Cogito? Ergo? Sum?

An Onto-Logical Quandary Free of Existence Assumptions

"To think and to be are the same"[1]. The notion that intellection and existence are inseparably intertwined concepts dates back to the dawn of philosophy. Despite notable variations on this theme such as *"If I am mistaken, I exist"*[2] and *"the intellect knows itself through its acts"*[3], the most memorable formulation remains the Cartesian *"Cogito, Ergo Sum."*[4] *(I think, therefore I am)* – henceforth referred to as *"The Cogito"*. Though this initial phrasing implies an inferential structure, thus suggesting a logical step from thinking to existing, its interpretation as a formal argument would go down in philosophical history as a highly controversial topic.

One important point of contention concerns the ontological commitment of the singular term "I" in the premise, which appears to assume rather than prove existence. Gassendi claimed that this existential implication rendered the argument circular, thereby diminishing the importance of the intellectual predicate *"Cogito"*. He argued that Descartes' formulation allowed any predicate in the antecedent to grant validity to the consequent, famously exclaiming *"Ambulo, Ergo Sum" (I walk, therefore I am)*[5]. This issue becomes most apparent when we attempt to formalize this inferential version of the Cogito in classical and negative free logic.

Another reoccurring point of contention – although hardly dissimilar – relates to the assumption that it is an "I" that is doing the thinking. Some have argued that thinking does not necessarily entail a unified self, and this aspect has been criticized at length by successors of Descartes. Lichtenberg claimed that *"an unjustified I was smuggled in the cogito"*, suggesting the alternative formulation *"Es denkt" (It thinks)*[6]. Sartre similarly held the belief that selfhood is not a precondition for thought[7]. Other notable criticisms were presented by Kierkegaard[8] and Nietzsche[9] respectively. We will examine whether this criticism can be formally addressed by Russellian definite descriptions or a more central role of predicates in second-order free logic.

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[1] Parmenides, On Nature, Fragment 3, in Early Greek Philosophy, trans. Jonathan Barnes (London: Penguin Classics, 2001), 140.

[2] Augustine, On the Trinity, trans. Stephen McKenna (Washington, D.C.: Catholic University of America Press, 1963), XV.12.21.

[3] Thomas Aquinas, Summa Theologica, trans. Fathers of the English Dominican Province (New York: Benziger Bros., 1947), I, Q.87, A.1.

[4] René Descartes, Discourse on the Method, trans. Ian Maclean (Oxford: Oxford University Press, 2006), Part IV.

[6] Georg Christoph Lichtenberg, The Waste Books, trans. R. J. Hollingdale (New York: New York Review Books, 2000), Notebook E, Aphorism 177. [7] Søren Kierkegaard, Philosophical Fragments, trans. Howard V. Hong and Edna H. Hong (Princeton: Princeton University Press, 1985), Chapter 1.

[8] Jean-Paul Sartre, The Transcendence of the Ego: An Existentialist Theory of Consciousness, trans. Forrest Williams and Robert Kirkpatrick (New York: Noonday Press, 1957), Section II.

[9] Friedrich Nietzsche, Beyond Good and Evil, trans. Judith Norman (Cambridge: Cambridge University Press, 2002), §16.

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In response to his contemporaries' objections, Descartes reintroduced the idea in the form of *"Ego sum, ego existo" (I am, I exist)* and *"Res Cogitans"* (*I am a thinking thing*)[10], shifting his narrative towards the Cogito being an intuition of indubitable certainty rather than an argument. Despite this, logicians such as Jaako Hintikka and Gordon Brittan have given the inferential Cogito a go in free logics, arriving at different conclusions for its justification as an argument. This essay's thesis is that despite appearances, both logicians in fact share the same insight when validating the Cogito, namely that self-reference is at the core of the Cogito's necessity.

Classical Logic

Our initial formal framework is standard FOL with Tarskian semantics[11]

Examining the Cogito in its inferential formulation, that is, *"Cogito, Ergo Sum"*, the closest thing to a one-to-one formalization in our language yields $C(a) \rightarrow S(a)$. However, since classical logic treats existence not as an ordinary predicate, but as an implicit assumption for singular terms, and an explication of Quine's dictum, "To be is to be the value of a bound variable"[12] for variables, the statement $C(a) \rightarrow (\exists x)\ (x = a)$ resembles the Cogito more closely.

