A Locally Weighted Metric for Measuring the Perceptual Quality of 3D Objects

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Abstract. The increasing use of 3D models in many areas lead us to think about the impact of the different distortions that can affect the 3D object during the rendering process. These deformations are usually evaluated using geometric metrics, which have not a good correlation with human judgment, while the visual perceptual quality of 3D models is necessary. In this context, a new full-reference metric denoted LWRMS is defined in this work. It can predict the distortion score between the original object and its damaged version without taking into account the constraint of connectivity. The proposed metric is defined in order to get a good correlation with the human visual perception coming from a subjective measurement. The numerical experiments are carried out on a known database LIRIS/EPFL General-Purpose database. The quantitative results show a good performance of the proposed metric in comparison with methods from the literature.

Keywords: Computer graphics · Shape processing · 3D quality metric · Objective measurement · Human visual perception · Image processing

1 Introduction

Currently, 3D models are created or acquired in various ways: 3D scanners, reconstruction from 2D images, etc. Therefore, these 3D data are presented in several forms such as point clouds and triangular meshes. These are the most used because of their algebraic properties and their integration in computer graphics. They are used in several application areas. The medical industry uses them for the analysis of organs to represent chemical components in a detailed way [10]. The architectural domain uses 3D meshes to model and to visualize buildings, bridges, etc. The automotive industry uses 3D meshes to represent new concepts and designs. 3D models also find their place in 3D printing, which is now enjoying considerable growth as well as in video games, cinema, fashion, reconstruction, etc. [1]. Other applications such as hospitalization and consumer 3D photography take advantage of 3D meshes [15]. In this context, it is obvious that the mass

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and exchanges of 3D meshes increase and lead to new challenges concerning the implementation of effective tools to manipulate, protect, compress, present and evaluate the final quality of 3D meshes while taking into account human vision. In this context, the paper presents a new 3D perceptual quality metric, denoted LWRMS (Locally Weighted Root Mean Square). It integrates the sensitivity of the human visual system to the level of details of 3D object. In fact, the weighting of details level allows the regions where the error is perceptually sensitive to be detected and on which the human visual attention is focused. This paper is organized as follows: Sect. 2 presents the state of the art, Sect. 3 describes the mathematical expression of the LWRMS and the perceptual properties which are integrated and Sect. 4 presents the results of experiments which are used to validate and to compare the performance of the LWRMS with the state of the art methods using the Pearson and Spearman correlations between the subjective and objective measurements.

2 Related Work

With the development of 3D scanners, large quantities of 3D objects are acquired and represented in the form of 3D triangular meshes. These are used in several applications taking into account the properties of the human visual system such as compression [17], medical imaging, restoration, watermarking, etc. A 3D model can be subjected to various treatments (smoothing, compression, watermarking, etc.) before being exposed to a human observer, which can introduce distortions that affect the visual rendering [13]. The first degradation that can affect 3D mesh during its acquisition is the noise of the sensor. Then, for optimal transmission, the mesh can be simplified using smoothing and compression for size and bandwidth reduction purposes. Also, for reasons of protection of copyright and intellectual property, a watermarking process can be applied to a mesh which can also degrade the geometry of the 3D object.

Therefore, assuming that one or more of the previously enumerated distortions are applied, an assessment of the perceptual quality becomes necessary to quantify the visual impact of these distortions on the geometry of the 3D model that will be presented to the final observer [21].

2.1 Subjective Versus Objective Metrics for 3D Models

Nowadays, 3D meshes represent an emerging content which are used in several fields and applications such as medical industry, automotive industry, 3D printing, video games and other more applications [14]. In this context, it is obvious that the quantity and the frequency of 3D meshes exchanges will increase in an exponential way. This leads to new challenges where the objective is to evaluate the quality of 3D models. A first approach to evaluate such models is done by collecting the quality scores (subjective measures) provided by human observers. However, this method is restrictive, slow, and inadequate for some applications [3]. An alternative approach can be done by using objective measures designed for this purpose. In the literature, these objective metrics are classified into three distinct categories: metrics with reference (the reference version of the degraded mesh is available) [5], metrics with reduced reference (partial information of the reference mesh and the degraded mesh are available) [12] and metrics without reference (no reference mesh information is available [24]. In this paper, the proposed metric LWRMS is classified as a full reference metric. Such metrics with reference or full reference can also be categorized in two classes: those integrating geometrical properties and those integrating human visual system properties [18].

