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Limb-darkening and exoplanets II: choosing the right law for minimum bias retrieval of transit parameters from transit lightcurves

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ABSTRACT

It has recently been demonstrated that the parametrization of the limb-darkening effect on a star has a direct impact on the parameters retrieved from transit lightcurves such as the exoplanet's radius, scaled semi-major axis and inclination. However, studies regarding this issue have only focused on the widely used quadratic limb-darkening law, leaving outside of the analysis other proposed laws that in fact are better descriptors of model intensity profiles. In this work, we show that in fact laws such as the logarithmic, square-root and three-parameter do a better job at retrieving the mentioned parameters from transit lightcurves than the widely used quadratic law and, as such, we recommend using those instead. We also detail when to use each of those, which we note has a dependence on both stellar and transit parameters. In addition, we demonstrate that the exponential law should not be used as it fundamentally non-physical, typically producing a divergence towards negative intensities at the limb.

Key words: stellar astrophysics – limb darkening – exoplanets: transits.

1 INTRODUCTION

In the past decade, the study of transiting exoplanets has been evolving from discovery to very precise characterization of these systems, thanks to the exquisit precision allowed mainly by space-based observatories such as the Hubble Space Telescope (HST) and the Kepler mission. This in turn has allowed the exoplanet community to study precise mass-radius diagrams (see, e.g., Weiss & Marcy 2014, and references therein), and in turn raise questions about the possibility of not only obtaining a precise determination of the internal composition of small, rocky exoplanets based on mass-radius measurements (Dorn et al. 2015) or of their atmospheric fractions from radius measurements alone (Wolfgang & Lopez 2015), but also has allowed for the determination of derived parameters of the systems that even allow for the obtention of stellar parameters directly from transit lightcurves through techniques such as asterodensity profiling (Seager & Mallén-Ornelas 2003; Kipping 2014), or of the detection of atmospheric features in exoplanet atmospheres through the technique of transmission spectroscopy. All these studies and techniques rely on the retrieval of precise and accurate transit parameters from transit lightcurves.

In a recent study (Espinoza & Jordán 2015, hereafter referred as EJ15), we showed that the accuracy is actually catching up with the precision of these measurements due to our poor understanding of the limb-darkening effect. In particular, we showed that there are important biases both when fixing and fitting the limb-darkening coefficients in the light curve fitting process, with the latter arising from the fact that the popular and widely used quadratic law is unable to model the complex intensity profile of real stars and the former arising for this same reason plus the fact that different methods of fitting the model intensity profiles give rise to different limb-darkening coefficients and from the fact that we have an imperfect knowledge of the real intensity profiles of stars. This showed that fixing the limb-darkening coefficients is actually the worst option if one is willing to obtain precise *and* accurate transit parameters, because the biases arise from three different sources, with the last one (the fact that we do not have a perfect modelling of the stellar intensity profiles of real stars) having an unknown but possibly large impact on them.

Given the above, unless the data quality is really poor, there is actually no good reason why one would actually want to fix the limb-darkening coefficients in the transit fitting process (assuming computational resources are not an

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issue). However, although fitting the limb-darkening coefficients seems to be a good solution to the accuracy problem, this still has the issue of the low flexibility of the quadratic limb-darkening law which still causes important biases on the retrieved transit parameters according to EJ15, which can be a large as $\sim 1\%$ for the planet-to-star radius ratio R_p/R_*, $\sim 2\%$ for the scaled semi-major axis, a/R_*, and 2% for the inclination, where R_p is the planetary radius, R_* the stellar radius and a the semi-major axis. The importance for missions like Kepler and for future missions like the Transiting Exoplanet Survey Satellite (TESS, Ricker et al. 2014) is evident when looking at the most recent results from the Kepler mission: if we focus on the planet-to-star radius ratio alone, which according to EJ15 has an accuracy bias on the order of $\sim 0.2\%$, a query to the Nasa Exoplanet Archive[1] shows that out of 3063 planetary candidates, 933 (26%) have precisions better than 0.2%, and out of 1001 Kepler confirmed exoplanets, 463 (46%) do too. This means that at least[2] 26% of the planet candidates (from which, e.g., population studies, which rely on averaging out *random* and not *systematic* uncertainties like the ones introduced by limb-darkening) and almost half of the confirmed exoplanets (from which, e.g., characterization studies are based on) have important systematic errors.

An obvious solution to the above mentioned problem if one decides to fit the limb-darkening coefficients is to try to use *other* laws to describe the intensity profile of stars. Although the non-linear law proposed by Claret (2000) seems to be the most flexible, the fact that it has four free parameters does not make it a very attractive choice in this scheme if one is willing to replace the quadratic law. However, laws with fewer parameters have been proposed, with the two-parameter laws being the exponential proposed by Claret & Hauschildt (2003), the logarithmic proposed by Klinglesmith & Sobieski (1970) and the square-root proposed by Díaz-Cordovéz & Giménez (1992). In addition, a very flexible three-parameter law was also proposed by Sing et al. (2009). These lower parameter laws seem to be very attractive for transit fitting purposes, due to their flexibility at following different intensity profiles (see, e.g., Howarth 2011, for a comparison between the goodness of fit to ATLAS model atmospheres of the mentioned two-parameter laws which outperform the linear and quadratic laws in terms of following the intensity profiles) and their low number of parameters.

