On the evolution of cultural traits in social networks

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Contents

Abstract

In this research, we explore the evolution of cultural traits within a theoretical framework where each individual - or minority - is integrated into a social network. Individuals behave strategically and face a trade-off between acting like their observed peers or keeping faithful to their idiosyncrasy. Utilizing Finite Markov Processes, we characterize the Nash equilibrium of this interaction and identify a crucial statistic that encapsulates the impact of network topology on equilibrium outcomes. The structure of the social network is found to be instrumental in determining the equilibrium actions. Further, our analysis extends to an overlapping generations model, where parents' actions determine the traits of their offspring. We identify the Nash equilibrium in each period and show that, when the network is strongly connected, all society converges to the same trait. This long-run trait is inherently linked to the network topology. Lastly, we use the results to illustrate, with comparative dynamics, the consequences of the entry of a group of immigrants to a host country and discuss some policy implications of our findings.

Keywords: Social Networks, Assimilation, Cultural Traits, Immigration, Games on Networks.

Because identity is fundamental to behaviour, choice of identity may be the most important 'economic' decision people make.

George Akerlof and Rachel Kranton, *Economics and Identity* (2002)

1 Introduction

The observation that humans acquire valuable life skills and knowledge through copying others has been the focus of animal behaviourists' attention dating back to Darwin.

Culture is information that people acquire from others by teaching, imitation, and other forms of social learning. People acquire skills, beliefs, and values from those around them, shaping their behaviour. Humans, unlike any living creature, have cumulative cultural adaption. Humans hold their knowledge and learn things from others, improve them, pass this knowledge to the next generation, and so on (Boyd and Richerson, 2005). It is this unique mechanism which leads to the rapid non-genetic evolution of superbly designed adaptations to particular environments.[1] Hence, the underlying social structure where humans interact plays a prominent role in the evolution of cultural traits.

Cultural transmission is closely tied to the development of individual and group identity. Our identities are shaped by the cultural norms and values we absorb from our surroundings, including family, peers, and the wider society. As we interact within our social networks, we constantly pick up and adapt these cultural elements, which in turn mould our identities. This connection between how we form our identities and the way cultural traits are passed along is crucial. Cultural traits not only shape but are also shaped by, the identities within a society.

Preferences, beliefs, norms, and habits are formed as a result of heritable

[1] It is in this context that Dawkins (1989), when looking for a noun to convey the idea of unit of cultural transmission, coined the term *meme* as an inflexion of the greek word *mimeme* (something that is imitated) to make it sound like *gene*.

traits. They are transmitted from generation to generation and shaped by the social interactions of the individual. Thus, several social sciences pay attention to the role of vertical (parents), oblique (role models), and horizontal (peers) cultural transmission.

In the field of Economics, the inter-generational and peer-to-peer transmission of cultural traits is pivotal in shaping individual behavioural characteristics. This transmission influences a range of preference traits, including discounting, consumption patterns, patience, and risk aversion, as detailed in Robson and Samuelson (2011).

Furthermore, the transmission of cultural traits extends beyond individual preferences to influence broader societal norms and practices. As Guiso et al. (2008) elucidate, these traits play a critical role in determining attitudes and social norms related to family and community life. These elements are instrumental in forming and keeping social capital; a key driver of economic and social development. The intricate dynamics of cultural trait transmission thus not only shape individual preferences but also contribute to the broader fabric of social and economic life.

The seminal work that contains the quote at the start of this paper, Akerlof and Kranton (2000), comprehensively shows how personal identity and social affiliations profoundly influence economic choices, often leading to behaviours that deviate from traditional models of rationality and utility maximization. They particularly recognized that one identity category posing the dilemma between assimilation and idiosyncrasy was gender. For instance, women in law often feel pressured to adopt masculine characteristics to succeed, illustrating how professional norms can push individuals towards specific gender behaviours. They explain how, in general, external influences such as pressure from social identity, mould behaviour. George Akerlof and Rachel Kranton irreversibly linked identity to economics by revealing the evolving nature of behaviour as it aligns with changes in society and culture, thus affecting our preferences and choices.

Cultural transmission plays a key role in understanding various economic phenomena, particularly in the context of how individuals and groups interact within a society. This concept becomes particularly intriguing when we consider the dynamics of immigrant populations integrating into new cultures.

Assimilation with the local culture for immigrants has been positively correlated with their life satisfaction. Angelini et al. (2015) obtain conclusions in line with this view by using self-reported measures of well-being and panel data from migrants in Germany. Empirical work has looked at the impact of immigrants' social networks on their assimilation choices. Damm (2009) and Edin et al. (2003) use ethnic concentration/enclave as a proxy for immigrants' networks in a host country. The language group and proficiency of immigrants have been used as a proxy of the network effect, too (Bertrand et al., 2000). Besides, the language skills of immigrants influenced their performance in the labour market in the United States (Chiswick and Miller, 2002).

Research indicates a positive correlation between immigrants' assimilation into local cultures and their overall life satisfaction. This is supported by Angelini et al. (2015), who used self-reported well-being measures and panel data from migrants in Germany to demonstrate this relationship. Furthermore, the role of social networks in facilitating assimilation has been a focus of empirical studies. For instance, Damm (2009) and Edin et al. (2003) examined how the concentration of ethnic groups in certain areas, or enclaves, influences immigrants' choices to assimilate. Similarly, Bertrand et al. (2000) showed that language groups and proficiency are significant factors, affecting not only cultural integration but also economic outcomes, as evidenced by Chiswick and Miller (2002) study on the impact of language skills on immigrants' performance in the U.S. labour market.

Large inflows of immigration tend to carry a consequential impact on the labour market. Immigrants usually bring skills and know-how that differ from those appearing and required in the local markets. Moreover, individuals with low skills and low education levels may self-select themselves to migrate to a foreign country. Language barriers may be present as well. Cultural differences also play a role in how the newly arrived adjusts to the practices and customs of the working community.

This is of particular relevance in Chile, where the immigrant population has shown an accelerated increase during the last decade. According to Instituto Nacional de Estadísticas (2018), international immigrants rose from 1.27% of the total population in the 2002 Census to 4.35% of the total in the 2017 Census. Plus, around two-thirds of the total immigrants censused in 2017 arrived in the country between 2010 and 2017. Furthermore, 61% of the total migrated between 2015 and 2017.

By 31 December 2019, it is estimated that of the total population of immigrants in the territory, 30,5 % is from Venezuela, 15,8% from Peru, and 12,5 % from Haiti (Instituto Nacional de Estadísticas y Departamento de Extranjería y Migración, 2019).

Haitian immigrants, in particular, face significant barriers due to language differences and social segregation. The majority believe that it is not essential to know Spanish before they arrive in Chile, which hinders their integration into society and the labour market (Calderón and Saffirio, 2017).

On the other hand, Venezuelan immigrants in Chile have experienced a more seamless integration process, aided by their Spanish language proficiency and existing social networks within the country. Their assimilation experience has been more positive, as indicated by surveys and interviews conducted among the Venezuelan immigrant population (de la Hoz, 2018).

This contrasting experience between Haitian and Venezuelan immigrants in Chile underscores the importance of cultural, linguistic, and social factors in the assimilation process.

It also highlights the dynamic nature of cultural transmission. Not only do immigrants assimilate into the culture of the host country, but locals also adopt cultural traits and customs from the newly arrived culture. As observed in Santiago, the Chilean capital, dishes, and ingredients from Venezuelan cuisine are taking a central place in the main food market. Those of the Peruvian cuisine have already been there for long (Torres, 2018).

This is nothing new. In the United States, a country with a long history of immigration, social scientists argued that the conditions of life and opportunities in the country would eventually create a ”melting pot” between the different ethnic groups. However, after 1960, some began to question that view, observing that many minorities kept part of their distinctive economic, political, and cultural patterns long after their arrival to the United States (Bisin and Verdier, 2000).

These insights show the importance of understanding the dynamics of cultural transmission in the context of immigration. They encapsulate the need for modelling the intricate interplay of language, culture, and social networks in the process of cultural integration and assimilation. Elucidating these dynamics is crucial for designing policies that foster inclusive and harmonious multicultural societies. This sets the stage for our research, where we delve into understanding how these processes play out within the framework of social networks.

In this work, we move from these observational insights to a more formalized approach. We propose a model that reflects the decision-making processes of individuals who are embedded within a social network. Each individual behaves strategically optimizing a utility function that reflects the assimilation dilemma: to assimilate into the prevailing culture of the society or to maintain their distinct cultural identity. This choice is influenced by a variety of factors, including the structure of the social network they are part of and the interactions, both vertical (parental or ancestral) and horizontal (peer), they engage in.

We focus on the Nash equilibrium derived from this game and attempt to summarize the network's topology's effect on the transmission of cultural traits. We address the following questions that arise in this setting. How can we summarize the effect of the network's topology on the transmission of cultural traits? How can the topology of the network explain the transmission of characteristics across populations? Which types influence the long run, and which ones disappear?

