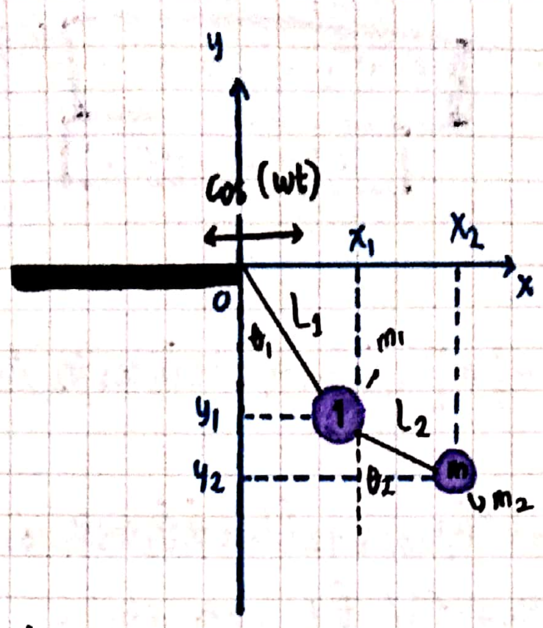

Ket :

l_1, l_2 = panjang
m_1, m_2 = massa
x, y = sistem koordinat

Tinjau m_1

$x_1 = l_1 \sin\theta_1 \rightarrow \dot{x} = l_1 \sin\theta_1$
$y_1 = l_1 \cos\theta_1$

$\dot{x} = l_1 \sin\theta_1$

$$= \frac{d}{dt}(l_1 \sin\theta_1) \cdot \frac{d\theta}{d\theta}$$
$$= l_1 \frac{d\theta}{dt} \cdot \frac{d}{d\theta}(\sin\theta_1)$$
$$= l_1 \dot{\theta}_1 \cos\theta_1$$

$\dot{y} = \frac{d}{dt}(l_1 \cos\theta_1) \frac{d\theta_1}{d\theta_1}$

$$= -l_1 \frac{d\theta_1}{dt} \frac{d}{d\theta_1}(\cos\theta_1)$$
$$= -l_1 \dot{\theta}_1 (-\sin\theta_1)$$
$$= l_1 \dot{\theta}_1 \sin\theta_1$$

Tinjau m_2

$x_2 = x_1 + l_2 \sin\theta_2$

$$= l_1 \sin\theta_1 + l_2 \sin\theta_2$$

$$\dot{x}_2 = \frac{d}{dt}(l_1 \sin\theta_1 + l_2 \sin\theta_2)$$
$$= l_1 \dot{\theta}_1 \cos\theta_1 + l_2 \dot{\theta}_2 \cdot \cos\theta_2$$

$y_2 = y_1 + l_2 \cos\theta_2$

$$= l_1 \cos\theta_1 + l_2 \cos\theta_2$$

$$\dot{y}_2 = \frac{d}{dt}(-l_1 \cos\theta_1 - l_2 \cos\theta_2)$$
$$= l_1 \dot{\theta}_1 \sin\theta_1 + l_2 \dot{\theta}_2 \sin\theta_2$$

$V = m_1 g y_1 + m_2 g y_2$

$$= m_1 g(-l_1 \cos\theta_1) + m_2 g(-l_1 \cos\theta_1 - l_2 \cos\theta_2)$$
$$= -m_1 g l_1 \cos\theta_1 - m_2 g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2$$
$$= -(m_1 + m_2) g \cdot l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2$$

$\therefore$
$$\dot{x}_1 = l_1 \dot{\theta}_1 \cos\theta_1$$
$$\dot{x}_2 = l_1 \dot{\theta}_1 \cos\theta_1 + l_2 \dot{\theta}_2 \cos\theta_2$$

$$\dot{y}_1 = l_1 \dot{\theta}_1 \sin\theta_1$$
$$\dot{y}_2 = l_1 \dot{\theta}_1 \sin\theta_1 + l_2 \dot{\theta}_2 \sin\theta_2$$

$$V = -(m_1 + m_2) g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2$$

Kel 3

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$$(x+y)^2 = x^2 + 2xy + y^2$$ NoTe!

