# Machine Learning

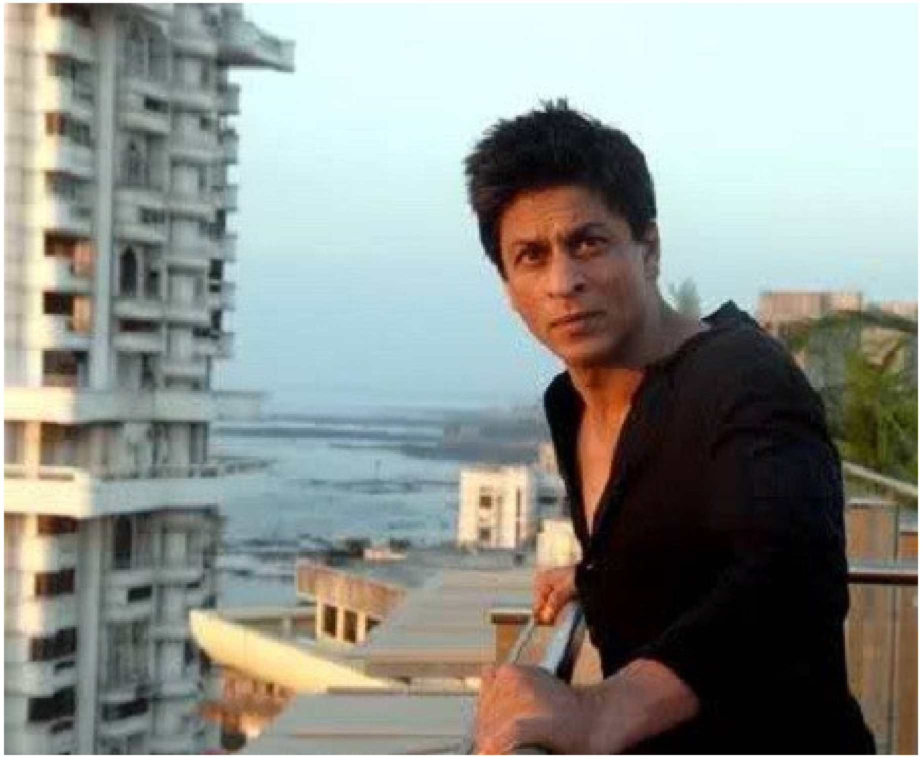
# CS60050

## Concept Learning

# Mannat: Waiting outside for an Autograph

# Which days does he come out to enjoy sports?

- Sky condition
- Humidity
- Temperature
- Wind
- Water
- Forecast



- Attributes of a day: takes on values

# Learning Task

- We want to make a hypothesis about the day on which SRK comes out ...
  - *May be it depends* in the form of a Boolean function on the attributes of the day.
  - Can we learn SRK's outing?

- Find the right hypothesis/function from historical data

# Training Examples for EnjoySport

| | Sky | Temp | Humid | Wind | Water | Forecst | | EnjoySpt |
|---|---|---|---|---|---|---|---|---|
| c( | Sunny | Warm | Normal | Strong | Warm | Same | )=1 | Yes |
| c( | Sunny | Warm | High | Strong | Warm | Same | )=1 | Yes |
| c( | Rainy | Cold | High | Strong | Warm | Change | )=0 | No |
| c( | Sunny | Warm | High | Strong | Cool | Change | )=1 | Yes |

- Negative and positive learning examples
- Concept learning:

  c is the target concept

  - Deriving a Boolean function from training examples
    - Many “hypothetical” boolean functions
      - ➢ Hypotheses; find h such that h = c.
  - – Other more complex examples:
    - ❖ Non-boolean functions
- Generate hypotheses for concept from TE’s

# Representing Hypotheses

- Task of finding appropriate set of hypotheses for concept given training data
- Represent hypothesis as Conjunction of constraints of the following form:
  - Values possible in any hypothesis
    - Specific value : Water = *Warm*
    - Don’t-care value: Water = ? (anything permissible value)
    - No value allowed : Water = ∅ (nothing permissible value)
  - Use vector of such values as hypothesis:
    - Attributes: < Sky AirTemp Humid Wind Water Forecast **>**
    - Example : < *Sunny* ? ? *Strong* ? *Same* **>**
- Idea of *satisfaction of hypothesis* by some example
  - “example satisfies hypothesis” defined by a function

$$h(x) = 1, \text{ if } h \text{ is true on } x$$
$$= 0, \text{ otherwise}$$

- Want hypothesis that best fits examples:
  - Can reduce learning to search problem over space of hypotheses

# Prototypical Concept Learning Task

- **TASK T**: predicting when person will enjoy sport
  - **Target function (concept)** **c**: *EnjoySport* : $X \rightarrow \{0,1\}$
  - Cannot, in general, know Target function c
    - ❖ Adopt hypotheses H about c
  - Form of hypotheses ***H***:

    Conjunctions of literals $< ?\ ,\ Cold\ ,\ High\ ,\ ?\ ,\ ?\ ,\ ?\ >$
- **EXPERIENCE E**
  - **Instances *X***: possible days described by attributes

    *Sky*, *AirTemp*, *Humidity*, *Wind*, *Water*, *Forecast*
  - **Training examples *D***: Positive/negative examples of target function

    $$\{ <x_1, c(x_1)> , \ldots , <x_m, c(x_m)> \}$$
- **PERFORMANCE MEASURE P**: Hypotheses *h* in *H* such that

  $$h(x) = c(x) \text{ for all } x \text{ in } D$$
  - There may exist several alternative hypotheses that fit examples

