CMPE 362
Digital Image Processing

Spatial Domain Operations I: Intensity Transformations

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Image processing techniques

1. Spatial domain operations
   - operate on the pixels of an image
2. Frequency domain operations
   - operate on the Fourier transform of an image
     1. Compute the Fourier transform of an image
     2. Operate
     3. Take the inverse transform

Spatial domain operations

$$g(x,y) = T[f(x,y)]$$

$f(x,y)$: input image
$g(x,y)$: output image
T : operator defined over a neighborhood of (x,y)

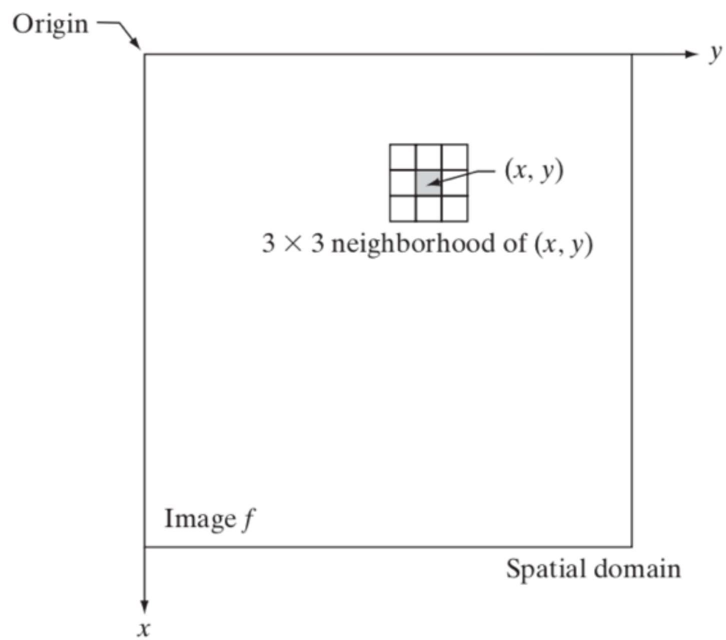


FIGURE 3.1 A 3 × 3 neighborhood about a point (x, y) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

Intensity transformations

- When the neighborhood size is 1x1, each pixel is processed independently of other pixels.
- The output image g at a pixel (x, y) depends only on the value of input image f at the same pixel (x, y).
- The transformation function T remaps each intensity value:

$$s = T(r)$$

r is the value in the input

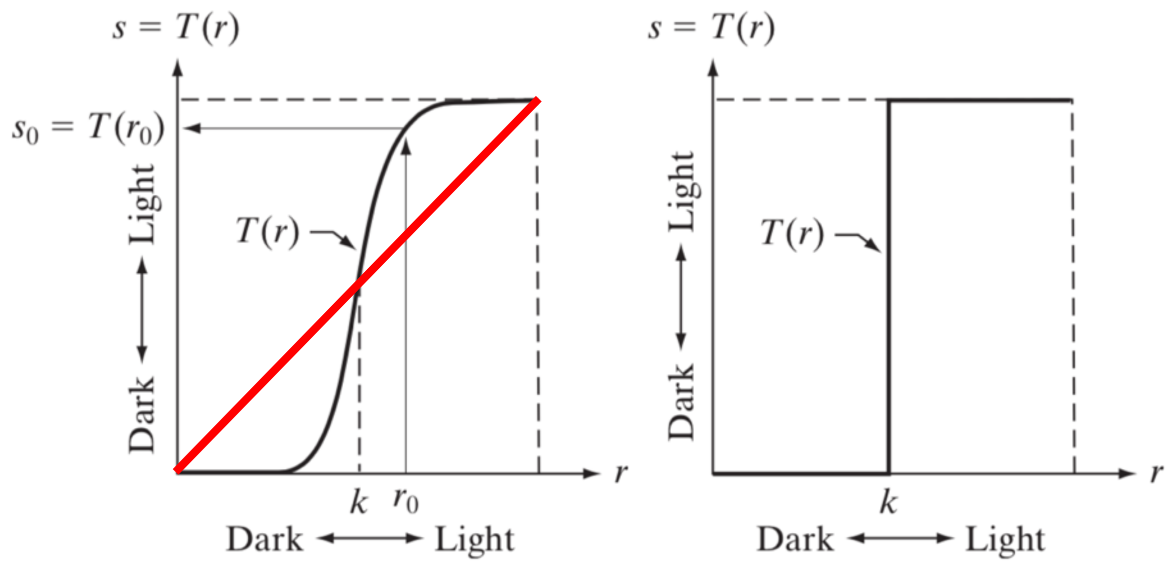
s is the new value in the output

T is an intensity transformation function

Intensity transformation examples

a b

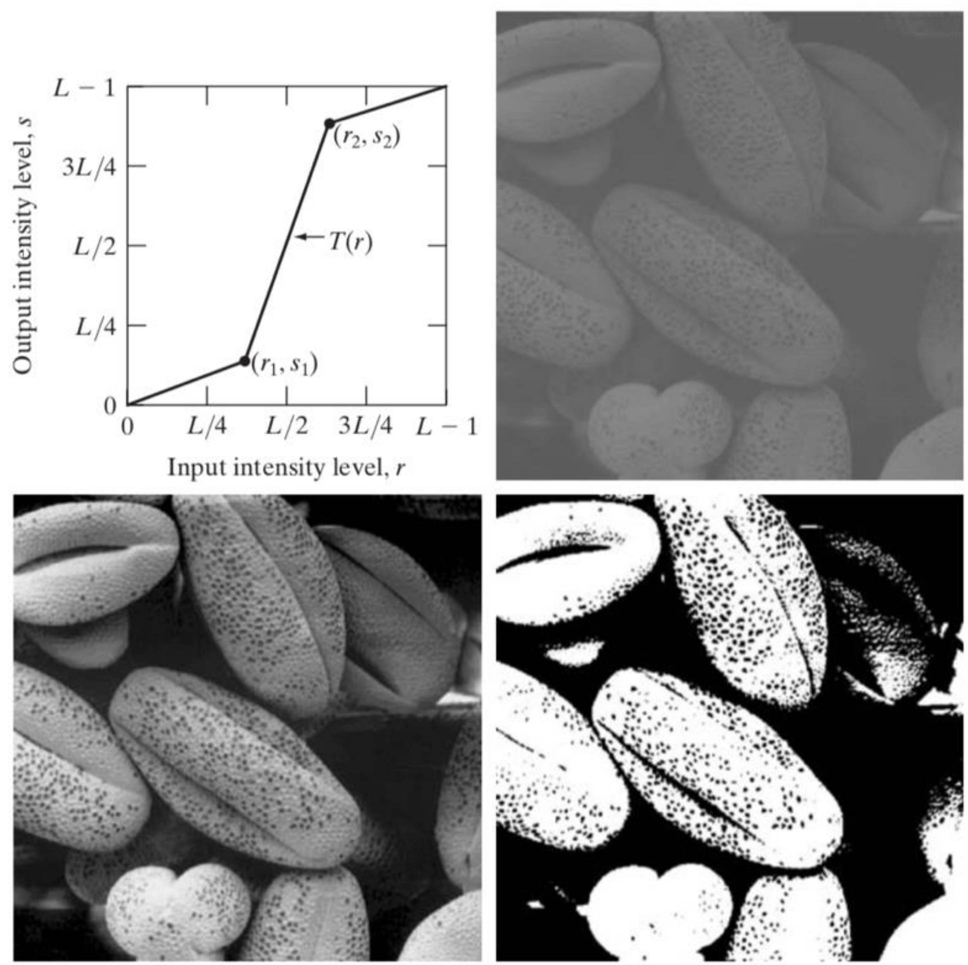
FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.



