities for arbitrary price profiles (including static monopoly prices); second, settings where the distribution of consumption values is such that static monopoly prices happen to leave no trading-up opportunities.

Lemma 2 *The dynamic setting is characterized by the tuple $(\bar{x}, \Gamma, \mathcal{F})$. There are no trading-up opportunities for the seller*

(i) for arbitrary price profiles p, if the initial state $\bar{x}$ is absorbing.

(ii) for arbitrary price profiles p, if the initial state $\bar{x}$ is the (weakly) most-preferred state for all buyers, and all other accessible states are absorbing (positive selection).

(iii) for any price profile p such that the lowest-value consumer obtains a strictly positive utility in at least one of the accessible states, if the initial state $\bar{x}$ is the (weakly) least-preferred state for all buyers and all accessible states are absorbing (Board and Pycia, 2014).

(iv) for arbitrary price profiles p, if all buyers have the same preference ranking over all accessible states and only transitions from a preferred to a (weakly) less-preferred state are admissible (trading down).

Otherwise, there exist trading-up opportunities at static monopoly prices, except if $\mathcal{F}$ is such that $\pi(p^m) = \pi(\bar{p})$.

Lemma 2 characterizes the settings in which Proposition 2 applies. Case (i) is trivial in the sense that no transition out of the initial state is admissible. In all other cases, the initial state $\bar{x}$ is assumed to be non-absorbing. Case (ii) describes a positive-selection setting where the initial state is the most-preferred state for all consumers who can transition to less-preferred absorbing states only. Tirole (2016) provides an in-depth analysis of such a setting with a single non-absorbing variety as the initial state and the outside option as an absorbing state.[15] Lemma 2 shows that we can extend this setting to allow for a second variety while ensuring that $p^m = \bar{p}$ continues to apply by requiring that the second variety is less-preferred and absorbing. This arguably is the essence of positive selection: all buyers start in the most-preferred state and can only transition to less-preferred states in which they

[15] See Figure 4b for an illustration of positive selection with a single variety a.

are "captured", that is, they can only trade down but never back up. With a single variety, this is ensured if the outside option is absorbing. Case (iii) describes a setting in which the initial state is the least-preferred state for all consumers who can transition to more-preferred absorbing states and obtain a strictly positive utility in at least one of them. Board and Pycia (2014) provide a detailed analysis of such a setting where the initial state is the non-absorbing outside option, and the seller offers a single absorbing variety, but consumers can also choose a second absorbing outside option with a strictly positive value for all buyers. We can embed this analysis in our framework by restricting the price for one durable variety to be strictly below the lowest value in the support. Case (iv) describes settings in which the initial state is allowed to be any of the three states, but the admissible transitions and value profiles of buyers are such that they may only ever trade down.

In any other setting, static monopoly prices will typically leave trading-up opportunities in the static game. For example, consider a mixed setting with rental variety a and durable variety b, and assume that the initial state is the non-absorbing outside option (see Figure 4c). For a price profile to leave no trading-up opportunities, we must have that the market clears and that all types allocating themselves to the non-absorbing variety prefer it to the absorbing one. Thus, we need to check if $\pi(p^m)$ happens to satisfy these conditions for the given measure $\mathcal{F}$ and associated support V of consumers.

5 Dynamic Pricing With Trading Up

In this section, we study dynamic pricing if there are trading-up opportunities at the static monopoly outcome ($\pi(p^m) \neq \pi(\bar{p})$) and the seller cannot commit to future prices. Our next result shows that a profit-maximizing seller will engage in dynamic pricing after any history at which trading-up opportunities exist by repeatedly lowering prices until all trading-up opportunities are exhausted. This dynamic ends at prices $\bar{p}$, and may only be played out in finite time, if the static profit associated with these prices, $\pi(\bar{p})$, is strictly positive.

Proposition 3 *In any PBE,*

(i) for any history h^t at which there exist trading-up opportunities, the seller trades up a strictly positive measure of types along the equilibrium path.

(ii) the seller will never set a price for a variety i below $\bar{p}_i$ at any history h^t at which the transition to state i is admissible.

(iii) the seller's (present discounted) profit satisfies $\Pi \geq \pi(\bar{p})\Delta$.

(iv) for all trading-up opportunities to be exhausted in finite time, $\pi(\bar{p}) > 0$ is required.

Proposition 3 shows that trading-up opportunities are the driving force behind pricing dynamics. For a seller who faces trading-up opportunities and lacks commitment ability, it is strictly profit-maximizing to trade up (some) buyers to a higher-valued consumption option. By doing so, the seller can extract a larger surplus from these consumers. However, in order to induce buyers to trade up, the seller must lower the prices relative to the prices previously offered. Thus, as the game progresses, the seller is lowering prices to trade up more and more consumers. Since any price profile $p \in \Omega$ leaves no trading-up opportunities in the dynamic game, either, the seller will not want to set prices below $\bar{p}$. Hence, the dynamics come to an end at prices $\bar{p}$, provided that transitions to the respective consumption options are admissible.[16] This implies that the seller's profit in the absence of commitment ability is bounded below at $\pi(\bar{p})\Delta$. The time it takes for price dynamics to play out depends on the setting under study, but it is a necessary condition that the seller can obtain positive profits from playing prices $\bar{p}$ in the static game.

The proof of statement (iv) in Proposition 3 identifies the conditions under which trading up all types at once at a given history is the profit-maximizing choice for the seller and highlights that while $\pi(\bar{p}) > 0$ is always required, it further depends on the lowest values in the support. In particular, we need to distinguish between the case in which the lowest value in the support for a given variety is strictly positive (the "gaps" case), or zero (the "no gaps" case). The following corollary uses these findings to further clarify the time it takes for all pricing dynamics to play out for a number of settings.

[16] The latter qualification is required for price dynamics to end at $\bar{p}$, because otherwise any price is a best-response (including prices below $\bar{p}$)

Corollary 3.1 *In any PBE, all trading-up opportunities are exhausted*

(i) in finite time if only one variety is accessible and the lowest value is strictly positive (1 variety gaps).

(ii) in infinite time if only one variety is accessible and the lowest value is zero (1 variety no gaps).

(iii) in finite time if both varieties are accessible and both lowest values are strictly positive (2 varieties 2 gaps).

(iv) in finite or infinite time if both varieties are accessible and both lowest values are zero (2 varieties no gaps).

