CMPE 362
Digital Image Processing

Frequency Domain Techniques – Part II

Frequency Domain Filters

Filtering in Frequency Domain

- If we want to do a spatial filtering operation

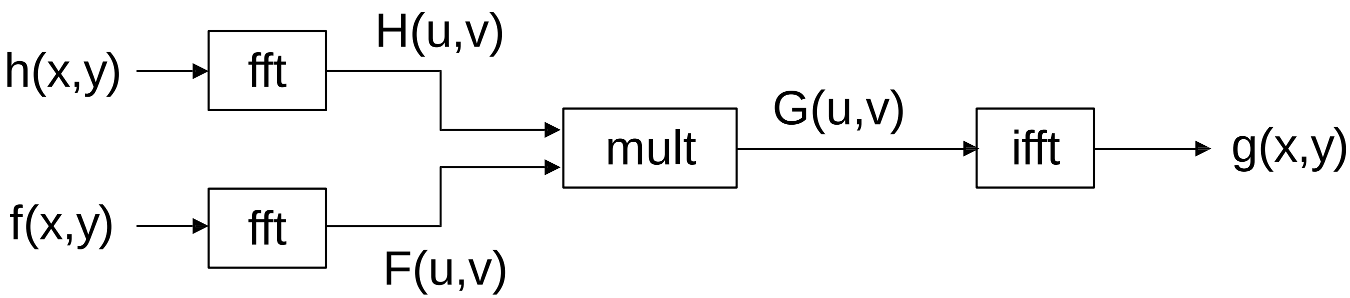
$$g(x,y) = h(x,y) * f(x,y)$$

- By the convolution theorem, we can transform the filter and the image to the frequency domain and do the operation there

$$\mathrm{F}\left\{h(x,y) * f(x,y)\right\} = H(u,v)F(u,v)$$

  – then

$$g(x,y) = \mathrm{F}^{-1}\left\{H(u,v)F(u,v)\right\}$$



- Notes:
  – The convolution filter is the same size as the image (you have to pad the filter with zeros if necessary)
  – Multiplication is point-by-point, of complex numbers

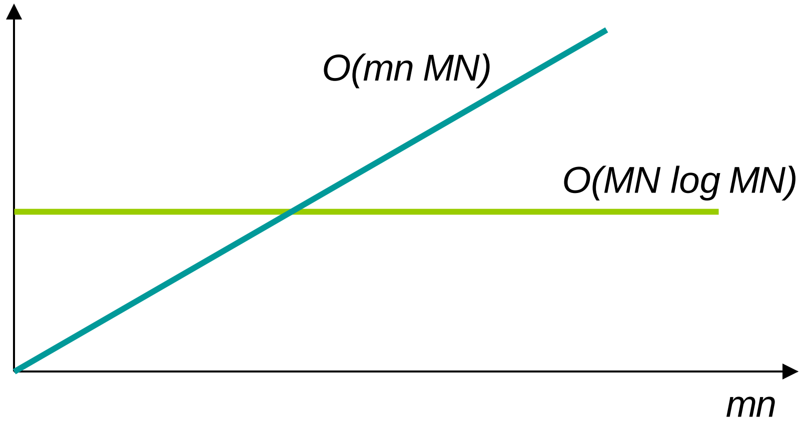
Advantages of Filtering in Frequency Domain

- Cost (number of operations) of the computation of Fast Fourier Transform is

  O(*MN* log *MN*)

  where *MN* = number of points in image
- The total cost of filtering in the frequency domain is dominated by FFT
- Compare this to convolution in spatial domain - it is O((*mn*)(*MN*))

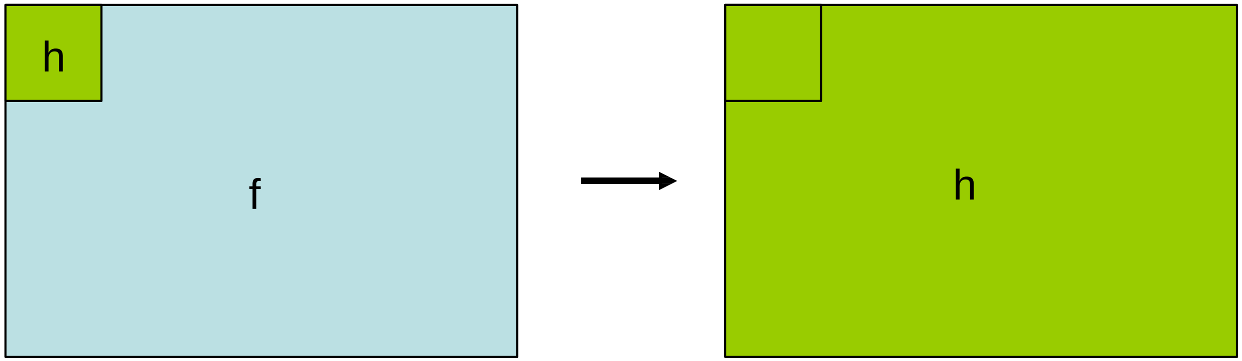
Plot of cost vs mn, with image size MN fixed