Our main findings are, for a static model: a sufficient statistic to summarize how the topology of social networks shapes the transmission of behaviour in equilibrium, a novel decomposition of equilibrium behaviour, and an intuitive interpretation of the influence of each type on the population. In the dynamic model, we find conditions for convergence in behaviour, how each trait influences in the long run, and which traits disappear in the long run. Crucially, the *shape* of the social network plays a central role in all results.

The remainder of this paper is structured as follows: The subsequent section provides an overview of related literature, contextualizing our study's contribution to the existing body of knowledge. In section 3, we present the model and the preliminaries for our analysis. Then, we show the Nash equilibrium for the static model and decompose the network's effect in this equilibrium. Subsequently, we extend this result to a dynamic setting, obtain convergence results, and characterize the long-term Nash equilibrium for a strongly connected network. In section 4, we perform some comparative dynamics to exemplify the explaining capabilities of the presented model and comment on the implications of these results. Lastly, we offer our conclusions.

2 Related Literature

2.1 On evolution of cultural traits

The first formal treatments to the modelling of transmission of cultural traits come from the area of evolutionary biology, and date back to Cavalli-Sforza and Feldman (1973, 1981) and Boyd and Richerson (1985). The stylized model presented in both papers and their terminology is largely adopted by the literature that followed them.

Especially influential has been the seminal work of Alberto Bisin and Thierry Verdier. Bisin and Verdier (1998) introduce a setup for the study of inter-generational transmission of preferences for status in a two types model. Bisin and Verdier (2000, 2001) introduce the parental socialization choice on the model in Cavalli-Sforza and Feldman (1981), and conclude that the distribution of cultural traits converges to a heterogeneous distribution, explaining that immigrants' assimilation to the majority culture depends on parents' preferences for cultural traits.

Verdier and Zenou (2015, 2018) further studies the dynamics of a two-types model when there is inter-generational transmission and with a community leader, to explore the role of cultural leaders in the integration of immigrants in a host country.

Recent literature in Economics moved from types (discrete variables) to (continuous) traits. While the literature on discrete traits is well-established, the one on continuous traits is still in development. Bisin and Verdier (2011) offers a comprehensive survey on both. Bisin and Topa (2003) propose a model of endogenous transmission of preferences, cognitive and psychological continuous traits, and then put it to data to identify their mechanisms of transmission. Verdier and

Zenou (2015, 2018) and Prummer and Siedlarek (2017) study the dynamics of cultural traits with a community leader in a continuous traits model.

2.2 On games and opinions on networks with continuous actions

Ballester et al. (2006) provide the workhorse model of games in networks with continuous actions. Calvó-Armengol and Beltran (2009) study an organization as a coordination game in which all players face a common task, and use a network to model the communication structure within the organization. Bramoullé et al. (2014) obtain conditions on convergence when there are strategic interactions in networks.

The literature on opinion dynamics, introduced most notoriously in DeGroot (1974), is related to this work as well. In the DeGroot model, agents want to decide on a common parameter, and the opinion of each agent on the true value of this parameter is a weighted average of other agents' expected opinions plus his own. The works of DeMarzo et al. (2003) and Golub and Jackson (2010, 2012) are also central in this branch by extending the standard DeGroot model. The former proves that persuasion bias leads to the phenomenon of social influence, while the latter shows that the opinion of every agent in the network converges to the truth if, and only if, the influence of the most influential agent eventually vanishes. Our framework shares the linear updating of actions but with strategical behaviour from the agents.

2.3 Mixing both

Lately, the literature on cultural traits in Economics has moved from random encounters between agents to interactions in social networks.

Buechel et al. (2014) uses an overlapping generations model to study the transmission of continuous cultural traits from parents to children, where parents are also immersed in a social network. Panebianco (2014) examine the dynamics of inter-ethnic attitudes with a setup of inter-generational transmission of continuous cultural traits. They consider that children are exposed to their parents and a network of non-parental socialization, and obtain conditions for the convergence of these traits. Olcina et al. (2017) use a model in which each ethnic group is part of a social network, and their utility is determined by their agreement with their values and by their assimilation with their peers. They show that, in their setup, there is always convergence to a steady state. Ushchev and Zenou (2020) provide a micro foundation and a discussion of the linear-in-means model, the workhorse model for empirical work on social interactions and peer effects.

Förster et al. (2014) present a model of opinion formation and evolution based on DeGroot (1974) in which agents behave strategically. They explore the chance of manipulation between agents in the form of random encounters where the agents can decide to influence each other. The framework in Rapanos et al. (2019) introduces uncertainty on agents' private and social utilities when they are part of a network, and proves that there is always a unique Bayesian Nash equilibrium.

Our work is a contribution to the branch of transmission of continuous cultural traits in social networks. We consider a network model and utility function similar to those in Ushchev and Zenou (2020) and Olcina et al. (2017). Our OLG setup is taken from Buechel et al. (2014), but differs in two things: individuals directly inherit their parents' traits in the previous period, and these do not consider their offspring trait in their utility function; and descendants are exposed to vertical and horizontal influence, instead of vertical and oblique.

We will solve this model using a novel result. In similar settings, previous literature - such as Del Bello et al. (2016) and Verdier and Zenou (2017) - shows that equilibrium actions are a function of how *central* each agent is in the network.

Centrality is measured using the (weighted) *Bonacich centrality index*. This statistic summarizes the interaction between networks' topology and distribution of traits. Yet with this centrality index, results are not straightforward to present and understand. Furthermore, centrality is but one part of the picture.

In this work, we present a different approach, based on a *hidden* stochastic process defined by the network's topology. This yields concise equilibrium results which capture topology in a more structural sense through the stationary distribution of the Markov process. This leads to a straightforward presentation of the interpretations and formulas.

Our central findings are mainly derived using results from the Theory of Finite Markov Chains, which can be revised in Karlin and Taylor (1975), Kemeny and Snell (1976), and Hartfiel (2006).

3 The Model

3.1 General setting

Let $N = \{1, ..., n\}$ be the set of *agents* or *players*. Each agent $i \in N$ here may represent either individuals or a minority in the society.

Let $T : N \to \mathbb{R}$ be a function that uniquely identifies each agent $i \in N$ with a type $T(i) = \gamma_i$. The type of each agent is exogenous and represents, in one variable, its set of intrinsic characteristics, habits, values, and quirks.

Agents *choose* actions $a_i \in \mathbb{R}$ and *observe* actions of a subset of agents $N_i \subseteq N$ exogenous to the model, with $N_i \neq \varnothing$. This action represents the assimilation choice to the majority of the individual or group and their *observable behaviour* for other agents. The set N_i contains those other players that the agent i directly listens to or cares for. They may, for example, represent the community to which the agents aim to be included, the citizens of the country to which the agent immigrated, their new classmates or colleagues. Thus, $\forall j \in N_i$ we say that agent j *influences* directly agent i. We refer to N_i as the *influence* of agent i.

To avoid redundancy later, we always assume $i \notin N_i$. Let $\mathbf{a}_{N_i} = (a_j)_{j \in N_i}$. We define the payoff of agent $i \in N$ as given by

$$\mathcal{U}_i(a_i; \mathbf{a}_{N_i}) = -(1-\theta)(a_i - \gamma_i)^2 - \theta \sum_{j \in N} \sigma_{ij}(a_i - a_j)^2, \tag{3.1}$$

where $\theta \in (0, 1)$, $\sigma_{ij} \geq 0$, $\sigma_{ij} = 0 \iff j \notin N_i$, and

$$\sum_{j \in N} \sigma_{ij} = 1. \tag{3.2}$$

To understand this payoff function, observe first that the payoffs have two components: the $-(a_i - \gamma_i)^2$ component is strictly decreasing in $|a_i - \gamma_i|$ and represents how agents do not want to deviate from their type; the $-\sigma_{ij}(a_i - a_j)^2$ component, strictly decreasing in $|a_i - a_j|$, represents how the agents want to behave like their observed peers. The payoff of i is defined completely by its action and those of its influence, players not observed by i have no direct effect on its payoff. Thus, when the trait of an agent is different from the actions of other agents in his influence, he faces a trade-off between choosing a a_i not too far from γ_i nor too far from the j's terms. This portrays how, in reality, new members of a community face the dilemma between wanting to assimilate into the new culture, but staying faithful to their personal beliefs, family values, or idiosyncratic customs.

Agents may give different relative importance to the players in their influence. Hence, the exogenous parameter σ_{ij} measures the *importance* that agent j has for agent i.

Lastly, the exogenous parameter θ measures how important are peers concerning family or idiosyncrasy. For model tractability, and because we are focused on the broad network topology, we consider a single θ for all agents.