(v) in finite time if both varieties are accessible and one of the two lowest values is strictly positive (2 varieties 1 gap) and $\pi(\bar{p}) > 0$.

Proposition 3 highlights the connections among the different strands of literature on dynamic monopoly pricing. In the canonical setting with a single durable good and non-absorbing outside option as the initial state, trading-up opportunities exist for non-buyers only, since buyers of the durable good are captured in an absorbing state (see Figure 4a). Thus, if the seller lacks commitment ability, profit-maximizing prices for non-buyers are falling over time due to Coasian dynamics (Fudenberg et al., 1987; Coase, 1972). Since all consumers value the good more than the outside option, trading-up opportunities exist whenever a positive measure of non-buyers remains. A similar result emerges in the setting with two durables and the non-absorbing outside option as the initial state. Once again, the seller lowers the prices of the varieties until the market clears (Nava and Schiraldi, 2019). However, it is possible to clear the market with only one of the two prices set to zero, which allows the seller to still obtain a strictly positive profit. In a behavior-based pricing setting with a single rental good and the non-absorbing outside option as the initial state, in turn, trading-up opportunities exist for non-buyers only, whereas loyal buyers cannot be traded up. Hence, prices fall only for non-buyers (Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Tirole, 2016; Buehler and Eschenbaum, 2020).

This result implies that the seller is eventually forced to reduce prices to zero in many settings. Conversely, there are settings in which the seller can avoid the zero-profit lower bound. For example, in a mixed setting with one durable and one rental variety, the seller can earn a positive profit if the rental variety provides a higher value to all buyers (see Figure 4c). Thus, a seller of a rental good may want to introduce an absorbing low-quality variety to avoid the zero-profit lower bound. Hence, Nava and Schiraldi (2019)'s insight for two durable varieties that intra-temporal price discrimination can shield the seller from zero profits that arise from inter-temporal price discrimination extends to a setting with mixed varieties.

Our analysis shows that for a broad class of dynamic monopoly pricing problem, the pricing dynamics can be characterized based on the comparison of the monopoly outcome and the optimal outcome for the seller that leaves no trading-up opportunities in the static game. Specifically, depending on the existence of trading-up opportunities at the monopoly outcome in the static game, we can classify dynamic monopoly pricing problems into one of two categories, namely "Coasian" and "Non-Coasian". In Coasian settings the two profits do not coincide ($\pi(p^m) \neq \pi(\bar{p})$), so that there are trading-up opportunities, and prices for buyers who can be traded up are falling over time if the seller lacks commitment ability (Proposition 3). In Non-Coasian settings the profits do coincide ($\pi(p^m) = \pi(\bar{p})$), so that there are no trading-up opportunities, and the seller cannot do better than obtain the repeated monopoly outcome of the static game (Proposition 2).

Table 1 classifies a selection of dynamic monopoly pricing settings into Non-Coasian and Coasian. Clearly, the four settings listed in Lemma 2 are Non-Coasian in nature. The Coasian settings are further divided based on whether or not $\pi(\bar{p})$ is strictly positive.[17] For example, the seller of a single variety must set the price at the lowest value in the support in order to leave no trading-up opportunities (see Figure 4a for the case of an absorbing variety). Thus, the profit is strictly positive only if there is a "gap" (i.e., the lowest value in the support is strictly positive). In the two rentals setting, in turn, the seller must ensure that all types choose their preferred variety in order to leave no trading-up opportunities, implying that the profit is strictly positive only if there are two gaps. The seller of two durables or of mixed varieties, however, can obtain a strictly positive profit irrespective of the lowest values in the support.

[17] All Coasian settings presume that there are strictly positive measures of buyers for each accessible variety.

Table 1: Classification of dynamic monopoly pricing settings

Non-Coasian $\pi(p^m) = \pi(\bar{p})$	Coasian $\pi(p^m) \neq \pi(\bar{p})$	
	$\pi(\bar{p}) > 0$	$\pi(\bar{p}) = 0$
Absorbing initial state	1 variety, 1 gap	1 variety, no gap
Positive selection (Tirole 2016)	2 rentals, 2 gaps	2 rentals w/out 2 gaps
Board & Pycia (2014)	2 durables*	
Trading down	Mixed varieties*	

* If there are no 2 gaps we assume that there exist consumers with $v_i \neq v_j$ in the support and consumers that prefer the rental variety with mixed varieties.

6 Conclusion

This paper employs a unified analytical framework to study a broad class of dynamic monopoly pricing problems, including various unexplored settings (e.g., multiple rental varieties). We show that pricing dynamics emerge due to the seller's incentive to trade up buyers to higher-valued consumption options. Consequently, the dynamics in equilibrium can be characterized by studying two solutions of the static game: i) the monopoly outcome, and ii) the optimal outcome for the seller that leaves no trading-up opportunities. In Coasian settings, the two outcomes do not coincide, so that there are trading-up opportunities in the static monopoly outcome, and in equilibrium dynamics will arise until all consumers cannot be traded up any further. In Non-Coasian settings, in turn, the two outcomes coincide, so that there are no trading-up opportunities in the static monopoly outcome, and in the essentially unique equilibrium no dynamics will emerge. Examples for Coasian settings that are covered by our framework include settings with one or multiple durable or rental varieties. Examples for Non-Coasian settings include applications with positive selection (Tirole, 2016) with one or multiple varieties or a single durable variety with another durable outside option with strictly positive value (Board and Pycia, 2014).

Our analysis extends the logic of Coasian dynamics, which apply to the pricing of a durable good to non-buyers, to the pricing of durable or rental goods for buyers who can select a more valuable consumption option. It underlines the seminal contribution of Coase's original insight: Pricing dynamics emerge only in settings that are Coasian in nature.

Our findings can be given a simple intuitive interpretation, as trading up can be understood as exploiting (additional) gains from trade. Thus, trading-up opportunities in the static monopoly outcome means that there are unexploited gains from trade for the seller in the static monopoly outcome. If there are no trading-up opportunities in the static monopoly outcome, then the seller can exhaust all gains from trade in the first round by playing static optimal prices and has no incentive to change prices over the course of the game. Hence, no dynamics will arise. Otherwise, the seller has an incentive to exploit additional gains from trade by lowering prices over time.

References

Acquisti, A. and Varian, H. R. (2005). Conditioning prices on purchase history. *Marketing Science*, 24(3):367–381.