The geometric metrics are not correlated with the human visual perception such as the Hausdorff distance (Hd) [2] or the root mean squared error (RMS) [6]. These metrics are widely used because of their simplicity and timeliness contrary to metrics that integrate human visual perception which are obviously more complex [8].

2.2 Integration of the Human Visual Perception in 3D Metrics

The metrics integrating the human visual system are well correlated with the subjective measures [8]. In this context, Karni and Gotsman [16] introduced the Geometric Laplacian measurement (GL) which allows to measure and compare the roughness of 3D mesh surfaces. This measure was later improved by Sorkine and the new version called GL2 [25]. Gelasca and al. propose a metric named 3DWPM based on the variations of global roughness [11]. Corsini et al. proposes an extension of the 3DWPM metric named 3DWPM2 in this version the roughness is calculated with the variance of the dihedral angles [7]. Lavoué et al. proposes an extension of the 2D SSIM metric to the 3D models [20]. In 2011 Lavoué et al. proposes an improvement of the MSDM metric named MSDM2 [18]. In this version, the multi-scale aspect is taken into account and a corresponding step between vertices is integrated into the metric pipeline to evaluate the quality of the 3D meshes without taking into account the connectivity constraint. To the best of our knowledge, the state-of-the-art methods do not take into account the level of details which represents an important property for the human visual perception.

3 The Proposed Metric

The proposed approach is firstly based on the Root Mean Square metric which calculates the deformation in a purely mathematical way [6]. Thereafter, the properties of the human visual system are integrated to obtain scores that can be correlated with the human vision. In the literature, it is proved that metrics integrating the human visual properties have good correlation scores with the subjective measures, such as the geometric metrics [19]. In addition, many studies prove that the distortion value depends on the level of details and the rendering condition of the 3D object [27]. Existing metrics do not take into account this level of details. This specificity is a very important property of the human visual perception [18]. If the distortion is located in area that contains a high level of details, the distortion is more significant for the HVS. To reduce this limitation, a new 3D metric is here defined for measuring the perceptual quality of different distortion types that can affect the geometry of 3D objects.

3.1 Mathematical Definition

A new perceptual quality metric, denoted Locally Weighted Root Mean Square (LWRMS), is here proposed. It is based on the geometric Root Mean Square error and integrates human visual system properties. The LWRMS is defined as follows:

$$LWRMS(A,B) = \frac{1}{D_{max}(A)} \times \frac{1}{M} \sum_{i=1}^{M} \frac{\left\| P_A^i - P_B^j \right\|_2^2}{d_{Locally}(P_A^i)} \tag{1}$$

where, A is the reference object and B its damaged version, $D_{max}(Z)$ represents the maximum distance between two points within the object Z, M is the number of vertices in the 3D object, $\|.\|_2$ represents the Euclidean norm, $P_.^i$ corresponds to the point coordinates of the i-th vertex, $d_{Locally}(P_A^i)$ is the distance between the point i and its neighbors which is calculated as follows:

$$d_{Locally}(P_A^i) = \frac{1}{N} \sum_{\substack{i=1 \\ j \in vois_i}}^{N} \left\| P_A^i - P_A^j \right\|_2 \tag{2}$$

N is the number of neighbors of the selected vertex i and $vois_i$ corresponds to the indices j such that P^i and P^j are connected by an edge. In the following, the properties of the LWRMS metric are presented.

3.2 Properties

Consistency with the Human Visual System. There are several supercities in the human visual system that can be integrated with mathematical measures to improve the visual quality assessment of 3D objects. The human visual system is very sensitive to the level of details [23] and rendering conditions of 3D objects [26]. The 3D models are mainly characterized by two elements: vertices and faces, the higher the number of vertices is, the higher the number of faces is and therefore the higher level of details is. The degradation that affects the 3D model generally modifies the vertex positions [9]. In an area containing a high level of details (close vertices and small faces) a local deformation is more significant to the human observer compared to an area with a low level of details.