# Inductive Learning Hypothesis

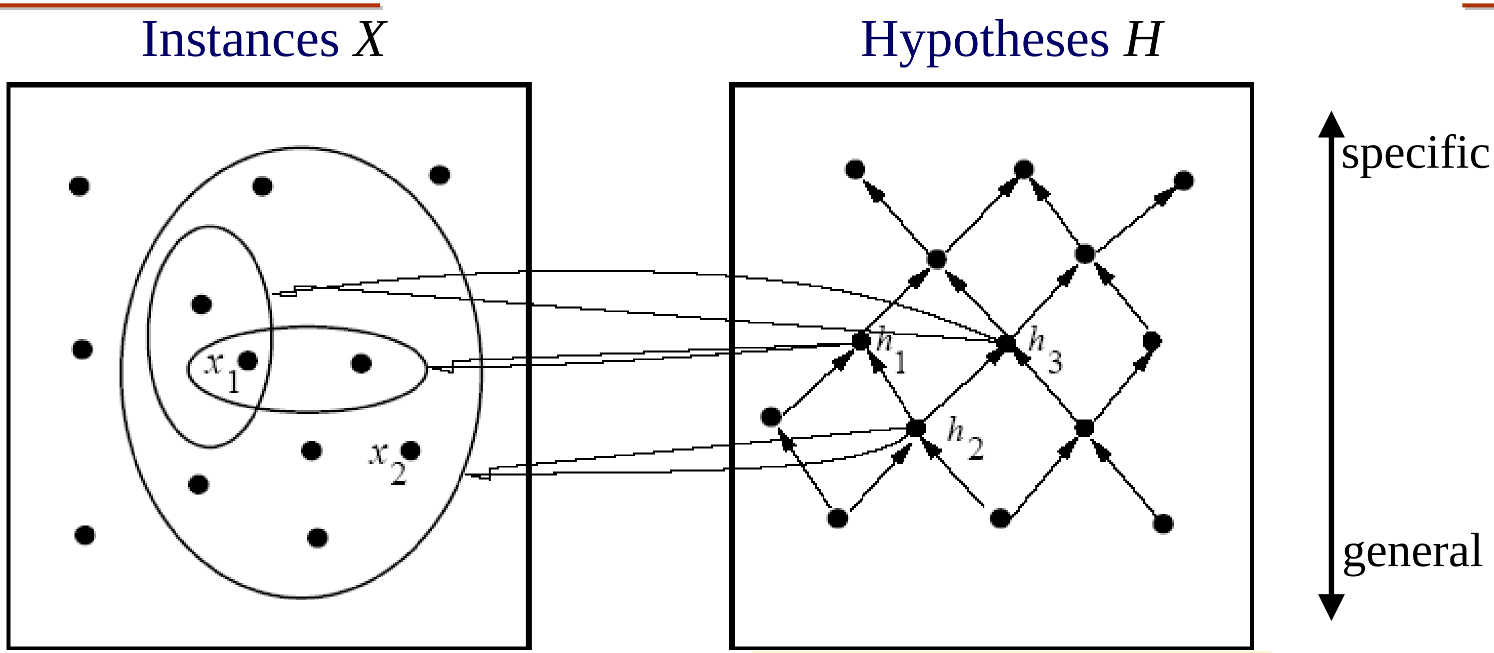
*Any hypothesis found to approximate the target function well over a sufficiently large set of training examples will also approximate the target function well over other unobserved examples*

# Approaches to Larning Algorithms

**The choice of the hypothesis space reduces the number of hypotheses.**

- Brute force search
  - Enumerate all possible hypotheses and evaluate
  - Highly inefficient even for small *EnjoySport* example
    - |X| = 3.2.2.2.2= 96 distinct *instances*
    - Large number of *syntactically distinct* hypotheses (0's, ?'s)
      - EnjoySport: |H| = 5.4.4.4.4.4=5120
      - Actually Fewer when consider h's with 0's
        - Semantically distinct h's is: |H| = 1+ (4.3.3.3.3.3) = 973
    - EnjoySport is VERY small problem compared to many
- Hence use other search procedures
  - *Approach 1:* Search based on ordering of hypotheses
  - *Approach 2:* Search based on finding all possible hypotheses using a good representation of hypothesis space
    - All hypotheses that fit data

# Ordering on Hypotheses




$x_1$=⟨*Sunny Warm High Strong Cool Same*⟩

$x_2$=⟨*Sunny Warm High Light Warm Same*⟩

$h_1$=⟨*Sunny ? ? Strong ? ?*⟩

$h_2$=⟨*Sunny ? ? ? ? ?*⟩

$h_3$=⟨*Sunny ? ? ? Cool ?*⟩

- $h$ is more general than $h'$ ( $h \geq_g h'$ ) if for each instance $x$,

$$h'(x) = 1 \rightarrow h(x) = 1$$

- Which is the most general/most specific hypothesis?