Armstrong, M. (2006). *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress: volume II*, chapter Recent developments in the economics of price discrimination. Cambridge University Press.

Ausubel, L. M. and Deneckere, R. J. (1989). Reputation in bargaining and durable goods monopoly. *Econometrica*, 57(3):511–31.

Aydin, G. and Ziya, S. (2008). Pricing promotional products under upselling. *Manufacturing & Service Operations Management*, 10(3):360–376.

Blattberg, R. C., Kim, B.-D., and Neslin, S. A. (2008). *Cross-Selling and Up-Selling*, pages 515–547. Springer New York, New York, NY.

Board, S. and Pycia, M. (2014). Outside options and the failure of the coase conjecture. *American Economic Review*, 104(2):656–71.

Bond, E. W. and Samuelson, L. (1984). Durable good monopolies with rational expectations and replacement sales. *The RAND Journal of Economics*, 15(3):336–345.

Buehler, S. and Eschenbaum, N. (2020). Explaining escalating prices and fines: A unified approach. *Journal of Economic Behavior & Organization*, 171:153 – 164.

Bulow, J. I. (1982). Durable-goods monopolists. *Journal of Political Economy*, 90(2):314–32.

Coase, R. (1972). Durability and monopoly. *Journal of Law and Economics*, 15(1):143–49.

Fuchs, W. and Skrzypacz, A. (2010). Bargaining with arrival of new traders. *American Economic Review*, 100(3):802–36.

Fudenberg, D., Levine, D. K., and Tirole, J. (1985). *Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information*, pages 73–98. Cambridge University Press, Cambridge, UK and New York.

Fudenberg, D., Levine, D. K., and Tirole, J. (1987). Incomplete information bargaining with outside opportunities. *The Quarterly Journal of Economics*, 102(1):37–50.

Fudenberg, D. and Villas-Boas, J. M. (2007). *Behavior-Based Price Discrimination and Customer Recognition*. Elsevier Science, Oxford.

Gul, F., Sonnenschein, H., and Wilson, R. (1986). Foundations of dynamic monopoly and the coase conjecture. *Journal of Economic Theory*, 39:155–190.

Hahn, J.-H. (2006). Damaged durable goods. *The RAND Journal of Economics*, 37(1):121–133.

Hart, O. D. and Tirole, J. (1988). Contract renegotiation and coasian dynamics. *The Review of Economic Studies*, 55(4):509–540.

Inderst, R. (2008). Durable goods with quality differentiation. *Economics Letters*, 100(2):173–177.

Kahn, C. (1986). The durable goods monopolist and consistency with increasing costs. *Econometrica*, 54(2):275–294.

Nava, F. and Schiraldi, P. (2019). Differentiated durable goods monopoly: A robust coase conjecture. *American Economic Review*, 109(5):1930–68.

Rochet, J.-C. and Thanassoulis, J. (2019). Intertemporal price discrimination with two products. *The RAND Journal of Economics*, 50(4):951–973.

Sobel, J. (1991). Durable goods monopoly with entry of new consumers. *Econometrica*, 59(5):1455–85.

Stokey, N. L. (1981). Rational expectations and durable goods pricing. *The Bell Journal of Economics*, 12(1):112–128.

Takeyama, L. N. (2002). Strategic vertical differentiation and durable goods monopoly. *The Journal of Industrial Economics*, 50(1):43–56.

Taylor, C. R. (2004). Consumer privacy and the market for customer information. *The Rand Journal of Economics*, 35(4):631–650.

Tirole, J. (2016). From bottom of the barrel to cream of the crop: Sequential screening with positive selection. *Econometrica*, 84(4):1291–1343.

Wilkie, W. L., Mela, C. F., and Gundlach, G. T. (1998). Does “bait and switch” really benefit consumers? *Marketing Science*, 17(3):273–282.

Appendix

Proof of Lemma 3

(i) Since type v obtains a higher total indirect utility along path $\mathbf{x}_k^t$ than along path $\mathbf{x}_l^t$ by assumption, we must have

$$\nu(v, \mathbf{x}_k^t) - \rho(\mathbf{x}_k^t) \geq \nu(v, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t).$$

Now, consider some type $\tilde{v} \neq v$. Then, we have

$$\begin{aligned}\nu(\tilde{v}, \mathbf{x}_k^t) - \rho(\mathbf{x}_k^t) &= \nu(v, \mathbf{x}_k^t) - \rho(\mathbf{x}_k^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t) \\ &\geq \nu(v, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t),\end{aligned}$$

since type v obtains a higher total indirect utility along path $\mathbf{x}_k^t$ than along path $\mathbf{x}_l^t$ by assumption. For type $\tilde{v}$ to obtain a higher total indirect utility along path $\mathbf{x}_k^t$, we must thus have

$$\nu(v, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t) \geq \nu(\tilde{v}, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t),$$

which can be rearranged to yield the result in (1).
(ii) To show (2), we follow a similar line of argument. Denoting the continuation valuation of a type v following choice x by $U(v, x)$ (suppressing h^t for brevity) and considering the choices x and x', respectively, we can write

$$\begin{aligned}(\tilde{v} - p) \cdot x + \delta U(\tilde{v}, x) &= (v - p) \cdot x + \delta U(v, x) + (\tilde{v} - v) \cdot x + \delta[U(\tilde{v}, x) - U(v, x)] \\ &\geq (v - p) \cdot x' + \delta U(v, x') + (\tilde{v} - v) \cdot x + \delta[U(\tilde{v}, x) - U(v, x)] \\ &\geq (\tilde{v} - p) \cdot x' + \delta U(\tilde{v}, x')\end{aligned}$$

or

$$(\tilde{v} - v) \cdot (x - x') + \delta[U(\tilde{v}, x) - U(v, x)] - \delta[U(\tilde{v}, x') - U(v, x')] \geq 0. \tag{3}$$