Convolution in frequency domain faster for large kernels (when mn gets much larger than log(MN))

Fourier-Domain Filtering

- Need to pad filter to be same size as image



- The inverse Fourier Transform (`ifft2`) should yield a real image
  - But take `real` of final result (to get rid of tiny imaginary values)

Example filtering_example1.ipynb

Ideal Low Pass Filter in 2D

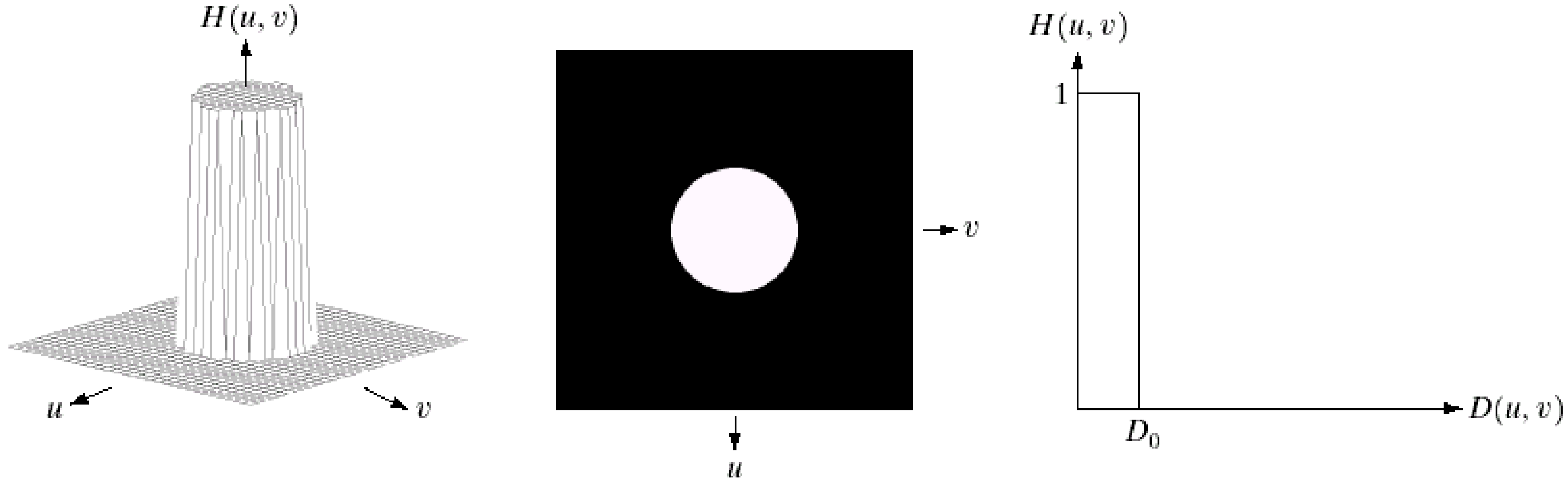
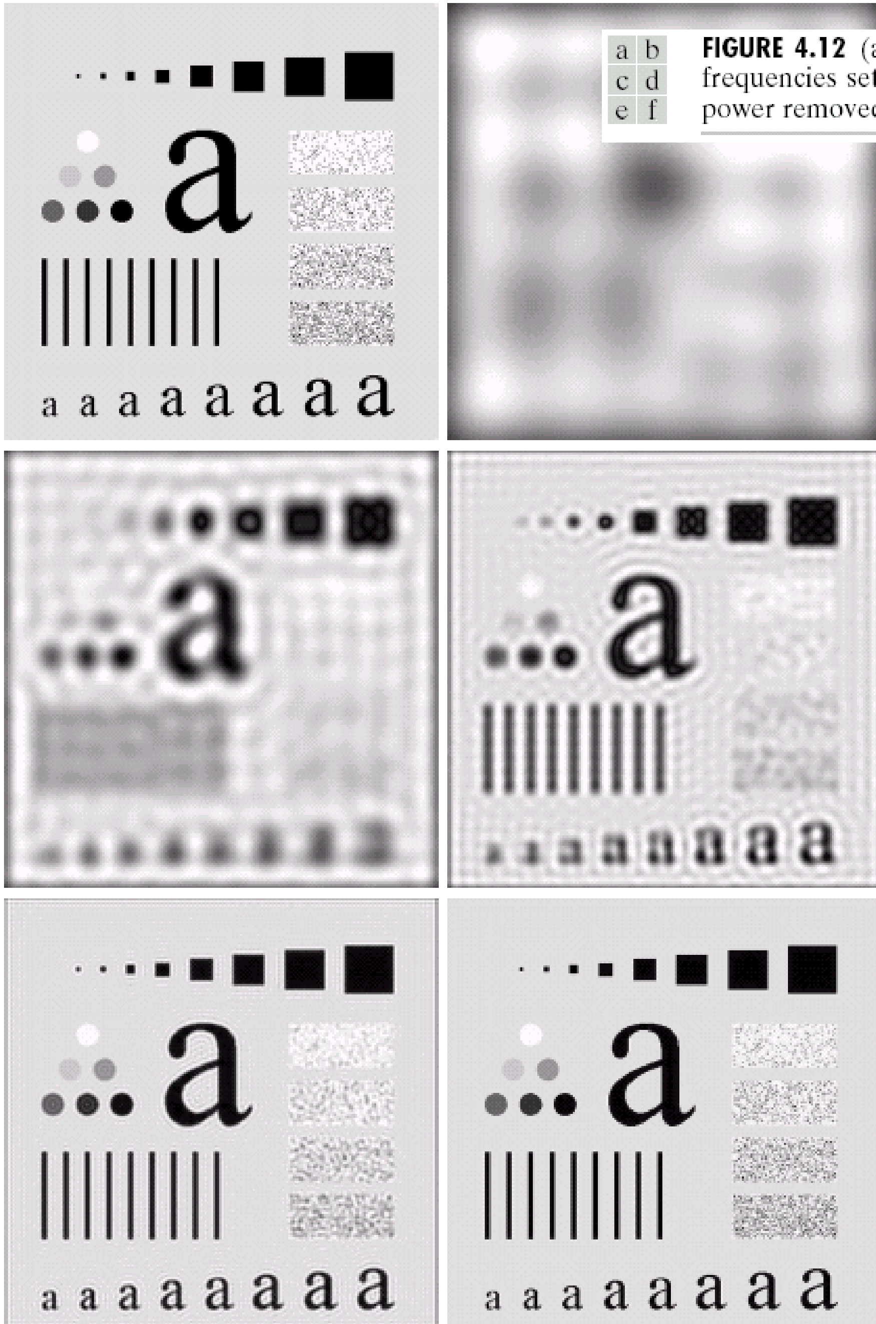


FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.



a b
c d
e f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

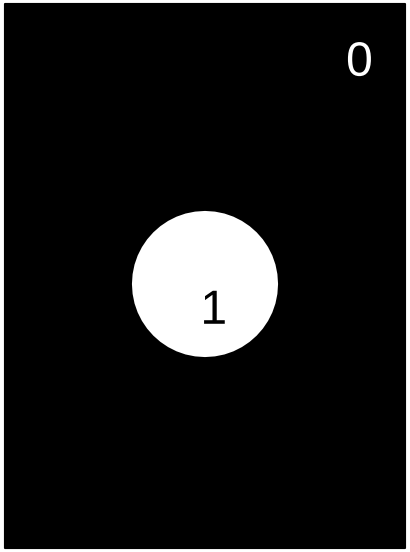
Note “ringing” near sharp edges

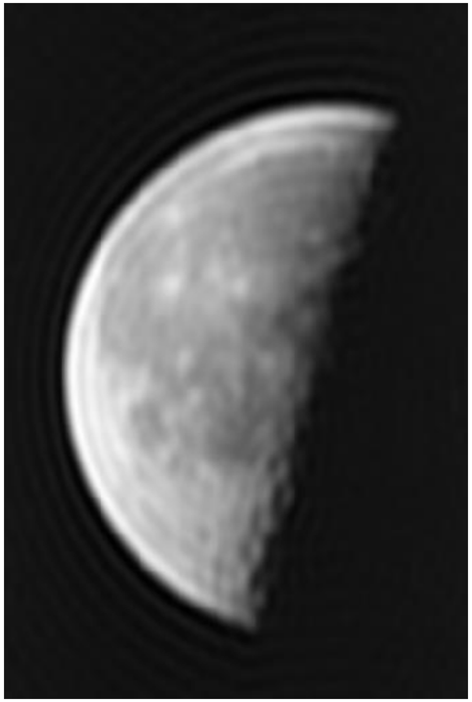
Example

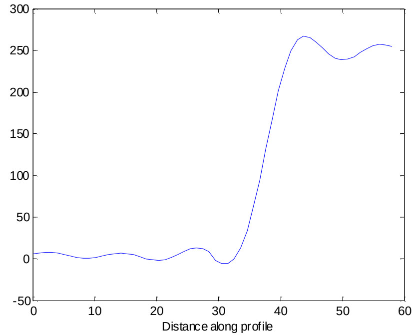
- Create image `H` of a disk in center
- Multiplication in freq domain

    `G = H * F`

- `ifft2(G)`
- Note ringing (do improfile)