Despite the attractive nature of the mentioned limb-darkening laws, testing their performance at retrieving transit parameters from transit lightcurves was a problem until recently for two reasons. First, there was no published algorithm that was capable of efficiently generating fast and accurate transit lightcurves using all of these non-standard laws. However, recently Kreidberg (2015) published an algorithm that does exactly this called `batman`, enabling one then to generate transit lightcurves very efficiently with any arbitrary limb-darkening law. The second problem was that it was not clear how to sample limb-darkening coefficients in an informative (i.e., sampling all the physically possible parameter space) and efficient way for all of these laws. Kipping (2015) recently derived an algorithm to sample parameters from the three-parameter law by imposing physically plausible constrains on the intensity profiles which derived in constrains on the limb-darkening coefficients being fitted, while Kipping (2013), using the same principle, derived an algorithmic way of doing this for two-parameter laws. However, he found that his algorithm was not applicable to the exponential law and thus no constrains for this law could be derived. In addition to this, the form of the logarithmic law used in Kipping (2013) differs from the standard one proposed by Klinglesmith & Sobieski (1970), which is actually the one used in `batman`. Thus, in order to sample coefficients from this limb-darkening law, one first has to derive an efficient informative sampling scheme for this standard form of the law.

In this work, we aim at testing how well do these non-standard laws perform at retrieving transit parameters from transit lightcurves, and compare them to the retrievals done by the standard quadratic limb-darkening law. For this, we first follow Kipping (2013) in order to derive an efficient sampling scheme for the standard form of the logarithmic and exponential laws, and show that the latter is actually *always* non-physical in the sense proposed by Kipping (2013). Then, we simulate transit lightcurves and perform a simulation study similar to the one done in EJ15 using the derived sampling schemes for the logarithmic law and the ones detailed in Kipping (2013) for the other two-parameter laws and Kipping (2015) for the three-parameter law, detailing when one law should be preferred over the other.

This work is organised as follows. In §2, we first revisit the logarithmic and exponential laws in order to try to derive an efficient informative sampling strategy following the methods in Kipping (2013) for the standard forms of these laws. In §3 we use those results in addition to the ones published by Kipping (2013) and Kipping (2015) in order to simulate transit lightcurves to test how well these laws perform in retrieving transit parameters. In §4 we present a discussion of our work and present our conclusions in §5.

2 EFFICIENT SAMPLING OF COEFFICIENTS FROM THE LOGARITHMIC AND EXPONENTIAL LAWS REVISITED

As explained in the introduction, an informative and efficient sampling of limb-darkening coefficients in order to fit them to transit lightcurves is fundamental. Motivated by this, Kipping (2013) introduced an algorithm which first imposes physically plausible constrains on the intensity profiles, namely, an everywhere positive intensity profile ($I(\mu) > 0$) and a decreasing intensity profile from center to limb ($\partial I(\mu)/\partial\mu > 0$), which in turn impose constrains on the parameters of the studied two-parameter law. With these constrains at hand, Kipping (2013) devised a triangular sampling strategy which only require sampling two uniformly distributed numbers between 0 and 1, q_1 and q_2, which through a transformation can be converted in order to

[1] `http://exoplanetarchive.ipac.caltech.edu/`; query done on 29/09/2015.

[2] It is important to note that this is an underestimate of the number of planet candidates with estimation errors, due to the fact that the Kepler pipeline makes use of fixed limb-darkening coefficients using the quadratic law (Rowe et al. 2015), which as explained here and estimated in EJ15, gives rise to more severe biases than the one used for this estimation

sample coefficients that follow the derived constrains. This was done in that work for the most popular two-parameter limb-darkening laws with the exception of the exponential limb-darkening law, whose derived constrains on the coefficients were not enough to use the triangular sampling technique, and for the typical form of the logarithmic law, which is different to the one used in Kipping (2013).

We here first derive an informative and efficient sampling strategy for the typical form of the logarithmic law and then we study the exponential law, for which we conclude that deriving an efficient sampling strategy is actually *impossible*; this implies that the law is fundamentally non-physical and, therefore, should not be used.