# Find-S Algorithm

Assumption: Everything except the positive examples is negative

**Assumes**

- There is hypothesis h in H describing target function c
- There are no errors in the TEs

**Procedure**

1. Initialize *h* to the most specific hypothesis in *H (what is this?)*
2. For each *positive* training instance *x*

   For each attribute constraint $a_i$ in *h*

   If the constraint $a_i$ in *h* is satisfied by *x*, do nothing

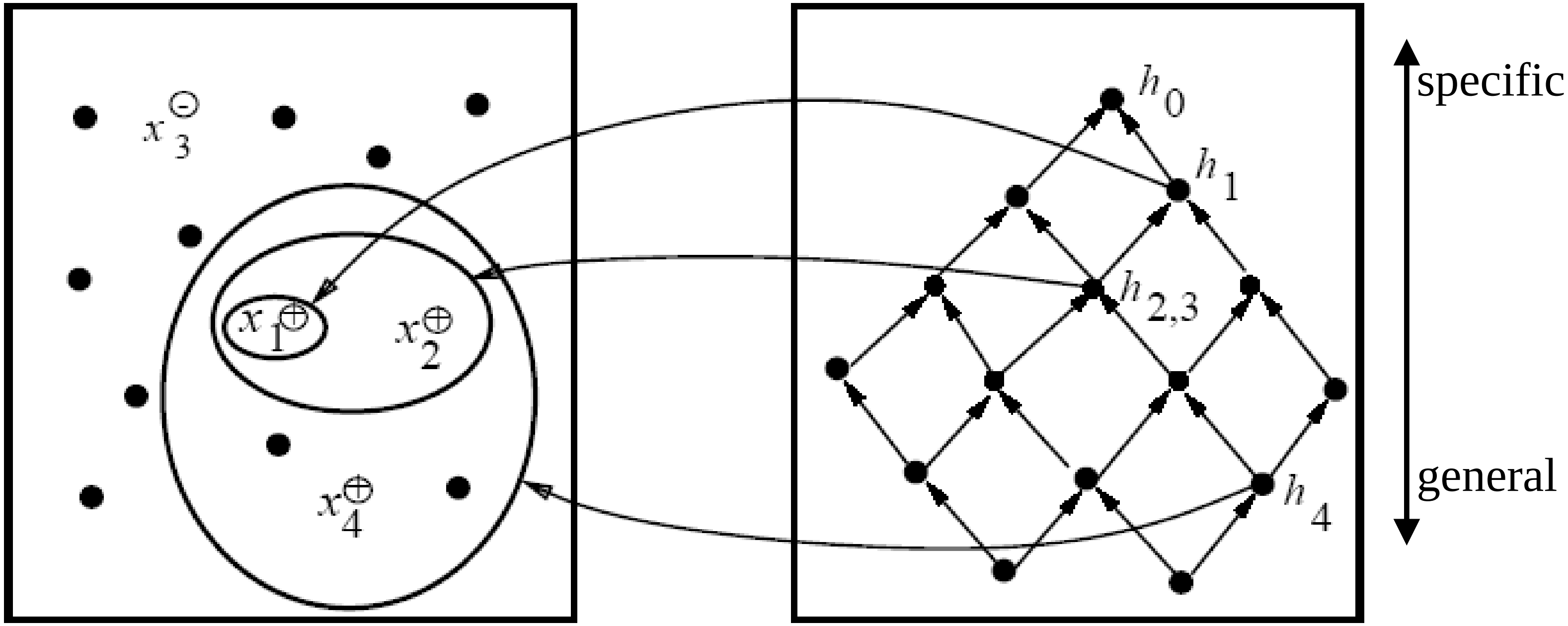
   Else, replace $a_i$ in *h* by next more general constraint that is satisfied by *x*
3. Output hypothesis *h*

**Note**

- There is no change for a negative example, so they are ignored.
- This follows from assumptions that there is h in H describing target function c *(ie, for this h, h=c) and* that there are no errors in data.
- It follows that hypothesis at any stage cannot be changed by neg example.

# Example of Find-S

Instances *X* Hypotheses *H*




$x_1$=<*Sunny Warm Normal Strong Warm Same*> **+**

$x_2$=<*Sunny Warm High Strong Warm Same*> **+**

$x_3$=<*Rainy Cold High Strong Warm Change*> **-**

$x_4$=<*Sunny Warm High Strong Cool Change*> **+**

$h_0$=<$\emptyset$ $\emptyset$ $\emptyset$ $\emptyset$ $\emptyset$ $\emptyset$>

$h_1$=<*Sunny Warm Normal Strong Warm Same*>

$h_2$=<*Sunny Warm ? Strong Warm Same*>

$h_3$=<*Sunny Warm ? Strong Warm Same*>

$h_4$=<*Sunny Warm ? Strong ? ?*>

# Problems with Find-S

- Problems:
  - Throws away information!
    - Negative examples
  - Can’t tell whether it has learned the concept
    - Depending on *H*, there might be several h’s that fit TEs!
    - Picks a maximally specific *h* (why?)
  - Can’t tell when training data is inconsistent
    - Since ignores negative TEs
- But
  - It is very simple ...
  - Outcome is independent of order of examples
    - Why?
- What alternative overcomes these problems?
  - Keep *all* consistent hypotheses!
    - *Candidate Elimination Algorithm*

