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“Sobre la evolución de los rasgos culturales en las redes sociales”

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On the evolution of cultural traits in social networks

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Abstract

Cultural traits' evolution is studied in a theoretical setup, where each individual (or minority) is part of a social network. Individuals behave strategically and face a trade-off between acting like their observed peers or keeping faithful to their idiosyncrasy. We characterize the Nash equilibrium of this game using the theory of Finite Markov Processes and find a sufficient statistic that summarizes the effect of the topology of the network in this equilibrium. We extend this to an overlapping generations model, where parents' actions determine the trait of their offspring. We identify the Nash equilibrium in each period and show that, when the network is strongly connected, all society converges to the same trait. Lastly, we use the results to illustrate, with comparative static, the consequences of the entry of a group of immigrants to a host country and discuss some policy implications of our findings.

Keywords: Social Networks, Assimilation, Cultural Traits, Immigration, Games on Networks.

1 Introduction

The observation that humans acquire valuable life skills and knowledge through copying others has been the focus of animal behaviorists' attention dating back to Darwin.

Culture is information that people acquire from others by teaching, imitation, and other forms of social learning. People acquire skills, beliefs, and values from those around them, shaping their behavior. Humans, unlike any living creature, have cumulative cultural adaption. Humans hold their knowledge and learn things from others, improve them, pass this knowledge to the next generation, and so on (Boyd and Richerson, 2005). It is this unique mechanism which leads to the rapid non-genetic evolution of superbly designed adaptations to particular environments [1] . Hence, the underlying social structure where humans interact plays a prominent role in the evolution of cultural traits.

Preferences, beliefs, norms, and habits are formed as a result of heritable traits. They are transmitted from generation to generation and shaped by the social interactions of the individual. Thus, several social sciences pay attention to the role of vertical (parents), oblique (role models), and horizontal (peers) cultural transmission. For Economics, the transmission of cultural traits between generations and peers plays an essential role in determining the individuals' preference traits, such as discounting, purchasing patterns, patience, and risk aversion (Robson and Samuelson, 2011). Furthermore, it plays a crucial part in determining fertility practices, and the attitudes and social norms towards family and community that shape social capital (Guiso et al., 2008).

While the cultural transmission is convenient for the understanding of varied phenomena in the field of Economics, it is of particular interest for illustrating the assimilation process of immigrants in the host country and their effect on the local inhabitants.

Assimilation with the local culture for immigrants has been positively correlated with their life satisfaction. Angelini et al. (2015) obtain conclusions in line with this view by using self-reported measures of well-being and panel data from migrants in Germany. Empirical work has looked at the impact of immigrants' social networks on their assimilation choice. Damm (2009) and Edin et al. (2003) use ethnic concentration/enclave as a proxy for immigrants' networks in a host country. The language group and proficiency of immigrants have been used as a proxy of the network effect, too (Bertrand et al., 2000). Besides, the language skills of immigrants influenced their performance in the

[1] It is in this context that Dawkins (1989), when looking for a noun to convey the idea of unit of cultural transmission, coined the term *meme* as an inflexion of the greek word *mimeme* (something that is imitated) to make it sound like *gene*

labor market in the United States (Chiswick and Miller, 2002).

Large inflows of immigration tend to carry a consequential impact on the labor market. Immigrants usually bring skills and know-how that differ from those appearing and required in the local markets. Moreover, individuals with low skills and low education levels may self-select themselves to migrate to a foreign country. Language barriers may be present as well. Cultural differences also play a role in how the newly-arrived adjust to the practices and customs of the working community.

This is of particular relevance in Chile, where the immigrant population has shown an accelerated increase during the last decade. According to Instituto Nacional de Estadísticas (2018), international immigrants rose from 1.27% of the total population in the 2002 Census to 4.35% of the total in 2017 Census.[2] Plus, around two-thirds of the total immigrants censused in 2017 arrived in the country between 2010 and 2017. Furthermore, 61% of the total migrated between 2015 and 2017.

By 31 December of 2019, it is estimated that of the total population of immigrants in the territory, 30,5 % is from Venezuela, 15,8% from Peru, and 12,5 % from Haiti (Instituto Nacional de Estadísticas y Departamento de Extranjería y Migración, 2019).

Of all of the newly arrived communities, the Haitian is a singular one and has been particularly visible for the Chilean people, mainly for two reasons. Firstly, it is the only one of the big groups arriving from America for which the native language is not Spanish (Chile's national language), but the French/Creole. Secondly, it is the only dominantly black community that arrived in the country.

A series of natural disasters in the Caribbean country and a stagnant economy have propelled the exodus from Haiti, a country now widely regarded as a failed state (Torgman, 2012). Most of those arriving from the Creole-speaking nation turned to Chile because of its stable high-income economy, and its lenient policy in handling visas compared to the alternatives. Indeed, around 186,600 Haitians lived in Chile by the end of last year, roughly 1.7 % of Haiti's population.

However, Haitians find it arduous to get absorbed in the society and the labor market. The ethnic and cultural differences between Haitians and the local population, and the language barrier have contributed to this. Moreover, Haitians present the lowest percentage of educational level and the highest of individuals without formal education of all the rest of countries of origins

[2]Censuses in Chile usually take place every 10 years, as in most countries. But in what was a public national scandal, the data collected during the 2012 Census was deemed "unfit to the standards required for the operative to be denominated as a census" (Instituto Nacional de Estadísticas, 2014), and thus discarded. The procedure was then repeated on 2017

of the immigrant population (see Figure 4 in Aldunate et al. (2019)). All is combined with the massive inflow of immigrants recently arrived. Natives of the Caribbean nation face racism, social exclusion, lack of job opportunities, abuse and underpayment by their employers, overcrowding in segregated areas of the city, and resentment from the Chileans.

Most Haitians arrive in Chile with no local connections and insert into already segregated communities of migrants. Their lack of Spanish language proficiency further aggravates the problem. The majority believes that it is not essential to know Spanish before their arrival in Chile. Thus, they come with no skills in the local language, hindering their integration in society and the labor market (Calderón and Saffirio, 2017).

The Haitian experience contrasts with Venezuelan immigrants' experience in Chile, for which integration has been more successful. Unlike Haitians, Venezuelans arrive in Chile speaking the same native language, they blend better given they are mostly of white or mestizo race, and many of them already have social connections in the country. In a survey conducted by de la Hoz (2018) on Venezuelan immigrants in Chile, most respondents, when asked whether they felt welcomed in Chile, replied their experience was "overwhelmingly positive." Furthermore, most respondents claimed not having experienced any discrimination. Even though many had to begin working in low-paid jobs and some declare abuse from their employers, most report having progressed in their professional lives, even managing to obtain work in their profession. Moreover, regarding their connections in Chile, most of the interviewed subjects indicated they already had contact with people in the country, usually with a friend or a family member.

Not only do immigrants assimilate to the culture of the host country, but locals also adopt cultural traits and customs from the newly-arrived culture. For instance, in Santiago, the Chilean capital, dishes, and ingredients from Venezuelan cuisine are taking a central place in the main food market. Those of the Peruvian cuisine have already been there for long (Torres, 2018). It is nothing new. In the United States, a country with a long history of immigration, social scientists argued that the conditions of life and opportunities in the country would eventually create a "melting pot" between the different ethnic groups. However, after 1960, some began to question that view, observing that many minorities kept part of their distinctive economic, political, and cultural patterns long after their arrival to the United States (Bisin and Verdier, 2000).

