

Question 5.1

```
In [29]: import numpy as np
import math
from functools import reduce
from matplotlib import pyplot as plt

def multivariate_normal_pdf(x, mean, sigma):
    l = x.shape[0]
    det_S = np.linalg.det(sigma)
    norm_const = 1.0/((2.0*np.pi)**(l/2.0)*np.sqrt(det_S))
    inv_S = np.linalg.inv(sigma)
    a1 = np.dot(np.dot((x-mean), inv_S), (x-mean))

    return norm_const*np.exp(-(1.0/2.0)*a1)

def multivariate_normal_pdf_v2(x, mean, sigma):
    l = x.shape[1]
    det_S = np.linalg.det(sigma)
    norm_const = 1.0/((2.0*np.pi)**(l/2.0)*np.sqrt(det_S))
    inv_S = np.linalg.inv(sigma)
    a1 = np.sum(np.dot(x-mean, inv_S)*(x-mean), axis = 1)

    return norm_const*np.exp(-0.5*a1)
```

i

```
In [30]: #Generation and plotting of training and testing sets

N = 1500 # Number of data points per class
m1 = np.array([0, 2]) #mean for first class
m2 = np.array([0, 0]) #mean for second class
S = np.array([[4, 1.8], [1.8, 1]]) #covariance matrix
p = 2*N

X_class1 = np.random.multivariate_normal(m1,S,N)
X_class2 = np.random.multivariate_normal(m2, S,N)

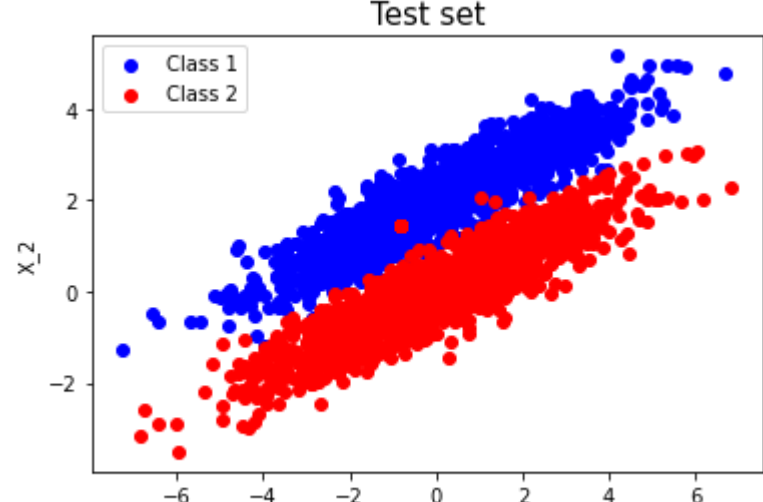
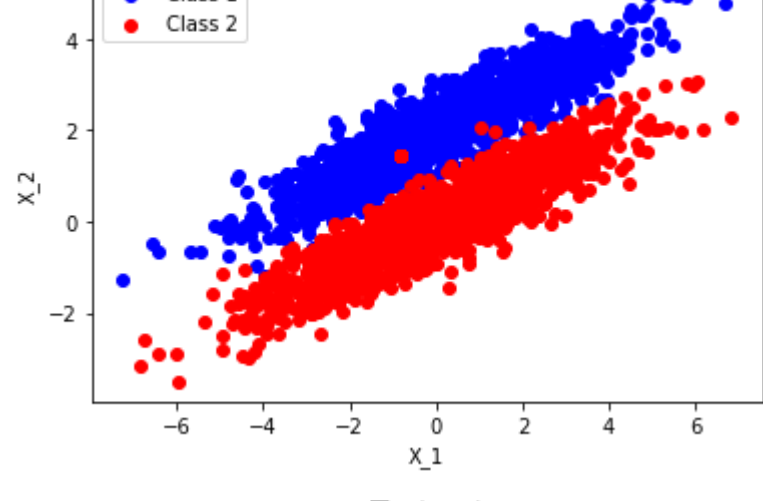
X_1 = np.concatenate((X_class1, X_class2), axis = 0) #data_set
Y_1 = np.concatenate((np.zeros((N, 1)), 1*np.ones((N, 1))), axis = 0)

plt.figure(1)
plt.scatter(X_1[np.nonzero(Y_1 == 0),0], X_1[np.nonzero(Y_1 == 0),1], color = "b",label = "Class 1")
plt.scatter(X_1[np.nonzero(Y_1 == 1),0], X_1[np.nonzero(Y_1 == 1),1], color = "r",label = "Class 2")
plt.title("Training set", fontsize=15)
plt.legend(loc = 0)
plt.xlabel("X_1");
plt.ylabel("X_2");

np.random.seed(4)

X_2 = np.concatenate((X_class1, X_class2), axis = 0) #data_set
Y_2 = np.concatenate((np.zeros((N, 1)), 1*np.ones((N, 1))), axis = 0)

plt.figure(2)
plt.scatter(X_2[np.nonzero(Y_2 == 0),0], X_2[np.nonzero(Y_2 == 0),1], color = "b",label = "Class 1")
plt.scatter(X_2[np.nonzero(Y_2 == 1),0], X_2[np.nonzero(Y_2 == 1),1], color = "r",label = "Class 2")
plt.title("Test set", fontsize=15)
plt.legend(loc = 0)
plt.xlabel("X_1");
plt.ylabel("X_2");
```



ii

```
In [31]: # Estimation of priori probabilities
# Classification using bayesian rule

P2 = P1 = 0.5
p1 = np.zeros(p)
p2 = np.zeros(p)

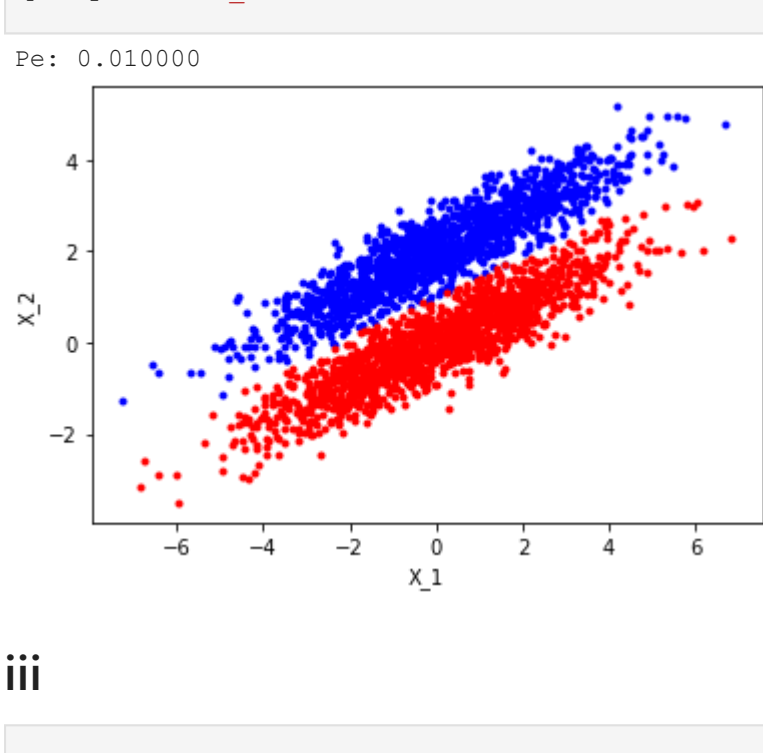
p1 = multivariate_normal_pdf_v2(X_2,m1 ,S);
p2 = multivariate_normal_pdf_v2(X_2,m2 ,S);
classes = np.zeros(p)

for i in range(0, p):
    if P1 * p1[i] > P2 * p2[i]:
        classes[i] = 0
    else:
        classes[i] = 1

Pe = 0
for i in range(0, p):
    if classes[i] != Y_2[i][0]:
        Pe += 1

Pe /= p
print('Pe: %f' % Pe)

plt.figure(1)
plt.plot(X_2[np.nonzero(classes == 0),0], X_2[np.nonzero(classes == 0),1], '.b')
plt.plot(X_2[np.nonzero(classes == 1),0], X_2[np.nonzero(classes == 1),1], '.r')
```



iii

```
In [32]: def model(X_train, Y_train, X_test, Y_test, num_iterations , learning_rate , print_cost):

    # perform parameter initialization
    dim = X_train.shape[0]
    w = np.zeros((dim, 1))
    b = 0

    m = X_train.shape[1]

    costs = []

    # training
    for i in range(num_iterations):

        # calculate cost and gradients - forward propagation
        A = 1/(1+np.exp(-(np.dot(w.T,X_train)+b)))
        cost = (-1/m)*(Y_train*np.log(A)+(1-Y_train)*np.log(1-A)).sum()

        # calculate gradients - backward propagation
        dw = np.dot(X_train,(A-Y_train).T)/m
        db = (A-Y_train).sum()/m

        # perform the update
        w = w - learning_rate*dw
        b = b - learning_rate*db

    A_test = 1/(1+np.exp(-(np.dot(w.T,X_test)+b)))
    Y_predict_test = np.around(A_test)

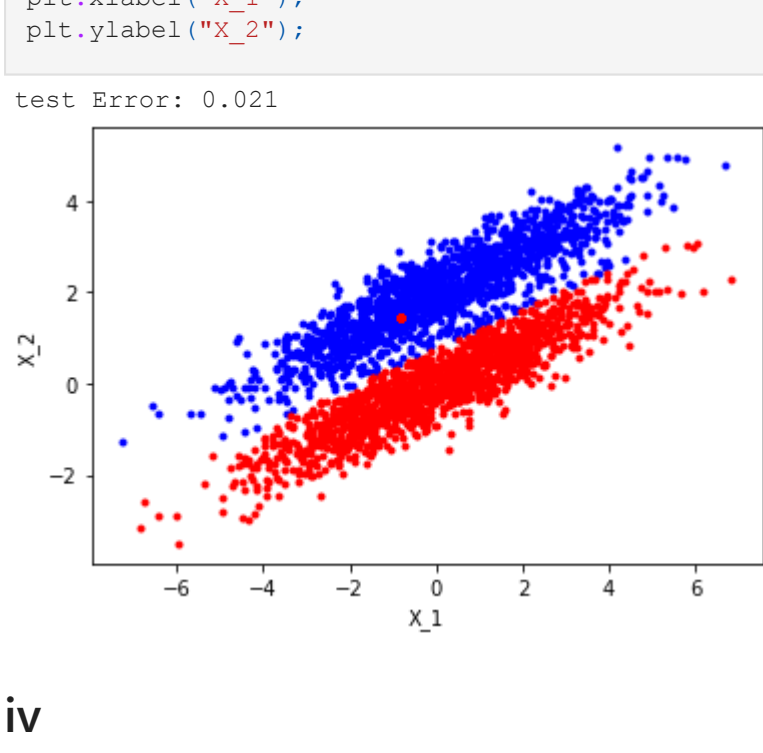
    # Print test Errors
    print("test Error: {} ".format( np.mean(np.abs(Y_predict_test - Y_test))))

    return Y_predict_test
```

```
In [33]: #Logistic regression

ypredict=model(X_1.T, Y_1.T, X_2.T, Y_2.T, num_iterations=20000 , learning_rate=0.001 , print_cost=True)

plt.figure(1)
plt.plot(X_2[np.nonzero(ypredict == 0),0], X_2[np.nonzero(ypredict == 0),1], '.b')
plt.plot(X_2[np.nonzero(ypredict == 1),0], X_2[np.nonzero(ypredict == 1),1], '.r')
```