Figure 1 shows that two vertices, with the same error deviation but located in different areas of details, do not represent the same visual difference while they have the same error value with a classical metric such as RMS. On the contrary, LWRMS takes into account the difference between two 3D models according to its level of details. This parameter is modeled with the average of the locally distance measured between each 3D vertex and its neighbor vertices. Therefore a low value means that the studied point is located in an area with an important level of details and the distortion will be more significant for the human observer.

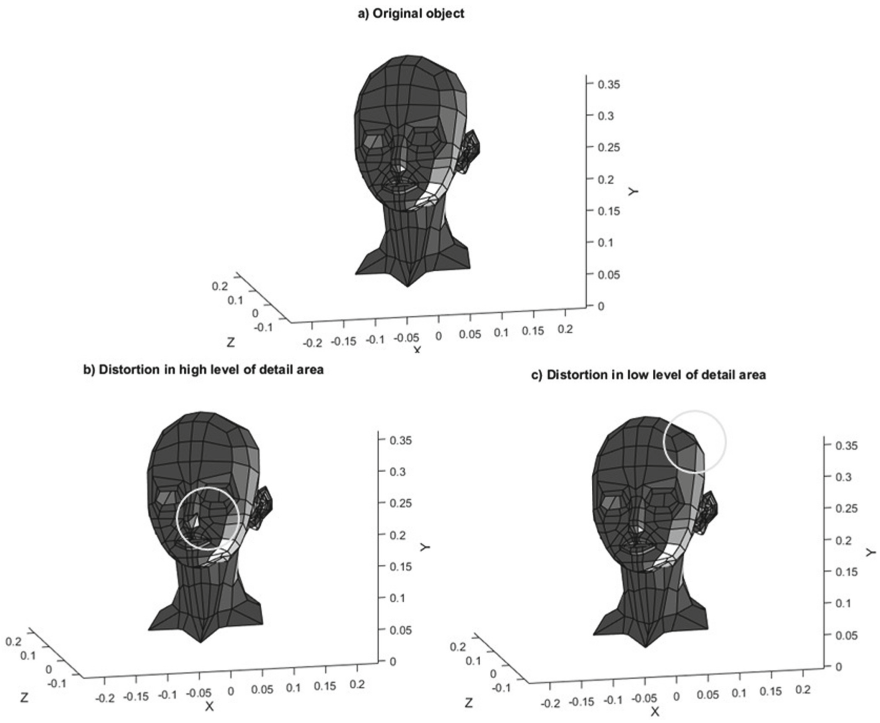

Fig. 1. Example of distortion applied on Head model in different local areas.

Figure 1a represent the reference model A. Figure 1b represents the distorted model B1 with a deformation in an area with a high level of details. Figure 1c represent a distorted model B2 with a deformation in an area with a low level of details.

We measured the distortion shown in Fig. 1 with the two measurements RMS and LWRMS, we obtained the following results: [LWRMS (A, B1) = **4.35** $\gg$ LWRMS (A,B2) =**1.30**] while [RMS(A, B1) = **0,0173** RMS(A,B2) = **0,0173**].

The obtained result shows that the proposed metric is sensitive to the level of details.

Scale Invariance. Three-dimensional models can be viewed via different screen sizes. In addition, once the 3D models are created, their appearance depends not only on the geometry, but also on the size of the object [4]. As an example, a deformation of 3 mm in a 3D object with a global size that equals 10 mm does not have the same perceptual importance as another of 3 mm in a 3D object with a global size that equals 100 mm. In this way, a perceptual quality metric should take into account the size of the 3D object as it has an impact on the human judgement as shown in Fig. 2.

We measured the distortion shown in Fig. 2 with the two measurements RMS and LWRMS, we obtained the following results: [LWRMS (a,b) = **1,3** LWRMS (c,d) =**1.3**] while [RMS(a,b) = **0,0173** RMS(c,d) = **0,0087**].

These results shows that the proposed metric takes into account the size of the object.