These very same existential presuppositions render the Cogito non-inferential, in the sense that its provability rests solely on them, and not on the nature of the intellection predicate, as Gassendi showed. Whether we take the antecedent to be true or not is of little importance. One may choose any interpretation of the truth value of $C(a)$, since $a = a$ is true due to the reflexivity of the identity relation. Due to existential generalization being a valid inference rule, it follows that $(\exists x)\ (x = a)$. In other words, the Cogito proves to be tautological.

In classical logic, non-existent terms are generally not well-formed. Therefore, not assuming the antecedent's existence is not an obvious option. Russell's Theory of Definite Descriptions is the most well-known attempt at addressing non-existent terms within classical logic[13]. While it decouples existence from well-formedness, it does not decouple existence assumptions from truth, which always renders the antecedent false and thus trivializes the inference anew:

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[10] René Descartes, Meditations on First Philosophy, in The Philosophical Writings of Descartes, vol. 2, trans. John Cottingham, Robert Stoothoff, and Dugald Murdoch (Cambridge: Cambridge University Press, 1984), 17, 19.

[11] Peter Smith, An Introduction to Formal Logic (Cambridge: Cambridge University Press, 2013).

[12] Quine, Willard Van Orman. "On What There Is." In From a Logical Point of View, 1–19. Cambridge, MA: Harvard University Press, 1953.

[13] "On Denoting." Mind 14, no. 56 (October 1905): 479–493. Reprinted in Logic and Knowledge: Essays 1901–1950, edited by Robert C. Marsh, 41–56. London: George Allen & Unwin, 1956.

Definite Descriptions

The Cogito interpreted not as "*I think*", but "*the thinking*", or "*the entity that thinks*", does not only allow for a syntactically permissible formalization of the inference without the need to presume existence in the premise, addressing the first criticism we presented. It also addresses the second criticism by ensuring that the inference does not illicitly assume a distinct subject, remaining within the bounds of what is strictly given: the occurrence of thought itself.

Whether we adopt the iota operator Russell introduced in Principia Mathematica[14], formalizing the Cogito as $C(\iota x C(x)) \rightarrow (\exists y)\ (y=\iota x C(x))$ (where $\iota x C(x)$ is whatever refers to the entity that thinks), or the earlier formulation using quantifiers, namely $(\exists x)\ (C(x) \wedge (\forall y)(C(y) \rightarrow x=y))$ for individual and $(\exists x)(C(x))$ for generic definite descriptions, the same problem persists: definite descriptions are based on quantification of variables, which entails existence. If we adopt Descartes' position of radical doubt, then individual definite descriptions align best with his assumption that only the unique thinking referent is indubitable. Their caveat is that statements containing them can only be true if the predicate forming their basis is true of exactly one existent[15]. This means that the antecedent of the inferential Cogito can still not be true without existential assumptions. As we can see, Russell did not fully solve the issue at hand, he merely relocated it by creating his own meta-condition, where truth remains dependent on existence.

Accepting that more instances of thinking can occur simultaneously reveals similar implicit existential presuppositions. Generic definite descriptions permitting an entire domain of thinking things to satisfy the antecedent predicate proves equally problematic, since existential presuppositions persist, albeit in a different form. For if the domain of thinking things happens to be empty, then the description fails to refer, rendering the antecedent false and collapsing the inferential structure of the Cogito. In this sense, Russell does not weaken classical existence. He preserves the existential weight of quantifiers, merely ensuring syntactical well formedness and semantical coherence, thus quarantining non-existence inside classical logic.

While definite descriptions do not validate the inference of the Cogito, they do manage to address multiple aspects of its nature in an existentially restrictive formal environment. Though we will not return to them in the free logics part – more pressing matters await us there – they can be used as effectively in free logic to capture the essence of the Cogito, among other things.

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[14] Whitehead, Alfred North, and Bertrand Russell. Principia Mathematica, 2nd ed. (Cambridge: Cambridge University Press, 1927), Vol. I, ✶14.

[15] Karel Lambert. Free Logics: Their Foundations, Character, and Some Applications Thereof (Sankt Augustin: Academia Verlag, 1997), 97

Hintikka expressed similar concerns regarding existential presuppositions and commitments[16]. He, too, addressed the issue of linguistically meaningful terms like "Homer" failing to refer in classical logic, noting that Russell's theory of definite descriptions, while attempting to resolve reference issues, ultimately preserved the existential assumptions of traditional quantification. Additionally, he demonstrated that the Quinean framework presupposes existence rather than proving it, making it inadequate as a foundation for quantification. He subsequently proposed an alternative approach to quantification that explicitly rejects existential presuppositions, akin to an early logic free of existence assumptions with respect to their general and singular terms.