iv

By using Bayesian classification with logistic regression, the probability error is 0.021333333333333333 this is bigger for logistic regression while comparing with Bayesian classification.

v

```
In [34]: #Different covariance

S1 = np.array([[4, 1.8], [1.8, 1]])
S2 = np.array([[4, -1.8], [-1.8, 1]])

X_class1 = np.random.multivariate_normal(m1,S1,N)
X_class2 = np.random.multivariate_normal(m2,S2,N)

X_1 = np.concatenate((X_class1, X_class2), axis = 0)
Y_1 = np.concatenate((0*np.ones((N, 1)), 1*np.ones((N, 1))), axis = 0)

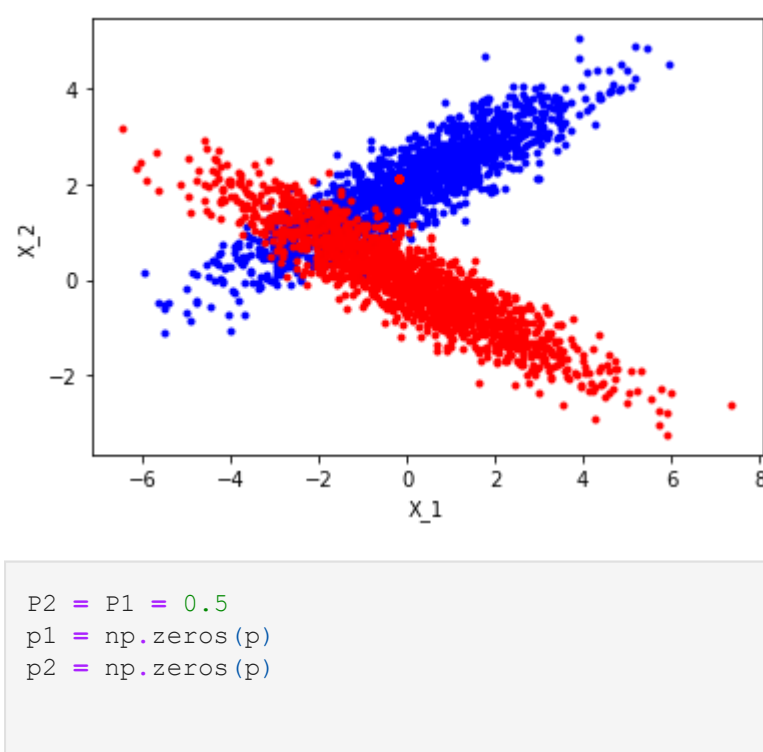
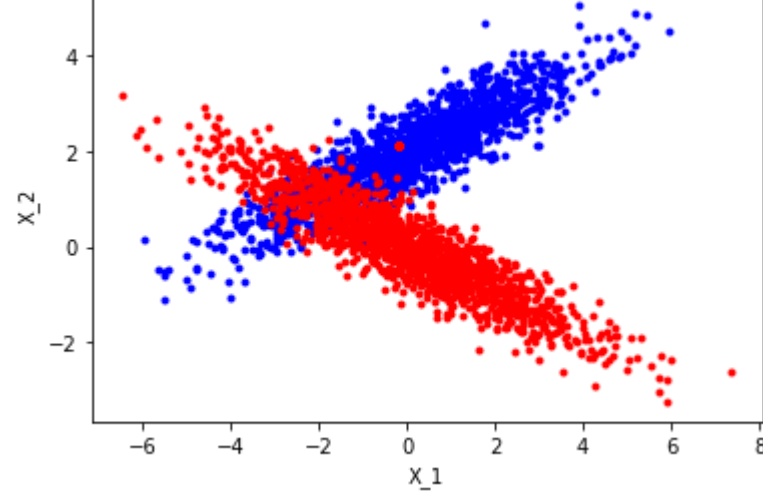
plt.figure(1)
plt.plot(X_1[np.nonzero(Y_1 == 0),0], X_1[np.nonzero(Y_1 == 0),1], '.b')
plt.plot(X_1[np.nonzero(Y_1 == 1),0], X_1[np.nonzero(Y_1 == 1),1], '.r')

plt.xlabel("X_1");
plt.ylabel("X_2");

np.random.seed(5)

X_2 = np.concatenate((X_class1, X_class2), axis = 0)
Y_2 = np.concatenate((0*np.ones((N, 1)), 1*np.ones((N, 1))), axis = 0)

plt.figure(2)
plt.plot(X_2[np.nonzero(Y_2 == 0),0], X_2[np.nonzero(Y_2 == 0),1], '.b')
plt.plot(X_2[np.nonzero(Y_2 == 1),0], X_2[np.nonzero(Y_2 == 1),1], '.r')
```



```
In [35]: P2 = P1 = 0.5
p1 = np.zeros(p)
p2 = np.zeros(p)

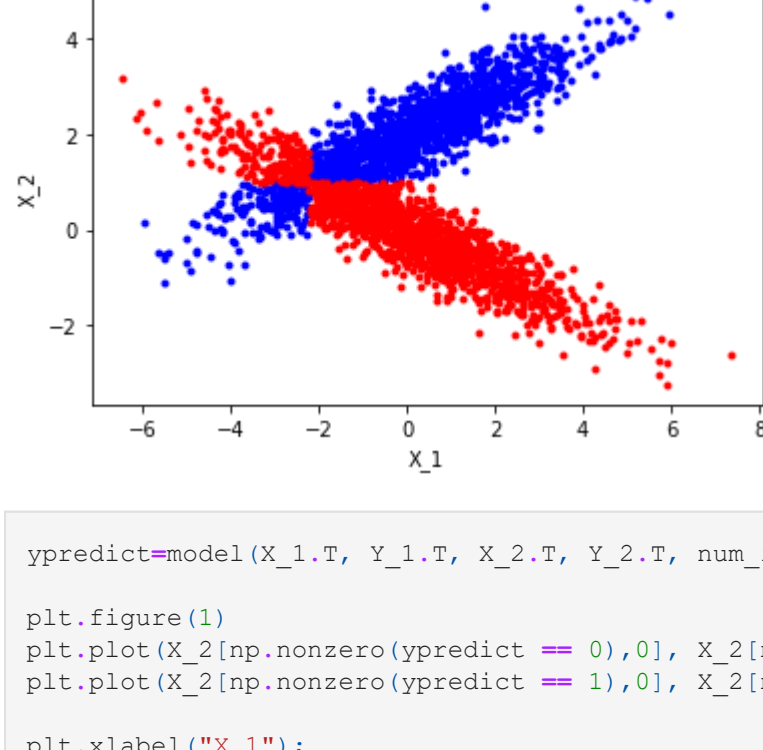
p1=multivariate_normal_pdf_v2(X_2,m1 ,S1);
p2=multivariate_normal_pdf_v2(X_2,m2 ,S2);
classes = np.zeros(p)

for i in range(0, p):
    if P1*p1[i] > P2*p2[i]:
        classes[i] = 0
    else:
        classes[i] = 1

Pe = 0
for i in range(0, p):
    if classes[i] != Y_2[i][0]:
        Pe += 1

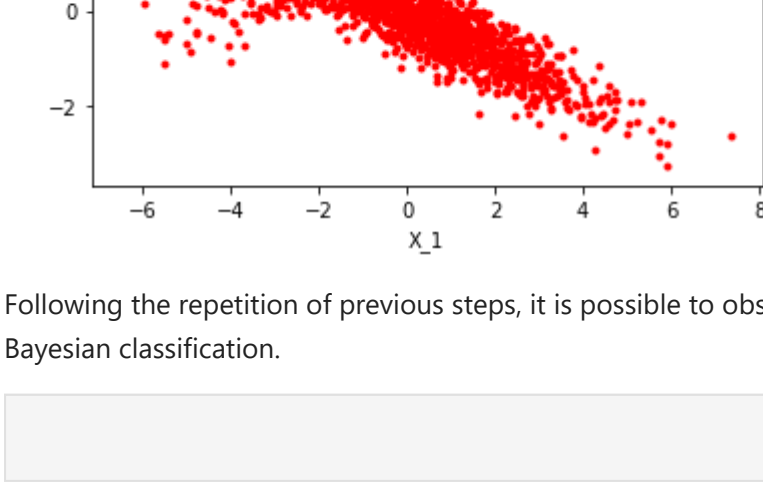
Pe /= p
print('Pe: %f' % Pe)

plt.figure(1)
plt.plot(X_2[np.nonzero(classes == 0),0], X_2[np.nonzero(classes == 0),1], '.b')
plt.plot(X_2[np.nonzero(classes == 1),0], X_2[np.nonzero(classes == 1),1], '.r')
```



```
In [36]: ypredict=model(X_1.T, Y_1.T, X_2.T, Y_2.T, num_iterations=20000 , learning_rate=0.001 , print_cost=True)

plt.figure(1)
plt.plot(X_2[np.nonzero(ypredict == 0),0], X_2[np.nonzero(ypredict == 0),1], '.b')
plt.plot(X_2[np.nonzero(ypredict == 1),0], X_2[np.nonzero(ypredict == 1),1], '.r')
```