Quality evaluation metrics estimate the perceptual quality of a 3D model as it appears on the screen (Fig. 2), independently of its scale. The proposed LWRMS metric integrates this property by calculating the maximum distance between point pairs of the 3D object. This latter can be efficiently calculated by only analyzing points on the convex hull of the 3D model [22].

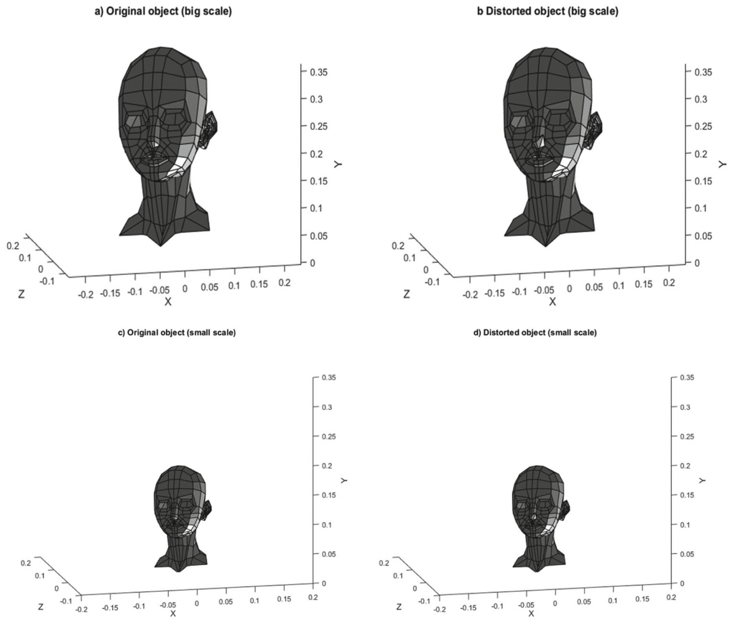

Fig. 2. Exemple of 3D object rendered with different scales. a and c represent the original object in different scales, b and d represent a distorted version with another scale (homothety ratio: 0.5).

Algorithmic Pipeline of the Proposed Metric LWRMS. The visual distortion score, using LWRMS, is computed in three steps:

- (Algorithm 1) A local distance is calculated for each vertex of the original model $d_{Locally}(P_A^i)$. It enables to know the level of details of the local area that include the selected point.
- (Algorithm 2) The maximal distance between pairs of points in the reference object is calculated to further get invariance to scaling.
- (Algorithm 3) The Euclidean distance between all corresponding 3D vertices $\left\| P_A^i - P_B^j \right\|_2^2$ in A and B objects is computed. The final score of the LWRMS is thereafter both normalized by $d_{Locally}(P_A^i)$ (integrating the level of details) and by the size of the object $D_{max}(A)$ (for scale invariance).

Algorithm 1: CalculateLocallyDistance (input: A)

```
Result: distance vector d_locally for each point in A
Length_edges(i) = 0 ;
Nb_edges(i) = 0;
for i ∈ A do
    for j ∈ A do
        if A(i) is_connected_to A(j) then
            Nb_edges(i) += 1;
            Length_edges(i) += length(edge(A(i),A(j)));
        else
            d_locally(i) = Length_edges(i) / Nb_edges(i);
        end
    end
end
```

Algorithm 2: CalculateDmax (input: A)

```
Result: Maximal distance between point pairs of A
CH_A = convex_hull(A) ;
D_max = 0;
for i ∈ CH_A do
    for j ∈ CH_A do
        D_max = max(D_max, dist(A(i), A(j));
    end
end
```

Algorithm 3: CalculateLWRMS (input: reference object A and distorted object B)

```
Result: Distance LWRMS between A and B
Dmax = CalculateDmax(A) ;
distAB = 0;
M = nb_points(A);
for i ∈ A do
    Dlocal = CalculateLocallyDistance(A(i));
    distAB += dist(A(i), B(i)) ∧2 / Dlocal;
end
LWRMS = (distAB / M) / Dmax;
```

4 Results and Discussion

4.1 Database and Experiments

To validate and compare the results of our LWRMS metric with state-of-the-art methods, the Liris/Epfl General-Purpose 3D database (subjectively evaluated) is used. It contains four reference objects [20]. These objects are affected by two types of distortions: additive noise and smoothing. These distortions are applied according to three degrees of intensity on different regions of the mesh: 1) uniformly on the surface of the mesh, 2) specifically on the rough or smooth zones of the mesh and 3) specifically on the zones of transitions between the rough zones and smooth areas. In total, twenty-two degraded 3D models of each reference are generated and evaluated by twelve human observers.