Since type v can always mimic the actions of type $\tilde{v}$ (and vice versa) by making the same consumption choices in every future period, the difference in continuation values from period $t+1$ onward must satisfy

$$\begin{aligned}&\min\left\{(\tilde{v} - v) \cdot \sum_{\tau=t+1}^{T} \delta^{\tau-t} \varphi^\tau(\tilde{v})\right\} \\ &\qquad \leq U(\tilde{v}, x) - U(v, x) \leq \max\left\{(\tilde{v} - v) \cdot \sum_{\tau=t+1}^{T} \delta^{\tau-t} \varphi^\tau(\tilde{v})\right\},\end{aligned} \tag{4}$$

where $\varphi^\tau(\tilde{v}) = (\varphi_a^\tau(\tilde{v}), \varphi_b^\tau(\tilde{v}))$ indicates the probabilities associated with type $\tilde{v}$ selecting variety a and variety b, respectively, in period τ. For given types $(\tilde{v}, v)$, the min and the max must exclude randomization by consumers, so that (4) becomes

$$\min_{\mathbf{x}^{t+1}\in\{\mathbf{X}^{t+1}|x^t\}} \left\{(\tilde{v}-v)\cdot\chi(\mathbf{x}^{t+1})\right\}$$
$$\leq U(\tilde{v}, x^t) - U(v, x^t) \leq \max_{\mathbf{x}^{t+1}\in\{\mathbf{X}^{t+1}|x^t\}} \left\{(\tilde{v}-v)\cdot\chi(\mathbf{x}^{t+1})\right\}.$$

Substituting the boundaries into (3) and reorganizing yields (2).

Proof of Proposition 1

We proceed in three steps. First, we establish that the repeated supremum of the one-shot game profit, $\pi^*(h^t)$, is the highest profit the seller can obtain from buyers with common history h^t if all buyers choose their preferred variety at $\pi^*(h^t)$. Next, we establish that the pure strategies of posting the price $p^*(h^t)$ associated with $\pi^*(h^t)$ in every period and purchasing variety i in every period form a PBE. Finally, we show that whenever any other prices are posted, the seller cannot obtain the highest profit.

We begin by characterizing the seller's present discounted profit at history h^t. Since any strategy profile $\{\sigma, \hat{\sigma}\}$ gives rise to sequences of prices and consumption choices, we can define the seller's present discounted profit at history h^t in terms of the payments made along the admissible consumption paths $\mathbf{x}_k^t \in \mathbf{X}^t$,

$$\Pi(h^t) = \sum_{\mathbf{x}_k^t\in\mathbf{X}^t} \rho(\mathbf{x}_k^t, h^t)\mathcal{F}\left(v \in V(h^t)|\ \mathbf{x}_k^t\right),$$

where the shorthand notation $\mathcal{F}\left(v \in V(h^t)|\ \mathbf{x}_k^t\right)$ indicates the measure of types on path $\mathbf{x}_k^t$, and $\mathbf{X}^t$ is the set of admissible consumption paths at history h^t.

We now derive an auxiliary result that allows us to conveniently rewrite the seller's profit $\Pi(h^t)$. Denote the set of consumers at history h^t who are indifferent between two distinct consumption paths $\mathbf{x}_k^t$ and $\mathbf{x}_l^t$ by

$$V_{k,l}(h^t) \equiv \{v : U(v, \mathbf{x}_k^t, h^t) = U(v, \mathbf{x}_l^t, h^t)\},$$

and the difference in the present discounted values obtained by indifferent consumers with value profile $v \in V_{k,l}(h^t)$ along consumption paths $\mathbf{x}_k^t$ and $\mathbf{x}_l^t$, respectively, by

$$\Delta\nu_{k,l}^t \equiv \nu(v, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_l^t) = \rho(\mathbf{x}_k^t, h^t) - \rho(\mathbf{x}_l^t, h^t).$$

Then, the following result holds.

Lemma 3 *Consider a set of buyers with common history, $V(h^t) \subseteq V$. The seller's present discounted profit at history h^t can be written as*

$$\Pi(h^t) = \rho(\mathbf{x}_0^t, h^t)\mathcal{F}\big(v \in V(h^t)\big) + \sum_{k=1}^{K} \Delta\nu_{k,k-1}^t \mathcal{F}\left(v \in V(h^t)| \cup_{j \geq k} \mathbf{x}_j^t\right),$$

where the set of admissible paths $\mathbf{X}^t = \{\mathbf{x}_0^t, ..., \mathbf{x}_K^t\}$ is ordered by the payments such that $\rho(\mathbf{x}_0^t, h^t) \leq \rho(\mathbf{x}_1^t, h^t) \leq ... \leq \rho(\mathbf{x}_K^t, h^t)$, and $\Delta\nu_{k,k-1}^t$ is the difference in the present discounted values obtained by indifferent consumers along consumptions paths $\mathbf{x}_k^t$ and $\mathbf{x}_{k-1}^t$, respectively.

Proof. Let $V_{1,0}(h^t)$ denote the set of value profiles of consumers with common history h^t who are indifferent between $\mathbf{x}_1^t$ and $\mathbf{x}_0^t$. Then for any $v \in V_{1,0}(h^t)$, we have

$$\nu(v, \mathbf{x}_1^t) - \rho(\mathbf{x}_1^t, h^t) = \nu(v, \mathbf{x}_0^t) - \rho(\mathbf{x}_0^t, h^t)$$

or

$$\rho(\mathbf{x}_1^t, h^t) = \rho(\mathbf{x}_0^t, h^t) + \Delta\nu_{1,0}^t$$

by construction. Next, let $V_{2,1}(h^t)$ denote the set of value profiles of consumers with common history h^t who are indifferent between $\mathbf{x}_2^t$ and $\mathbf{x}_1^t$. Then for all $v \in V_{2,1}(h^t)$ we have $\rho(\mathbf{x}_2^t, h^t) = \rho(\mathbf{x}_1^t, h^t) + \Delta\nu_{2,1}^t$, and thus

$$\rho(\mathbf{x}_2^t, h^t) = \rho(\mathbf{x}_0^t, h^t) + \Delta\nu_{1,0}^t + \Delta\nu_{2,1}^t.$$

Iterating this procedure for all consumption paths up to $\mathbf{x}_K^t$ yields

$$\rho(\mathbf{x}_k^t, h^t) = \rho(\mathbf{x}_0^t, h^t) + \sum_{k=1}^{K} \Delta\nu_{k,k-1}^t.$$

Adding up the total payments made by buyers in $V(h^t)$ along the admissible consumption paths, the present discounted profit at history h^t is given by

$$\Pi(h^t) = \rho(\mathbf{x}_0^t, h^t)\mathcal{F}\big(v \in V(h^t)\big) + \sum_{k=1}^{K} \Delta\nu_{k,k-1}^t \mathcal{F}\left(v \in V(h^t)| \cup_{j \geq k} \mathbf{x}_k^t\right).$$

