Spelling Correction: Edit Distance

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- How to define 'closest'?
- Need a **distance metric**

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- How to define ‘closest’?
- Need a **distance metric**
- The simplest metric: **edit distance**

Edit Distance

- The minimum edit distance between two strings

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- Is the minimum number of editing operations

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- Is the minimum number of editing operations
  - Insertion
  - Deletion
  - Substitution

Minimum Edit Distance

Example

Edit distance from ‘intention’ to ‘execution’

What is the minimum number of edit operations needed to go from ‘intention’ to ‘execution’?

Minimum Edit Distance

Example

Edit distance from ‘intention’ to ‘execution’

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

1. Delete I
2. Substitute N with E
3. Substitute T with X
4. Insert C
5. Substitute N with U

Minimum Edit Distance

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
d s s   i s
```

One delete (d), one insert (i), 3 substitution (s) operations.

What is the cost of each operation?

Minimum Edit Distance

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
d s s   i s
```

- If each operation has a cost of 1 (Levenshtein)
  - Distance between these is 5

Minimum Edit Distance

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- If substitution costs 2 (alternate version)
  - Distance between these is 8

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Weighted versions of edit distance also exist, where different operations, especially different substitutions, are weighted differently — look up details

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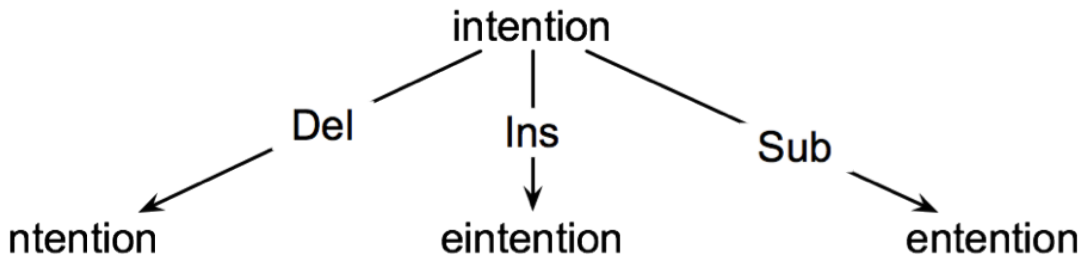

A naive method — try all possible edit operations starting with the first character of initial state

Minimum Edit as Search

How to navigate?

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- The space of all edit sequences is huge
- Lot of distinct paths end up at the same state
- Don't have to keep track of all of them

Ideally, we want to keep track of only those paths that take us 'towards' the goal state.

Minimum Edit as Search

How to navigate?

- The space of all edit sequences is huge
- Lot of distinct paths end up at the same state
- Don't have to keep track of all of them
- Keep track of the shortest path to each state

Defining Minimum Edit Distance Matrix

For two strings

- X of length n
- Y of length m

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We define $D(i,j)$

- the edit distance between $X[1..i]$ and $Y[1..j]$
- i.e., the first i characters of X and the first j characters of Y

Defining Minimum Edit Distance Matrix

For two strings

- X of length n
- Y of length m

We define $D(i,j)$

- the edit distance between $X[1..i]$ and $Y[1..j]$
- i.e., the first i characters of X and the first j characters of Y

Thus, the edit distance between X and Y is $D(n,m)$

Computing Minimum Edit Distance

Dynamic Programming

- A tabular computation of $D(n, m)$

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  - Compute $D(i,j)$ for small i,j
  - Compute larger $D(i,j)$ based on previously computed smaller values

Computing Minimum Edit Distance

Dynamic Programming

- A tabular computation of $D(n,m)$
- Solving problems by combining solutions to subproblems
- Bottom-up
  - Compute $D(i,j)$ for small i,j
  - Compute larger $D(i,j)$ based on previously computed smaller values
  - Compute $D(i,j)$ for all i and j till you get to $D(n,m)$