Thus, just as economists and social scientists have debated during the last half-century whether the different minorities would merge into one culture, it becomes a question for Chile where this sharp incorporation of different migrant cultures is leading.

In this work, we study how the conflicting choice between assimilation and idiosyncrasy effect, when facing vertical and horizontal interactions, the behaviour of minorities and locals after a new migrant group is inserted into society.

A formal model is proposed in which every individual is embedded into a social network, and behaves strategically optimizing a utility function that reflects the assimilation dilemma.

We focus on the Nash equilibrium derived from this game and attempt to summarize the network's topology's effect on the transmission of cultural traits. We address the following questions that arise in this setting. How can we summarize the effect of the network's topology on the transmission of cultural traits? How can the topology of the network explain the transmission of characteristics across populations? Which types influence the long run, and which ones disappear? Is the society converging to a "melting-pot" equilibrium?

Our main findings are, for a static model: a sufficient statistic to summarize how the topology of social networks shapes the transmission of behavior in equilibrium, a novel decomposition of equilibrium behavior, and an intuitive interpretation of the influence of each type on the population. While in the dynamic model we find conditions for convergence in behavior, each trait's influence in the long run, and which traits disappear in the long run.

The rest of this work is organized as follows. In the next section, part of the related literature is mentioned to explain the position of this work in said literature. In the proceeding section, we present the model and the preliminaries for our analysis. Then, we show the Nash equilibrium for the static model and decompose the network's effect in this equilibrium. Subsequently, we extend this result to a dynamic setting, obtain convergence results, and characterize the long-term Nash equilibrium for a strongly connected network. In the last sections, we perform some comparative statics to exemplify the explaining capabilities of the presented model and comment on the implications of these results.

2 Related Literature

2.1 On evolution of cultural traits

The first formal treatments to the modeling of transmission of cultural traits come from the area of evolutionary biology, and date back to Cavalli-Sforza and Feldman (1973, 1981) and Boyd and Richerson (1985). The stylized model presented in both papers, and their terminology, are largely adopted by the literature that followed them.

Especially influential has been the seminal work of Alberto Bisin and

Thierry Verdier. Bisin and Verdier (1998) introduce a setup for the study of inter-generational transmission of preferences for status in a two types model. Bisin and Verdier (2000, 2001) introduce the parental socialization choice on the model in Cavalli-Sforza and Feldman (1981), and conclude that the distribution of cultural traits converges to a heterogeneous distribution, explaining that immigrants assimilation to the majority culture depends on parents' preferences for cultural traits.

Verdier and Zenou (2015, 2018) further study the dynamics of a two-types model when there is inter-generational transmission and with a community leader, in order to explore the role of cultural leaders in the integration of immigrants in a host country.

Recent literature in Economics moved from types (discrete variable) to continuous traits. While the literature on discrete traits is well-established, the one on continuous traits is still on development (Bisin and Verdier (2011) offer a comprehensive survey on both). Bisin and Topa (2003) propose a model of endogenous transmission of preferences, cognitive and psychological continuous traits, and then put it to data to identify their mechanisms of transmission. Verdier and Zenou (2015, 2018) and Prummer and Siedlarek (2017) study the dynamics of cultural traits with a community leader in a continuous traits model.

2.2 On games and opinions on networks with continuous actions

Ballester et al. (2006) provide the workhorse model of games in networks with continuous actions. Calvó-Armengol and Beltran (2009) study an organization as a coordination game in which all players face a common task, and use a network to model the communication structure within the organization. Bramoullé et al. (2014) obtain conditions on convergence when there are strategic interactions in networks.

The literature on opinion dynamics, introduced most notoriously in DeGroot (1974), is related to this work as well. In the DeGroot model, agents want to decide on a common parameter, and the opinion of each agent on the true value of this parameter is a weighted average of other agents' expected opinions plus his own. The works of DeMarzo et al. (2003) and Golub and Jackson (2010, 2012) are also central in this branch by extending the standard DeGroot model. The former prove that persuasion bias lead to the phenomenon of social influence, while the latter show that the opinion of every agent in the network converges to the truth if, and only if, the influence of the most influential agent eventually vanishes. Our framework shares the linear updating of actions, but with strategical behaviour from the agents.

2.3 Mixing both

Lately, the literature of cultural traits (in Economics) has moved from random encounters between the agents, to interactions in social networks.

Buechel et al. (2014) use an overlapping generations model to study the transmission of continuous cultural traits from parents to children, where parents are also immersed in a social network. Panebianco (2014) examines the dynamics of inter-ethnic attitudes with a setup of inter-generational transmission of continuous cultural traits. They consider that children are exposed to their parents and a network of non-parental socialization, and obtains conditions for the convergence of these traits. Olcina et al. (2017) use a model in which each ethnic group is part of a social network, and their utility is determined by their agreement with their personal values and by their assimilation to their peers. They show that, in their setup, there is always convergence to a steady state. Ushchev and Zenou (2020) provide a microfoundation and a discussion for the linear-in-means model, the workhorse model for empirical work on social interactions and peer-effects.

Förster et al. (2014) present a model of opinion formation and evolution based on DeGroot (1974) in which agents behave strategically. They explore the chance of manipulation between agents in the form of random encounters where the agents can decide to influence each other. Rapanos et al. (2019) introduce uncertainty on agents' private and social utilities when they are part of a network, and prove that there is always a unique Bayesian Nash equilibrium.

Our work is a contribution to the branch of transmission of continuous cultural traits in social networks. We consider a network model and utility function similar to those in Ushchev and Zenou (2020) and Olcina et al. (2017). Our OLG setup is taken from Buechel et al. (2014), but differs in two things: individuals directly inherit their parents' traits in the previous period, and these do not consider their offspring trait in their utility function; and descendants are exposed to vertical and horizontal influence, instead of vertical and oblique. Our central findings are mainly derived using results from the Theory of Finite Markov Chains, which can be revised in Karlin and Taylor (1975), Kemeny and Snell (1976), and Hartfiel (2006).

3 The Model

3.1 General setting

Let $\mathcal{N} = \{1, ..., N\}$ be the set of *agents* or *players*. Each agent $i \in \mathcal{N}$ here may represent either individuals, or a minority in the society.

Let $T : \mathcal{N} \to \mathbb{R}$ be a function that uniquely identifies each agent $i \in \mathcal{N}$ with a type $T(i) = \gamma_i$. The type of each agent is exogenous and represents, in one variable, its set of intrinsic characteristics, habits, values, and idiosyncrasies.