Following the repetition of previous steps, it is possible to observe that the error in logistic regression is higher while comparing it with Bayesian classification.

```
In [ ]:
```


Question 5.2

```
In [10]: import numpy as np
import math
import soundfile as sf
import warnings
warnings.filterwarnings("ignore", category=DeprecationWarning)
import matplotlib.pyplot as plt
import sys
import os

In [11]: def py_awgn(input_signal, snr_dB, rate=1.0):
    """ Additive White Gaussian Noise (AWGN) Channel.

    Parameters
    -----
    input_signal : 1D ndarray of floats
        Input signal to the channel.

    snr_dB : float
        Output SNR required in dB.

    rate : float
        Rate of the a FEC code used if any, otherwise 1.

    Returns
    -----
    output_signal : 1D ndarray of floats
        Output signal from the channel with the specified SNR.
    """
    avg_energy = np.sum(np.dot(input_signal.conj().T, input_signal)) / input_signal.shape[0]
    snr_linear = 10 ** (snr_dB / 10.0)
    noise_variance = avg_energy / (2 * rate * snr_linear)

    if input_signal.dtype is np.complex:
        noise = np.array(np.sqrt(2 * noise_variance) * np.random.randn(input_signal.shape[0]) * (1 + 1j)), ndmin=2)
    else:
        noise = np.array(np.sqrt(2 * noise_variance) * np.random.randn(input_signal.shape[0])), ndmin=2)

    output_signal = input_signal + noise.conj().T

    return output_signal

def Kernel_Ridge(Xtrain, Ytrain, Xtest, sigma, lambda_):
    ' use of a kernel based solution to calculate the output Y_out'

    from scipy.spatial.distance import pdist, cdist, squareform

