

CAUSAL INFERENCE FROM NETWORK DATA



Presenters:

David Arbour, Adobe Research @darbour

Elena Zheleva, University of Illinois at Chicago @elenadata

KDD 2021 Tutorial
August 14, 2021

<https://netcause.github.io>

TUTORIAL LOGISTICS

Website: <https://netcause.github.io>

- All materials, slides & references
- Our contact information

You can ask David and Elena questions during the tutorial over chat

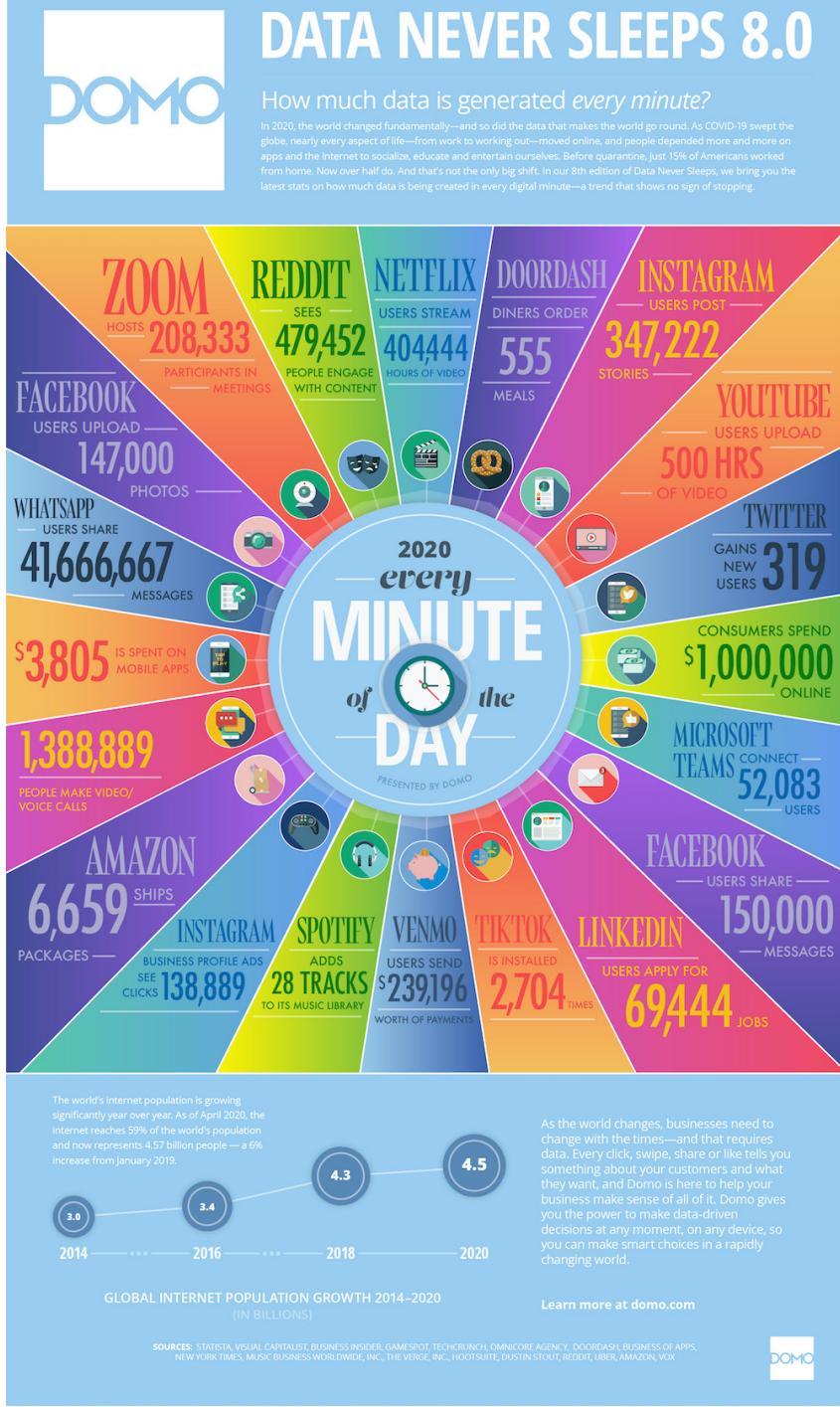
There will be a short break half-way through the tutorial

Note: the tutorial uses images from the papers it covers

CAUSAL INFERENCE

Causal inference is the study of how actions, interventions, or treatments affect outcomes of interest

Increasing interest in studying social phenomena and extracting causal insights from large amounts of “found” data





What messages in online support groups
cause people to feel more empathy?

Can social media
interactions **make** users
more “hateful” and **why?**





What social **interventions** can facilitate
the viral spread of a product?

CAUSAL INFERENCE AND INTERFERENCE

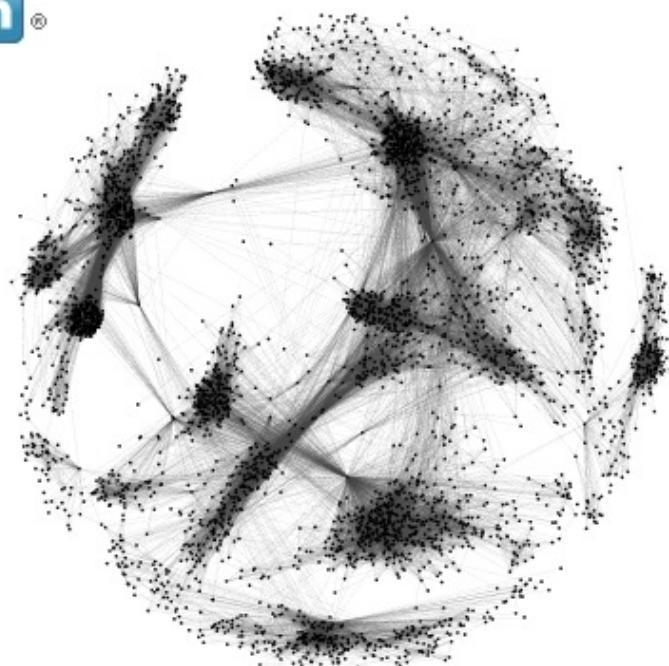
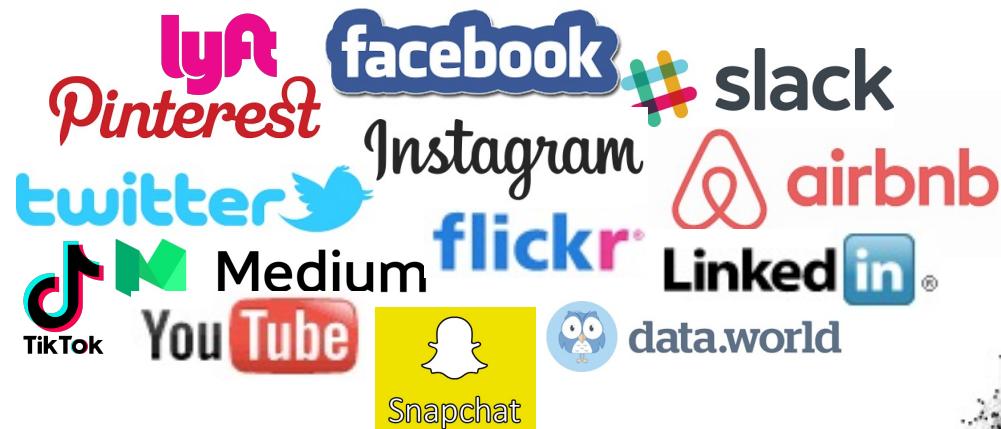
Common among these questions:

- 1) They are concerned with causes and effects
- 2) There is data from digital platforms that may help with answering them
- 3) Interference: the actions of one user can affect the actions of others



When and how can we answer causal questions of interest while accounting for interference?

INTERFERENCE



TUTORIAL OUTLINE

Background

- Motivation
- Causal inference 101
- Causal effects in networks

1/3 of tutorial

Interventions and network experiment design

Counterfactuals & causal effects in observational data

- Representation, identification, estimation
 - Block representation
 - 10-minute BREAK ---
 - Representation challenges
 - Chain and segregated graphs
 - Multi-relational data and abstract ground graphs
- Discovery

2/3 of tutorial

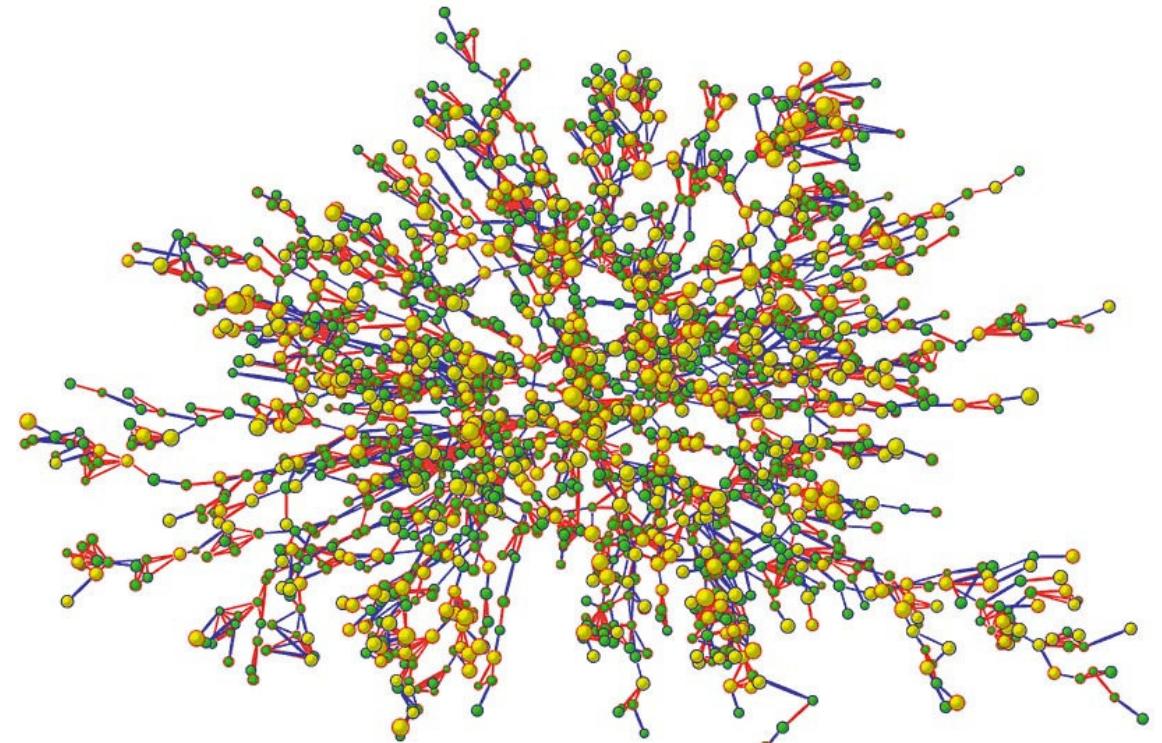
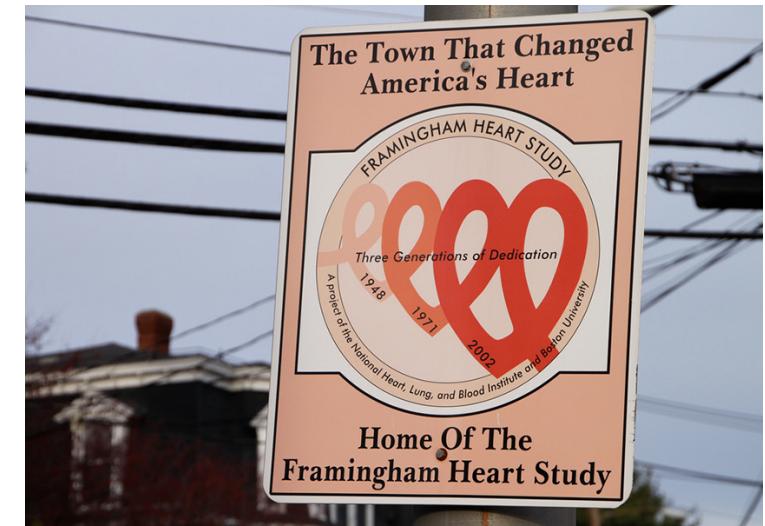
EXAMPLE: SPREAD OF OBESITY

Analyzed person-to-person spread of obesity

"A person's chances of becoming obese increased by 57% if he or she had a friend who became obese in a given interval"

Similar studies on spread of smoking and happiness

These studies may suffer from spurious associations due to network dependence**



Christakis & Fowler. The Spread of Obesity in a Large Social Network Over 32 Years. New England Journal of Medicine. 2007.

**Lee & Ogburn. Network Dependence Can Lead to Spurious Associations and Invalid Inference. Journal of American Statistical Association. 2020.

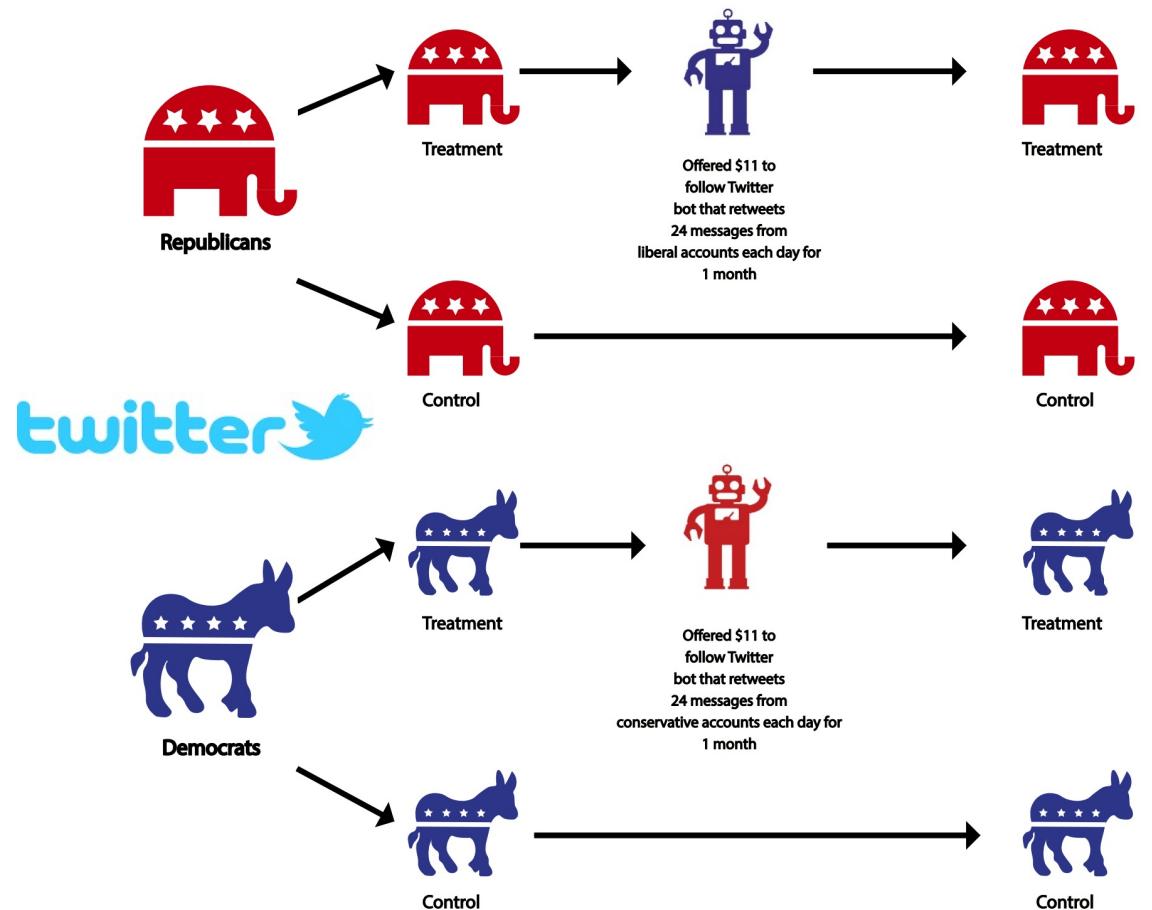
EXAMPLE: SOCIAL MEDIA AND POLARIZATION

Expose people to opposite views =>
get along better, hate each other more?

Block randomization at level of party
attachment and interest in current events

Answered questions before and after 1
month of following bot of opposite view

Republicans became significantly more
conservative and Democrats slightly more
liberal



EXAMPLE: VIRAL MARKETING

Customers can choose:

1. Product to share with friends
2. Share recipient

Company can vary the rest of the message

Endorsement effect

Incentive effect

| Added info | Referred purchases | Follow-up referrals |
|--------------------|--------------------|---------------------|
| Sharer purchase | 15% lift | No effect |
| Referral incentive | No effect | 65% lift |
| Both | No effect | No effect |

what are friends for? 

