Research Group Presentation

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Overview of Bayesian analysis

Motivating example: Likelihood

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- Go Bayesian

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- Stan [Stan development team, 2012] improves on those packages by implementing HMC and NUTS
- Still need to becareful about your prior and even your model structure

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- BRMS [Buckner, 2017] is a high level interface for Stan on R
- It uses the same formula syntax as `lme4`
- Under the hood, it automatically generates and compiles Stan code, which we can retrieve

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General Framework

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General Framework

- OLS regression: $E[Y|\mathbf{X}] = \mathbf{X}^T\boldsymbol{\beta}$
- Quantile regression: $q_p(Y|\mathbf{X}) = \mathbf{X}^T\boldsymbol{\beta}(p)$
- Frequentist approach:

$$\widehat{\boldsymbol{\beta}(p)} = \mathrm{argmin}_{\boldsymbol{\beta}} \sum_i \rho_p(y_i - \mathbf{x}_i^T\boldsymbol{\beta})$$

where $\rho_p(u) = u(p - \mathbf{1}_{\{u<0\}})$

Bayesian quantile regression with AL

- It can be shown that the previous minimization problem is equivalent to maximizing the likelihood associated to the distribution

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- This leads to a proper posterior distribution (only after some additional post-analysis steps)

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  - We can write

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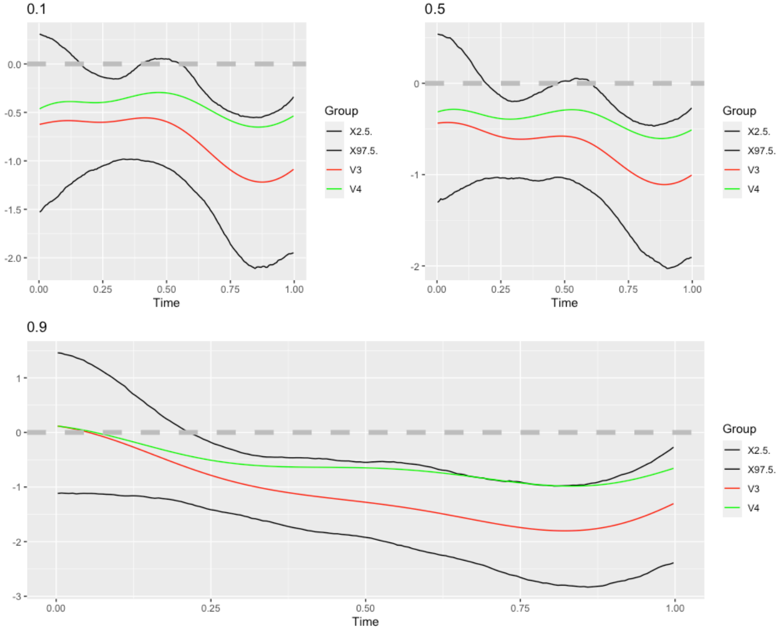
  - We even allow for a mixture of GAL as a likelihood (Dirichlet process mixture)


0.1
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-0.5
-1.0
-1.5
-2.0
0.00
0.25
0.50
0.75
1.00
Time
Group
X2.5.
X97.5.
V3
V4
0.5
0
-1
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Project we are working on

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- A Bayesian medation model with multiple outcome, latent mediator and a function covariate measured with error