produces an image of higher contrast than the original by darkening the intensity levels below k and brightening intensities above k

produces a binary (two-intensity level) image

Contrast stretching and Thresholding



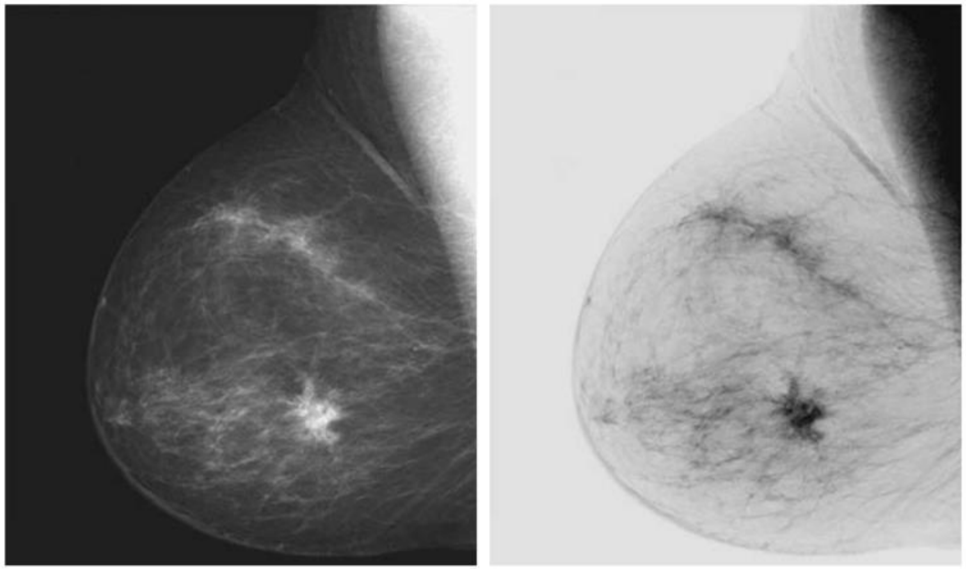
a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

$$(r_1, s_1) = (r_{min}, 0)$$
$$(r_2, s_2) = (r_{max}, L-1)$$

$$(r_1, s_1) = (m, 0)$$
$$(r_2, s_2) = (m, L-1)$$

Image negative



a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

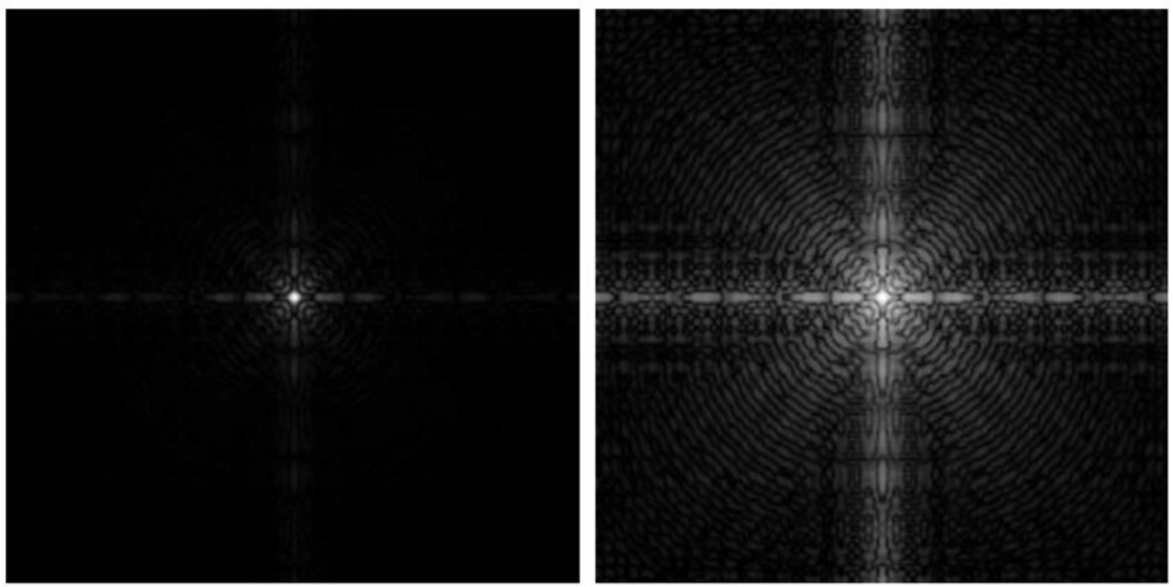
$$s = (L - 1) - r$$

Log transformations

$$s = c \log(1 + r)$$

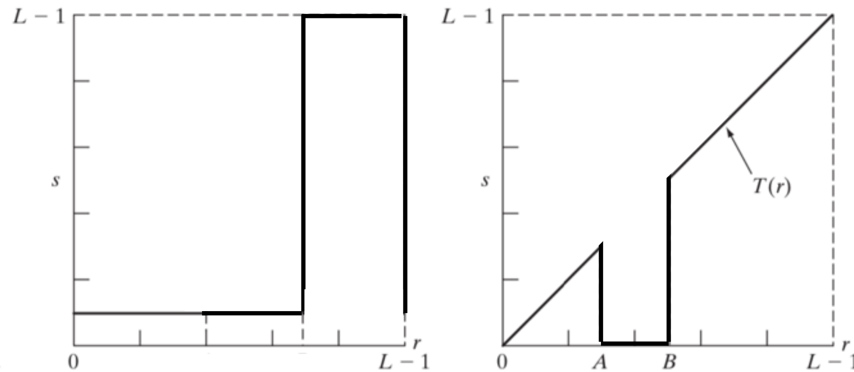
a b

FIGURE 3.5
(a) Fourier spectrum.
(b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.



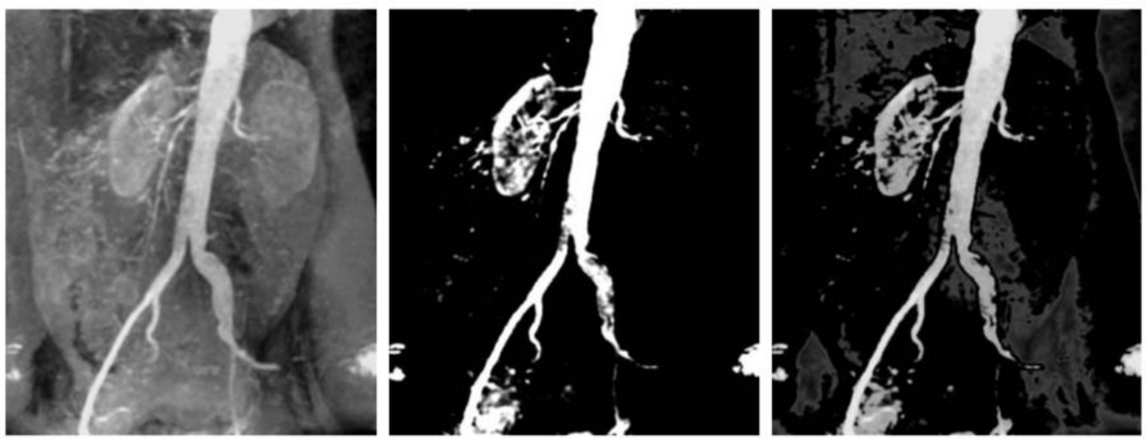
values in range 0 to 1.5×10^6 values in range 0 to 6.2

Intensity level slicing



a b

FIGURE 3.11 (a) This transformation highlights intensity range [A, B] and reduces all other intensities to a lower level. (b) This transformation highlights range [A, B] and preserves all other intensity levels.



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Power-Law (Gamma) Transformations

