Machine Learning

CS60050

Artificial Neural Networks

Learning From Data

Yaser S. Abu-Mostafa
California Institute of Technology

Lecture 10: Neural Networks

Sponsored by Caltech's Provost Office, E&AS Division, and IST • Thursday, May 3, 2012

Outline

- Stochastic gradient descent

- Neural network model

- Backpropagation algorithm

Stochastic gradient descent

GD minimizes:

$$E_{\text{in}}(\mathbf{w}) = \frac{1}{N} \sum_{n=1}^{N} \underbrace{\mathrm{e}\left(h(\mathbf{x}_n), y_n\right)}_{\ln\left(1+e^{-y_n \mathbf{w}^{\mathsf{T}} \mathbf{x}_n}\right)} \quad \longleftarrow \text{ in logistic regression}$$

by iterative steps along $-\nabla E_{\text{in}}$:

$$\Delta \mathbf{w} = - \eta \, \nabla E_{\text{in}}(\mathbf{w})$$

∇E_{in} is based on all examples $(\mathbf{x}_n, y_n)$

"**batch**" GD

The stochastic aspect

Pick <u>one</u> $(\mathbf{x}_n, y_n)$ at a time. Apply GD to $\mathbf{e}\left(h(\mathbf{x}_n), y_n\right)$

"Average" direction:

$$\mathbb{E}_n\left[-\nabla\mathbf{e}\left(h(\mathbf{x}_n), y_n\right)\right] = \frac{1}{N}\sum_{n=1}^{N} -\nabla\mathbf{e}\left(h(\mathbf{x}_n), y_n\right)$$

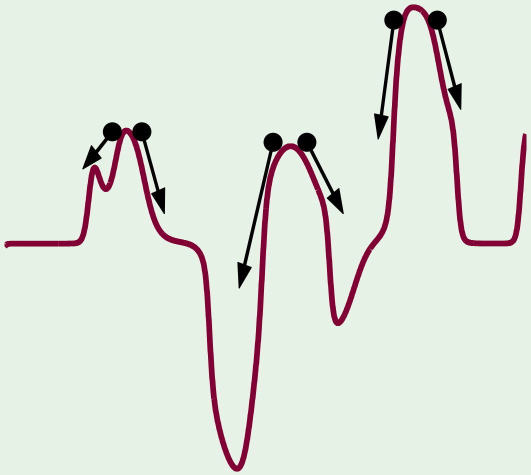
$$= -\nabla\ E_{\text{in}}$$

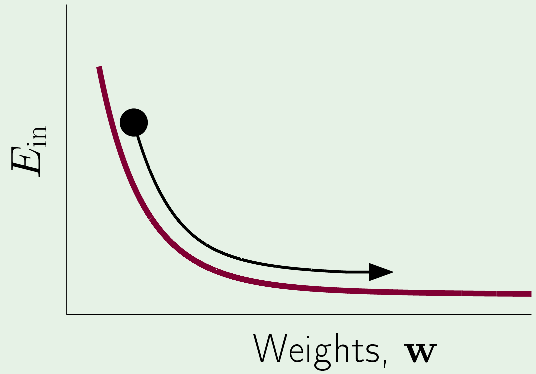
randomized version of GD

stochastic gradient descent (SGD)

Benefits of SGD

1. cheaper computation
2. randomization
3. simple





Rule of thumb:

