CMPE 362
Digital Image Processing

Image Interpolation
Affine Transformations of Images

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Image interpolation

- Interpolation is a basic tool used in tasks such as
  - Zooming
  - Shrinking
  - Rotating

- Interpolation is the process of using known data to estimate values at unknown locations.

Slide credit: Following slides are mostly adapted from Prof. Sinisa Todorovic.

Image interpolation

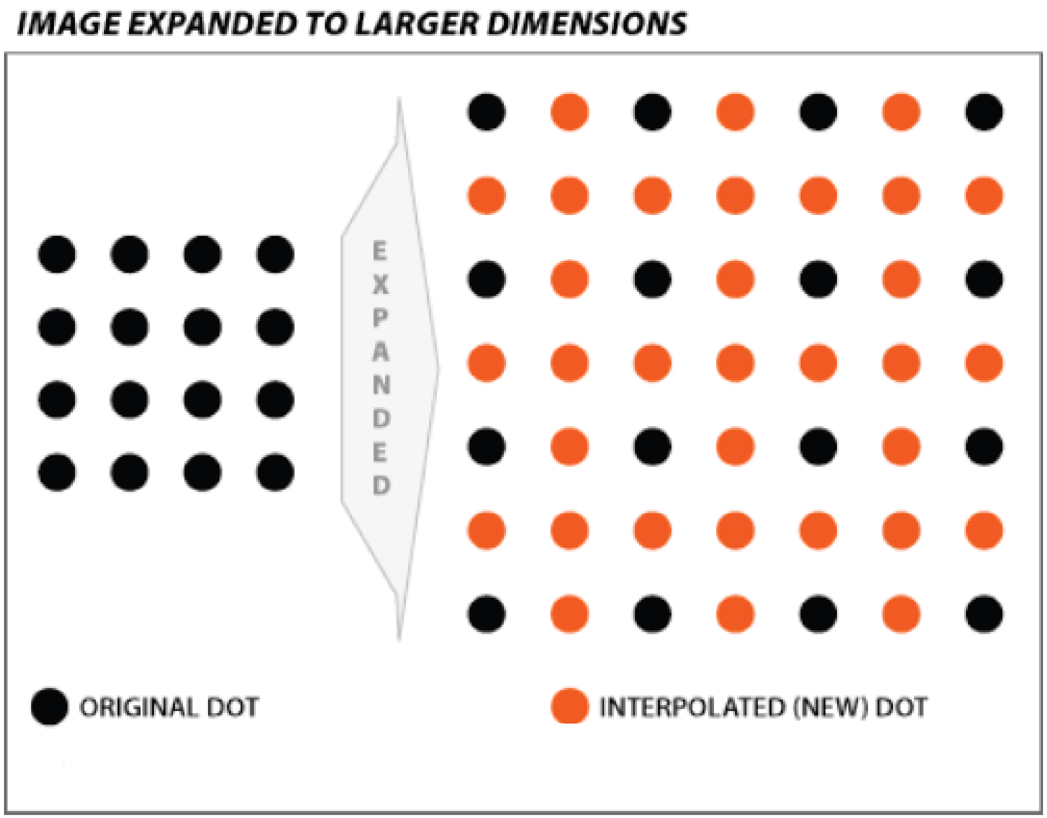
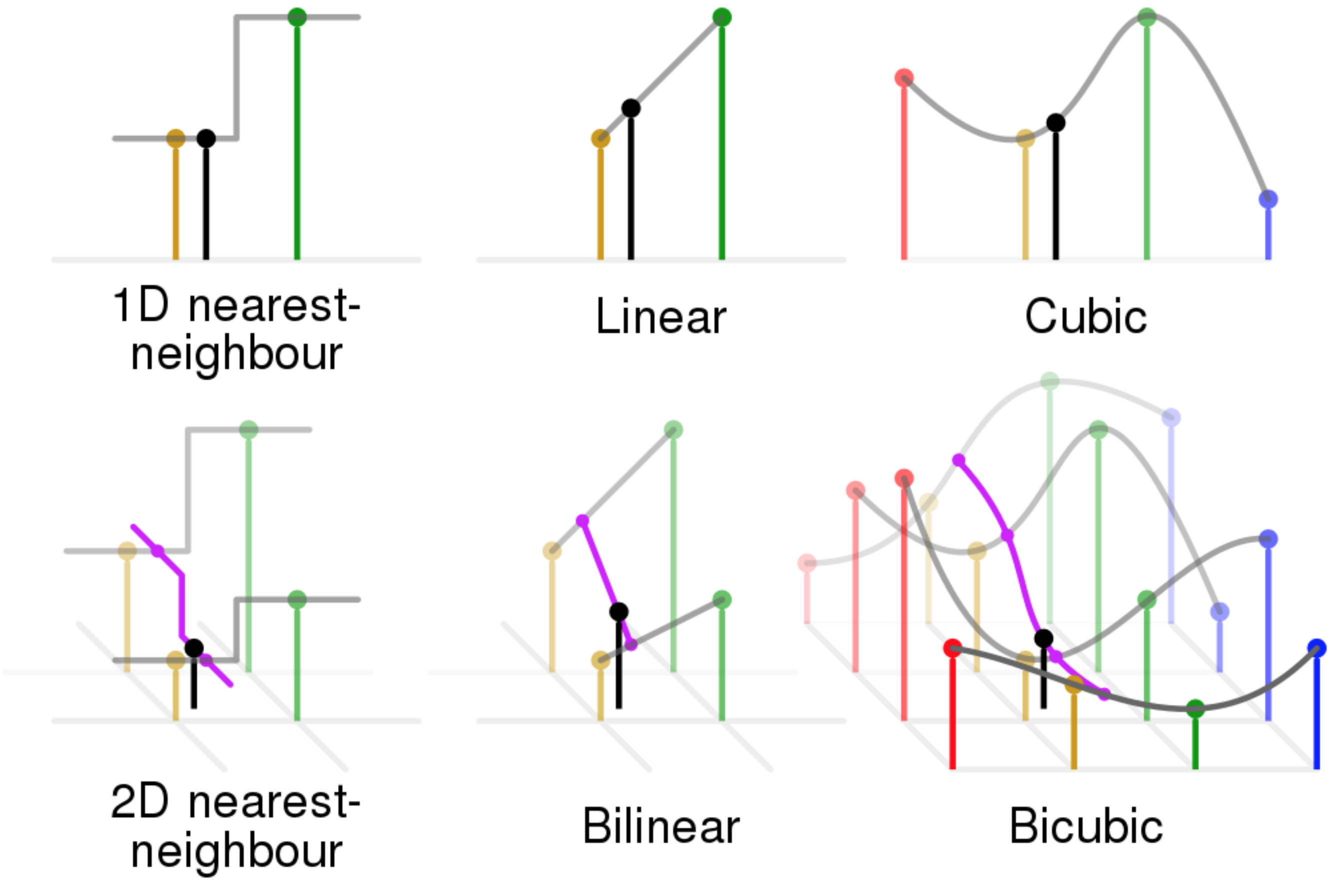