While Hintikka's treatment remained semantic, Lambert extended these insights by providing a syntactic foundation[17]. He observed that even seemingly trivial identity statements such as "everything identical with Pegasus is Pegasus" can illegitimately imply existence in Hintikka's treatment when classical rules are applied. Instantiating $(\forall x)\ (x=p \rightarrow G(x))$ with p (for Pegasus) yields $(p=p \rightarrow G(p))$, whereupon the trivial $p = p$ and Modus Ponens give us $G(p)$ and the introduction of existential generalization results in $(\exists x)\ (x=p \wedge G(x))$. In sum, $(\forall x)\ (x=p \rightarrow G(x)) \rightarrow (\exists x)\ (x=p \wedge G(x))$. In response, $E!$ (E-shriek) was introduced to explicitly express existence in the total absence of presuppositions for singular terms, laying the groundwork for an official free logic framework and marking a clear break from traditional quantification. Universal instantiation thus becomes $(\forall x)\ (G(x) \wedge E!p \rightarrow G(p))$, while existential generalization becomes $G(p) \wedge E!(p) \rightarrow (\exists x)\ (x = p \wedge G(p))$. It is now time to examine the Cogito in free logics.

Free Logics

The following segment assumes an extension of our language by the symbol $E!$[18]. Using this existence predicate, the Cogito is expressed as $C(a) \rightarrow E!(a)$, which is equivalent to the quantificational formulation $C(a) \rightarrow (\exists x)\ (x = a)$. Out of the three prominent types of free logics, neutral free logics are of the least interest to our inquiry, since the fact that all statements containing at least one singular term not referring to an existent (except, perhaps, for statements of the type $E!(a)$) are truthvalueless[19], means that the antecedent cannot be true.

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[16] Hintikka, Jaakko. "Existential Presuppositions and Existential Commitments." The Journal of Philosophy 56, no. 3 (January 29, 1959): 125–137.

[17] Karel Lambert. "Existential Import Revisited." Notre Dame Journal of Formal Logic 4, no. 4 (October 1963): 288–294.

[18] Karel Lambert. Free Logics: Their Foundations, Character, and Some Applications Thereof (Sankt Augustin: Academia Verlag, 1997), 39.

[19] Karel Lambert. Free Logics: Their Foundations, Character, and Some Applications Thereof (Sankt Augustin: Academia Verlag, 1997), 62.

Negative Free Logics on the other hand consider reference a sufficient condition for existence. In this regard, they are very similar to classical logic. In an axiomatization[20] by Karel Lambert, "*A(a/x) → E!(a), if A(a/x) is a simple statement, x is free in A*" annuls any prospect of the Cogito not being a tautology. This demonstrates that their main function is to formulate existence assumptions explicitly, not to break away from said assumptions, which is also reflected in the semantic condition "$V_M(E!(a)) = T$ *just in case f(a) is defined, otherwise* $V_M(E!(a)) = F$ ".

This leaves us, then, with positive free logics. While their axiomatic treatment of variables is identical, they allow for non-existent terms to refer and therefore to be imbedded in true statements, foregoing the above-mentioned axiom of negative free logics[21]. In reference to his criticism on the existential generalization in identity statements, Lambert's axiomatization swaps $(\forall x)\ (x=x)$ from negative free logics for $a=a$ in positive free logics, now that referential terms are not existentially committed. What might however already be clear by looking at these axioms, is that there is no way to ever conclude that something exists without explicitly assuming that it does. This ends up rendering the Cogito either trivial, if existence is presumed in the premise, or simply non-valid if it is not, for it cannot be inferred in this system.

This becomes even clearer when we look at the two most prominent semantic structures: In Meinongian Inner-Domain-Outer-Domain semantics[22], the only condition for existence is "$V_M(E!(a)) = T$ *iff* $f(a) \in D_I$, *otherwise* $V_M(E!(a)) = F$ ". Unless we know for certain that our thinking thing is an element of the inner domain, meaning the set of existent objects, the Cogito's consequent will be false. Knowing the Cogito to exist would again be tautological. In Bencivengian supervaluational semantics[23], the clause "$V_M(E!(a)) = F$ *iff f(a) is undefined* " (1st valuation function) renders our consequent unequivocally false, regardless of the antecedent. If, just for the sake of it, we also examine our antecedent's truth value, we find out the following: arguing via case distinction and assuming first that the thinking thing exists, the Cogito is tautological. Assuming that it does not, the supervaluational thought experiment (3rd valuation function) cannot produce an imagined referent, since in Descartes' radical doubt, there could only possibly be oneself. Even if it did, the best outcome we could hope for is that the antecedent be classified as truthvalueless. Either way, the antecedent remains non-true.