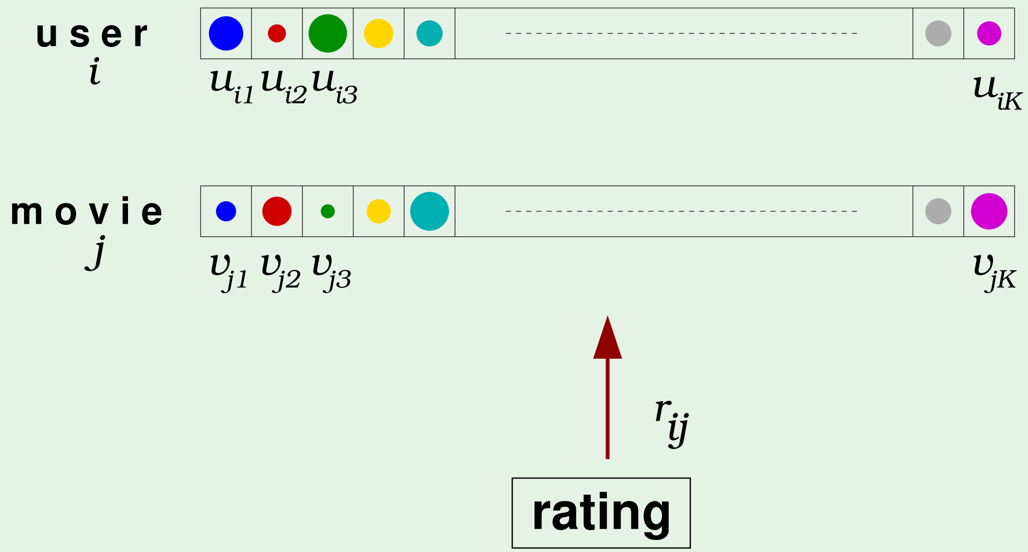
$\eta = 0.1$ works

randomization helps

SGD in action

Remember movie ratings?