This setup naturally defines a weighted directed network $G = (N, E, w)$: the set of nodes corresponds to the set of agents N; the set of directed edges (or out-links) E is the ordered pairs (i, j) such that agent i observes agent j, formally $E = \{(i,j)|(i,j) \in N^2 \wedge j \in N_i\}$; and $w : E \rightarrow (0, 1]$ is the weight function, which maps each pair (i, j) to the importance that agent i gives to agent j, hence $w(i, j) = \sigma_{i,j}$. We refer to a network of this kind as a *social network*.

In a social network G a *walk* from i to j is any sequence of connected directed edges that starts in i and ends in j. A *path* from i to j is a walk that goes at most once through each node. If there exists a path from i to j we say that i *hears* j, and we denote it by $i \rightarrow j$. If there is a path from i to j and vice-versa we say that i and j *communicate*, and we denote this by $i \leftrightarrow j$.We say the social network G is

- *Strongly Connected* if $\forall(i, j) \in N^2, i \leftrightarrow j$;
- *Connected* if $\forall(i, j) \in N^2, i \rightarrow j$ or $i \leftarrow j$;
- *Disconnected* if $\exists(i, j) \in N^2$, such that neither $i \rightarrow j$ or $i \leftarrow j$.

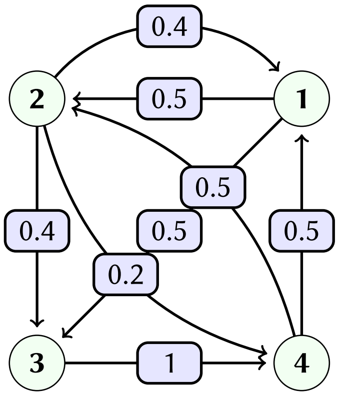

Figure 1: Diagram of Social Network G_1

Figure 1 illustrates the structure of one such social network. In this depiction, circles mean agents, arrows indicate the direction of out-links, and the weights of these out-links are represented by the values inside the rounded rectangles.

In G_1 we we observe that Agent 1 observes Agents 2 and 3. Conversely, agent 1 is observed by agents 2 and 4. The only mutually influential pair is Agents 1 and 2, while Agent 3 exclusively observes Agent 4. A notable feature of this network is the absence of a dominant agent; there is no clear hierarchy, and no one is marginalized. Each agent both observes and is observed; G_1 defines a strongly connected network. The agents in this network seem to be "well integrated" with each other. This balanced interaction suggests a network evocative of a close group of friends, a cohesive family, or minority groups seamlessly integrated into a community.

Now, examine the structure of network G_2 as depicted in Figure 2. There are two distinct clusters of agents: the first group, located on the left side of the figure, consists of agents 1 to 5, while the second group, positioned on the right side, includes agents 6 to 8. There is almost no interaction between agents of the left group and those of the right, except for agent 8 from the right group who observes agent 5 from the left group with a significant weight of 0.4. Despite this, each group exhibits strong internal cohesion. This configuration could represent various social dynamics, such as two different political factions, separate families, distinct religious communities, or a group of recent immigrants (right group) integrating into the existing society.

Furthermore, in the right group, each agent displays to be of equal impor-

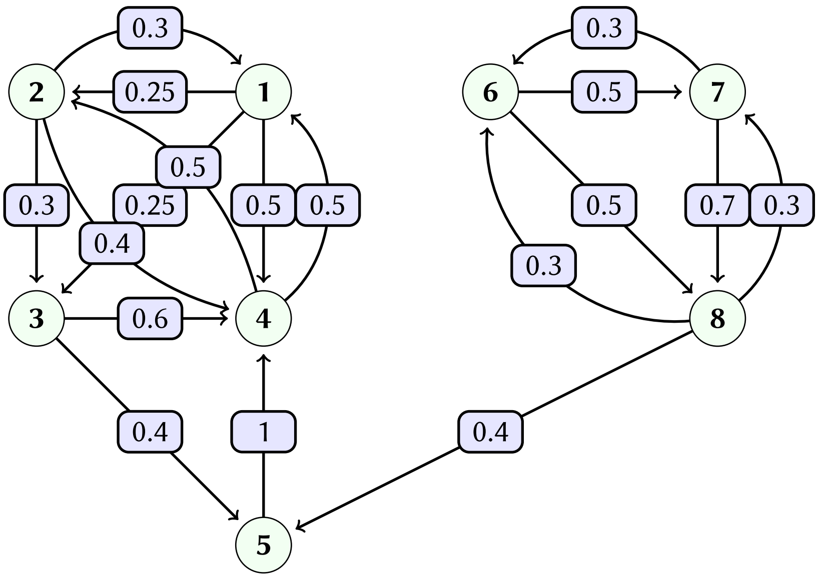

Figure 2: Diagram of Social Network G_2

tance to the rest. In contrast, the left group shows a clear hierarchical structure. For instance, agent 5 seems somewhat marginalized, being observed only by agent 3 with minimal importance. Conversely, agent 4 emerges as the influential figure or 'leader' within this group, observed by all other members, each attributing high importance to him.

This presents the model's flexibility in capturing manifold real social network characteristics and features within society. By carefully selecting the appropriate values for σ_{ij}'s, θ, and γ, the model can effectively represent various social dynamics such as cultural leaders, segregated minorities, groups in conflict, companies' or industries' agents. In the realm of immigration, this framework can adapt to different scenarios: unobserved immigrant communities (such as Haitians in Chile), communities under observation (such as Venezuelans), efforts by institutions in the host country to facilitate the integration of newcomers (through strategic link formation), and varying degrees of cultural disparity with the host society (reflected in the choice of γ), among others.

In our model, a disconnected social network essentially functions as a collection of independent societies, with its disjoint components exerting no influence on one another. The behaviour of a network of this kind can be explored by studying each of these components on their own.

Therefore, we restrict our attention to connected networks.

Let us now proceed to derive and enunciate the key results necessary to characterize the behaviour of our setup.

3.2 Preliminary remarks

Let $\mathbf{\Sigma} = [\sigma_{ij}]_{i,j \in N}$, where we define $\sigma_{ij} = 0$ if $j \notin N_i$. From (3.2) we have that $\mathbf{\Sigma}$ is a row stochastic (or simply stochastic) matrix that defines a Markov Process (MP). Here, agents would be the states of the MP and σ_{ij} the probability of moving from i to j. Powers $\mathbf{\Sigma}^k = [\sigma_{ij}^{(k)}]_{i,j \in N}$ of $\mathbf{\Sigma}$ are all stochastic matrices, and give the probability of moving from i to j in different numbers of steps.

$\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$ are the matrices defined by G_1 and G_2 respectively.

$$\mathbf{\Sigma}_1 = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 \\ 0.4 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0 & 1 \\ 0.5 & 0.5 & 0 & 0 \end{bmatrix}$$

$$\mathbf{\Sigma}_2 = \begin{bmatrix} 0 & 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.7 \\ 0 & 0 & 0 & 0 & 0.4 & 0.3 & 0.3 & 0 \end{bmatrix}$$

Let i and j be two states of the Markov Process defined by $\mathbf{\Sigma}$. As with the network equivalent, in the context of Markov Theory, it is said that i and j communicate if the probability of going from one of these states to another is positive (not necessarily in one step) and again we denote this by $i \leftrightarrow j$.

The relation defined by $\leftrightarrow$ is an equivalence relation. Therefore it defines a partition of the states in $\mathbf{\Sigma}$ - and therefore of the nodes in G - into equivalence classes. States communicating with each other form an equivalence class. These

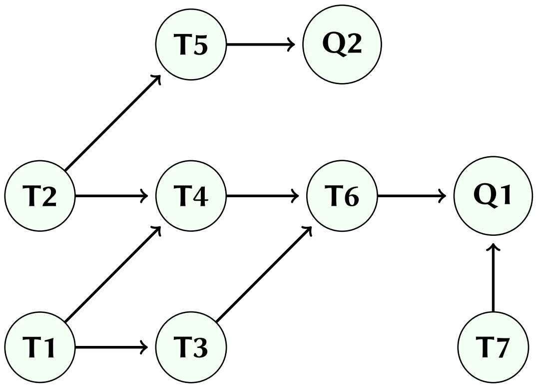

Figure 3: Diagram of a network of Transient and Ergodic Classes

classes correspond to the strongly connected components of the network.

For any of these equivalent classes one, and only one, of two conditions always applies: either a process that enters the class never leaves, or a process that leaves the class never returns. A class of the first kind is called an *ergodic set*, while one of the second kind is called a *transient set*. A Markov Process does not need to have a transient set. However, there always is at least one ergodic set (Kemeny and Snell, 1976).

For two communicating classes C_1 and C_2, we say that $C_1 \prec C_2$ if there exist $c_1 \in C_1$ and $c_2 \in C_2$ such that $c_1 \to c_2$. When the opposite happens, it would imply both classes are the same, since they communicate with each other. Then, the relation $\prec$ defines a partial order in $\mathcal{C}$. Maximal elements are equivalents to the ergodic sets and minimal elements to transient sets that are not heard by any other set.

