Debrief from presentation

Nico

Big Picture

- Before we run any simulations, we need to think this through
- Lan had some fundamental objections to our set up (not even about the functional part)
- Her comments made me think about what this paper is trying to say

Lan's first point

- Simplified setting:
  - Covariates $\mathbf{x}_i$
  - Response Y_i
- Suppose the true distribution is

$$Y_i = \beta_0 + \mathbf{x}_i^T \boldsymbol{\beta}_1 + \delta_i$$

  where $\delta_i \sim \mathrm{N}(0, 1)$
- If we model this as

$$Y_i \sim \lambda_{0,\tau} + \mathbf{x}^T \boldsymbol{\lambda}_{1,\tau} + \epsilon_i$$

  where $\epsilon_i \sim \mathrm{AL}_\tau(0, \sigma, \gamma)$ then if we choose *flat* priors, the posterior mode of $\boldsymbol{\lambda}|\mathbf{x}$ coincides with the frequentist QR estimates.

- In formulas

$$E[\mathrm{Mode}(\lambda_{0,\tau}|\mathbf{x},\mathbf{y})] = \beta_0 + z_\tau$$

  and

$$E[\mathrm{Mode}(\boldsymbol{\lambda}_{1,\tau}|\mathbf{x},\mathbf{y})] = \boldsymbol{\beta}_{1,\tau}$$

- If we move to

$$Y_i \sim \lambda'_{0,\tau} + \mathbf{x}^T \boldsymbol{\lambda}'_{1,\tau} + \epsilon'_i$$

  where $\epsilon'_i \sim \mathrm{GAL}_\tau(0, \sigma, \gamma)$, we loose this property, especially if we use priors

- This is bad!
- We seem to be saying that the second model fits the data better because it's more flexible
- But what's the connection between $\boldsymbol{\lambda}'_{1,\tau}|\mathbf{x}, \mathbf{y}$?
- The benefit of the Bayesian framework is that it gives us a distribution. But what is that distribution telling us?

A possible answer

Model the likelihood!

- BQR is completely agnostic about the true likelihood
- In reality, we have some idea about it. Or at least we try to make assumptions about it
- Our assumption is a perfectly reasonable one:

$$Y_i = \beta_0 + \mathbf{x}_i^T \boldsymbol{\beta}_1 + \exp\left(\mathbf{x}_i^T \boldsymbol{\eta}\right) \delta_i$$

  where $\delta_i \sim \mathrm{N}(0, 1)$
- For small η

$$Q_\tau(Y_i|\mathbf{x}_i) \approx \beta_0 + z_\tau + \mathbf{x}_i^T(\boldsymbol{\beta}_1 + z_\tau \boldsymbol{\eta})$$