

# **“Statistical Mechanics, Integrability and Combinatorics”**

## **Schedule for Week 4, June 1-5**

### **Monday June 1, Room B, 11.30**

#### **Exact enumeration of alternating sign matrices**

Roger Behrend (*Cardiff University*)

I will review a range of classical and recent results for the exact enumeration of alternating sign matrices with prescribed values of certain statistics (in particular, the numbers of inversions and -1's, and the positions of the 1's in the first and last rows and columns), and in some cases with certain symmetry conditions imposed. The results can typically be obtained by using connections with the statistical mechanical six-vertex model with domain-wall (or related) boundary conditions.

### **Monday June 1, Cloister, 17.30**

Wine and Cheese “garden party”  
Accompanying persons are welcome.

### **Tuesday June 2**

June 2 is National holiday in Italy (Festa della Repubblica).

GGI is closed but you are anyway authorized to come to the Institute.  
Note that lunch is not available.

You can access GGI area through the small gate  
in Via G. Righini 2, using the pink key.

### **Wednesday June 3, Room B, 11.30**

#### **Combinatorial aspects of correlation functions of integrable models**

Nikolay Bogoliubov (*St.-Petersburg Department of V. A. Steklov Mathematical Institute and ITMO University*)

We discuss the connection between quantum integrable and some aspects of enumerative combinatorics and the theory of partitions. As a basic example, we consider the spin XXZ Heisenberg chain in the limiting cases of zero and infinite anisotropy. The representation of the Bethe wave functions via the Schur functions allows to apply the well-developed theory of the symmetric functions to the calculation of the thermal correlation functions as well as of the form-factors. The determinantal expressions of the form-factors and of the thermal correlation functions

are obtained. We provide a combinatorial interpretation of the formula for the correlation functions in terms of nests of the self-avoiding lattice paths. The interpretation proposed is in turn related to enumeration of the boxed plane partitions. The asymptotical behavior of the thermal correlation functions is studied in the limit of small temperature provided that the characteristic parameters of the system are large enough. The leading asymptotics of the correlation functions are found to be proportional to the squared numbers of boxed plane partitions.

#### **Thursday June 4, Room B, 11.30**

##### **Non-linear integral equation approach to $sl(2|1)$ integrable network models**

Andreas Klümper (*University of Wuppertal*)

An integrable  $sl(2|1)$  invariant network model with alternating 3 and  $\bar{3}$  representations on vertical and horizontal lines is considered. This system can be formulated equivalently as a superspin chain. The model is 'solvable' by nested Bethe ansatz which yields two sets of coupled equations for the Bethe roots.

The model was introduced by Gade (1998) and was extensively investigated by Essler, Frahm, Saleur (2005) most notably by numerical techniques. There are different motivations for the study of this kind of models. One reason is due to the appearance of a non-compact degree of freedom in the continuum limit of lattice models with staggering and (by definition) compact local quantum space.

I will focus on the possibility of derivation and solution of well posed non-linear integral equations (NLIE) for suitable, finitely many auxiliary functions. This is indeed possible by an ansatz that has been developed for the thermodynamics of the supersymmetric  $tJ$  model. For this model a set of three auxiliary functions were shown to satisfy a closed set of NLIE. For the network model two copies of these functions satisfy a closed set of six NLIE. These NLIE can be written as NLIE for two 'weakly coupled'  $su(2)$  spin-1 chains of Takhtajan-Babujian type. With some tweaks the equations can be solved numerically. This is work in progress.

#### **Friday June 5, Room B, 11.30**

##### **Application of the hidden fermionic structure to the CFT**

Hermann Boos (*University of Wuppertal*)

We discuss the scaling limit of the fermionic operators that were originally constructed for the lattice six vertex model. We relate these operators to the usual Virasoro generators modulo the integrals of motion. Also the OPE in the fermionic basis and the recursion relations for the conformal blocks discovered by Al. Zamolodchikov in eighties as well as some other aspects are discussed.