In this paper, scores are calculated by distortion type. The general LIRIS/EPFL database is divided into two parts: objects affected by a general noise distortion and secondly by a smoothing distortion.

4.2 Evaluation Criteria

The performance of the LWRMS is measured by two correlation coefficients: Spearman (Rs: Spearman Correlation Coefficient) and Pearson (Rp: Pearson Correlation Coefficient).

- Pearson Correlation Coefficient

This coefficient makes it possible to detect the presence or absence of a linear relationship between two continuous quantitative characters. To calculate this coefficient one must first calculate the covariance. Covariance is the average of the product of deviations from the mean.

$$Cov\left(X,Y\right) = \frac{1}{N}\ \sum_{i=1}^{N} \left(X_i - \overline{X}\right) \times\ \left(Y_i - \overline{Y}\right) \tag{3}$$

The linear correlation coefficient of two characters X and Y is equal to the covariance of X and Y divided by the product of standard deviations of X and Y.

$$Rp\,(X,Y) = \frac{Cov\,(X,Y)}{\sigma_X \times \sigma_Y} \tag{4}$$

This coefficient varies between −1 and +1; the higher the intensity of the linear relation is, the closer to +1 the value of the coefficient Rp is [26].

– Spearman Correlation Coefficient

The rank correlation coefficient (called the Spearman coefficient) examines whether there is a relationship between the rank of the observations for two feature vectors X and Y, which makes it possible to detect the existence of monotonic relations (increasing or decreasing), whatever either their precise shape (linear, exponential, power, ...). Spearman's coefficient is based on the study of the rank difference between the attributes of individuals for the two feature vectors X and Y. The correlation coefficient of Spearman's ranks is then calculated by:

$$Rs\,(X,Y) = 1 - \frac{\sum_{i=1}^{n} \left(Rg(X_i) - Rg(Y_i)\right)^2}{n \times (n^2 - 1)} \tag{5}$$

Where $Rg\,(X_i)$ represents the rank of each score of the studied metric, $Rg(Y_i)$ represents the rank of each score of the MOS et n is the total number of values. This coefficient varies between −1 and 1. The higher the monotony of the functional relation is, the closer to +1 the value of the coefficient Rs is [26].

4.3 Quantitative Results and Discussion

To compare the proposed method with state-of-the-art methods, we used the Liris/Ep database that contains eighty-eight objects. Distorted objects are affected by two types of distortions: additive noise and smoothing. The performance of LWRMS is measured by Spearman and Pearson Correlation. The correlation values of different metrics are presented in Table 1.

Looking at Table 1, several observations can be done. Firstly, regarding the geometrical metrics, the RMS gives better results over all objects and also on the whole database comparing to Hd metric. Secondly, regarding metrics that integrate the human visual system, the LWRMS outperform the state-of-the-art metrics with a good performance for each 3D object separately. Also LWRMS has a very good correlation score over the whole database. LWRMS presents a competitive score with the GL1, GL2 and MSDM metrics on the Armadillo and Dyno objects respectively (Rs = **81**%, Rp = **87**%) and (Rs = **85**%, Rp = **79**%). On the other hand, GL2 present a good linear correlation on Venus model with (Rs = **92**%). GL1 also presents a good result in term of the Pearson correlation with Rocker model (Rp = **90**%). Finally, looking at the whole database, LWRMS presents the best performance with Rs = **86**% and Rp = **85**%, compared to all other metrics (geometric and HVS metrics).

Table 1. Spearman (Rs) and Pearson (Rp) correlation values between Mean Opinion Scores and metrics values applied on LIRIS/EPFL General-purpose database.