$$s = cr^{\gamma}$$

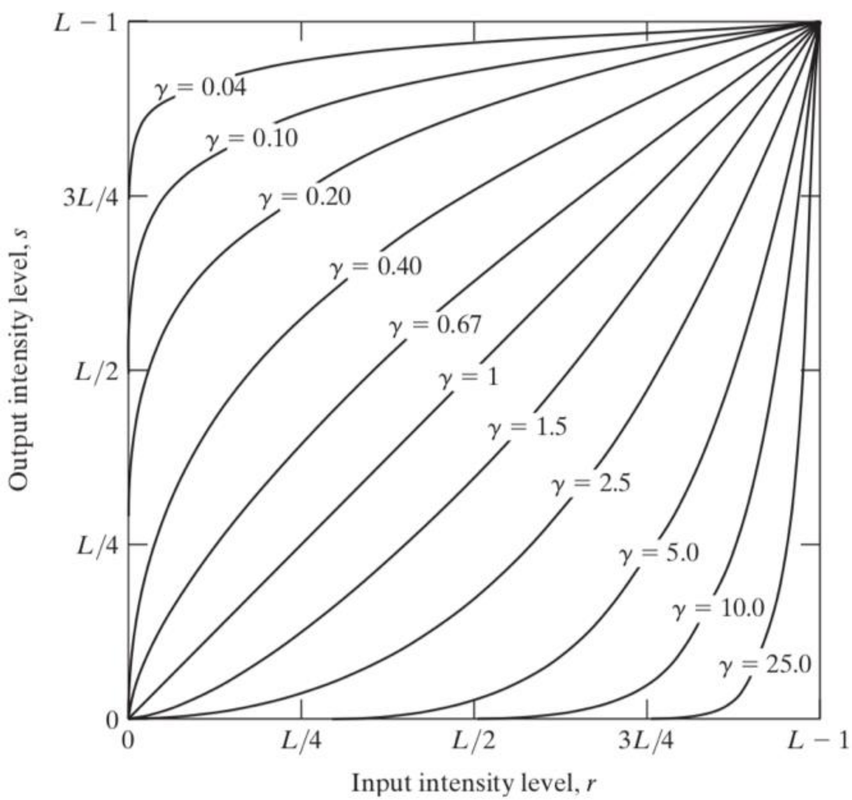
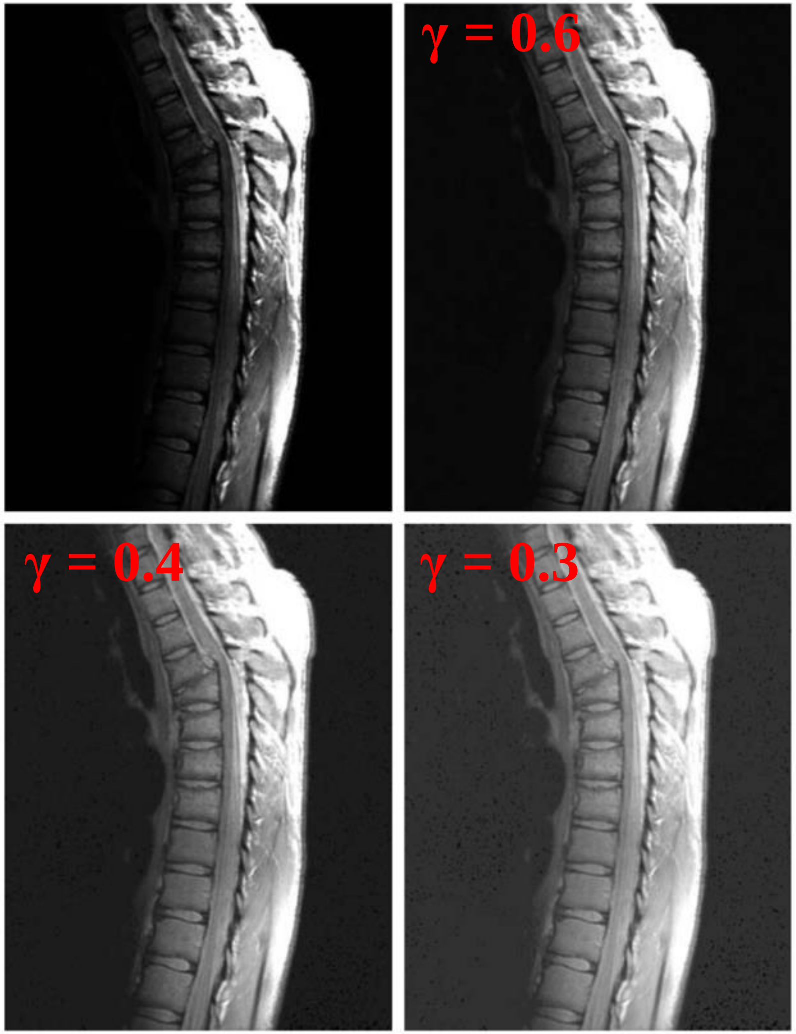


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

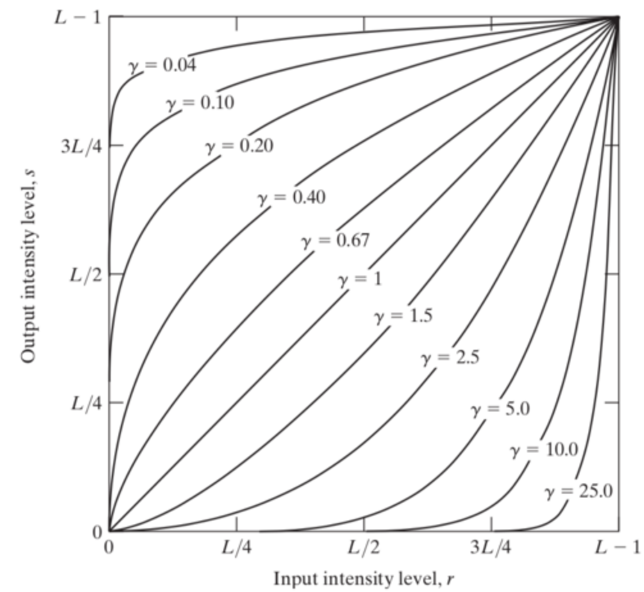
Power-Law (Gamma) Transformations



a b
c d

FIGURE 3.8
(a) Magnetic resonance image (MRI) of a fractured human spine.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

$$s = cr^{\gamma}$$



Power-Law (Gamma) Transformations

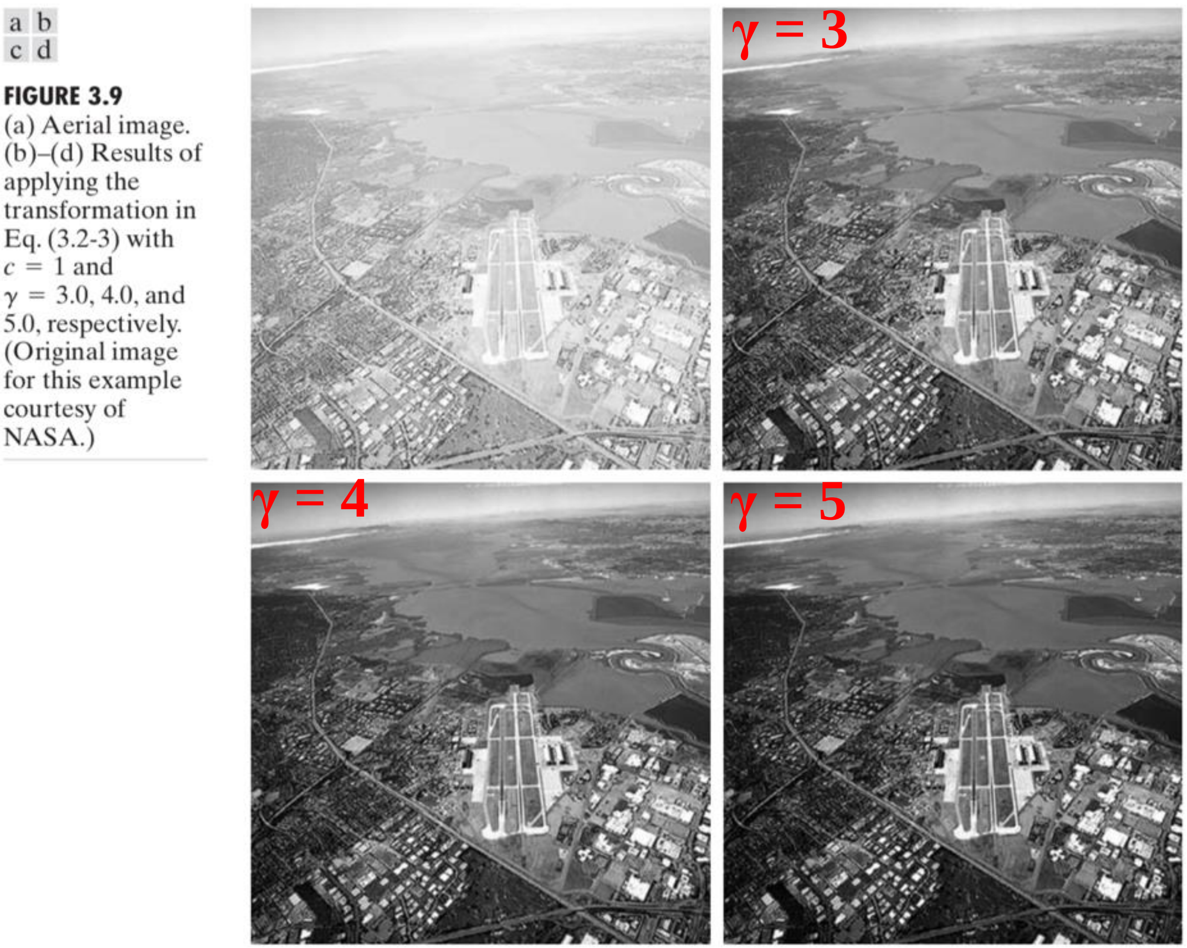
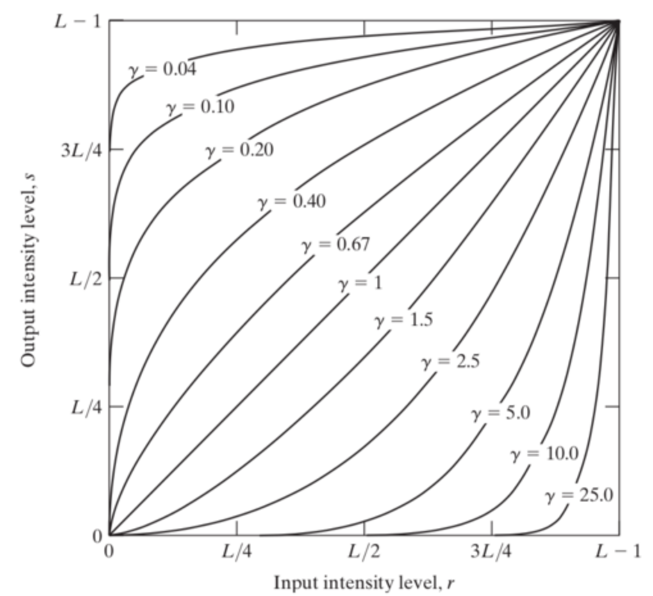


FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0$, 4.0, and 5.0, respectively. (Original image for this example courtesy of NASA.)