$$\mathbf{e}_{ij} = \left(r_{ij} - \sum_{k=1}^{K} u_{ik} v_{jk} \right)^2$$

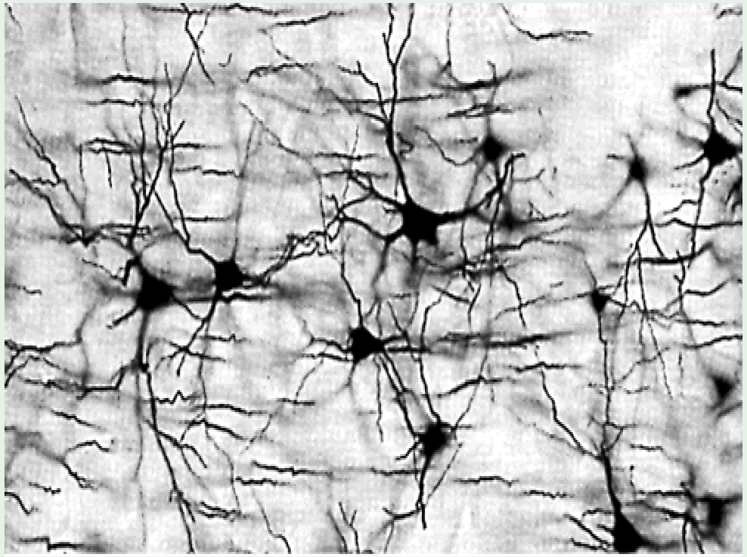


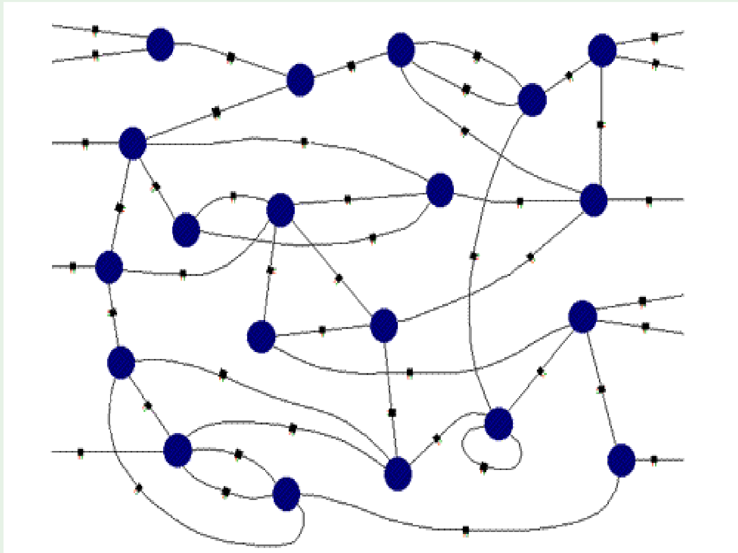
Outline

- Stochastic gradient descent
- Neural network model
- Backpropagation algorithm

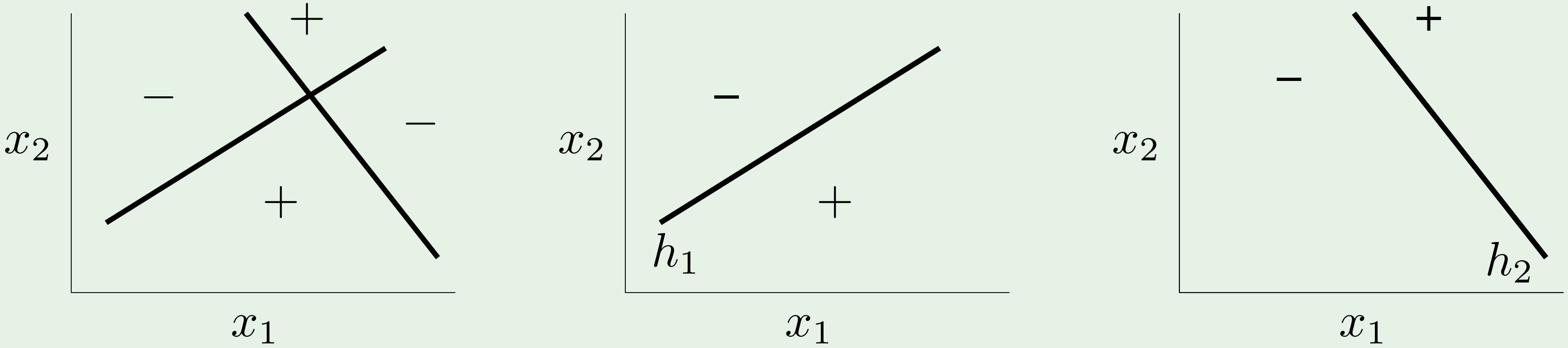
Biological inspiration

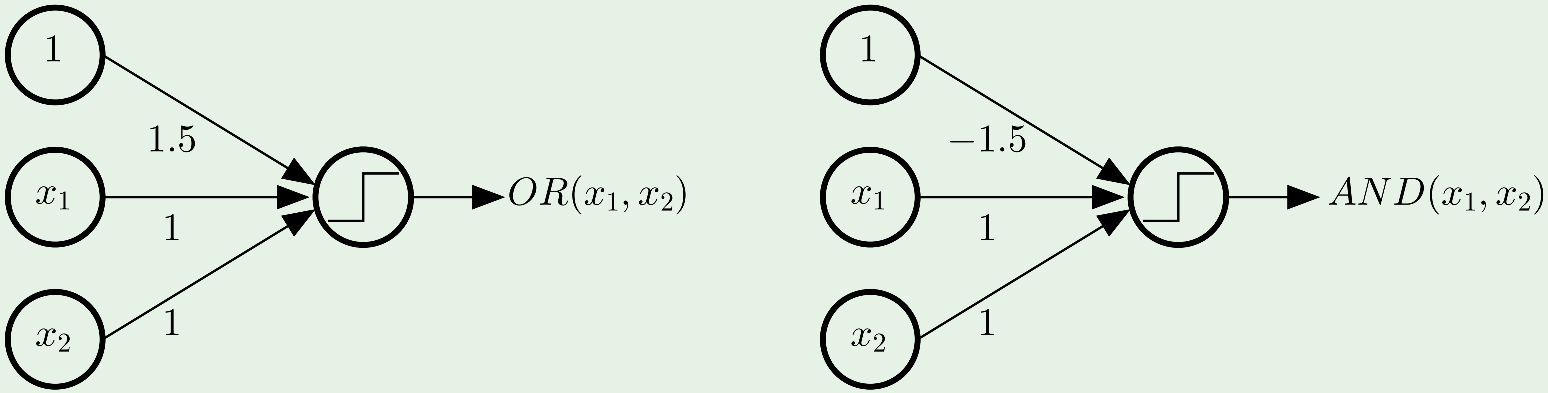
biological function $\longrightarrow$ biological structure