Darrell Rivera has just purchased this great offer, and thought you might be interested as well.

Hey! I found this LivingSocial deal from River Expeditions and thought you may be interested in it too. Check it out!

River Expeditions
Whitewater Rafting and Camping Trip

Immersion yourself in a wild adventure through some of the most breathtaking scenery in the region as you take on the rapids rolling through West Virginia's New River Gorge National Park, also known as "The Grand Canyon..."

Earn REWARDS by sharing with FRIENDS

[view deal »](#)

[Check out other deals](#)



HOMOPHILY VS. CONTAGION



A large, abstract network graph is positioned at the top of the slide, consisting of numerous small, semi-transparent nodes connected by thin, light-colored lines.

Motivation

Causal inference 101

Causal effects in networks

Interventions and network experiment design

Counterfactuals & causal effects in observational data

Representation, identification, estimation

Blocks

Representation challenges

Chain and segregated graphs

Multi-relational data and abstract ground graphs

Discovery

CAUSAL INFERENCE 101

RELATED TUTORIALS

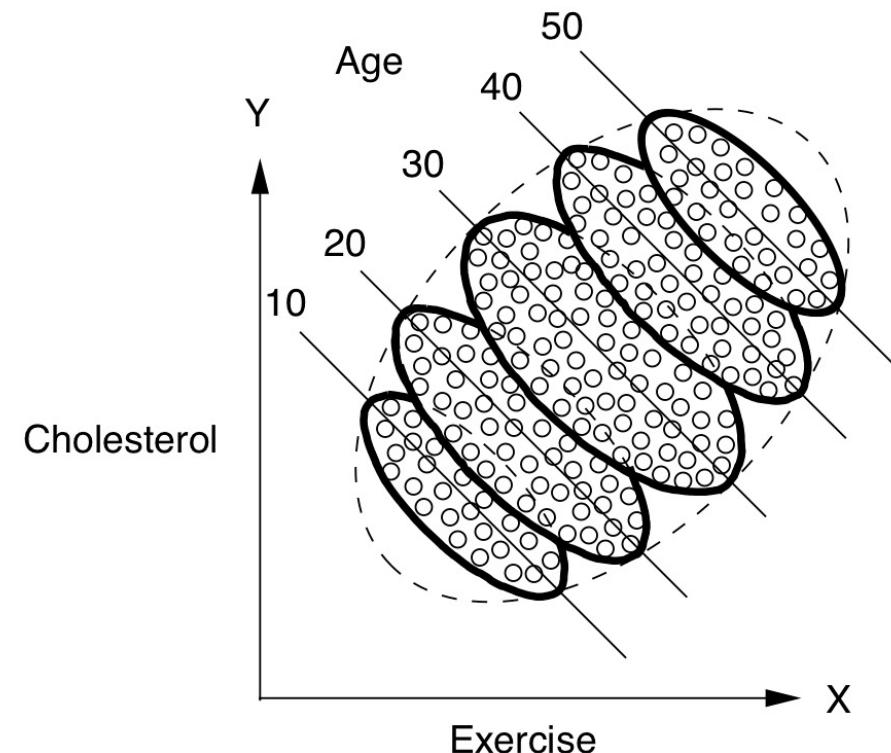
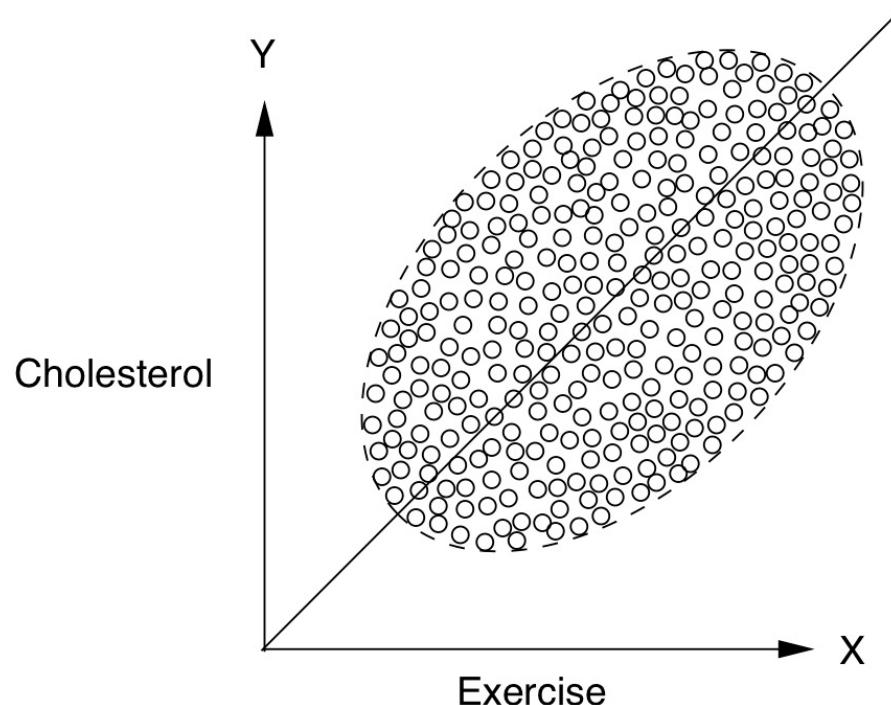
Shalit & Sontag. Causal Inference for Observational Studies. ICML 2016

- <https://shalit.net.technion.ac.il/homepage/causal-inference-tutorial-icml-2016/>

Kiciman, Sharma. Causal Inference and Counterfactual Reasoning. KDD 2018.

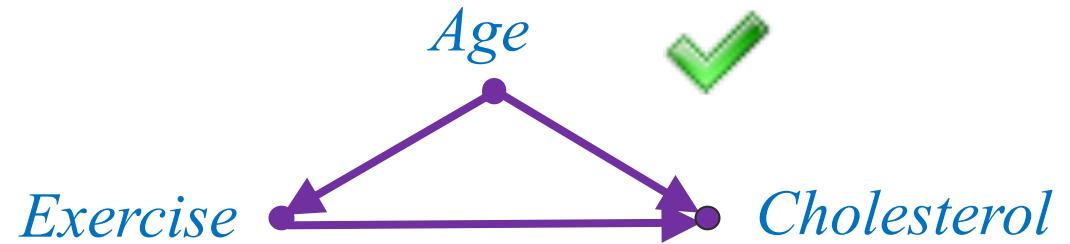
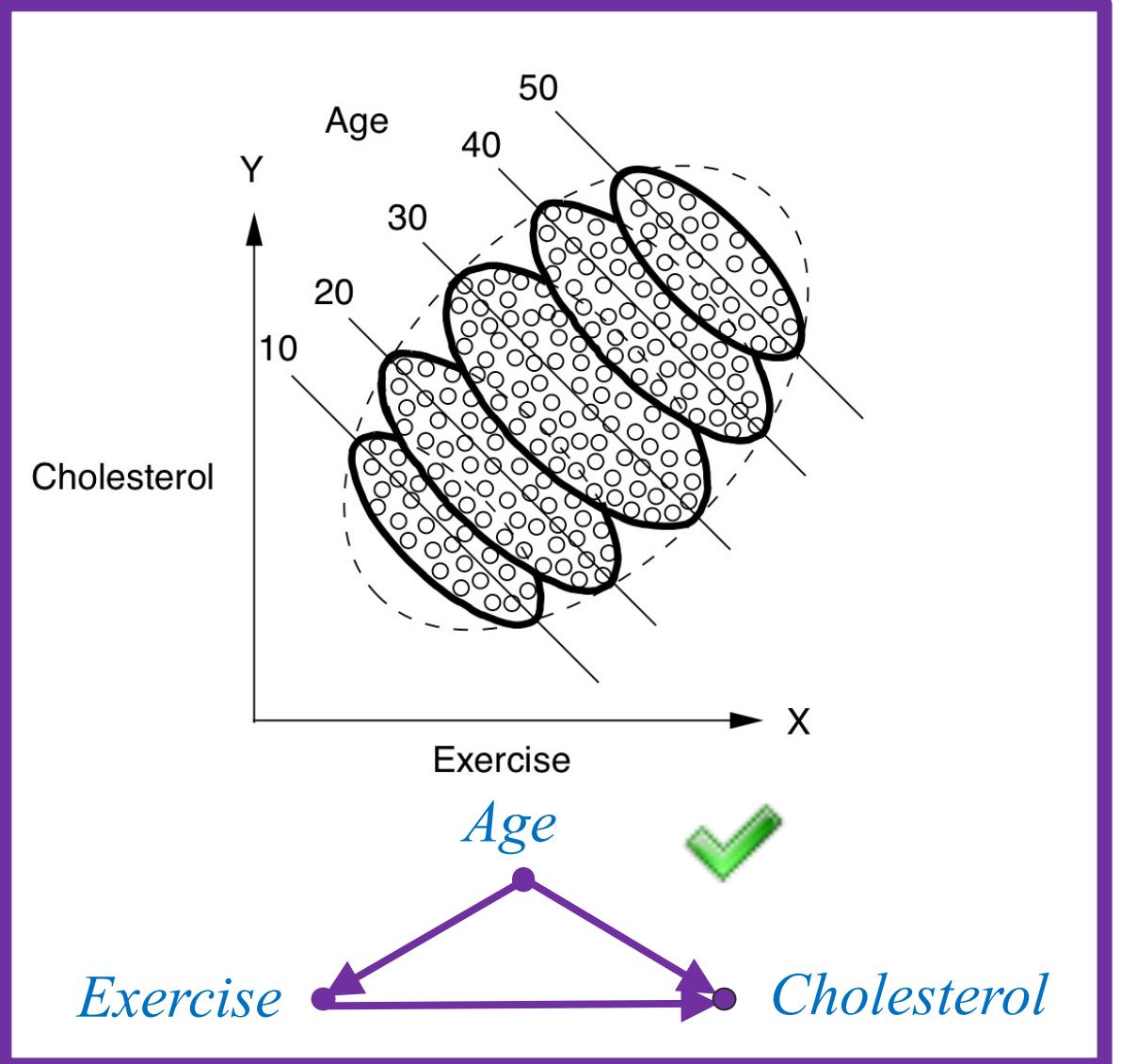
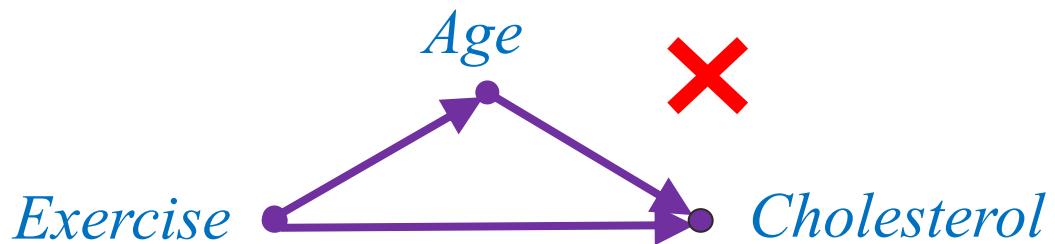
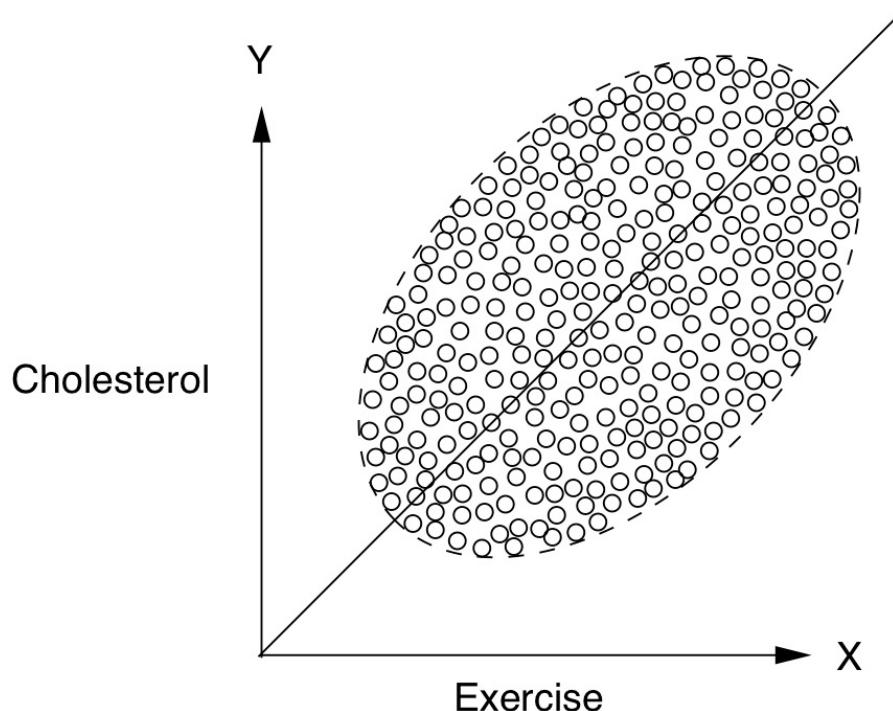
- <https://causalinference.gitlab.io/kdd-tutorial/>

SIMPSON'S PARADOX



Same data can have different causal explanations!

SIMPSON'S PARADOX



POTENTIAL OUTCOMES AND COUNTERFACTUALS

Treatment (Z): something administered to experimental units; a cause of interest (e.g., received vaccine or not)

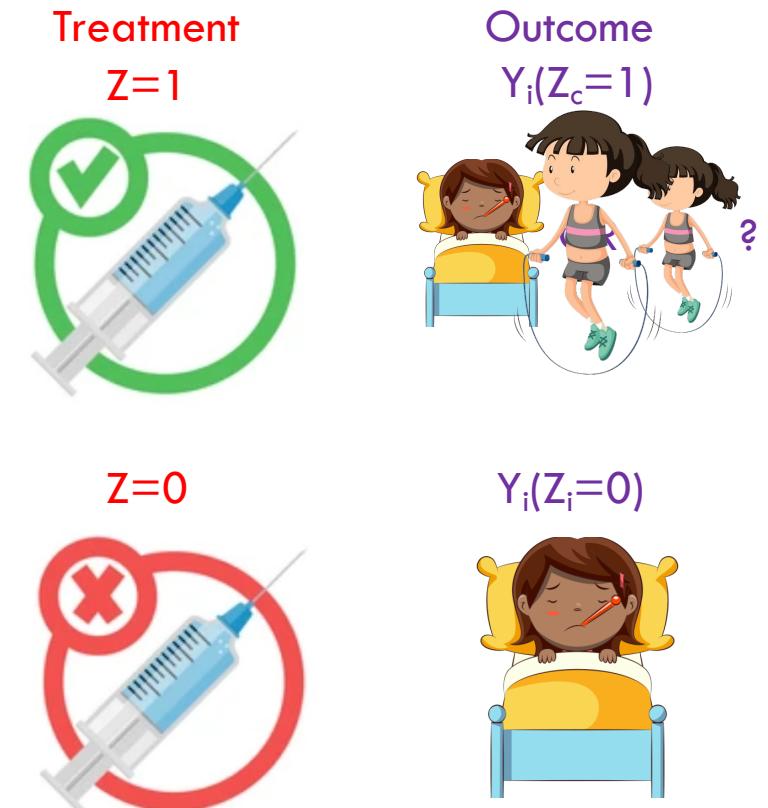
Potential outcome: the outcome $Y_i(z)$ that would be realized if an individual i received a specific treatment z (e.g., got sick or not)

Counterfactual: the outcome $Y_i(z_c)$ that would have been realized had an individual had a different treatment z_c than the observed z_i

Individual causal effect: $Y_i(Z=1) - Y_i(Z=0) = Y_i(1) - Y_i(0)$

Fundamental law of causal inference: $Y_i(0)$ can never be observed at the same time as $Y_i(1)$ and the causal effect cannot be measured

How do we estimate causal effects then?