# Consistent Hypotheses and Version Space

- A hypothesis $h$ is consistent with a set of training examples $D$ of target concept $c$ if
  - $h(x) = c(x)$ for each training example $<x, c(x)>$ in $D$
  - Note that consistency is with respect to specific D.
  - Notation:

$$Consistent(h, D) \equiv \forall <x, c(x)> \in D, h(x) = c(x)$$

- The version space, $VS_{H,D}$, with respect to hypothesis space $H$ and training examples $D$, is the subset of hypotheses from $H$ consistent with $D$
  - Notation:

$$VS_{H,D} = \{h \mid h \in H \text{ and } Consistent(h, D)\}$$

# List-Then-Eliminate Algorithm

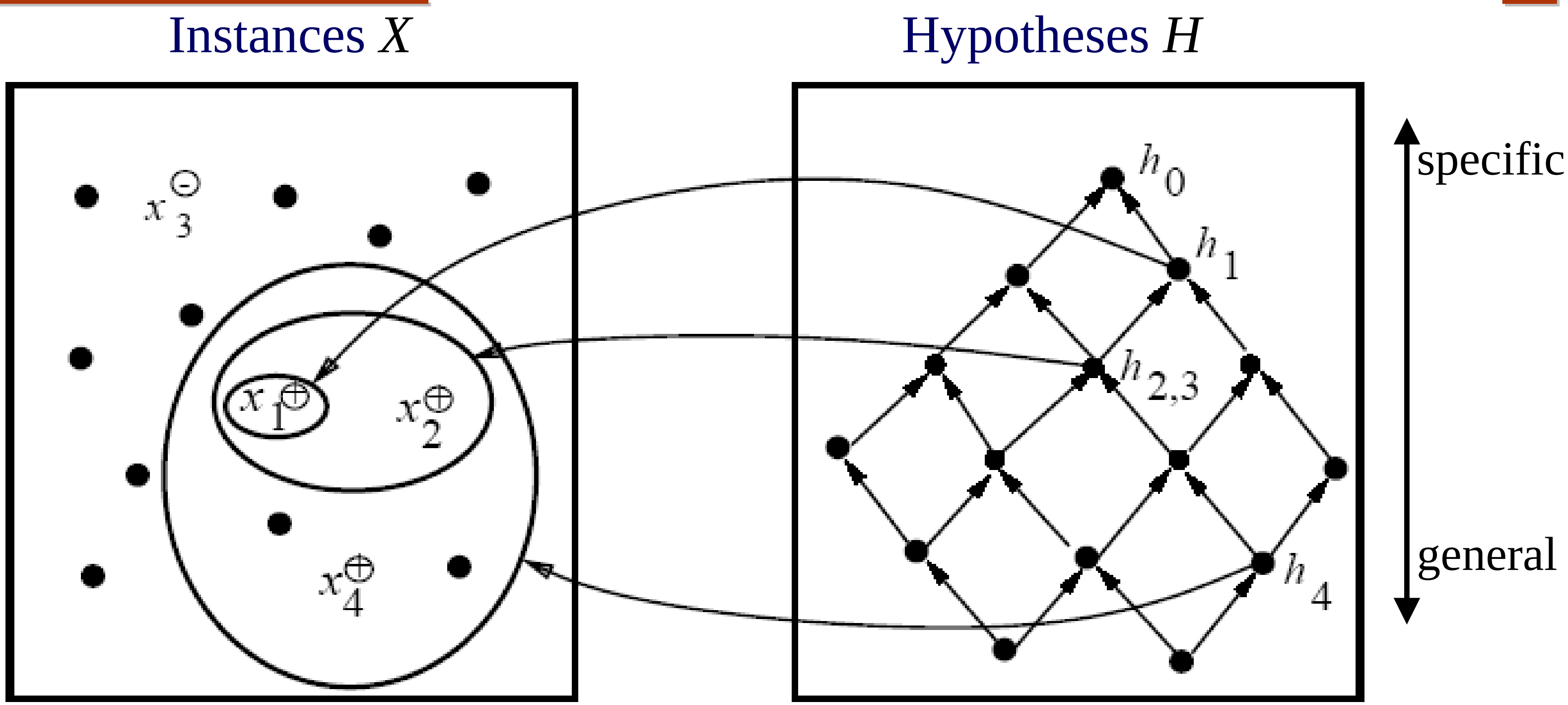
1. *VersionSpace* $\leftarrow$ list of all hypotheses in *H*
2. For each training example $<x, \ c(x)>$