[20] Karel Lambert. Free Logics: Their Foundations, Character, and Some Applications Thereof (Sankt Augustin: Academia Verlag, 1997), 85.

[21] Karel Lambert. Free Logics: Their Foundations, Character, and Some Applications Thereof (Sankt Augustin: Academia Verlag, 1997), 41.

[22] Karel Lambert. Free Logics: Their Foundations, Character, and Some Applications Thereof (Sankt Augustin: Academia Verlag, 1997), 65.

[23] Karel Lambert. Free Logics: Their Foundations, Character, and Some Applications Thereof (Sankt Augustin: Academia Verlag, 1997), 71.

While free logics have undoubtably provided a sharper formulation of the inferential Cogito, a certain inescapable dichotomy seems to have arisen. Whenever the existence of referents was guaranteed, either implicitly in the case of classical logics, or explicitly in the case of negative free logic, the Cogito has collapsed into tautology. When existence and reference are separated in positive free logics, the Cogito becomes invalid. It begs the question, if there can ever be an in-between to those two positions, and if so, whether this in-between can be postulated within the confounds of formal logic or requires an auxiliary conceptual framework to be conceived.

For Hintikka, it was clear that the latter is the case. Arguing against a purely logical reading of the Cogito, he claims that Descartes himself was fully aware of the above presented problems leading to an unsatisfactory dichotomy, but did not yet possess the necessary insights to validate the Cogito as a performative speech act, instead retreating to treating the Cogito as self-evident. In his seminal paper, "*Cogito, Ergo Sum: Inference or Performance?*"[24], Hintikka introduces the concept of existential inconsistency that arises when self-referential acts of speech contain a contradiction between the nature of their contents and their performer. For example, sentences of the type "*Descartes does not exist*" are not in and of themselves logically invalid. If, however, it so happens that Descartes himself were to utter this sentence, the result would be absurd. One can then reduce ad absurdum, that "*Descartes*" must refer to an existent entity.

This, Hintikka argues, is why the Cogito should be understood as a performance, rather than an inference. Juxtaposing inconsistent sentences, whose enactment (statement) leads to absurdity and, in principle, self-defeat – akin to Gödel sentences[25] and the Parmenidean paradox – with their negation ("*It is not the case that I do not exist*"), he uses the term "*self verifying*", which is akin to Henkin sentences, for the latter. The simplest example of such a sentence (removing the double negation of the previous sentence) is a revised formulation of the Cogito that Descartes himself proposed, namely "*Ego sum, Ego Existo*" ("*I am, I exist*"), bringing the argument full circle. Although our antecedent is not necessary for the self verification of this statement, it does capture that the essence of the prerequisite of speech acts is intellect (for, even though "public" speech acts might require more than intellect depending on one's ontological view, it is sufficient for a "private" thought act such as Descartes' Cogito).

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[24] Hintikka, Jaakko. 1962. "Cogito, Ergo Sum: Inference or Performance?" The Philosophical Review 71, no. 1 (January): 3–32.

[25] Douglas R. Hofstadter, "Gödel, Escher, Bach: An Eternal Golden Braid", 20th Anniversary ed. (New York: Basic Books, 1999), 541.

One possible objection to Hintikka's performative reading is that it commits a category mistake: it conflates the success of an utterance with the ontological status of the subject. Similarly to Ryle's critique that Descartes treats mental phenomena such as thinking as though they belong to the category of physical objects[26], it can be argued that performative self verification only demonstrates the success of an utterance, without demonstrating the existence of a thinker as a metaphysical entity. Hintikka acknowledged this critique in his follow-up, "*Cogito, Ergo Quis Est*?" ("*I think, therefore who is?*"), where he contemplates what kind of entity Descartes might hope to demonstrate the existence of. While Hintikka agrees with Quine that "*There is no entity without identity*"[27], he also recognizes the merit of the Ryleian criticism, suggesting that different modes of reference may entail different criteria of identity.