COMMON CAUSAL ESTIMANDS

Individual effects are hard to estimate. Instead:

Average treatment effect (ATE)

$$E[Y_i(1) - Y_i(0)] \cong \frac{1}{n} \sum_{i=1}^n (Y_i(1) - Y_i(0)) \cong \frac{1}{n} \sum_{i=1}^n (Y_i(1)Z_i - Y_i(0)(1 - Z_i))$$

Under certain assumptions

Conditional average treatment effect (CATE)

$$E[Y_i(1) - Y_i(0)|\mathbf{X}_i = \mathbf{x}]$$

| i | Z | $Y(Z_1)$ | $Y(Z_0)$ | Sex | Education |
|-----|-------|----------|----------|-----|-------------|
| 1 | 注射器图标 | Healthy | ? | F | High School |
| 2 | 注射器图标 | ? | Sick | F | Bachelors |
| 3 | 注射器图标 | ? | Healthy | M | High School |
| ... | | | | | |
| n | 注射器图标 | Healthy | ? | M | Masters |

COMMON ASSUMPTIONS

Consistency: $Y_i(z_i) = y_i$ when $Z = z_i$

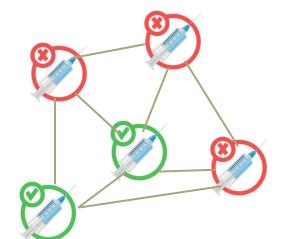
Positivity/overlap: a unit could have received any treatment $P(Z_i = z|X = x_i) > 0, \forall z, x_i$

No unmeasured confounders/Ignorability/Exchangeability: $(Y(0), Y(1)) \perp Z|X$

Stable unit treatment value assumption (SUTVA). $Y_i(z) = Y_i(z_i)$, the outcome of unit i depends only on the treatment it receives and not on the treatment other units receive

- This is violated in the presence of interference

Interference assumption: $Y_i(z) = Y_i(z_i; z_{Ni})$, a unit's response can be affected by the treatment it receives and by the treatments received by its neighbors/peers



Counterfactuals

What if I had done X?
Why?



Intervention

What if I do X?

Reinforcement learning,
A/B testing



Associations

What is?

Machine learning

LADDER OF CAUSATION*

Associations: $P(y | z)$ [Level 1]

- Example question: Is working in academia (z) correlated with happiness (y)?

Interventions: $P(y | \text{do}(z), x)$ [Level 2]

- Example: If Alice takes a job in industry, would she be happier than taking one in academia?
- Treatment z , outcome y , context x

Counterfactuals: $P(y_z | z', y')$ [Level 3]

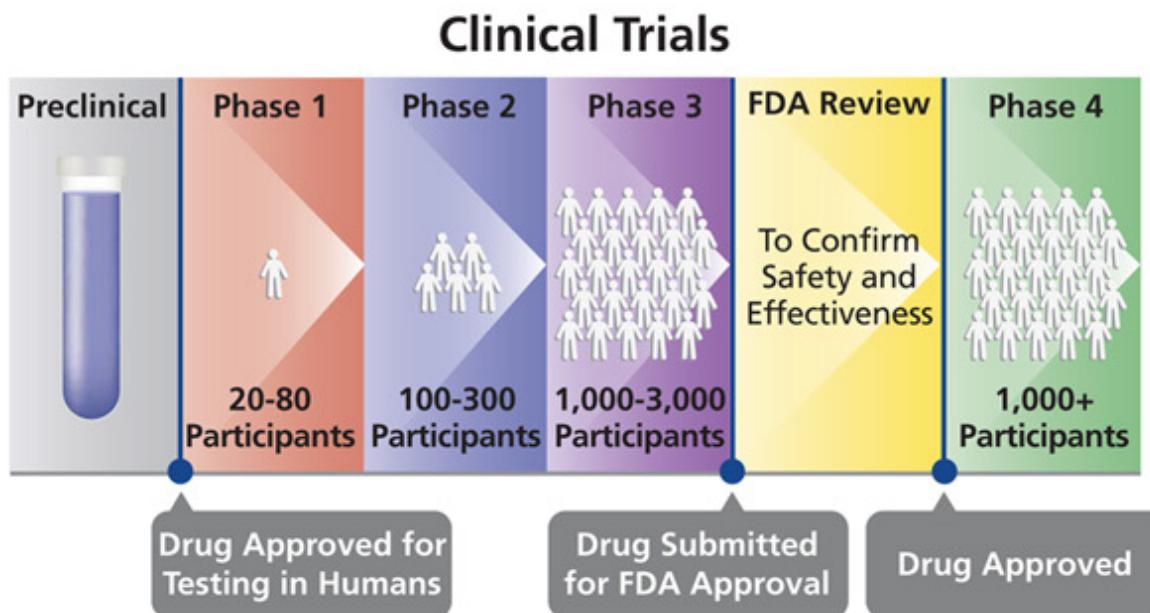
- Example: If Alice stayed in industry (z), would Alice have been happier, given that she took a job in academia (z')?

Counterfactual queries require different tools from associational ones!

Questions from level j can be answered if you have information from a higher level but not the other way around

INTERVENTIONS

- Randomized controlled trials required for drug approval by FDA
 - A random group given the drug is compared to a random group given the placebo



Science

Contents ▾ News ▾ Careers ▾ Journals ▾

SHARE

f 92K
t 283
in 283
a 283
e 283



Vaccination with bacillus Calmette-Guérin leads to a small pustule that can develop into a scar.
KWANGMOOZAA/ISTOCK

Can a century-old TB vaccine steel the immune system against the new coronavirus?

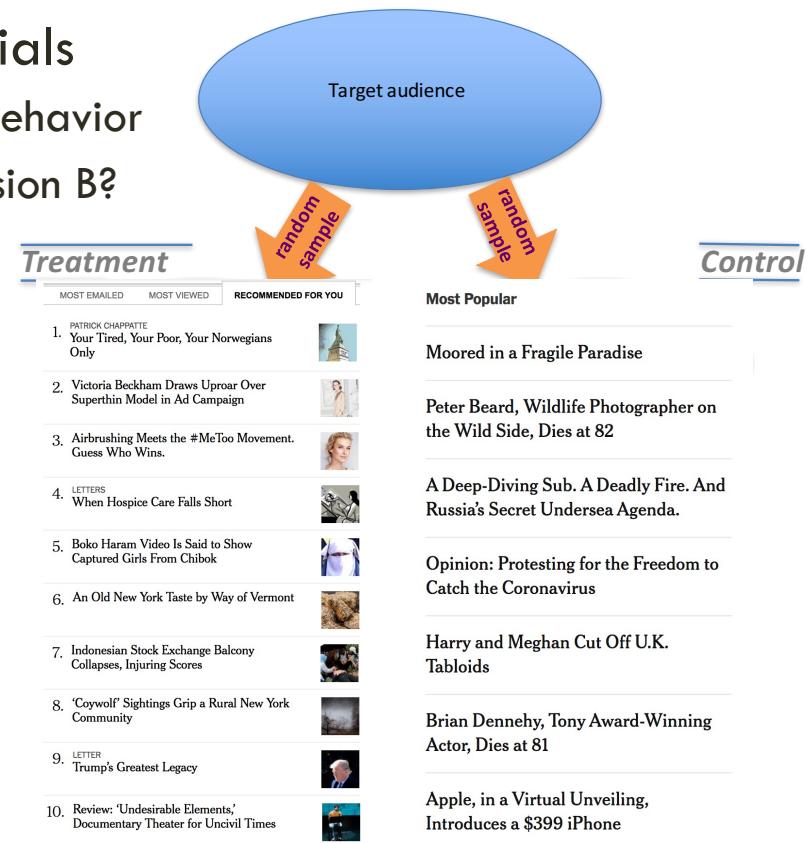
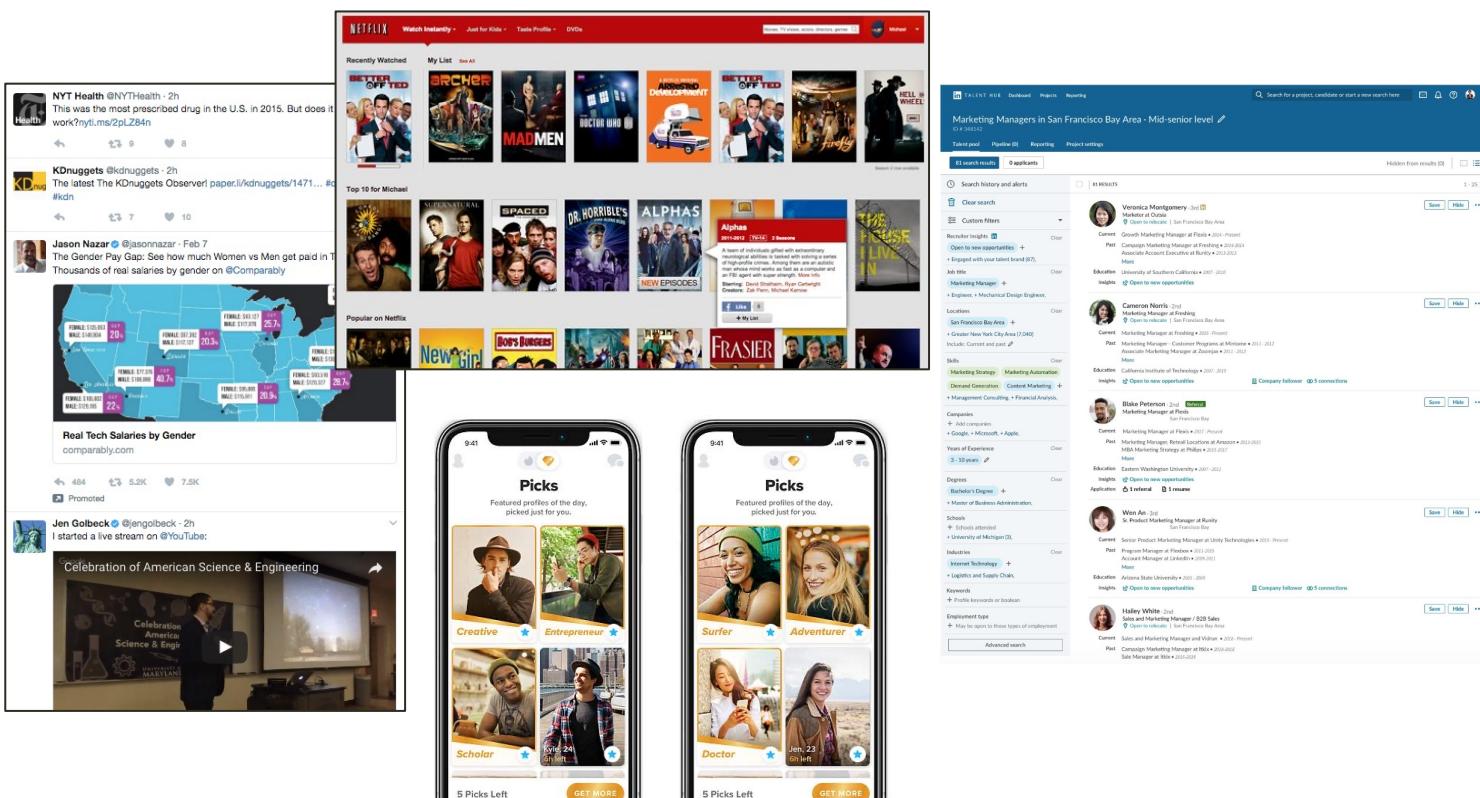
By Jop de Vrieze | Mar. 23, 2020, 6:25 AM

Researchers in four countries will soon start a clinical trial of an unorthodox approach to the new

WHICH RECOMMENDATION ALGORITHM IS BETTER?

A/B testing = controlled experiment = randomized controlled trials

- Best scientific design for establishing **causality** between a change and user behavior
- Is the outcome better on average for people “treated with” version A or version B?



$$ATE = E[Y(Z_1)] - E[Y(Z_0)]$$

INTERVENTIONS NOT ALWAYS POSSIBLE

Ethical concerns

The New York Times

OKCupid Plays With Love in User Experiments



Mingling at an event in Manhattan sponsored by OKCupid, which on Monday published the results of three experiments. Yana Paskova for The New York Times

Too expensive

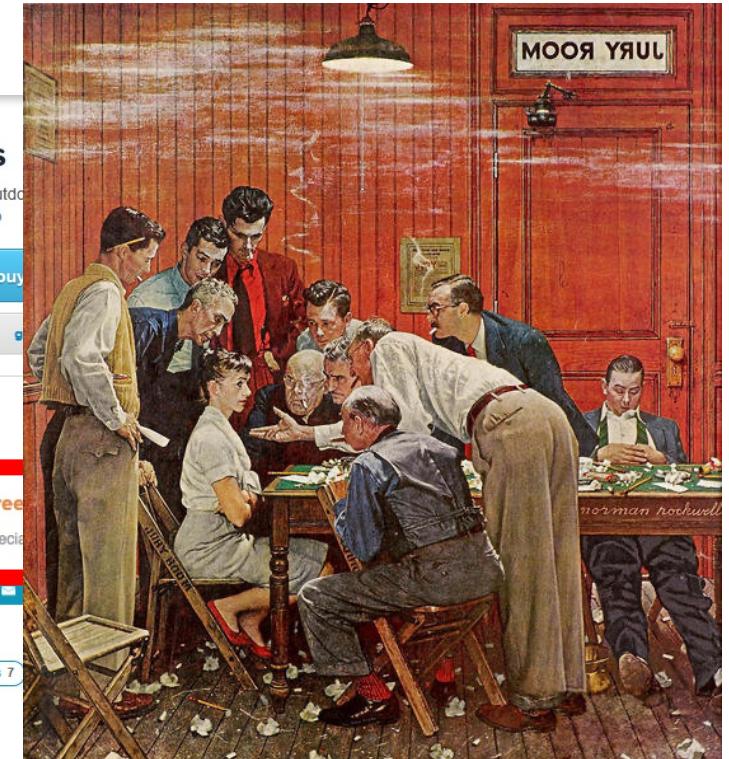


details

Immerse yourself in a wild adventure through some of the most breathtaking scenery in the region as you take on the rapids rolling through West Virginia's New River Gorge National Park, also known as "the Grand Canyon of the East."

- \$69 (\$140 value) for a two-night rafting trip for one (valid Monday to Friday)
- Includes one day of rafting, two nights of camping, breakfast, and beverages
- You also get round-trip river transportation

Immutable characteristics



STRUCTURAL CAUSAL MODELS (SCM)

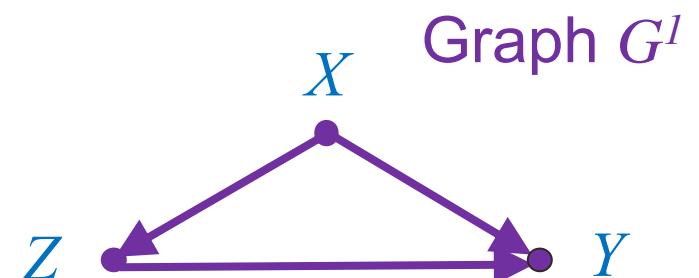
SCM describes how nature assigns values to variables of interest

- **Variables:** U (exogenous) and V (endogenous)
- **Functions:** assign each variable in V a value based on other variables
 - Direct cause: X is direct cause of Y if X is in the function assigning Y
 - Cause: X is a cause of Y if it is a direct cause of Y or of any cause of Y
- **Graphical causal model:** nodes represent variables, edges represent functional dependences
 - Also referred to as graph or graphical model or causal diagram
 - Allows us to reason about exchangeability through d-separation

Do-calculus: Provides rules for estimating causal effects from observational data when identification possible, given an SCM

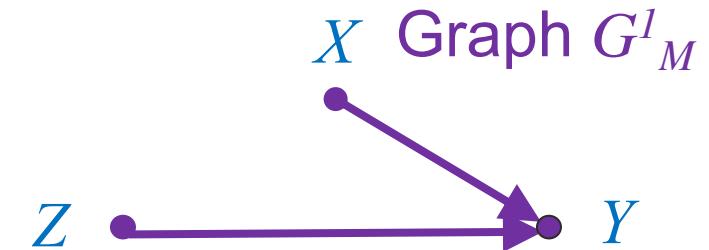
- Works even when some variables are latent

$$X = f_X(U_X); Z = f_Z(X, U_Z); Y = f_Y(X, Z, U_Y)$$



$$P(Y = y | do(Z = z)) = ?$$

Causal model under intervention



BACKDOOR CRITERION

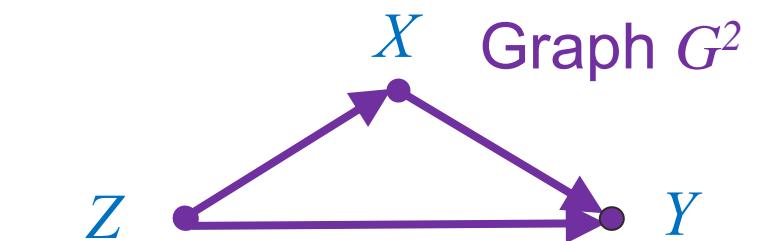
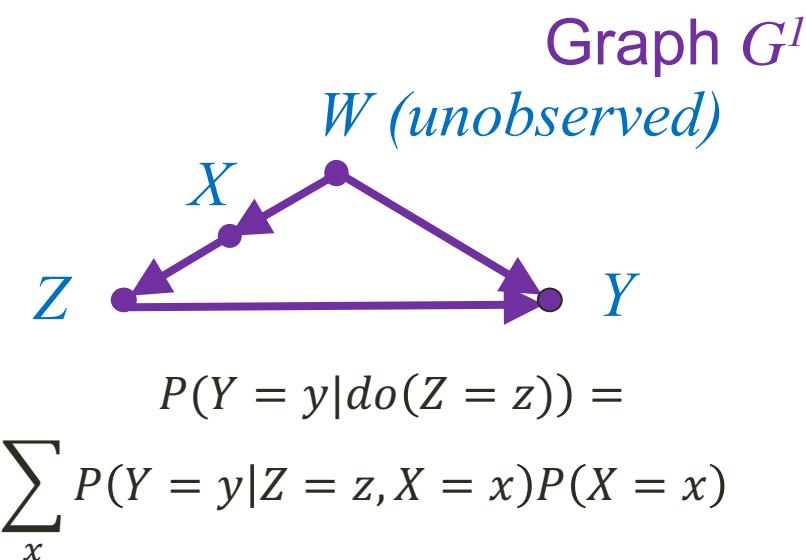
A common rule for deriving a valid causal estimand from observational data