    # Design matrix K
    pairwise_sq_dists = squareform(pdist(Xtrain, 'seuclidean'))
    K = np.exp(-pairwise_sq_dists / sigma**2)
    A = K + lambda_ * np.identity(len(K))
    kx = np.exp(-cdist(Xtrain, Xtest, 'seuclidean')/sigma**2)
    A_inv = np.linalg.inv(A + lambda_ * np.identity(len(K)))
    B = np.matmul(kx.T, A_inv)
    Y_out = np.matmul(B, Ytrain)

    return Y_out

In [12]: # Reading wav file. x corresponds to time instances (i.e., x_i in [0,1])
# fs is the sampling frequency
# Replace the name "BladeRunner.wav" with the name of the file
# you intend to use.

np.random.seed(6)

N = 2000
samples = 20000
ind = range(0, samples, int(samples/N))
strt = 150000
[data, fs] = sf.read('BladeRunner.wav')
sound = np.array(data[strt:strt+samples+1], :1), dtype=np.float32)
y = np.reshape(sound[ind, 0], newshape=(len(sound), 1))

Ts = 1/fs #sampling period

x = np.array(range(0, samples)).conj().transpose()*Ts
x = x[ind]
x = np.reshape(x, newshape=(x.shape[0], 1))

# Add white Gaussian noise
snr = 10 # dB
y = py_awgn(y, snr)

# add outliers
O = 0.8*np.max(np.abs(y))
percent = 0.1
M = int(np.floor(percent*N))
out_ind = np.random.choice(N, M, replace=False)
outs = np.sign(np.random.randn(M, 1))*O
y[out_ind] = y[out_ind] + outs

i
```

```
In [13]: lambda_ = 0.0001
sigma = 0.004

Y_test = Kernel_Ridge(x,y, x, sigma, lambda_) #Predicted Y values using Kernel Ridge

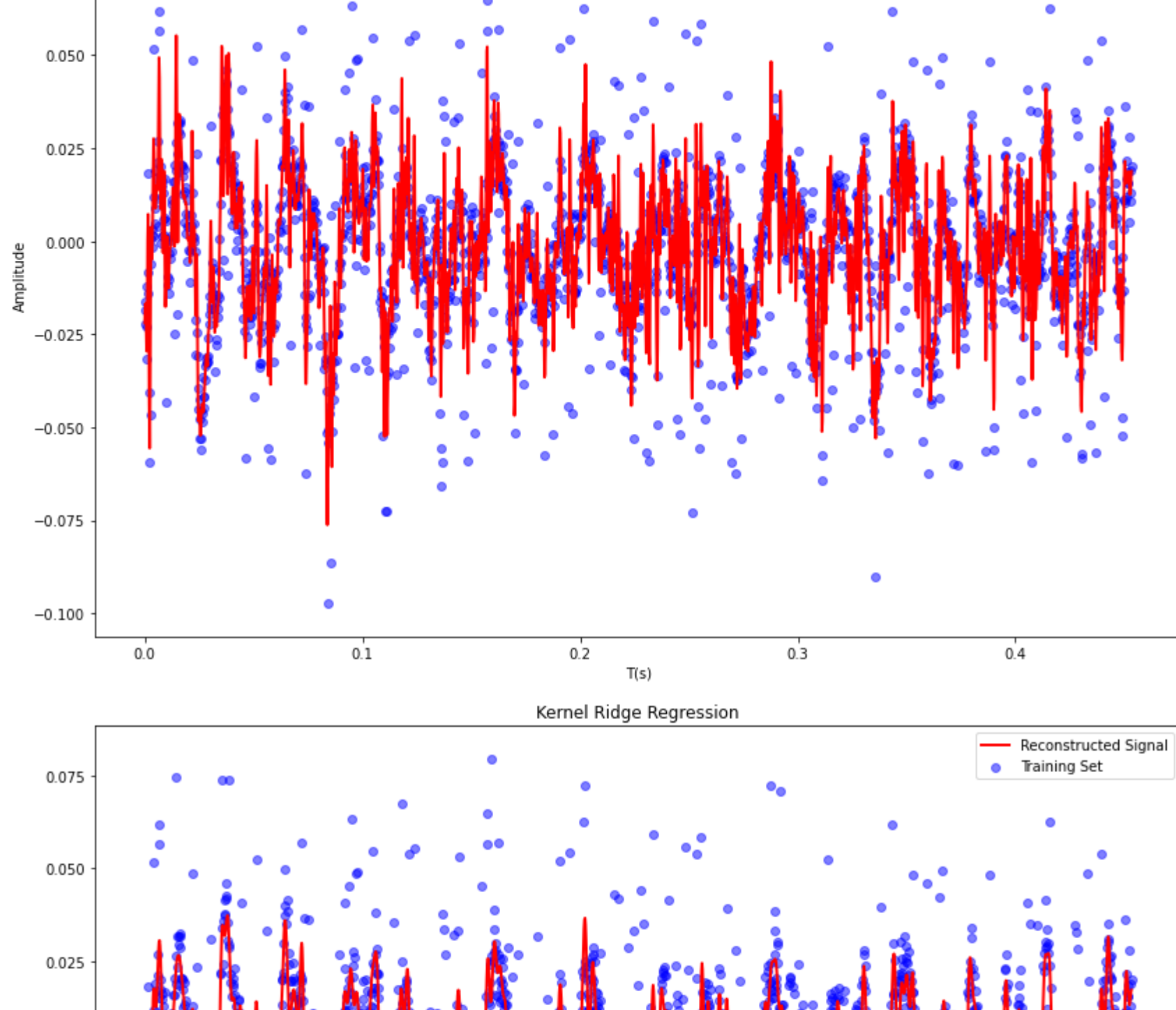
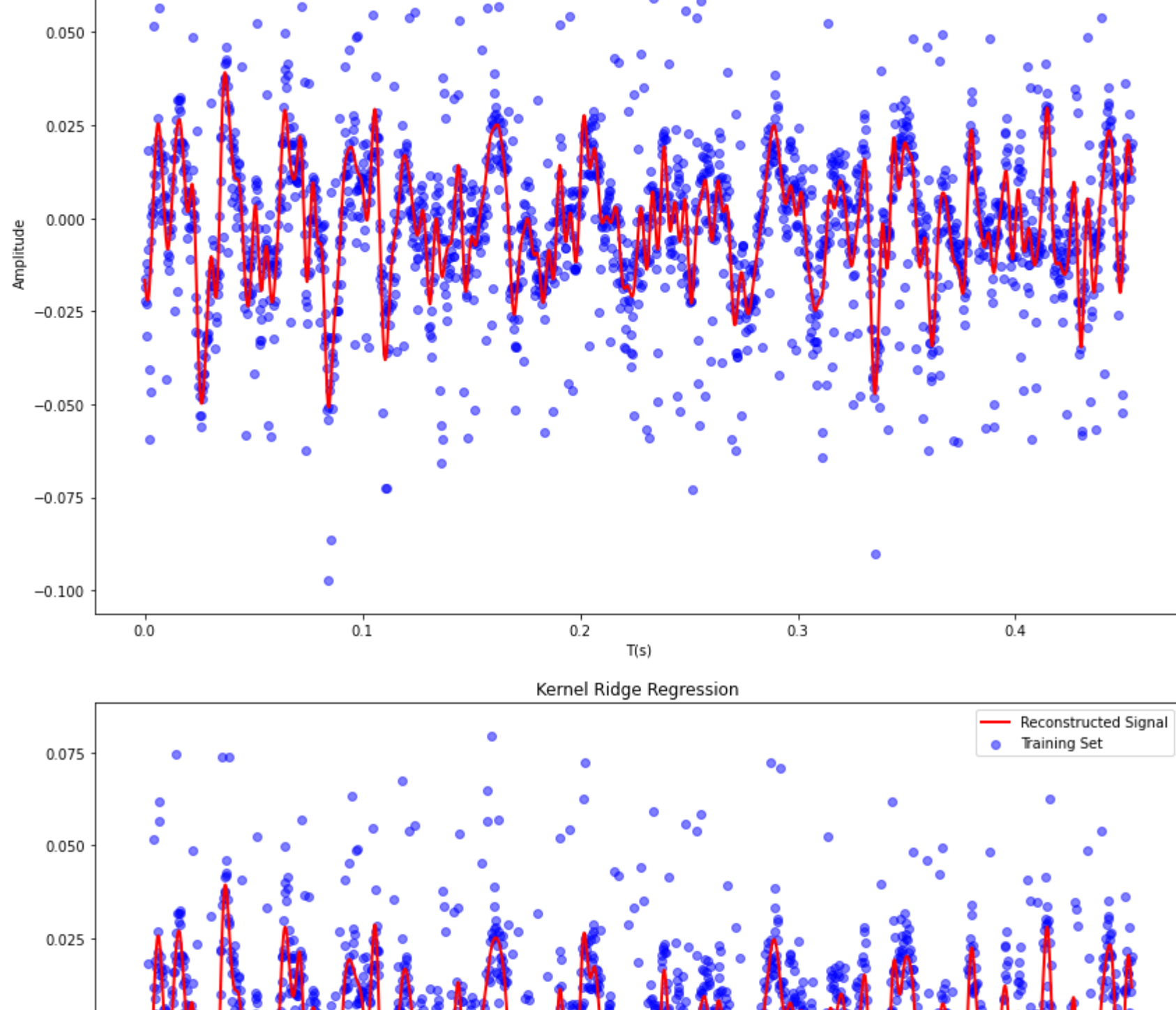
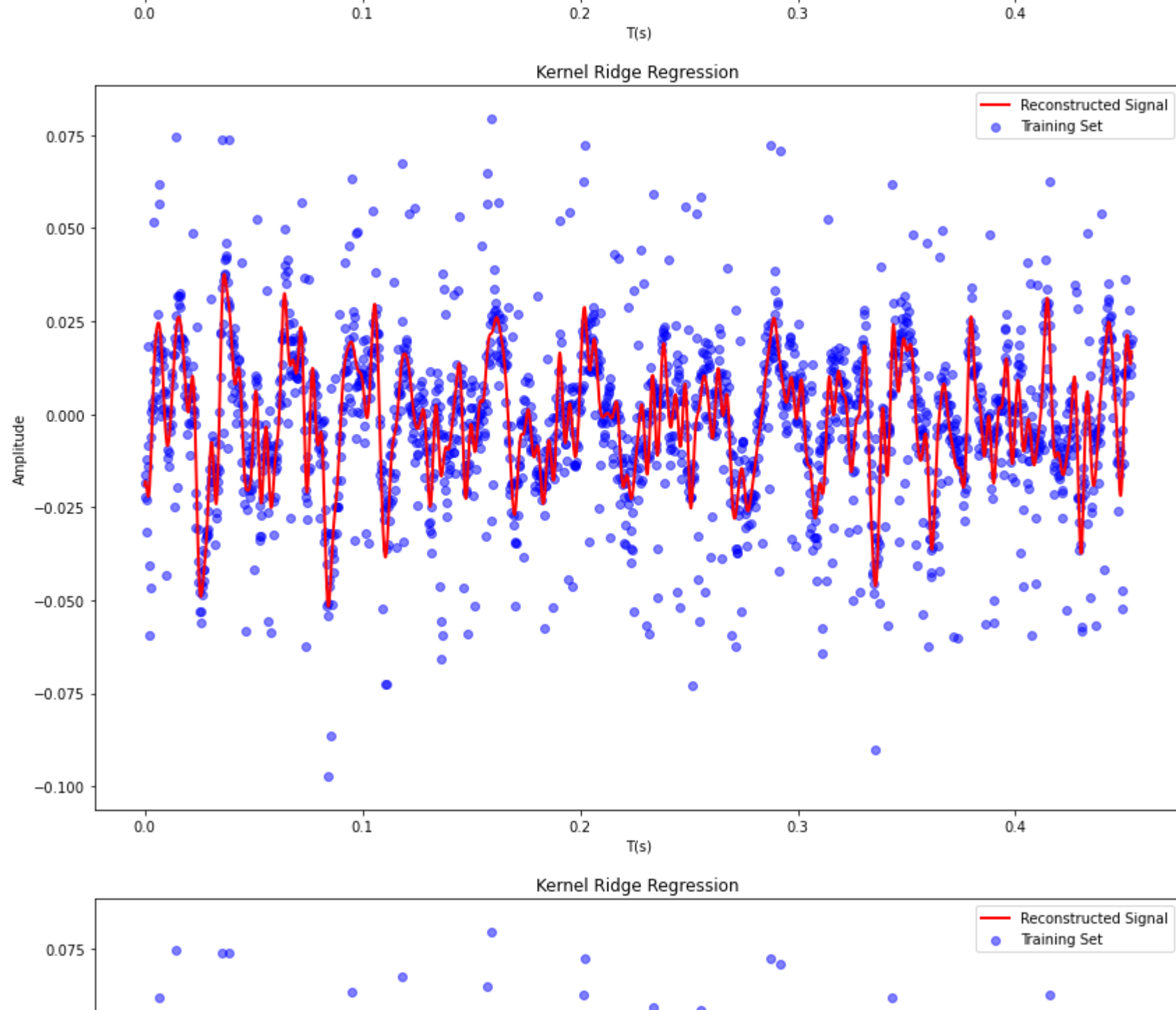
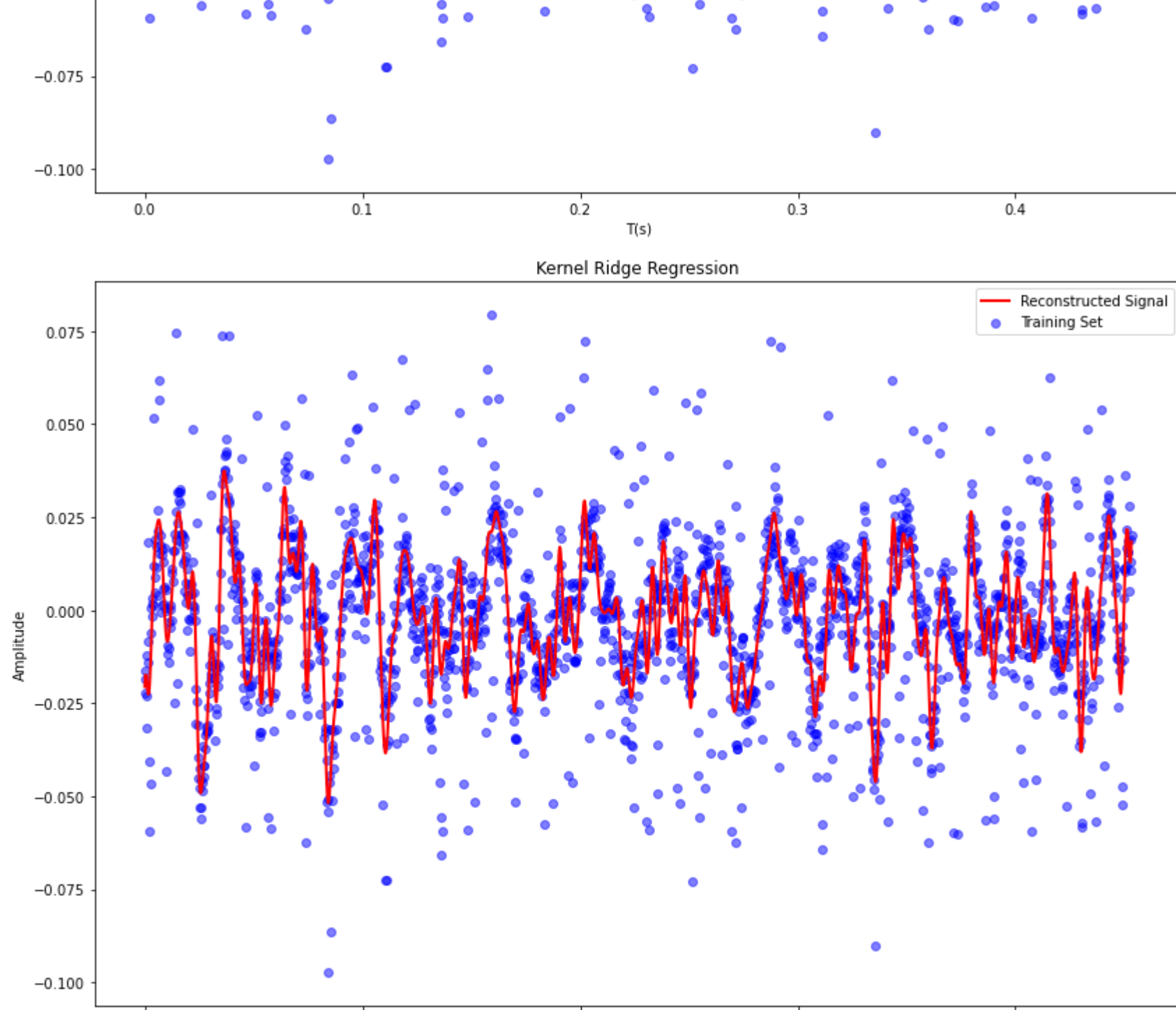
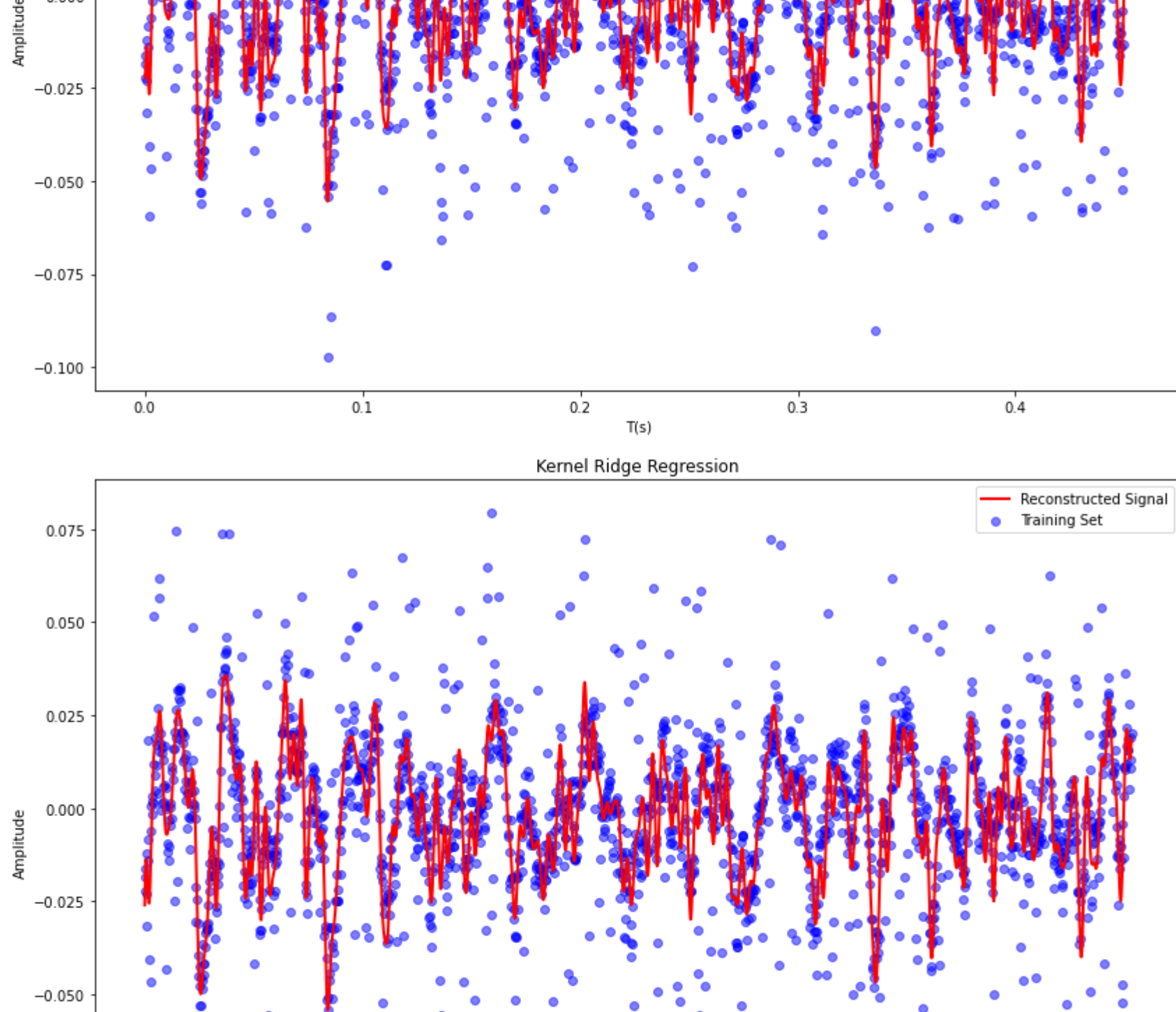