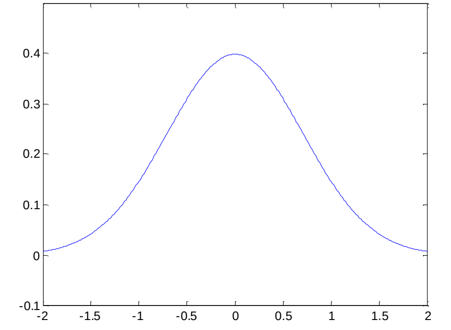


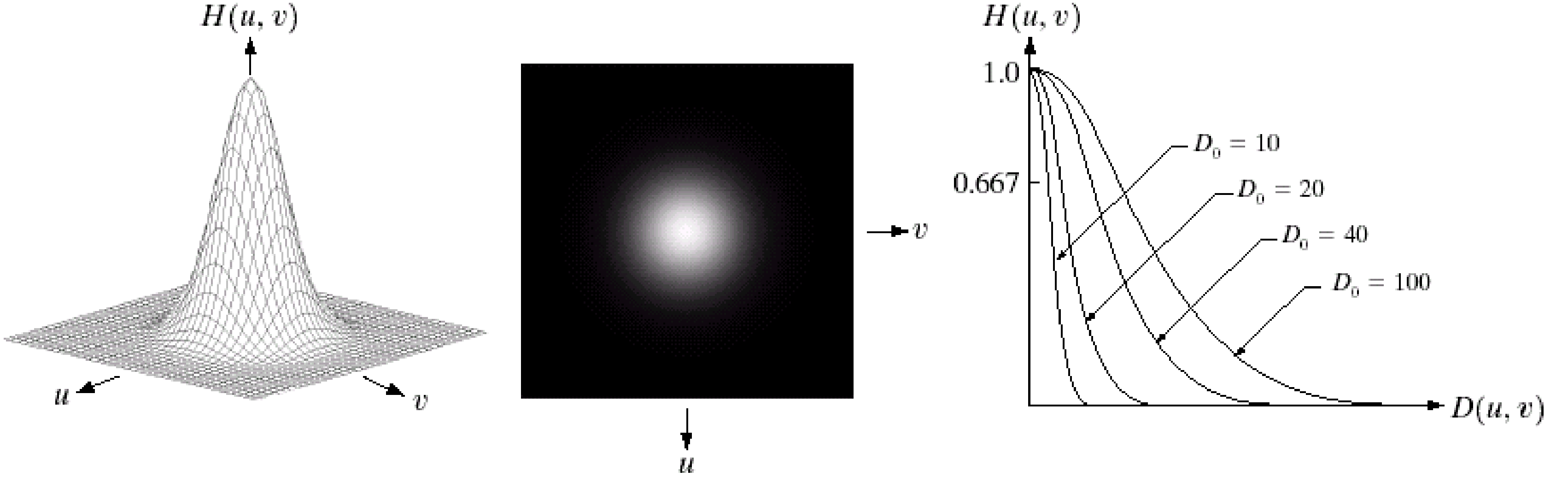


Example filtering_example2.ipynb

Gaussian Lowpass Filter

- A Gaussian in the spatial domain also has the form of a Gaussian in the frequency domain
- No ringing, but allows high frequencies to pass