■

Now, let σ^* denote the seller's pure strategy of setting the prices $p^*(h^t)$ at every subsequent period $\tau \geq t$ and history h^τ, and suppose that the seller commits to σ^* at history h^t. Then all buyers $v \in V(h^t)$ behave as if they were myopic because they cannot gain from behaving strategically. In addition, let $\hat{\sigma}^*$ denote the buyers' pure strategy of purchasing variety i at prices $p^*(h^t)$. Then, the seller's present discounted profit for the strategy profile $\{\sigma^*, \hat{\sigma}^*\}$ is

$$\Pi(h^t) = p_i^*(h^t)\mathcal{F}(v \in V(h^t))\Delta^t, \tag{5}$$

since all buyers will follow the "always-i path" from t on, $\mathbf{x}_i^t = (i, i, ..., i)$. Lemma 3 implies that this is the maximum profit attainable for the seller. To see this, fix without loss of generality the first path in the order of paths as the always-i-path, $\mathbf{x}_0^t = \mathbf{x}_i^t$. As $v_i > v_j \; \forall v \in V(h^t)$ by assumption, there exists no alternative path $\mathbf{x}_k^t$ with $\Delta\nu_{k,i}^t > 0$. Hence, (5) is the most the seller can obtain at history h^t.

Next, we show that the strategy profile $\{\sigma^*, \hat{\sigma}^*\}$ constitutes a PBE. Suppose that the seller adopts strategy σ^* at h^t and consider the deviation incentives of buyers $v \in V(h^t)$. Buyers who deviate to variety j or the outside option o, respectively, must obtain a lower instantaneous utility if all buyers $v \in V(h^t)$ purchase variety i at prices $p^*(h^t)$ in the one-shot game at history h^t. This instantaneous loss in utility cannot be compensated in the future, because the highest utility any type $v \in V(h^t)$ can obtain from period $\tau \geq t+1$ onward is $(v_i - p_i^*(h^t))\Delta^\tau$. And since the seller obtains the maximum attainable profit by playing σ^* when buyers play strategy $\hat{\sigma}^*$, the strategy profile $\{\sigma^*, \hat{\sigma}^*\}$ is a PBE.

Finally, to prove that the only PBE is one in which $p_i^\tau = p_i^*(h^t)$ and $x^\tau = i$ for all buyers, note that any strategy that results in sequences of prices that ensure all types purchase variety i in every period such that the sum of prices is equal to $p_i^*(h^t)\Delta^t$ is a candidate PBE. In addition, consider that if all types play $x = i$ in the one-shot game at prices $p^*(h^t)$, then it must be that $p_i^*(h^t)$ is the highest price at which all types with a positive density in the support $V(h^t)$ play $x = i$, as otherwise the seller would leave rent on the table. Then it follows that if the seller plays a strategy that results in a price $p_i^\tau > p_i^*(h^t)$, then some types $v_i < p_i^\tau$ must make an instantaneous loss if they accept. As the seller will leave no rent to the lowest type in any future period at any history, this loss can never be recouped. Thus, some types will not purchase at price $p_i^\tau > p_i^*(h^t)$, which must result in a lower profit by Lemma 3. Similarly, any price $p_i^\tau < p_i^*(h^t)$ must be strictly sub-optimal, as the seller will obtain a smaller profit at time τ that can only be recouped by setting a $p_i^\tau > p_i^*(h^t)$ in a future period, at which point some types will not accept. Thus, playing a strategy that results in constant prices at $p_i^*(h^t)$ and sufficiently high prices for $p_j^\tau(h^t)$ to ensure all types play i at every $\tau \geq t$ is the only PBE.

Proof of Proposition 2

(i) Let σ^m be the seller's (pure) monopoly strategy that implements fixed prices $p^m = (p_a, p_b)$ at every $t \geq 0$ and every history h^t. If the seller commits to σ^m at $t = 0$, we know that all buyers behave as if they were myopic, and the seller's profit is

$$\Pi = p_a^m \mathcal{F}\big(v \in V \big| \arg\max_{\mathbf{x}_l \in \mathbf{X}} \{\nu(v, \mathbf{x}_l) - \rho(\mathbf{x}_l)\} = \mathbf{x}_a\big)\Delta$$
$$+ p_b^m \mathcal{F}\big(v \in V \big| \arg\max_{\mathbf{x}_l \in \mathbf{X}} \{\nu(v, \mathbf{x}_l) - \rho(\mathbf{x}_l)\} = \mathbf{x}_b\big)\Delta,$$

where $\mathbf{x}_a = \{a, a, ..., a\}$ is the always-a path and $\mathbf{x}_b = \{b, b, ..., b\}$ is the always-b path, respectively. Applying Lemma 3, we cannot find a path $\mathbf{x}_k \neq \mathbf{x}_a, \mathbf{x}_b$ such that $\Delta\nu_{k,i} > 0$, with $i \in \{a, b\}$, by the assumption of no trading-up opportunities. Therefore, the seller cannot do better than obtain the repeated monopoly profit $\pi(p^m)$ in every period $t = 0, ..., T$, which establishes the result.

(ii) It is straightforward to check that monopoly strategies with non-absorbing varieties form a PBE: if buyers behave as if they were myopic, then setting the monopoly prices p^m in every period $t \geq 0$ yields the maximum profit; similarly, if the seller repeatedly plays p^m, then myopic behavior is optimal. To complete the proof for absorbing varieties, consider that any sequence of prices that results in buyers of i consuming i from the first period on and paying $p_i^m\Delta$ yields the maximum profit by (i). Thus, if the seller increases the first price paid for consuming i to $p_i^m\Delta^t$ for every h^t then since all future prices are set to zero by construction, the sum of prices satisfies the condition. As all consumers know for sure that future prices will always be zero, it is easy to see that myopic behavior continues to be optimal and thus this continues to be a PBE.

(iii) Part (i) shows that the highest profit the seller can achieve is the repeated monopoly profit. Part (ii) shows that the seller can obtain this profit in a PBE.

Then it follows that the seller will never choose a strategy in PBE that does not deliver the repeated monopoly profit.

Proof of Proposition 3

We prove the four statements in turn.