2.1 Efficient sampling from the logarithmic law

As described in Klinglesmith & Sobieski (1970), the logarithmic law is given by

$$I(\mu) = 1 - l_1(1-\mu) - l_2\mu \ln \mu. \qquad (1)$$

However, Kipping (2013) derived the constrains for a law similar (but not the same) to the original logarithmic law, given by

$$I_K(\mu) = 1 - A(1-\mu) - B\mu(1-\ln \mu),$$

which can be viewed as the original logarithmic law with an extra linear term. Given that this is not the original law many authors have adopted both to obtain limb-darkening coefficients and to model transit lightcurves, we here derive a sampling strategy for the original logarithmic limb-darkening law given in eq. (1). To do this, we start by imposing physically plausible constrains on the intensity profiles. The constrain of an everywhere positive intensity profile implies

$$l_1(1-\mu) + l_2\mu \ln \mu < 1 \quad \forall \quad 0 < \mu < 1.$$

In order to ensure this condition, we need to find the extrema of the expression in the left-hand side. We note that the maximum of the expression is obtained only if $l_2 < 0$, while the minimum of the expression is obtained if $l_2 > 0$. In both cases, this happens at $\mu_e = \exp[(l_1 - l_2)/l_2]$, where we have assumed $l_2 \neq 0$. If $l_2 = 0$, then we obtain the inequality $l_1(1-\mu) < 1$, which implies $l_1 < 1$, constraint we will see is valid for all the possible values of l_2. In the case $l_2 > 0$, because μ_e gives a minimum, $\mu \to 1$ and $\mu \to 0+$ maximise the expression in the desired range. The first limit gives the trivial constrain $1 > 0$, while the second limit gives the constrain $l_1 < 1$. In the case $l_2 < 0$, replacing μ_e in the expression leads to the constrain

$$\frac{l_1 - 1}{\mu_e} < l_2,$$

which, because $l_2 < 0$, again implies $l_1 < 1$. The condition for a decreasing intensity profile from center to limb, on the other hand, leads to the condition

$$l_1 - l_2(1 + \ln \mu) > 0 \quad \forall \quad 0 < \mu < 1.$$

The left-hand side has, again, a division depending on the sign of l_2. If $l_2 > 0$, then the expression is convex and has no absolute minimum. It can be seen that then the expression is always satisfied for $0 < \mu < 1$. In the limit $\mu \to 1$, the expression leads to the condition $l_1 > l_2$. For the $\mu \to 0^+$ limit, however, no condition can be derived because the expression diverges. If $l_2 < 0$, on the other hand, the expression is concave, and it does not have an absolute maximum. It can be seen, however, that the expression in the limit $\mu \to 1$, leads to $l_1 > l_2$ just as in the first case, but in this case the expression is not valid for $\mu \to 0^+$, because it goes to $-\infty$. This implies that for $l_2 < 0$ the inequality will never be satisfied in the desired range and, thus, we need $l_2 > 0$ in order for the profile to be everywhere decreasing. In summary, the derived condition for an everywhere positive intensity profile is

$$l_1 < 1,$$

while the conditions for an everywhere decreasing intensity profile from center to limb are

$$l_2 > 0,$$
$$l_1 > l_2.$$

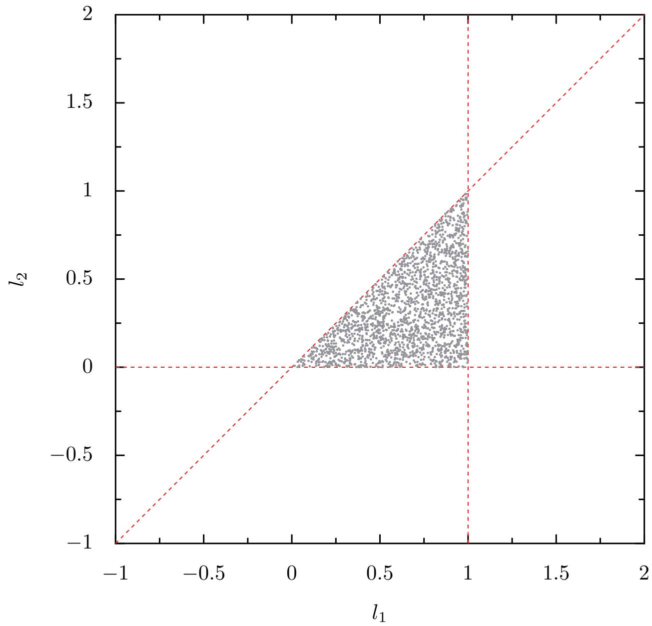

Figure 1. Samples of the logarithmic limb-darkening coefficients that satisfy the derived constrains in this section out of 10^6 uniformly sampled points between $-1 < l_1 < 2$ and $-1 < l_2 < 2$. Only $\sim 5\%$ of those samples satisfy such relations, shown here with red dashed lines.

Figure 1 show these constrains geometrically. For illustration, 10^6 points were uniformly sampled between $-1 < l_1 < 2$ and $-1 < l_2 < 2$ and a sample of those that satisfy these constrains were plotted. Only 5% of them did satisfy such relations which demonstrates, as Kipping (2013) showed, the inefficiency of using such sampling strategy to draw physically plausible limb-darkening coefficients. This is the reason why we need to use the triangular sampling technique described in Kipping (2013).