   remove from *VersionSpace* any hypothesis *h* for which

   $$h(x) \neq c(x)$$

3. Output the list of hypotheses in *VersionSpace*
4. This is essentially a brute force procedure

# Example of Find-S (revisited)




$h_0$=<∅ ∅ ∅ ∅ ∅ ∅>

*$x_1$=<Sunny Warm Normal Strong Warm Same>* **+**

*$h_1$=<Sunny Warm Normal Strong Warm Same>*

*$x_2$=<Sunny Warm High Strong Warm Same>* **+**

*$h_2$=<Sunny Warm ? Strong Warm Same>*

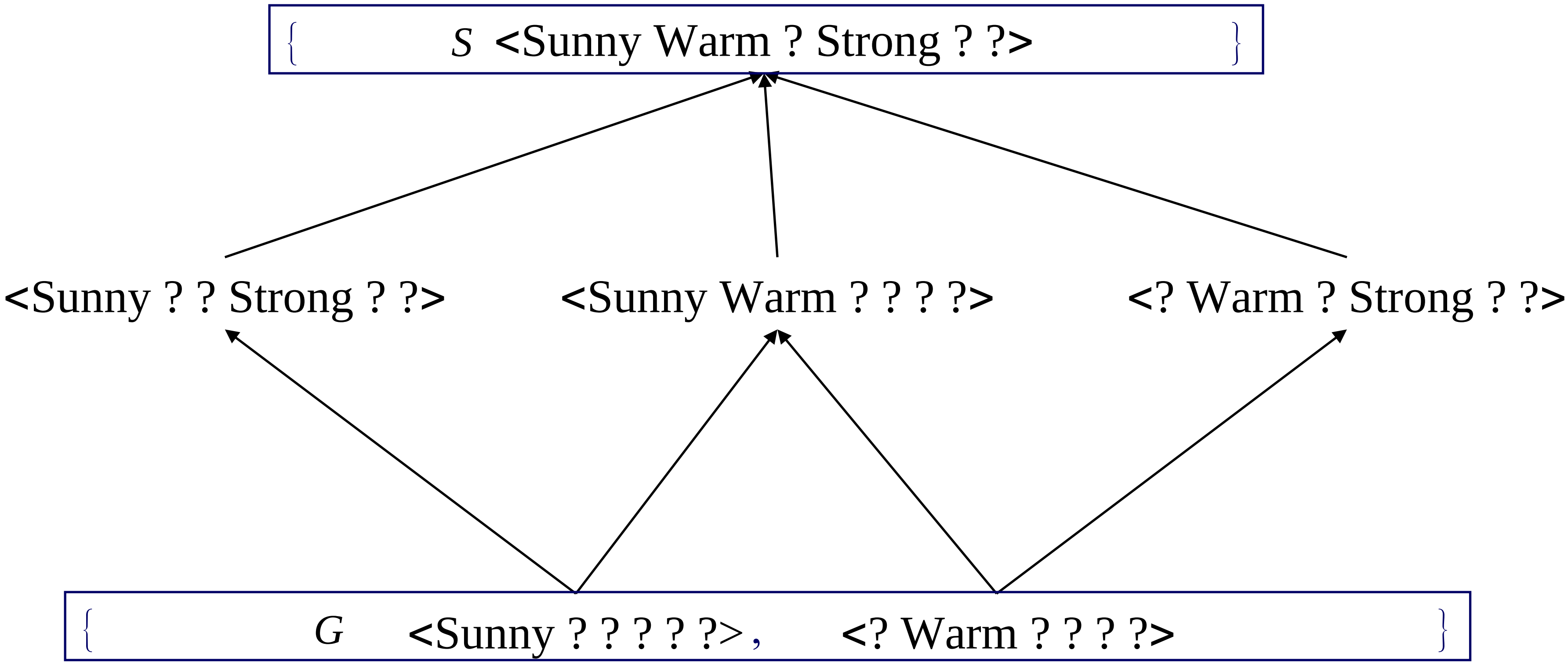
*$x_3$=<Rainy Cold High Strong Warm Change>* **-**

*$h_3$=<Sunny Warm ? Strong Warm Same>*

*$x_4$=<Sunny Warm High Strong Cool Change>* **+**

*$h_4$=<Sunny Warm ? Strong ? ?>*

# Version Space for this Example



# Representing Version Spaces

- Want more compact representation of VS
  - Store most/least general boundaries of space
  - Generate all intermediate h's in VS
  - Idea that any h in VS must be consistent with all TE's
    - Generalize from most specific boundaries
    - Specialize from most general boundaries
- The general boundary, $G$, of version space $VS_{H,D}$ is the set of its maximally general members consistent with D
  - Summarizes the negative examples; anything more general will cover a negative TE
- The specific boundary, $S$, of version space $VS_{H,D}$ is the set of its maximally specific members consistent with D
  - Summarizes the positive examples; anything more specific will fail to cover a positive TE

# Theorem

- Every member of the version space lies between the S,G boundary

$$VS_{H,D} = \{h \mid h \in H \text{ and } \exists s \in S\ \exists g \in G\ (g \geq h \geq s)\}$$

- Must prove:
  - (1) every h satisfying RHS is in $VS_{H,D;}$
  - (2) every member of $VS_{H,D}$ satisfies RHS.
- For (1), let g, h, s be arbitrary members of G, H, S respectively with g>h>s
  - s must be satisfied by all + TEs and so must h because it is more general;
  - g cannot be satisfied by any – TEs, and so nor can h
  - h is in $VS_{H,D}$ since satisfied by all + TEs and no – TEs
- For (2),
  - Since h satisfies all + TEs and no – TEs, h ≥ s, and $g \geq h$.

# Candidate Elimination Algorithm

$G \leftarrow$ maximally general hypotheses in $H$
$S \leftarrow$ maximally specific hypotheses in $H$

For each training example $d$, do

- If $d$ is **positive example**
  - Remove from $G$ every hypothesis inconsistent with $d$
  - For each hypothesis $s$ in $S$ that is inconsistent with $d$
    - Remove $s$ from $S$
    - Add to $S$ all minimal generalizations $h$ of $s$ such that

      1. $h$ is consistent with $d$, and

      2. some member of $G$ is more general than $h$
  - Remove from $S$ every hypothesis that is more general than another hypothesis in $S$

# Candidate Elimination Algorithm (continued)

- If *d* is a **negative example**
  - Remove from *S* every hypothesis inconsistent with *d*
  - For each hypothesis *g* in *G* that is inconsistent with *d*
    - Remove *g* from *G*
    - Add to *G* all minimal specializations *h* of *g* such that