Image interpolation

- Nearest neighbor interpolation
- Bilinear interpolation
- Bicubic interpolation

Image interpolation



Nearest neighbor interpolation

$$\hat{I}(x_0, y_0) = I(u_0, v_0),$$

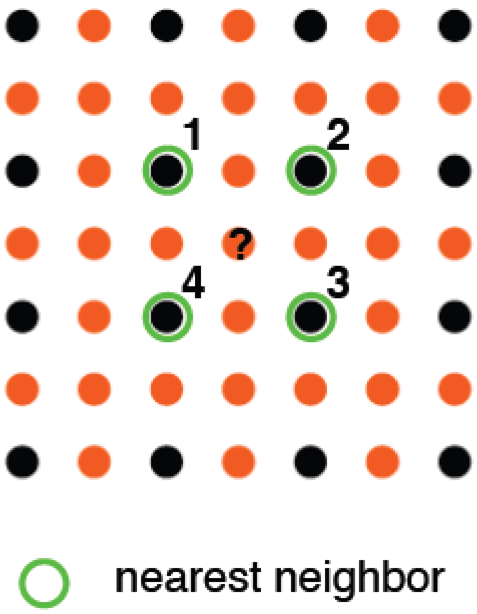
with

$$\begin{aligned} u_0 &= \text{round}(x_0) = \lfloor x_0 + 0.5 \rfloor \\ v_0 &= \text{round}(y_0) = \lfloor y_0 + 0.5 \rfloor. \end{aligned}$$

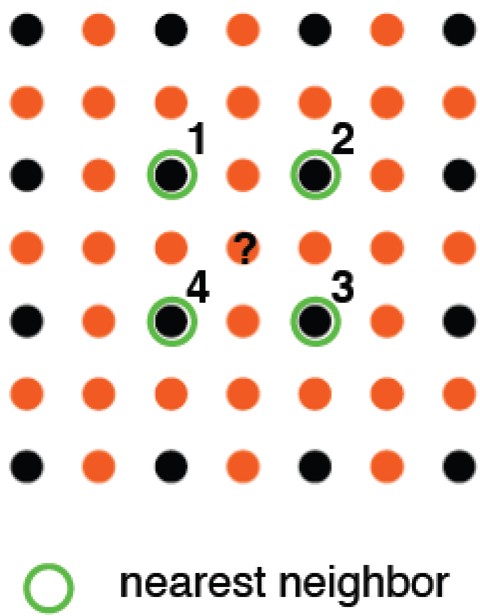
Bilinear interpolation

For the given interpolation point (x0,y0) , find the four closest (surrounding) pixels in image I