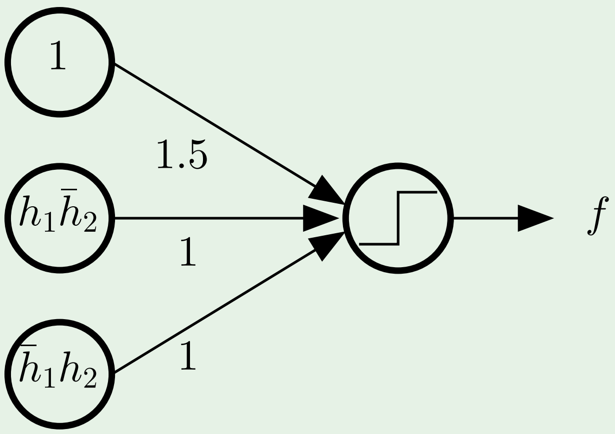


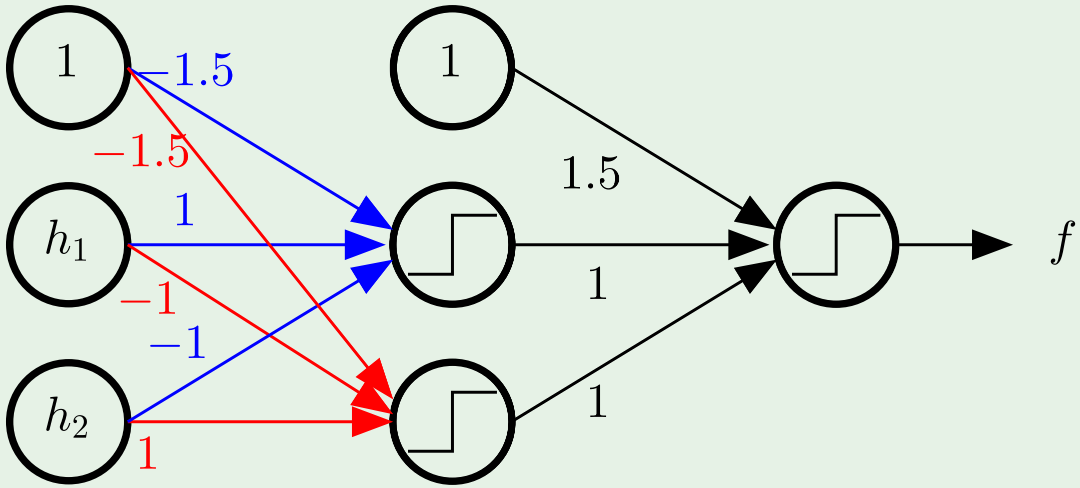
Combining perceptrons



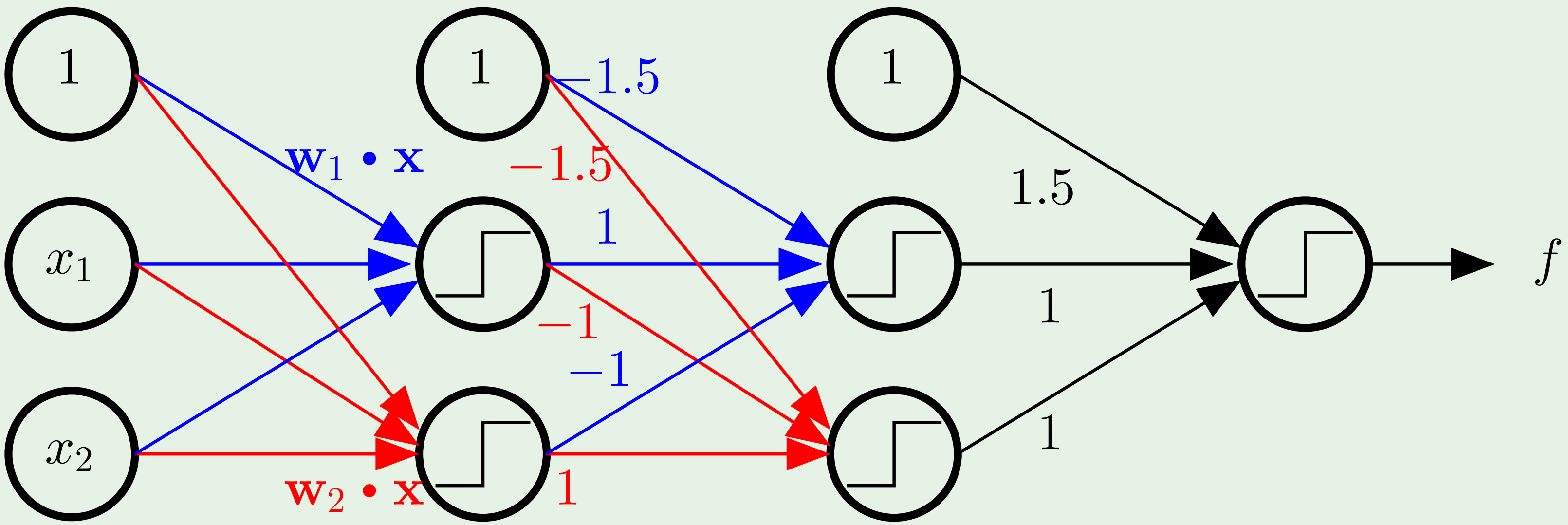