Given an ordered pair of variables (Z, Y) in a directed acyclic graph G , a set of variables $\textcolor{red}{X}$ satisfies the **backdoor criterion** relative to (Z, Y) if no node in X is a descendant of Z , and X blocks every path between Z and Y that contains an arrow into Z (X d-separates Z and Y on these paths)

$$\begin{aligned} P(Y = y|do(Z = z)) &= \sum_x P(Y = y|Z = z, X = x)P(\textcolor{red}{X} = x) \\ &= \sum_x \frac{P(Y = y, Z = z, \textcolor{red}{X} = x)}{P(Z = z|\textcolor{red}{X} = x)} \end{aligned}$$

propensity score

The adjustment formula is “controlling” for X

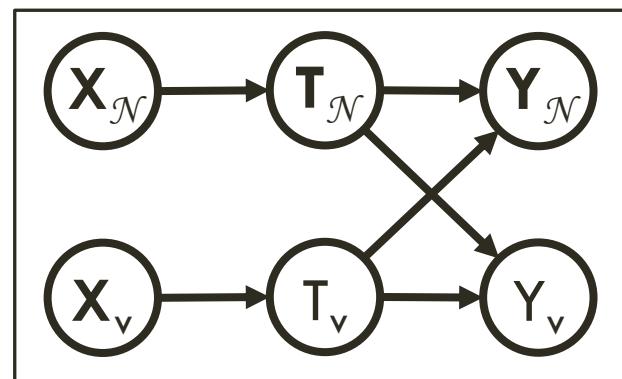


$$P(Y = y|do(Z = z)) = P(Y = y|Z = z)$$

CAUSAL INFERENCE ENGINE

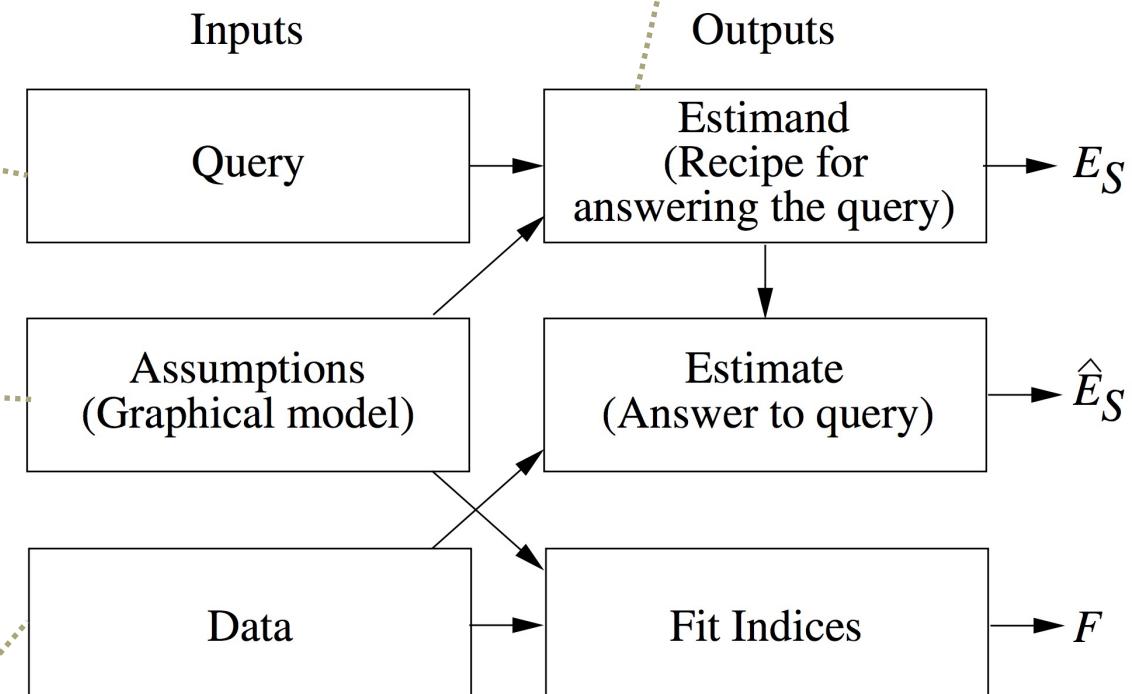
Input examples:

If Mia **read Flo's tweets**, would she have **vaccinated** herself?



| Name | Age | Gender | Race | VaccineView | ... | Vaccinated |
|----------|-----|--------|-------|-------------|-----|------------|
| Mia | 50 | F | Asian | ? | ... | No |
| Flo | 34 | F | Black | ? | ... | Yes |
| LotusOak | 42 | F | White | Yes | ... | No |

Based on do-calculus rules
e.g., $P(y_v|do(t_v)) = P(y_v|t_v)$



CAUSAL EFFECTS IN NETWORKS

Motivation

Causal inference 101

Causal effects in networks

Interventions and network experiment design

Counterfactuals & causal effects in observational data

Representation, identification, estimation

Blocks

Representation challenges

Chain and segregated graphs

Multi-relational data and abstract ground graphs

Discovery



CAUSAL ESTIMANDS UNDER INTERFERENCE

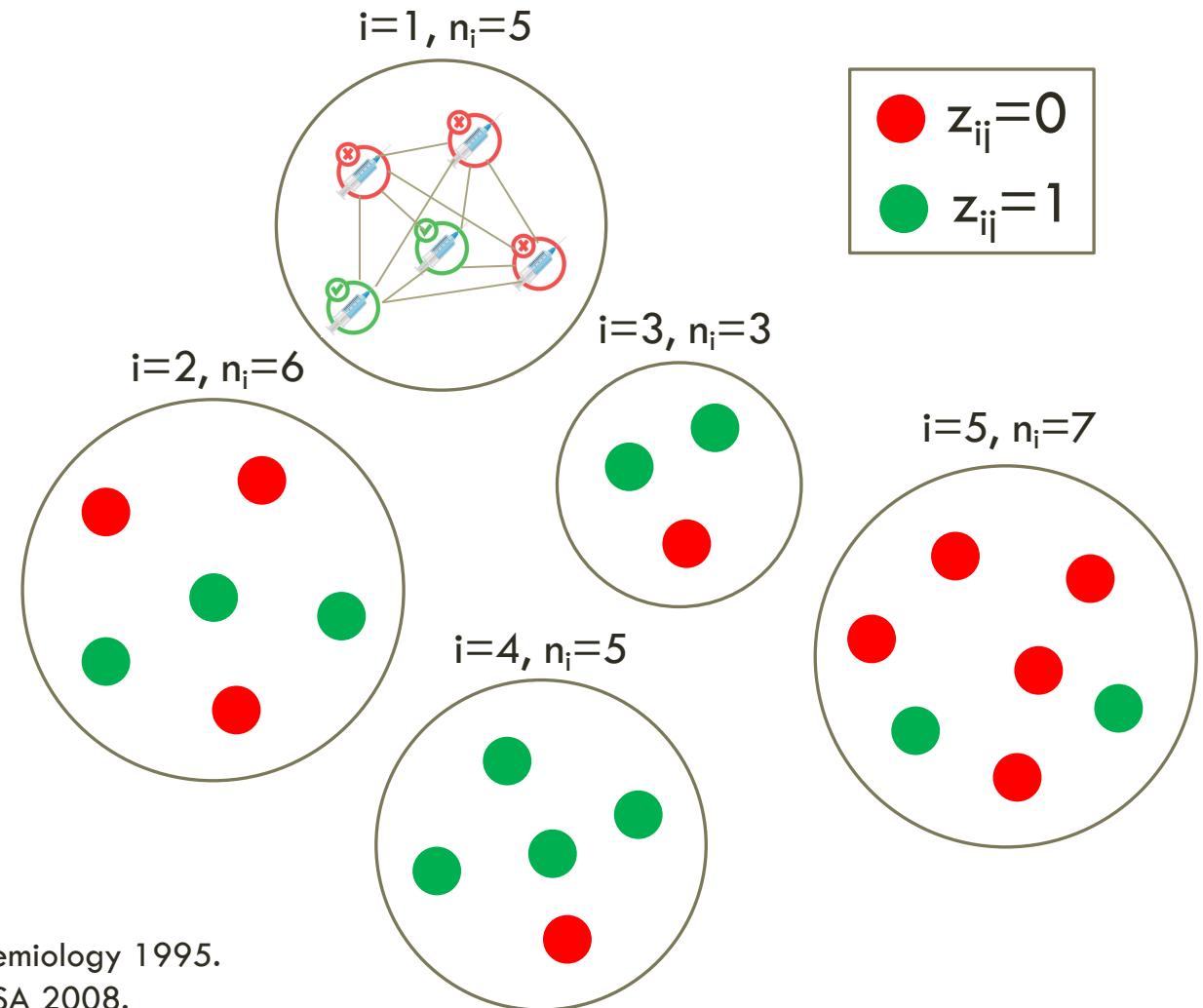
Start with simplifying assumptions:

Multiple non-overlapping groups

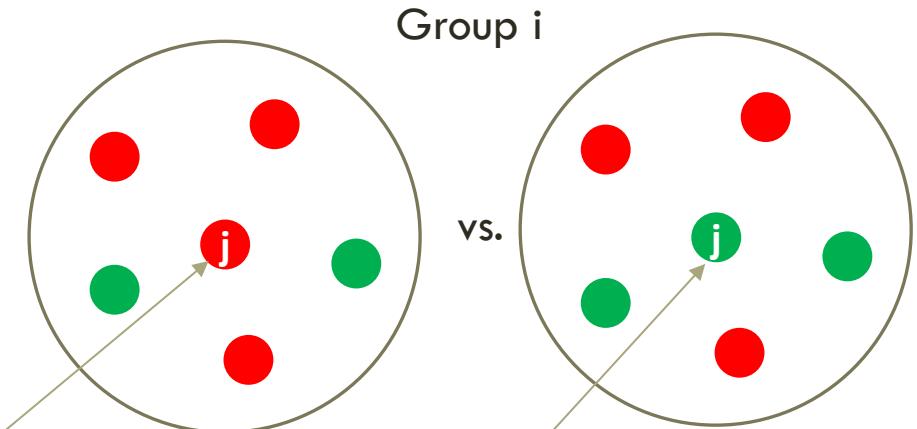
Partial interference: interference occurs within but not across groups

Treatment assignment within each group has treatment regime

$$P(Z=1) = \psi$$



DIRECT CAUSAL EFFECT



Individual Direct Causal Effect (DCE): the difference in outcome due to the treatment alone

- e.g., effect of getting vaccinated on getting sick

$$CE_{ij}^D(\mathbf{z}_{i(j)}) \equiv Y_{ij}(\mathbf{z}_{i(j)}, z_{ij} = 0) - Y_{ij}(\mathbf{z}_{i(j)}, z_{ij} = 1)$$

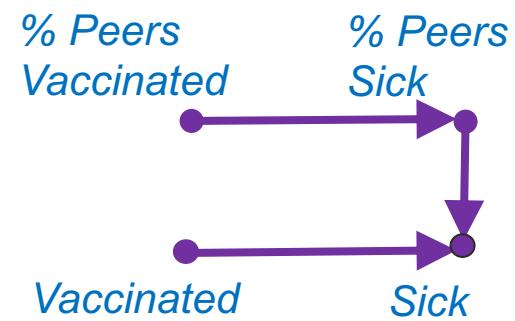
$\mathbf{z}_{i(j)}$: treatment assignment of units in j's group i

z_{ij} : treatment assignment of unit j in group i

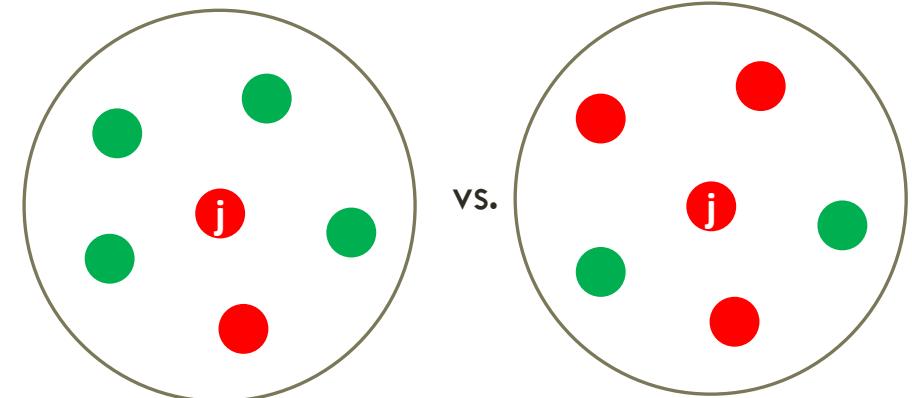
Individual Avg. DCE: difference of expected values of the marginal distributions under treatment regime ψ of group i $\overline{CE}_{ij}^D(\psi) \equiv \overline{Y}_{ij}(0; \psi) - \overline{Y}_{ij}(1; \psi)$

Group Avg. DCE: $\overline{CE}_i^D(\psi) \equiv \overline{Y}_i(0; \psi) - \overline{Y}_i(1; \psi) = \sum_{j=1}^{n_i} \overline{CE}_{ij}^D(\psi) / n_i$

Population Avg. DCE: $\overline{CE}^D(\psi) \equiv \overline{Y}(0; \psi) - \overline{Y}(1; \psi) = \sum_{i=1}^N \overline{CE}_i^D(\psi) / N$



INDIRECT/PEER EFFECT



Individual indirect causal effect (ICE): the effect of the treatment received by others in the group on an individual outcome

- e.g., effect of % vaccinated people on getting sick

$\mathbf{z}_{i(j)}$: treatment assignment of unit i's neighbors (group j)

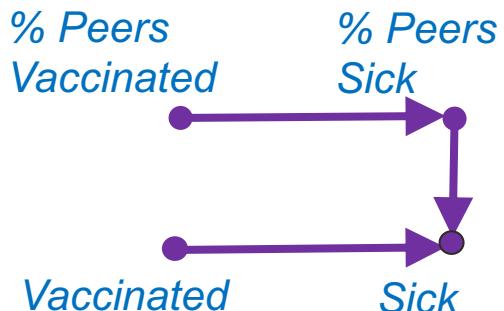
z_{ij} : treatment assignment of unit i

$$CE_{ij}^I(\mathbf{z}_{i(j)}, \mathbf{z}'_{i(j)}) \equiv Y_i(\mathbf{z}_{i(j)}, z_{ij} = 0) - Y_i(\mathbf{z}'_{i(j)}, z'_{ij} = 0)$$

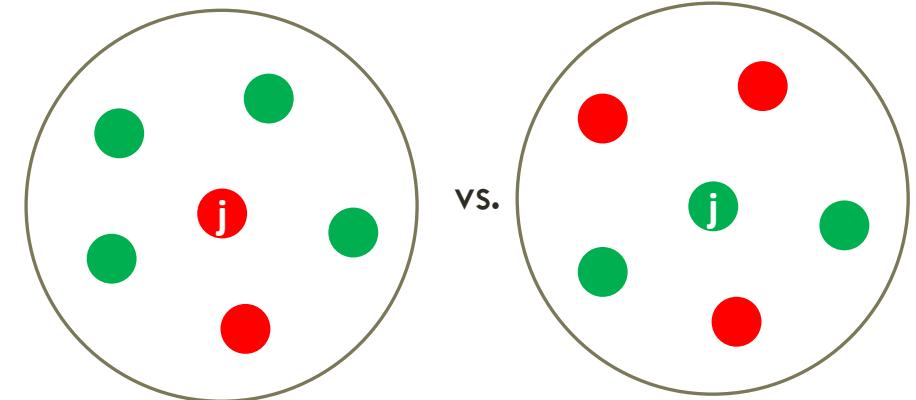
Individual Avg. ICE: difference of expected values of the marginal distributions under two different treatment regimes ψ and ϕ of group i $\overline{CE}_{ij}^I(\phi, \psi) \equiv \overline{Y}_{ij}(0; \phi) - \overline{Y}_{ij}(0; \psi)$

