On the evolution of cultural traits in social networks

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Abstract

Cultural traits' evolution is studied in a theoretical setup, where each individual (or minority) is part of a social network. Individuals behave strategically and face a trade-off between acting like their observed peers or keeping faithful to their idiosyncrasy. We characterize the Nash equilibrium of this game using the theory of Finite Markov Processes and find a sufficient statistic that summarizes the effect of the topology of the network in this equilibrium. We extend this to an overlapping generations model, where parents' actions determine the traits of their offspring. We identify the Nash equilibrium in each period and show that, when the network is strongly connected, all society converges to the same trait. Lastly, we use the results to illustrate, with comparative static, the consequences of the entry of a group of immigrants to a host country and discuss some policy implications of our findings.

Keywords: Social Networks, Assimilation, Cultural Traits, Immigration, Games on Networks.

1 Introduction

The observation that humans acquire valuable life skills and knowledge through copying others has been the focus of animal behaviourists' attention dating back to Darwin.

Culture is information that people acquire from others by teaching, imitation, and other forms of social learning. People acquire skills, beliefs, and values from those around them, shaping their behaviour. Humans, unlike any living creature, have cumulative cultural adaption. Humans hold their knowledge and learn things from others, improve them, pass this knowledge to the next generation, and so on (Boyd and Richerson, 2005). It is this unique mechanism which leads to the rapid non-genetic evolution of superbly designed adaptations to particular environments.[1] Hence, the underlying social structure where humans interact plays a prominent role in the evolution of cultural traits.

Preferences, beliefs, norms, and habits are formed as a result of heritable traits. They are transmitted from generation to generation and shaped by the social interactions of the individual. Thus, several social sciences pay attention to the role of vertical (parents), oblique (role models), and horizontal (peers) cultural transmission. For Economics, the transmission of cultural traits between generations and peers plays an essential role in determining the individuals' preference traits, such as discounting, purchasing patterns, patience, and risk aversion (Robson and Samuelson, 2011). It is a determinant for fertility practices and the attitudes and social norms towards family and community that shape social capital (Guiso et al., 2008).

While cultural transmission is convenient for the understanding of varied phenomena in the field of Economics, it is of particular interest for illustrating the assimilation process of immigrants in the host country and their effect on the local inhabitants.

Assimilation with the local culture for immigrants has been positively correlated with their life satisfaction. Angelini et al. (2015) obtain conclusions in line

[1] It is in this context that Dawkins (1989), when looking for a noun to convey the idea of unit of cultural transmission, coined the term *meme* as an inflexion of the greek word *mimeme* (something that is imitated) to make it sound like *gene*.

with this view by using self-reported measures of well-being and panel data from migrants in Germany. Empirical work has looked at the impact of immigrants' social networks on their assimilation choices. Damm (2009) and Edin et al. (2003) use ethnic concentration/enclave as a proxy for immigrants' networks in a host country. The language group and proficiency of immigrants have been used as a proxy of the network effect, too (Bertrand et al., 2000). Besides, the language skills of immigrants influenced their performance in the labour market in the United States (Chiswick and Miller, 2002).

Large inflows of immigration tend to carry a consequential impact on the labour market. Immigrants usually bring skills and know-how that differ from those appearing and required in the local markets. Moreover, individuals with low skills and low education levels may self-select themselves to migrate to a foreign country. Language barriers may be present as well. Cultural differences also play a role in how the newly arrived adjusts to the practices and customs of the working community.

This is of particular relevance in Chile, where the immigrant population has shown an accelerated increase during the last decade. According to Instituto Nacional de Estadísticas (2018), international immigrants rose from 1.27% of the total population in the 2002 Census to 4.35% of the total in the 2017 Census.[2] Plus, around two-thirds of the total immigrants censused in 2017 arrived in the country between 2010 and 2017. Furthermore, 61% of the total migrated between 2015 and 2017.

By 31 December 2019, it is estimated that of the total population of immigrants in the territory, 30,5 % is from Venezuela, 15,8% from Peru, and 12,5 % from Haiti (Instituto Nacional de Estadísticas y Departamento de Extranjería y Migración, 2019).

Of all of the newly arrived communities, the Haitian is a singular one and has been particularly visible for the Chilean people, mainly for two reasons. Firstly, it is the only one of the big groups arriving from America for which the native

[2] Censuses in Chile usually take place every 10 years, as in most countries. But in what was a public national scandal, the data collected during the 2012 Census was deemed "unfit to the standards required for the operative to be denominated as a census" (Instituto Nacional de Estadísticas, 2014), and thus discarded. The procedure was then repeated in 2017

language is not Spanish (Chile's national language), but French/Creole. Secondly, it is the only dominantly black community that arrived in the country.

A series of natural disasters in the Caribbean country and a stagnant economy have propelled the exodus from Haiti, a country now widely regarded as a failed state (Torgman, 2012). Most of those arriving from the Creole-speaking nation turned to Chile because of its stable high-income economy, and its lenient policy in handling visas compared to the alternatives. Indeed, around 186,600 Haitians lived in Chile by the end of last year, roughly 1.7 % of Haiti's population.

However, Haitians find it arduous to get absorbed into society and the labour market. The ethnic and cultural differences between Haitians and the local population, and the language barrier have contributed to this. Moreover, Haitians present the lowest percentage of educational level and the highest of individuals without formal education of all the rest of the countries of origin of the immigrant population (see Figure 4 in Aldunate et al. (2019)). All is combined with the massive inflow of immigrants recently arrived. Natives of the Caribbean nation face racism, social exclusion, lack of job opportunities, abuse and underpayment by their employers, overcrowding in segregated areas of the city, and resentment from the Chileans.

Most Haitians arrive in Chile with no local connections and are inserted into already segregated communities of migrants. Their lack of Spanish language proficiency further aggravates the problem. The majority believe that it is not essential to know Spanish before they arrive in Chile. Thus, they come with no skills in the local language, hindering their integration into society and the labour market (Calderón and Saffirio, 2017).

The Haitian experience contrasts with Venezuelan immigrants' experience in Chile, for which integration has been more successful. Unlike Haitians, Venezuelans arrive in Chile speaking the same native language, they blend better given they are mostly of white or mestizo race, and many of them already have social connections in the country. In a survey conducted by de la Hoz (2018) on Venezuelan immigrants in Chile, most respondents, when asked whether they felt welcomed in Chile, replied their experience was "overwhelmingly positive." Furthermore, most respondents claimed not to have experienced any discrimination. Even though many had to begin working in low-paid jobs and some declare abuse from their employers, most report having progressed in their professional lives, even managing to obtain work in their profession. Moreover, regarding their connections in Chile, most of the interviewed subjects indicated they already had contact with people in the country, usually with a friend or a family member.

Not only do immigrants assimilate into the culture of the host country, but locals also adopt cultural traits and customs from the newly arrived culture. For instance, in Santiago, the Chilean capital, dishes, and ingredients from Venezuelan cuisine are taking a central place in the main food market. Those of the Peruvian cuisine have already been there for long (Torres, 2018). It is nothing new. In the United States, a country with a long history of immigration, social scientists argued that the conditions of life and opportunities in the country would eventually create a "melting pot" between the different ethnic groups. However, after 1960, some began to question that view, observing that many minorities kept part of their distinctive economic, political, and cultural patterns long after their arrival to the United States (Bisin and Verdier, 2000).

Thus, just as economists and social scientists have debated during the last half-century whether the different minorities would merge into one culture, it becomes a question for Chile where this sharp incorporation of different migrant cultures is leading.

In this work, we study how the conflicting choice between assimilation and idiosyncrasy effect, when facing vertical and horizontal interactions, the behaviour of minorities and locals after a new migrant group is inserted into society.

A formal model is proposed in which every individual is embedded into a social network, and behaves strategically optimizing a utility function that reflects the assimilation dilemma. Our purpose is to offer a framework that captures these interactions and to deliver results that characterize the behaviour of its agents.

We focus on the Nash equilibrium derived from this game and attempt to summarize the network's topology's effect on the transmission of cultural traits. We address the following questions that arise in this setting. How can we summarize the effect of the network's topology on the transmission of cultural traits? How can the topology of the network explain the transmission of characteristics across populations? Which types influence the long run, and which ones disappear? Is the society converging to a ”melting-pot” equilibrium?