Agents *choose* actions $a_i \in \mathbb{R}$ and *observe* actions of a subset of agents $\mathcal{N}_i \subseteq \mathcal{N}$ exogenous to the model, with $\mathcal{N}_i \neq \varnothing$. This action represents the assimilation choice to the majority of the individual or group and their *observable behaviour* for other agents. The set $\mathcal{N}_i$ contains those other players that the agent i directly listens to or care for. They may, for example, represent the community to which the agents aim to be included, the citizens of the country to which the agent immigrated, their new classmates or colleagues, or the family of their new spouse. Thus, $\forall j \in \mathcal{N}_i$ we say that agent j *influences* directly agent i. We refer to $\mathcal{N}_i$ as the *influence* of agent i. In order to avoid redundancy later, we always assume $i \notin \mathcal{N}_i$. Let $\mathbf{a}_{\mathcal{N}_i} = (a_j)_{j \in \mathcal{N}_i}$. We define the payoff of agent $i \in \mathcal{N}$ as given by

$$\mathcal{U}_i(a_i; \mathbf{a}_{\mathcal{N}_i}) = -(1-\theta)(a_i - \gamma_i)^2 - \theta \sum_{j \in \mathcal{N}} \sigma_{ij}(a_i - a_j)^2, \tag{3.1}$$

where $\theta \in (0, 1)$, $\sigma_{ij} \geq 0$, $\sigma_{ij} = 0 \iff j \notin \mathcal{N}_i$, and

$$\sum_{j \in \mathcal{N}} \sigma_{ij} = 1. \tag{3.2}$$

In order to understand this utility function, observe first that the payoffs have two components: the $-(a_i - \gamma_i)^2$ component is strictly decreasing in $|a_i - \gamma_i|$ and represents how agents do not want to deviate from their type; the $-\sigma_{ij}(a_i - a_j)^2$ component, strictly decreasing in $|a_i - a_j|$, represents how the agents want to behave like their observed peers. The payoff of i is defined completely by its own action and those of its influence, players not observed by i have no direct effect on its payoff. Thus, when the trait of an agent is different from the actions of other agents in his influence, he faces a trade-off between choosing a a_i not too far from γ_i nor too far from the j's terms. This portrays how, in reality, new members in a community face the dilemma between wanting to assimilate to the new culture, but staying faithful to their personal beliefs, family values, or idiosyncratic customs.

Agents may give different relative importance to the players in their influence. Hence, the parameter σ_{ij} measures the *importance* that agent j has for agent i. Lastly, the parameter θ measures how important are peers with respect to family or idiosyncrasy. Both are exogenous to the model.

This setup naturally defines a weighted directed network $\mathcal{G} = (\mathcal{N}, E, w)$: the set of nodes corresponds to the set of agents $\mathcal{N}$; the set of directed edges (or out-links) E is the ordered pairs (i, j) such that agent i listens to agent j, formally $E = \{(i, j)|(i, j) \in \mathcal{N}^2 \vee j \in \mathcal{N}_i\}$; and $w : E \to (0, 1]$ is the weight function, which maps each pair (i, j) to the importance that agent i gives to agent j, hence $w(i, j) = \sigma_{i,j}$. We refer to networks of this kind as a *social network*.

In a social network $\mathcal{G}$ a *walk* from i to j is any sequence of connected directed edges that starts in i and ends in j . A *path* from i to j is a walk that goes at most once through each node. If there exists a path from i to j we say that i *hears* j, and we denote it by $i \to j$. If there is a path from i to j and vice-versa we say that i and j communicate, and we denote this by $i \leftrightarrow j$.

Figure 1: Diagram of a Social Network $\mathcal{G}_1$

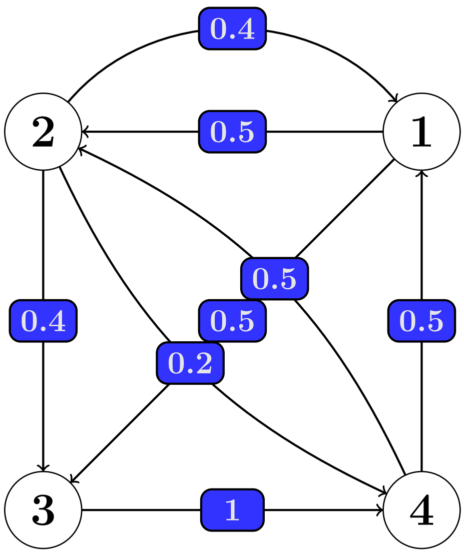
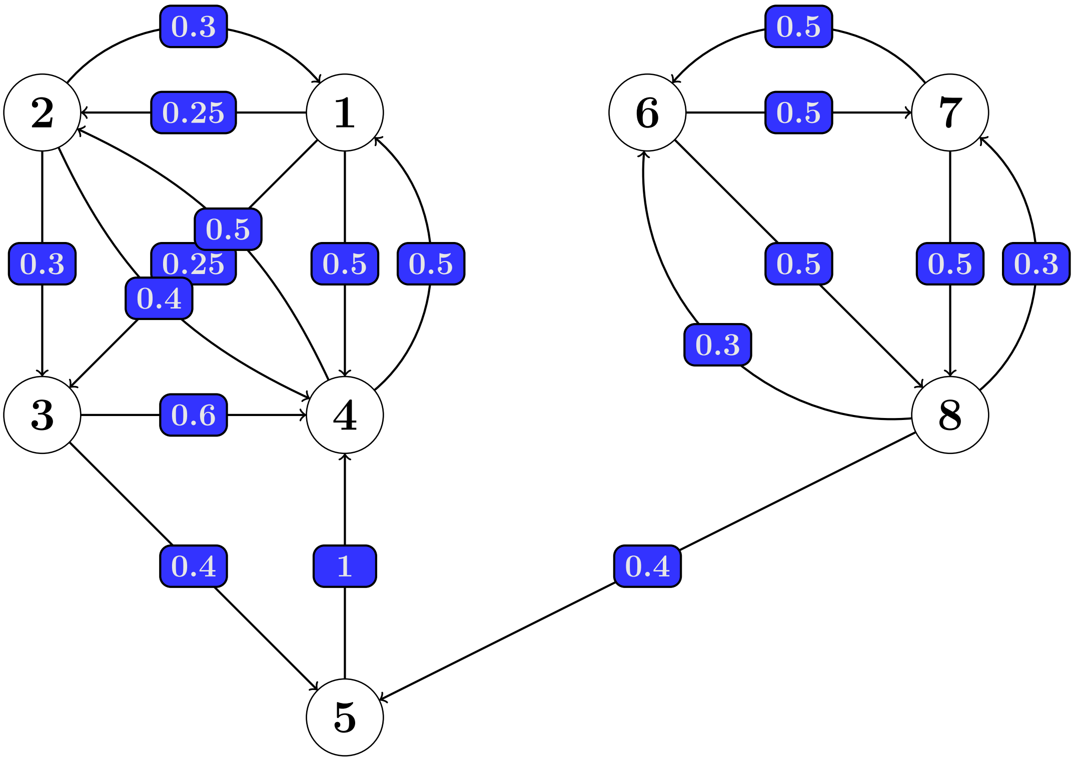


Figure 1 shows how the diagram of one of these social networks looks. Circles represent agents, arrows represent out-links, and values inside the rounded rectangles show the weight of each out-link.

In $\mathcal{G}_1$ we have, for instance, that agent 1 only observes agents 2 and 3. On the other side, agent 1 is observed by agents 2 and 4. The only pair of agents which gives importance to each other's opinion is agents 1 and 2, and 3 only listens to agent 4. Overall, no agent in this networks appears to be more important than the others, nor anyone seems to be left out of the network. Everyone observe and everyone is being observed; all agents communicate with

Figure 2: Diagram of a Social Network $\mathcal{G}_2$



each other. The agents in this network seem to be "well integrated" with each other. Thus, this network may represent, for example, a group of friends, a family, or minority groups in an integrated community.