$$\sin^2\theta + \cos^2\theta = 1$$

Energi kinetik (T) sistem

$$T = \frac{1}{2} m v_1^2 + \frac{1}{2} m V_2^2$$

$$= \frac{1}{2} m (\dot{x}_1 + \dot{y}_1)^2 + \frac{1}{2} m (\dot{x}_2 + \dot{y}_2)^2$$

$$= \frac{1}{2} m\left[(l_1 \dot{\theta}_1 \cos\theta_1)^2 + (l_1 \dot{\theta}_1 \sin\theta_1)^2\right] + \frac{1}{2} m\left[(l_1 \dot{\theta}_1 \cos\theta_1 + l_2 \dot{\theta}_2 \cos\theta_2)^2 + (l_1 \dot{\theta}_1 \sin\theta_1 + l_2 \dot{\theta}_2 \sin\theta_2)^2\right]$$

$$= \frac{1}{2} m\left[l_1^2 \dot{\theta}_1^2 \cos^2\theta_1 + l_1^2 \dot{\theta}_1^2 \sin^2\theta_1 + l_1^2 \dot{\theta}_1^2 \cos^2\theta + 2(l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos\theta_1 . \cos\theta_2) + l_2^2 \dot{\theta}_2^2 \cos^2\theta_2 + l_1^2 \dot{\theta}_1^2 \sin^2\theta_1 + 2(l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_1 \sin\theta_2) + l_2^2 \dot{\theta}_2^2 \sin^2\theta_2\right]$$

$$= \frac{1}{2} m\left[\left[l_1^2 \dot{\theta}_1^2 (\sin^2\theta_1 + \cos^2\theta_1)\right] + \left[l_1^2 \dot{\theta}_1^2 (\sin^2\theta_1 + \cos^2\theta_1) + l_2^2 \dot{\theta}_2^2 (\sin^2\theta_2 + \cos^2\theta_2) + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 (\cos\theta_1 \cos\theta_2 + \sin\theta_1 - \sin\theta_2)\right]\right]$$

$$T = \frac{1}{2} m_1 \left[l_1^2 \dot{\theta}_1^2 \right] + \frac{1}{2} m(l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2))$$

$$\boxed{\begin{array}{l} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0 \end{array}}$$

NoTe!

$$L = T - V$$

$$= \frac{1}{2} m_1 (l_1^2 \dot{\theta}_1^2) + \frac{1}{2} m_2 (l_1^2 \dot{\theta}_1^2 + l_2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)) + ((m_1 + m_2) g l_1 \cos\theta_1 - m g l_2 \cos\theta_2)$$

Turunkan Fungsi terhadap $(\dot{\theta}_1, \dot{\theta}_2, \theta_1, \theta_2)$

$$\frac{\partial L}{\partial \dot{\theta}_1} = m_1 l_1^2 \dot{\theta}_1 + m_2 l_1^2 \dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial L}{\partial \theta_1} = - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g l_1 \sin\theta_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_2 \sin(\theta_1 - \theta_2) . (\dot{\theta}_1 - \dot{\theta}_1)$$

$$m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_2 \sin(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2) + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g l_1 \sin\theta_1 = 0$$

$$(m_1 + m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2)$$

$$= m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g l_1 \sin\theta_1)$$

#$(m_1+m_2) l_1\ddot{\theta}_1 + m_2 l_2\ddot{\theta}_2 \cos(\theta_1-\theta_2) - m_2 l_2 \dot{\theta}_1\dot{\theta}_2 \sin(\theta_1-\theta_2) + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1-\theta_2)$

$+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_2) - (m_1+m_2) g \sin\theta_1) = 0$

$(m_1+m_2) l_1\ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1-\theta_2) + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1-\theta_2) - (m_1+m_2) g \sin\theta_1 = 0$

$2ml_1\ddot{\theta}_1 + ml_2\ddot{\theta}_2 - (2m)g\theta_1 = 0$

$2ml\ddot{\theta}_1 + ml\ddot{\theta}_2 - 2mg\theta_1 = 0$

$L = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \left(l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1-\theta_2) + (m_1+m_2)\right.$

$g l_1 \sin\theta_1 + m_2 g l_2 \sin\theta_2 \longrightarrow \left| \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0 \right|$

#$\frac{\partial}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1-\theta_2)$

$\frac{\partial L}{\partial \theta_2} = -m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_2) + m_2 g l_2 \cos\theta_2$

#$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2 l_2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 l_2 \dot{\theta}_1 \sin(\theta_1-\theta_2) . (\dot{\theta}_1-\dot{\theta}_2)$

$m_2 l_2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1-\theta_2) + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_2)$

$+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_2) - m_2 g l_2 \sin\theta_2 = 0$

$m_2 l_2 \ddot{\theta}_2 + m_2 l_1 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 \dot{\theta}_1^2 \sin(\theta_1-\theta_2) + m_2 l_1 \cancel{\dot{\theta}_1 \dot{\theta}_2} \sin(\theta_1-\theta_2)$

$+ m_2 l_1 \cancel{\dot{\theta}_1 \dot{\theta}_2} \sin(\theta_1-\theta_2) - m_2 g \sin\theta_2 = 0$

$m_2 l_2 \ddot{\theta}_2 + m_2 l_1 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 \dot{\theta}_1^2 \sin(\theta_1-\theta_2) - m_2 g \sin\theta_2 = 0$