"Statistical Mechanics, Integrability and Combinatorics"
Schedule for Week 5, June 8-12

<u>Monday June 8, Room B, 14.30</u>

Spin chains with generic boundaries
Nicolai Kitanine (Université de Bourgogne)

In this seminar I'll consider the XXZ spin chain with the most general boundary term. It will be shown that the separation of variables method permits to construct the complete set of eigenstates and to characterize the spectrum in terms of the solutions of the inhomogeneous version of the Baxter T-Q equation. In the constrained case this equation reduces to the usual homogeneous T-Q equation. Finally I'll discuss the ways to compute the correlation functions and the form factors for the models solvable by the separation of variables technique.

<u>Tuesday June 9, Room B, 11.30</u>

Correlations and inhomogeneous field theory inside the arctic circle
Jean-Marie Stéphan *(Max Planck Institute for the Physics of Complex Systems, Dresden)*

A one-dimensional toy-model of fermionic particles evolving in imaginary time from a domain-wall initial state is introduced, and solved. The main interest of this toy-model is that it exhibits the "arctic-circle phenomenon" originally discovered in dimer models on the Aztec diamond, namely a spatial phase separation between a critically fluctuating region and a frozen region.
The purpose of the talk is to study the critical region from a field-theoretical perspective. Large-scale correlations inside the disk are expressed in terms of correlators in a (euclidean) massless Dirac theory. It is observed that this theory is inhomogeneous: contrary to better understood models or geometries the metric is position-dependent, so it is in fact a Dirac theory in curved two-dimensional space. The technique used to solve the toy-model can be extended to deal with the transfer matrices of other models: dimers on the honeycomb lattice, on the square lattice, and the six-vertex model at the free fermion point ($\Delta = 0$). In all cases, the underlying action is Dirac in curved space.

<u>Wednesday June 10, Room B, 11.30</u>

Yang-Baxter Maps, Discrete Integrable Equations and Quantum Groups
Vladimir Bazhanov *(Australian National University, Canberra)*

For every quantized Lie algebra there exists a map from the tensor square of the algebra to itself, which by construction satisfies the set-theoretic Yang-Baxter equation. This map allows one to define an integrable discrete quantum evolution system on quadrilateral lattices, where local degrees of freedom (dynamical variables) take values in a tensor power of the quantized Lie algebra. The corresponding equations of motion admit the zero curvature representation. The commuting Integrals of Motion are defined in the standard way via the Quantum Inverse Problem Method, utilizing Baxter's famous commuting transfer matrix approach. All elements of the above construction have a meaningful quasi-classical limit. As a result one obtains an integrable discrete Hamiltonian evolution system, where the local equation of motion are determined by a classical Yang-Baxter map and the action functional is determined by the quasi-classical asymptotics of the universal R-matrix of the underlying quantum algebra. In this paper we present detailed considerations of the above scheme on the example of the algebra $U_q(sl_2)$ leading to discrete Liouville equations, however the approach is rather general and can be applied to any quantized Lie algebra.

Thursday June 11, Room B
Mini-Workshop

10.00-10.45

The entropy of six-vertex model with variety of different boundary conditions
Thiago Silva Tavares *(Universidade Federal de São Carlos, São Paolo)*

In this work we study the dependence of the six-vertex model intensive properties on boundary conditions. In the first part I shall talk mainly on toroidal boundary conditions which mixed periodic and anti-periodic closings. We argue that these kinds of boundary conditions cannot produce different results from completely periodic boundary. In second part we address the fixed boundaries types and show some examples of boundaries agreeing or disagreeing with periodic's value. In particular we introduce the Néel boundary condition whose number of configurations we believe to be maximal among all fixed boundaries. By means of boundary merge we show that the residual entropy may take any value between zero and domain-wall's value. Then we provide some numerical results supporting the idea that the entropy may take any value between zero and periodic's value.

10.45-11.30

Bruhat and Tamari orders in integrable systems
Folkert Müller-Huissen *(Max-Planck-Institute for Dynamics and Self-Organization, Göttingen)*

A (weak) Bruhat order is a natural partial order on a symmetric group. It appears, e.g., in the scattering of KdV solitons in a "tropical limit". Tamari orders are partially ordered sets (actually lattices) based on the associativity law. They are physically realized in terms of (at fixed time tree-shaped) soliton solutions of the famous KP equation (Dimakis and M-H 2010). Addressing the combinatorics underlying simplex equations, which generalize the Yang-Baxter equation, Manin and Schechtman introduced in 1986 higher Bruhat orders. Corresponding higher (Stasheff-) Tamari orders can be obtained from the higher Bruhat orders via a kind of projection. In the same way as higher Bruhat orders encode the structure of simplex equations, higher Tamari orders determine generalizations ("polygon equations") of the pentagon equation (Dimakis and M-H, arXiv:1409.7855, to appear in SIGMA). In this talk we will try to explain all this in a fairly elementary way.

11.45-12.30

Spontaneous Breaking of *U(N)* symmetry in invariant Matrix Models
Fabio Franchini *(INFN, Sezione di Firenze)*

Matrix Models successfully capture the behavior of many strongly interacting systems in a variety of contexts, while being a wonderful playground of integrability and analytical methods. Traditionally, the requirement of base invariance has lead to the conclusion that these models describe only extended systems.

We show that deviations of the eigenvalue statistics from the Wigner-Dyson universality reflects itself on the eigenvector distribution and that gaps in the eigenvalue density break U(N) symmetry to a smaller one. This spontaneous symmetry breaking means that egeinvectors become localized to a sub-manifold of the Hilbert space (and, physically, that one describes a system which lacks ergodicity). This realization means that random matrix techniques can be lent to the study of new observables, which have not been examined before.
We also consider models with log-normal weight, such as those emerging in Chern-Simons and ABJM theories. They can be solved through q-deformed orthogonal polynomials and their eigenvalue distribution is intermediate between Wigner-Dyson and Poissonian, which candidates these models for describing a phase intermediate between the extended and the localized ones. We show that they have a much richer energy landscape than expected, with their partition functions decomposable in a large number of equilibrium configurations. We argue that this structure is a reflection of the non-trivial (multi-fractal) eigenvector statistics and comment on the implications of these results.

- F. Franchini; "On the Spontaneous Breaking of U(N) symmetry in invariant Matrix Models"; arXiv:1412.6523.
- F. Franchini; "Toward an invariant matrix model for the Anderson Transition"; arXiv:1503.03341.

Friday June 12, Room B, 11.30

Observables at combinatorial points of solvable models.
Bernard Nienhuis *(University of Amsterdam)*

Since the discovery of Razumov and Stroganov of remarkable properties of the ground states of the XXZ model at Delta=-1/2, I have been interested in obtaining observables of this kind of solvable models for finite size geometries. By 'this kind of models' I mean 1D quantum chains or 2D statistical models of which the ground state energy or free energy respectively has no finite size corrections. It has been useful to represent these models as loop models: sums of configurations of paths (or a single path) a 2D lattice, typically on a L x infinite geometry. In the talk I will discuss attempts (succesful or not) to obtain closed forms of observables or the relation of such quantities to objects in enumerative combinatorics or other seemingly unrelated fields.