Now look at the diagram of network $\mathcal{G}_2$ in Figure 2. Notice first how there are two clear groups of agents, namely: one group on the left of the figure comprised by agents 1 through 5, and another group right of the figure comprised by agents 6 through 8. No agent from the group on the left observes another from the right group, and only agent 8 in the right group observes 5 on the left group, but with a relatively high weight of 0.4. Yet, each group looks as it is well integrated within itself. This may be portraying, for example, two different political parties, two families, a couple of religious groups, or a newly arrived group of migrants (right group) inserting into the society. Notice also how, in the rightmost group, every agent seems to be of equal importance for the rest. But, in the leftmost group there are clear hierarchical differences in the importance each agent has for the rest. Agent 5, for example, seems to be excluded; only agent 3 observes him with low relative importance. Agent 4, on the other hand, appears as the "leader" of the group; he is observed by every other agent of the group and all of them assign him the highest importance.

This serves to present how, by specifying the right σ_{ij}'s, θ, and γ, the characteristics and features of real social networks in society that this setting allows to capture are manifold: cultural leaders, segregated minorities, conflicting groups, tight or disperse families. In the context of immigration: unobserved communities (like the Haitians in Chile), observed communities (such as the Venezuelans), an institution in the host country attempting to integrate the newly-arrived (with an accurate choice of links), larger or smaller disparity with the host society (in the election of γ), et cetera.

3.2 Preliminary remarks

Let $\mathbf{\Sigma} = [\sigma_{ij}]_{i,j\in\mathcal{N}}$, where we define $\sigma_{ij} = 0$ if $j \notin \mathcal{N}_i$. From (3.2) we have that $\mathbf{\Sigma}$ is a row stochastic (or simply stochastic) matrix that defines a Markov Process (MP). Here, agents would be the states of the MP and σ_{ij} the probability of moving from i to j. Powers $\mathbf{\Sigma}^k = [\sigma_{ij}^{(k)}]_{i,j\in\mathcal{N}}$ of $\mathbf{\Sigma}$ are all stochastic matrices, and give the probability of moving from i to j in different numbers of steps.

$\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$ would, for example, be the matrices defined by $\mathcal{G}_1$ and $\mathcal{G}_2$ respectively.

$$\mathbf{\Sigma}_1 = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 \\ 0.4 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0 & 1 \\ 0.5 & 0.5 & 0 & 0 \end{bmatrix}$$

$$\mathbf{\Sigma}_2 = \begin{bmatrix} 0 & 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.4 & 0.3 & 0.3 & 0 \end{bmatrix}$$

Let i and j be two states of the Markov Process defined by $\mathbf{\Sigma}$. As with the network equivalent, in the context of Markov Theory it is said that i and j communicate if the probability of going from one of these states to another is positive (not necessarily in one step) and again we denote this by $i \leftrightarrow j$. States communicating with each other conform an equivalence class. For any of these classes one, and only one, of two conditions always applies: either a process that enters the class never leaves, or a process that leaves the class never returns. A class of the first kind is called an *ergodic set*, while one of the second kind is called a *transient set*. A MP need not have a transient set; there always is at least one ergodic set (Kemeny and Snell, 1976).

A state i has period k if any walk (in the network sense) that begins in i and ends in i occurs in a multiple of k-steps. Formally, the period of i is

$$k = \text{gdc}\{n > 0 : \Pr(X_n = i | X_0 = i) > 0\}. \tag{3.3}$$

If $k = 1$ the state i is said to be aperiodic. A Markov Process with only aperiodic states is called aperiodic.

An aperiodic MP consisting of only one ergodic set is called a *regular Markov process*. Let us express the following classic result central to the theory of regular Markov processes. The proof is omitted given that it can be found in any introductory book of Markov Theory.

Theorem 3.1. *If* $\mathbf{\Sigma}$ *defines a regular MP of* n *states, then there exists a unique (row) stochastic vector* $\eta = (\eta_1, \eta_2, ..., \eta_n) \in \mathbb{R}^n$ *such that*

(i) $\eta \mathbf{\Sigma}^k = \eta$, $\forall k \in \mathbb{N}$.

(ii) The powers of $\mathbf{\Sigma}$ *converge to a matrix* $\mathbf{H}$, *for which each row equals* η.

(iii) The vector η *has full support. This is,* $\eta_i > 0$ *for each* $1 \leq i \leq n$.

Such vector is known as the unique stationary distribution *of* $\mathbf{\Sigma}$.

In the theory of Finite Markov Processes, η holds a probabilistic interpretation, and it is the expected amount of times that the process goes through each state.

A social network is said to be *strongly connected* if there is a path connecting any two pairs of agents in the network, and it is acyclical in the MP sense. Then, if $\mathcal{G}$ is strongly connected, it defines a regular MP by $\mathbf{\Sigma}$.

Furthermore, consider a MP that consists of one ergodic set and a finite number of transient sets, and in which there is a walk (in the network sense) from a state in each of the transient sets to one in the ergodic set (and not vice-versa, otherwise they would be part of the ergodic set). Then, the process is expected to eventually leave the transient sets and enter the ergodic set with a probability of 1. Let us call a MP of this kind (and its network equivalent) *segregated*. As the name suggests, with this class we want to capture segregated social networks, in which there are minorities' groups that hear society but are not heard by any member of it.

We enunciate the following theorem which will be useful below, the proof can be found in Theorem 1.5 of Hartfiel (2006).

Theorem 3.2. *Let* $\mathbf{\Sigma}$ *be a stochastic matrix defining a segregated MP. Let* $\mathbf{Q}$ *denote the bottom right submatrix of* $\mathbf{\Sigma}$ *associated with the transient states. Then, as* $k \to \infty$, $\mathbf{Q} \to 0$.

The key property of stochastic matrices in the context of Markov Theory is their averaging effect. If $\mathbf{\Sigma}$ is a stochastic matrix of size n and $\mathbf{w}$ any (column) vector of size n, then $\min_j \mathbf{w}_j \leq \sum_j \sigma_{ij}\mathbf{w}_j \leq \max_j \mathbf{w}_j$. This effect is condensed in the *coefficient of ergodicity* $\mathcal{T}(\mathbf{\Sigma})$ defined as

$$\mathcal{T}(\mathbf{\Sigma}) = \frac{1}{2}\max_{i,j}\sum_k |\sigma_{ik} - \sigma_{jk}| = 1 - \min_{i,j}\sum_k \min\{\sigma_{ik}, \sigma_{jk}\}, \tag{3.4}$$

which fulfills

$$\max_j(\mathbf{\Sigma w})_j - \min_j(\mathbf{\Sigma w})_j \leq \mathcal{T}(\mathbf{\Sigma})(\max_j \mathbf{w}_j - \min_j \mathbf{w}_j). \tag{3.5}$$

It is clear that $0 \leq \mathcal{T}(\mathbf{\Sigma}) \leq 1$, and $\mathcal{T}(\mathbf{\Sigma}) < 1$ if, and only if, every pair of rows i,j of $\mathbf{\Sigma}$ have a common position k such that $\sigma_{ik} > 0$ and $\sigma_{jk} > 0$. In particular, if one column of $\mathbf{\Sigma}$ has only positive entries, then it is guaranteed that $\mathcal{T}(\mathbf{\Sigma}) < 1$ (Hartfiel, 2006). This coefficient also fulfills

$$\mathcal{T}(\mathbf{\Sigma}^k) \leq \mathcal{T}(\mathbf{\Sigma})^k. \tag{3.6}$$

For $\rho \in (0,1)$ define $\mathbf{\Pi}(\rho)$ as

$$\mathbf{\Pi}(\rho) = \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}} \rho^k \Sigma^k. \tag{3.7}$$

This matrix is central for what ensues. We will need the following results.