      1. *h* is consistent with *d*, and

      2. some member of *S* is more specific than *h*
  - Remove from *G* every hypothesis that is less general than another hypothesis in *G*

- Essentially use: +ve TEs to generalize S / -ve TEs to specialize G
- Independent of order of TEs
- Convergence guaranteed if:
  - ***no errors* in TEs and *there is h in H describing c*.**

# Example

***Recall*:** If *d* is positive

$S_0$ $\boxed{\{<\varnothing\ \varnothing\ \varnothing\ \varnothing\ \varnothing\ \varnothing>\}}$

$G_0$ $\boxed{\{<?\ ?\ ?\ ?\ ?\ ?>\}}$

Remove from *G* every hypothesis inconsistent with *d*
For each hypothesis *s* in *S* that is inconsistent with *d*

- Remove *s* from *S*
- Add to *S* all minimal generalizations *h* of *s* that are specializations of a hypothesis in G
- Remove from *S* every hypothesis that is more general than another hypothesis in *S*

*<Sunny Warm Normal Strong Warm Same>* **+**

$S_1$ $\boxed{\{<\text{Sunny Warm Normal Strong Warm Same}>\}}$

$G_1$ $\boxed{\{<?\ ?\ ?\ ?\ ?\ ?>\}}$

# Example (contd)

$S_1$ {<Sunny Warm Normal Strong Warm Same>}

$G_1$ {<? ? ? ? ? ?>}

*<Sunny Warm High Strong Warm Same>* **+**

$S_2$ {<Sunny Warm ? Strong Warm Same>}

$G_2$ {<? ? ? ? ? ?>}

# Example (contd)

*Recall:* If *d* is a negative example

- Remove from S every hypothesis inconsistent with d
- For each hypothesis *g* in *G* that is inconsistent with *d*
  - ❖ Remove *g* from *G*
  - ❖ Add to *G* all minimal specializations *h* of *g* that generalize some hypothesis in S
  - ❖ Remove from *G* every hypothesis that is less general than another hypothesis in *G*

$S_2$ {<Sunny Warm ? Strong Warm Same>}

$G_2$ {<? ? ? ? ? ?>}

Current G boundary is incorrect
So, need to make it more specific.

*<Rainy Cold High Strong Warm Change>* **-**

$S_3$ {‹Sunny Warm ? Strong Warm Same›}

$G_3$ {<Sunny ? ? ? ? ?> , <? Warm ? ? ? ?> , <? ? ? ? ? Same>}

# Example (contd)

- Why are there no hypotheses left relating to

$$< Cloudy \;\; ? \;\; ? \;\; ? \;\; ? \;\; ? >$$

- The following specialization using the third value

$$< ? \;\; ? \;\; Normal \;\; ? \;\; ? \;\; ? >,$$

is not more general than the specific boundary

$$\boxed{\{ \; \text{<Sunny Warm ? Strong Warm Same>} \; \}}$$

- The specializations are also inconsistent with S

$$< ? \;\; ? \;\; ? \;\; Weak \;\; ? \;\; ? >, < ? \;\; ? \;\; ? \;\; ? \;\; Cool \;\; ? >$$

# Example (contd)

$S_3$ $\boxed{\{\text{<Sunny Warm ? Strong Warm Same>}\}}$

$G_3$ $\boxed{\{\text{<Sunny ? ? ? ? ?>, <? Warm ? ? ? ?>, <? ? ? ? ? Same>}\}}$

*⟨Sunny Warm High Strong Cool Change⟩* **+**

$S_4$ $\boxed{\{\text{<Sunny Warm ? Strong ? ?>}\}}$

$G_4$ $\boxed{\{\text{<Sunny ? ? ? ? ?>, <? Warm ? ? ? ?>}\}}$

# Example (contd)

*⟨Sunny Warm High Strong Cool Change⟩* **+**

- Why does this example remove a hypothesis from G?:

  *<? ? ? ? ? Same>*

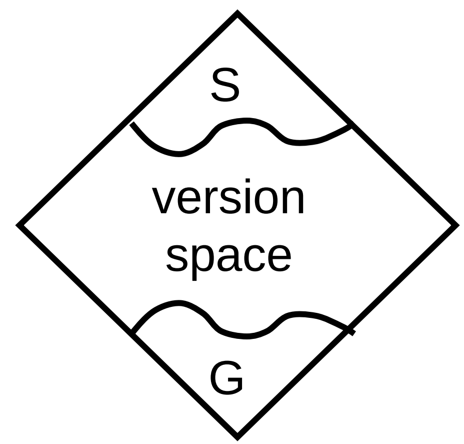
- This hypothesis
  - Cannot be specialized, since would not cover new TE
  - Cannot be generalized, because more general would cover negative TE.
  - Hence must drop hypothesis.