(i) Fix a PBE. Consider a history h^t on the equilibrium path and denote the state that consumers are in by $x^{t-1} \in X$. Suppose that there exist trading-up opportunities, so that there exists a consumption option $x \in \{a, b\}$ for which some types $v \in V(h^t)$ satisfy $v \cdot x > v \cdot x^{t-1}$ and $(x^{t-1}, x) \in \Gamma$. Denote the set of types that satisfy these conditions by $V^{TU}(h^t) \subseteq V(h^t)$. Let the highest value for variety $i \in \{a, b\}$ for types $v \in V(h^t)$ be $\bar{v}_i$, the lowest value $\underline{v}_i$, and analogously for types $v \in V^{TU}(h^t)$ we have $\bar{v}_i^{TU}$ and $\underline{v}_i^{TU}$. We similarly define $\bar{v}_j, \bar{v}_j^{TU}, \underline{v}_j, \underline{v}_j^{TU}$ for $j \in \{a, b\}, j \neq i$.

In addition, denote the measure of types that are traded up by $\mathcal{F}(v \in V(h^t)|TU)$ and the remaining measure of types that are not traded up by $\mathcal{F}(v \in V(h^t)|NTU)$. By definition, $\mathcal{F}(v \in V(h^t)) = \mathcal{F}(v \in V(h^t)|TU) + \mathcal{F}(v \in V(h^t)|NTU)$. There are four cases to distinguish.

Case 1: $x^{t-1} = j$ and $(j, o) \notin \Gamma$, or $x^{t-1} = o$.
If $x^{t-1} = j$ and $(j, o) \notin \Gamma$, $p_j^t(h^t)$ is set to zero by assumption, and the existence of trading up opportunities implies that (some) types $v \in V^{TU}(h^t)$ will purchase variety i at a strictly positive price $p_i^t(h^t)$, resulting in a profit increase. Similarly, if $x^{t-1} = o$, then inducing (some) types $v \in V^{TU}(h^t)$ to choose $x^t \in \{a, b\}$ constitutes trading up. As the seller earns no profit from types in the outside option, inducing consumers to purchase variety i (or variety j) at a strictly positive price is profit-increasing.

Case 2: $x^{t-1} = j$ and $(j, o) \in \Gamma$, and $\bar{v}_i^{TU} > \bar{v}_j$.
The equilibrium profit of the seller if she decides not to trade up any buyers, $\hat{\Pi}(h^t)$, satisfies

$$\hat{\Pi}(h^t) < \bar{v}_j \mathcal{F}(v \in V(h^t)) \Delta^t, \tag{6}$$

as the seller cannot extract the full surplus of types with a linear price. However, if the seller trades up (some) types $v \in V^{TU}(h^t)$, then the equilibrium profit obtained from trading up, $\Pi^*(h^t)$, satisfies

$$\Pi^*(h^t) \geq v_i^* \mathcal{F}(v \in V(h^t)|TU)\Delta^t, \tag{7}$$

where v_i^* denotes the lowest value v_i of the cutoff types who are indifferent to trading up to i, as the seller can always obtain at least the value of the lowest type in the set. The equilibrium profit obtained from types not traded up, $\Pi^\circ(h^t)$, satisfies

$$\Pi^\circ(h^t) < \bar{v}_j \mathcal{F}(v \in V(h^t)|NTU)\Delta^t, \tag{8}$$

because as before the seller cannot extract the full surplus using a linear price. As $\bar{v}_i^{TU} > \bar{v}_j$ by assumption, there exists a v_i^* that satisfies $\bar{v}_i^{TU} > v_i^* > \bar{v}_j$. Therefore, (6), (7), (8), and $\mathcal{F}(v \in V(h^t)) = \mathcal{F}(v \in V(h^t)|TU) + \mathcal{F}(v \in V(h^t)|NTU)$ together imply that

$$\Pi^*(h^t) + \Pi^\circ(h^t) > \hat{\Pi}(h^t).$$

Case 3: $x^{t-1} = j$ and $(j, o) \in \Gamma$, and $\bar{v}_i^{TU} < \bar{v}_j$; non-absorbing outside option. Suppose the seller does not trade up any types to variety i along the equilibrium path. Then, for any types that play $x^t = o$, we have that Case 1 applies at time $t+1$, and thus trading up is profit-increasing. Specifically, suppose all types $v \in V^{TU}(h^t)$ play $x^t = o$. Since consumers only ever purchase at a price at which they earn a (weakly) positive utility over the course of the game, we know that if the seller never trades up any types $v \in V^{TU}(h^t)$ to i along equilibrium play after $x^t = o$, then since Case 1 applies at any history at which the state is the outside option, it is profit-increasing for the seller to set $p_j^\tau(h^\tau) \leq \bar{v}_j^{TU}$ for some $\tau > t$. But then we can find a $p_i^\tau(h^\tau) > p_j^\tau(h^\tau)$ such that $v_i^*(h^\tau) > \min\{v_j\} \in V(h^t)$, which implies that inducing some types $v \in V^{TU}(h^t)$ to play $x^\tau = i$ is strictly profit-increasing, since the equilibrium profit of the seller when trading up satisfies

$$\Pi^*(h^\tau) \geq v_i^* \mathcal{F}(v \in V(h^\tau)|TU)\Delta^\tau.$$

Now suppose instead (some) types $v \in V^{TU}(h^t)$ play $x^t = j$. Then again, we must have $p_j^\tau(h^\tau) \leq v_j^{TU}(h^\tau)$ for the types $v \in V^{TU}(h^t)$ who are willing to play $x^\tau = j$, such that trading up to variety i is profit-increasing by the above argument. For those types who play $x^\tau = o$, the above argument applies. Thus, trading up must be strictly profit-increasing.

Case 4: $x^{t-1} = j$ and $(j, o) \in \Gamma$, and $\bar{v}_i^{TU} < \bar{v}_j$; absorbing outside option. Suppose that (some) types $v \in V^{TU}(h^t)$ play $x^t = j$. If the seller never trades up any types to variety i, then we know from Case 3 that $p_j^t(h^t) \leq \bar{v}_j^{TU}$, and we can thus find a $p_i^t(h^t) > p_j^t(h^t)$ which ensures that trading up to variety i is strictly profitable. Suppose instead now that all types $v \in V^{TU}(h^t)$ play $x^t = o$. As $x^{t-1} = j$ by assumption, at time $t-1$ we must have had $p_j^{t-1}(h^{t-1}) < p_i^{t-1}(h^{t-1})$ by incentive compatibility, since the indifference condition at $t-1$ is

$$v_i - p_i^{t-1}(h^{t-1}) + \delta U(v, h^{t-1}, x^{t-1} = i) = v_j - p_j^{t-1}(h^{t-1}) + 0, \tag{9}$$