a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

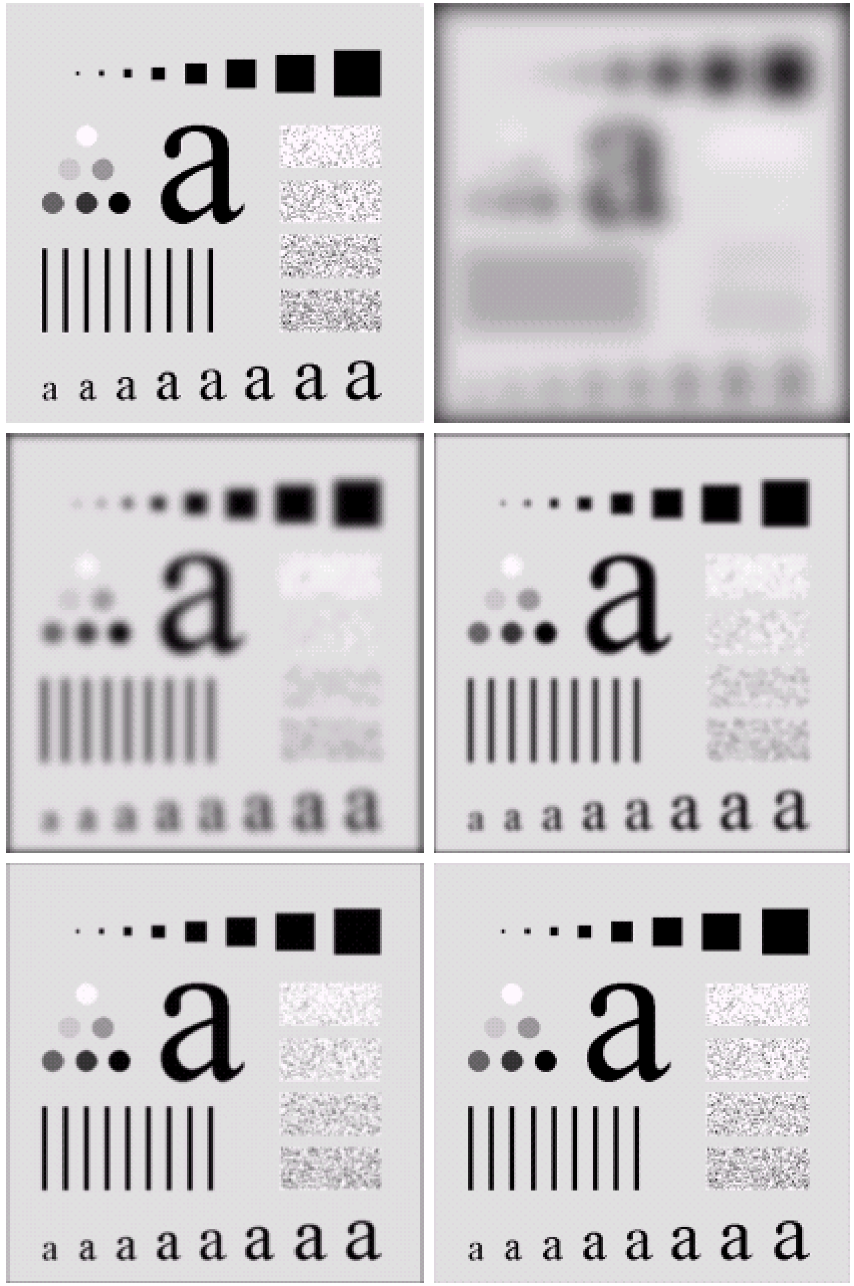


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

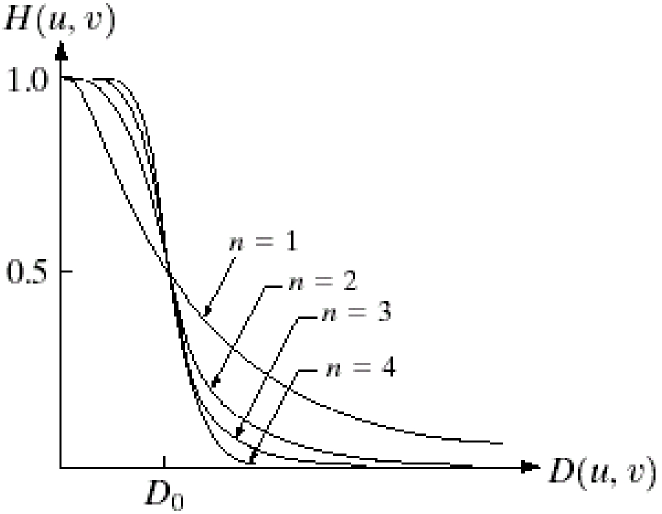
a b
c d
e f

Butterworth Lowpass Filter

- Definition

$$H(u,v)=\frac{1}{1+\left[D(u,v)/D_0\right]^{2n}}$$

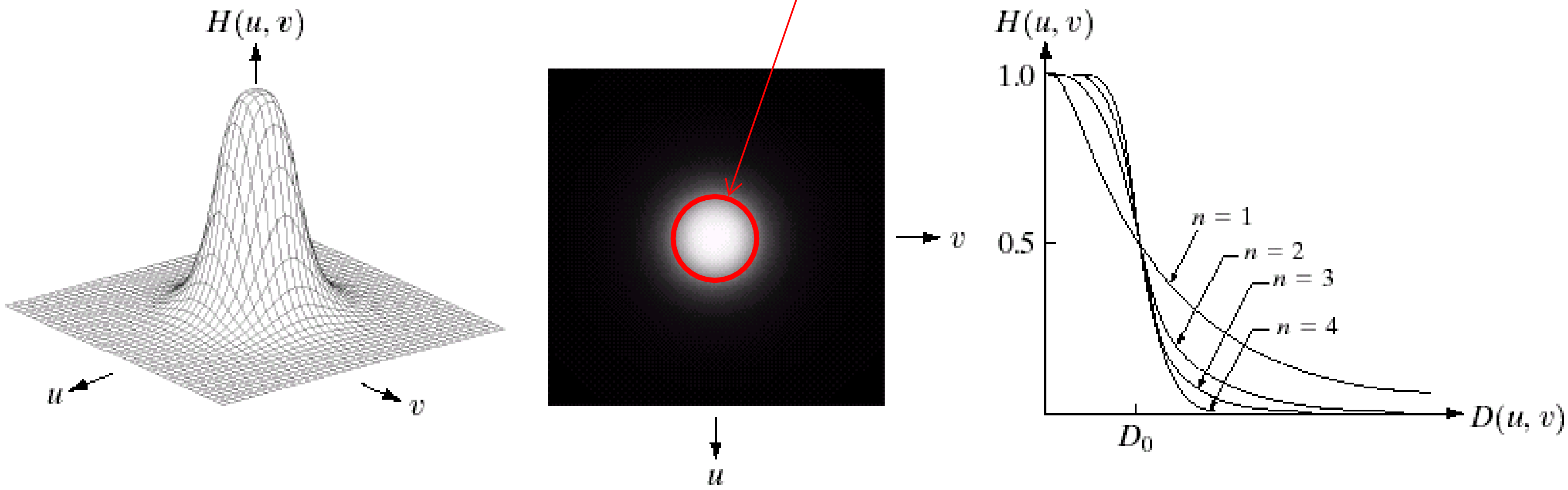
  - $D(u,v)$ is distance from (0,0) to (u,v)
  - D_0 is cutoff frequency
  - n is the "order" of the filter



- Properties
  - For $D(u,v) << D_0$, $H \approx 1$
  - For $D(u,v) >> D_0$, $H \approx 0$
  - At $D(u,v) = D_0$, $H = 1/2$

- Advantages
  - Reduces "ringing" while keeping clear cutoff
  - Tradeoff between amount of ringing and sharpness of cutoff