In order to use the triangular sampling technique, one needs to re-parametrize the constrains in order for them to be sampled on a right-angled triangle with this angle posed in the origin. A parametrisation that does this is the one with

$$v_1 \quad = 1 - l_1, \qquad (2)$$

$$v_2 \quad = l_2. \tag{3}$$

If we now consider the transformations (see Kipping 2013)

$$v_1 \quad = \sqrt{q_1}q_2, \tag{4}$$
$$v_2 \quad = 1 - \sqrt{q_1}, \tag{5}$$

sampling q_1 and q_2 from uniform distributions between $(0, 1)$ leads to a sampling with the desired constrains between l_1 and l_2. Replacing the expressions in eqs. (4) and (5) in eqs. (2) and (3) gives

$$l_1 \quad = \quad 1 - \sqrt{q_1}q_2$$
$$l_2 \quad = \quad 1 - \sqrt{q_1}$$

which leads to the inverse equations

$$q_1 \quad = \quad (1 - l_2)^2,$$
$$q_2 \quad = \quad \frac{1 - l_1}{1 - l_2}.$$

We note that, as expected, these relations return limb-darkening coefficients that are physically plausible for this law (i.e., they follow the derived constrains). Furthermore, we note these relations are different to the ones derived in Kipping (2013); these are only applicable for his form of the logarithmic law.

2.2 The exponential law is non-physical

We now study the exponential limb-darkening law. This law was introduced by Claret & Hauschildt (2003), and is given by

$$I(\mu) \quad = \quad 1 - e_1(1-\mu) - e_2/(1-e^{\mu}).$$

Kipping (2013) tried to apply the same methods used for the other two-parameter laws in order to derive constrains and hence an efficient sampling scheme for the coefficients of this law, but observed that the triangular sampling technique was not applicable in this case because the imposed physical constrains were not able to yield a sufficient number of relations between the coefficients for it to be used. However, apparently overlooked by Kipping (2013) is the fact that the exponential law will *never* yield physically plausible coefficients for $0 < \mu < 1$, because the conditions of an everywhere positive and decreasing intensity profile from center to limb cannot be satisfied at the same time, as we now show.

We start by imposing an everywhere positive intensity profile for this law. This leads to the relation

$$e_1(1-\mu) + e_2/(1-e^{\mu}) < 1 \quad \forall \quad 0 < \mu < 1,$$

and the objective is to study the left-hand side. We note that this expression has a minimum if $e_2 < 0$, while it has a maximum when $e_2 > 0$. We see that in the first case, as $\mu \to 0^+$, the expression tends to ∞, and therefore the expression is never satisfied; this implies that e_2 cannot be less than zero in order to have an everywhere positive intensity profile. On the other hand, if we impose an everywhere decreasing intensity profile for this law from center to limb, one can note this leads to the constrain:

$$e_1 - e_2\frac{e^{\mu}}{(1-e^{\mu})^2} > 0 \quad \forall \quad 0 < \mu < 1.$$

The form of the left-hand side expression again depends on the value of e_2 in the same way as before; however, in this case there is no absolute maximum or minimum and, thus, it suffices to study the expression at the borders. We note that in order for the expression to be satisfied for all the values of μ, $e_2 < 0$, because only in this case the expression goes to ∞ when $\mu \to 0^+$. Because in order for the exponential law to have an everywhere positive intensity profile this condition was ruled out, this implies that both conditions cannot be met at the same time and, thus, there is no combination of coefficients (e_1, e_2) that satisfy the physically plausible conditions suggested by Kipping (2013) for this law: in this sense, the exponential law introduced by Claret & Hauschildt (2003) is non-physical.

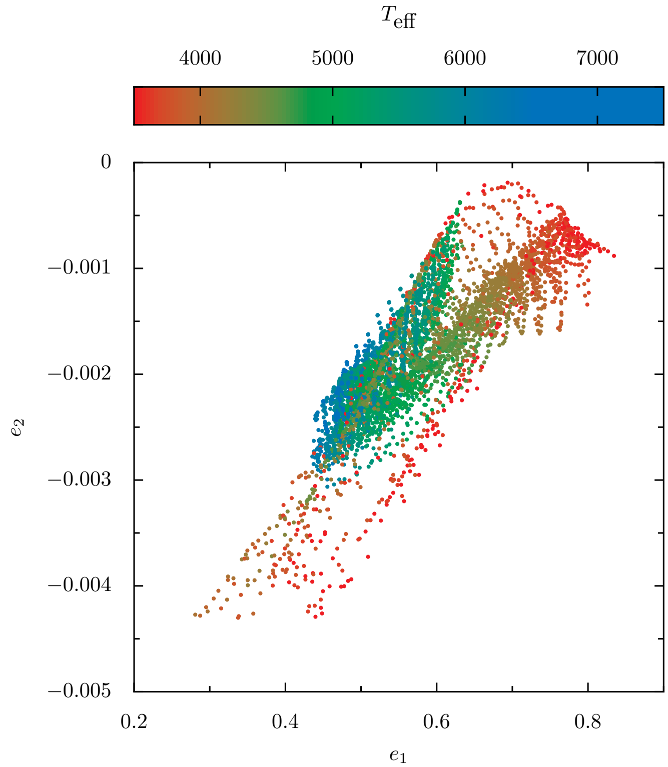

Figure 2. Limb-darkening coefficients for the exponential law obtained using the methods described in EJ15 with the Kepler bandpass for all the stars in the ATLAS models with $T_{\rm eff} < 9000$ K. As can be seen, $e_2 < 0$, which implies that all the fitted intensity profiles are not everywhere positive and, thus, non-physical.