Our main findings are, for a static model: a sufficient statistic to summarize how the topology of social networks shapes the transmission of behavior in equilibrium, a novel decomposition of equilibrium behavior, and an intuitive interpretation of the influence of each type on the population. In the dynamic model, we find conditions for convergence in behaviour, how each trait influences in the long run, and which traits disappear in the long run. Crucially, topology plays a central role in all results.

The remainder of this paper is structured as follows: The subsequent section provides an overview of related literature, contextualizing our study's contribution to the existing body of knowledge. In section 3, we present the model and the preliminaries for our analysis. Then, we show the Nash equilibrium for the static model and decompose the network's effect in this equilibrium. Subsequently, we extend this result to a dynamic setting, obtain convergence results, and characterize the long-term Nash equilibrium for a strongly connected network. In section 4, we perform some comparative dynamics to exemplify the explaining capabilities of the presented model and comment on the implications of these results. Lastly, we offer our conclusions.

2 Related Literature

2.1 On evolution of cultural traits

The first formal treatments to the modelling of transmission of cultural traits come from the area of evolutionary biology, and date back to Cavalli-Sforza and Feldman (1973, 1981) and Boyd and Richerson (1985). The stylized model presented in both papers and their terminology is largely adopted by the literature that followed them.

Especially influential has been the seminal work of Alberto Bisin and Thierry Verdier. Bisin and Verdier (1998) introduce a setup for the study of inter-generational transmission of preferences for status in a two types model. Bisin and Verdier (2000, 2001) introduce the parental socialization choice on the model in Cavalli-

Sforza and Feldman (1981), and conclude that the distribution of cultural traits converges to a heterogeneous distribution, explaining that immigrants' assimilation to the majority culture depends on parents' preferences for cultural traits.

Verdier and Zenou (2015, 2018) further studies the dynamics of a two-types model when there is inter-generational transmission and with a community leader, to explore the role of cultural leaders in the integration of immigrants in a host country.

Recent literature in Economics moved from types (discrete variables) to continuous traits. While the literature on discrete traits is well-established, the one on continuous traits is still in development. Bisin and Verdier (2011) offers a comprehensive survey on both). Bisin and Topa (2003) propose a model of endogenous transmission of preferences, cognitive and psychological continuous traits, and then put it to data to identify their mechanisms of transmission. Verdier and Zenou (2015, 2018) and Prummer and Siedlarek (2017) study the dynamics of cultural traits with a community leader in a continuous traits model.

2.2 On games and opinions on networks with continuous actions

Ballester et al. (2006) provide the workhorse model of games in networks with continuous actions. Calvó-Armengol and Beltran (2009) study an organization as a coordination game in which all players face a common task, and use a network to model the communication structure within the organization. Bramoullé et al. (2014) obtain conditions on convergence when there are strategic interactions in networks.

The literature on opinion dynamics, introduced most notoriously in DeGroot (1974), is related to this work as well. In the DeGroot model, agents want to decide on a common parameter, and the opinion of each agent on the true value of this parameter is a weighted average of other agents' expected opinions plus his own. The works of DeMarzo et al. (2003) and Golub and Jackson (2010, 2012) are also central in this branch by extending the standard DeGroot model. The former proves that persuasion bias leads to the phenomenon of social influence, while the latter shows that the opinion of every agent in the network converges to the truth if, and only if, the influence of the most influential agent eventually vanishes. Our framework shares the linear updating of actions but with strategical behaviour from the agents.

2.3 Mixing both

Lately, the literature on cultural traits in Economics has moved from random encounters between the agents to interactions in social networks.

Buechel et al. (2014) uses an overlapping generations model to study the transmission of continuous cultural traits from parents to children, where parents are also immersed in a social network. Panebianco (2014) examine the dynamics of inter-ethnic attitudes with a setup of inter-generational transmission of continuous cultural traits. They consider that children are exposed to their parents and a network of non-parental socialization, and obtain conditions for the convergence of these traits. Olcina et al. (2017) use a model in which each ethnic group is part of a social network, and their utility is determined by their agreement with their values and by their assimilation with their peers. They show that, in their setup, there is always convergence to a steady state. Ushchev and Zenou (2020) provide a micro foundation and a discussion of the linear-in-means model, the workhorse model for empirical work on social interactions and peer effects.

Förster et al. (2014) present a model of opinion formation and evolution based on DeGroot (1974) in which agents behave strategically. They explore the chance of manipulation between agents in the form of random encounters where the agents can decide to influence each other. Rapanos et al. (2019) introduce uncertainty on agents' private and social utilities when they are part of a network, and prove that there is always a unique Bayesian Nash equilibrium.

Our work is a contribution to the branch of transmission of continuous cultural traits in social networks. We consider a network model and utility function similar to those in Ushchev and Zenou (2020) and Olcina et al. (2017). Our OLG setup is taken from Buechel et al. (2014), but differs in two things: individuals directly inherit their parents' traits in the previous period, and these do not consider their offspring trait in their utility function; and descendants are exposed to vertical and horizontal influence, instead of vertical and oblique. Our central findings are mainly derived using results from the Theory of Finite Markov Chains, which can be revised in Karlin and Taylor (1975), Kemeny and Snell (1976), and Hartfiel (2006).

3 The Model

3.1 General setting

Let $N = \{1, ..., N\}$ be the set of *agents* or *players*. Each agent $i \in N$ here may represent either individuals or a minority in the society.

Let $T : N \to \mathbb{R}$ be a function that uniquely identifies each agent $i \in N$ with a type $T(i) = \gamma_i$. The type of each agent is exogenous and represents, in one variable, its set of intrinsic characteristics, habits, values, and idiosyncrasies.

Agents *choose* actions $a_i \in \mathbb{R}$ and *observe* actions of a subset of agents $N_i \subseteq N$ exogenous to the model, with $N_i \neq \varnothing$. This action represents the assimilation choice to the majority of the individual or group and their *observable behaviour* for other agents. The set N_i contains those other players that the agent i directly listens to or cares for. They may, for example, represent the community to which the agents aim to be included, the citizens of the country to which the agent immigrated, their new classmates or colleagues. Thus, $\forall j \in N_i$ we say that agent j *influences* directly agent i. We refer to N_i as the *influence* of agent i.

To avoid redundancy later, we always assume $i \notin N_i$. Let $\mathbf{a}_{N_i} = (a_j)_{j \in N_i}$. We define the payoff of agent $i \in N$ as given by

$$\mathcal{U}_i(a_i; \mathbf{a}_{N_i}) = -(1-\theta)(a_i - \gamma_i)^2 - \theta \sum_{j \in N} \sigma_{ij}(a_i - a_j)^2, \tag{3.1}$$

where $\theta \in (0, 1)$, $\sigma_{ij} \geq 0$, $\sigma_{ij} = 0 \iff j \notin N_i$, and

$$\sum_{j \in N} \sigma_{ij} = 1. \tag{3.2}$$

To understand this utility function, observe first that the payoffs have two components: the $-(a_i - \gamma_i)^2$ component is strictly decreasing in $|a_i - \gamma_i|$ and represents how agents do not want to deviate from their type; the $-\sigma_{ij}(a_i - a_j)^2$ component, strictly decreasing in $|a_i - a_j|$, represents how the agents want to behave like their observed peers. The payoff of i is defined completely by its action and those of its influence, players not observed by i have no direct effect on its payoff. Thus, when the trait of an agent is different from the actions of other agents in his influence, he faces a trade-off between choosing a a_i not too far from γ_i nor too far from the j's terms. This portrays how, in reality, new members of a community face the dilemma between wanting to assimilate into the new culture, but staying faithful to their personal beliefs, family values, or idiosyncratic customs.

Agents may give different relative importance to the players in their influence. Hence, the exogenous parameter σ_{ij} measures the *importance* that agent j has for agent i.

Lastly, the exogenous parameter θ measures how important are peers concerning family or idiosyncrasy. For model tractability, and because we are focused on the broad network topology, we consider a single θ for all agents.

This setup naturally defines a weighted directed network $G = (N, E, w)$: the set of nodes corresponds to the set of agents N; the set of directed edges (or out-links) E is the ordered pairs (i, j) such that agent i observes agent j, formally $E = \{(i, j)|(i, j) \in N^2 \wedge j \in N_i\}$; and $w : E \to (0, 1]$ is the weight function, which maps each pair (i, j) to the importance that agent i gives to agent j, hence $w(i, j) = \sigma_{i,j}$. We refer to a network of this kind as a *social network*.