fig = plt.figure(figsize = (12, 8))
axes = fig.add_axes([0.1, 0.1, 0.9, 0.9])
axes.scatter(x, y, color = 'blue', alpha = 0.5, label = "Training Set")
axes.plot(x, Y_test, "r-", linewidth = 2, label = 'Reconstructed Signal')
axes.set_xlabel("T(s)");
axes.set_ylabel("Amplitude");
axes.set_title("Kernel Ridge Regression")
axes.legend(loc=0);
```



```
ii

In [14]: sigma = 0.004
lambda_ = np.array([10**-6, 10**-5, 0.0005, 0.001, 0.01, 0.05])

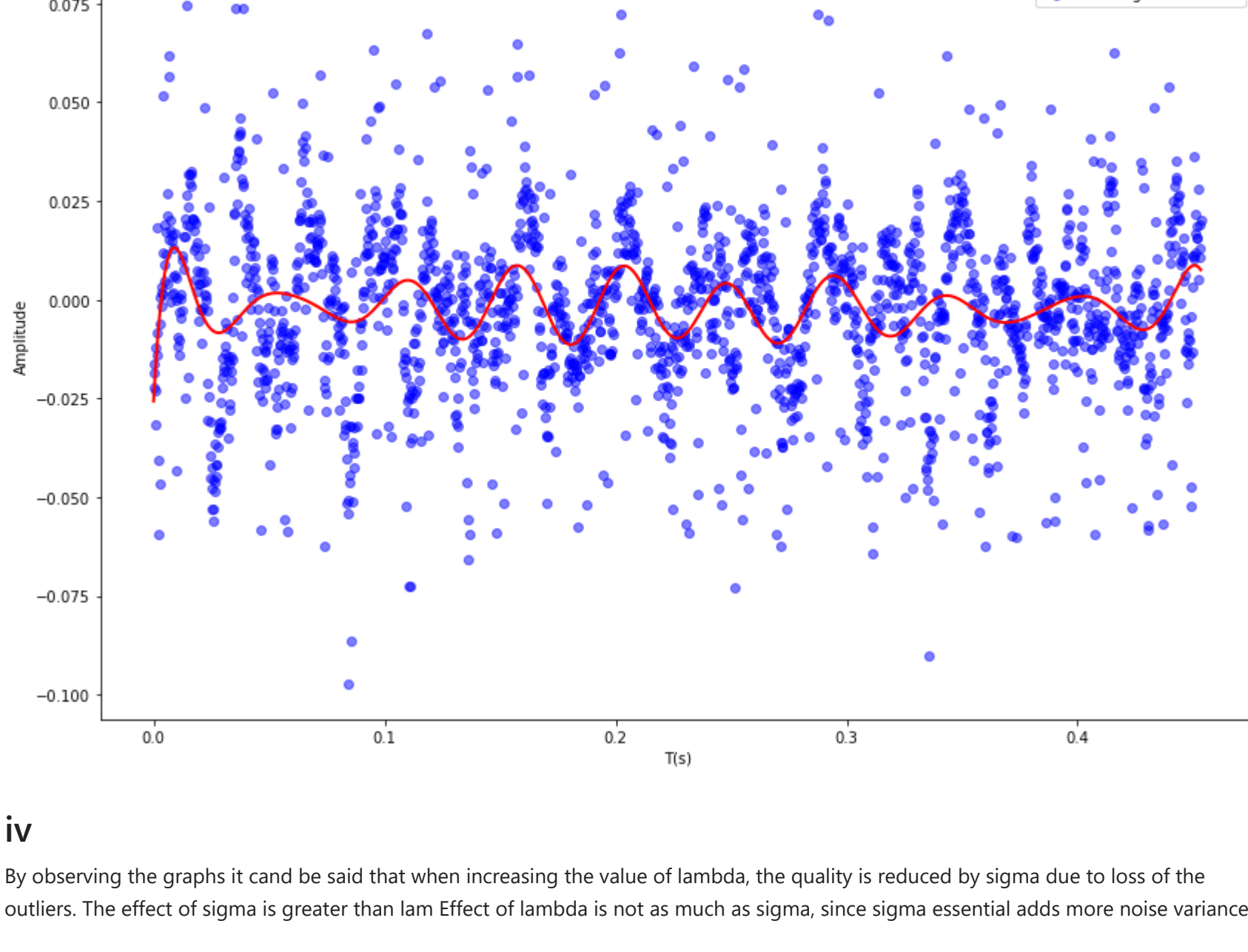
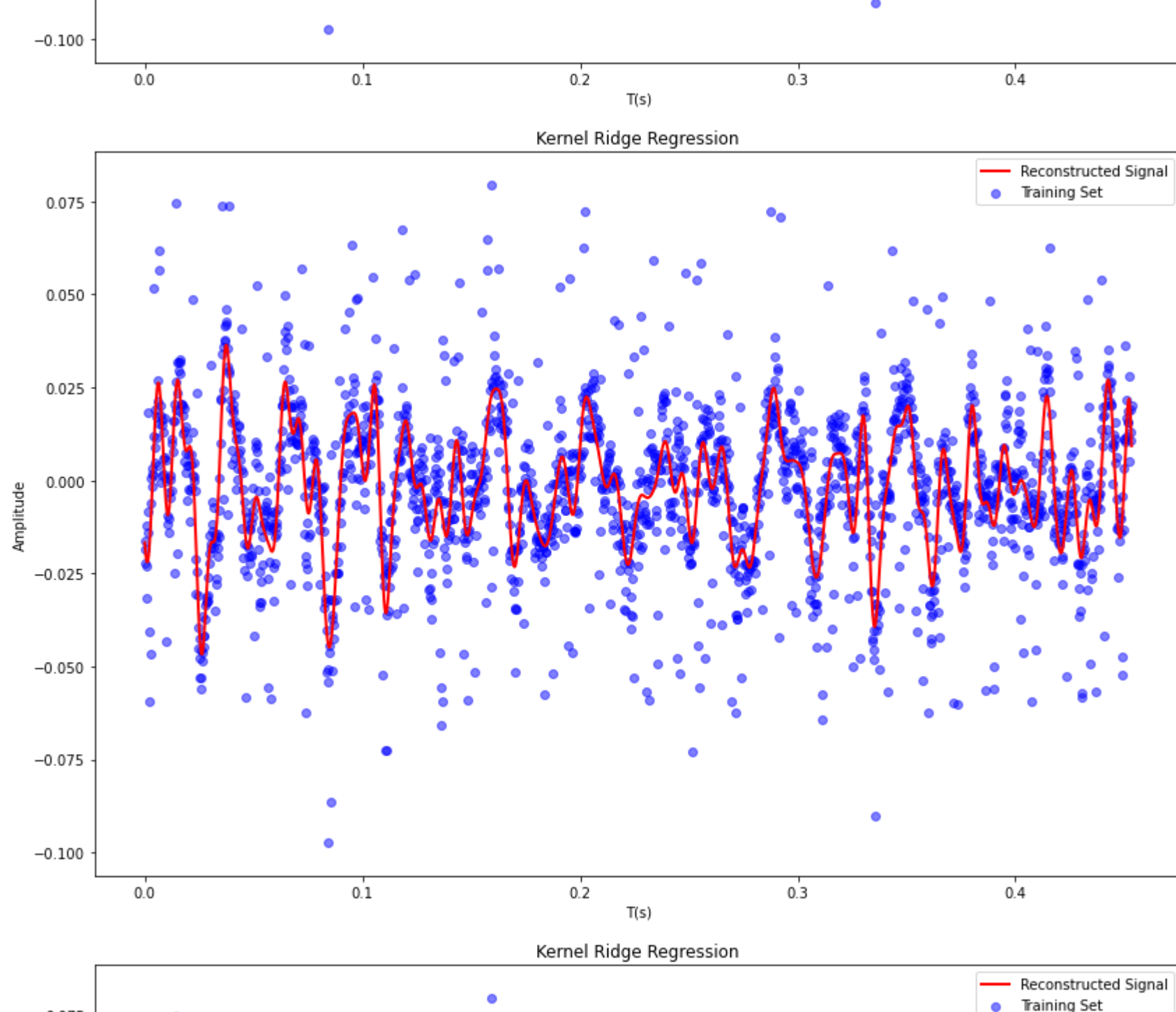
for i in range (len(lambda_)):
    Y_test = Kernel_Ridge(x,y, x, sigma, lambda_[i]) #Predicted Y values using Kernel Ridge
    fig = plt.figure(figsize = (12, 8))
    axes = fig.add_axes([0.1, 0.1, 0.9, 0.9])
    axes.scatter(x, y, color = 'blue', alpha = 0.5, label = "Training Set")
    axes.plot(x, Y_test, "r-", linewidth = 2, label = 'Reconstructed Signal')
    axes.set_xlabel("T(s)");
    axes.set_ylabel("Amplitude");
    axes.set_title("Kernel Ridge Regression")
    axes.legend(loc=0);
```



```
iii

In [15]: lambda_ = 0.0001
sigma = np.array([0.001, 0.003, 0.008, 0.05])

for i in range (len(sigma)):
    Y_test = Kernel_Ridge(x,y, x, sigma[i], lambda_) #Predicted Y values using Kernel Ridge
    fig = plt.figure(figsize = (12, 8))
    axes = fig.add_axes([0.1, 0.1, 0.9, 0.9])
    axes.scatter(x, y, color = 'blue', alpha = 0.5, label = "Training Set")
    axes.plot(x, Y_test, "r-", linewidth = 2, label = 'Reconstructed Signal')
    axes.set_xlabel("T(s)");
    axes.set_ylabel("Amplitude");
    axes.set_title("Kernel Ridge Regression")
    axes.legend(loc=0);
```