We note that typical values obtained by fitting the exponential law to model atmospheres give $e_2 < 0$, which means that most of the profiles fitted to model atmospheres do not have an everywhere positive intensity profile. To illustrate this, in Figure 2 we plot all the coefficients fitted using the exponential law following the methods of EJ15 using the Kepler bandpass for stars of interest for exoplanet studies ($T_{\rm eff} < 9000$ K) obtained with the ATLAS models. In order to see at which value of μ the profile starts to be negative, we note that this happens as $\mu \to 0^+$. Here $1/(1-e^{\mu}) \approx -1/\mu$, which gives

$$I(\mu) \quad \approx \quad 1 - e_1(1-\mu) + e_2/\mu.$$

The intensity profile then, touches $I(\mu) = 0$ around

$$\mu_0 \approx \frac{e_1 - 1 \pm \sqrt{(1-e_1)^2 - 4e_1 e_2}}{2e_1}$$

According to Figure 2, $e_1 \sim 0.6$, while $e_2 \sim -2 \times 10^{-3}$, which gives $\mu_0 \sim 0.005$, so the law starts giving negative values very close to (but not at) the limb, as we predicted. We note that typical transit observations are capable of sampling these points by chance, with high cadence observations as the ones done by the Kepler mission and the ones that will be done with the future TESS mission (Ricker et al. 2014) having an almost certain chance of sampling points inside those ranges. Furthermore, as the generation of transit lightcurves for laws like the exponential requiere either numerical integration or the usage of clever approximations that usually sample more points close to the limb (which is the case for e.g., the `batman` code, Kreidberg 2015), this law is doomed to produce numerical errors in transit lightcurves due to this fact. Because of these reasons, it is advisable not to use this law in real applications.

3 BIASES IN TRANSIT PARAMETERS USING TWO-PARAMETER LAWS

Having solved the issues regarding the sampling of limb-darkening coefficients from the logarithmic and exponential laws, we now turn to the study of the biases introduced on retrieved transit parameters in the transit fitting process introduced by using different limb-darkening laws. In order to perform this study, we perform a similar study as the one done in EJ15, but now (1) using the `batman` transit code described in Kreidberg (2015), (2) using three different two-parameter laws: the quadratic, logarithmic and square-root laws (with the exponential law left out of our analysis because, as was shown in §2.2, it's fundamentally non-physical) and the three-parameter law and (3) smaller steps in the planet-to-star radius ratio, $p = R_p/R_*$, in order to illustrate the evolution of the biases with this parameter in a cleaner way: small ratios ($p = 0.01$), medium ratios ($p = 0.07$) and large ones ($p = 0.13$) .

As in EJ15, we simulate transit lightcurves for planets orbiting around host stars with solar metallicity, gravity and microturbulent velocity between 3500 K and 9000 K, with the mentioned planet-to-star radius ratios, values of the scaled semi-major axis of $a_R = a/R_* = \{3.27, 3.92, 4.87, 6.45, 9.52, 18.18, 200\}$ and different impact parameters. The transits are simulated using a non-linear law in order to emulate "real" intensity profiles, with the coefficients generated using the ATLAS models and the same methods as the ones used in EJ15 with the Kepler bandpass. The transits were then fitted using free limb-darkening coefficients with the mentioned laws.

For the simulations, the applied strategies in this work to sample physically plausible coefficients from two-parameter laws are as follows, all of which use numbers $q_1 \in (0,1)$ and $q_2 \in (0,1)$:

- For the quadratic limb-darkening law, we fit for the numbers $q_1 = (u_1+u_2)^2$ and $q_2 = u_1/2(u_1+u_2)$. To obtain the coefficients from those fitted numbers, we use the relations $u_1 = 2\sqrt{q_1}q_2$ and $u_2 = \sqrt{q_1}(1-2q_2)$, which are derived in Kipping (2013).
- For the square-root law, we fit for the numbers $q_1 = (s_1+s_2)^2$ and $q_2 = s_2/2(s_1+s_2)$. To obtain the coefficients from those fitted numbers, we use the relations $s_1 = \sqrt{q_1}(1-2q_2)$ and $s_2 = 2\sqrt{q_1}q_2$, which are derived in Kipping (2013).
- For the logarithmic law, we fit for the numbers $q_1 = (1-l_2)^2$ and $q_2 = (1-l_1)/(1-l_2)$. To obtain the coefficients from those fitted numbers, we use the relations $l_1 = 1 - \sqrt{q_1}q_2$ and $l_2 = 1 - \sqrt{q_1}$, which are derived in §2.1.