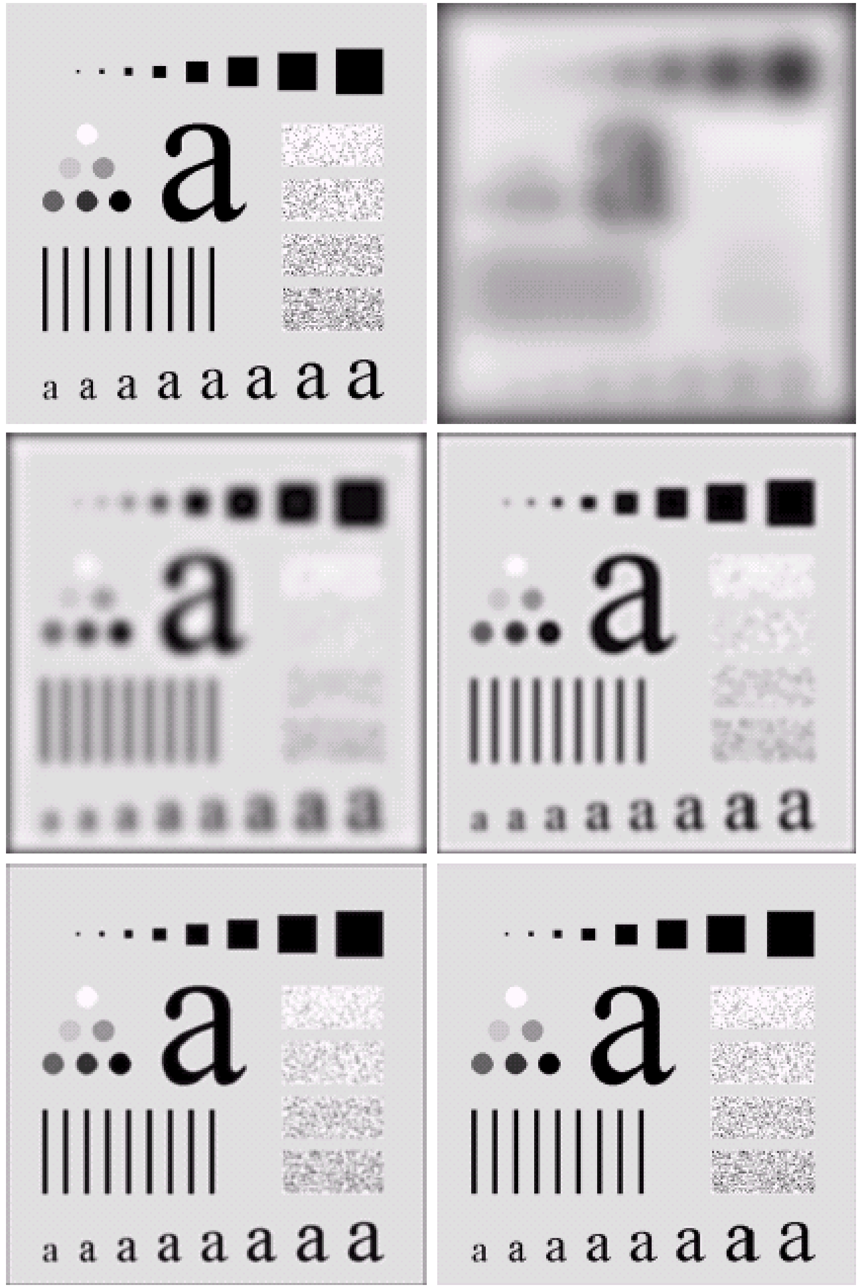
D_0 is the radius where the magnitude drops to 0.5