Let us consider the bi-partition of $\mathcal{C}$ into $\mathcal{T}$ and $\mathcal{Q}$, the sets of transient classes and ergodic classes respectively. All classes in $\mathcal{Q}$ are disconnected.

Figure 3 shows the diagram of a network where each circle represents one of the communicating classes. In this example T_5, T_6, and T_7 are the maximal transient classes, while T_1, T_2, and T_3 are minimal classes.

This classification helps to elucidate the process of trait transition from ergodic to transient classes.

A state i has period k if any walk (in the network sense) that begins in i and ends in i occurs in a multiple of k-steps. Formally, the period of i is

$$k = \text{gdc}\{n > 0 : \Pr(X_n = i | X_0 = i) > 0\}. \tag{3.3}$$

If $k = 1$ the state i is said to be aperiodic. A Markov Process with only aperiodic states is called aperiodic.

An aperiodic MP consisting of only one ergodic set is called a *regular Markov process*. Let us express the following classic result central to the theory of regular Markov processes. The proof is found in any introductory book of Markov Theory.

Theorem 3.1. *If Σ defines a regular MP of n states, then there exists a unique (row) stochastic vector $\eta = (\eta_1, \eta_2, ..., \eta_n) \in \mathbb{R}^n$ such that*

(i) $\eta\boldsymbol{\Sigma}^k = \eta$, $\forall k \in \mathbb{N}$.

(ii) The powers of $\boldsymbol{\Sigma}$ converge to a matrix $\mathbf{H}$, for which each row equals η.

(iii) The vector η has full support. This is, $\eta_i > 0$ for each $1 \leq i \leq n$.

Such vector is known as the unique stationary distribution *of $\boldsymbol{\Sigma}$.*

In the theory of Finite Markov Processes, η holds a probabilistic interpretation, and it is the expected amount of times that the process goes through each state.

Furthermore, consider an MP that consists of one ergodic set and a finite number of transient sets, and in which there is a walk (in the network sense) from a state in each of the transient sets to one in the ergodic set (and not vice-versa, or they would be part of the ergodic set). Then, the process is expected to eventually leave the transient sets and enter the ergodic set with a probability of 1. Let us call an MP of this kind (and its network equivalent) *segregated*. As the name suggests, with this class we want to capture segregated social networks, in which some minorities' groups hear society but are not heard by any member of it.

We enunciate the following theorem which will be useful below, the proof can be found in Theorem 1.5 of Hartfiel (2006).

Theorem 3.2. *Let $\mathbf{\Sigma}$ be a stochastic matrix defining a segregated MP. Let $\mathbf{Q}$ denote the bottom right sub-matrix of $\mathbf{\Sigma}$ associated with the transient states.*

Then, as $k \to \infty$, $\mathbf{Q} \to 0$.

The key property of stochastic matrices in the context of Markov Theory is their averaging effect. If $\mathbf{\Sigma}$ is a stochastic matrix of size n and $\mathbf{w}$ any (column) vector of size n, then $\min_j \mathbf{w}_j \leq \sum_j \sigma_{ij}\mathbf{w}_j \leq \max_j \mathbf{w}_j$. This effect is condensed in the *coefficient of ergodicity* $\mathcal{T}(\mathbf{\Sigma})$ defined as

$$\mathcal{T}(\mathbf{\Sigma}) = \frac{1}{2}\max_{i,j}\sum_k |\sigma_{ik} - \sigma_{jk}| = 1 - \min_{i,j}\sum_k \min\{\sigma_{ik}, \sigma_{jk}\}, \tag{3.4}$$

which fulfils

$$\max_j(\mathbf{\Sigma w})_j - \min_j(\mathbf{\Sigma w})_j \leq \mathcal{T}(\mathbf{\Sigma})(\max_j \mathbf{w}_j - \min_j \mathbf{w}_j). \tag{3.5}$$

It is clear that $0 \leq \mathcal{T}(\mathbf{\Sigma}) \leq 1$. Strictly $\mathcal{T}(\mathbf{\Sigma}) < 1$ if, and only if, there is at least one row in Σ that is orthogonal to every other row. In particular, if one column of $\mathbf{\Sigma}$ has only positive entries, then it is guaranteed that $\mathcal{T}(\mathbf{\Sigma}) < 1$ (Hartfiel, 2006). This coefficient also fulfils

$$\mathcal{T}(\mathbf{\Sigma}^k) \leq \mathcal{T}(\mathbf{\Sigma})^k. \tag{3.6}$$

For $\rho \in (0,1)$ define $\mathbf{\Pi}(\rho)$ as

$$\mathbf{\Pi}(\rho) = \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}} \rho^k \Sigma^k. \tag{3.7}$$

This matrix is central to what ensues. We will need the following results.

Lemma 3.3. *Consider a stochastic matrix $\mathbf{\Sigma}$ of dimension n and $\rho \in (0,1)$, $\mathbf{\Pi}(\rho)$, as defined in (3.7), is a stochastic matrix. Furthermore, if $\mathbf{\Sigma}$ defines a regular MP, so does $\mathbf{\Pi}(\theta)$, and both have the same stationary distribution.*

Proof. Let $e = (1, 1, ..., 1) \in \mathbb{R}^n$ be the column vector with n 1's. A matrix $\mathbf{S}$ of dimension n is stochastic if, and only if, $Se = e$. Thus, let us compute

$$
\begin{aligned}
\mathbf{\Pi}(\rho)e &= \left(\frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\Sigma^k\right)e \\
&= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\Sigma^k e \\
&= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k e \\
&= \frac{1-\rho}{\rho}\cdot\frac{\rho}{1-\rho}e \\
&= e.
\end{aligned}
\tag{3.8}
$$

Now, $\mathbf{\Pi}(\rho)$ is defined as a weighted summation of the powers of $\mathbf{\Sigma}$. The weighted powers contain all non-negative entries. Hence, a pair of states communicate in $\mathbf{\Sigma}$ if, and only if, they communicate in $\mathbf{\Pi}(\rho)$. Particularly, if $\mathbf{\Sigma}$ defines a regular MP, $\mathbf{\Pi}(\rho)$ does too.

Lastly, assume $\mathbf{\Sigma}$ defines a regular Markov Chain and let η be its stationary distribution. Then,

$$
\begin{aligned}
\eta\mathbf{\Pi}(\rho) &= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\eta\mathbf{\Sigma}^k \\
&= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\eta \\
&= \frac{1-\rho}{\rho}\cdot\frac{\rho}{1-\rho}\eta \\
&= \eta.
\end{aligned}
\tag{3.9}
$$

■

$\mathbf{\Pi}(\rho)$ has the same stationary distribution as Σ. $\mathbf{\Pi}(\rho)$ defines a *decaying* MP where each step k occurs with probability $(1-\rho)\rho^{k-1}$. Longer steps are less likely to occur. In the context of Markov theory, $\pi_k^i := \mathbf{\Pi}_{ij}(\rho)$ is the probability that, starting in i, the process ends in j.

Consider one minimal transient class defined as before. A row that represents a node in this class is orthogonal to no other row. By contradiction, if it were, it would mean there is one other class that has no path from the first class. By definition, there is no path from another class to a minimal transient class. Hence, the network would be disconnected, which we have discarded. Thus, strictly $\mathcal{T}(\mathbf{\Pi}(\rho)) < 1$ in our setup.

Additionally, periods disappear in $\mathbf{\Pi}(\rho)$ since the length of a path from any i to j is either 1 or 0 in this matrix. Therefore, regardless of the periodicity of $\mathbf{\Sigma}$, $\mathbf{\Pi}(\rho)$ is aperiodic.

Let $\pi^i = (\pi^i_1, ..., \pi^i_N)$, $\forall i \in N$. $\pi^i(\rho)$ is a probability measure that's agent specific. Hence, for any $\nu \in \mathbb{R}^N$ we can define a random variable (r.v.) ν^i as

$$v^i = \begin{cases} \nu_1 & \text{with prob. } \pi^i_1(\rho) \\ \vdots & \vdots \\ \nu_N & \text{with prob. } \pi^i_N(\rho) \end{cases}, \ \forall i \in N$$

Then, given the r.v. ν^i, its expected value is

$$\mathbb{E}_{\pi^i(\rho)}[\nu] = \sum_{j \in N} \pi^i_j(\rho)\nu_j.$$

We are now prepared to find the equilibria of the model.

3.3 Static equilibrium

Let $\mathbf{\Gamma}$ be the simultaneous move N-player game defined by $\mathbf{\Sigma}$, with agents in N, payoffs as in (3.1), and strategy spaces $\mathbb{R}$. We are first interested in computing the Nash equilibrium of this game.

