

# 1 High-level Algorithm

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## Algorithm 1: High-level Quantifier Instantiation

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**Input:** Quantified axioms:  $\forall \mathbf{x}. A_1, \dots, \forall \mathbf{x}. A_n$   
**Input:** Quantifier free:  $\varphi$   
**Output:** Instantiations  $A_{i_1}[\mathbf{t}_1], \dots, A_{i_m}[\mathbf{t}_m]$  s.t.  $\varphi \wedge \bigwedge_j A_{i_j}[\mathbf{t}_j]$  is unsat  
1  $\mathcal{M} := \emptyset$  // Initially, empty set of models  
2  $\mathcal{I} := \emptyset$  // and empty set of instantiations  
3 **while**  $\varphi \wedge \mathcal{I}$  is sat **do**  
4     Get model for  $\varphi \wedge \mathcal{I}$  and add to  $\mathcal{M}$   
5     Find a small\* set of instantiations  $\mathcal{I}'$  s.t.  $\forall M \in \mathcal{M}. \exists I \in \mathcal{I}'. M \not\models I$   
6      $\mathcal{I} := \mathcal{I} \cup \mathcal{I}'$   
7     **if**  $\mathcal{M}$  is too large and line 5 takes too long **then**  
8         Reset  $\mathcal{M} \leftarrow \emptyset$  or optimize  $\mathcal{M} \leftarrow \mathcal{M}'$  where  $\mathcal{M}' \subset \mathcal{M}$   
9     **end**  
10 **end**  
11 **return**  $\mathcal{I}$

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## 2 Finding Instantiations

Let there be an axiom  $A = \forall \mathbf{x}. \psi$ . An instantiation of that quantifier is a tuple of ground terms  $\mathbf{t}$ :  $\psi[\mathbf{t}]$ . Given a model  $M$ , violated instantiations are determined by checking  $M \models \psi[\mathbf{t}]$ . We can think of a quantifier as a relation  $R_A$ , and then violations are  $\mathbf{t}^M \notin R_A^M$ . We can define the set of violations as  $\text{Violations}(M, A) := \{\mathbf{t} \mid \mathbf{t}\}$ .

Given a set of axioms  $A_1, \dots, A_n$  we can define the set of all violations as  $\text{Violations}(M) := \{(A_i, \mathbf{t}) \mid \mathbf{t} \in \text{Violations}(M, A_i)\}$ .

Given a set of models  $M_1, \dots, M_m$  we want a set of

## 3 Points

- Lemma pollution solution: models hitting sets to choose “good” lemmas.
- Running CVC5 with Dafny examples.