For the three-parameter law, we use the formalism and codes described in Kipping (2015). As explained before, analyses for the exponential law were not made as it is fundamentally non-physical. The code used to generate these simulations has been published in GitHub[3], and can be used to perform analyses in a finer grid to the ones published here.

3.1 The case of central transits

The results of the simulations for central transits (i.e., $b = 0$) are shown in Figure 3, with the sizes of the points representing the input values of p. The upper panels showing the biases introduced on the planet-to-star radius ratio, p, and the lower panels showing the biases introduced on the scaled semi-major axes, a_R. From left to right, the panels show the biases introduced by using a quadratic law, a logarithmic law, a square-root law and a three-parameter law in the transit fitting procedures leaving the corresponding limb-darkening coefficients as free parameters. The dependence with the input values of a_R are not shown as the bias seem to be almost independent of this parameter in this case (this can be also seen by looking at the lower panels of Figure 10 in EJ15, which give the same results as the ones shown here for the quadratic law, showing a nice agreement between the simulations done with our implementation of the Mandel & Agol (2002) codes for the transit light curve generation and the `batman` code).

As can be seen from the figure, different limb-darkening laws have different behaviours in retrieving p and a_R, with a dependence on the input value of p, and also depending on the temperature of its host star. First, it is interesting to note that the logarithmic law does a very good job at retrieving the parameters of small planet-to-star radius ratios, even at the level of the three-parameter law for stars with $T_{\rm eff} < 6000$ K, and also for medium and large planet-to-star radius ratios with host-stars with $T_{\rm eff} < 4000$ K (which makes this law perfect for present and future transit detection and characterisation around M-dwarfs). The quadratic law, on the other hand, is only the best among the options for host-stars between $4250 < T_{\rm eff} < 5500$ K, with the logarithmic law being the best in the $5500 < T_{\rm eff} < 6250$ K range, and the square-root law being the best for planets around host stars with temperatures between $6250 < T_{\rm eff} < 8000$ K. Hotter than this, the best option in terms of accuracy is the three-parameter law.

It is interesting to emphasise that although the biases introduced by the quadratic law might seem to be small, as mentioned in the introduction and in EJ15 they are significant for several candidate and confirmed exoplanets. However, choosing the right law in this case can help to achieve

[3] `http://www.github.com/nespinoza/ld-simulations/`

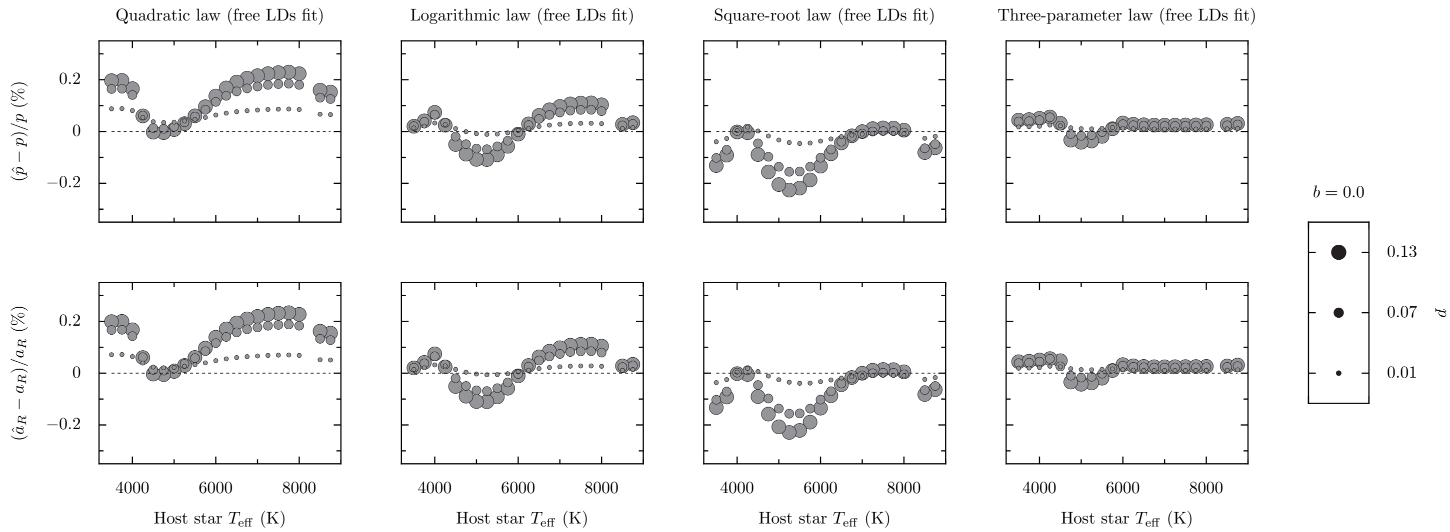

Figure 3. Results of fitting transit lightcurves as described in §3.1 with the limb-darkening coefficients as free parameters. We only plot the size of the points which represent the input values of p.

the same level of accuracy than the level of precision that Kepler achieves for most planets. In this case, for example, accuracies on the order of $\sim 0.01\%$ can be achieved for p with a careful selection of a two-parameter law, and exoplanets with precisions better than that according to a query to the Nasa Exoplanet Archive[4] form only 0.58% of all candidate exoplanets and 1.2% of the confirmed exoplanets. For the remaining exoplanets requiring a better accuracy, a new law might have to be created, as this precision is also the limit of the three-parameter law (which has a precision on p around $\sim 0.01\%$ for stars with temperatures larger than $T_{\rm eff} = 6000$ K). Studying new limb-darkening laws, however, is out of the scope of the present work.