Creating layers



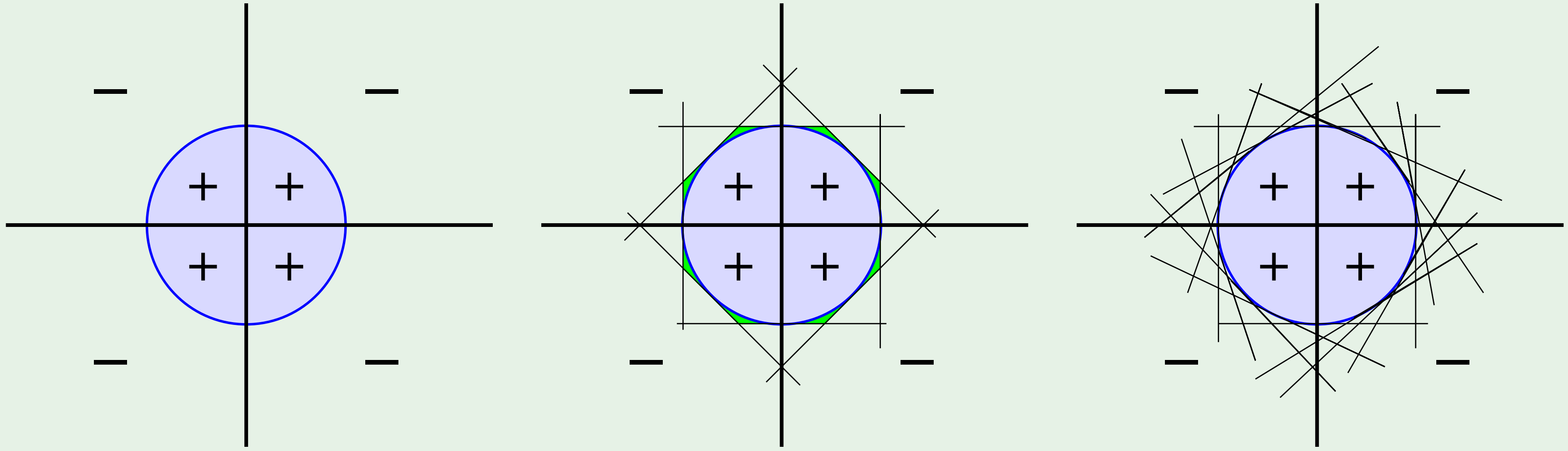


The multilayer perceptron



3 layers “feedforward”