Bilinear interpolation



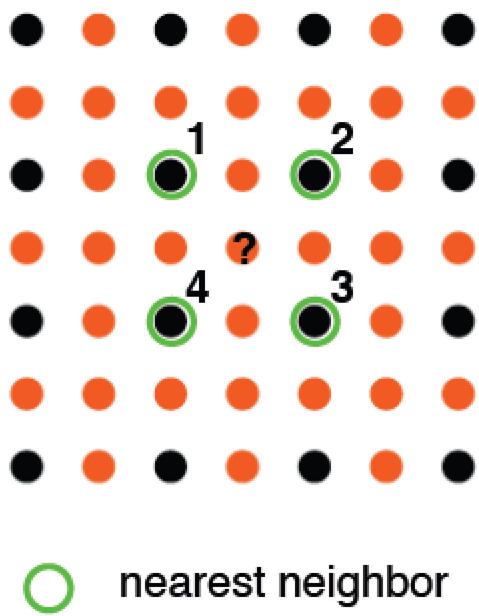
Bilinear interpolation



$$f(x, y) = ax + by + cxy + d$$

coefficients that need to be estimated

Bilinear interpolation



$$f(x, y) = ax + by + cxy + d$$

coefficients that need to be estimated

$$ax_1 + by_1 + cx_1y_1 + d = f(x_1, y_1)$$

$$ax_2 + by_2 + cx_2y_2 + d = f(x_2, y_2)$$

$$ax_3 + by_3 + cx_3y_3 + d = f(x_3, y_3)$$

$$ax_4 + by_4 + cx_4y_4 + d = f(x_4, y_4)$$

$$\Downarrow$$

$$a,\ b,\ c,\ d$$

Bicubic interpolation

Poll 2

- Bilinear N = 1
- Bicubic N = 3

$$f(x, y) = \sum_{i=0}^{N} \sum_{j=0}^{N} a_{ij} x^i y^j$$

new locations

new locations

estimated from the known neighboring locations

Image Interpolation


Nearest neighbor

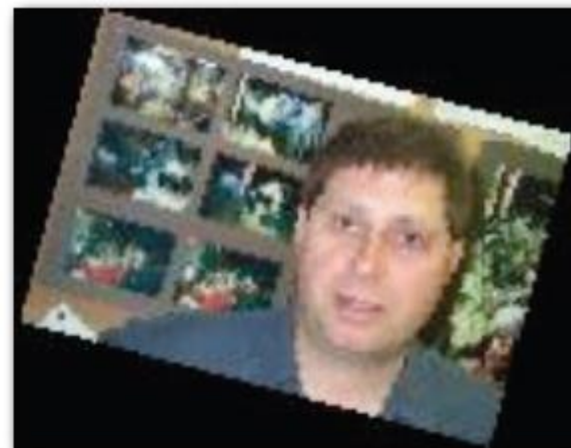

Bilinear


Bicubic

Image transformation

- Affine transforms
    - translation
    - scaling
    - rotation
    - shear

Image transformation

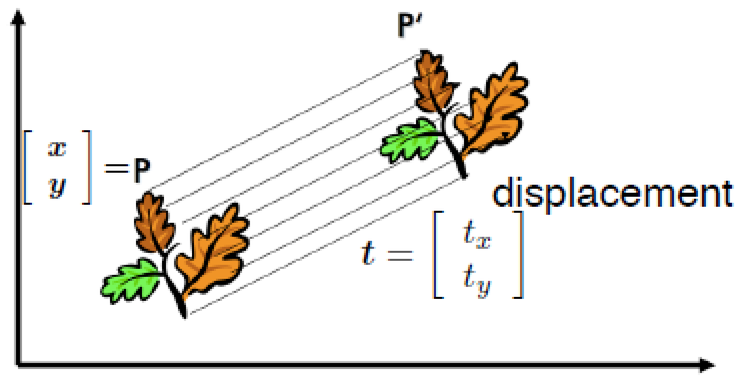
$$(x', y') = T\{(x, y)\}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous coordinates

Image transformation

2D Translation



homogeneous coordinates

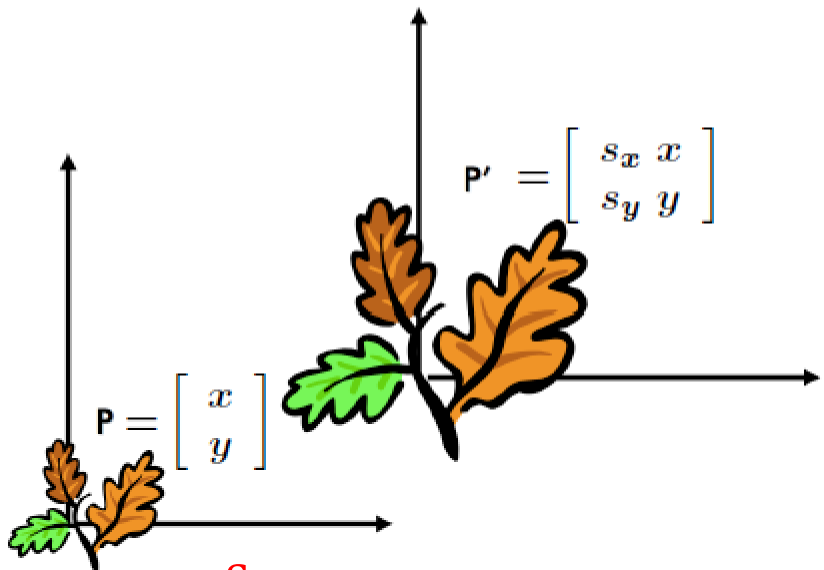
$$P' = P + \boldsymbol{t} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad P' \rightarrow \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translation matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Slide credit: Silvio Savarese