Lemma 3.3. *Consider a stochastic matrix $\mathbf{\Sigma}$ of dimension n and $\rho \in (0,1)$, $\mathbf{\Pi}(\rho)$, as defined in (3.7), is a stochastic matrix. Furthermore, if $\mathbf{\Sigma}$ defines a regular MP, so does $\mathbf{\Pi}(\theta)$, and both have the same stationary distribution.*

Proof. Let $e = (1, 1, ..., 1) \in \mathbb{R}^n$ be the column vector with n 1's. A matrix $\mathbf{S}$ of dimension n is stochastic if, and only if, $Se = e$. Thus, let us compute

$$
\begin{aligned}
\mathbf{\Pi}(\rho)e &= \left(\frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\Sigma^k\right)e \\
&= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\Sigma^k e \\
&= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k e \\
&= \frac{1-\rho}{\rho}\cdot\frac{\rho}{1-\rho}e \\
&= e
\end{aligned}
\tag{3.8}
$$

Now, $\mathbf{\Pi}(\rho)$ is defined as a weighted summation of the powers of $\mathbf{\Sigma}$. Since the weighted powers contain all non-negative entries, any pair of states communicating in $\mathbf{\Sigma}$ also communicate in $\mathbf{\Pi}(\rho)$ (possibly more). Therefore, if $\mathbf{\Sigma}$ defines a regular MP (where all states communicate with each other), $\mathbf{\Pi}(\rho)$ does too.

Lastly, assume $\mathbf{\Sigma}$ defines a regular Markov Chain and let η be its stationary distribution. Then,

$$
\begin{aligned}
\eta\mathbf{\Pi}(\rho) &= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\eta\mathbf{\Sigma}^k \\
&= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\eta \\
&= \frac{1-\rho}{\rho}\cdot\frac{\rho}{1-\rho}\eta \\
&= \eta.
\end{aligned}
\tag{3.9}
$$

■

$\mathbf{\Pi}(\rho)$ defines a *decaying* MP where each step k occurs with probability $(1-\rho)\rho^{k-1}$. Longer steps are less likely to occur. In the context of Markov theory, $\pi_k^i := \mathbf{\Pi}_{ij}(\rho)$ is the probability that, starting in i, the process ends in j.

Then, notice that states that are transient in $\mathbf{\Sigma}$ are transient in $\mathbf{\Pi}(\rho)$ as well.

Let $\pi^i = (\pi_1^i, ..., \pi_N^i)$, $\forall i \in \mathcal{N}$. $\pi^i(\rho)$ is a probability measure that's agent specific. Hence, for any $\nu \in \mathbb{R}^N$ we can define a random variable (r.v.) ν^i as

$$v^i = \begin{cases} \nu_1 & \text{with prob. } \pi_1^i(\rho) \\ \vdots & \vdots \\ \nu_N & \text{with prob. } \pi_N^i(\rho) \end{cases}, \ \forall i \in \mathcal{N}$$

Then, given the r.v. ν^i, its expected value is

$$\mathbb{E}_{\pi^i(\rho)}[\nu] = \sum_{j \in \mathcal{N}} \pi_j^i(\rho)\nu_j$$

3.3 Static equilibrium

Let $\mathbf{\Gamma}$ be the simultaneous move N-player game defined by $\mathbf{\Sigma}$, with agents in $\mathcal{N}$, payoffs as in (3.1), and strategy spaces $\mathbb{R}$. We are first interested in computing the Nash equilibrium of this game.

Proposition 3.4. *For any Σ, a Nash equilibrium exists, is unique and is given by*

$$a_i^* = (1-\theta)\gamma_i + \theta\mathbb{E}_{\pi^i(\theta)}[\gamma], \ \forall i \in \mathcal{N} \tag{3.10}$$

Proof. Let $i \in \mathcal{N}$. The first derivative of $\mathcal{U}_i$ with respect to a_i is

$$\begin{aligned} \frac{\partial \mathcal{U}_i}{\partial a_i} &= -2(1-\theta)(a_i - \gamma_i) - 2\theta \sum_{j \in \mathcal{N}} \sigma_{ij}(a_i - a_j) \\ &= -2(1-\theta)(a_i - \gamma_i) - 2\theta \sum_{j \in \mathcal{N}} \sigma_{ij} a_i + 2\theta \sum_{j \in \mathcal{N}} \sigma_{ij} a_j \\ &= -2(1-\theta)(a_i - \gamma_i) - 2\theta a_i + 2\theta \sum_{j \in \mathcal{N}} \sigma_{ij} a_j \\ &= 2(1-\theta)\gamma_i - 2a_i + 2\theta \sum_{j \in \mathcal{N}} \sigma_{ij} a_j \end{aligned} \tag{3.11}$$

A vector of actions $\mathbf{a}^* = (a_1^*, a_2^*, ..., a_N^*) \in \mathbb{R}^N$ is a Nash equilibrium in pure strategies of the game if, and only if, $\forall i \in \mathcal{N}$ it solves

$$\begin{aligned} & \frac{\partial \mathcal{U}_i}{\partial a_i}(a_i^*; \mathbf{a}^*_{-\mathbf{i}}) = 0 \\ \iff & 2(1-\theta)\gamma_i - 2a_i^* + 2\theta \sum_{j \in \mathcal{N}} \sigma_{ij} a_j^* = 0 \\ \iff & a_i^* - \theta \sum_{j \in \mathcal{N}} \sigma_{ij} a_j^* = (1-\theta)\gamma_i. \end{aligned} \tag{3.12}$$

The last line in (3.12), since it is true $\forall i \in \mathcal{N}$, defines the set of N equations

$$\begin{aligned}
(\mathbf{I} - \theta\boldsymbol{\Sigma})\mathbf{a} &= (1-\theta)\gamma \\
\iff \mathbf{a} &= (1-\theta)\big[\sum_{k=0}^{\infty}\theta^k\boldsymbol{\Sigma}^k\big]\gamma \\
&= (1-\theta)\big[\mathbf{I} + \sum_{k=1}^{\infty}\theta^k\boldsymbol{\Sigma}^k\big]\gamma \\
&= (1-\theta)\gamma + (1-\theta)\big[\sum_{k=1}^{\infty}\theta^k\boldsymbol{\Sigma}^k\big]\gamma \\
&= (1-\theta)\gamma + \theta\Big[\frac{1-\theta}{\theta}\sum_{k=1}^{\infty}\theta^k\boldsymbol{\Sigma}^k\Big]\gamma \\
&= (1-\theta)\gamma + \theta\boldsymbol{\Pi}(\theta)\gamma,
\end{aligned} \tag{3.13}$$

and this last expression yields (3.10). ■

Notice first that, if there is no interaction between agents, i.e. $\theta = 0$, then $a_i^* = \gamma_i$. Hence, with no interaction, equilibrium actions are agents' types. Second, let $\tilde{\gamma} \in \mathbb{R}$, if $\forall i \in \mathcal{N}$ $\gamma_i = \tilde{\gamma}$, then $a_i^* = \tilde{\gamma}$. Therefore, if agents are homogeneous, the network's topology has no effect on equilibrium actions. Thus, is the heterogeneity in the population what affects equilibrium actions through the network's topology and the distribution of γ across players.