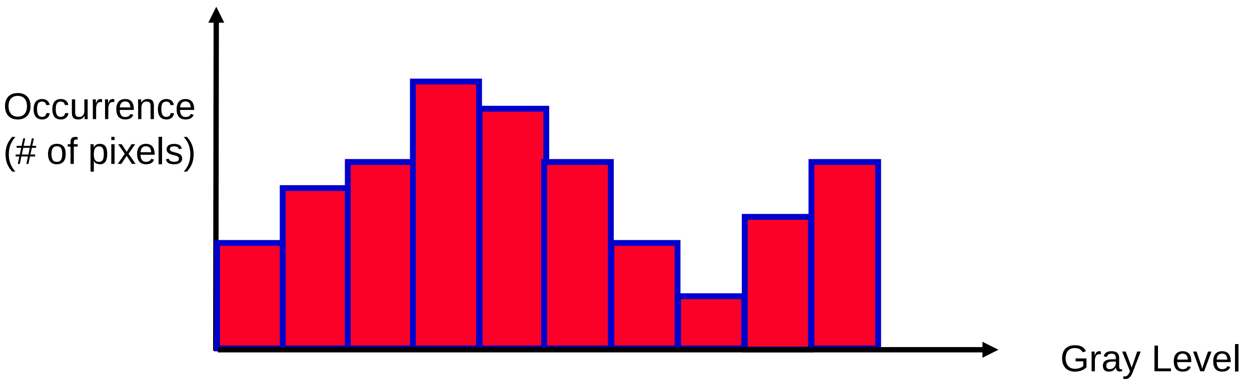
$$s = cr^{\gamma}$$



Histogram equalization

- Find a transformation function that creates an output image with a uniform histogram
  - Every intensity level is equally likely to occur in an image

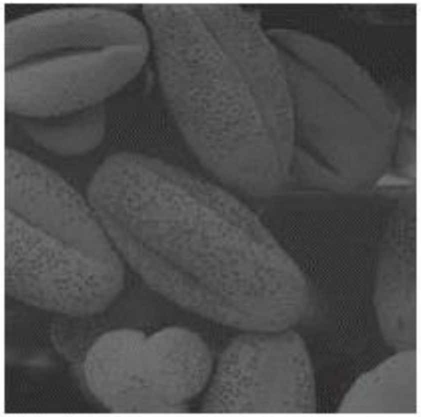
Image Histogram



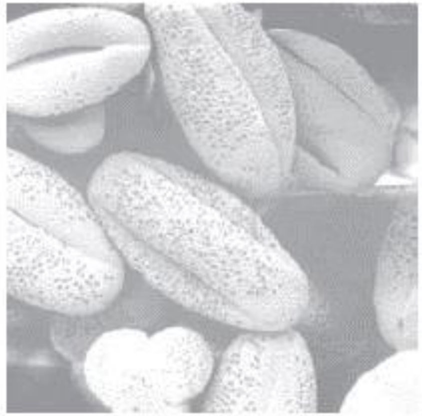
- $h(r_k)$ specifies the # of pixels with gray-value r_k