iv

By observing the graphs it can be said that when increasing the value of lambda, the quality is reduced by sigma due to loss of the outliers. The effect of sigma is greater than lam Effect of lambda is not as much as sigma, since sigma essential adds more noise variance

Question 5.3

```
In [1]: import numpy as np
import scipy
import matplotlib.pyplot as plt

def nn_model(X, Y, n_h, num_iterations, learning_rate, print_cost):

    np.random.seed(3)

    ## NN definition
    n_x = X.shape[0]    # size of input layer
    n_h = n_h            # size of hidden layer
    n_y = Y.shape[0]    # size of output layer

    # parameter initialization
    W1 = np.random.randn(n_h, n_x)*0.1
    b1 = np.zeros((n_h, 1))
    W2 = np.random.randn(n_y, n_h)*0.1
    b2 = np.zeros((n_y, 1))

    # gradient descent loop
    for i in range(0, num_iterations):

        # Forward propagation
        A1 = np.dot(W1, X) + b1
        Z1 = np.tanh(A1)
        A2 = np.dot(W2, Z1) + b2
        Z2 = A2

        # compute the cost
        m = Y.shape[1]
        cost = np.sum((Y-Z2)**2)/m

        # perform back-propagation
        dA2 = Z2 - Y
        dB2 = np.matmul(dA2, Z1.T)/m
        dB1 = np.sum(dB2, axis = 1, keepdims = True)/m
        dA1 = np.multiply(np.matmul(W2.T, dA2), (1-np.power(Z1, 2)))
        dW1 = np.matmul(dA1, X.T)/m
        dB1 = np.sum(dA1, axis = 1, keepdims = True)/m

        # Parameter update
        W1 = W1-learning_rate*dW1
        b1 = b1-learning_rate*dB1
        W2 = W2-learning_rate*dW2
        b2 = b2-learning_rate*dB2

        # Print the cost every 1000 iterations
        if print_cost and i % 1000 == 0:
            print ("Cost after iteration %i: %f" % (i, cost))

    parameters = {"W1": W1, "b1": b1, "W2": W2, "b2": b2}

    return parameters

def nn_predict(parameters, X):

    """
    Arguments:
    parameters -- python dictionary containing trained parameters
    X -- input data of size (n_x, m)

    Returns:
    predictions -- vector of predictions of our model (red: 0 / blue: 1)
    """

    # unpack the parameters
    W1 = parameters["W1"]
    b1 = parameters["b1"]
    W2 = parameters["W2"]
    b2 = parameters["b2"]

    # Forward propagation
    A1 = np.dot(W1,X) + b1
    Z1 = np.tanh(A1)
    A2 = np.dot(W2,Z1) + b2
    Z2 = A2    # output latyer's sigmoid transfer function

    predictions = np.around(Z2)

    return predictions

def plot_decision_boundary(model, X, y):

    # set min and max values and give it some padding
    x_min, x_max = X[0, :].min() - 1, X[0, :].max() + 1
    y_min, y_max = X[1, :].min() - 1, X[1, :].max() + 1
    h = 0.01

    # Generate a grid of points with distance h between them
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))

    # predict the function value of the whole grid
    Z = model(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)

    # plot the contour and training examples
    plt.contour(xx, yy, Z, cmap = plt.cm.Spectral)
    plt.ylabel("x2")
    plt.xlabel("x1")
    plt.scatter(X[0, :], X[1, :], c = y, cmap = plt.cm.Spectral)

def mixt_model(m, S, N, seed):

    """
    m : matrix for means of the subclasses : (1, c), where c is number of classes, 1 dimensions of input space
    S : matrix for covariances of the subclasses : (1, 1, c)
    N : array with the number of elements per class : (1, c)
    """

    np.random.seed(seed)
    i = m.shape[0]    # dimension of the space
    c = m.shape[1]    # number of gaussian sub-classes
    Ntotal = np.sum(N)

    X = []

    for i in range(0,c):
        Xc = np.random.multivariate_normal(np.array(m[:, i]).T, np.array(S[:, :, i]).T, N[i]).T
        X.append(Xc)
    X = np.hstack(X)

    return X[:, :].random.permutation(Ntotal)]
```

```
In [2]: seed = 10
D = 2 # Dimension
m1 = np.array([[1-5, 5], [5, -5]]).T #mean for class one
m2 = np.array([[1-5, -5], [0, 0], [5, 5]]).T #mean for class two
c1 = m1.shape[1] # number of gaussians per class 1
c2 = m2.shape[1] # number of gaussians per class 2

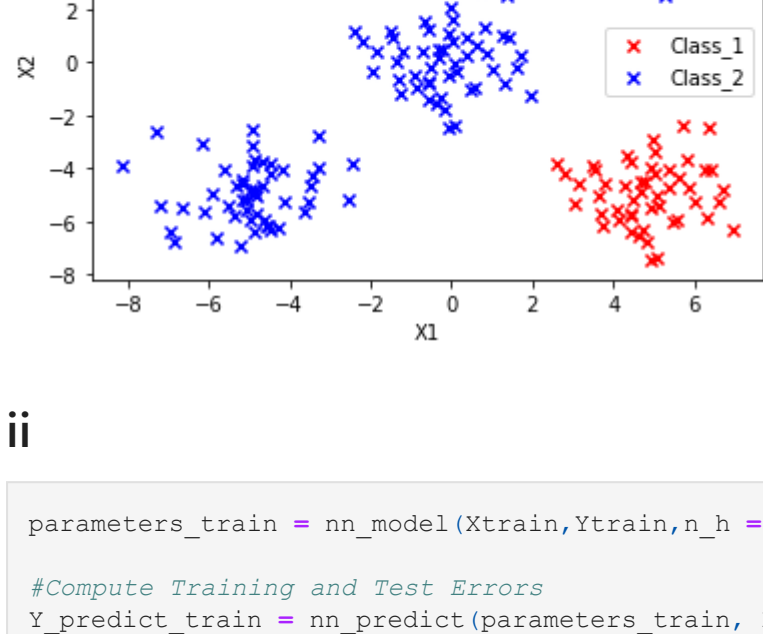
#Training Set Generation
N1 = np.array([50,50])
S1 = np.zeros(shape=(D,D, c1))
s_1=1 #Variance
for i in range(0, c1):
    S1[:, :, i] = np.array(s_1*np.eye(2))

w1training = mixt_model(m1, S1, N1, seed) #training set for first class
N2 = np.array([50,50,50])
S2 = np.zeros(shape=(D, D, c2))
for i in range(0, c2):
    S2[:, :, i] = s_1 * np.eye(D)
w2training = mixt_model(m2, S2, N2, seed) #training set for second class
Xtrain = np.concatenate((w1training, w2training), axis=1)
Ytrain = np.concatenate((np.zeros(shape=(1, np.sum(N1))), np.ones(shape=(1, np.sum(N2))))), axis=1)

#Test Set Generation
seed = 8
w1test = mixt_model(m1, S1, N1, seed)#test set for first class
w2test = mixt_model(m2, S2, N2, seed)#test set for second class
Xtrain = np.concatenate((w1test, w2test), axis=1)
Ytest=np.concatenate((np.zeros(shape=(1, np.sum(N1))), np.ones(shape=(1, np.sum(N2))))), axis=1)

plt.figure(1)
plt.scatter(Xtrain[0,np.nonzero(Ytrain == 0)], Xtrain[1,np.nonzero(Ytrain ==0)]
            , marker = "x", color = "r", label = "Class_1");
plt.scatter(Xtrain[0,np.nonzero(Ytrain == 1)], Xtrain[1,np.nonzero(Ytrain ==1)]
            , marker = "x", color = "b", label = "Class_2");
plt.legend(loc=0);
plt.xlabel("X1");
plt.ylabel("X2");
plt.title("Training Set");

plt.figure(2)
plt.scatter(Xtest[0,np.nonzero(Ytrain == 0)], Xtest[1,np.nonzero(Ytrain ==0)]
            , marker = "x", color = "r", label = "Class_1");
plt.scatter(Xtest[0,np.nonzero(Ytrain == 1)], Xtest[1,np.nonzero(Ytrain ==1)]
            , marker = "x", color = "b", label = "Class_2");
plt.legend(loc=0);
plt.xlabel("X1");
plt.ylabel("X2");
plt.title("Testing Set");
```