We can separate additively the individual effect of their own type from the network effect on every player's equilibrium actions in (3.10).

$$a_i^* = \underbrace{\gamma_i}_{\text{No interaction}} + \theta\underbrace{\left(\mathbb{E}_{\pi^i(\theta)}[\gamma] - \gamma_i\right)}_{\text{Network effect}} \tag{3.14}$$

With this notation, we have a clear interpretation of *spillover* effects:

$$\begin{aligned}
\forall i, j \in \mathcal{N},\ i \neq j :\ \frac{\partial a_i^*}{\partial \gamma_i} &= (1-\theta) + \theta\pi_i^i(\theta), \\
\text{and}\ \frac{\partial a_i^*}{\partial \gamma_j} &= \theta\pi_j^i(\theta).
\end{aligned} \tag{3.15}$$

Moreover, we have the following result:

Corollary 3.4.1. *Regarding how agents affect the behavior of others, we have:*

- *If i does not hear j, then $\pi_j^i(\theta) = 0$ and the trait of j has no effect on i's behaviour.*

- *If $\forall i, j \in \mathcal{N}$, $i \leftrightarrow j$, then $\pi^i_j(\theta) > 0 \; \forall i, j \in \mathcal{N}$, and each agent's trait has an effect on the behaviour of the rest.*

In a society where every pair of agents communicates, each of the agents' traits persists in equilibrium. More central agents, in the sense that the MP defined by the network passes by them more often, are the ones whose traits are the most persistent.

Once we have identified how the network's topology shapes the equilibrium, we can go a little deeper.

When $\mathbf{\Sigma}$ is a strongly connected network, we have the following decomposition.

Proposition 3.5. *Let $\mathbf{\Sigma}$ be a strongly connected network. Then, for each $i \in \mathcal{N}$ the network effect $\nu_i = \mathbb{E}_{\pi^i(\theta)}[\gamma] - \gamma_i$ can be uniquely decomposed as*

$$\nu_i = \underbrace{\mathbb{E}_\eta[\gamma]}_{\textit{Structural effect}} + \underbrace{\Delta_i(\gamma)}_{\textit{Local effect}}, \tag{3.16}$$

where η is the stationary distribution of Σ, and the local effects *are bounded as function of the size of θ, the ergodicity of the chain, and the position of the agents in the network.*

Proof. Let $d_{ij} = \pi^i_j - \eta_j$. Since η is unique, d_{ij} is well defined and unique for each i and j. Let $\mathbf{d}_i = (d_{i1}, d_{i2}, ..., d_{iN})$ we can then decompose ν_i as

$$\mathbb{E}_{\pi^i(\theta)}[\gamma] - \gamma_i = \mathbb{E}_\eta[\gamma] + \mathbf{d}_i\gamma - \gamma_i \tag{3.17}$$

And we define $\Delta^i_\theta(\gamma) = \mathbf{d}_i\gamma - \gamma_i$ as the local effect. Now, any stochastic matrix has the averaging effect described above. Let $M_i = \max_{j\in\mathcal{N}}\{\sigma_{ij}\}$ and $m_i = \min_{j\in\mathcal{N}}\{\sigma_{ij}\}$.

$$\begin{aligned}
|\sigma_{ij} - \eta_j| &\leq M_i - m_i \\
|\sigma^{(2)}_{ij} - \eta_j| &\leq \mathcal{T}(\mathbf{\Sigma})(M_i - m_i) \\
|\sigma^{(3)}_{ij} - \eta_j| &\leq \mathcal{T}(\mathbf{\Sigma^2})(M_i - m_i) \\
&\vdots \\
|\sigma^{(k)}_{ij} - \eta_j| &\leq \mathcal{T}(\mathbf{\Sigma}^{k-1})(M_i - m_i) \\
&\vdots
\end{aligned} \tag{3.18}$$

We can then perform the following rearrangement after multiplying the k-th inequality in (3.18) by the k-th power of θ.

$$
\begin{aligned}
|\theta\sigma_{ij} - \theta\eta_j| &\leq \theta(M_i - m_i) \\
|\theta^2\sigma_{ij}^{(2)} - \theta^2\eta_j| &\leq \theta^2\mathcal{T}(\mathbf{\Sigma})(M_i - m_i) \\
|\theta^3\sigma_{ij}^{(3)} - \theta^3\eta_j| &\leq \theta^3\mathcal{T}(\mathbf{\Sigma^2})(M_i - m_i) \\
&\vdots \\
|\theta^k\sigma_{ij}^{(k)} - \theta^k\eta_j| &\leq \theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1})(M_i - m_i) \\
&\vdots
\end{aligned}
\tag{3.19}
$$

Adding all inequalities in (3.19) and using the Triangular Inequality on the terms inside absolute values on each left side, yields:

$$
\begin{aligned}
\Big|\sum_{k=1}^{\infty}\theta^k\sigma_{ij}^{(k)} - \sum_{k=1}^{\infty}\theta^k\eta_j\Big| &\leq \sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1})(M_i - m_i) \\
\Rightarrow \frac{1-\theta}{\theta}\Big|\sum_{k=1}^{\infty}\theta^k\sigma_{ij}^{(k)} - \frac{\theta}{1-\theta}\eta_j\Big| &\leq \frac{1-\theta}{\theta}\Big(\sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1})(M_i - m_i)\Big) \\
\Rightarrow |\pi_j^i - \eta_j| &\leq \frac{1-\theta}{\theta}(M_i - m_i)\sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1}) \\
&\leq \frac{1-\theta}{\theta}(M_i - m_i)\sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma})^{k-1} \\
&= (1-\theta)(M_i - m_i)\sum_{k=0}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma})^{k} \\
&= \frac{(1-\theta)(M_i - m_i)}{1-\theta\mathcal{T}(\mathbf{\Sigma})}
\end{aligned}
\tag{3.20}
$$

Hence,

$$
\begin{aligned}
|\Delta_i(\gamma)| &= |\mathbf{d}_i\gamma - \gamma_i| \\
&\leq |\mathbf{d}_i\gamma| + \gamma_i \\
&\leq \frac{(1-\theta)(M_i - m_i)}{1-\theta\mathcal{T}(\mathbf{\Sigma})}\sum_{\mathcal{N}}\gamma_i + \gamma_i
\end{aligned}
\tag{3.21}
$$

■

Thus, every agent's actions fluctuate around a common structural effect derived from the topology of the network. If their actions are above or below that level, depends on their position on the network. The size of the deviation from this level for agent i is bounded by: the size of θ, the differences on the absolute value of the importance other agents give to i, and the ergodicity of $\mathbf{\Sigma}$. Thus, the tighter and more stationary is the network, the smaller the local effect relative to the structural one.