Poll 4

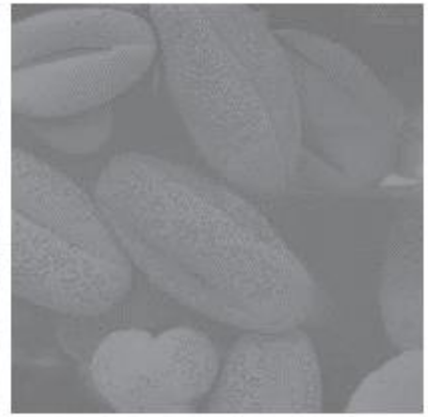
A



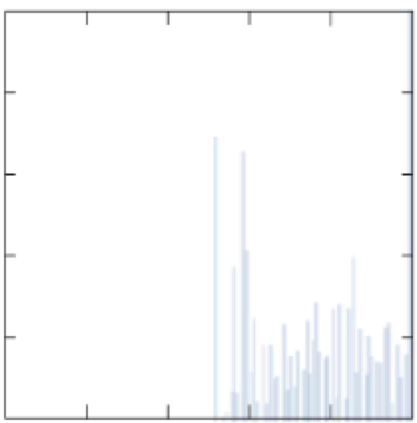
B



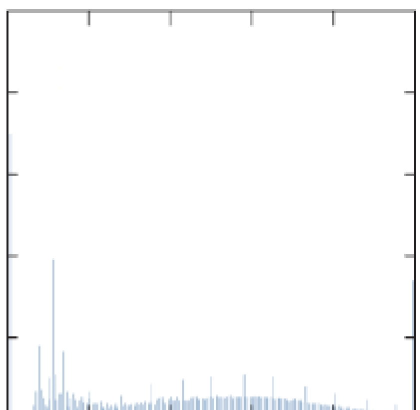
C



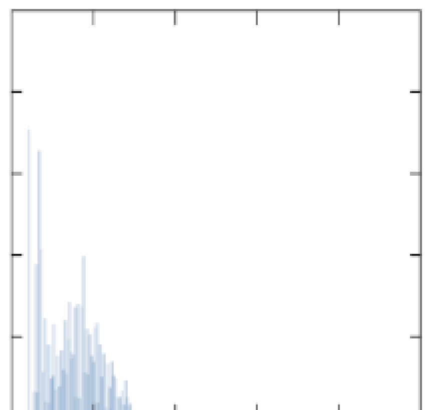
D



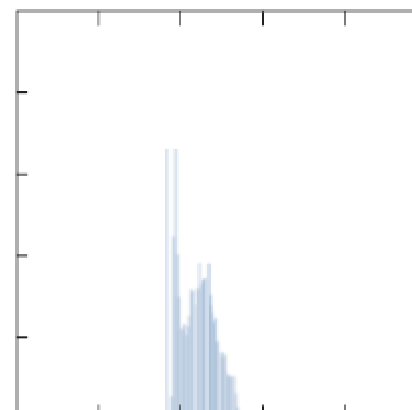
1



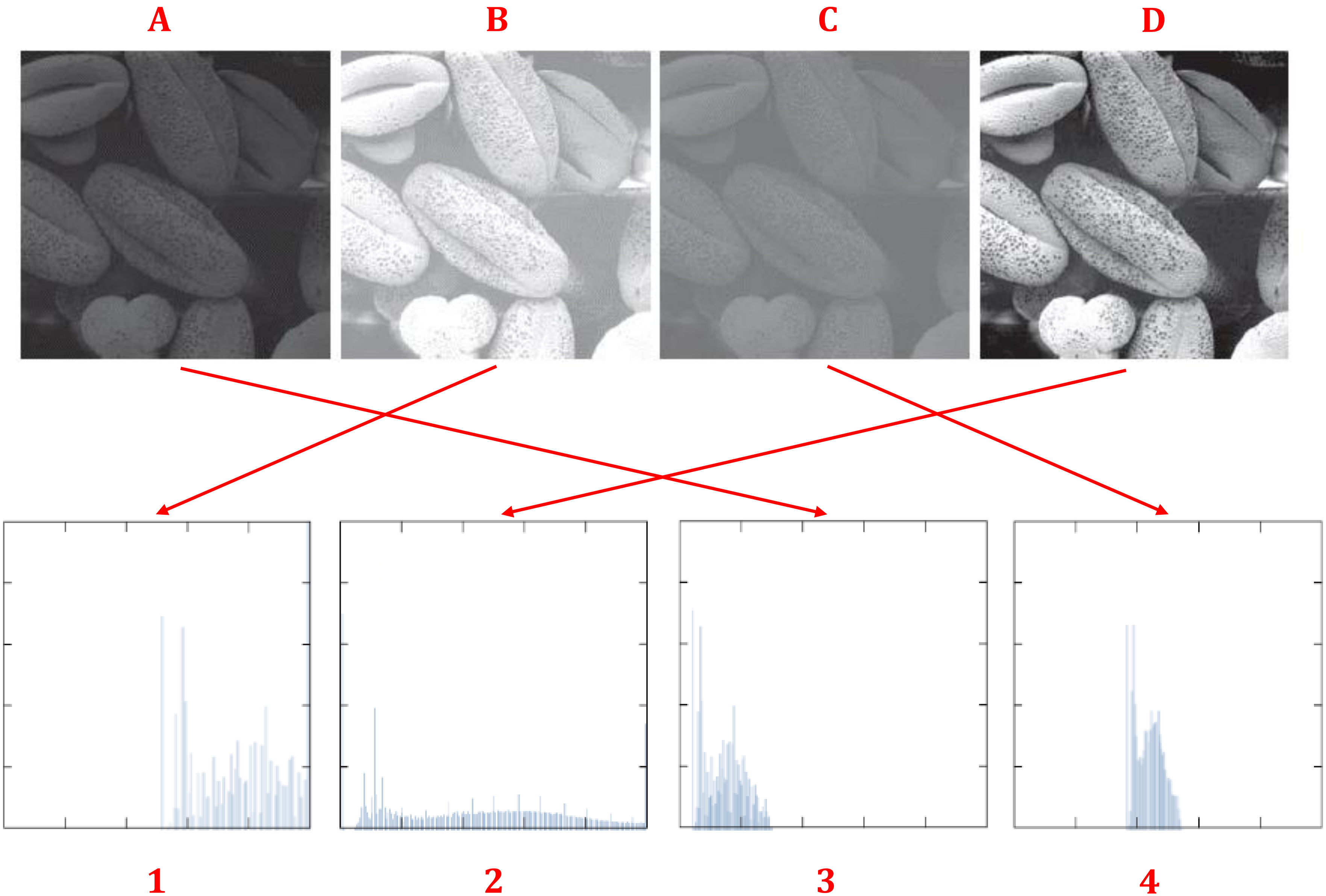
2



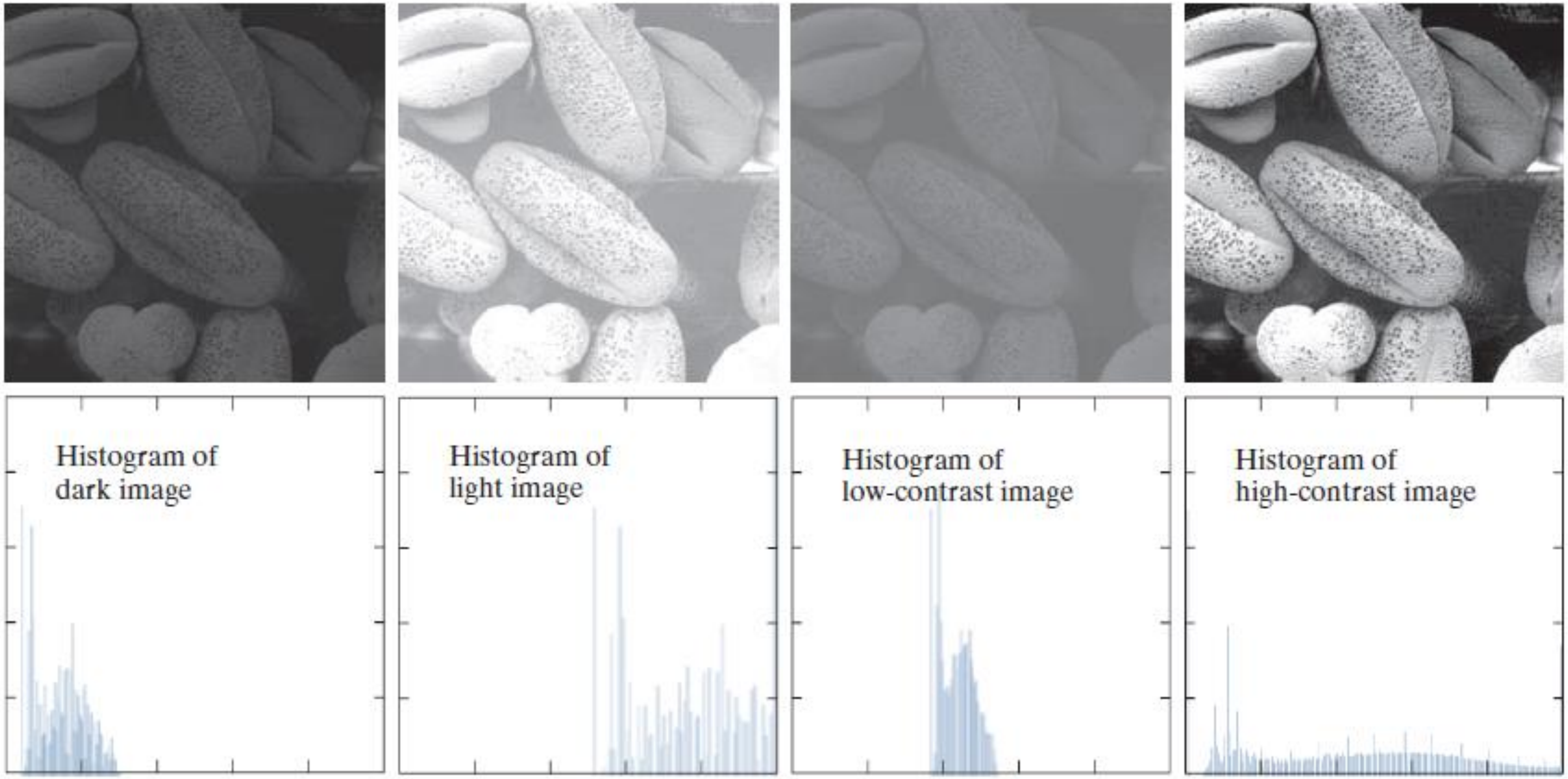
3



4


A
B
C
D
1
2
3
4

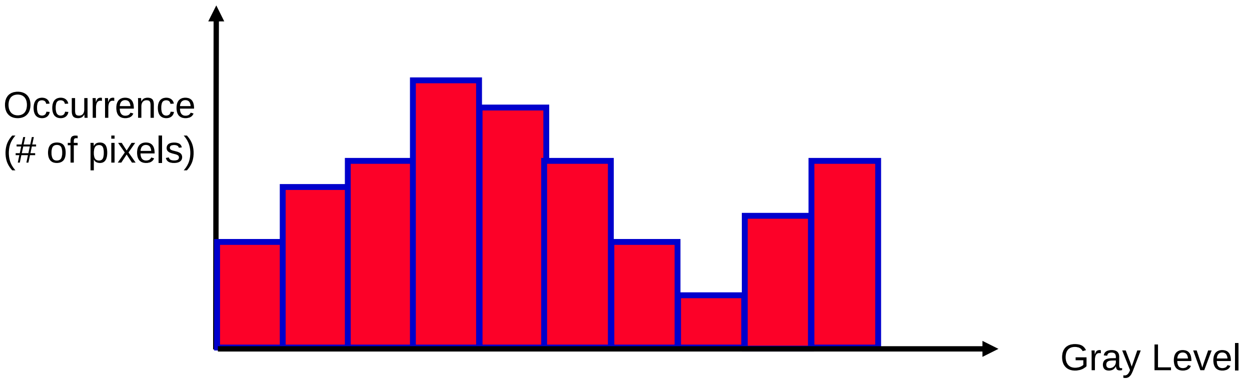
dark light low-contrast high-contrast



a b c d

FIGURE 3.16 Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.

