Bayesian Methods for Measurement Error Problems

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Project 1: BQR for Scalar-on-Function Regression with Heteroskedastic Data

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Background

General Introduction

- Quantile regression (QR) has gained popularity as a robust alternative to ordinary regression
- QR is formulated as an optimization problem using a loss function ρ_τ, where τ is the desired quantile
- Bayesian Quantile Regression (BQR) offers the usual adventages of the Bayesian framework, like quantifying uncertainty
- BQR is based on a likelihood function called the Asymmetric Laplace (AL) distribution whose kernel includes ρ_τ

GAL Distribution

- AL turns out to be too restrictive
- Yan and Kottas (2017) proposed the Generalized Asymmetric Laplace distribution (GAL)
- It can be reparametrized with:
  - A location parameter μ
  - A scale parameter σ
  - A shape parameter γ
- Closed forms for the pdf and cdf can be derived, but they are cumbersome

Sampling from the posterior

- In Bayesian statistics, every parameter has a distribution
- We sample from the distribution of μ, σ and γ using MCMC
- In recent years, Hamiltonian Monte Carlo (HMC) has gained popularity
- Implemented in the brms library in R
- Uses the same formula syntax as lme4
- Supports non-parametric methods through mgcv

Low dimensional ilustration of our problem

- Consider data generated according to

$$y_i = x_i^2 + \epsilon_i$$

where $\epsilon_i \sim N(0, \exp(4x))$

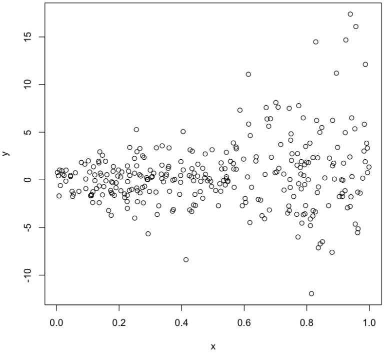

Figure 1: Heteroskedastic data

Warning

- Quantile regression should be able to handle this kind of heteroskedasticity
- Turns out you have to be careful:

```
model1 <- brm(y ~ s(x), family = 'GAL')
model2 <- brm(y ~ s(x), sigma ~ s(x),
                        family = 'GAL')
```