In a social network G a *walk* from i to j is any sequence of connected directed edges that starts in i and ends in j. A *path* from i to j is a walk that goes at most once through each node. If there exists a path from i to j we say that i *hears* j, and we denote it by $i \to j$. If there is a path from i to j and vice-versa we say that i and j *communicate*, and we denote this by $i \leftrightarrow j$.We say the social network G is

- *Strongly Connected* if $\forall(i, j) \in N^2$, $i \leftrightarrow j$;
- *Connected* if $\forall(i, j) \in N^2$, $i \to j$ or $i \leftarrow j$;

- *Disconnected* if $\exists(i,j) \in N^2$, such that neither $i \to j$ or $i \leftarrow j$.

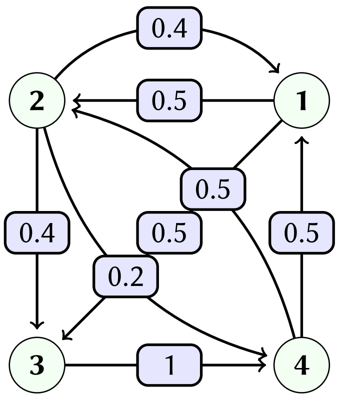

Figure 1: Diagram of Social Network G_1

Figure 1 illustrates the structure of one such social network. In this depiction, circles mean agents, arrows indicate the direction of out-links, and the weights of these out-links are represented by the values inside the rounded rectangles.

In G_1 we we observe that Agent 1 observes Agents 2 and 3. Conversely, agent 1 is observed by agents 2 and 4. The only mutually influential pair is Agents 1 and 2, while Agent 3 exclusively observes Agent 4. A notable feature of this network is the absence of a dominant agent; there is no clear hierarchy, and no one is marginalized. Each agent both observes and is observed; G_1 defines a strongly connected network. The agents in this network seem to be ”well integrated” with each other. This balanced interaction suggests a network evocative of a close group of friends, a cohesive family, or minority groups seamlessly integrated into a community.

Now, examine the structure of network G_2 as depicted in Figure 2. There are two distinct clusters of agents: the first group, located on the left side of the figure, consists of agents 1 to 5, while the second group, positioned on the right side, includes agents 6 to 8. There is almost no interaction between agents of the left group and those of the right, except for agent 8 from the right group who observes agent 5 from the left group with a significant weight of 0.4. Despite this, each group exhibits strong internal cohesion. This configuration could represent various social dynamics, such as two different political factions, separate families, distinct religious communities, or a group of recent immigrants (right group) integrating into the existing society.

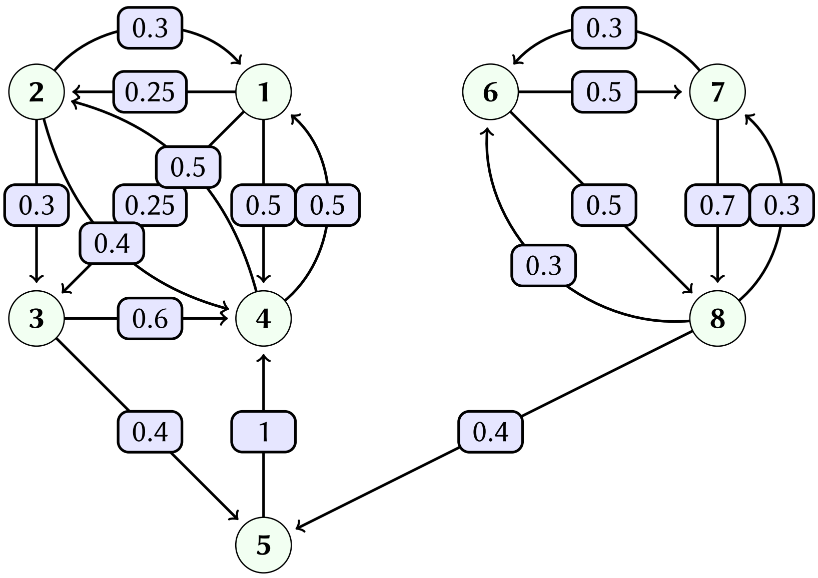

Figure 2: Diagram of Social Network G_2

Furthermore, in the right group, each agent displays to be of equal importance to the rest. In contrast, the left group shows a clear hierarchical structure. For instance, agent 5 seems somewhat marginalized, being observed only by agent 3 with minimal importance. Conversely, agent 4 emerges as the influential figure or 'leader' within this group, observed by all other members, each attributing high importance to him.

This presents the model's flexibility in capturing manifold real social network characteristics and features within society. By carefully selecting the appropriate values for σ_{ij}'s, θ, and γ, the model can effectively represent various social dynamics such as cultural leaders, segregated minorities, groups in conflict, companies' or industries' agents. In the realm of immigration, this framework can adapt to different scenarios: unobserved immigrant communities (such as Haitians in Chile), communities under observation (such as Venezuelans), efforts by institutions in the host country to facilitate the integration of newcomers (through strategic link formation), and varying degrees of cultural disparity with the host society (reflected in the choice of γ), among others.

Let us now proceed to derive and enunciate the key results necessary to characterize the behaviour of our setup. In our model, a disconnected social network essentially functions as a collection of independent societies, with its disjoint components exerting no influence on one another. The behaviour of a network of this kind can be explored by studying each of these components on their own.

Therefore, we restrict our attention to connected networks.

3.2 Preliminary remarks

Let $\mathbf{\Sigma} = [\sigma_{ij}]_{i,j\in N}$, where we define $\sigma_{ij} = 0$ if $j \notin N_i$. From (3.2) we have that $\mathbf{\Sigma}$ is a row stochastic (or simply stochastic) matrix that defines a Markov Process (MP). Here, agents would be the states of the MP and σ_{ij} the probability of moving from i to j. Powers $\mathbf{\Sigma}^k = [\sigma_{ij}^{(k)}]_{i,j\in N}$ of $\mathbf{\Sigma}$ are all stochastic matrices, and give the probability of moving from i to j in different numbers of steps.

$\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$ are the matrices defined by G_1 and G_2 respectively.

$$\mathbf{\Sigma}_1 = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 \\ 0.4 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0 & 1 \\ 0.5 & 0.5 & 0 & 0 \end{bmatrix}$$

$$\mathbf{\Sigma}_2 = \begin{bmatrix} 0 & 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.7 \\ 0 & 0 & 0 & 0 & 0.4 & 0.3 & 0.3 & 0 \end{bmatrix}$$

Let i and j be two states of the Markov Process defined by $\mathbf{\Sigma}$. As with the network equivalent, in the context of Markov Theory, it is said that i and j communicate if the probability of going from one of these states to another is positive (not necessarily in one step) and again we denote this by $i \leftrightarrow j$.

The relation defined by $\leftrightarrow$ is an equivalence relation. Therefore it defines a partition of G into equivalence classes. States communicating with each other form an equivalence class. These classes correspond to the strongly connected components of the network.

For any of these equivalent classes one, and only one, of two conditions always applies: either a process that enters the class never leaves, or a process that leaves the class never returns. A class of the first kind is called an *ergodic set*, while one of the second kind is called a *transient set*. A Markov Process does not need to have a transient set. However, there always is at least one ergodic set (Kemeny and Snell, 1976).

For two communicating classes C_1 and C_2, we say that $C_1 \prec C_2$ if there exist $c_1 \in C_1$ and $c_2 \in C_2$ such that $c_1 \to c_2$. When the opposite happens, it would imply both classes are the same, since they communicate with each other. The relation $\prec$ defines a partial order in $\mathcal{C}$. Maximal elements are equivalents to the ergodic sets and minimal elements to transient sets that are not heard by any other set.

Let us consider the bi-partition of $\mathcal{C}$ into $\mathcal{T}$ and $\mathcal{Q}$, the sets of transient classes and ergodic classes respectively. All classes in $\mathcal{Q}$ are disconnected.

A maximal element in the subset $\mathcal{T}$ is a transient class that only hears ergodic sets and a minimal element is a transient class that is heard by no other class.

Figure 3 shows the diagram of a network where each circle represents one of the communicating classes. In this example T_5, T_6, and T_7 are the maximal transient classes, while T_1, T_2, and T_3 are minimal classes.

This classification helps to elucidate the process of trait transition from ergodic to transient classes.

A state i has period k if any walk (in the network sense) that begins in i and ends in i occurs in a multiple of k-steps. Formally, the period of i is

$$k = \text{gdc}\{n > 0 : \text{Pr}(X_n = i | X_0 = i) > 0\}. \tag{3.3}$$

If $k = 1$ the state i is said to be aperiodic. A Markov Process with only aperiodic states is called aperiodic.