Image Histogram

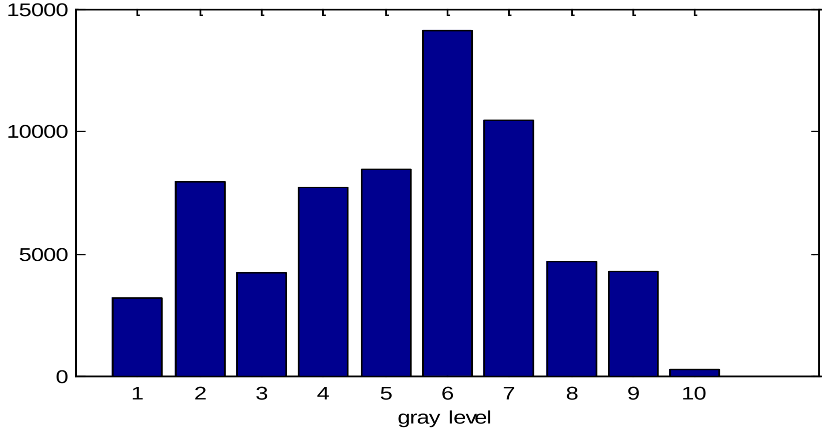


- $h(r_k)$ specifies the # of pixels with gray-value r_k

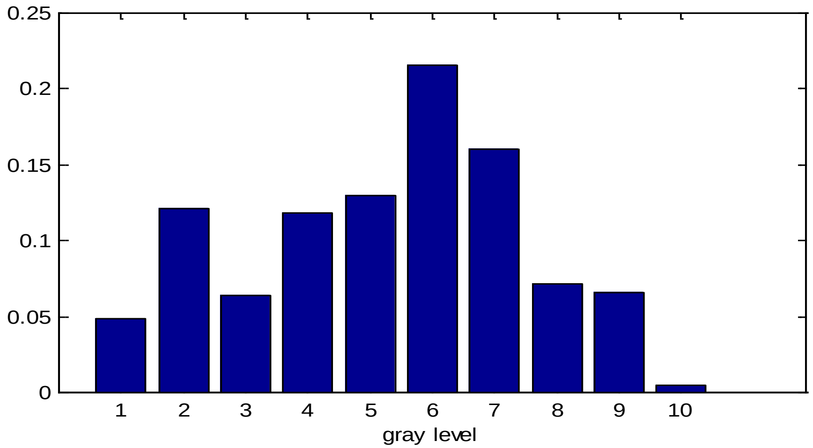
- Let $\mathrm{M} \times N$ be the number of pixels

- $p(r_k) = h(r_k)/(M \times N)$ defines the normalized histogram

measures the probability of occurrence of an intensity level r_k in an image



Histogram



Normalized Histogram

Histogram equalization: Continuous domain

$$\boxed{s = T(r) = (L-1)\int_0^r p_r(w)\,dw \qquad (3.3\text{-}4)}$$

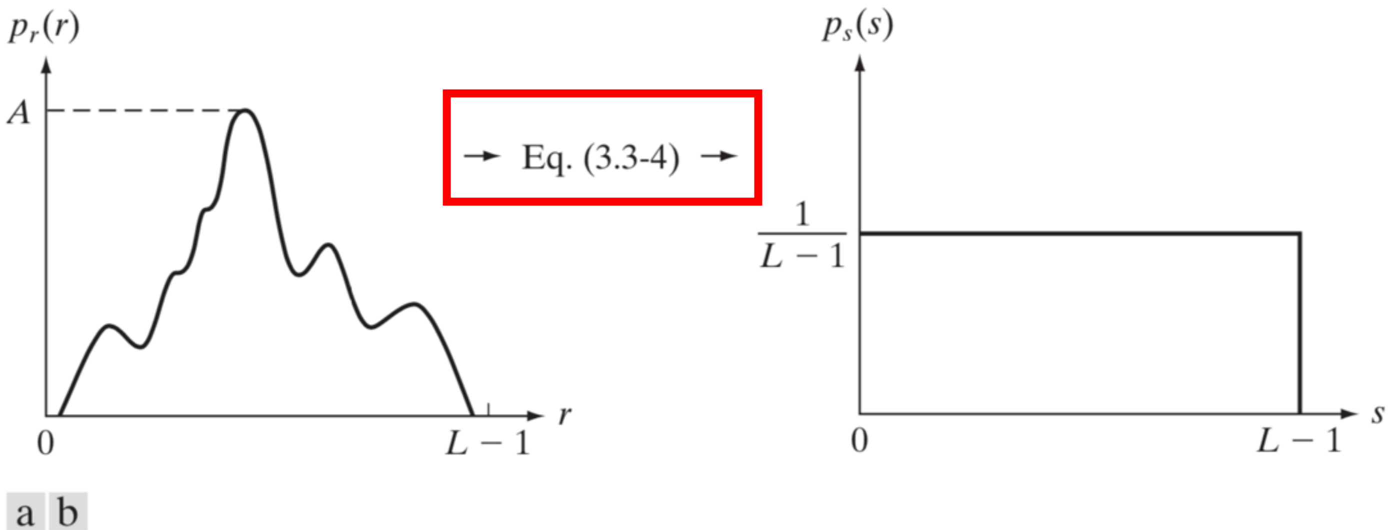


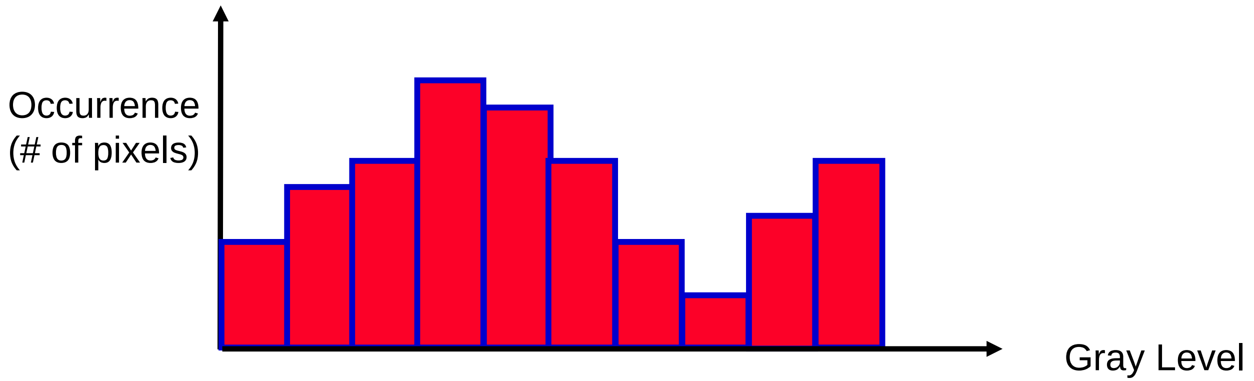
FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram equalization: Discrete domain

$$s_k = T(r_k) = (L - 1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L - 1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots, L - 1$$

Image Histogram

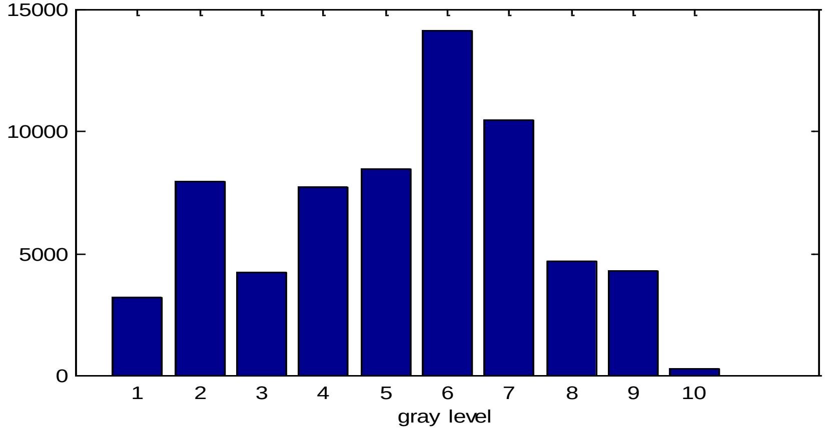


- $h(r_k)$ specifies the # of pixels with gray-value r_k

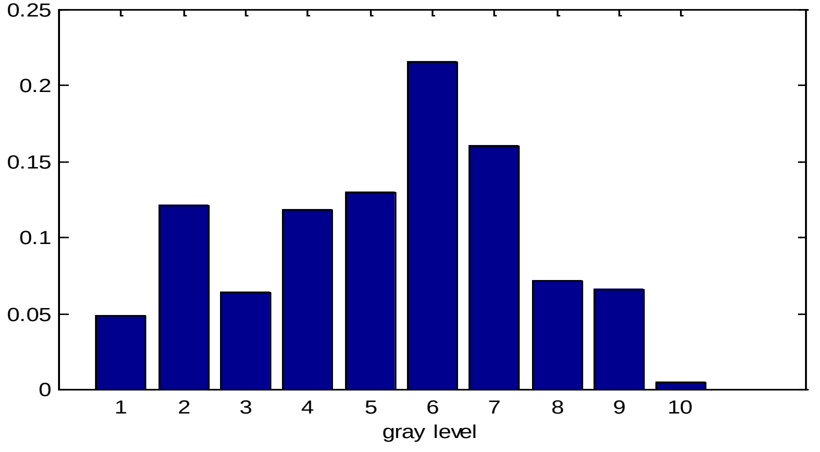
- Let $\mathrm{M} \times N$ be the number of pixels