Dynamic Programming Algorithm

Initialization

```
D(i,0) = i
D(0,j) = j
```

Recurrence Relation:

```
For each  i = 1…M
      For each  j = 1…N
                          D(i-1,j) + 1
          D(i,j)= min     D(i,j-1) + 1
                          D(i-1,j-1) +   2; if X(i) ≠ Y(j)
                                         0; if X(i) = Y(j)
```

The X[i] character is deleted

The Y[j] character is inserted

substitute X[i] by Y[j] if they are not same

Termination:

```
D(N,M) is distance
```

The Edit Distance Table

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

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N	9									
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$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

The Edit Distance Table

N	9	8	9	10	11	12	11	10	9	**8**
O	8	7	8	9	10	11	10	9	8	9
I	7	6	7	8	9	10	9	8	9	10
T	6	5	6	7	8	9	8	9	10	11
N	5	4	5	6	7	8	9	10	11	10
E	4	3	4	5	6	7	8	9	10	9
T	3	4	5	6	7	8	7	8	9	8
N	2	3	4	5	6	7	8	7	8	7
I	1	2	3	4	5	6	7	6	7	8
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

Edit distance: 8

Computing Alignments

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- Computing edit distance may not be sufficient for some applications

Computing Alignments

- Computing edit distance may not be sufficient for some applications
  - We often need to align characters of the two strings to each other

Which parts of the two strings are aligned?

What edit operations were required to go from one string to the other?

Computing Alignments

- Computing edit distance may not be sufficient for some applications
  - We often need to align characters of the two strings to each other
- We do this by keeping a “backtrace”
- Every time we enter a cell, remember where we came from
- When we reach the end,
  - Trace back the path from the upper right corner to read off the alignment

The Edit Distance Table

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

The Edit Distance Table

N	9									
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$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

As we are filling up each cell, remember which operation was used at each step (the lowest-cost operation)

Minimum Edit with Backtrace

n	9	↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙←↓ 12	↓ 11	↓ 10	↓ 9	↙ **8**
o	8	↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↓ 10	↓ 9	↙ **8**	← 9
i	7	↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9	↙ **8**	← 9	← 10
t	6	↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙ **8**	← 9	← 10	←↓ 11
n	5	↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ **8**	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙↓ 10
e	4	↙ 3	← 4	↙← **5**	← **6**	← 7	←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9
t	3	↙←↓ 4	↙←↓ **5**	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙ 7	←↓ 8	↙←↓ 9	↓ 8
n	2	↙←↓ **3**	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↓ 7	↙←↓ 8	↙ 7
i	**1**	↙←↓ 2	↙←↓ 3	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙ 6	← 7	← 8
#	**0**	1	2	3	4	5	6	7	8	9
	#	**e**	**x**	**e**	**c**	**u**	**t**	**i**	**o**	**n**

Left arrow implies insertion
Down arrow implies deletion
Diagonal arrow implies substitution

Adding Backtrace to Minimum Edit

Base conditions:

$D(i,0) = i$ $\quad$ $D(0,j) = j$

Termination:

$D(N,M)$ is distance

Recurrence Relation:

For each $i = 1 \ldots M$

For each $j = 1 \ldots N$

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 & \text{deletion} \\ D(i,j-1) + 1 & \text{insertion} \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} & \text{substitution} \end{cases}$$

$$ptr(i,j) = \begin{cases} LEFT & \text{insertion} \\ DOWN & \text{deletion} \\ DIAG & \text{substitution} \end{cases}$$

Performance

Time

Performance

Time

$O(nm)$

Space

Performance

Time

$O(nm)$

Space

$O(nm)$

Backtrace

Performance

Time

$O(nm)$

Space

$O(nm)$

Backtrace

$O(n+m)$ Worst case complexity (all left arrows, followed by all down arrows)

So, now we know one method for spelling correction (there are others too).

Do we need to know the English language in order to apply this method?

So, now we know one method for spelling correction (there are others too).

Do we need to know the English language in order to apply this method?

Are there any situations where this method will fail, but a knowledge of the English language could have helped to identify spelling mistakes?