An aperiodic MP consisting of only one ergodic set is called a *regular Markov*

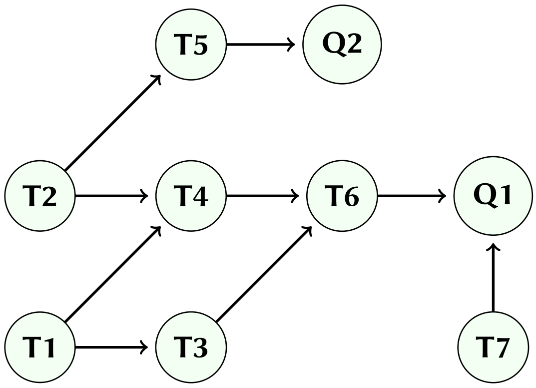

Figure 3: Diagram of a network of Transient and Ergodic Classes

process. Let us express the following classic result central to the theory of regular Markov processes. The proof is found in any introductory book of Markov Theory.

Theorem 3.1. *If Σ defines a regular MP of n states, then there exists a unique (row) stochastic vector $\eta = (\eta_1, \eta_2, ..., \eta_n) \in \mathbb{R}^n$ such that*

(i) $\eta\mathbf{\Sigma}^k = \eta, \forall k \in \mathbb{N}$.

(ii) The powers of $\mathbf{\Sigma}$ converge to a matrix $\mathbf{H}$, for which each row equals η.

(iii) The vector η has full support. This is, $\eta_i > 0$ for each $1 \leq i \leq n$.

Such vector is known as the unique stationary distribution *of $\mathbf{\Sigma}$.*

In the theory of Finite Markov Processes, η holds a probabilistic interpretation, and it is the expected amount of times that the process goes through each state.

Furthermore, consider an MP that consists of one ergodic set and a finite number of transient sets, and in which there is a walk (in the network sense) from a state in each of the transient sets to one in the ergodic set (and not vice-versa, or they would be part of the ergodic set). Then, the process is expected to eventually leave the transient sets and enter the ergodic set with a probability of 1. Let us call an MP of this kind (and its network equivalent) *segregated.* As the name suggests, with this class we want to capture segregated social networks, in which some minorities' groups hear society but are not heard by any member of it.

We enunciate the following theorem which will be useful below, the proof can be found in Theorem 1.5 of Hartfiel (2006).

Theorem 3.2. *Let Σ be a stochastic matrix defining a segregated MP. Let $\mathbf{Q}$ denote the bottom right sub-matrix of Σ associated with the transient states.*

Then, as $k \to \infty$, $\mathbf{Q} \to 0$.

The key property of stochastic matrices in the context of Markov Theory is their averaging effect. If $\mathbf{\Sigma}$ is a stochastic matrix of size n and $\mathbf{w}$ any (column) vector of size n, then $\min_j \mathbf{w}_j \leq \sum_j \sigma_{ij}\mathbf{w}_j \leq \max_j \mathbf{w}_j$. This effect is condensed in the *coefficient of ergodicity* $\mathcal{T}(\Sigma)$ defined as

$$\mathcal{T}(\mathbf{\Sigma}) = \frac{1}{2}\max_{i,j}\sum_k |\sigma_{ik} - \sigma_{jk}| = 1 - \min_{i,j}\sum_k \min\{\sigma_{ik}, \sigma_{jk}\}, \tag{3.4}$$

which fulfils

$$\max_j(\mathbf{\Sigma w})_j - \min_j(\mathbf{\Sigma w})_j \leq \mathcal{T}(\mathbf{\Sigma})(\max_j \mathbf{w}_j - \min_j \mathbf{w}_j). \tag{3.5}$$

It is clear that $0 \leq \mathcal{T}(\Sigma) \leq 1$. Strictly $\mathcal{T}(\Sigma) < 1$ if, and only if, there is at least one row in Σ that is orthogonal to every other row. In particular, if one column of $\mathbf{\Sigma}$ has only positive entries, then it is guaranteed that $\mathcal{T}(\mathbf{\Sigma}) < 1$ (Hartfiel, 2006). This coefficient also fulfils

$$\mathcal{T}(\mathbf{\Sigma}^k) \leq \mathcal{T}(\mathbf{\Sigma})^k. \tag{3.6}$$

For $\rho \in (0, 1)$ define $\mathbf{\Pi}(\rho)$ as

$$\mathbf{\Pi}(\rho) = \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}} \rho^k \Sigma^k. \tag{3.7}$$

This matrix is central to what ensues. We will need the following results.

Lemma 3.3. *Consider a stochastic matrix $\mathbf{\Sigma}$ of dimension n and $\rho \in (0, 1)$, $\mathbf{\Pi}(\rho)$, as defined in (3.7), is a stochastic matrix. Furthermore, if $\mathbf{\Sigma}$ defines a regular MP, so does $\mathbf{\Pi}(\theta)$, and both have the same stationary distribution.*

Proof. Let $e = (1, 1, ..., 1) \in \mathbb{R}^n$ be the column vector with n 1's. A matrix $\mathbf{S}$ of dimension n is stochastic if, and only if, $Se = e$. Thus, let us compute

$$
\begin{aligned}
\mathbf{\Pi}(\rho)e &= \left(\frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\Sigma^k\right)e \\
&= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\Sigma^k e \\
&= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k e \\
&= \frac{1-\rho}{\rho}\cdot\frac{\rho}{1-\rho}e \\
&= e
\end{aligned}
\tag{3.8}
$$

Now, $\mathbf{\Pi}(\rho)$ is defined as a weighted summation of the powers of $\mathbf{\Sigma}$. The weighted powers contain all non-negative entries. Hence, a pair of states communicate in $\mathbf{\Sigma}$ if, and only if, they communicate in $\mathbf{\Pi}(\rho)$. Particularly, if $\mathbf{\Sigma}$ defines a regular MP, $\mathbf{\Pi}(\rho)$ does too.

Lastly, assume $\mathbf{\Sigma}$ defines a regular Markov Chain and let η be its stationary distribution. Then,

$$
\begin{aligned}
\eta\mathbf{\Pi}(\rho) &= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\eta\mathbf{\Sigma}^k \\
&= \frac{1-\rho}{\rho}\sum_{k\in\mathbb{N}}\rho^k\eta \\
&= \frac{1-\rho}{\rho}\cdot\frac{\rho}{1-\rho}\eta \\
&= \eta.
\end{aligned}
\tag{3.9}
$$

■

$\mathbf{\Pi}(\rho)$ has the same stationary distribution as Σ. $\mathbf{\Pi}(\rho)$ defines a *decaying* MP where each step k occurs with probability $(1-\rho)\rho^{k-1}$. Longer steps are less likely to occur. In the context of Markov theory, $\pi_k^i := \mathbf{\Pi}_{ij}(\rho)$ is the probability that, starting in i, the process ends in j.

Consider one minimal transient class defined as before. A row that represents a node in this class is orthogonal to no other row. By contradiction, if it were, it would mean there is one other class that has no path from the first class. By definition, there is no path from another class to a minimal transient class. Hence, the network would be disconnected, which we have discarded. Hence, $\mathcal{T}(\mathbf{\Pi}(\rho)) < 1$ in our setup.

Additionally, periods disappear in $\mathbf{\Pi}(\rho)$ since the length of a path from any i to j is either 1 or 0 in this matrix. Therefore, regardless of the periodicity of $\mathbf{\Sigma}$, $\mathbf{\Pi}(\rho)$ is aperiodic.

Let $\pi^i = (\pi^i_1, ..., \pi^i_N)$, $\forall i \in N$. $\pi^i(\rho)$ is a probability measure that's agent specific. Hence, for any $\nu \in \mathbb{R}^N$ we can define a random variable (r.v.) ν^i as

$$v^i = \begin{cases} \nu_1 & \text{with prob. } \pi^i_1(\rho) \\ \vdots & \vdots \\ \nu_N & \text{with prob. } \pi^i_N(\rho) \end{cases}, \ \forall i \in N$$

Then, given the r.v. ν^i, its expected value is

$$\mathbb{E}_{\pi^i(\rho)}[\nu] = \sum_{j \in N} \pi^i_j(\rho)\nu_j$$

3.3 Static equilibrium

Let $\mathbf{\Gamma}$ be the simultaneous move N-player game defined by $\mathbf{\Sigma}$, with agents in N, payoffs as in (3.1), and strategy spaces $\mathbb{R}$. We are first interested in computing the Nash equilibrium of this game.

