Week 8

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1 Bayesian Quantile Regression with BRMS

Consider the following model:

$$x \sim \text{Unif}\,(0, 1)$$
$$y \sim \text{N}\,(0, \exp(2x))$$

It is illustrated in Figure 1.

We want to perform Bayesian quantile regression on this data. Specifically, we want to model the .25, .5, and .75 quantiles of y as a function of x.

The literature would suggest using the following model:

```
GAL2 <- custom_family(
"GAL2",
dpars = c("mu", "sigma", "ligam", "tau"),
links = c("identity", "log", "identity", "identity"),
lb = c(NA, 0, Bd[1] * .9, 0), ub = c(NA, NA, Bd[2] * .9, 1),
type = "real"
)
q25n <- brm(bf(y ~ exp(x), tau = .25), data = synthetic, family = GAL2,
stanvars = stanvars2, chains = 2, iter = 2000, control = list(adapt_delta = 0.99),
cores = 4, seed = 123)
```

However, we suggest modeling the variance separately, as follows:

```
q25 <- brm(bf(y ~ exp(x), sigma ~ x, tau = .25), data = synthetic,
family = GAL2, stanvars = stanvars2, chains = 2, iter = 2000,
control = list(adapt_delta = 0.99), cores = 4, seed = 123)
```

We found that our suggestion improves the model fit considerably, as illustrated in Figure 2.

In the previous models, we told `brms` explicitely that the location parameter was an exponential function of x. We further explored this avenue and told `brms` only that the location parameter was a smooth function of x:

```
q25 <- brm(bf(y ~ s(x), sigma ~ x, tau = .25), data = synthetic,
family = GAL2, stanvars = stanvars2, chains = 2, iter = 2000,
control = list(adapt_delta = 0.99), cores = 4, seed = 123)
```

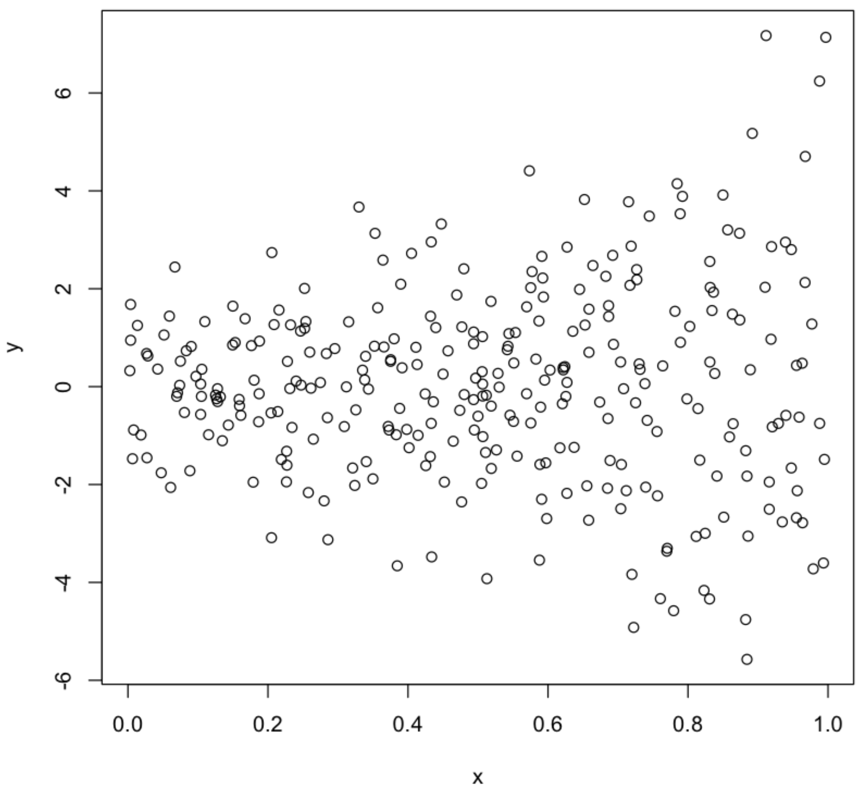


Figure 1: Raw data

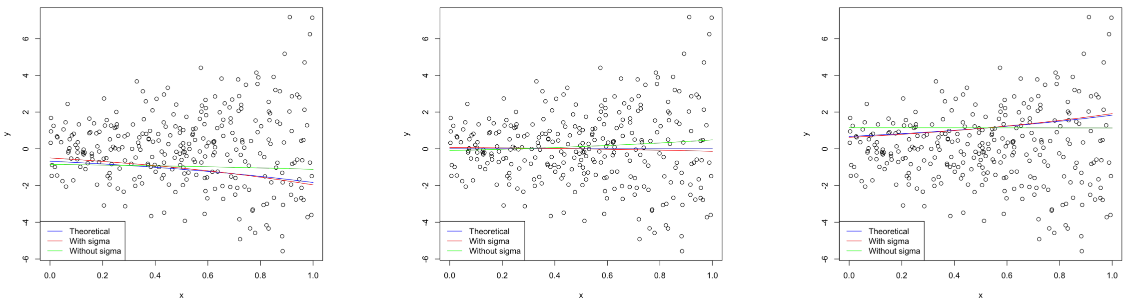
BRMS was able to recover the appropriate shape of the location parameter, as illustrated in Figure 3.

