“Statistical Mechanics, Integrability and Combinatorics”
Schedule for Week 3, May 25-29

<u>Monday May 25, Room A, 14.30</u>

From conormal bundles of Schubert varieties to loop models
Paul Zinn-Justin *(CNRS & UMPC-Jussieu, Paris)*

In this work in collaboration with A. Knutson, we investigate the correspondence between algebraic geometry and quantum integrable systems (which has been recently popularized by the work of Maulik and Okounkov, among others) from the point of view of Grobner degenerations. The latter is very combinatorial in nature and works equally well for cohomology and K-theory. Following Knutson and Miller, I shall recall the simplest framework in which one can develop this approach, namely (matrix) Schubert varieties and Schubert and Grothendieck polynomials. After that, I shall formulate a broad extension of these results which will naturally lead us to loop models on general lattices: first noncrossing loops (Temperley-Lieb model), then, if time allows, crossing loops (Brauer model).

<u>Tuesday May 26, Room A, 14.30</u>

Airy diffusions and $N^{1/3}$ fluctuations in the 2D and 3D Ising models
Senya Shlosman *(Aix-Marseille Université)*

Consider the 3D Ising model at a low temperature. We will look at the level lines of the Ising droplet near its edge. I will explain that their fluctuations are of the order of $N^{1/3}$. When scaled by $N^{1/3}$, their limiting behavior for large N is given by the Airy diffusion process. This diffusion has appeared earlier in a paper by Ferrari and Spohn, where the Brownian motion above the parabolic barrier is considered.
Work in progress with D. Ioffe and Y. Velenik.

<u>Wednesday May 27, Room A, 11.30</u>

Elliptic cohomology and real life
Andrei Okounkov *(Columbia University, New York)*

In recent year, there has been a surge of interests in relating quantum integrable systems to questions of enumerative geometry. I will review some aspects of such connections, which prove to be very fruitful for both fields. A natural challenge is push this connection to the extreme and understand elliptic R-matrices and related integrable system from geometric perspective. This will be the topic of my talk, based on joint work with Mina Aganagic.

Thursday May 28, Room A
Mini-workshop

Morning Session

9.30-10.10
Counting perfect matching in graphs with application in monomer-dimer models
Afshin Behmaram *(University of Tabriz)*

In this talk, we introduce the Pfaffian method for counting perfect matching (monomer-dimer) in graphs. Using this methods we calculate the number of perfect matching in some class of lattice models. Also, we give upper and lower bound for the number of perfect matching in some class of graphs.

10.10-10.50
Quasi-invariants of 2-knots and integrable models
Dmitry Talalaev *(Moscow State University, and ITEP, Moscow)*

I'll talk about the tetrahedron equation, some natural problems associated with it, the problem of describing the invariants of 2-knots using the theory quandles cohomology and some new construction of quasi-invariants based on the 3-dimensional lattice statistical models with special properties. A few words I would say about a purely combinatorial question of describing the so-called n-simplicial complexes. I will also emphasize the relation of this structure with the theory of quantum integrable systems on two-dimensional lattices.

11.10-11.50
The Scalar product of XXZ spin chain. Application to the ground state at $\Delta = -1/2$
Alexander Garbali *(CNRS & UPMC Jussieu, Paris)*

The computation of the correlation functions of the integrable XXZ spin chain can be done using the form factor approach. This approach relies on the scalar products of Bethe states. We study the ground state scalar product S_n of the inhomogeneous XXZ $\mathrm{s}=1/2$ spin chain of length $2\mathrm{n}$ at its combinatorial point $\Delta = -1/2$ with twisted periodic boundary conditions. At this point the ground state eigenvalue τ_n of the transfer matrix is known and has a simple form that does not contain the Bethe roots. We use the knowledge of τ_n and the Slavnov determinant written in a suitable form to obtain a closed expression for the scalar product S_n and the norm of the ground state.

11.50-12.30
Sum rule for a mixed boundary qKZ equation
Caley Finn *(The University of Melbourne)*

I will describe a graphical construction giving a generalised sum rule for components of the solution of the Temperley-Lieb qKZ equation with mixed boundaries. Our construction uses the fact that the solutions of the qKZ equation can be written as factorised products of Baxterized Hecke generators. I will also discuss the connection to the Kazhdan-Lusztig basis and other bases of the Hecke algebra of type B_N.