for types $v \in V^{TU}(h^t)$. As all types $v \in V^{TU}(h^t)$ play $x^t = o$ in period t, we know that $p_j^t(h^t) \geq \bar{v}_j^{TU}(h^t)$ and that $p_j^t(h^t) > p_j^{t-1}(h^{t-1})$. But if it is profit-increasing to induce $x^t = o$ for all types by setting $p_j^t(h^t)$ in period t, the seller could have increased profit by playing $p_j^{t-1}(h^{t-1}) = p_i^{t-1}(h^{t-1})$ in period $t-1$. To see this, note that by setting these prices in $t-1$, the seller assures that no type plays $x^{t-1} = j$, while the profit from types playing $x^{t-1} = i$ at h^{t-1} must be increasing, as the price $p_i^{t-1}(h^{t-1})$ remains unchanged, and the measure of types playing $x^{t-1} = i$ increases. As the seller can always induce these additional types to play $x^t = o$ at time t, the continuation profit is unaffected. Thus, a history where the seller induces $x^t = o$ after $x^{t-1} = j$ for all types cannot arise in equilibrium.

Then in conjunction statement (i) follows.

(ii) Denote by Λ the set of price profiles p that leave no trading-up opportunities for any history h^t in the dynamic game. We will show that $\Omega \setminus \Lambda = \emptyset$ and $\bar{p} \in \Lambda$. Consider the price profile $\bar{p} = (\bar{p}_a, \bar{p}_b)$. Note first, that because $\bar{p} \in \Omega$ by assumption, a price profile $\tilde{p}$ on the diagonal through the type space, with $\eta \geq 0$, satisfies

$$\tilde{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\}) - (\eta, \eta) \implies \tilde{p} \in \Omega, \tag{10}$$

as all types willing to purchase at prices $\tilde{p}$ choose their most-preferred variety, and all types choosing the outside option will also do so at prices $\bar{p}$. Similarly, a price profile $\tilde{\tilde{p}}$ on the (vertical or horizontal) line between $\bar{p}$ and the diagonal, with $\eta \in [0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$, satisfies

$$\tilde{\tilde{p}} = \begin{cases} (\bar{p}_a, \bar{p}_b) - (0, \eta), & \text{if } \bar{p}_b > \bar{p}_a \\ (\bar{p}_a, \bar{p}_b) - (\eta, 0), & \text{if } \bar{p}_b < \bar{p}_a \end{cases} \implies \tilde{\tilde{p}} \in \Omega, \tag{11}$$

as all types purchasing a different variety at prices $\tilde{\tilde{p}}$ than at prices $\bar{p}$ must now choose their most-preferred variety, and all types switching from the outside option to consumption must now choose their most-preferred variety. Second, observe that the price profile $p^\circ = (-\Delta^{t+1}, -\Delta^{t+1})$ is contained in Λ. To see this, recall from the proof of Lemma 1 that types can always mimic each other's behavior (i.e., make the same choices from t onward), so that we have

$$U(\tilde{v}, h^t, x^t) - U(v, h^t, x^t) \leq \max_{i \in \{a,b\}} \{\tilde{v}_i - v_i\} \Delta^{t+1}, \quad v \neq \tilde{v},$$

where U denotes the continuation valuation following choice x^t. Since the maximum value difference satisfies $\max_{i \in \{a,b\}} \{\tilde{v}_i - v_i\} = 1$, all types purchase their most-preferred variety when facing prices p°. In addition, by (10) we also have that $p^\circ \in \Omega$.

Now pick a price profile $\hat{p}$ that satisfies $\hat{p} = p^\circ + (\varepsilon, \varepsilon)$ for some $0 \leq \varepsilon \leq \Delta^{t+1} + \min\{\bar{p}_a, \bar{p}_b\}$. By (10) we know $\hat{p} \in \Omega$. Denote by x° the choice that buyers make in the static game when facing prices p°. By (10) we then have

$$x^\circ \cdot (v - p^\circ) \geq x' \cdot (v - p^\circ), \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \tag{12}$$

where we know that $x^\circ \in \{a, b\}$ as $p_a^\circ = p_b^\circ < 0$. Since $p^\circ \in \Lambda$, we also have that

$$\begin{aligned} &x^\circ \cdot (v - p^\circ) + \delta U^\circ(v, h^t) \\ &\qquad \geq x' \cdot (v - p^\circ) + \delta U'(v, h^t), \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \end{aligned} \tag{13}$$

where U° and U' denote the continuation valuations associated with the choices x° and x' respectively, given history h^t. By the definition of p° and (13) it then follows that

$$\delta(U' - U^\circ) \leq (x^\circ - x') \cdot v, \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \tag{14}$$

which also implies that

$$\begin{aligned} &x^\circ \cdot (v - p^\circ - \varepsilon) + \delta U^\circ(v, h^t) \\ &\qquad \geq x' \cdot (v - p^\circ - \varepsilon) + \delta U'(v, h^t), \qquad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V. \end{aligned} \tag{15}$$

Thus, for any $0 \leq \varepsilon \leq \Delta^{t+1} + \min\{\bar{p}_a, \bar{p}_b\}$, only types $v < \min\{\bar{p}_a, \bar{p}_b\}$ prefer o to x°, which continues to leave no trading-up opportunities as $\bar{p} \in \Omega$ by assumption, and thus $\hat{p} \in \Lambda$. Hence, for any $\tilde{p}$ that satisfies (10) we have $\tilde{p} \in \Lambda$.

Now fix the price profile $\hat{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\})$. By (11) we have $\hat{p} \in \Omega$, and as shown above we also have $\hat{p} \in \Lambda$. Consider a price profile $p' = \hat{p} + (0, \varepsilon)$ if $\bar{p}_b > \bar{p}_a$ and $p' = \hat{p} + (\varepsilon, 0)$ if $\bar{p}_b < \bar{p}_a$ where $\varepsilon \in (0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$. Then by the same logic as above, for any $\max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\} \geq \varepsilon > 0$, we find that the only types that now prefer the outside option to consumption also prefer the outside option at prices $\bar{p}$ and the only types now preferring the other variety also do so at prices $\bar{p}$. Thus, we find $p' \in \Lambda$ or equally that any $\tilde{p}$ that satisfies (11) satisfies $\tilde{p} \in \Lambda$ and therefore $\bar{p} \in \Lambda$.