3.4 Dynamic equilibrium

Now, in order to represent the ongoing nature of cultural traits over time, we model cultural traits as a continuous variable in discrete time $\mathcal{T} = \{0, 1, 2, 3...\}$. For this, we use an overlapping generations model (OLG). In this setup, agents are called *dynasties*. Each dynasty is represented with one agent who reproduces asexually on each period. Thus, in every period there is a new generation representing each dynasty.

Time starts at $t = 0$, where the initial generation has an exogenous type $\gamma_0 = (\gamma_{1,0}, .., \gamma_{N,0})$. The main feature of OLG is that, from then on, dynasties types are endogenous and are given by parents' behavior in the previous period. This is:

$$\gamma_{i,t} = a^*_{i,t-1}, \ \forall i \in \mathcal{N}, \ t = 1, 2, 3...$$

Dynasties' payoff in each period is:

$$\mathcal{U}(a_{i,t}, \mathbf{a}_{\mathcal{N}_i,t}) = -(1-\theta)(a_{i,t} - \gamma_{i,t})^2 - \theta \sum_{j \in \mathcal{N}_i} \sigma_{ij}(a_{i,t} - a_{j,t})^2$$

Now, we want to check the convergence of behavior (i.e., cultural traits) as $t \to \infty$, this is, check the existence of

$$\mathbf{a}^* = \lim_{t \to \infty} \mathbf{a}^*_t$$

where $\mathbf{a}^*_t = (a^*_{1,t}, ..., a^*_{N,t})$.

Let us first show the following result.

Lemma 3.6. *The Nash equilibrium of the game at each period is given by*

$$\mathbf{a}^*_t(\theta) = \mathbf{\Omega}^t(\theta)\gamma_0, \ t \in \mathcal{T} \backslash \{0\}, \tag{3.22}$$

where $\mathbf{\Omega}(\theta) := (1-\theta)\mathbf{I} + \theta\mathbf{\Pi}(\theta)$

Proof. From (3.13) we have

$$\begin{aligned} \mathbf{a}^*_t(\theta) &= (1-\theta)\gamma_t + \theta\mathbf{\Pi}(\theta)\gamma_t \\ &= ((1-\theta)\mathbf{I} + \theta\mathbf{\Pi}(\theta))\mathbf{a}^*_{t-1}. \end{aligned}$$

By applying recursion on this last term, (3.22) follows. ■

Hence, convergence of cultural traits depends entirely on $\mathbf{\Omega}(\theta)$.

Regard the contracting effect of stochastic matrices condensed in $\mathcal{T}$ and discussed above. Notice how (3.6) implies that $\mathcal{T}(\mathbf{\Omega}(\theta)^k) \leq \mathcal{T}(\mathbf{\Omega}(\theta))$, and this is true for every class of stochastic matrix. Therefore, $\mathbf{a}_t^*(\theta)$ can only become more contracted over time. This implies that the traits of individuals do not diversify as time passes. Whenever $\mathcal{T}(\mathbf{\Omega}(\theta)^k) < 1$ for some $k \in \mathbb{N}$, the process of homogenization is pervasive; the traits become strictly less and less diverse as time passes.

Furthermore, consider a segregated social network as defined above. From Theorem 3.2 and Lemma 3.22, the traits of the segregated agents disappear in the long-run. Thus, when groups of immigrants are isolated from society, the influence of their idiosyncrasy and culture on society fades as time passes. They become completely absorbed into the cultural traits of local inhabitants.

With a strongly connected network, we have convergence to the same point.

Proposition 3.7. *If $\mathbf{\Sigma}$ defines a strongly connected network, then*

$$a_i^* = \mathbb{E}_\eta[\gamma_0], \ \forall i \in \mathcal{N} \tag{3.23}$$

where η is the unique stationary distribution of $\mathbf{\Sigma}$.

Proof. If $\mathbf{\Sigma}$ is a regular MP, then so is $\mathbf{\Pi}(\theta)$ by Lemma 3.3, and it follows immediately that $\mathbf{\Omega}(\theta)$ is regular. Let η be the stationary distribution of $\mathbf{\Sigma}$. Then,

$$\begin{aligned} \eta\mathbf{\Omega}(\theta) &= (1-\theta)\eta\mathbf{I} + \theta\eta\mathbf{\Pi}(\theta) \\ &= (1-\theta)\eta + \theta\eta \\ &= \eta \end{aligned} \tag{3.24}$$

Hence, η is also the unique stationary distribution of $\mathbf{\Omega}$. This implies:

$$\begin{aligned} \mathbf{a}^* &= \lim_{t\to\infty} \mathbf{a}_t^* \\ &= \lim_{t\to\infty} \mathbf{\Omega}^t(\theta)\gamma_0 \\ &= \mathbf{H}\gamma_0, \end{aligned} \tag{3.25}$$

where $\mathbf{H}$ is as defined in Theorem 3.1. Computing the last expression for each component of $\mathbf{a}^*$, yields (3.22).

■

Because $\mathbf{\Sigma}$ is strongly connected, we know that η has full support. The influence on asymptotic behavior of each dynasty is given by the (inverse) of mean times the MP passes through it. Notice how, in the long run, the parameter θ does not affect the equilibrium value, since η is directly derived from $\mathbf{\Sigma}$. Thus, no matter the relative importance that agents give to their values relative to their peers' actions, society will eventually converge to a common trait underlying structurally in the topology of the network through η.

4 Comparative Statics

This setup allows us to do different comparative statics. As a mode of illustration, let us revise some cases related to the migration described in the introduction.

4.1 Tightly connected network with heterogeneity among the agents

Let us begin with a simple example by computing the equilibrium in network $\mathcal{G}_1$ defined above. We define the set of traits $\gamma_1 = (0, 0, 1, 1)$, i.e., there are two sharply different traits in the social networks.

- $\theta = 0.2$.

  On the first period, the equilibrium is given by $(0.10, 0.11, 0.96, 0.82)$. After 5 periods, the traits of the group are $(0.37, 0.38, 0.73, 0.52)$. In $t = 10$: (0.48,0.47,0.56,0.48). Only in $t = 18$, the group appears to converge to an average trait of $(0.5, 0.5, 0.5, 0.49)$

- $\theta = 0.5$.

  On the first period, equilibrium actions are $(0.27, 0.28, 0.81, 0.63)$. In $t = 3$: $(0.27, 0.28, 0.85, 0.61)$ After 6 periods, society converges to an average trait of 0.5

Convergence was considerably faster when the value of θ was increased to 0.5. As noted above, the long-run common trait was the same in both cases, independent of θ. Given the quasi-symmetry of the network, this trait was a normal average of both traits in the society.

4.2 Arrival of an unobserved homogeneous group of migrants

Let us now turn to the network in $\mathcal{G}_2$. We consider that the leftmost group is the host society, which is already in stationary state; therefore sharing the same trait equal to 1. The rightmost group is the just arrived group of migrants, which we consider to be homogeneous and share a trait equal to 0. We always consider from now on $\theta = 0.5$.