# Version Space of the Example

*S* {<Sunny Warm ? Strong ? ?>}

<Sunny ? ? Strong ? ?>     <Sunny Warm ? ? ? ?>     <? Warm ? Strong ? ?>

*G* {<Sunny ? ? ? ? ?> , <? Warm ? ? ? ?>}



# Convergence of algorithm

- Convergence guaranteed if:
  - ***no errors***
  - ***there is h in H describing c***.
- Ambiguity removed from VS when S = G
  - Containing single h
  - When have seen enough TEs
- If have false negative TE, algorithm will remove every h consistent with TE, and hence will remove correct target concept from VS
  - If observe enough TEs will find that S, G boundaries converge to empty VS

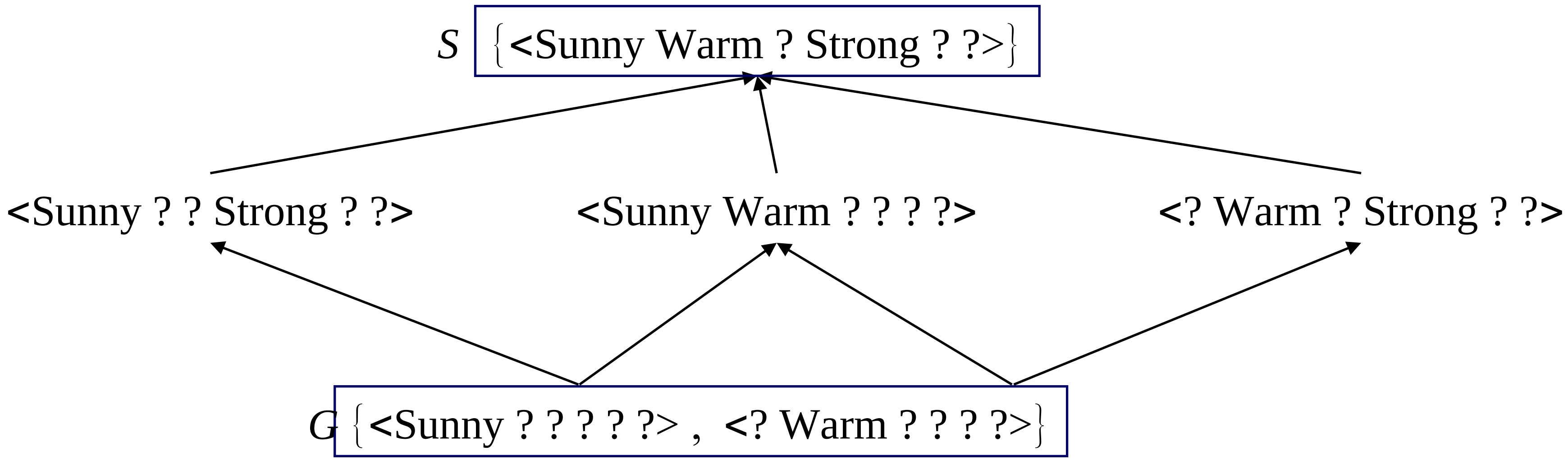
# Let us try this (Homework)

| Origin | Mfg. | Color | Decade | Type | Outcome |
|---|---|---|---|---|---|
| Japan | Honda | Blue | 1980 | Economy | + |
| Japan | Toyota | Green | 1970 | Sports | - |
| Japan | Toyota | Blue | 1990 | Economy | + |
| USA | Chrysler | Red | 1980 | Economy | - |
| Japan | Honda | White | 1980 | Economy | + |

# And this ...

| Origin | Mfg. | Color | Decade | Type | Outcome |
|---|---|---|---|---|---|
| Japan | Honda | Blue | 1980 | Economy | + |
| Japan | Toyota | Green | 1970 | Sports | - |
| Japan | Toyota | Blue | 1990 | Economy | + |
| USA | Chrysler | Red | 1980 | Economy | - |
| Japan | Honda | White | 1980 | Economy | + |
| Japan | Toyota | Green | 1980 | Economy | + |
| Japan | Honda | Red | 1990 | Economy | - |