Group Avg. ICE: $\overline{CE}_i^I(\phi, \psi) \equiv \overline{Y}_i(0; \phi) - \overline{Y}_i(0; \psi) = \sum_{j=1}^{n_i} \overline{CE}_{ij}^I(\phi, \psi) / n_i$

Population Avg. ICE: $\overline{CE}^I(\phi, \psi) \equiv \overline{Y}(0; \phi) - \overline{Y}(0; \psi) = \sum_{i=1}^N \overline{CE}_i^I(\phi, \psi) / N$



TOTAL EFFECT



Individual total causal effect (TCE): both direct and indirect effect of treatment assignment

- e.g., effect of % vaccinated people and getting vaccinated on getting sick

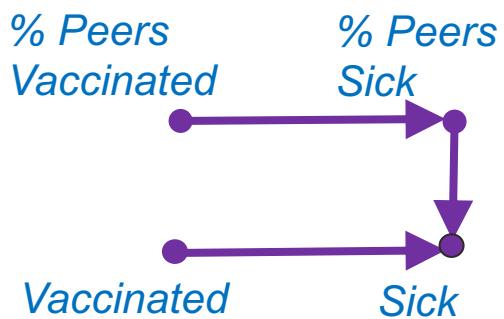
$$CE_{ij}^T(\mathbf{z}_{i(j)}, \mathbf{z}'_{i(j)}) \equiv Y_{ij}(\mathbf{z}_{i(j)}, z_{ij} = 0) - Y_{ij}(\mathbf{z}'_{i(j)}, z'_{ij} = 1)$$

Individual Avg. TCE: difference of expected values of the marginal distributions under two different treatment regimes $0; \psi$ and $1; \phi$ of group i $\overline{CE}_{ij}^T(\phi, \psi) \equiv \overline{Y}_{ij}(0; \phi) - \overline{Y}_{ij}(1; \psi)$

Group Avg. TCE: $\overline{CE}_i^T(\phi, \psi) \equiv \overline{Y}_i(0; \phi) - \overline{Y}_i(1; \psi) = \sum_{j=1}^{n_i} \overline{CE}_{ij}^T(\phi, \psi)/n_i$

Population Avg. ICE:

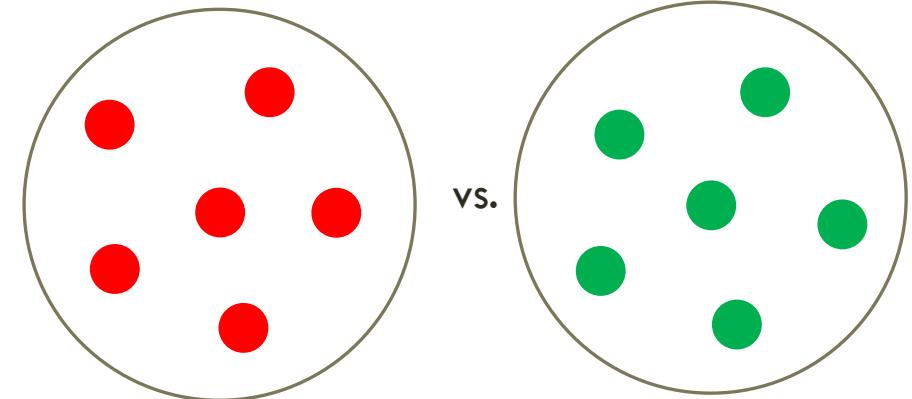
$$\overline{CE}^T(\phi, \psi) \equiv \overline{Y}(0; \phi) - \overline{Y}(1; \psi) = \sum_{i=1}^N \overline{CE}_i^T(\phi, \psi)/N$$



Halloran, Struchiner. *Causal inference in infectious diseases*. Epidemiology 1995.

Hudgens, Halloran. *Toward causal inference with interference*. JASA 2008.

TOTAL EFFECT: ALTERNATIVE ESTIMAND



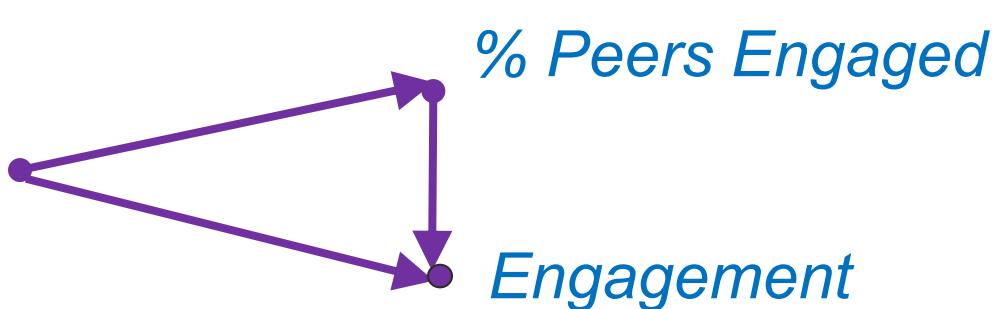
Total treatment effect (TTE): both direct and indirect effect of treatment assignment

- e.g., effect of vaccinating everyone

$$TTE = \frac{1}{N} \sum_{v_i \in V} (v_i.Y(\mathbf{Z_1}) - v_i.Y(\mathbf{Z_0}))$$

Applications: recommender systems

New news feed algorithm



Motivation

Causal inference 101

Causal effects in networks

Interventions and network experiment design

Counterfactuals & causal effects in observational data

Representation, identification, estimation

Blocks

Representation challenges

Chain and segregated graphs

Multi-relational data and abstract ground graphs

Discovery



INTERVENTIONS AND NETWORK EXPERIMENT DESIGN

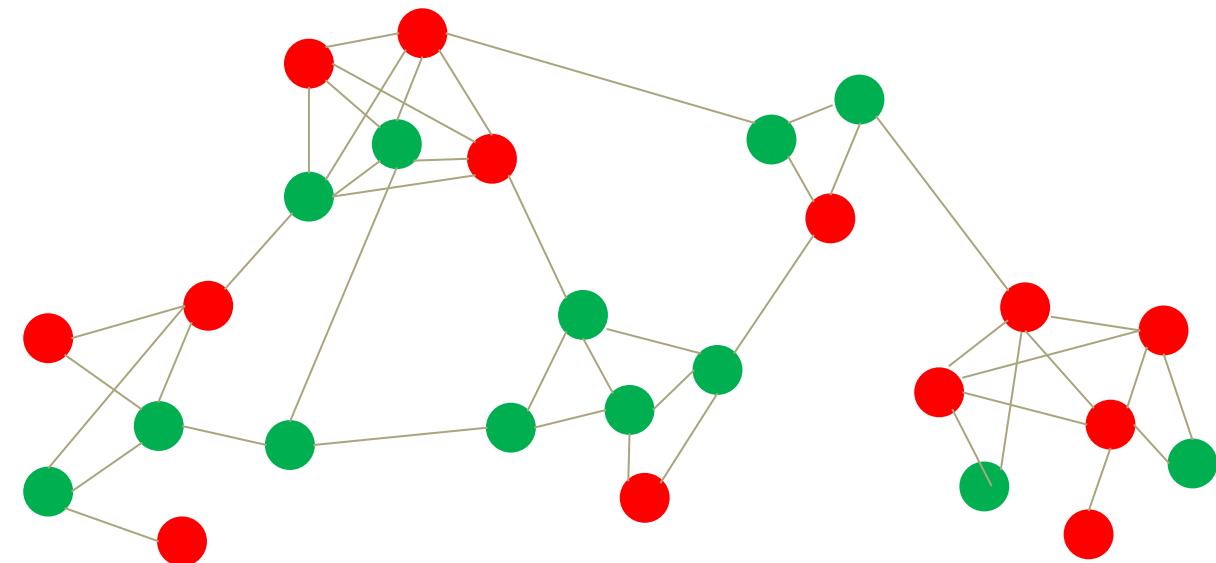
RANDOMIZATION IN NETWORKS

Network experiment design:

Design for randomized controlled trials that take into consideration interactions and potential interference between units of interest

Randomization at the node level

- High variance of estimators
- Need additional assumptions

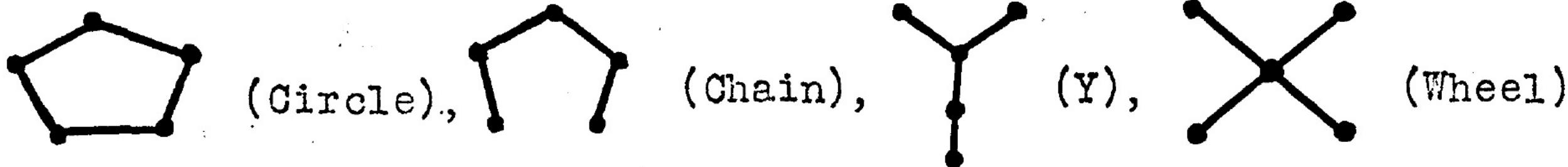


The choice of randomization design depends on the causal effect of interest!

NETWORK EXPERIMENT DESIGN

Early network experiments in 1940s were performed in labs at a small scale

Leavitt: solve a data collation task using only one of four randomly assigned communication patterns



“The Circle was erratic, active (message-wise), unorganized, and leaderless, but satisfying to its members. The Wheel was less erratic, required few messages, was well organized, and had a definite leader, but was less satisfying to most of its members”

NETWORK EXPERIMENT DESIGN

Network experiments nowadays are often large-scale and use digital platforms with millions of users



Can peers influence voter turnout? [Bond et al. 2012]

Can product endorsements from friends increase ad clicks? [Bakshy et al. 2012]

Can emotional states be transferred via contagion? [Kramer et al 2014]

TWO-STAGE RANDOMIZATION DESIGN UNDER PARTIAL INTERFERENCE

Two-stage randomization

1. Assign groups to treatment and control with prob. ν

2. For each group i:

If group in treatment ($S_i=1$), assign each unit to treatment with probability ψ

Else group in control ($S_i=0$), assign each unit to treatment with probability θ

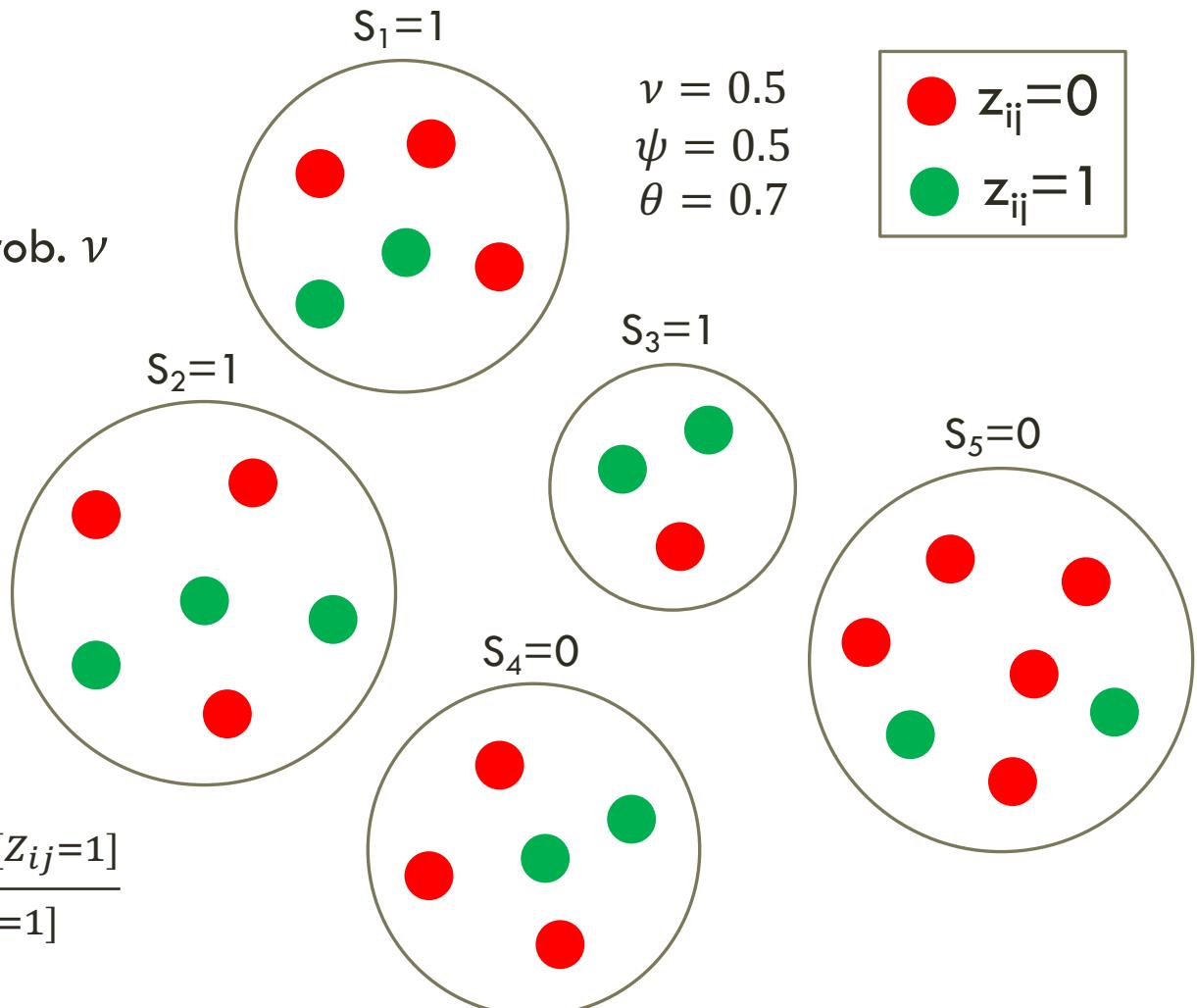
E.g., Group Average Direct Causal Effect estimator

Estimand

$$\overline{CE}_i^D(\psi) = \frac{1}{n_i} \sum_{j=1}^{n_i} (\bar{Y}_{ij}(0, \psi) - \bar{Y}_{ij}(1, \psi))$$

Unbiased estimator

$$\widehat{CE}_i^D(\psi) = \frac{\sum_{j=1}^{n_i} Y_{ij}(Z_i) I[Z_{ij}=0]}{\sum_{j=1}^{n_i} I[Z_{ij}=0]} - \frac{\sum_{j=1}^{n_i} Y_{ij}(Z_i) I[Z_{ij}=1]}{\sum_{j=1}^{n_i} I[Z_{ij}=1]}$$



INSULATED NEIGHBOR RANDOMIZATION DESIGN FOR K-LEVEL PEER EFFECT ESTIMATION

A potential outcome is defined based on the treatment assignment of neighbors

K-level treatment: a node is k -exposed to peer influence effects if exactly k of its neighbors are treated

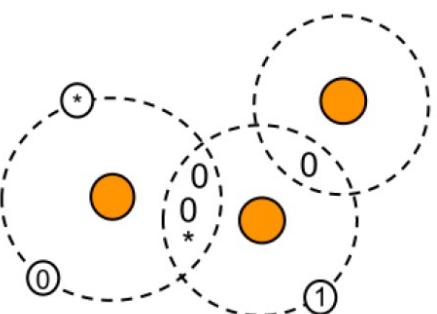
Estimand for k-level peer effects: neighbors are treated

$$\delta_k = \frac{1}{|V_k|} \sum_{i \in V_k} \left[\binom{n_i}{k}^{-1} \sum_{\mathbf{z} \in \mathbf{Z}(N_i; k)} Y_i(0, \mathbf{z}) - Y_i(\mathbf{0}) \right]$$

V_k : nodes with $\geq k$ neighbors

possible combinations with exactly k treated neighbors

Outcome when k
neighbors are treated
but ego is not
treated



INR Design: nodes from V_k are sequentially assigned to either be k -exposed or 0-exposed

- Estimator bias depends on network topology and whether shared neighbors are as influential as non-shared ones