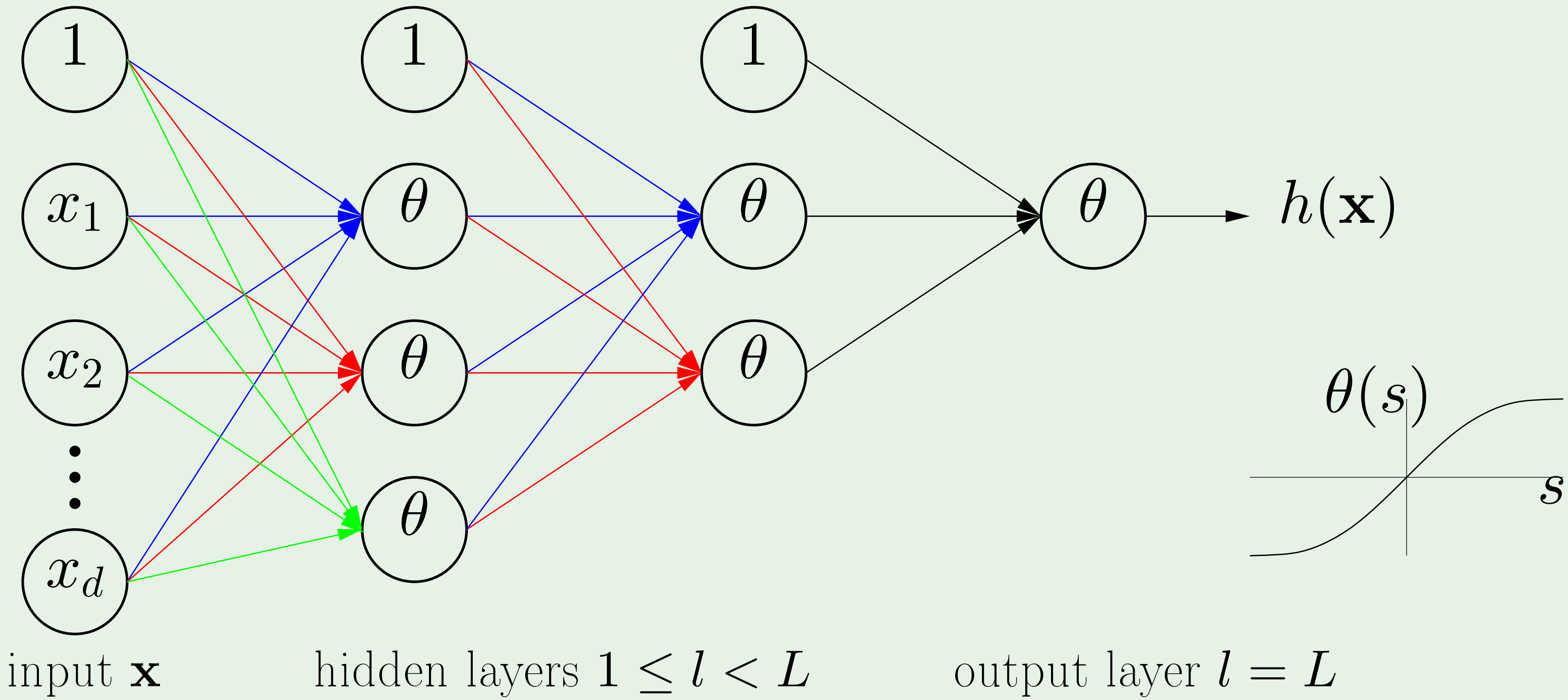
A powerful model



Target | 8 perceptrons | 16 perceptrons

2 red flags for **generalization** and **optimization**

The neural network

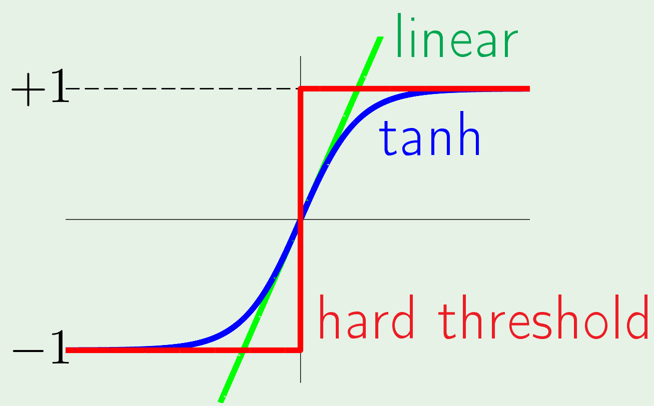


How the network operates

$$w_{ij}^{(l)} \quad \begin{cases} 1 \le l \le L & \text{layers} \\ 0 \le i \le d^{(l-1)} & \text{inputs} \\ 1 \le j \le d^{(l)} & \text{outputs} \end{cases}$$

$$x_j^{(l)} = \theta(s_j^{(l)}) = \theta\left(\sum_{i=0}^{d^{(l-1)}} w_{ij}^{(l)} \, x_i^{(l-1)} \right)$$