- $p(r_k) = h(r_k)/(M \times N)$ defines the normalized histogram

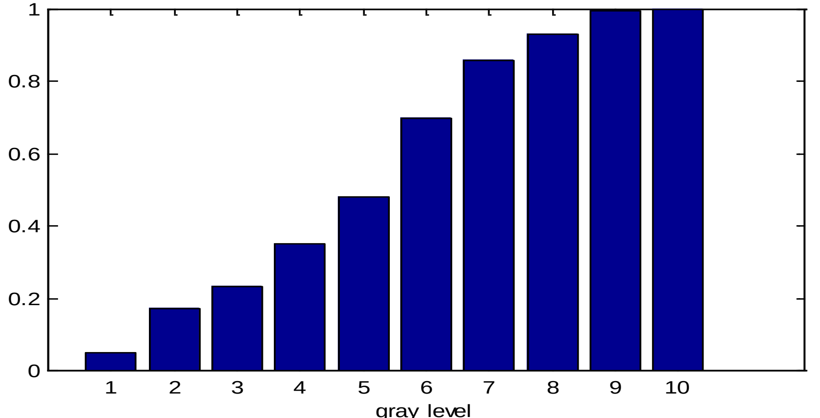
- $\mathrm{C}(r_k) = \sum_{i=0}^{k} p(r_k)$ defines the cumulative histogram



Histogram



Normalized Histogram



Cumulative Normalized Histogram

Histogram equalization: Discrete domain

Example:

Suppose that a 3-bit image (L = 8) of size 64x64 pixels (MN = 4096) has the intensity distribution shown below

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Histogram equalization: Discrete domain

Example:

Suppose that a 3-bit image (L = 8) of size 64x64 pixels (MN = 4096) has the intensity distribution shown below

r_k	n_k	$p_r(r_k) = n_k/MN$	
$r_0 = 0$	790	0.19	0.19
$r_1 = 1$	1023	0.25	0.19+0.25
$r_2 = 2$	850	0.21	0.19+0.25+0.21
$r_3 = 3$	656	0.16	0.19+0.25+0.21+0.16
$r_4 = 4$	329	0.08	0.19+0.25+0.21+0.16+0.08
$r_5 = 5$	245	0.06	0.19+0.25+0.21+0.16+0.08+0.06
$r_6 = 6$	122	0.03	0.19+0.25+0.21+0.16+0.08+0.06+0.03
$r_7 = 7$	81	0.02	0.19+0.25+0.21+0.16+0.08+0.06+0.03+0.02

Histogram equalization: Discrete domain

Example:

Suppose that a 3-bit image ($L = 8$) of size 64x64 pixels ($MN = 4096$) has the intensity distribution shown below

r_k	n_k	$p_r(r_k) = n_k/MN$	
$r_0 = 0$	790	0.19	0.19
$r_1 = 1$	1023	0.25	0.44
$r_2 = 2$	850	0.21	0.65
$r_3 = 3$	656	0.16	0.81
$r_4 = 4$	329	0.08	0.89
$r_5 = 5$	245	0.06	0.95
$r_6 = 6$	122	0.03	0.98
$r_7 = 7$	81	0.02	1.00

Histogram equalization: Discrete domain

Example:

Suppose that a 3-bit image (L = 8) of size 64x64 pixels (MN = 4096) has the intensity distribution shown below

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

0.19*7 = 1.33 → 1
0.44*7 = 3.08 → 3
0.65*7 = 4.55 → 5
0.81*7 = 5.67 → 6
0.89*7 = 6.23 → 6
0.95*7 = 6.65 → 7
0.98*7 = 6.86 → 7
1.00*7 = 7.00 → 7

Histogram equalization: Discrete domain

$s_0 = 1.33 \rightarrow 1$	$s_4 = 6.23 \rightarrow 6$
$s_1 = 3.08 \rightarrow 3$	$s_5 = 6.65 \rightarrow 7$
$s_2 = 4.55 \rightarrow 5$	$s_6 = 6.86 \rightarrow 7$
$s_3 = 5.67 \rightarrow 6$	$s_7 = 7.00 \rightarrow 7$

Example:

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

$s_2 = 4.55,\ s_3 = 5.67,\ s_4 = 6.23,\ s_5 = 6.65,\ s_6 = 6.86,\ s_7 = 7.00.$

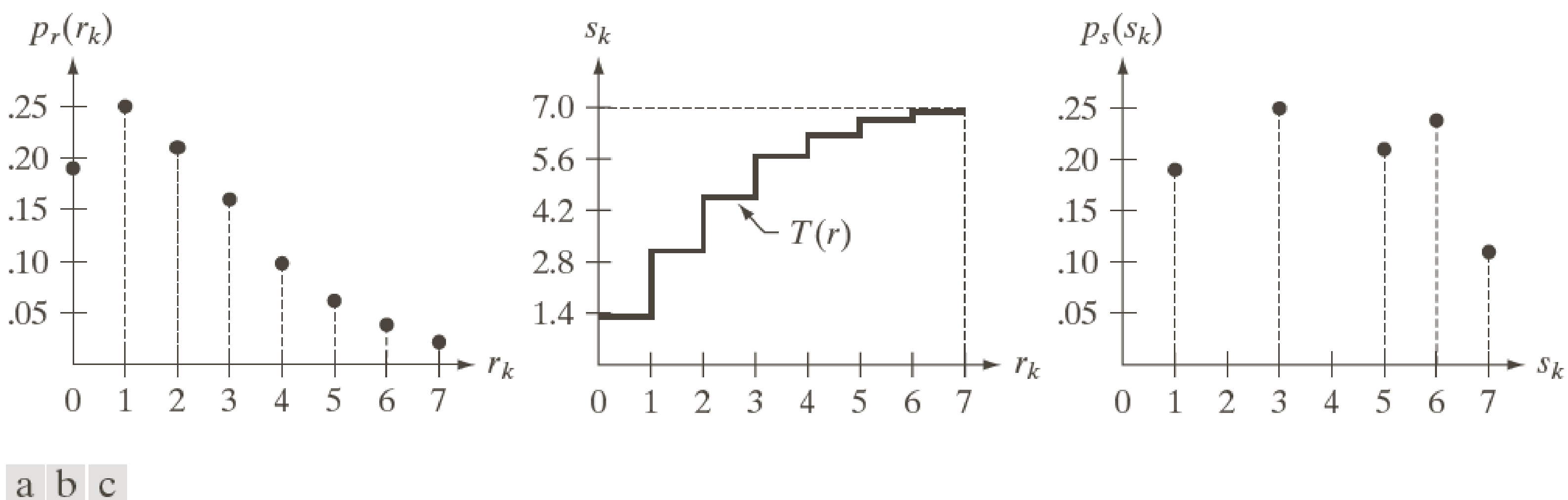
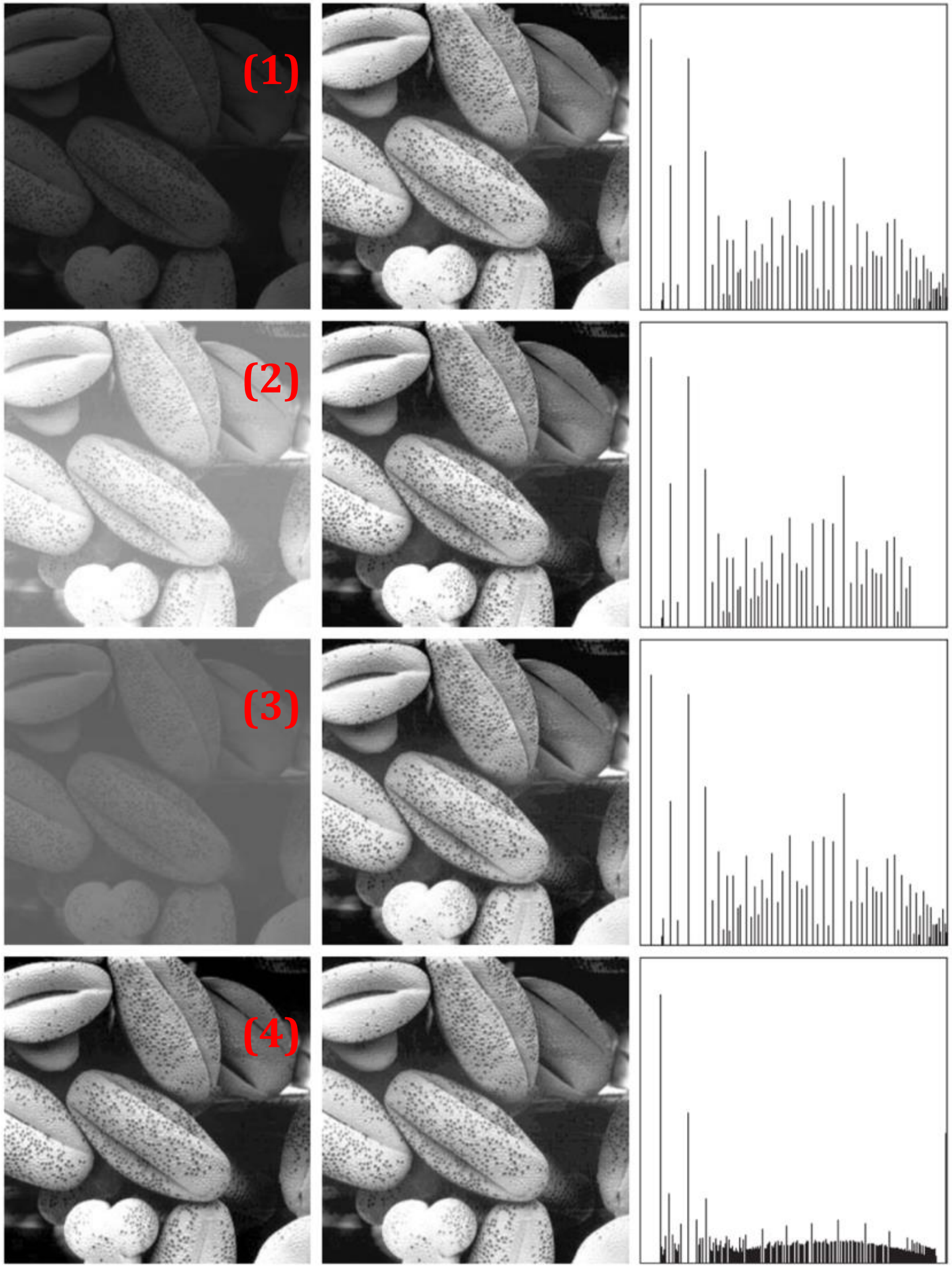
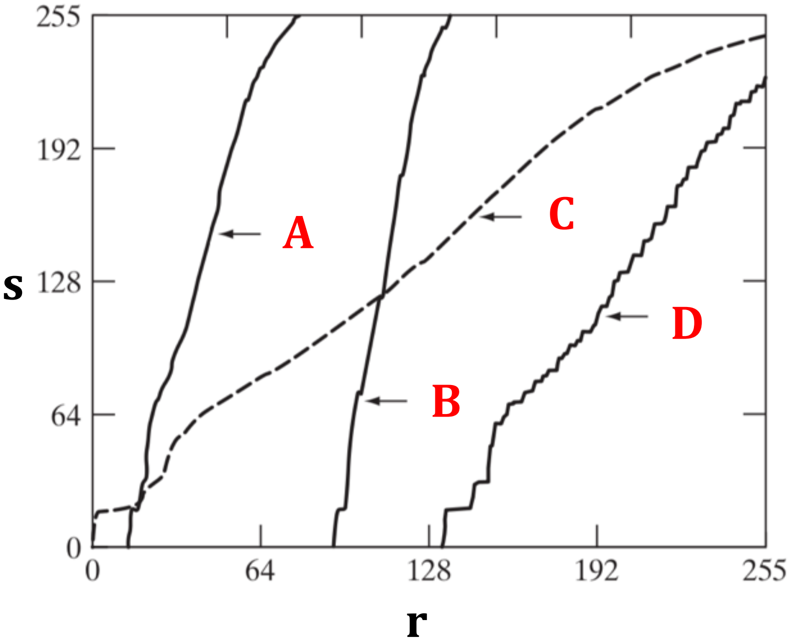
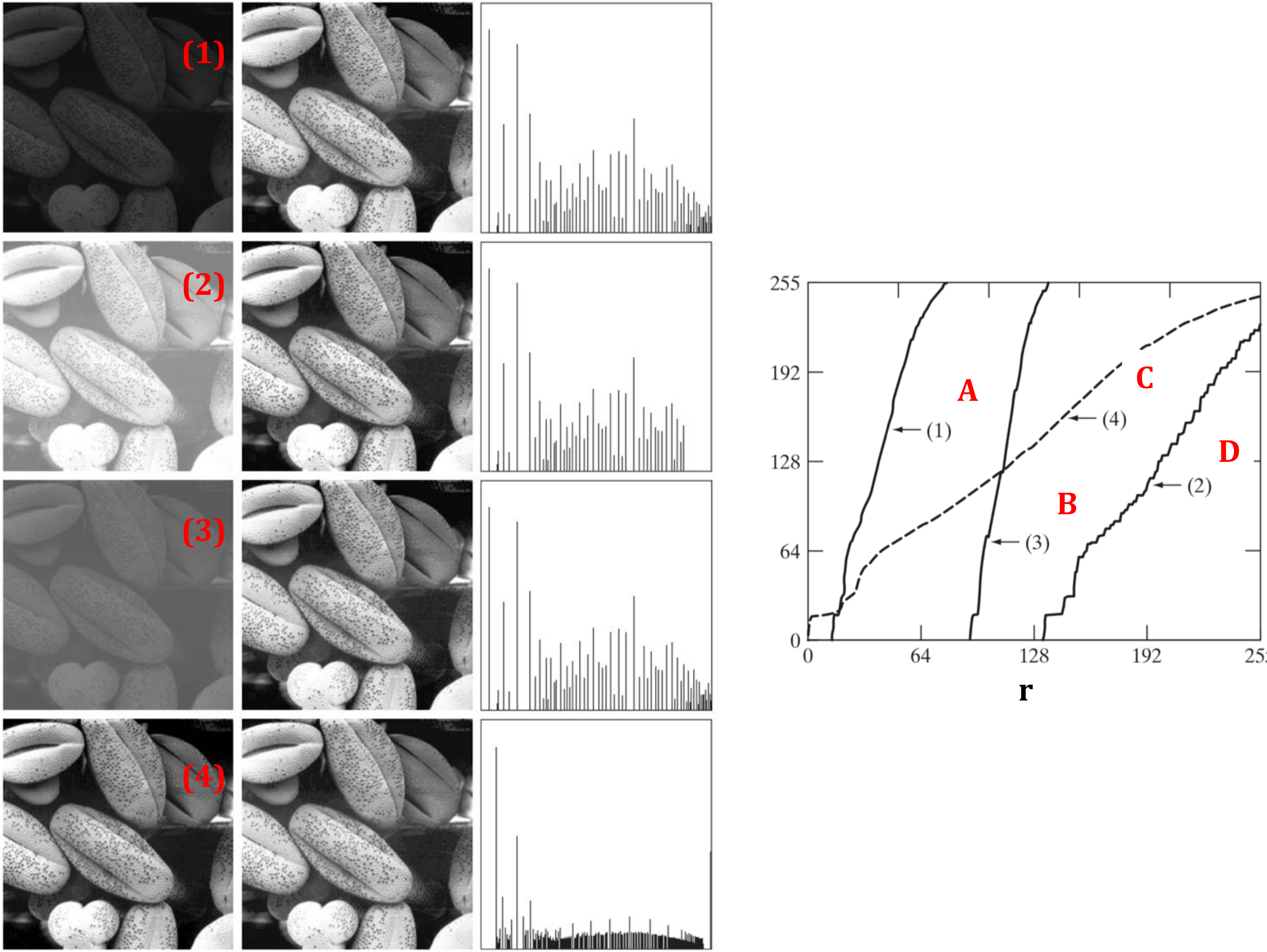


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Poll 5







(1)
(2)
(3)
(4)
255
192
128
64
0
0
64
128
192
255
r
A
B
C
D
(1)
(2)
(3)
(4)

Week 03 – Hands on activity

- Prepare and submit a Jupyter Notebook file containing the code and the results for the following Task

Task

- Read snow.png and display it.
- Convert the image into HSV color space.
- Display V channel and its histogram.
- Apply histogram equalization to the V channel and display the histogram of the result.
- By using H and S channel from the original image and V channel after histogram equalization, go back to RGB color space.
- Display the result