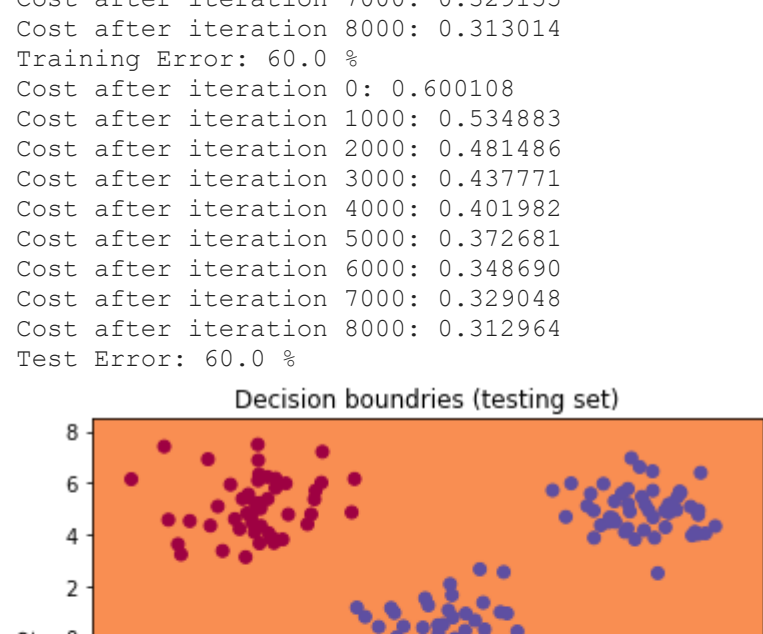
```
In [3]: parameters_train = nn_model(Xtrain,Ytrain,n_h = 2,num_iterations = 9000,learning_rate = 0.01,print_cost = True)

#Compute Training and Test Errors
Y_predict_train = nn_predict(parameters_train, Xtrain)
print("Training Error: {} %".format(np.mean(np.abs(Y_predict_train-Ytrain))*100))

parameters_test = nn_model(Xtest,Ytest,n_h = 2,num_iterations = 9000,learning_rate = 0.01,print_cost = True)
Y_predict_test = nn_predict(parameters_test, Xtest)
print("Test Error: {} %".format(np.mean(np.abs(Y_predict_test-Ytest))*100))

#Plotting Decision Boundaries generated from the network
plot_decision_boundary(lambda x:nn_predict(parameters_test,x.T),Xtest,Ytest.ravel());
plt.title("Decision boundaries (testing set)", fontsize=15);

Cost after iteration 0: 0.601382
Cost after iteration 1000: 0.239997
Cost after iteration 2000: 0.239996
Cost after iteration 3000: 0.239995
Cost after iteration 4000: 0.239995
Cost after iteration 5000: 0.239994
Cost after iteration 6000: 0.239993
Cost after iteration 7000: 0.239991
Cost after iteration 8000: 0.239990
Training Error: 40.0 %
Cost after iteration 0: 0.600108
Cost after iteration 1000: 0.239995
Cost after iteration 2000: 0.239993
Cost after iteration 3000: 0.239992
Cost after iteration 4000: 0.239990
Cost after iteration 5000: 0.239987
Cost after iteration 6000: 0.239983
Cost after iteration 7000: 0.239977
Cost after iteration 8000: 0.239966
Test Error: 40.0 %
```



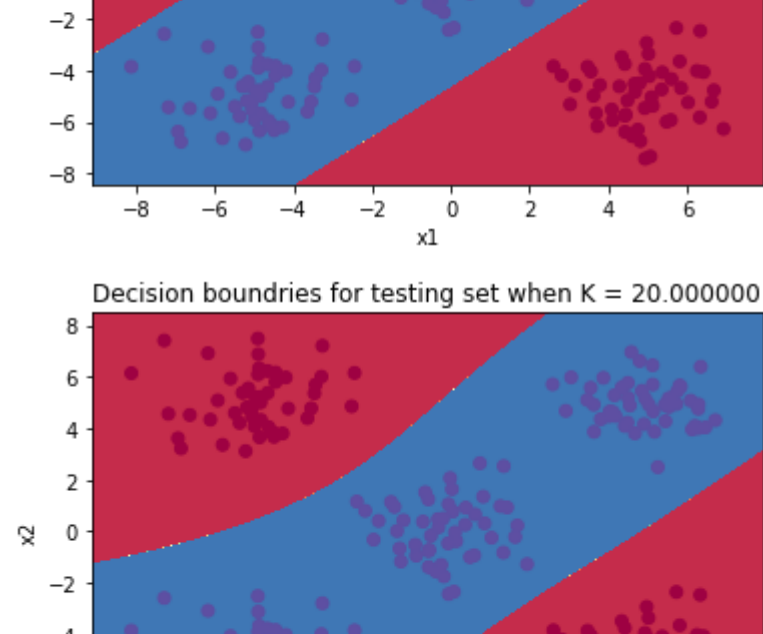
```
In [4]: parameters_train = nn_model(Xtrain,Ytrain,n_h = 2,num_iterations = 9000,learning_rate = 0.0001,print_cost = True)

#Compute Training and Test Errors
Y_predict_train = nn_predict(parameters_train, Xtrain)
print("Training Error: {} %".format(np.mean(np.abs(Y_predict_train-Ytrain))*100))

parameters_test = nn_model(Xtest,Ytest,n_h = 2,num_iterations = 9000,learning_rate = 0.0001,print_cost = True)
Y_predict_test = nn_predict(parameters_test, Xtest)
print("Test Error: {} %".format(np.mean(np.abs(Y_predict_test-Ytest))*100))

#Plotting Decision Boundaries generated from the network
plot_decision_boundary(lambda x:nn_predict(parameters_test,x.T),Xtest,Ytest.ravel());
plt.title("Decision boundaries (testing set)", fontsize=12);

Cost after iteration 0: 0.601382
Cost after iteration 1000: 0.535850
Cost after iteration 2000: 0.482213
Cost after iteration 3000: 0.438309
Cost after iteration 4000: 0.402371
Cost after iteration 5000: 0.372953
Cost after iteration 6000: 0.348870
Cost after iteration 7000: 0.329155
Cost after iteration 8000: 0.313014
Training Error: 60.0 %
Cost after iteration 0: 0.600108
Cost after iteration 1000: 0.534882
Cost after iteration 2000: 0.481486
Cost after iteration 3000: 0.437771
Cost after iteration 4000: 0.401982
Cost after iteration 5000: 0.372681
Cost after iteration 6000: 0.348690
Cost after iteration 7000: 0.329048
Cost after iteration 8000: 0.312964
Test Error: 60.0 %
```