Proposition 3.4. *For any $\mathbf{\Sigma}$, a Nash equilibrium exists, is unique and is given by*

$$a^*_i = (1-\theta)\gamma_i + \theta\mathbb{E}_{\pi^i(\theta)}[\gamma], \ \forall i \in N \tag{3.10}$$

Proof. Let $i \in N$. The first derivative of $\mathcal{U}_i$ with respect to a_i is

$$
\begin{aligned}
\frac{\partial \mathcal{U}_i}{\partial a_i} &= -2(1-\theta)(a_i-\gamma_i) - 2\theta \sum_{j\in N} \sigma_{ij}(a_i - a_j) \\
&= -2(1-\theta)(a_i-\gamma_i) - 2\theta \sum_{j\in N} \sigma_{ij} a_i + 2\theta \sum_{j\in N} \sigma_{ij} a_j \\
&= -2(1-\theta)(a_i-\gamma_i) - 2\theta a_i + 2\theta \sum_{j\in N} \sigma_{ij} a_j \\
&= 2(1-\theta)\gamma_i - 2a_i + 2\theta \sum_{j\in N} \sigma_{ij} a_j
\end{aligned}
\tag{3.11}
$$

A vector of actions $\mathbf{a}^* = (a_1^*, a_2^*, ..., a_N^*) \in \mathbb{R}^N$ is a Nash equilibrium in pure strategies of the game if, and only if, $\forall i \in N$ it solves

$$
\begin{aligned}
&\frac{\partial \mathcal{U}_i}{\partial a_i}(a_i^*; \mathbf{a}^*_{-\mathbf{i}}) = 0 \\
\iff &\; 2(1-\theta)\gamma_i - 2a_i^* + 2\theta \sum_{j\in N} \sigma_{ij} a_j^* = 0 \\
\iff &\; a_i^* - \theta \sum_{j\in N} \sigma_{ij} a_j^* = (1-\theta)\gamma_i.
\end{aligned}
\tag{3.12}
$$

The last line in (3.12), since it is true $\forall i \in N$, defines the set of N equations

$$
\begin{aligned}
&(\mathbf{I} - \theta\mathbf{\Sigma})\mathbf{a} = (1-\theta)\gamma \\
\iff &\; \mathbf{a} = (1-\theta)\Big[\sum_{k=0}^{\infty} \theta^k \mathbf{\Sigma}^k\Big]\gamma \\
&= (1-\theta)\Big[\mathbf{I} + \sum_{k=1}^{\infty} \theta^k \mathbf{\Sigma}^k\Big]\gamma \\
&= (1-\theta)\gamma + (1-\theta)\Big[\sum_{k=1}^{\infty} \theta^k \mathbf{\Sigma}^k\Big]\gamma \\
&= (1-\theta)\gamma + \theta\Big[\frac{1-\theta}{\theta}\sum_{k=1}^{\infty} \theta^k \mathbf{\Sigma}^k\Big]\gamma \\
&= (1-\theta)\gamma + \theta\mathbf{\Pi}(\theta)\gamma,
\end{aligned}
\tag{3.13}
$$

and this last expression yields (3.10). ■

Notice first that, if there is no interaction between agents, i.e. $\theta = 0$, then $a_i^* = \gamma_i$. Hence, with no interaction, equilibrium actions are agents' types. Second, let $\tilde{\gamma} \in \mathbb{R}$, if $\forall i \in N$ $\gamma_i = \tilde{\gamma}$, then $a_i^* = \tilde{\gamma}$. Therefore, if agents are homogeneous, the network's topology has no effect on equilibrium actions. Thus, is the heterogeneity in the population what affects equilibrium actions through the network's topology and the distribution of γ across players.

We can separate additively the individual effect of their own type from the network effect on every player's equilibrium actions in (3.10).

$$a_i^* = (1-\theta)\underbrace{\gamma_i}_{\substack{\text{No}\\\text{interaccion}}} + \theta \underbrace{\mathbb{E}_{\pi^i(\theta)}[\gamma]}_{\text{Network effect}} \tag{3.14}$$

With this notation, we have a clear interpretation of *spillover* effects:

$$\begin{aligned} \forall i, j \in N,\ i \neq j:\ \frac{\partial a_i^*}{\partial \gamma_i} &= (1-\theta) + \theta \pi_i^i(\theta), \\ \text{and } \frac{\partial a_i^*}{\partial \gamma_j} &= \theta \pi_j^i(\theta). \end{aligned} \tag{3.15}$$

Moreover, we have the following result:

Corollary 3.4.1. *Regarding how agents affect the behavior of others, we have:*

- *If i does not hear j, then $\pi_j^i(\theta) = 0$ and the trait of j has no effect on i's behaviour.*
- *If $\forall i, j \in N$, $i \leftrightarrow j$, then $\pi_j^i(\theta) > 0$ $\forall i, j \in N$, and each agent's trait affects the behaviour of the rest.*

In a society where every pair of agents communicates, each of the agents' traits persists in equilibrium. More central agents, in the sense that the MP defined by the network passes by them more often, are the ones whose traits are the most persistent.

Once we have identified how the network's topology shapes the equilibrium, we can go a little deeper.

When $\mathbf{\Sigma}$ is a strongly connected network, we have the following decomposition.

Proposition 3.5. *Let $\mathbf{\Sigma}$ be a strongly connected network. Then, for each $i \in N$ the network effect $\nu_i = \mathbb{E}_{\pi^i(\theta)}[\gamma]$ can be uniquely decomposed as*

$$\nu_i = \underbrace{\mathbb{E}_\eta[\gamma]}_{\textit{Structural effect}} + \underbrace{\Delta_i(\gamma)}_{\textit{Local effect}}, \tag{3.16}$$

where η is the stationary distribution of Σ, and the local effects *are bounded as function of the size of θ, the ergodicity of the chain, and the position of the agents in the network.*

Proof. Let $d_{ij} = \pi^i_j - \eta_j$. Since η is unique, d_{ij} is well defined and unique for each i and j. Let $\mathbf{d}_i = (d_{i1}, d_{i2}, ..., d_{iN})$ we can then decompose ν_i as

$$\mathbb{E}_{\pi^i(\theta)}[\gamma] = \mathbb{E}_\eta[\gamma] + \mathbf{d}_i\gamma \tag{3.17}$$

And we define $\Delta^i_\theta(\gamma) = \mathbf{d}_i\gamma$ as the local effect. Now, any stochastic matrix has the averaging effect described above. Let $M_i = \max_{j\in N}\{\sigma_{ij}\}$ and $m_i = \min_{j\in N}\{\sigma_{ij}\}$.

$$\begin{aligned}
|\sigma_{ij} - \eta_j| &\leq M_i - m_i \\
|\sigma^{(2)}_{ij} - \eta_j| &\leq \mathcal{T}(\mathbf{\Sigma})(M_i - m_i) \\
|\sigma^{(3)}_{ij} - \eta_j| &\leq \mathcal{T}(\mathbf{\Sigma^2})(M_i - m_i) \\
&\vdots \\
|\sigma^{(k)}_{ij} - \eta_j| &\leq \mathcal{T}(\mathbf{\Sigma}^{k-1})(M_i - m_i) \\
&\vdots
\end{aligned} \tag{3.18}$$

We can then perform the following rearrangement after multiplying the k-th inequality in (3.18) by the k-th power of θ.

$$
\begin{aligned}
|\theta\sigma_{ij} - \theta\eta_j| &\leq \theta(M_i - m_i) \\
|\theta^2\sigma_{ij}^{(2)} - \theta^2\eta_j| &\leq \theta^2\mathcal{T}(\mathbf{\Sigma})(M_i - m_i) \\
|\theta^3\sigma_{ij}^{(3)} - \theta^3\eta_j| &\leq \theta^3\mathcal{T}(\mathbf{\Sigma^2})(M_i - m_i) \\
&\vdots \\
|\theta^k\sigma_{ij}^{(k)} - \theta^k\eta_j| &\leq \theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1})(M_i - m_i) \\
&\vdots
\end{aligned}
\tag{3.19}
$$