Proposition 3.4. *For any $\mathbf{\Sigma}$, a Nash equilibrium exists, is unique and is given by*

$$a^*_i = (1-\theta)\gamma_i + \theta\mathbb{E}_{\pi^i(\theta)}[\gamma], \ \forall i \in N. \tag{3.10}$$

Proof. Let $i \in N$. The first derivative of $\mathcal{U}_i$ with respect to a_i is

$$\begin{aligned}
\frac{\partial \mathcal{U}_i}{\partial a_i} &= -2(1-\theta)(a_i-\gamma_i) - 2\theta \sum_{j\in N} \sigma_{ij}(a_i - a_j) \\
&= -2(1-\theta)(a_i-\gamma_i) - 2\theta \sum_{j\in N} \sigma_{ij} a_i + 2\theta \sum_{j\in N} \sigma_{ij} a_j \\
&= -2(1-\theta)(a_i-\gamma_i) - 2\theta a_i + 2\theta \sum_{j\in N} \sigma_{ij} a_j \\
&= 2(1-\theta)\gamma_i - 2a_i + 2\theta \sum_{j\in N} \sigma_{ij} a_j.
\end{aligned} \tag{3.11}$$

A vector of actions $\mathbf{a}^* = (a_1^*, a_2^*, ..., a_N^*) \in \mathbb{R}^N$ is a Nash equilibrium in pure strategies of the game if, and only if, $\forall i \in N$ it solves

$$\begin{aligned}
& \frac{\partial \mathcal{U}_i}{\partial a_i}(a_i^*; \mathbf{a}^*_{-\mathbf{i}}) = 0 \\
\iff & 2(1-\theta)\gamma_i - 2a_i^* + 2\theta \sum_{j\in N} \sigma_{ij} a_j^* = 0 \\
\iff & a_i^* - \theta \sum_{j\in N} \sigma_{ij} a_j^* = (1-\theta)\gamma_i.
\end{aligned} \tag{3.12}$$

The last line in (3.12), since it works $\forall i \in N$, defines the set of N equations

$$\begin{aligned}
& (\mathbf{I} - \theta\mathbf{\Sigma})\mathbf{a} = (1-\theta)\gamma \\
\iff & \mathbf{a} = (1-\theta)\Big[\sum_{k=0}^{\infty} \theta^k \mathbf{\Sigma}^k\Big]\gamma \\
& = (1-\theta)\Big[\mathbf{I} + \sum_{k=1}^{\infty} \theta^k \mathbf{\Sigma}^k\Big]\gamma \\
& = (1-\theta)\gamma + (1-\theta)\Big[\sum_{k=1}^{\infty} \theta^k \mathbf{\Sigma}^k\Big]\gamma \\
& = (1-\theta)\gamma + \theta\Big[\frac{1-\theta}{\theta}\sum_{k=1}^{\infty} \theta^k \mathbf{\Sigma}^k\Big]\gamma \\
& = (1-\theta)\gamma + \theta\mathbf{\Pi}(\theta)\gamma,
\end{aligned} \tag{3.13}$$

and this last expression yields (3.10). ■

Notice first that, if there is no interaction between agents, i.e. $\theta = 0$, then $a_i^* = \gamma_i$. Hence, with no interaction, equilibrium actions are agents' types. Second, let $\tilde{\gamma} \in \mathbb{R}$, if $\forall i \in N$ $\gamma_i = \tilde{\gamma}$, then $a_i^* = \tilde{\gamma}$. Therefore, if agents are homogeneous, the network's topology does not affect equilibrium actions. Thus, is the heterogeneity in the population what affects equilibrium actions through the network's topology and the distribution of γ across players.

We can separate additively the individual effect of their own type from the network effect on every player's equilibrium actions in (3.10).

$$a_i^* = (1-\theta)\underbrace{\gamma_i}_{\substack{\text{No}\\\text{interaccion}}} + \theta \underbrace{\mathbb{E}_{\pi^i(\theta)}[\gamma]}_{\text{Network effect}} \tag{3.14}$$

With this notation, we have a clear interpretation of *spillover* effects:

$$\begin{aligned} \forall i, j \in N,\ i \neq j : \frac{\partial a_i^*}{\partial \gamma_i} &= (1-\theta) + \theta \pi_i^i(\theta), \\ \text{and } \frac{\partial a_i^*}{\partial \gamma_j} &= \theta \pi_j^i(\theta). \end{aligned} \tag{3.15}$$

Moreover, we have the following result.

Corollary 3.4.1. *Regarding how agents affect the behaviour of others, we have:*

- *If i does not hear j, then $\pi_j^i(\theta) = 0$ and the trait of j has no effect on i's behaviour.*
- *If $\forall i, j \in N$, $i \leftrightarrow j$, then $\pi_j^i(\theta) > 0$ $\forall i, j \in N$, and each agent's trait affects the behaviour of the rest.*

In a society where every pair of agents communicates, each of the agents' traits persists in equilibrium. More central agents, in the sense that the MP defined by the network passes by them more often, are the ones whose traits are the most persistent.

Once we have identified how the network's topology shapes the equilibrium, we can go a little deeper.

When $\mathbf{\Sigma}$ is a strongly connected network, we have the following decomposition.

Proposition 3.5. *Let $\mathbf{\Sigma}$ be a strongly connected network. Then, for each $i \in N$ the network effect $\nu_i = \mathbb{E}_{\pi^i(\theta)}[\gamma]$ can be uniquely decomposed as*

$$\nu_i = \underbrace{\mathbb{E}_\eta[\gamma]}_{\textit{Structural effect}} + \underbrace{\Delta_i(\gamma)}_{\textit{Local effect}}, \tag{3.16}$$

where η is the stationary distribution of $\mathbf{\Sigma}$, and the local effects *are bounded as a function of the size of θ, the ergodicity of the chain, and the position of the agents in the network.*

Proof. Let $d_{ij} = \pi^i_j - \eta_j$. Since η is unique, d_{ij} is well-defined and unique for each i and j. Let $\mathbf{d}_i = (d_{i1}, d_{i2}, ..., d_{iN})$ we can then decompose ν_i as

$$\mathbb{E}_{\pi^i(\theta)}[\gamma] = \mathbb{E}_\eta[\gamma] + \mathbf{d}_i\gamma \tag{3.17}$$

And we define $\Delta^i_\theta(\gamma) = \mathbf{d}_i\gamma$ as the local effect. Now, any stochastic matrix has the averaging effect described above. Consider, for each pair of agents i and j, the expression

$$|\sigma_{ij} - \eta_j| \leq |\sigma_{ij} - \eta_j|.$$

A self-evident tautology. Yet, when combined with the aforementioned averaging effect of $\mathbf{\Sigma}$ it yields the following set of inequalities.

$$\begin{aligned}
|\sigma_{ij} - \eta_j| &\leq |\sigma_{ij} - \eta_j| \\
|\sigma^{(2)}_{ij} - \eta_j| &\leq \mathcal{T}(\mathbf{\Sigma})|\sigma_{ij} - \eta_j| \\
|\sigma^{(3)}_{ij} - \eta_j| &\leq \mathcal{T}(\mathbf{\Sigma^2})|\sigma_{ij} - \eta_j| \\
&\vdots \\
|\sigma^{(k)}_{ij} - \eta_j| &\leq \mathcal{T}(\mathbf{\Sigma}^{k-1})|\sigma_{ij} - \eta_j| \\
&\vdots
\end{aligned} \tag{3.18}$$

We can then perform the following rearrangement after multiplying the k-th inequality in (3.18) by the k-th power of θ.

$$
\begin{aligned}
|\theta\sigma_{ij} - \theta\eta_j| &\leq \theta|\sigma_{ij} - \eta_j| \\
|\theta^2\sigma_{ij}^{(2)} - \theta^2\eta_j| &\leq \theta^2\mathcal{T}(\mathbf{\Sigma})|\sigma_{ij} - \eta_j| \\
|\theta^3\sigma_{ij}^{(3)} - \theta^3\eta_j| &\leq \theta^3\mathcal{T}(\mathbf{\Sigma^2})|\sigma_{ij} - \eta_j| \\
&\vdots \\
|\theta^k\sigma_{ij}^{(k)} - \theta^k\eta_j| &\leq \theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1})|\sigma_{ij} - \eta_j| \\
&\vdots
\end{aligned}
\tag{3.19}
$$