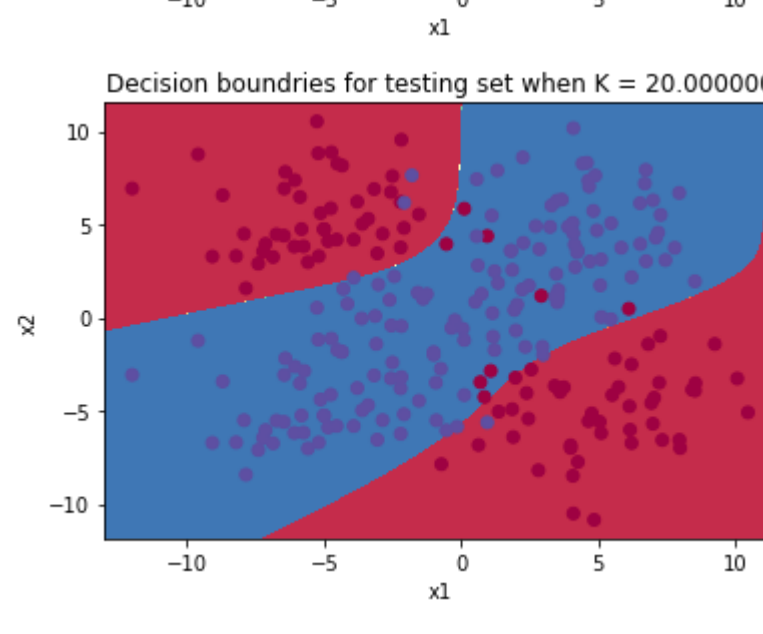
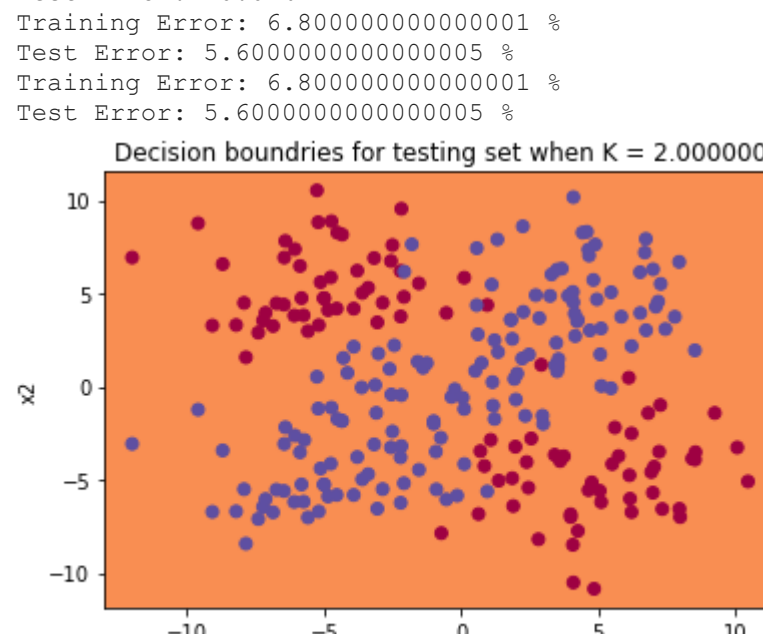
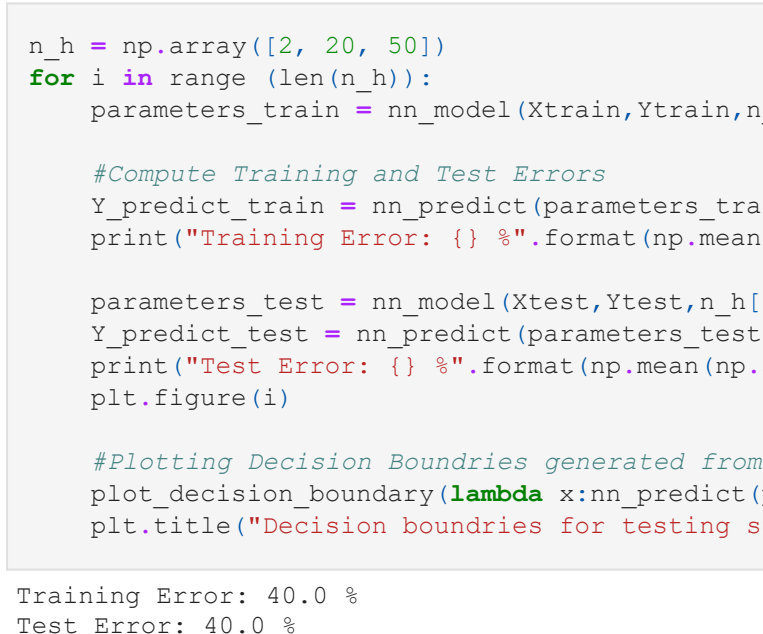
As evident from the graph above, with the parameters used this time, the model is not classifying, all of the data points are place with the same background color meaning that it did not classify.

```
In [5]: n_h = np.array([1, 4, 20])
for i in range (len(n_h)):
    parameters_train = nn_model(Xtrain,Ytrain,n_h[i],num_iterations = 9000,learning_rate = 0.01,print_cost = False)
    Y_predict_train = nn_predict(parameters_train, Xtrain)
    print("Training Error: {} %".format(np.mean(np.abs(Y_predict_train-Ytrain))*100))

    parameters_test = nn_model(Xtest,Ytest,n_h[i],num_iterations = 9000,learning_rate = 0.01,print_cost = False)
    Y_predict_test = nn_predict(parameters_test, Xtest)
    print("Test Error: {} %".format(np.mean(np.abs(Y_predict_test-Ytest))*100))
    plt.figure(i)

    #Plotting Decision Boundaries generated from the network
    plot_decision_boundary(lambda x:nn_predict(parameters_test,x.T),Xtest,Ytest.ravel());
    plt.title("Decision boundaries for testing set when K = %f" %n_h[i], fontsize=12);

Training Error: 40.0 %
Test Error: 40.0 %
Training Error: 0.0 %
Test Error: 0.0 %
Training Error: 0.0 %
Test Error: 0.0 %
```



It can be seen from the graphs that when the learning rate is low, the model can not classify the data pointsbecause it is harder to reach a conclusion of what class the points belong to.

```
In [6]: seed =10
D = 2 # Dimension
m1 = np.array([[1-5, 5], [5, -5]]).T #mean for class one
m2 = np.array([[1-5, -5], [0, 0], [5, 5]]).T #mean for class two
c1 = m1.shape[1] # number of gaussians per class 1
c2 = m2.shape[1] # number of gaussians per class 2

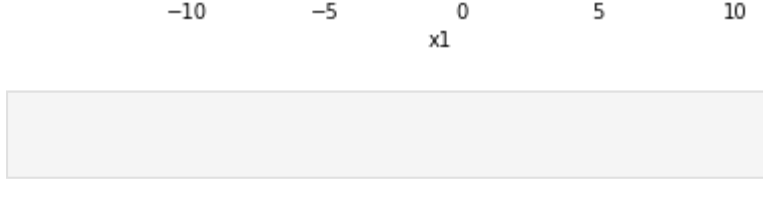
#Training Set Generation
N1 = np.array([50,50])
S1 = np.zeros(shape=(D,D, c1))
s_1=6 #Variance
for i in range(0, c1):
    S1[:, :, i] = np.array(s_1*np.eye(2))

w1training = mixt_model(m1, S1, N1, seed) #training set for first class
N2 = np.array([50,50,50])
S2 = np.zeros(shape=(D, D, c2))
for i in range(0, c2):
    S2[:, :, i] = s_1 * np.eye(D)
w2training = mixt_model(m2, S2, N2, seed) #training set for second class
Xtrain = np.concatenate((w1training, w2training), axis=1)
Ytrain = np.concatenate((np.zeros(shape=(1, np.sum(N1))), np.ones(shape=(1, np.sum(N2))))), axis=1)

#Test Set Generation
seed = 5
w1test = mixt_model(m1, S1, N1, seed)#test set for first class
w2test = mixt_model(m2, S2, N2, seed)#test set for second class
Xtrain = np.concatenate((w1test, w2test), axis=1)
Ytest=np.concatenate((np.zeros(shape=(1, np.sum(N1))), np.ones(shape=(1, np.sum(N2))))), axis=1)

plt.figure(1)
plt.scatter(Xtrain[0,np.nonzero(Ytrain == 0)], Xtrain[1,np.nonzero(Ytrain ==0)]
            , marker = "x", color = "r", label = "Class_1");
plt.scatter(Xtrain[0,np.nonzero(Ytrain == 1)], Xtrain[1,np.nonzero(Ytrain ==1)]
            , marker = "x", color = "b", label = "Class_2");
plt.legend(loc=0);
plt.xlabel("X1");
plt.ylabel("X2");
plt.title("Training Set");

plt.figure(2)
plt.scatter(Xtest[0,np.nonzero(Ytrain == 0)], Xtest[1,np.nonzero(Ytrain ==0)]
            , marker = "x", color = "r", label = "Class_1");
plt.scatter(Xtest[0,np.nonzero(Ytrain == 1)], Xtest[1,np.nonzero(Ytrain ==1)]
            , marker = "x", color = "b", label = "Class_2");
plt.legend(loc=0);
plt.xlabel("X1");
plt.ylabel("X2");
plt.title("Testing Set");
```



```
In [7]: n_h = np.array([2, 20, 50])
for i in range (len(n_h)):
    parameters_train = nn_model(Xtrain,Ytrain,n_h[i],num_iterations = 9000,learning_rate = 0.01,print_cost = False)
    Y_predict_train = nn_predict(parameters_train, Xtrain)
    print("Training Error: {} %".format(np.mean(np.abs(Y_predict_train-Ytrain))*100))

    parameters_test = nn_model(Xtest,Ytest,n_h[i],num_iterations = 9000,learning_rate = 0.01,print_cost = False)
    Y_predict_test = nn_predict(parameters_test, Xtest)
    print("Test Error: {} %".format(np.mean(np.abs(Y_predict_test-Ytest))*100))
    plt.figure(i)

    #Plotting Decision Boundaries generated from the network
    plot_decision_boundary(lambda x:nn_predict(parameters_test,x.T),Xtest,Ytest.ravel());
    plt.title("Decision boundaries for testing set when K = %f" %n_h[i], fontsize=12);

Training Error: 40.0 %
Test Error: 40.0 %
Training Error: 31.6 %
Test Error: 5.6000000000000005 %
Training Error: 19.6 %
Test Error: 5.6000000000000005 %
```



```
In [8]: n_h = np.array([2, 20, 50])
for i in range (len(n_h)):
    parameters_train = nn_model(Xtrain,Ytrain,n_h[i],num_iterations = 9000,learning_rate = 0.001,print_cost = False)
    Y_predict_train = nn_predict(parameters_train, Xtrain)
    print("Training Error: {} %".format(np.mean(np.abs(Y_predict_train-Ytrain))*100))

    parameters_test = nn_model(Xtest,Ytest,n_h[i],num_iterations = 9000,learning_rate = 0.01,print_cost = False)
    Y_predict_test = nn_predict(parameters_test, Xtest)
    print("Test Error: {} %".format(np.mean(np.abs(Y_predict_test-Ytest))*100))
    plt.figure(i)

    #Plotting Decision Boundaries generated from the network
    plot_decision_boundary(lambda x:nn_predict(parameters_test,x.T),Xtest,Ytest.ravel());
    plt.title("Decision boundaries for testing set when K = %f" %n_h[i], fontsize=12);

Training Error: 40.0 %
Test Error: 40.0 %
Training Error: 31.6 %
Test Error: 5.6000000000000005 %
Training Error: 19.6 %
Test Error: 5.6000000000000005 %
```