a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

For large n, H(u,v) approaches the ideal low pass filter



a b
c d
e f

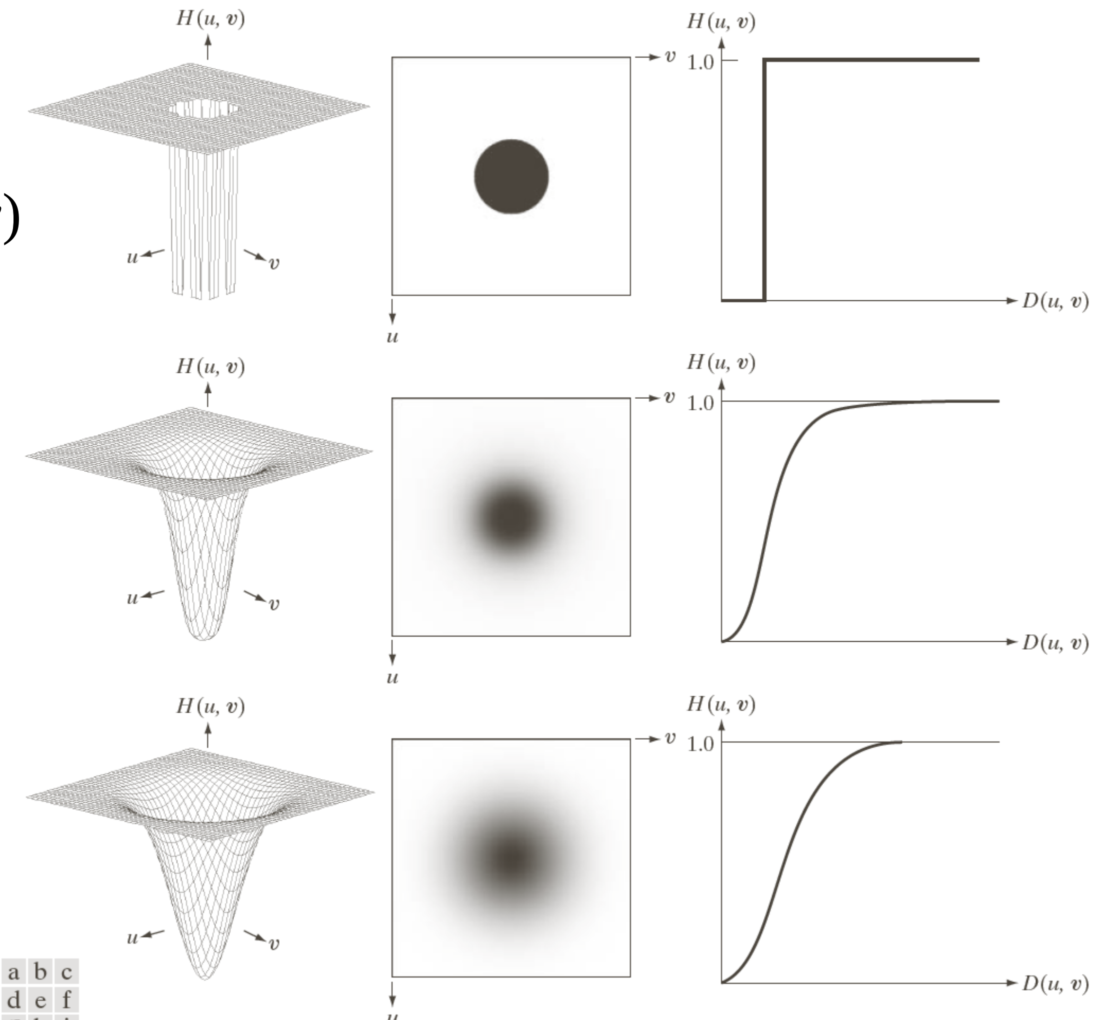
FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Sharpening Filters

- Can obtain by

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

- Types
  - Ideal high pass
  - Butterworth high pass
  - Gaussian high pass



a b c
d e f
g h i

FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60$, and 160.

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Notch Filters

- A filter that rejects (or passes) specific frequencies
- Example: periodic noise corresponds to spikes or lines in the Fourier domain
- Can design a filter with zeros at those frequencies … this will remove the noise
- Examples:
  - Image mosaics
  - Scan line noise
  - Halftoning noise (moire patterns)

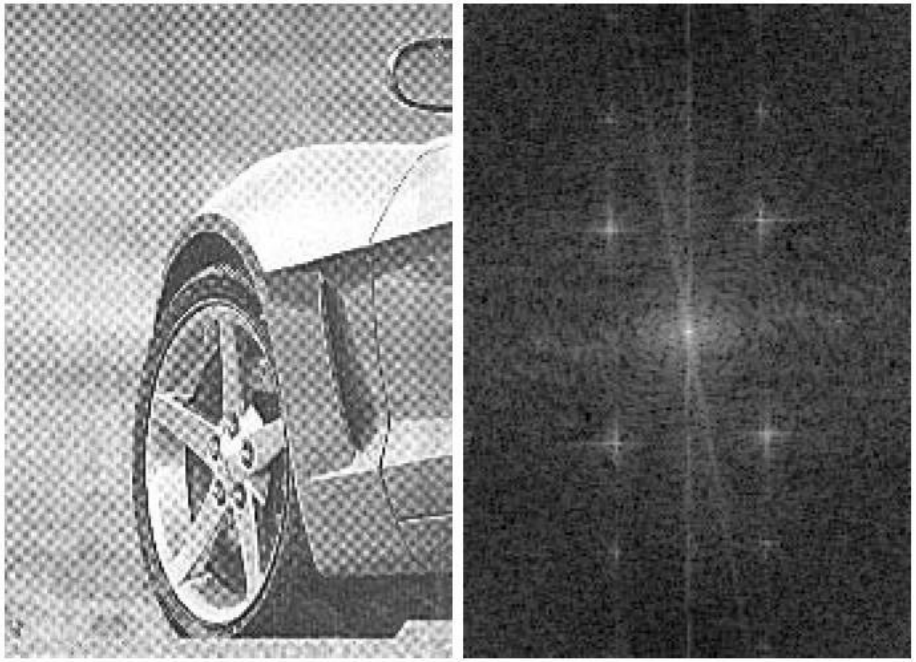


FIGURE 4.21 A newspaper image of size 246 × 168 pixels sampled at 75 dpi showing a moiré pattern. The moiré pattern in this image is the interference pattern created between the ±45° orientation of the halftone dots and the north–south orientation of the sampling grid used to digitize the image.