Adding all inequalities in (3.19) and using the Triangular Inequality on the terms inside absolute values on each left side, yields:

$$
\begin{aligned}
\Big|\sum_{k=1}^{\infty}\theta^k\sigma_{ij}^{(k)} - \sum_{k=1}^{\infty}\theta^k\eta_j\Big| &\leq \sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1})|\sigma_{ij} - \eta_j| \\
\Rightarrow \frac{1-\theta}{\theta}\Big|\sum_{k=1}^{\infty}\theta^k\sigma_{ij}^{(k)} - \frac{\theta}{1-\theta}\eta_j\Big| &\leq \frac{1-\theta}{\theta}\Big(\sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1})|\sigma_{ij} - \eta_j|\Big) \\
\Rightarrow |\pi_j^i - \eta_j| &\leq \frac{1-\theta}{\theta}|\sigma_{ij} - \eta_j|\sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1}) \\
&\leq \frac{1-\theta}{\theta}|\sigma_{ij} - \eta_j|\sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma})^{k-1} \\
&= (1-\theta)|\sigma_{ij} - \eta_j|\sum_{k=0}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma})^k \\
&= \frac{(1-\theta)|\sigma_{ij} - \eta_j|}{1-\theta\mathcal{T}(\mathbf{\Sigma})}
\end{aligned}
\tag{3.20}
$$

Hence,

$$
\begin{aligned}
|\Delta_i(\gamma)| &= |\mathbf{d}_i\gamma| \\
&\leq \frac{(1-\theta)|\sigma_{ij} - \eta_j|}{1-\theta\mathcal{T}(\mathbf{\Sigma})}\sum_{N}\gamma_i
\end{aligned}
\tag{3.21}
$$

■

Thus, every agent's actions fluctuate around a common structural effect derived from the topology of the network. If their actions are above or below that level, depends on their position on the network. The size of the deviation from this level for agent i is bounded by: the size of θ, how far the underlying Markov Process -through the perspective of i - is from its stationary state, and the ergodicity of $\mathbf{\Sigma}$. Thus, the tighter and more stationary is the network, the smaller the local effect relative to the structural one.

Let $M_i = \max_{j\in N}\{\sigma_{ij}\}$ and $m_i = \min_{j\in N}\{\sigma_{ij}\}$. These terms provide a more directly observable -yet less strict- bound for the local effect through the expression $|\sigma_{ij} - \eta_j| \leq M_i - m_i$, derived (once more) thanks to the averaging effect of stochastic matrices.

As we will see in the dynamic model, these bounds tend to become stricter as time passes, shrinking the local effect until it disappears.

3.4 Dynamic equilibrium

Now, to represent the ongoing nature of cultural traits over time, we model cultural traits as a continuous variable in discrete time $\mathcal{T} = \{0, 1, 2, 3...\}$. For this, we use an overlapping generations model (OLG). In this setup, agents are called *dynasties*. Each dynasty is represented by one agent who reproduces asexually in each period. Thus, in every period there is a new generation representing each dynasty.

Time starts at $t = 0$, where the initial generation has an exogenous type $\gamma_0 = (\gamma_{1,0}, .., \gamma_{N,0})$. The main feature of OLG is that, from then on, dynasties' types are endogenous and are given by parents' behaviour in the previous period. This is:

$$\gamma_{i,t} = a^*_{i,t-1}, \ \forall i \in N, \ t = 1, 2, 3...$$

Dynasties' payoff in each period is:

$$\mathcal{U}(a_{i,t}, \mathbf{a}_{N_i,t}) = -(1-\theta)(a_{i,t} - \gamma_{i,t})^2 - \theta \sum_{j\in N_i} \sigma_{ij}(a_{i,t} - a_{j,t})^2$$

Now, we want to check the convergence of behaviour (i.e., cultural traits) as $t \to \infty$, this is, check the existence of

$$\mathbf{a}^* = \lim_{t\to\infty} \mathbf{a}^*_t$$

where $\mathbf{a}_t^* = (a_{1,t}^*, ..., a_{N,t}^*)$.

Let us first show the following result.

Lemma 3.6. *The Nash equilibrium of the game at each period is given by*

$$\mathbf{a}_t^*(\theta) = \mathbf{\Omega}^t(\theta)\gamma_0,\ t \in \mathcal{T}\backslash\{0\}, \tag{3.22}$$

where $\mathbf{\Omega}(\theta) := (1-\theta)\mathbf{I} + \theta\mathbf{\Pi}(\theta)$

Proof. From (3.13) we have

$$\begin{aligned}\mathbf{a}_t^*(\theta) &= (1-\theta)\gamma_t + \theta\mathbf{\Pi}(\theta)\gamma_t \\ &= ((1-\theta)\mathbf{I} + \theta\mathbf{\Pi}(\theta))\mathbf{a}_{t-1}^*.\end{aligned}$$

By applying recursion on this last term, (3.22) follows. ∎

Hence, the convergence of cultural traits depends entirely on $\mathbf{\Omega}(\theta)$.

Regard the contracting effect of stochastic matrices condensed in $\mathcal{T}$ and discussed above. Notice how (3.6) implies that $\mathcal{T}(\mathbf{\Omega}(\theta)^k) < \mathcal{T}(\mathbf{\Omega}(\theta))$, and this is true for every class of stochastic matrix. Therefore, $\mathbf{a}_t^*(\theta)$ can only become more contracted over time. This implies that the traits of individuals do not diversify as time passes. Since $\mathcal{T}(\mathbf{\Omega}(\theta)) < 1$, the process of homogenization is pervasive; the traits become strictly less diverse as time passes.

Furthermore, consider a segregated social network as defined above. From Theorem 3.2 and Lemma 3.22, the traits of the segregated agents disappear in the long run. Thus, when groups of immigrants are isolated from society, the influence of their idiosyncrasy and culture on society fades as time passes. They become completely absorbed into the cultural traits of local inhabitants.

Therefore, we need to characterize what occurs in ergodic sets. With a strongly connected network, we have convergence to the same point.

Proposition 3.7. *If* $\mathbf{\Sigma}$ *defines a strongly connected network, then*

$$a_i^* = \mathbb{E}_\eta[\gamma_0],\ \forall i \in N \tag{3.23}$$

where η *is the unique stationary distribution of* $\mathbf{\Sigma}$.

Proof. If $\mathbf{\Sigma}$ is a regular MP, then so is $\mathbf{\Pi}(\theta)$ by Lemma 3.3, and it follows immediately that $\mathbf{\Omega}(\theta)$ is regular. Let η be the stationary distribution of $\mathbf{\Sigma}$. Then,

$$\begin{aligned} \eta\mathbf{\Omega}(\theta) &= (1-\theta)\eta\mathbf{I} + \theta\eta\mathbf{\Pi}(\theta) \\ &= (1-\theta)\eta + \theta\eta \\ &= \eta \end{aligned} \tag{3.24}$$

Hence, η is also the unique stationary distribution of $\mathbf{\Omega}$. This implies:

$$\begin{aligned} \mathbf{a}^* &= \lim_{t\to\infty} \mathbf{a}_t^* \\ &= \lim_{t\to\infty} \mathbf{\Omega}^t(\theta)\gamma_0 \\ &= \mathbf{H}\gamma_0, \end{aligned} \tag{3.25}$$

where $\mathbf{H}$ is as defined in Theorem 3.1. Computing the last expression for each component of $\mathbf{a}^*$, yields (3.22).

■

Because $\mathbf{\Sigma}$ is strongly connected, we know that η has full support. The influence on the asymptotic behaviour of each dynasty is given by the inverse of the mean times the MP passes through it. Notice how, in the long run, the parameter θ does not affect the equilibrium value, since η is directly derived from $\mathbf{\Sigma}$. Thus, no matter the relative importance that agents give to their values relative to their peers' actions, society will eventually converge to a common trait underlying structurally in the topology of the network through η.

Moreover, recall the formula in equation 3.21 for the local effect. The term $|\sigma_{ij} - \eta_j|$ tends to 0 as society tends to equilibrium. Hence, the bounds inside which local effects lie strictly diminish over time – the structural effect's sway grows for all agents.

We have turned our focus to where the equilibrium lies, and not to how far the society is from it. For a closing commentary on this, the coefficient of ergodicity proves insightful yet another time. Let us denote $\lambda_1, \lambda_2, \lambda_3, \ldots$ as the spectral values of a row-stochastic matrix $\mathbf{\Sigma}$, ordered in decreasing module magnitude. Given that the sum of each row in $\mathbf{\Sigma}$ equals 1, it follows that $\lambda_1 = 1$. Hence, the spectral gap of a stochastic matrix is $|1 - \lambda_2|$, where $|\lambda_k|$ denote the modulus of possibly complex $|\lambda_k|$. The crucial property of the coefficient of ergodicity is that $|\lambda_2| < \mathcal{T}(\Sigma)$, as outlined in Hartfiel (2006). Matrices with a large spectral gap mix faster, which in turn means that matrices with low $\mathcal{T}(\Sigma)$ move quicker to their steady state. Therefore $\mathcal{T}(\Sigma)$ is an adequate bound for quantifying the distance of the network from its stationary state.

4 Comparative Dynamics

This setup allows us to study a myriad of combinations of networks, weights and traits. It allows us to delve into a variety of economic challenges and phenomena where preferences play a role. Its computational tractability permits the simulation of large sets of nodes and links.