	Armadillo		Venus		Dyno		Rocker		Whole database	
	Rs	RP	Rs	RP	Rs	RP	Rs	RP	Rs	RP
Hd [5]	0.74	0.61	0.78	0.74	0.70	0.64	0.74	0.78	0.74	0.69
RMS [6]	0.80	0.80	0.89	0.86	0.83	0.72	0.89	0.91	0.85	0.82
3DWPM1 [25]	0.77	0.56	0.77	0.65	0.56	0.50	0.87	0.76	0.74	0.62
3DWPM2 [25]	0.77	0.61	0.84	0.63	0.69	0.55	0.86	0.81	0.79	0.65
GL1 [11]	0.77	0.78	0.91	0.84	0.83	0.70	0.90	**0.90**	0.85	0.81
GL2 [7]	0.77	0.77	**0.92**	0. 81	0.82	0.67	0.90	0.89	0.85	0.78
MSDM [20]	0.81	0.71	0.86	0.80	0.79	0.69	0.84	0.86	0.82	0.77
LWRMS	**0.81**	**0.87**	0.89	**0.88**	**0.85**	**0.79**	**0.91**	0.86	**0.86**	**0.85**

Geometric measurements can have scores correlated with human vision when applied to deformed 3D objects with the same type of distortion. These measurements fail when applied to a set of models generated from a series of reference objects that include different types of distortion, the task becomes much more difficult and this is due to the absence of human visual properties, which is verified by the LWRMS measurement.

5 Conclusion and Prospects

In this paper a new full reference perceptual quality metric for measuring the visual quality of 3D models is proposed. This new metric, denoted LWRMS, compares the content of the information of a reference 3D mesh with its damaged version. LWRMS is based on the RMS geometrical metric by integrating two properties of the human visual system: the sensitivity to the level of details and the scale invariance to provide a score quantifying the visual similarity between two 3D models. Experimental results have shown a good correlation between LWRMS scores and the quality scores provided by human observers (MOS). Compared to the state-of-the-art methods, the proposed metric shows the best performance. Our future work will focus on integrating the masking effect that affects vertices and that participate in the visual measurement of 3D objects.

References

1. Agathos, A., Azariadis, P.: 3D reconstruction of skeletal mesh models and human foot biomodel generation using semantic parametric-based deformation. Int. J. Comput. Appl. **42**(2), 127–140 (2020)
2. Aspert, N., Santa-Cruz, D., Ebrahimi, T.: Mesh: Measuring errors between surfaces using the hausdorff distance. In: Proceedings. IEEE International Conference on Multimedia and Expo, vol. 1, pp. 705–708. IEEE (2002)