In the case of the Cogito, Hintikka sides with many of Descartes' critics, arguing that Descartes assumes too much when he claims that the Cogito establishes the existence of a unified thinker. Instead, what the Cogito truly secures is not a fixed metaphysical self, but the existence of a standpoint from which thinking occurs. This is formally achieved by introducing perspectival criteria of identification with the quantifiers E and A, as well as a context-dependent indexical expression I instead of a singular term a (or a definite description for that matter). While recognizing that integrating different perspectival frameworks into a public one is a daunting task, Hintikka suggests the following formulation $C(I) \rightarrow (Ex)\,(x = I)$, which is equivalent to $C(I) \rightarrow (E!I)$. Kaplan's Theory of Indexicals[28] might especially benefit from a logical framework free of existence assumptions, though such an attempt has not yet been documented.

In summary, Hintikka addresses both central criticisms of the Cogito: it being a tautology and the nature of the thinker being overstated. First, he takes inspiration from Speech Act Theory to defend Descartes' intuition that the act of thinking does validate the existence of the thinker, and then from Pragmatics to conclude that the thinker is – at least contextually – well-defined.

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[26] Ryle, Gilbert. "The Concept of Mind". Chicago: University of Chicago Press, 1949.

[27] Hintikka, Jaakko. "Cogito, Ergo Quis Est?" Revue Internationale de Philosophie 50, no. 195 (1996): 5–21, 9.

[28] Kaplan, David. "Demonstratives." In Themes from Kaplan, edited by Joseph Almog, John Perry, and Howard Wettstein, 481–563. New York: Oxford University Press, 1989.

While Hintikka's views on the Cogito have been widely influential, Brittan argued that the Cogito could still be validated as a purely logical argument within free logic[29]. His line of reasoning hinges on two central claims: first, the antecedent ("*I think*") is necessarily true by virtue of its nature in an alethic, perhaps modal sense and second, the consequent ("*I exist*") always follows axiomatically, since Brittan choses to abide by the rules of negative free logic.

Brittan agrees with Descartes' response to Gassendi, that the truth of the proposition "*I think*" – contrary to propositions such as "*I walk*" – does not rest in one way or the other on inferences. The complementary principles he uses to motivate this are self-intimation and incorrigibility. He does not attempt to justify them, asserting them as foundational epistemic axioms instead. Their exclusivity to intellection is based on the Cartesian metaphysics of mind-body-dualism[30].

Self-intimation: "If I am thinking, then necessarily I believe that I am thinking"
$T(I) \rightarrow B_I(T(I))$
Incorrigibility: "If I believe that I am thinking, then necessarily I am thinking"
$B_I(T(I)) \rightarrow T(I)$

Despite the fact that Brittan uses these principles to defend the necessary truth of the premise, while Hintikka uses the principle of performativity to defend the necessary validity of the Cogito, both arguments share an underlying similarity as well as restriction at the heart of their justification: self-reference. Just as the statement "*Descartes does not exist*" is only self defeating if it is uttered or performed by Descartes himself, the statement "*Diego is thinking*" can only then be necessarily inferred, if Diego is the one holding the belief that he is thinking.

While Brittan does not feel the need to further formalize his line of thought, alethic modal standard free logic models with Kripke semantics that can accommodate his thoughts exist[31]. We will shortly discuss how existence might arise in self-referential systems in further detail. Assuming that the Cogito's premise is necessarily true, Brittan now turns to negative free logic. Seemingly without any motivation for it, other than it rendering the inferential Cogito valid, since the antecedent is always true, he only needs to call upon the axiom $A(a/x) \rightarrow E!a$.

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[29] Brittan, Gordon G., Jr. "The Cartesian Cogitos." In Existence and Explanation, edited by Wolfgang Spohn et al., 25–38. Dordrecht: Kluwer Academic Publishers, 1991.

[30] The Philosophical Writings of Descartes, Volume II". Translated by John Cottingham, Robert Stoothoff, and Dugald Murdoch. Cambridge: Cambridge University Press, 1984. 100-101.

[31] Nolt, John. "Free Logic." The Stanford Encyclopedia of Philosophy. Edited by Edward N. Zalta. Fall 2021 Edition. Stanford, CA: Metaphysics Research Lab, Stanford University, 2021, 33.

Brittan mentions an alternative argument that Descartes himself described, which might justify the entire inference at one fell swoop. However, he distances himself from it, since it relies on higher-order logics and predicate quantification, leading to their own set of existential claims.