In $t = 1$, equilibrium actions are $(1, 1, 1, 1, 1, 0.074, 0.074, 0.22)$. Since no member of the host society observes the migrants, it is expected that the traits of the latter have no influence on the locals. After period 5, traits in the society are $(1, 1, 1, 1, 1, 0.43, 0.43, 0.60)$. In $t = 10$: $(1, 1, 1, 1, 1, 0.71, 0.71, 0.80)$.

After period 30, traits are $(1, 1, 1, 1, 1, 0.982, 0.982, 0.987)$, and the immigrants are virtually assimilated.

The immigrant that observes the host society (agent 8) assimilates considerably faster in the first periods than the rest of the migrants, even when they observe 8, hence indirectly hearing the host society. In the long-run, when the host society does not observe the migrants and these do hear the locals, the traits of the migrants disappear, and society converges to the same stationary state it was in the beginning. The speed of convergence was slower than in the previous example, taking almost five times more periods to converge to a common trait. Even when society may be heading towards a melting pot, this process may be slow-moving.

Consider now the network $\mathcal{G}_3$, defined by

$$\mathbf{\Sigma}_3 = \begin{bmatrix} 0 & 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.8 & 0.1 & 0.1 & 0 \end{bmatrix}.$$

Now, agent 8 makes a higher assimilation effort than before.

In period 1, actions are $(1, 1, 1, 1, 1, 0.13, 0.13, 0.41)$. After $t = 15$ the immigrants virtually assimilated with traits $(1, 1, 1, 1, 1, 0.98, 0.98, 0.99)$. Making one individual in the group pay more attention to the host society, the speed of convergence reduced nearly by half. This hints the importance of peer effects in the assimilation process: if few members of a group of migrants are chosen to blend with the host society, the rest will soon follow.

4.3 Arrival of an observed homogeneous group of migrants

Let $\mathcal{G}_4$ be a social network defined by the matrix

$$\mathbf{\Sigma}_4 = \begin{bmatrix} 0 & 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.8 & 0.1 & 0.1 & 0 \end{bmatrix},$$

which is similar to $\mathcal{G}_3$ except that now agent 5 observes agent 8. Thus, the host society hears the group of immigrants, defining one strongly connected network. Let us take the same vector of traits as in the previous case.

In $t = 1$, actions are $(0.99, 0.99, 0.96, 0.99, 0.83, 0.11, 0.11, 0.34)$.

After $t = 5$, traits are $(0.95, 0.95, 0.91, 0.96, 0.82, 0.55, 0.55, 0.72)$. In period 15: $(0.92, 0.92, 0.91, 0.92, 0.90, 0.88, 0.88, 0.9)$, society is reaching its stationary state, which is heavily weighted towards the local culture. However, the migrants' culture persists within them, and the host country adopts part of their traits. Seemingly, convergence happens at about the same pace as when the immigrants were not observed.

4.4 Homogeneous group of migrants observed by a cultural leader

Within our interpretation of network $\mathcal{G}_4$, agent 5 does not exert much influence in the host society. What occurs instead, when cultural leaders attempt to desegregate the group of immigrants?

Let $\mathcal{G}_5$ be the social network defined by

$$\mathbf{\Sigma}_5 = \begin{bmatrix} 0 & 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0.8 & 0 & 0.1 & 0.1 & 0 \end{bmatrix}.$$

$\mathbf{\Sigma}_5$ is similar to $\mathcal{G}_4$, except that now is agent 4 (the leader of the host society) the one who directly communicates with agent 8.

In period 1, actions are $(0.93, 0.94, 0.92, 0.82, 0.91, 0.11, 0.11, 0.33)$. After $t = 5$: $(0.75, 0.76, 0.74, 0.7, 0.72, 0.5, 0.5, 0.64)$. On $t = 15$ society is seemingly converging, with traits $(0.692, 0.692, 0.692, 0.69, 0.69, 0.684, 0.69)$.

Even though convergence did not accelerate compared to the previous example, now the immigrants' traits more strongly persist in society. Thus, cultural leaders (in the form of individuals or institutions) could have a fundamental role in helping the immigrants keep their idiosyncrasy and integrating it in the host society.

4.5 Heterogeneous group of migrants observed by a cultural leader

Lastly, we explore the case when immigrants exhibit different traits in network $\mathcal{G}_5$. Let $\gamma_5 = (1, 1, 1, 1, 1, -0.5, 0.5, 0)$. The average trait of immigrants is still 0, and the one individual of this group that communicates with society maintains this trait. Heterogeneity is distributed equitably between the other two symmetric individuals.

After $t = 1$, traits are $(0.96, 0.96, 0.95, 0.89, 0.94, -0.06, 0.13, 0.611)$. For $t = 5$: $(0.83, 0.84, 0.82, 0.79, 0.81, 0.58, 0.58, 0.74)$.

When $t = 15$, traits are $(0.78, 0.78, 0.78, 0.78, 0.78, 0.78, 0.77, 0.77, 0.77)$. Convergence did not seem to slow down. However, the traits of the minorities now persist less in the long-term than in the previous case. Since there was no change in the network's topology, this is caused entirely by the heterogeneity in the minority group. Seemingly, in presence of diversity or conflict inside a minority, their features are adopted less by society.

As a final remark, in each of these simulations each agent's trait converged monotonically (without oscillating) towards equilibrium.

5 Concluding Remarks

In this work, we investigated a transmission model of continuous cultural traits when agents are embedded in a social network. This analysis was motivated by various examples in Economics, where cultural transmission plays a fundamental role. We were mainly interested in the potential use of this setup to explore the consequences of the sudden inflows of immigration that arrived in Chile during the last decade.

Our main findings are a succinct formula and decomposition that summarizes the effect of networks' topology on the equilibrium through the underlying Markov Process. Moreover, we derived explicit formulas for the spillover effect of each agent's type on the rest of the population. In the dynamic model, we obtained an expression for the equilibrium in each period. We found which agents have persistent traits in the long-run. Then, we concluded that, in a strongly connected network, society eventually converges to one trait, defined by a weighted average of every individual's trait and where the network's topology acts through the stationary distribution of the MP. Thus, supporting the "melting pot" view.

The results on the model and the simulations we performed allow us to make some comments regarding policy. Firstly, when society does not pay attention to a minority, its characteristic traits tend to disappear. It is generally desirable for a minority to keep part of their identity. Moreover, traits of native and ancient cultures usually want to be perpetuated in society for heritage value reasons. However, our results hint that society must make an active effort to assimilate these cultures' traits to keep them alive.

Second, the comparative statics hint some useful policy implications for the inclusion of migrants into society, such as the vital role of specialized institutions and the strength of the peer effect that few individuals of a minority who are assimilating have on the rest of their group.

We view this work as a starting point to elucidate the effect of different social networks' topologies on society's equilibrium behavior. We made several simplifying assumptions, and derived results on a setting that captures various of society's nuances. Lines for future research include: checking different topologies in more intricate examples than those presented, varying the distribution of links and traits; using empirical data on the model; testing the speed of convergence for different choices of parameters; or extending the model to dynamic or endogenous network formation.

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