Image transformation

2D Scaling



S

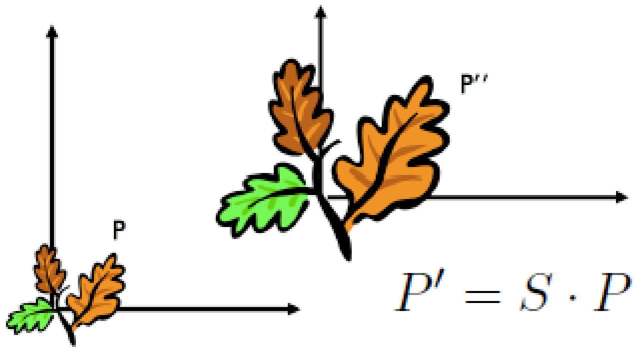
$$\begin{bmatrix} s_x\,x \\ s_y\,y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{scaling matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Slide credit: Silvio Savarese

Image transformation

Poll 3

2D Scaling + Translation



Is the ordering important?

$$P' = S \cdot P$$
$$P'' = T \cdot P' \quad \Rightarrow \quad P'' = (T \cdot S) \cdot P$$

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

scaling + translation matrix

Slide credit: Silvio Savarese

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

scaling + translation matrix

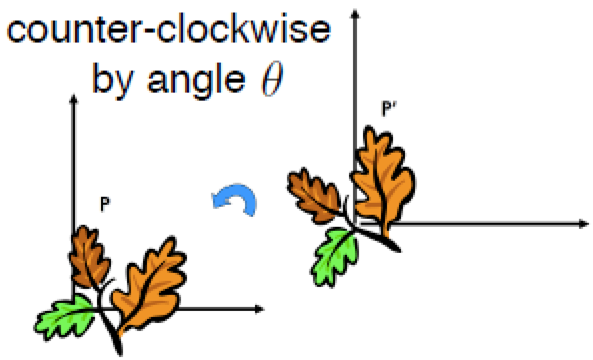
$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

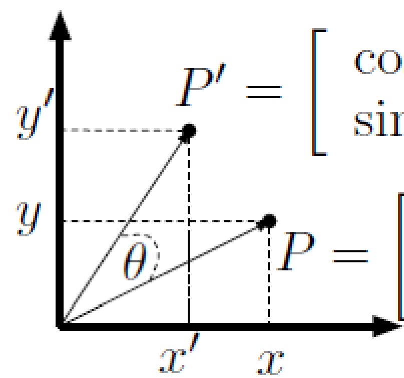
$$B = \begin{bmatrix} s_x & 0 & s_x * t_x \\ 0 & s_y & s_y * t_y \\ 0 & 0 & 1 \end{bmatrix}$$

translation + scaling matrix

Image transformation

2D Rotation





$$P' = \begin{bmatrix} \cos\theta \; x - \sin\theta \; y \\ \sin\theta \; x + \cos\theta \; y \end{bmatrix}$$

$$P = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \overbrace{\boxed{\begin{matrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{matrix}}}^{R} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{bmatrix}}_{\text{rotation matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

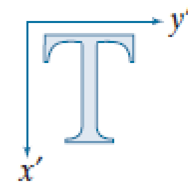
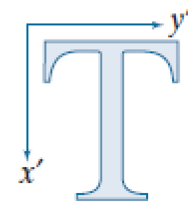
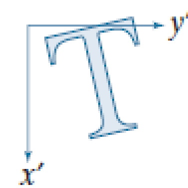
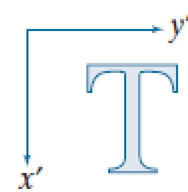
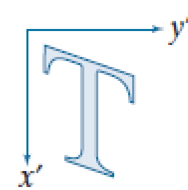
Slide credit: Silvio Savarese

Image transformation

2D Scaling + Rotation + Translation

$$
\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translation matrix}} \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{rotation matrix}} \underbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{scaling matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
$$

$$
\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R\,S & \boldsymbol{t} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
$$

Transformation Name	Affine Matrix, A	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = y$	
Scaling/Reflection (For reflection, set one scaling factor to –1 and the other to 0)	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = c_x x$ $y' = c_y y$	
Rotation (about the origin)	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x\cos\theta - y\sin\theta$ $y' = x\sin\theta + y\cos\theta$	
Translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + t_x$ $y' = y + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + s_v y$ $y' = y$	
Shear (horizontal)	$\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = s_h x + y$