# Which Next Training Example?

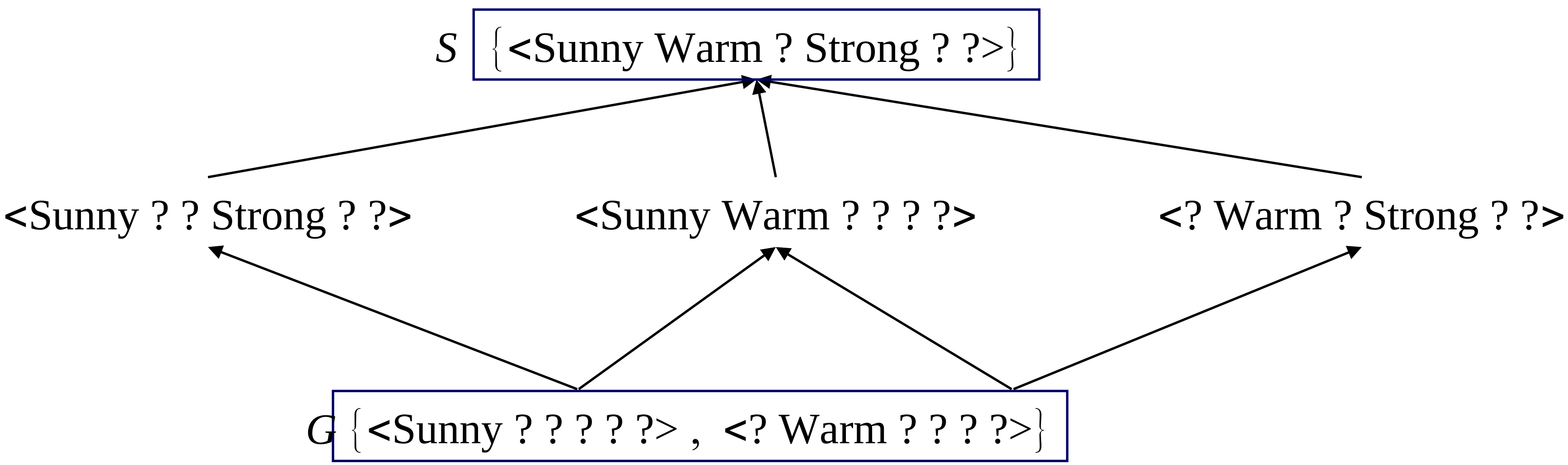



Order of examples matters for intermediate sizes of S,G; not for the final S, G

Assume learner can choose the next TE

- Should choose d such that
  - Reduces maximally the number of hypotheses in VS
  - Best TE: satisfies precisely 50% hypotheses;
    - Can’t always be done
  - Example: ‹Sunny Warm Normal Weak Warm Same〉 ?
    - If positive, generalizes S
    - If negative, specializes G

# Classifying new cases using VS




- Use *voting procedure* on following examples:
  - <Sunny Warm Normal Strong Cool Change>
  - <Rainy Cool Normal Weak Warm Same>
  - <Sunny Warm Normal Weak Warm Same>
  - <Sunny Cold Normal Strong Warm Same>

# Effect of Incomplete Hypothesis Space

- Preceding algorithms work if target function is in H
  - Will generally not work if target function *not* in H
- Consider following examples which represent target function

"sky = sunny or sky = cloudy":

- <Sunny Warm Normal Strong Cool Change>→Y
- **<**Cloudy Warm Normal Strong Cool Change>→Y
- **<**Rainy Warm Normal Strong Cool Change**>**→N

- If apply CE algorithm as before, end up with empty VS
  - After first two TEs, S= ‹? Warm Normal Strong Cool Change〉
  - New hypothesis is overly general
    - it covers the third negative TE!
- Our H does not include the appropriate c

Need more expressive hypotheses

# Incomplete Hypothesis Space

- If c not in H, then consider generalizing representation of H to contain c
  - For example, add disjunctions or negations to representation of hypotheses in H
- One way to avoid problem is to allow **all** possible representations of h's
  - Equivalent to allowing all possible subsets of instances as defining the concept of EnjoySport
    - Recall that there are 96 instances in EnjoySport; hence there are $2^{96}$ possible hypotheses in full space H
    - Can do this by using full propositional calculus with AND, OR, NOT
    - Hence H defined only by conjunctions of attributes is biased (containing only 973 h's)

# Unbiased Learners and Inductive Bias

- BUT if have no limits on representation of hypotheses (i.e., full logical representation: *and*, *or*, *not*), can only learn examples…no generalization possible!
  - Say have 5 TEs $\{x_1, x_2, x_3, x_4, x_5\}$, with $x_4$, $x_5$ negative TEs
- Apply CE algorithm
  - S will be disjunction of positive examples ($S=\{x_1 \vee x_2 \vee x_3\}$)
  - G will be negation of disjunction of negative examples ($G=\{!(x_4 \vee x_5)\}$)
  - Need to use all instances to learn the concept!
- Cannot predict usefully:
  - TEs have unanimous vote
  - other h's have 50/50 vote!
    - For every h in H that predicts +, there is another that predicts -

# Unbiased Learners and Inductive Bias

- Approach:
  - Place constraints on  representation of hypotheses
    - Example of limiting connectives to conjunctions
    - Allows learning of generalized hypotheses
    - Introduces bias that depends on hypothesis representation
- Need formal definition of inductive bias of learning algorithm

# Inductive System & Equivalent Deductive System

- Inductive bias made explicit in *equivalent deductive system*
  - *Logically represented system that produces same outputs (classification) from inputs (TEs, instance x, bias B) as CE procedure*
- Inductive bias (IB) of learning algorithm L is any minimal set of assertions B such that for any target concept c and training examples D, we can logically infer value c(x) of any instance x from B, D, and x
  - E.g., for rote learner, B = {}, and there is no IB
- Difficult to apply in many cases, but a useful guide

# Inductive Bias & Specific Learning Algorithms

- Role Learners:
  - no Inductive Bias
- Version space candidate elimination algorithm:
  - c can be represented in H
- Find-S:
  - c can be represented in H;
  - all instances that are not positive are negative

# Computational Complexity of VS

- The *S* set for conjunctive feature vectors and tree-structured attributes is linear in the number of features and the number of training examples.
- The *G* set for conjunctive feature vectors and tree-structured attributes can be exponential in the number of training examples.
- In more expressive languages, both *S* and *G* can grow exponentially.
- The order in which examples are processed can significantly affect computational complexity.

# Exponential size of G

- n Boolean attributes
- 1 positive example: (T, T, .., T)
- n/2 negative examples:
  - (F,F,T,..T)
  - (T,T,F,F,T..T)
  - (T,T,T,T,F,F,T..T)
  - ..
  - (T,..T,F,F)
- Every hypothesis in G needs to choose from n/2 2-element sets.
  - Number of hypotheses = $2^{n/2}$

# Summary

- Concept learning as search through *H*
- General-to-specific ordering over *H*
- Version space candidate elimination algorithm
- *S* and *G* boundaries characterize learner's uncertainty
- Learner can generate useful queries
- Inductive leaps possible only if learner is biased!
- Inductive learners can be modeled as equiv deductive systems
- Biggest problem is inability to handle data with errors
  - Overcome with procedures for learning decision trees

# Thank You!