As a mode of illustration, and in line with our introductory motivation, let us revise the following simple scenarios related to migration.

4.1 Tightly connected network with heterogeneity among the agents

Let us begin with a simple example by computing the equilibrium in network G_1 in Figure 1 defined above. We define the set of traits $\gamma_1 = (0, 0, 1, 1)$, i.e., there are two sharply different traits in the social network. We consider two values of theta: $\theta = 0.5$ and $\theta = 0.2$. Figures 4 and 5 show the evolution of the traits in both scenarios.

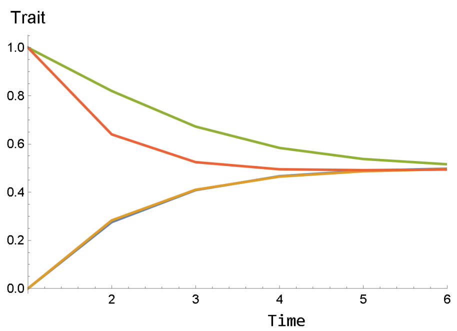

Figure 4: G_1 for $\theta = 0.5$

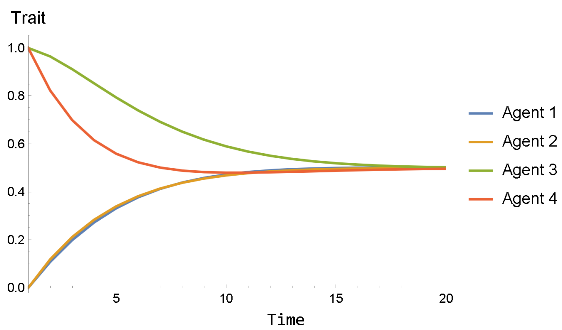

Figure 5: G_1 for $\theta = 0.2$

Convergence was considerably lower when the value of θ was decreased to 0.2. As noted above, the long-run common trait was the same in both cases, independently of θ. Yet, even in this simple strongly connected case, θ plays a significant role in the speed of convergence. Agent 3 takes the longest in converging, given he does not listen to any agent with trait 0. However, he converges not much later than the rest. Given the quasi-symmetry of the network, this trait was a normal average of both traits in this society.

4.2 Arrival of an unobserved homogeneous group of migrants

Let us now turn to the Network G_2 in Figure 2. We call (aptly) the leftmost group the *host society*. The average trait of this group is 1. Yet we consider two cases: one with some diversity around this trait that averages to 1, and another where the host is already in the stationary state when the migrant group arrives. Namely:

- Diverse host: $\gamma_d = (1.5, 1.3, 1, 0.7, 0.5, 0, 0, 0)$.
- Stationary host: $\gamma_s = (1, 1, 1, 1, 1, 0, 0, 0)$.

The rightmost group is the just-arrived group of migrants, which is homogeneous and shares a trait equal to 0. From now on $\theta = 0.5$. Figures 6 and 7 show the dynamics.

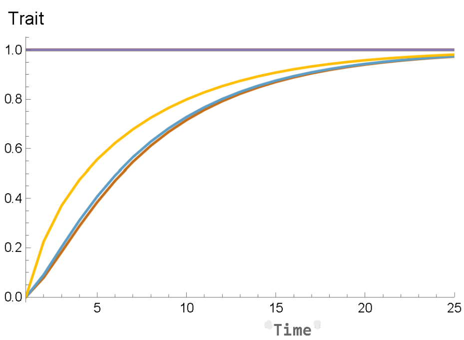

Figure 6: Stationary host. G_2.

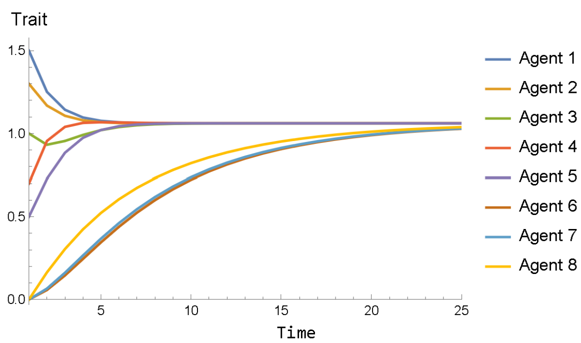

Figure 7: Diverse host. G_2.

The immigrant that observes the host society (agent 8) assimilates considerably faster in the first periods than the rest of the migrants, even when they observe 8. As expected, in the long run, when the host society -which is simply the purple line in Figure 6 - does not observe the migrants or these hear the locals, the traits of the migrants disappear, and society converges to the same stationary state it was in the beginning. The speed of convergence was slower than in the previous example, taking almost five times more periods to converge to a common trait. Even when society may be heading towards a melting pot, this process may be slow-moving for a large set of agents.

For the example where the host society was heterogeneous, the average trait of the entire network after 50 periods stabilizes at 1.06. When it is homogeneous, it stabilizes at 1, as we can expect. Even when the average trait of the host society was slightly perturbed around the average of 1, we see how the topology of the network affects the convergence result. The speed of convergence is similar in both cases.

Consider now the Network G_3, which is the same as Network G_2 except that agent 8 now does a greater assimilation effort with the host society, paying more attention to agent 5.

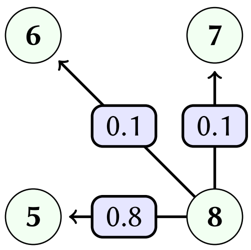

Figure 8: Change in behaviour from agent 8 in G_3

Now, agent 8 makes a higher assimilation effort than before. Figure 8 depicts this modification. Figures 9 and 10 show the resulting dynamics.

After $t = 12$ the immigrants virtually assimilated in both cases. Making one individual in the group pay more attention to the host society, the speed of convergence reduced nearly by half. This hints at the power of peer effects in the assimilation process: if even a few members of a group of migrants are induced to blend with the host society, the rest will soon follow.

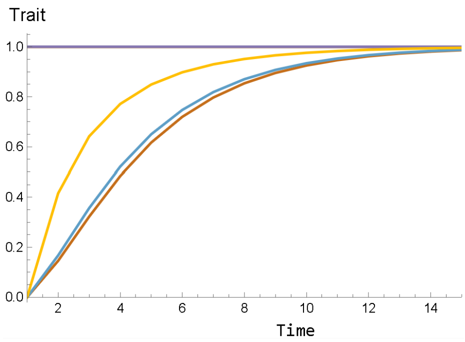

Figure 9: Stationary host. G_3.

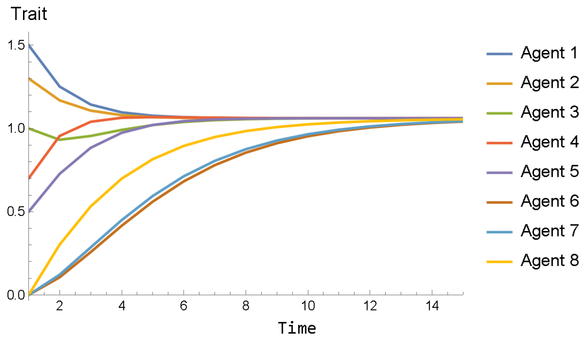

Figure 10: Diverse host. G_3.

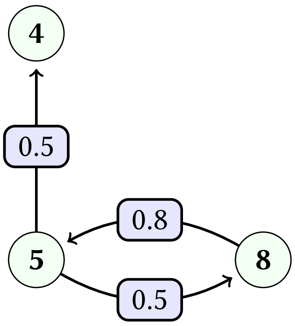

Figure 11: Change in behaviour from agent 5 in G_4

4.3 Arrival of an observed homogeneous group of migrants

Let G_4 be a social network identical to G_3, except that now agent 5 observes agent 8. Figure 11 depicts this modification.

Now the host society hears the group of immigrants, forming one strongly connected network. The vectors of traits are as in the previous case. Figure 14 charts the evolution of the average trait in both cases.

Society reaches its stationary state at around period 20. In both scenarios, the average trait is heavily skewed towards the local culture. However, now the migrants' culture persists within them, and the host country adopts part of their traits. In the diverse case, the interaction with the topology of the host society made its trait persist more robustly, with an equilibrium trait close to 0.95 against 0.9 in the stationary case. Convergence happens at about the same pace as when the immigrants were not observed.