Apply $\mathbf{x}$ to $x_1^{(0)} \cdots x_{d^{(0)}}^{(0)} \rightarrow \rightarrow x_1^{(L)} = h(\mathbf{x})$



$$\theta(s) = \tanh(s) = \frac{e^s - e^{-s}}{e^s + e^{-s}}$$

Outline

- Stochastic gradient descent

- Neural network model

- Backpropagation algorithm

Applying SGD

All the weights $\mathbf{w} = \{w_{ij}^{(l)}\}$ determine $h(\mathbf{x})$

Error on example $(\mathbf{x}_n, y_n)$ is

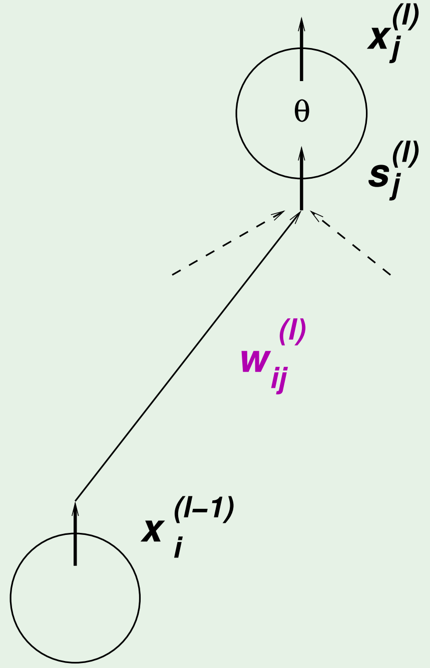
$$\mathbf{e}\left(h(\mathbf{x}_n), y_n\right) = \mathbf{e}(\mathbf{w})$$

To implement SGD, we need the gradient

$$\nabla\mathbf{e}(\mathbf{w}): \quad \frac{\partial\ \mathbf{e}(\mathbf{w})}{\partial\ w_{ij}^{(l)}} \quad \text{for all} \quad i, j, l$$

Computing $\dfrac{\partial\ \mathrm{e}(\mathbf{w})}{\partial\ w_{ij}^{(l)}}$

We can evaluate $\dfrac{\partial\ \mathrm{e}(\mathbf{w})}{\partial\ w_{ij}^{(l)}}$ one by one: analytically or numerically



A trick for efficient computation:

$$\frac{\partial\ \mathrm{e}(\mathbf{w})}{\partial\ w_{ij}^{(l)}} = \frac{\partial\ \mathrm{e}(\mathbf{w})}{\partial\ s_j^{(l)}} \times \frac{\partial\ s_j^{(l)}}{\partial\ w_{ij}^{(l)}}$$

We have $\dfrac{\partial\ s_j^{(l)}}{\partial\ w_{ij}^{(l)}} = x_i^{(l-1)}$ We only need: $\dfrac{\partial\ \mathrm{e}(\mathbf{w})}{\partial\ s_j^{(l)}} = \delta_j^{(l)}$

δ for the final layer

$$\delta_j^{(l)} = \frac{\partial\, \mathrm{e}(\mathbf{w})}{\partial\, s_j^{(l)}}$$

For the final layer $l = L$ and $j = 1$:

$$\delta_1^{(L)} = \frac{\partial\, \mathrm{e}(\mathbf{w})}{\partial\, s_1^{(L)}}$$