Afternoon Session

14.30-15.10

Boundary algebras and Kac modules for logarithmic minimal models

Alexi Morin-Duchesne *(Université Catholique de Louvain)*

Virasoro Kac modules were initially introduced indirectly as representations whose characters arise in the continuum scaling limits of certain transfer matrices in logarithmic minimal models, described using Temperley-Lieb algebras. However, the structure of the Virasoro Kac modules have remained largely unidentified. Here, we introduce the appropriate algebraic framework for the lattice analysis as a quotient of the one-boundary Temperley-Lieb algebra. The corresponding lattice modules are introduced and examined using invariant bilinear forms. The structures of the Virasoro Kac modules are inferred from these results and are found to be given by finitely generated submodules of Feigin-Fuchs modules. Additional evidence for this identification is obtained by comparing the formalism of lattice fusion with the fusion rules of the Virasoro Kac modules.

15.10-15.50

Non-equilibrium dynamics of the XXZ model

Jacopo de Nardis *(University of Amsterdam)*

We study quantum quenches in integrable spin-1/2 chains in which we unitary evolve the ground state of the antiferromagnetic Ising model with the XXZ Hamiltonian with Delta >= 1. For this non-equilibrium situation, an application of the first-principles-based quench action method allows us to give an exact description of the post-quench late time steady state in the thermodynamic limit. We show that a generalized Gibbs ensemble (GGE), implemented using all known local conserved charges, fails to reproduce the exact quench action steady state and to correctly predict post-quench equilibrium expectation values of physical (local) observables [Wouters et al., *Phys. Rev. Lett.* 113, 117202 (2014)]. This shows that the set of local conserved charges obtained from the expansion of the logarithm of the transfer matrix is not complete and more unknown conserved quantities intervene in the non-equilibrium dynamics of the XXZ spin chain in the Gapped phase and in the isotropic point Delta=1.

16.10-16.50

The Fibonacci family of dynamical universality classes

Gunter M Schütz *(Institute for Complex Systems II, Jülich)*

We use the universal nonlinear fluctuating hydrodynamics approach to study anomalous one-dimensional transport far from thermal equilibrium in terms of the dynamical structure function. Generically for more than one conservation law mode coupling theory is shown to predict a discrete family of dynamical universality classes with dynamical exponents which are consecutive ratios of neighboring Fibonacci numbers, starting with z = 2 (corresponding to a diffusive mode) or z = 3/2 (Kardar-Parisi-Zhang (KPZ) mode). If neither a diffusive nor a KPZ mode are present, all Fibonacci modes have as dynamical exponent the golden mean z=$(1+\sqrt{5})/2$. The scaling functions of the Fibonacci modes are asymmetric Lévy distributions which are completely fixed by the macroscopic current-density relation and compressibility matrix of the system. The theoretical predictions are confirmed by Monte-Carlo simulations of a three-lane asymmetric simple exclusion process.

<u>16.50-17.30</u>

Matrix Product Ansatz for nonequilibrium steady states of driven quantum systems: XXZ spin chain, Hubbard model and others

Vladislav Popkov *(University of Cologne)*

We review exact results from last 3 years, concerning one-dimensional open quantum systems, connected at the ends to a dissipative baths, which sustain global gradients of magnetization, energy, etc. across the system. Recent progress has allowed to push the concepts of integrability of quantum systems beyond the thermal equilibrium: exact nonequilibrium steady states (NESS) for several celebrated 1D quantum many-body systems models (XXZ spin chain, Hubbard model) were calculated analytically. The non-equilibrium integrability results from an additional degree of freedom, hidden in the Yang-Baxter structure for its `equilibrium' counterpart. This hidden degree of freedom (related to the amplitude of the dissipative term), gives rise to non-unitary representations of quantum symmetries of the models and allows to formulate a Matrix Product Ansatz for the NESS. A transfer matrix of the problem has an underlying Yang-Baxter structure, which hints at possible integrability of the full Liouvillean dynamics.

<u>Friday May 29, Room A, 11.30</u>

Littlewood-Richardson coefficients and integrable tilings

Michael Wheeler *(The University of Melbourne)*

Littlewood-Richardson coefficients are the expansion coefficients in the product of two Schur functions. The subject of this talk will be the Knutson-Tao puzzles, which are certain tilings of the triangular lattice which enumerate Littlewood-Richardson coefficients. I will describe how Knutson-Tao puzzles can be obtained using the framework of quantum integrability, and why this framework is natural for the study of symmetric functions. I will discuss various new generalizations of Knutson-Tao puzzles to other families of symmetric functions, including Grothendieck and Hall-Littlewood polynomials.