Steps in Notch Filtering

- Look at spectrum $|F(u,v)|$ of noisy image $f(x,y)$, find frequencies corresponding to the noise

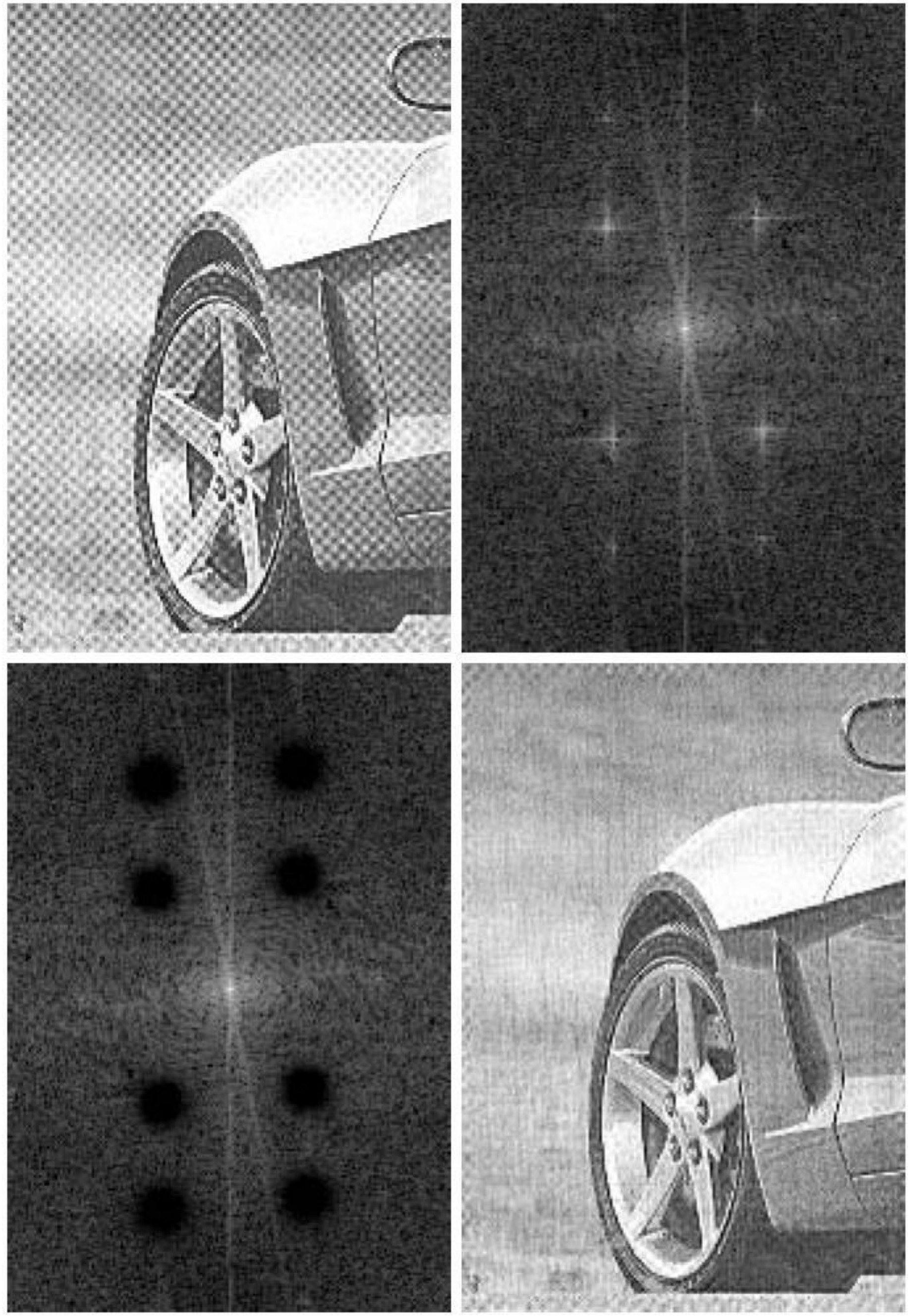
- Create a mask image $M(u,v)$ with notches (zeros) at those places, 1’s elsewhere

- Multiply mask with original image transform; this zeros out noise frequencies

  $$G(u,v) = M(u,v)\, F(u,v)$$

- Take inverse Fourier transform to get restored image

  $$g(x,y) = \mathrm{F}^{-1}(G(u,v))$$



a b
c d

FIGURE 4.64
(a) Sampled newspaper image showing a moiré pattern.
(b) Spectrum.
(c) Butterworth notch reject filter multiplied by the Fourier transform.
(d) Filtered image.

Summary

- The convolution theorem says that convolution in one domain (e.g., spatial) is equivalent to point-by-point multiplication in the other domain (e.g., frequency).
  - It gives us a way to understand the behavior of filters.

- Examples of some kinds of filters:
  - An "ideal lowpass filter" passes all frequencies with magnitudes below a specific level, and attenuates all frequencies above that level.
  - An "ideal highpass filter" does the opposite.
  - A "notch" filter rejects (or passes) frequencies at a specific point (the notch).