Adding all inequalities in (3.19) and using the Triangular Inequality on the terms inside absolute values on each left side, yields:

$$
\begin{aligned}
\left|\sum_{k=1}^{\infty}\theta^k\sigma_{ij}^{(k)} - \sum_{k=1}^{\infty}\theta^k\eta_j\right| &\leq \sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1})(M_i - m_i) \\
\Rightarrow \frac{1-\theta}{\theta}\left|\sum_{k=1}^{\infty}\theta^k\sigma_{ij}^{(k)} - \frac{\theta}{1-\theta}\eta_j\right| &\leq \frac{1-\theta}{\theta}\left(\sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1})(M_i - m_i)\right) \\
\Rightarrow |\pi_j^i - \eta_j| &\leq \frac{1-\theta}{\theta}(M_i - m_i)\sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma}^{k-1}) \\
&\leq \frac{1-\theta}{\theta}(M_i - m_i)\sum_{k=1}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma})^{k-1} \\
&= (1-\theta)(M_i - m_i)\sum_{k=0}^{\infty}\theta^k\mathcal{T}(\mathbf{\Sigma})^{k} \\
&= \frac{(1-\theta)(M_i - m_i)}{1-\theta\mathcal{T}(\mathbf{\Sigma})}
\end{aligned}
\tag{3.20}
$$

Hence,

$$
\begin{aligned}
|\Delta_i(\gamma)| &= |\mathbf{d}_i\gamma| \\
&\leq \frac{(1-\theta)(M_i - m_i)}{1-\theta\mathcal{T}(\mathbf{\Sigma})}\sum_{N}\gamma_i
\end{aligned}
\tag{3.21}
$$

■

Thus, every agent's actions fluctuate around a common structural effect derived from the topology of the network. If their actions are above or below that level, depends on their position on the network. The size of the deviation from this level for agent i is bounded by: the size of θ, the differences on the absolute value of the importance other agents give to i, and the ergodicity of Σ. Thus, the tighter and more stationary is the network, the smaller the local effect relative to the structural one.

3.4 Dynamic equilibrium

Now, to represent the ongoing nature of cultural traits over time, we model cultural traits as a continuous variable in discrete time $\mathcal{T} = \{0, 1, 2, 3...\}$. For this, we use an overlapping generations model (OLG). In this setup, agents are called *dynasties*. Each dynasty is represented with one agent who reproduces asexually on each period. Thus, in every period there is a new generation representing each dynasty.

Time starts at $t = 0$, where the initial generation has an exogenous type $\gamma_0 = (\gamma_{1,0}, .., \gamma_{N,0})$. The main feature of OLG is that, from then on, dynasties types are endogenous and are given by parents' behavior in the previous period. This is:

$$\gamma_{i,t} = a^*_{i,t-1},\ \forall i \in N,\ t = 1, 2, 3...$$

Dynasties' payoff in each period is:

$$\mathcal{U}(a_{i,t}, \mathbf{a}_{N_i,t}) = -(1-\theta)(a_{i,t} - \gamma_{i,t})^2 - \theta \sum_{j \in N_i} \sigma_{ij}(a_{i,t} - a_{j,t})^2$$

Now, we want to check the convergence of behavior (i.e., cultural traits) as $t \to \infty$, this is, check the existence of

$$\mathbf{a}^* = \lim_{t \to \infty} \mathbf{a}^*_t$$

where $\mathbf{a}^*_t = (a^*_{1,t}, ..., a^*_{N,t})$.

Let us first show the following result.

Lemma 3.6. *The Nash equilibrium of the game at each period is given by*

$$\mathbf{a}^*_t(\theta) = \mathbf{\Omega}^t(\theta)\gamma_0,\ t \in \mathcal{T}\backslash\{0\}, \tag{3.22}$$

where $\mathbf{\Omega}(\theta) := (1-\theta)\mathbf{I} + \theta\mathbf{\Pi}(\theta)$

Proof. From (3.13) we have

$$\begin{aligned}\mathbf{a}_t^*(\theta) &= (1-\theta)\gamma_t + \theta\mathbf{\Pi}(\theta)\gamma_t \\ &= ((1-\theta)\mathbf{I} + \theta\mathbf{\Pi}(\theta))\mathbf{a}_{t-1}^*.\end{aligned}$$

By applying recursion on this last term, (3.22) follows. ∎

Hence, the convergence of cultural traits depends entirely on $\mathbf{\Omega}(\theta)$.

Regard the contracting effect of stochastic matrices condensed in $\mathcal{T}$ and discussed above. Notice how (3.6) implies that $\mathcal{T}(\mathbf{\Omega}(\theta)^k) < \mathcal{T}(\mathbf{\Omega}(\theta))$, and this is true for every class of stochastic matrix. Therefore, $\mathbf{a}_t^*(\theta)$ can only become more contracted over time. This implies that the traits of individuals do not diversify as time passes. Since $\mathcal{T}(\mathbf{\Omega}(\theta)) < 1$, the process of homogenization is pervasive; the traits become strictly less diverse as time passes.

Furthermore, consider a segregated social network as defined above. From Theorem 3.2 and Lemma 3.22, the traits of the segregated agents disappear in the long run. Thus, when groups of immigrants are isolated from society, the influence of their idiosyncrasy and culture on society fades as time passes. They become completely absorbed into the cultural traits of local inhabitants.

Therefore, we need to characterize what occurs in ergodic sets. With a strongly connected network, we have convergence to the same point.

Proposition 3.7. *If* $\mathbf{\Sigma}$ *defines a strongly connected network, then*

$$a_i^* = \mathbb{E}_\eta[\gamma_0], \ \forall i \in N \tag{3.23}$$

where η *is the unique stationary distribution of* $\mathbf{\Sigma}$.

Proof. If $\mathbf{\Sigma}$ is a regular MP, then so is $\mathbf{\Pi}(\theta)$ by Lemma 3.3, and it follows immediately that $\mathbf{\Omega}(\theta)$ is regular. Let η be the stationary distribution of $\mathbf{\Sigma}$. Then,

$$\begin{aligned}\eta\mathbf{\Omega}(\theta) &= (1-\theta)\eta\mathbf{I} + \theta\eta\mathbf{\Pi}(\theta) \\ &= (1-\theta)\eta + \theta\eta \qquad (3.24) \\ &= \eta\end{aligned}$$

Hence, η is also the unique stationary distribution of $\mathbf{\Omega}$. This implies:

$$\begin{aligned}
\mathbf{a}^* &= \lim_{t\to\infty} \mathbf{a}_t^* \\
&= \lim_{t\to\infty} \mathbf{\Omega}^t(\theta)\gamma_0 \qquad (3.25)\\
&= \mathbf{H}\gamma_0,
\end{aligned}$$

where $\mathbf{H}$ is as defined in Theorem 3.1. Computing the last expression for each component of $\mathbf{a}^*$, yields (3.22).

■

Because $\mathbf{\Sigma}$ is strongly connected, we know that η has full support. The influence on the asymptotic behaviour of each dynasty is given by the inverse of the mean times the MP passes through it. Notice how, in the long run, the parameter θ does not affect the equilibrium value, since η is directly derived from $\mathbf{\Sigma}$. Thus, no matter the relative importance that agents give to their values relative to their peers' actions, society will eventually converge to a common trait underlying structurally in the topology of the network through η.

We have turned our focus to where the equilibrium lies, and not to how far the society is from it. For a closing commentary on this, the coefficient of ergodicity proves insightful yet another time. Let us denote $\lambda_1, \lambda_2, \lambda_3, \ldots$ as the spectral values of a row-stochastic matrix $\mathbf{\Sigma}$, ordered in decreasing module magnitude. Given that the sum of each row in $\mathbf{\Sigma}$ equals 1, it follows that $\lambda_1 = 1$. Hence, the spectral gap of a stochastic matrix is $|1 - \lambda_2|$, where $|\lambda_k|$ denote the modulus of possibly complex $|\lambda_k|$. The crucial property of the coefficient of ergodicity is that $|\lambda_2| < \mathcal{T}(\mathbf{\Sigma})$, as outlined in Hartfiel (2006). Matrices with a large spectral gap mix faster, which in turn means that matrices with low $\mathcal{T}(\mathbf{\Sigma})$ move quicker to their steady state. Therefore $\mathcal{T}(\mathbf{\Sigma})$ is an adequate bound for quantifying the distance of the network from its stationary state.

4 Comparative Dynamics

This setup allows us to study a myriad of combinations of networks, weights and traits. It allows us to delve into a variety of economic challenges and phenomena where preferences play a role. Its computational tractability permits the simulation of large sets of nodes and links.