3.2 The case of non-central transits

Figures 4 and 5 show the results for low ($b = 0.3$) and high ($b = 0.8$) input impact parameters respectively. In these cases, we see that the biases are in general worse for smaller values of p and a_R, i.e., for a given stellar radius, for smaller, close-in planets. We can also see that, again, a careful selection of the law to be used can allow one to minimize the biases on the retrieved transit parameters. For example, if the objective were to obtain p with minimum bias for a low impact-parameter transit, then for host stars colder than 4000 K the best option is to use either the logarithmic or the three-parameter law, which show biases on the order of $\sim$ 0.1%. In contrast, using the quadratic law for transits around those host stars the biases introduced on this parameter are on the order of $\sim 0.75\%$, i.e., almost an order of magnitude improvement in the accuracy.

Overall, as expected, the law that retrieves the parameters with minimum bias in low impact-parameter transits is the three-parameter law. However, for high impact-parameter transits of small planets, the logarithmic and square-root law seem to outperform this law in general if the objective is to retrieve a minimum bias estimator for p. This is probably due to the fact that the law is so flexible that it allows for small distortions of the transit lightcurve that, in this case, mimics variations that were actually attributable to p, adding a small but important bias on the retrieved parameter (**NE: maybe I should plot a transit fit showing this?**). However, for the other parameters (a_R and i) this law seems to be the best choice in terms of the achieved accuracy.

4 DISCUSSION

5 CONCLUSIONS

6 ACKNOWLEDGMENETS

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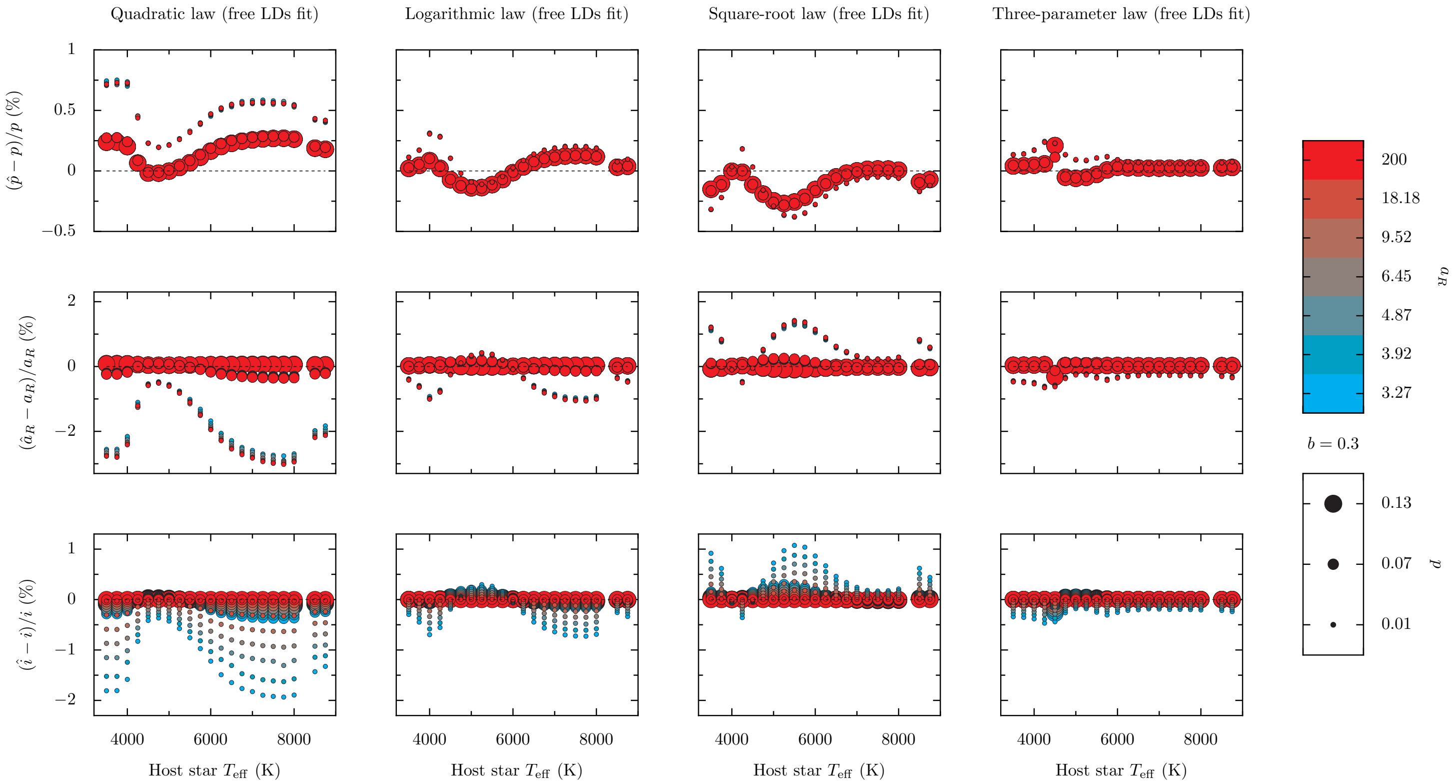