3. Booth, J., Roussos, A., Ponniah, A., Dunaway, D., Zafeiriou, S.: Large scale 3D morphable models. Int. J. Comput. Vision **126**(2–4), 233–254 (2018)
4. Bulbul, A., Capin, T., Lavoué, G., Preda, M.: Assessing visual quality of 3-D polygonal models. IEEE Signal Process. Mag. **28**(6), 80–90 (2011)
5. Chen, M.J., Su, C.C., Kwon, D.K., Cormack, L.K., Bovik, A.C.: Full-reference quality assessment of stereopairs accounting for rivalry. Signal Process. Image Commun. **28**(9), 1143–1155 (2013)
6. Cignoni, P., Rocchini, C., Scopigno, R.: Metro: measuring error on simplified surfaces. In: Computer Graphics Forum, vol. 17, pp. 167–174. Wiley Online Library (1998)
7. Corsini, M., Gelasca, E.D., Ebrahimi, T., Barni, M.: Watermarked 3-D mesh quality assessment. IEEE Trans. Multimedia **9**(2), 247–256 (2007)
8. Elloumi, N., Kacem, H.L.H., Bouhlel, M.S.: The efficiency of perceptual quality metrics 3D meshes based on the human visual system. In: International Workshop Soft Computing Applications, pp. 497–508. Springer (2016)
9. Elloumi, N., Kacem, H.L.H., Bouhlel, M.S.: Quality metric of 3d models using weber's law. In: Eleventh International Conference on Machine Vision (ICMV 2018), vol. 11041, p. 110410B. International Society for Optics and Photonics (2019)
10. Elloumi, N., Kacem, H.L.H., Dey, N., Ashour, A.S., Bouhlel, M.S.: Perceptual metrics quality: Comparative study for 3d static meshes. Int. J. Serv. Sci. Manage. Eng. Technol (IJSSMET) **8**(1), 63–80 (2017)
11. Gelasca, E.D., Ebrahimi, T., Corsini, M., Barni, M.: Objective evaluation of the perceptual quality of 3d watermarking. In: IEEE International Conference on Image Processing 2005, vol. 1, pp. I–241. IEEE (2005)
12. Hewage, C.T., Martini, M.G.: Reduced-reference quality assessment for 3D video compression and transmission. IEEE Trans. Consum. Electron. **57**(3), 1185–1193 (2011)
13. Itti, L., Koch, C., Niebur, E.: A model of saliency-based visual attention for rapid scene analysis. IEEE Trans. Pattern Anal. Mach. Intell. **11**, 1254–1259 (1998)
14. Izadi, S., Kim, D., Hilliges, O., Molyneaux, D., Newcombe, R., Kohli, P., Shotton, J., Hodges, S., Freeman, D., Davison, A., et al.: Kinectfusion: real-time 3d reconstruction and interaction using a moving depth camera. In: Proceedings of the 24th Annual ACM Symposium on User Interface Software and Technology, pp. 559–568. ACM (2011)
15. JG, T., Wolff, A., Torres, J., My, H.T.: Usability evaluation of the interactive 3D virtual reality cultural heritage museum display: Fountain of the lions software application. Int. J. Eng. Technol. **7**(2.28), 95–99 (2018)
16. Karni, Z., Gotsman, C.: Spectral compression of mesh geometry. In: Proceedings of the 27th Annual Conference on Computer Graphics and Interactive Techniques, pp. 279–286. ACM Press/Addison-Wesley Publishing Co. (2000)
17. Kaur, A., Sidhu, J.S., Bhullar, J.S.: Adaptive deblocking technique based on separate modes for removing compression effects in jpeg coded images. International Journal of Computers and Applications pp. 1–13 (2019)
18. Lavoué, G.: A multiscale metric for 3d mesh visual quality assessment. In: Computer Graphics Forum. vol. 30, pp. 1427–1437. Wiley Online Library (2011)
19. Lavoué, G., Corsini, M.: A comparison of perceptually-based metrics for objective evaluation of geometry processing. IEEE Trans. Multimedia **12**(7), 636–649 (2010)

20. Lavoué, G., Gelasca, E.D., Dupont, F., Baskurt, A., Ebrahimi, T.: Perceptually driven 3d distance metrics with application to watermarking. In: Applications of Digital Image Processing XXIX. vol. 6312, p. 63120L. International Society for Optics and Photonics (2006)
21. McNamara, A., Mania, K., Banks, M., Healey, C.: Perceptually-motivated graphics, visualization and 3d displays. In: ACM SIGGRAPH 2010 Courses. p. 7. ACM (2010)
22. Pérez-Lantero, P.: Area and perimeter of the convex hull of stochastic points. Comput. J. **59**(8), 1144–1154 (2016)
23. Reddy, M.: Perceptually optimized 3d graphics. IEEE Comput. Graphics Appl. **21**(5), 68–75 (2001)
24. Ryu, S., Sohn, K.: No-reference quality assessment for stereoscopic images based on binocular quality perception. IEEE Trans. Circuits Syst. Video Technol. **24**(4), 591–602 (2013)
25. Sorkine, O., Cohen-Or, D., Toledo, S.: High-pass quantization for mesh encoding. In: Symposium on Geometry Processing. vol. 42 (2003)
26. Vasa, L., Skala, V.: A perception correlated comparison method for dynamic meshes. IEEE Trans. Visual Comput. Graphics **17**(2), 220–230 (2010)
27. Zhu, Q., Zhao, J., Du, Z., Zhang, Y.: Quantitative analysis of discrete 3d geometrical detail levels based on perceptual metric. Computers & Graphics **34**(1), 55–65 (2010)