As a mode of illustration, and in line with our introductory motivation, let us revise the following simple scenarios related to migration.

4.1 Tightly connected network with heterogeneity among the agents

Let us begin with a simple example by computing the equilibrium in network G_1 in Figure 1 defined above. We define the set of traits $\gamma_1 = (0, 0, 1, 1)$, i.e., there are two sharply different traits in the social network. We consider two values of theta: $\theta = 0.5$ and $\theta = 0.2$. Figures 4 and 5 show the evolution of the traits in both scenarios.

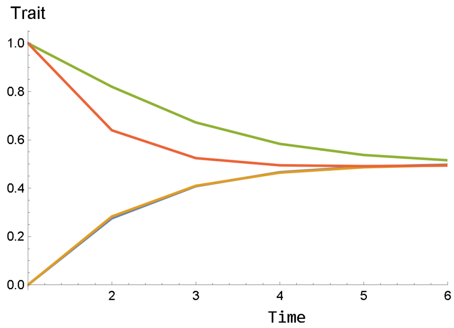

Figure 4: G_1 for $\theta = 0.5$

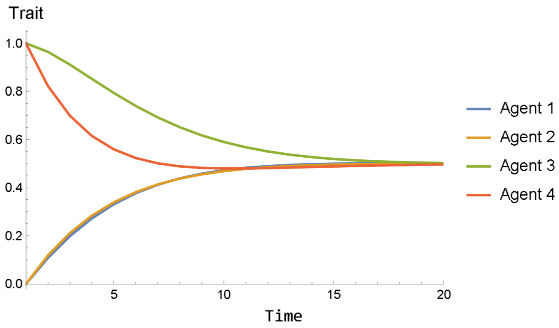

Figure 5: G_1 for $\theta = 0.2$

Convergence was considerably lower when the value of θ was decreased to 0.2. As noted above, the long-run common trait was the same in both cases, independently of θ. Yet, even in this simple strongly connected case, θ plays a significant role in the speed of convergence. Agent 3 takes the longest in converging, given he does not listen to any agent with trait 0. However, he converges not much later than the rest. Given the quasi-symmetry of the network, this trait was a normal average of both traits in this society.

4.2 Arrival of an unobserved homogeneous group of migrants

Let us now turn to the Network G_2 in Figure 2. We call (aptly) the leftmost group the *host society*. The average trait of this group is 1. Yet we consider two cases: one with some diversity around this trait that averages to 1, and another where the host is already in the stationary state when the migrant group arrives. Namely:

- Diverse host: $\gamma_d = (1.5, 1.3, 1, 0.7, 0.5, 0, 0, 0)$.
- Stationary host: $\gamma_s = (1, 1, 1, 1, 1, 0, 0, 0)$.

The rightmost group is the just-arrived group of migrants, which is homogeneous and shares a trait equal to 0. From now on $\theta = 0.5$. Figures 6 and 7 show the dynamics.

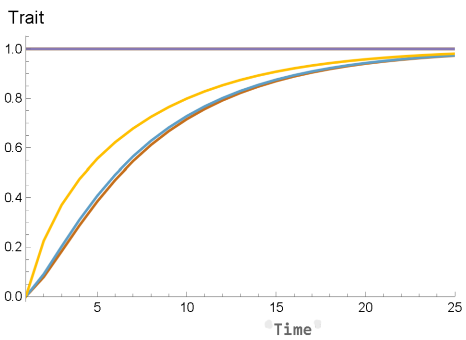

Figure 6: Stationary host. G_2.

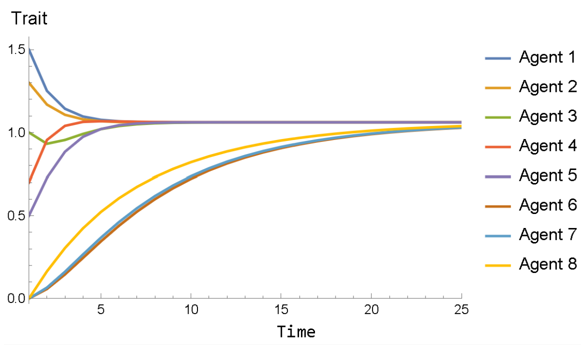

Figure 7: Diverse host. G_2.

The immigrant that observes the host society (agent 8) assimilates considerably faster in the first periods than the rest of the migrants, even when they observe 8. As expected, in the long run, when the host society does not observe the migrants or these hear the locals, the traits of the migrants disappear, and society converges to the same stationary state it was in the beginning. The speed of convergence was slower than in the previous example, taking almost five times more periods to converge to a common trait. Even when society may be heading towards a melting pot, this process may be slow-moving for a large set of agents.

For the example where the host society was heterogeneous, the average trait of the entire network after 50 periods stabilizes at 1.0613. When it is homogeneous, it stabilizes at 1, as we can expect. Even when the average trait of the host society was slightly perturbed around the average of 1, we see how the topology of the network affects the convergence result. The speed of convergence is similar in both cases.

Consider now the Network G_3, which is the same as Network G_2 except that agent 8 now does a greater assimilation effort with the host society, paying more attention to agent 5.

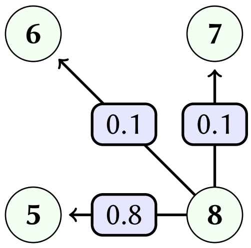

Figure 8: Change in behaviour from agent 8 in G_3

Now, agent 8 makes a higher assimilation effort than before. Figure 8 depicts this modification. Figures 9 and 10 show the resulting dynamics.

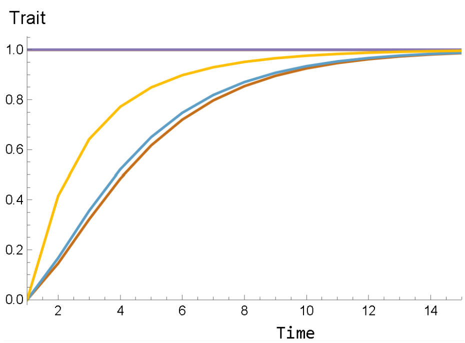

Figure 9: Stationary host. G_3.

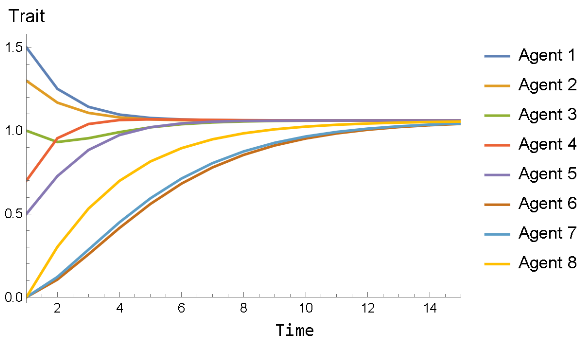

Figure 10: Diverse host. G_3.

After $t = 12$ the immigrants virtually assimilated in both cases. Making one individual in the group pay more attention to the host society, the speed of convergence reduced nearly by half. This hints at the power of peer effects in the assimilation process: if even a few members of a group of migrants are induced to blend with the host society, the rest will soon follow.

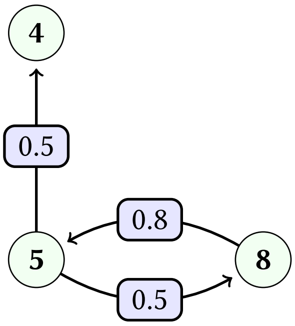

Figure 11: Change in behaviour from agent 5 in G_4

4.3 Arrival of an observed homogeneous group of migrants

Let G_4 be a social network identical to G_3, except that now agent 5 observes agent 8. Figure 11 depicts this modification.

Now the host society hears the group of immigrants, forming one strongly connected network. The vectors of traits are as in the previous case. Figure 14 charts the evolution of the average trait in both cases.