Figure 4. Results of fitting transit lightcurves as described in §3.1 with the limb-darkening coefficients as free parameters, but for low impact parameter transits ($b = 0.3$). The size of the points represent the input values of p, while their color represent the input values of a_R.

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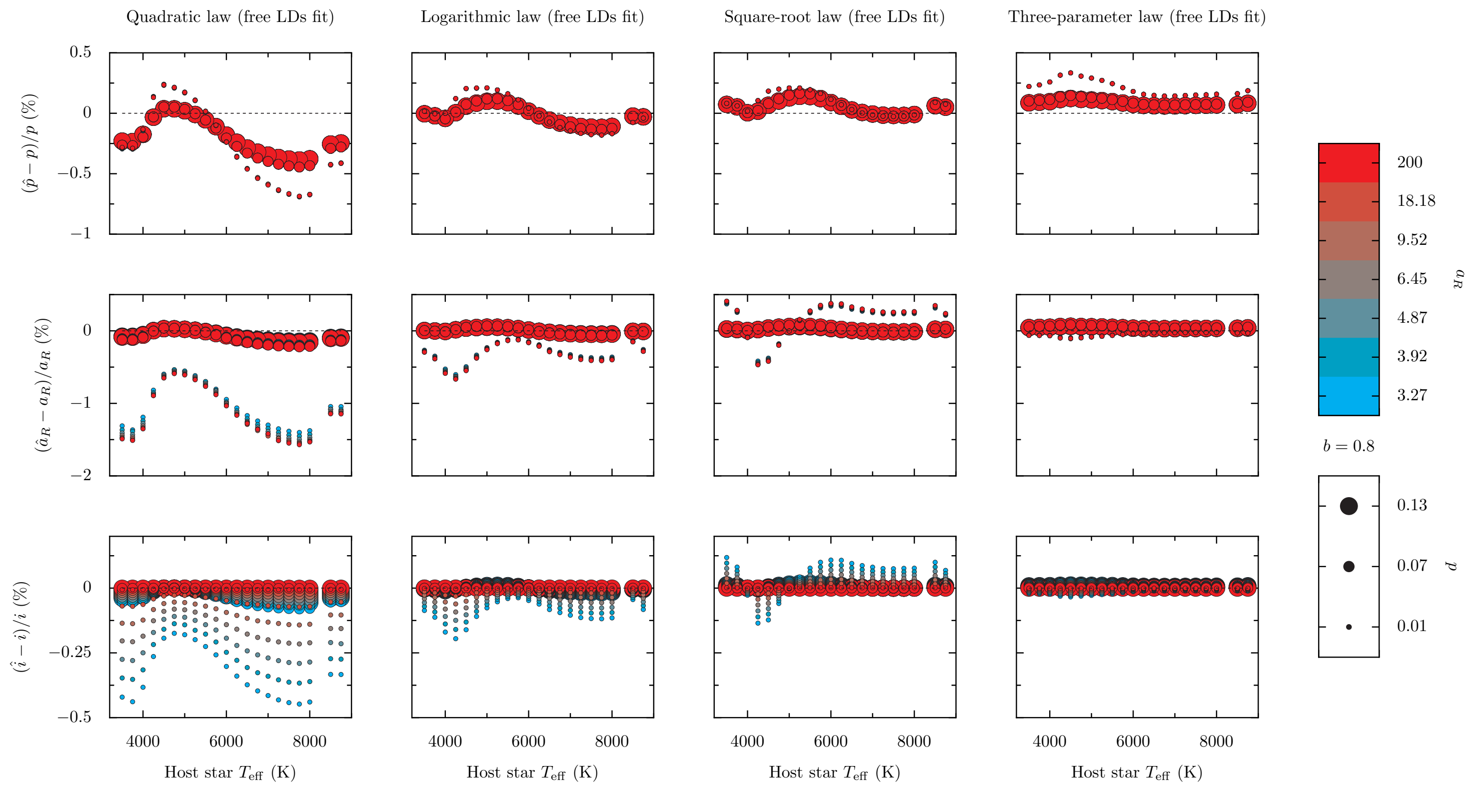

Figure 5. Results of fitting transit lightcurves as described in §3.1 with the limb-darkening coefficients as free parameters, but for high impact parameter transits ($b = 0.8$). The size of the points represent the input values of p, while their color represent the input values of a_R.