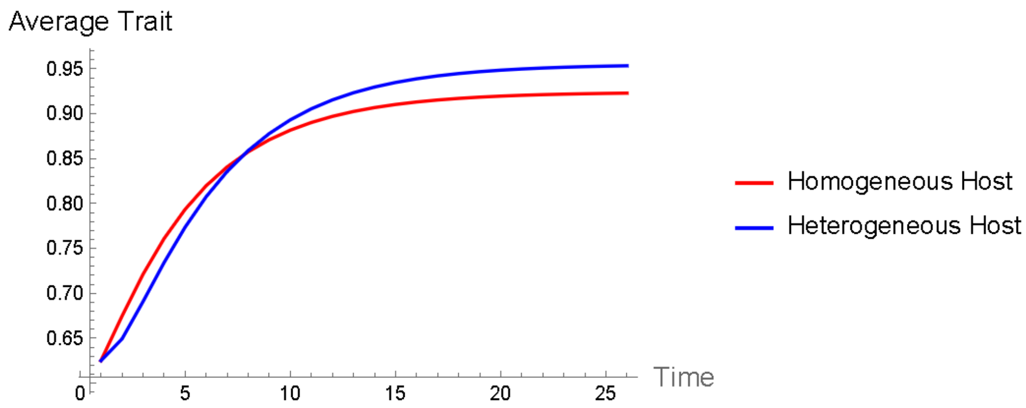

Figure 12: Evolution of average trait. G_4.

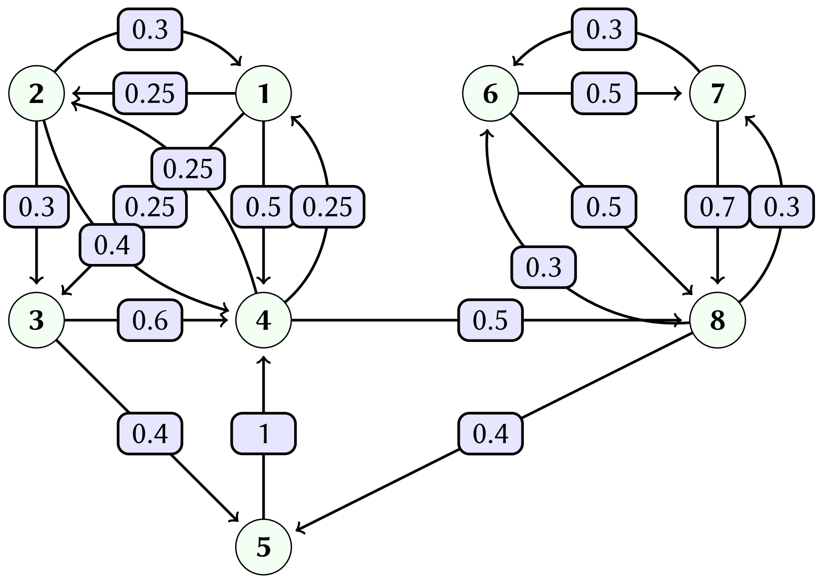

Figure 13: Diagram of Social Network G_5

4.4 Group of migrants observed by a cultural leader

Interpreting the roles of the agents in Network G_4, agent 5 has minimal influence on the host society. What occurs instead, when cultural leaders attempt to desegregate the immigrant group? Consider it is agent 4 (the leader of the host society) the one who directly communicates with agent 8. We alter the network G_4 to implement this effect: the direct link from agent 5 to agent 8 is eliminated, and instead, agent 4, who represents the leader of the host society, establishes a direct observational link with agent 8. Social Network G_5 outlined in Figure 13 accounts for this change.

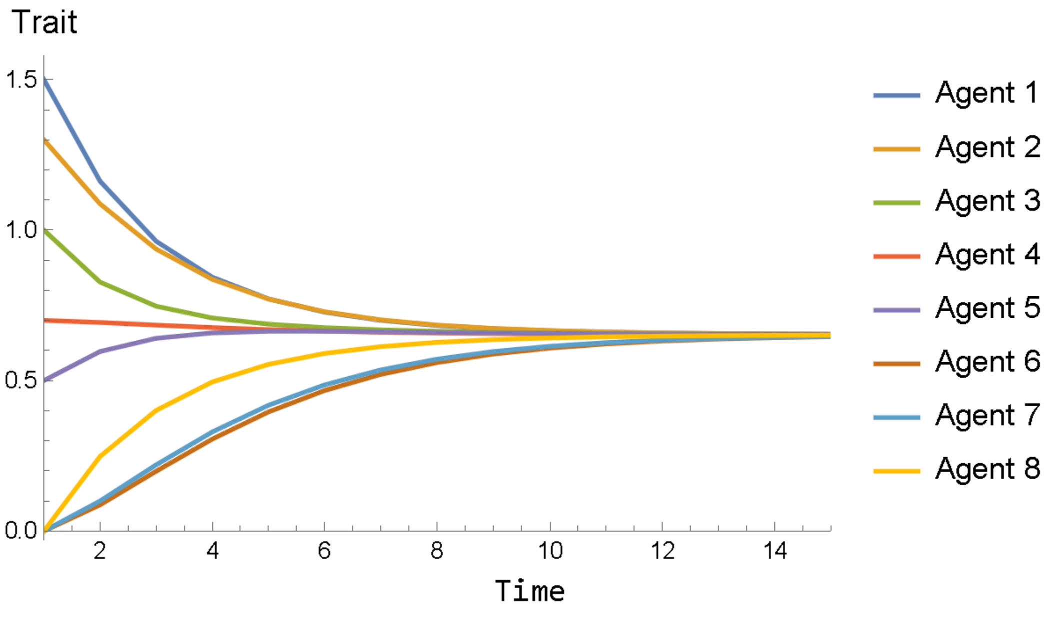

Figure 14: Diverse host. G_5.

We focus on the diverse host scenario. By $t = 12$, it appears that society has reached a state of convergence, with the long-run average cultural trait stabilizing at 0.65.

While the rate of convergence remains similar to that observed in the previous example, the persistence of immigrants' cultural traits within the society is notably stronger in this scenario, with a long-run average trait close to 0.65. This outcome suggests that cultural leaders, whether they are individuals or institutions, can play a crucial role not only in preserving the unique idiosyncrasies of immigrants but also in facilitating their integration into the host society.

In all our examples, convergence to the steady state is monotonic. This is determined by the matrix $\mathbf{\Omega}$, associated with each example, having real eigenvalues. However, since it is entirely possible for a stochastic matrix to have complex eigenvalues, convergence in more complex scenarios may introduce oscillatory dynamics.

5 Concluding Remarks

In this study, we explored a model of cultural trait transmission among agents within a social network context. This investigation was driven by numerous instances in Economics where the transmission of culture is crucial. The work was initially motivated by creating a theoretical setup that allowed exploring the consequences of sudden flows of immigration. However, the versatility of our model and its findings extend far beyond this specific scenario, offering broader applicability in various economic fields.

Our main findings are a succinct formula and decomposition that summarizes the effect of networks' topology on the equilibrium through the underlying Markov Process. Moreover, we derived explicit formulas for the spillover effect of each agent's type on the rest of the population. In the dynamic model, we obtained an expression for the equilibrium in each period. We found which agents have persistent traits in the long run. Then, we concluded that, in a strongly connected network, society eventually converges to one trait, defined by a weighted average of every individual's trait and where the network's topology acts through the stationary distribution of the MP.

A key insight from our research is the significant impact of network topology on the dynamics within our model. Much of the literature studying specific traits and leanings overlooks the spatial distribution of agents. Our findings highlight the critical importance of the network's geometry in these dynamics. Structure *matters*. This understanding is essential for a more accurate and holistic view of the dynamics at play in economic and social contexts, especially in scenarios involving cultural interactions and preference formation.

The outcomes of our model and simulations allow us to comment on some policy implications. When society does not pay attention to a minority, its characteristic traits tend to disappear. This is problematic given it is generally desirable for a minority to keep part of their identity. Moreover, traits of native and ancient cultures usually want to be perpetuated in society for heritage value reasons. However, our results hint that society must make an active effort to assimilate these cultures' traits to keep them alive.

The comparative dynamics hint at some useful policy implications for the inclusion of migrants into society, such as the vital role of specialized institutions and the strength of the peer effect that a few individuals of a minority who are assimilating have on the rest of their group. While these insights are illustrative, they highlight the need for more complex network analyses to robustly validate these assertions. The principal policy implication we find from our model is the necessity of accounting for the structure underlying society. To conduct effective policy, understanding and modelling societal network distributions is paramount and cannot be overlooked. Our model provides a succinct framework that can be employed as is or as a starting point for further exploration and situational adjustment.

We view this work as a starting point to elucidate the effect of different social networks' topologies on society's equilibrium behaviour. While our analysis is based on a set of simplified assumptions to capture societal complexities, it opens up a myriad of avenues for further research. Lines for future explorations include: checking different topologies representing particular real-world phenomena in more intricate setups than those presented; applying empirical data on the model; focusing on a granular characterization of convergence speed; or extending the model to dynamic or endogenous network formation.

This research not only sheds light on social network impacts but also lays the groundwork for more comprehensive studies that can significantly contribute to urban and community development, social strategy implementation and, consistent with the opening quote of this paper, economic policy development in general.

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