2 Latent Factor Models

Consider the following model in `lavaan` syntax:

```
mod <-
"
    f2 =~ x01 + x02
    x03 ~ f2
    "
fit_lavaan <- sem(mod, data = dat)
```

We would like to fit this model with `brms`. This is not possible with the current version of `brms`,



(a) .25 Quantile (b) .5 Quantile (c) .75 Quantile

Figure 2: Quantile Regression

but we can use the following workaround:

```
# Creating empty column for the mediator
dat$f1 <- as.numeric(NA)

# Defining the dependency of the mediator on observed variables:
bf1 <- bf(x01 ~ 0 + mi(f1))
bf2 <- bf(x02 ~ 0 + mi(f1))

# Telling brms that the mediator is a latent variable
bf3 <- bf(f1 | mi() ~ 0)

# Defining the dependency of the outcome on the mediator
bf4 <- bf(x03 ~ 0 + mi(f1))

# Fitting the brms object
fit_brms <- brm(bf1 + bf2 + bf3 + bf4 + set_rescor(FALSE), data = dat,
iter = 2000, chains = 2, cores = 4, seed = 123, prior = prior(normal(1, 0.00001),
coef = mif1, resp = x01))
```

The coefficients obtained this way are similar to the ones obtained with `lavaan`.

3 R Package

I started working on the R package for the grant. I will be working on the Corrected Score paper. I took a preliminary look at the code and refreshed my memory about how to write packages in R.

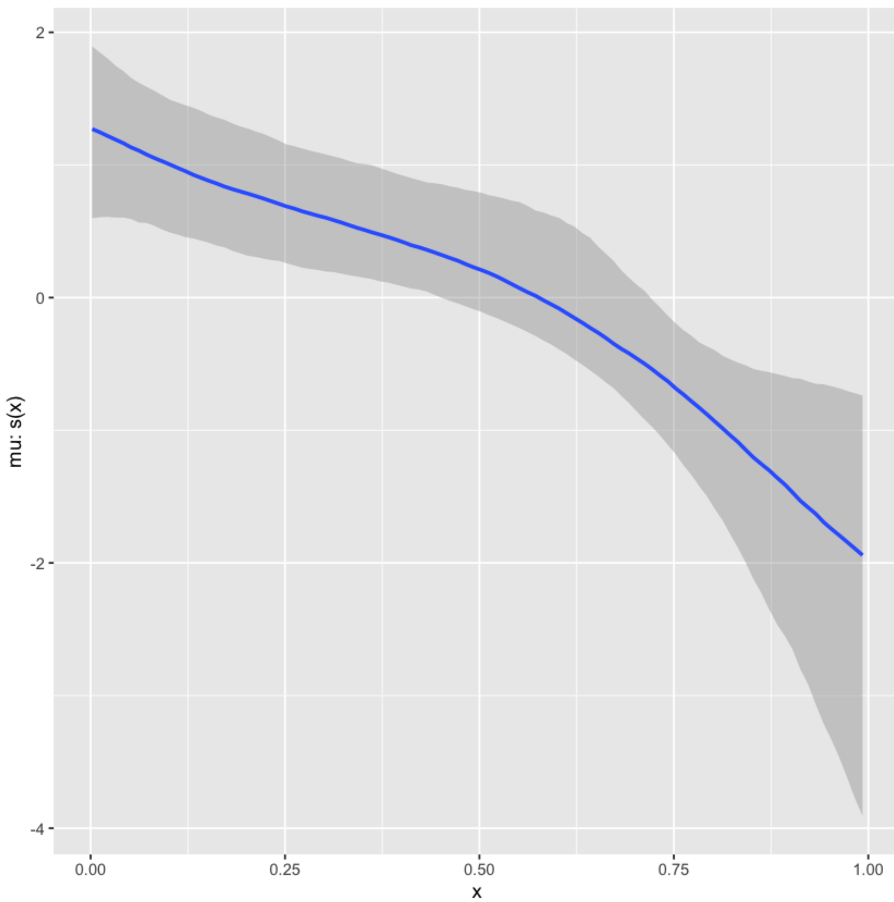


Figure 3: .25 Quantile