MECHANISM AND ENCOURAGEMENT DESIGNS FOR PEER EFFECT ESTIMATION

Randomizing peer behavior is not always realistic

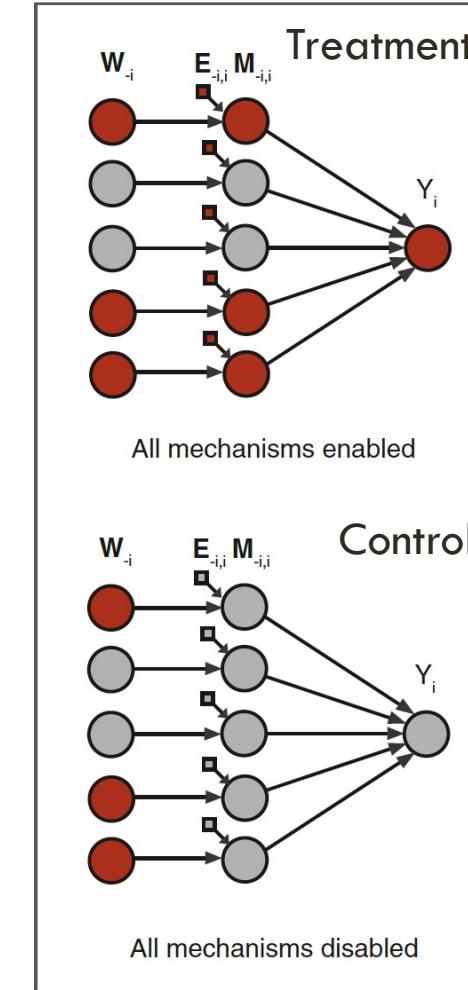
Mechanism designs: modulate the mechanism by which information about peer behavior is transmitted

Encouragement designs: measure peer effects of behaviors not directly controlled by the experimenter

Goal: Estimate effects of receiving feedback on how many posts egos make and how much feedback they give on others' posts



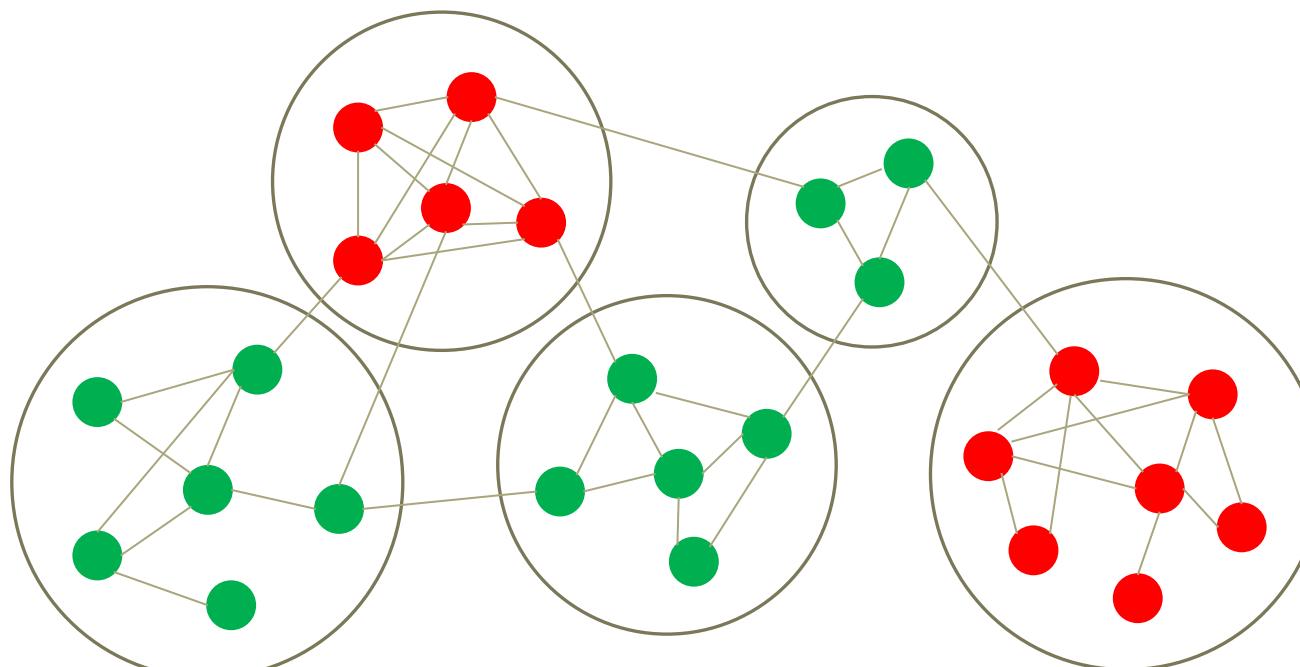
Mechanism design



CLUSTER-BASED RANDOMIZATION DESIGNS FOR TOTAL TREATMENT EFFECT ESTIMATION

Design for estimating total treatment effect

- Assumes partial interference: interference can occur within clusters but not across clusters
- Minimizes spillover between treatment and control



Estimand of interest:

$$TTE = \frac{1}{N} \sum_{v_i \in V} (v_i.Y(\mathbf{Z}_1) - v_i.Y(\mathbf{Z}_0))$$

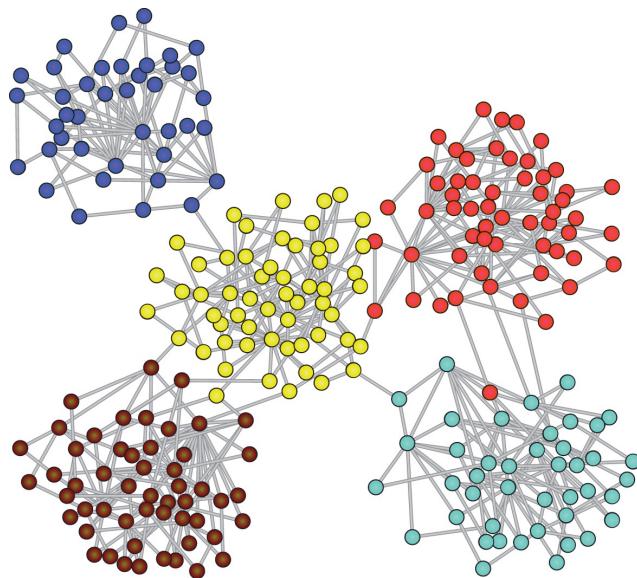
Horvitz-Thompson Estimator:

$$\hat{\tau}(Z) = \frac{1}{n} \sum_{i=1}^n \left(\frac{Y_i(Z)\mathbf{1}[Z \in \sigma_i^1]}{\Pr(Z \in \sigma_i^1)} - \frac{Y_i(Z)\mathbf{1}[Z \in \sigma_i^0]}{\Pr(Z \in \sigma_i^0)} \right)$$

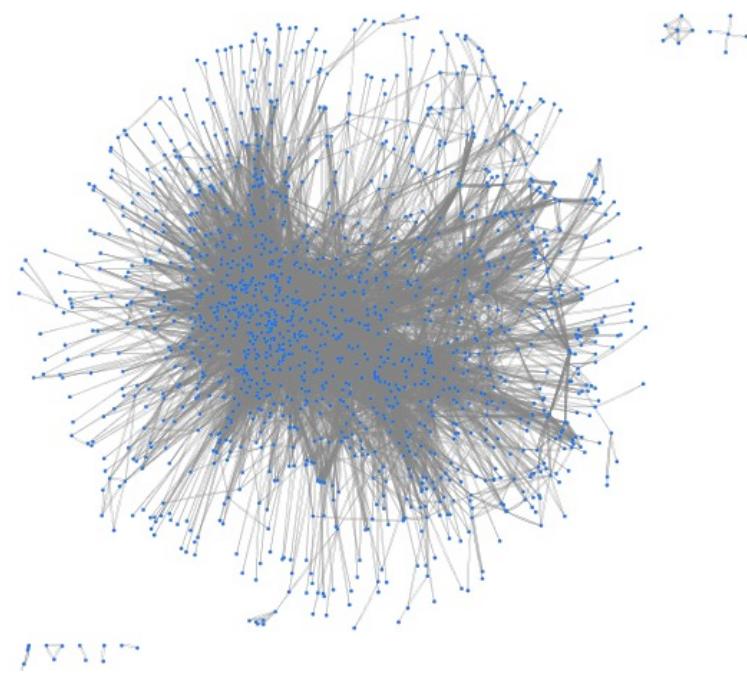
CHALLENGES WITH CLUSTER-BASED RANDOMIZATION

Challenge 1*: It can be hard to separate a real-world network into treatment and control clusters without leaving a lot of edges across

- E.g., LinkedIn graph clustering has 65-79% of inter-cluster edges**



Ideal network



Online social networks

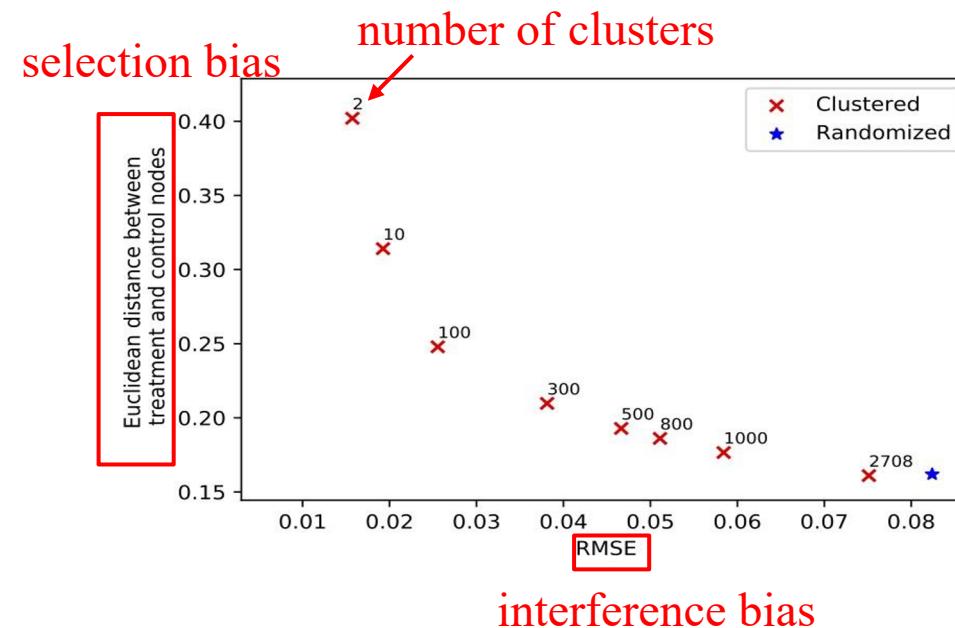
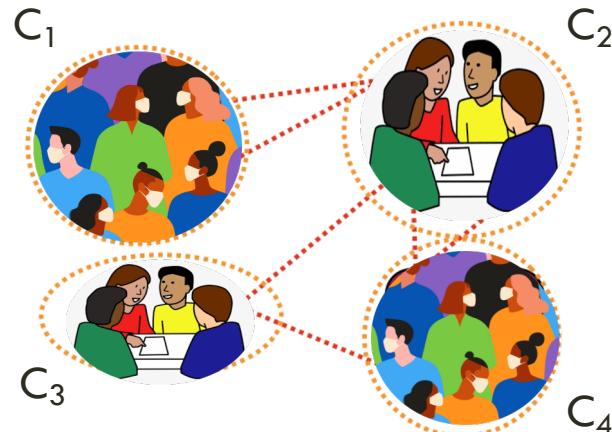
*Z. Fatemi, E. Zheleva. *Minimizing interference and selection bias in network experiment design*. ICWSM 2020.

**Saveski, Pouget-Abadie, Saint-Jacques, Duan, Ghosh, Xu, Airoldi. *Detecting network effects: Randomizing over randomized experiments*. KDD 2017.

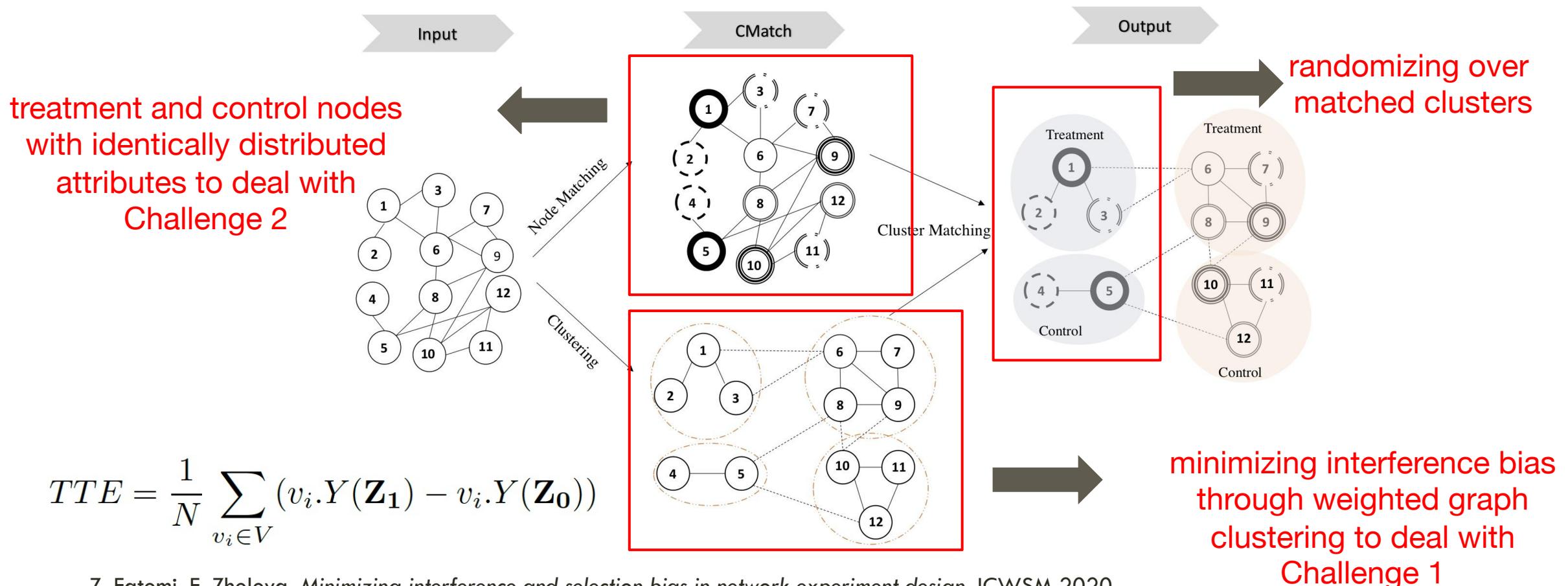
CHALLENGES WITH CLUSTER-BASED RANDOMIZATION

Challenge 2: Treatment and control clusters can have different covariate distributions

- Tradeoff between interference and selection bias based on number of clusters



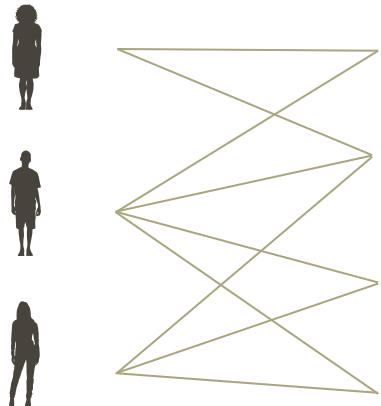
CMatch: CLUSTER-BASED RANDOMIZATION WITH CLUSTER MATCHING ON A WEIGHTED GRAPH



TWO-SIDED RANDOMIZATION FOR BIPARTITE GRAPH EXPERIMENTS

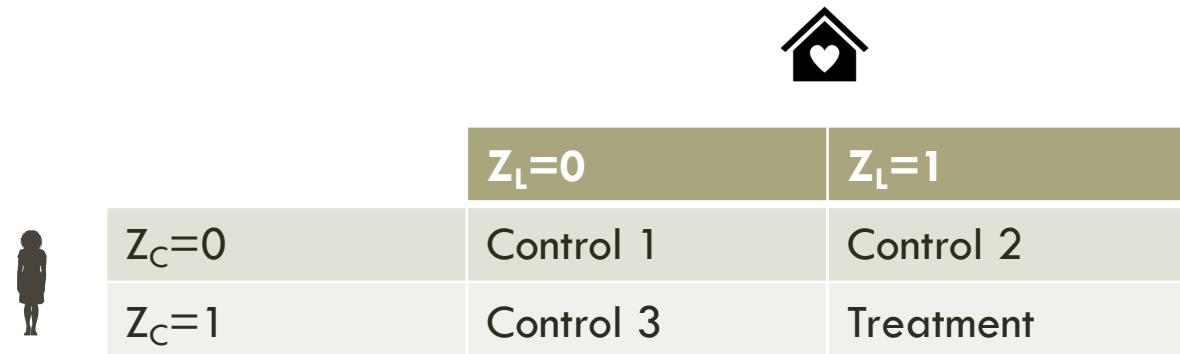
Two-sided markets

Customers



Listings

Lower bias than customer randomization or listing randomization alone
Bias goes to zero as relative demand goes to zero or infinity