$$\mathrm{e}(\mathbf{w}) = (\, x_1^{(L)} - y_n)^2$$

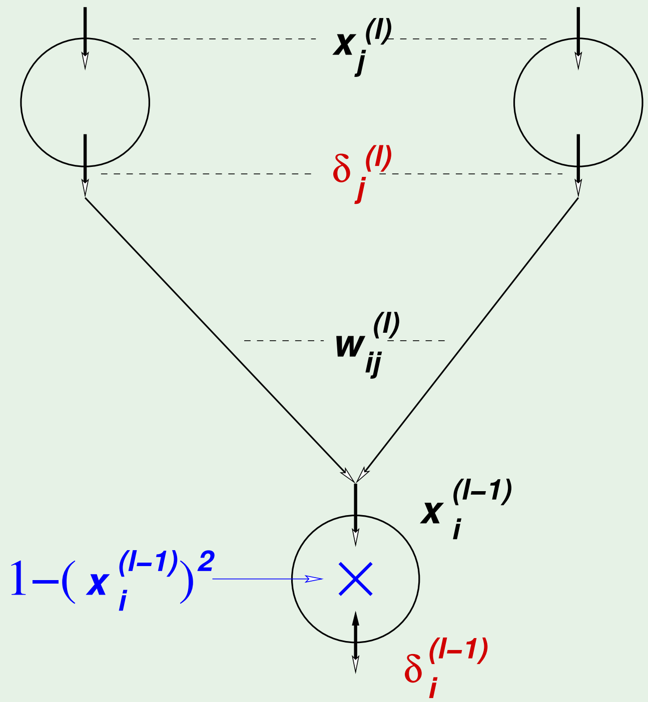
$$x_1^{(L)} = \theta(s_1^{(L)})$$

$$\theta'(s) = 1 - \theta^2(s)$$ for the tanh

Back propagation of δ

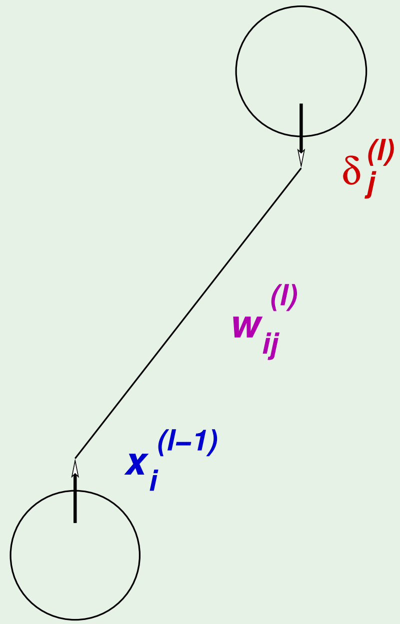
$$
\begin{aligned}
\delta_i^{(l-1)} &= \frac{\partial\, \mathrm{e}(\mathbf{w})}{\partial\, s_i^{(l-1)}} \\
&= \sum_{j=1}^{d^{(l)}} \frac{\partial\, \mathrm{e}(\mathbf{w})}{\partial\, s_j^{(l)}} \times \frac{\partial\, s_j^{(l)}}{\partial\, x_i^{(l-1)}} \times \frac{\partial\, x_i^{(l-1)}}{\partial\, s_i^{(l-1)}} \\
&= \sum_{j=1}^{d^{(l)}} \quad \delta_j^{(l)} \quad \times \quad w_{ij}^{(l)} \quad \times\, \theta'(s_i^{(l-1)})
\end{aligned}
$$

$$
\delta_i^{(l-1)} = (1 - (x_i^{(l-1)})^2) \sum_{j=1}^{d^{(l)}} w_{ij}^{(l)}\, \delta_j^{(l)}
$$



Backpropagation algorithm

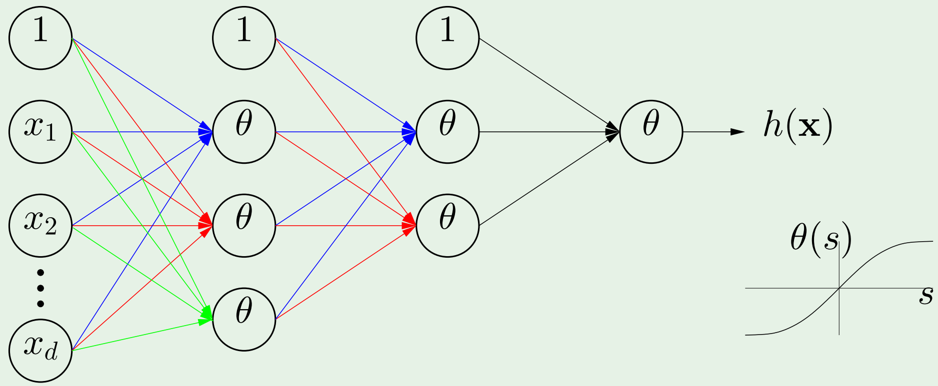
Initialize all weights $w_{ij}^{(l)}$ at random
for $t = 0, 1, 2, \ldots$ **do**
    Pick $n \in \{1, 2, \cdots, N\}$
    Forward: Compute all $x_j^{(l)}$
    Backward: Compute all $\delta_j^{(l)}$
    Update the weights: $w_{ij}^{(l)} \leftarrow w_{ij}^{(l)} - \eta \, x_i^{(l-1)} \delta_j^{(l)}$
    Iterate to the next step until it is time to stop
Return the final weights $w_{ij}^{(l)}$



Final remark: hidden layers

learned nonlinear transform

interpretation?



Thank You!