While Hintikka's dichotomy holds true in first-order logic devoid of existential presuppositions for singular terms, formalizing the Cogito in second-order logic with existential import for predicates and metaphysically justifying that predicates cannot be unsatisfied could render that dichotomy artificial. The latter point can be summarized by the cartesian "non-entis" principle:

$(\forall x)\ (\neg E!(x) \rightarrow (\forall C)\ (\neg C(x)))$ (What does not exist, has no properties)

Assuming that the Leibnizian principle of identity holds, namely:

$(\forall x)(\forall y)\ ((\forall P)\ (P(x) \leftrightarrow P(y)) \rightarrow x=y)$ (Objects are entirely described by their properties),

then the location of meaning is predicates, not objects. In Aristotelian metaphysics – the basis of Descartes' arguments – objects come ontologically before their properties[32]. A property cannot be empty, it must be the property of something. If this can be considered adequate justification of using the "non-entis" principle, the contraposition of its formalization yields:

$(\forall x)\ ((\exists C)\ (C(x)) \rightarrow E!(x))$ (What has properties, does exist)

For the Cogito, this means that at least someone needs to be thinking, if thinking is conceivable. It is therefore not so much the thinker that necessitates the existence of a thinking object, as it is the "thinking" itself. This line of argumentation may also better align with criticisms of Descartes assuming the nature of the thinking object. I recognize that the very same asymmetry required between singular terms and predicates to justify the latter's existence is an important part of Brittan's concern with quantifying predicates in the same manner as singular terms. Despite this, recent work on higher-order free logic has shown that not only does free logic provide the tools to handle non-existential quantification over predicates, but it also preserves intensional reasoning without collapsing into modal necessitism or paradox, possibly allowing self-referential structures like the Cogito to be modeled without presupposing the existence of the entity of the thinker and even integrating the "non-entis" with Brittan's modal argument[33].

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[32] Aristotle. "Categories", trans. J. L. Ackrill, in The Complete Works of Aristotle, ed. Jonathan Barnes (Princeton: Princeton University Press, 1984), 2a11–18.

[33] Bacon, Andrew, John Hawthorne, and Gabriel Uzquiano. "Higher-Order Free Logic and the Prior-Kaplan Paradox." Canadian Journal of Philosophy 46, no. 4 5 (2016): 493–541.

Much more has been said, and much more can still be said about almost every aspect of this essay. Perhaps this is precisely why the Cogito remains such a pivotal moment in philosophy – because it encapsulates, in a single stroke, the fundamental ambiguities of human experience: selfhood, existence, inference, and certainty. It can be argued that the very tension between valid viewpoints on these matters is inexplicably inseparable from the nature of human reality.

We have merely scraped the surface of this quandary. And yet a certain similarity has been detected: In both Hintikka's performative Cogito and Brittan's presuppositional reading, the inference's validity is restricted to the self-referential context. The Cogito's "*I think*" operates on two levels: One level is the subject thinking. The other is the subject referring to their thinking. Self-reference is what enables the ontological leap, allowing a system to move from representing existence to asserting it. The fact that self-reference is a meta-phenomenon that transcends systems such as classical FOL and free logic, is evident by the criticisms these systems have faced when used to formalize the Cogito, as well as the necessity admitted by the likes of Hintikka and Brittan to resort to realms external to model-theoretic logic to validate it.

Perhaps a more sophisticated logical framework is required to resolve this issue for the Cogito. More sophisticated frameworks like Peano Arithmetic are still driven to their limits by more complex self-referential statements, whether they be self-confirming like Löb's Theorem[34] (for any formula P, if it is provable in PA that "*if P is provable in PA, then P is true*", then P is provable in PA), or self-defeating like the meta-mathematically paradoxical Gödel sentence[35] "*This sentence is unprovable in PA*.", capturing the essence of his first incompleteness theorem. This suggests that self-reference is a consistently problematic phenomenon for logical systems.

Lastly, the role of existence as not merely another predicate, but the basis of quantification for variables, an implicit, self-evident truth in classical logic, and a special predicate in free logics in making the inferential Cogito worthy of such heated debate, cannot be overlooked. It is after all the de facto object of study of an entire philosophical discipline, namely that of ontology. While free logics might not be abled to carry out such a monumental task as solving the existential quandary that is the Cogito, they expose the lines where logic, language, and being intersect, reminding us that existence is not a given fact, but an assumption to be questioned.

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[34] Jason Gross, Adam Chlipala, and David I. Spivak, "Experience Implementing a Formally Verified Löb's Theorem in Coq," in Proceedings of the 21st ACM SIGPLAN International Conference on Functional Programming (ICFP 2016) (New York: ACM, 2016), 88.

[35] Juliet Floyd, "Gödel's Incompleteness Theorems," The Stanford Encyclopedia of Philosophy (Fall 2020 Edition), edited by Edward N. Zalta.

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I dedicate this essay to my mother, Kalliope.