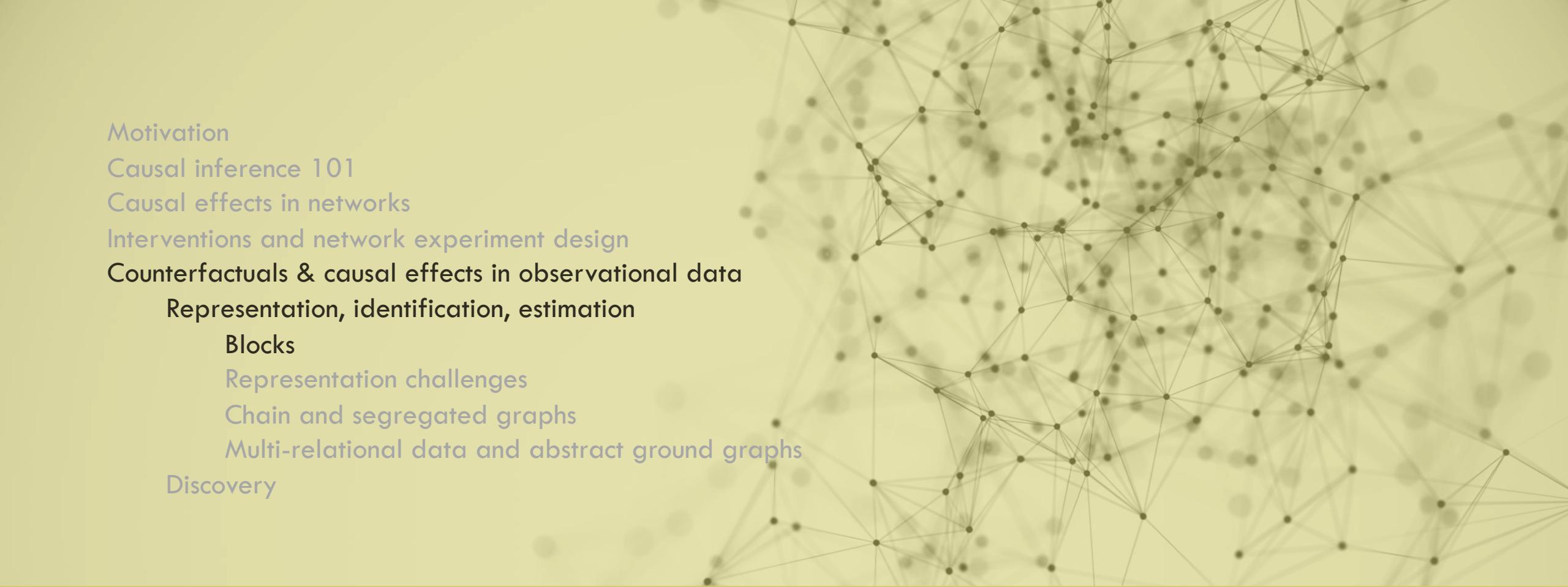


Interference due to competition:

- Making one listing more attractive makes others less attractive
- Making one customer more likely to book reduces supply for other customers

R. Johari, H. Li, I. Liskovic, G. Weintraub. *Experimental design in two-sided platforms: An analysis of bias*. Arxiv 2020.

P. Bajari, B. Burdick, G. Imbens, J. McQueen, T. Richardson, I. Rosen. *Multiple randomization designs for interference*. ASSA Annual Meeting 2020.

A complex network graph with numerous nodes represented by small dots and edges represented by thin lines, creating a dense web-like structure.

Motivation

Causal inference 101

Causal effects in networks

Interventions and network experiment design

Counterfactuals & causal effects in observational data

Representation, identification, estimation

Blocks

Representation challenges

Chain and segregated graphs

Multi-relational data and abstract ground graphs

Discovery

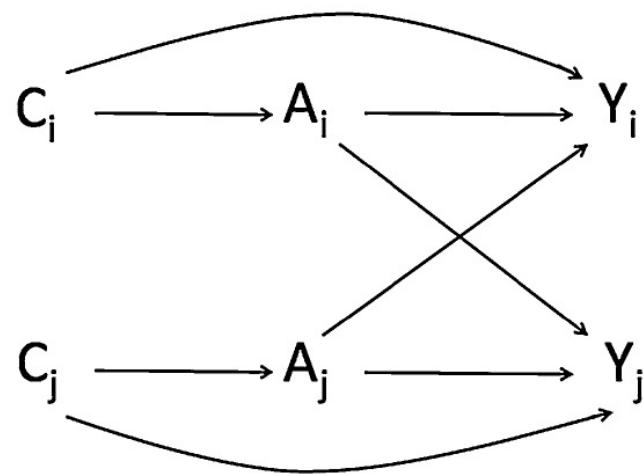
COUNTERFACTUALS & CAUSAL EFFECTS IN OBSERVATIONAL NETWORK DATA

Blocks

REPRESENTATION: GRAPHICAL MODELS

Blocks

Assume partial interference



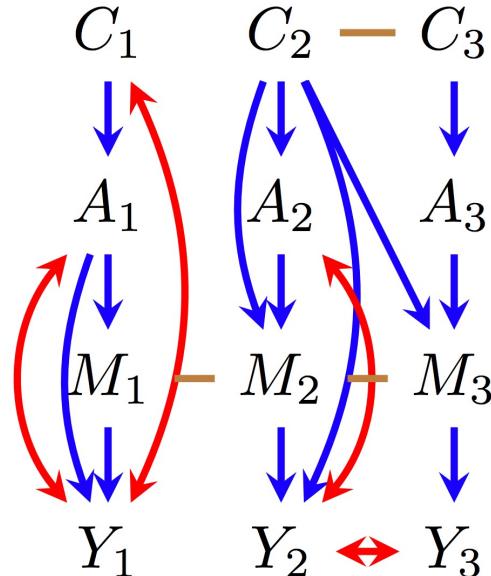
C-covariates

A-treatment

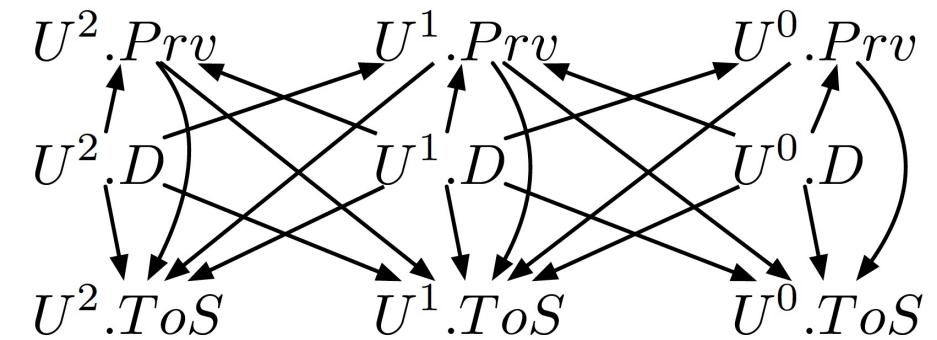
Y-outcome

Chain and segregated graphs

Can model more complex interference



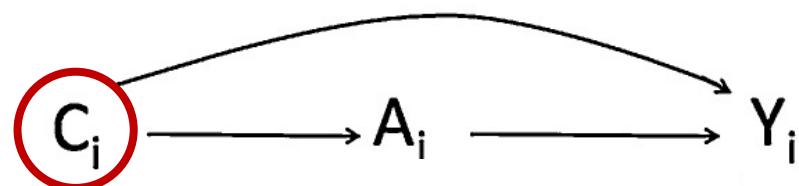
Abstract ground graphs



BLOCKS FOR DIRECT INTERFERENCE

Blocks: repeatable patterns of interference

Direct interference: treatments of peers/neighbors affect ego's outcome



Exchangeability holds and the effect of **A** on Y_i is identifiable:
 C_i blocks the backdoor paths* from A_i to Y_i and from A_i to Y_i

$$P(Y_i = y | do(A_i = a_i, A_j = a_j)) =$$

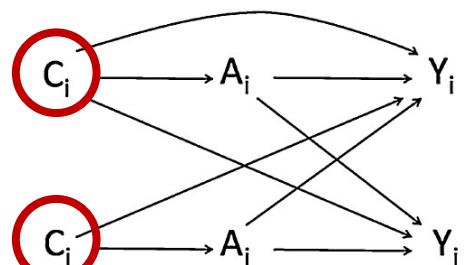
$$\sum_{c_i} P(Y_i = y | A_i = a_i, A_j = a_j, C_i = c_i) P(C_i = c_i)$$

*A set of variables **C** satisfies the backdoor criterion relative to (A, Y) if no node in C is a descendant of A, and C blocks every path between A and Y that contains an arrow into A

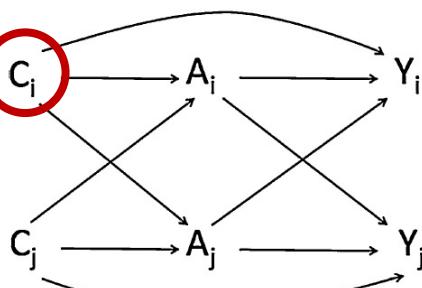
C-covariates A-treatment Y-outcome

BLOCKS FOR DIRECT INTERFERENCE

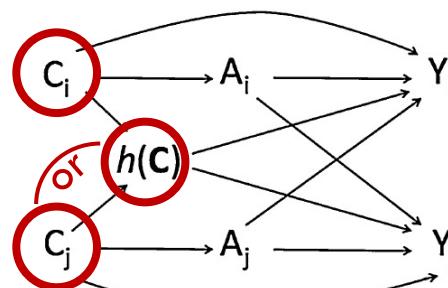
Identification of $E[Y_i|do(A = a_1)] - E[Y_i|do(A = a_2)]$ depends on the causal graph (domain knowledge) and which variables are available in the data



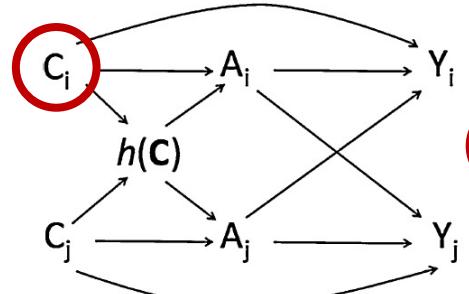
(a)



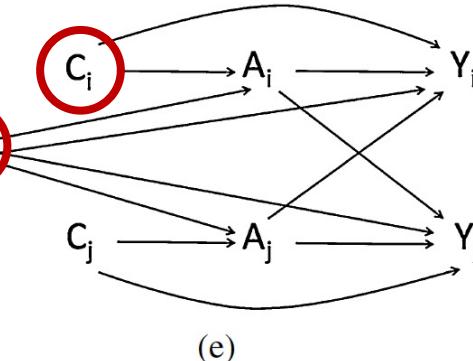
(b)



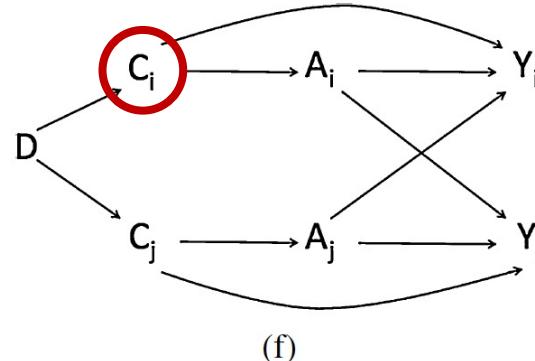
(c)



(d)



(e)



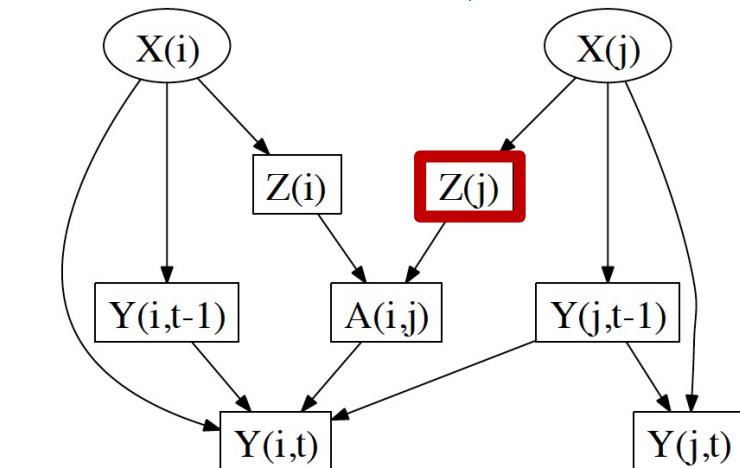
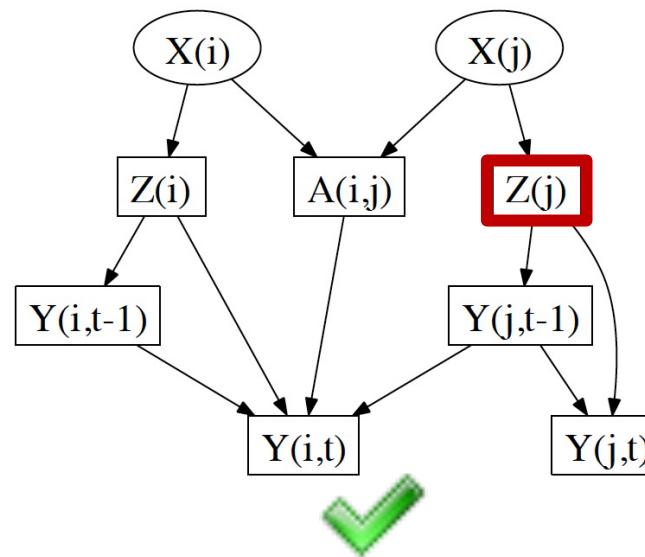
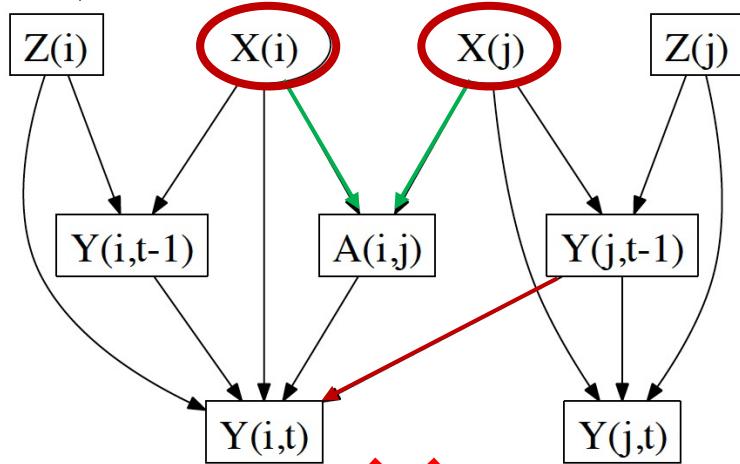
(f)

C-unit covariates
A-treatment
Y-outcome
D-common covariates
 $h(C)$ -function of C

*A set of variables C satisfies the backdoor criterion relative to (A, Y) if no node in C is a descendant of A , and C blocks every path between A and Y that contains an arrow into A

IDENTIFYING CONTAGION

Contagion $E[Y_{i,t} | do(Y_{j,t-1} = y_1)] - E[Y_{i,t} | do(Y_{j,t-1} = y_0)]$ may not be identifiable due to **latent homophily**



| Symbol | Meaning |
|--------|-------------------|
| i, j | Individuals |
| Z | Observed Traits |
| X | Latent Traits |
| Y | Observed Outcomes |
| A | Network Tie |

CAUSAL INFERENCE FROM NETWORK DATA

(10-MINUTE BREAK)



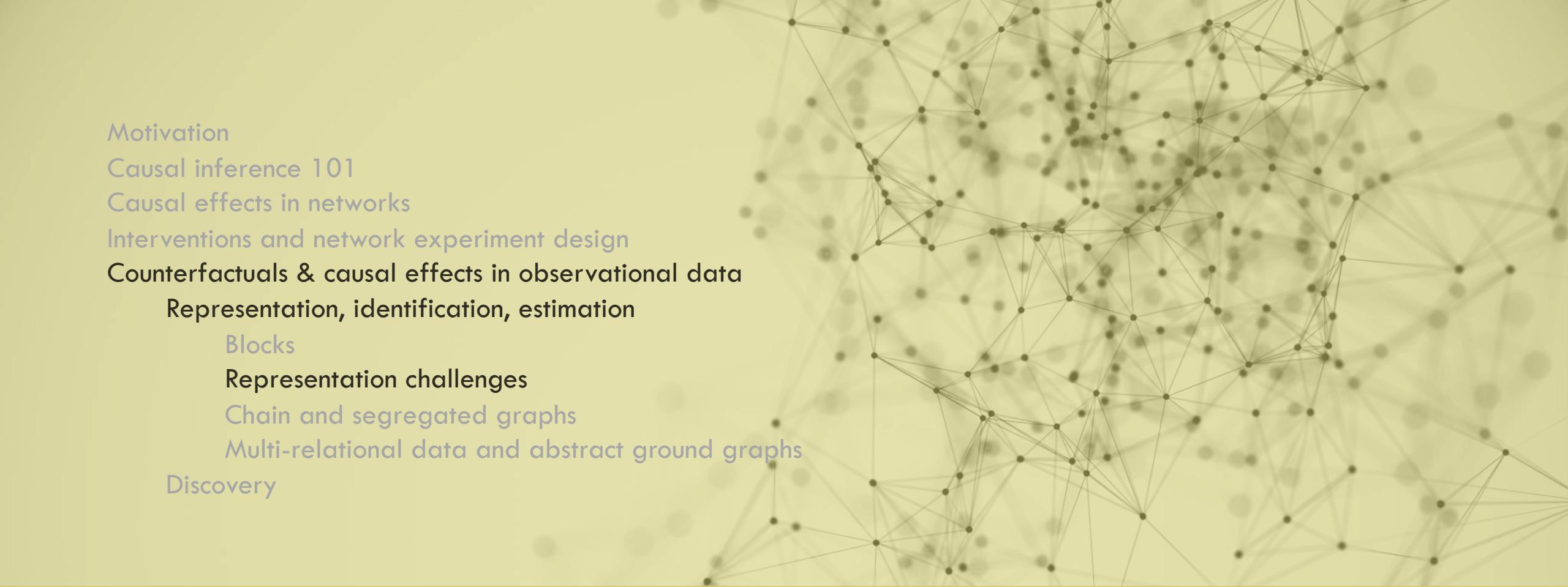
Presenters:

David Arbour, Adobe Research @darbour26

Elena Zheleva, University of Illinois at Chicago @elenadata

KDD 2021 Tutorial
August 14, 2021

<https://netcause.github.io>

A complex network graph composed of numerous small, semi-transparent nodes and a dense web of thin, light-colored lines representing connections between them, set against a yellow gradient background.