Finally, note that we can construct (10) and (11) for any price profile $p \in \Omega$ and thus we find that $\Omega \setminus \Lambda = \emptyset$. Then statement (ii) follows from the definition of $\bar{p}$.

(iii) From (ii), the seller's minmax profit per period is $\pi(\bar{p})$. The present discounted profit must therefore satisfy $\Pi \geq \pi(\bar{p})\Delta$.

(iv) Consider a history h^t and denote the state that consumers are in by $x^{t-1} \in X$. Suppose there are trading-up opportunities, so that there exists a consumption option $i \in \{a, b\}$ for which some types $v \in V(h^t)$ satisfy $v \cdot i > v \cdot x^{t-1}$ and $(x^{t-1}, i) \in \Gamma$. Denote the highest and lowest value among these types by $\bar{v}_i^{TU}(h^t)$ and $\underline{v}_i^{TU}(h^t)$, respectively. Define analogously $\bar{v}_j^{TU}, \underline{v}_j^{TU}$ for $j \in \{a, b\}, j \neq i$, if trading-up opportunities exist for j as well. Denote the set of types that can be traded up by $V^{TU}(h^t) \subseteq V(h^t)$, and the measure of types that can be traded up by $\omega(h^t) = \mathcal{F}\left(v \in V^{TU}(h^t)\right)$.

Let $\bar{v}^{TU}(h^t) = \max\{\bar{v}_i^{TU}(h^t), \bar{v}_j^{TU}(h^t)\}$ if the seller trades some types up to both i and j, and equal to the largest value of the variety the seller trades up to, if she trades up to only one variety. Now consider the difference of $\bar{v}^{TU}(h^t)$ to the lowest values in the support $V^{TU}(h^t)$ and assume without loss of generality that $\underline{v}_i^{TU}(h^t) \geq \underline{v}_j^{TU}(h^t)$,

$$\varepsilon(h^t) \geq \bar{v}^{TU}(h^t) - \min\{\underline{v}_i^{TU}(h^t), \underline{v}_j^{TU}(h^t)\} = \bar{v}^{TU}(h^t) - \underline{v}_i^{TU}(h^t).$$

As the seller trades up a positive measure of consumers at any history h^t with trading-up opportunities (see part (i)), by definition of $\bar{v}^{TU}(h^t)$ we have that $\bar{v}^{TU}(h^t) - \underline{v}_i^{TU}(h^t)$ must decrease with the length of a history by Lemma 1, such that a smaller $\varepsilon(h^t)$ will satisfy the above condition. We now show that for $\varepsilon(h^t)$ small enough there are three possible cases in which the seller strictly prefers to trade up all types. To ease notation, we henceforth suppress the conditioning of ω, ε, $\bar{v}^{TU}$ and $\underline{v}_i^{TU}$ on history h^t whenever possible.

Let $\Pi^*(h^t)$ denote the equilibrium profit for the seller obtained from trading up only some of the types $v \in V^{TU}(h^t)$. As the seller cannot extract the full surplus with a linear price or trade up the remaining types before time $t+1$, there exists a $\lambda \in (0,1)$ such that

$$\Pi^*(h^t) < \lambda\omega\bar{v}^{TU}\Delta^t + \delta(1-\lambda)\omega\bar{v}^{TU}\Delta^{t+1}.$$

In addition, let $\bar{\Pi}(h^t)$ denote the seller's equilibrium profit obtained from trading up all buyers at time t. Let $\varphi \in [0,1]$ be the share of types traded up to j. As the seller can always obtain at least the lowest value of the types in each period, we have that

$$\bar{\Pi}(h^t) \geq (1-\varphi)\omega\underline{v}_i^{TU}\Delta^t + \varphi\omega\underline{v}_j^{TU}\Delta^t.$$

Using these profits and noting that $\delta\Delta^{t+1} = \Delta^t - 1$, we can write

$$\begin{aligned}
\Pi^*(h^t) - \bar{\Pi}(h^t) &< \lambda\omega\bar{v}^{TU}\Delta^t + \delta(1-\lambda)\omega\bar{v}^{TU}\Delta^{t+1} - (1-\varphi)\omega\underline{v}_i^{TU}\Delta^t + \varphi\omega\underline{v}_j^{TU}\Delta^t \\
&= \left[(\Delta^t - 1 + \lambda)\bar{v}^{TU} - (1-\varphi)\underline{v}_i^{TU}\Delta^t - \varphi\underline{v}_j^{TU}\Delta^t\right]\omega \\
&\leq \left[(\Delta^t - 1 + \lambda)(\varepsilon + \underline{v}_i^{TU}) - (1-\varphi)\underline{v}_i^{TU}\Delta^t - \varphi\underline{v}_j^{TU}\Delta^t\right]\omega \\
&= \left[(\Delta^t - 1 + \lambda)\varepsilon - (1-\lambda)\underline{v}_i^{TU} - \varphi\Delta^t(\underline{v}_i^{TU} - \underline{v}_j^{TU})\right]\omega.
\end{aligned}$$

Therefore, $\bar{\Pi}(h^t) > \Pi^*(h^t)$ whenever

$$\varepsilon(h^t) \leq \frac{(1-\lambda)\underline{v}_i^{TU}(h^t) + \varphi\Delta^t(\underline{v}_j^{TU}(h^t) - \underline{v}_i^{TU}(h^t))}{\Delta^t - 1 + \lambda}.$$

That is, in PBE the seller eventually exhausts all trading-up opportunities at history h^t if $t \leq T$ is sufficiently large in the following three cases: (i) $\underline{v}_i^{TU}(h^t) > 0$ and there are no trading-up opportunities to j so that $\varphi = 0$; (ii) $\underline{v}_i^{TU}(h^t) > 0$ and there are trading-up opportunities to j; (iii) $\underline{v}_j^{TU}(h^t) > \underline{v}_i^{TU}(h^t) = 0$ and there are trading-up opportunities to j and the seller can induce a $\varphi > 0$.

Finally, note that for $\bar{\Pi}(h^t) > \Pi^*(h^t)$ to be satisfied we must have that $\bar{\Pi}(h^t) > 0$ since the definition of trading-up opportunities implies that $\Pi^*(h^t) > 0$, and that $\pi(\bar{p}) > 0$ ensures that $\bar{\Pi}(h^t) > 0$. Then statement (iv) and Corollary 3.1 follow.