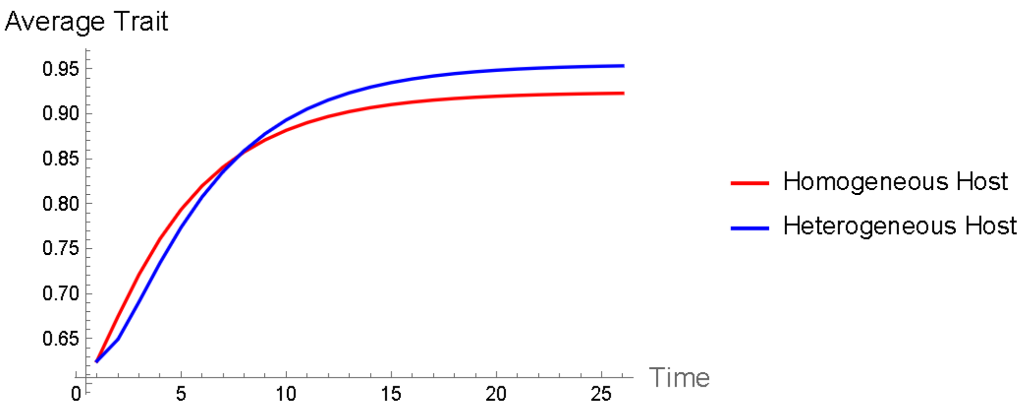

Figure 12: Evolution of average trait. G_4.

Society reaches its stationary state at around period 20. In both scenarios, the average trait is heavily skewed towards the local culture. However, now the migrants' culture persists within them, and the host country adopts part of their traits. In the diverse case, the interaction with the topology of the host society made its trait persist more robustly, with an equilibrium trait close to 0.95. Convergence happens at about the same pace as when the immigrants were not observed.

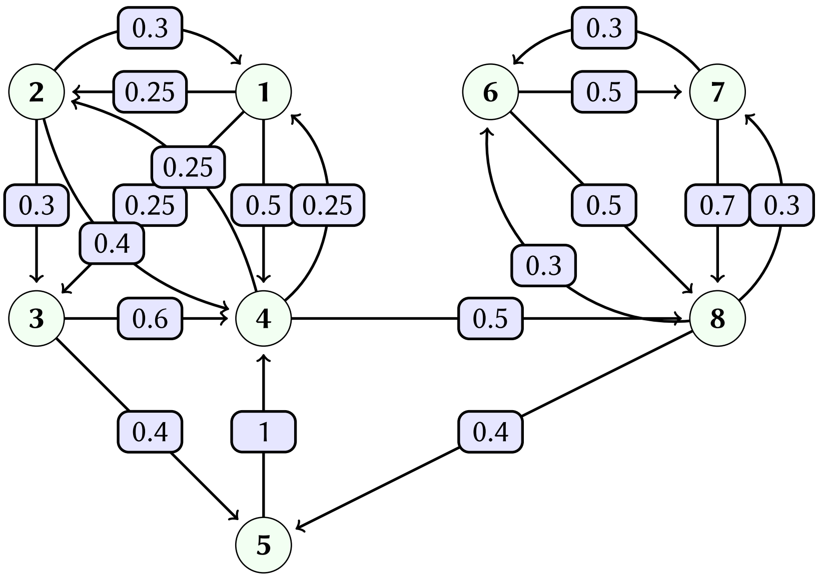

Figure 13: Diagram of Social Network G_5

4.4 Group of migrants observed by a cultural leader

Interpreting the roles of the agents in Network G_4, agent 5 has minimal influence on the host society. What occurs instead, when cultural leaders attempt to desegregate the immigrant group? Consider it is agent 4 (the leader of the host society) the one who directly communicates with agent 8. We alter the network G_4 to implement this effect: the direct link from agent 5 to agent 8 is eliminated, and instead, agent 4, who represents the leader of the host society, establishes a direct observational link with agent 8. Social Network G_5 outlined in Figure 13 accounts for this change.

We focus on the diverse host scenario. By $t = 12$, it appears that society has reached a state of convergence, with the long-run average cultural trait stabilizing at 0.65.

While the rate of convergence remains similar to that observed in the previous example, the persistence of immigrants' cultural traits within the society is notably stronger in this scenario, with a long-run average trait close to 0.65. This outcome suggests that cultural leaders, whether they are individuals or institutions, can play a crucial role not only in preserving the unique idiosyncrasies of immigrants but also in facilitating their integration into the host society.

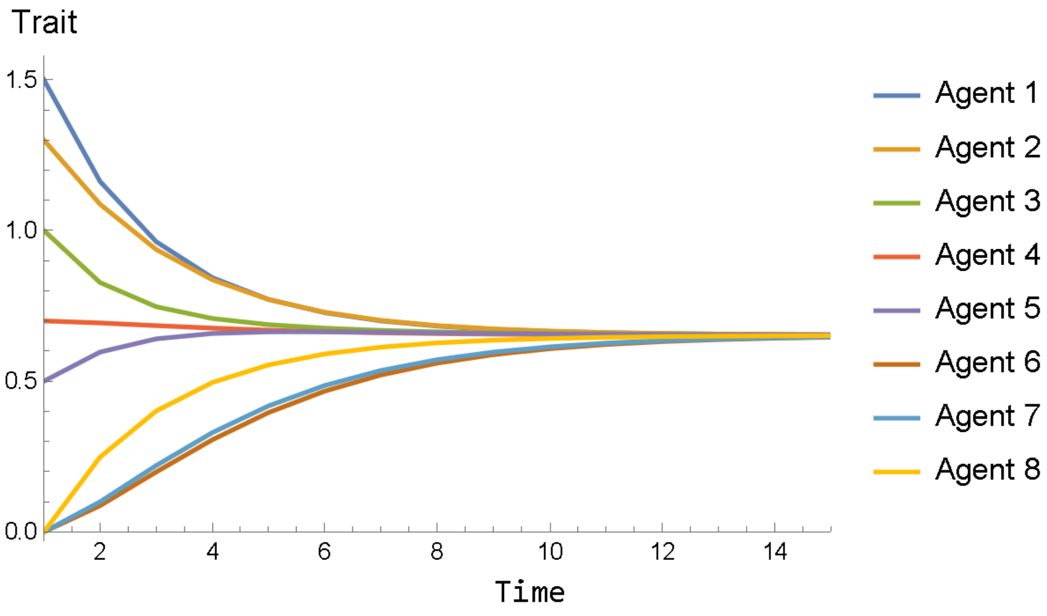

Figure 14: Diverse host. G_5.

In all our examples, convergence to the steady state is monotonic. This is determined by the matrix $\mathbf{\Omega}$ associated with each example has real eigenvalues. However, since it is entirely possible for a stochastic matrix to have complex eigenvalues, convergence in more complex scenarios may introduce oscillatory dynamics.

5 Concluding Remarks

In this study, we explored a model of cultural trait transmission among agents within a social network context. This investigation was driven by numerous instances in Economics where the transmission of culture is crucial. The work was initially motivated by creating a theoretical setup that allowed exploring the consequences of sudden flows of immigration. However, the versatility of our model and its findings extend far beyond this specific scenario, offering broader applicability in various economic fields.

Our main findings are a succinct formula and decomposition that summarizes the effect of networks' topology on the equilibrium through the underlying Markov Process. Moreover, we derived explicit formulas for the spillover effect of each agent's type on the rest of the population. In the dynamic model, we obtained an expression for the equilibrium in each period. We found which agents have persistent traits in the long run. Then, we concluded that, in a strongly connected network, society eventually converges to one trait, defined by a weighted average of every individual's trait and where the network's topology acts through the stationary distribution of the MP.

A key insight from our research is the significant impact of network topology on the dynamics within our model. Much of the literature studying specific traits and leanings overlooks the spatial distribution of agents. Our findings highlight the critical importance of the network's geometry in these dynamics. Structure *matters*. This understanding is essential for a more accurate and holistic view of the dynamics at play in economic and social contexts, especially in scenarios involving cultural interactions and preference formation.

The outcomes of our model and simulations allow us to comment on some policy implications. When society does not pay attention to a minority, its characteristic traits tend to disappear. This is problematic given it is generally desirable for a minority to keep part of their identity. Moreover, traits of native and ancient cultures usually want to be perpetuated in society for heritage value reasons. However, our results hint that society must make an active effort to assimilate these cultures' traits to keep them alive.

The comparative dynamics hint at some useful policy implications for the inclusion of migrants into society, such as the vital role of specialized institutions and the strength of the peer effect that a few individuals of a minority who are assimilating have on the rest of their group. While these insights are illustrative, they highlight the need for more complex network analyses to robustly validate these assertions.

We view this work as a starting point to elucidate the effect of different social networks' topologies on society's equilibrium behaviour. We made several simplifying assumptions and derived results on a setting that captures various of society's nuances. Lines for future research include: checking different topologies in more intricate examples than those presented, varying the distribution of links and traits; using empirical data on the model; testing the speed of convergence for different choices of parameters; or extending the model to dynamic or endogenous network formation.

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