Motivation

Causal inference 101

Causal effects in networks

Interventions and network experiment design

Counterfactuals & causal effects in observational data

Representation, identification, estimation

Blocks

Representation challenges

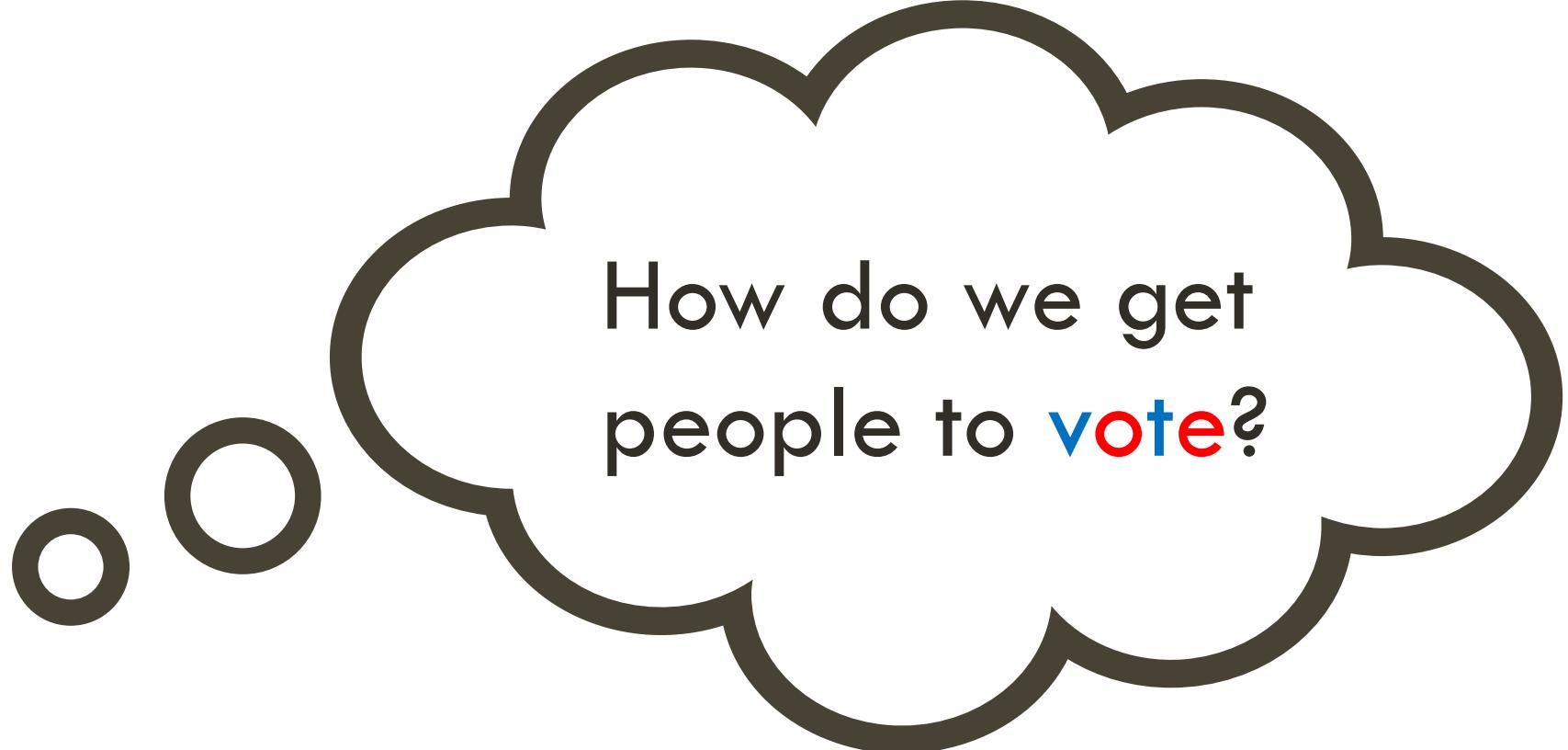
Chain and segregated graphs

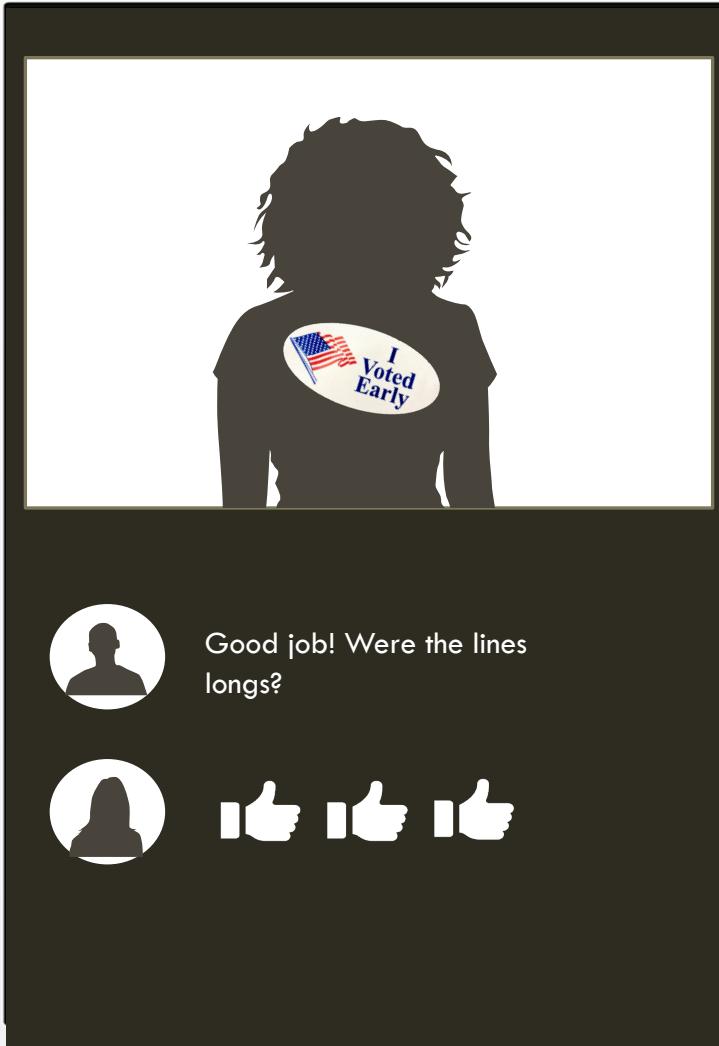
Multi-relational data and abstract ground graphs

Discovery

COUNTERFACTUALS & CAUSAL EFFECTS IN OBSERVATIONAL NETWORK DATA

Representation Challenges





Good job! Were the lines
longs?

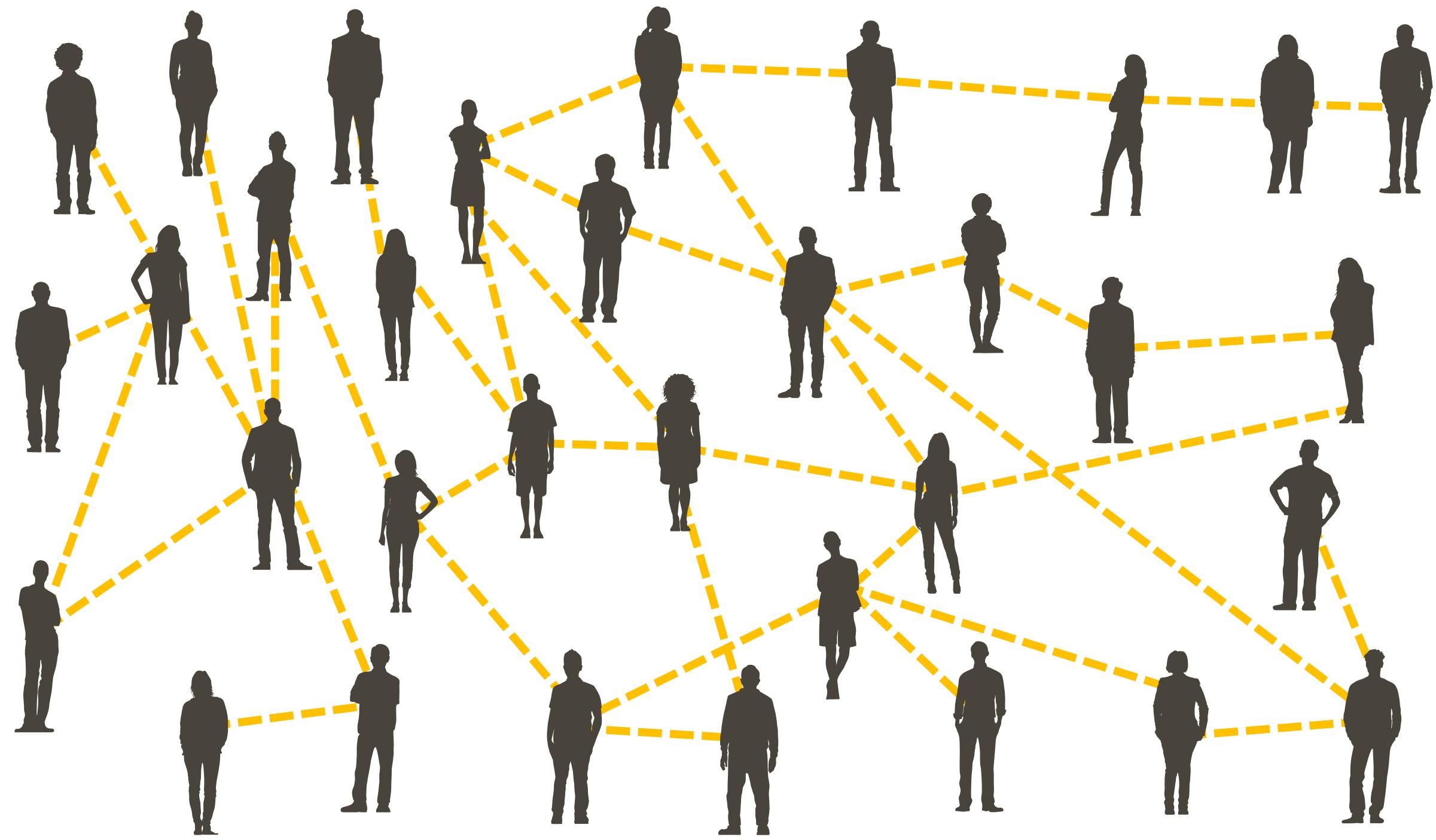


WHAT'S THE EFFECT?

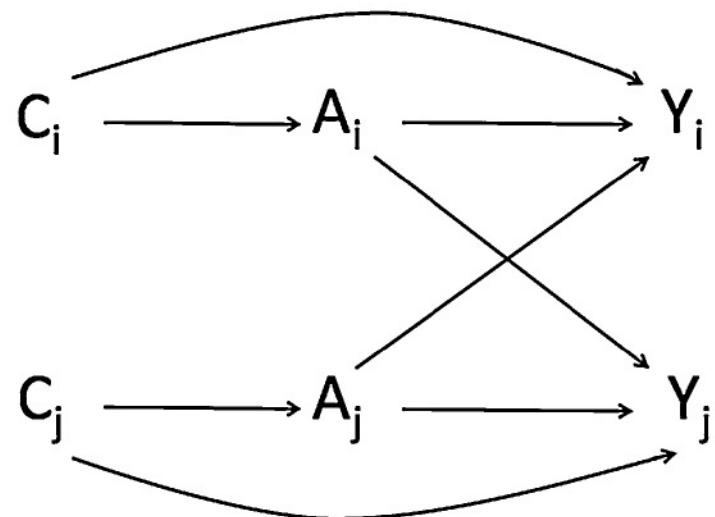


| Last Name | First Name | DOB | State | Voted (Y/N) |
|---|------------|-----|-------|-------------|
|  | | | | |
|  | | | | |
|  | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

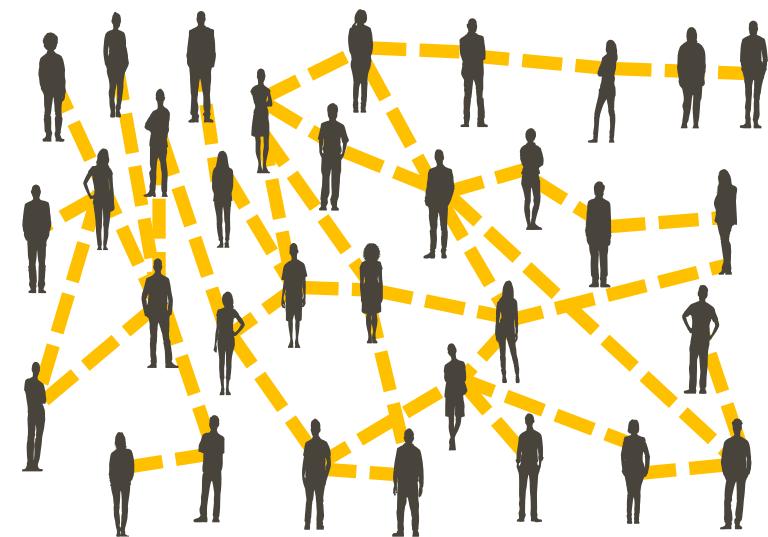
OBSERVED DATA



CHALLENGES

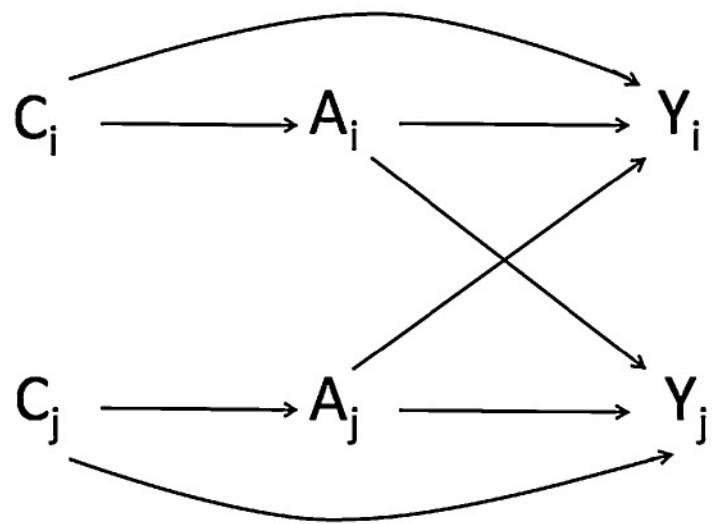


Causal



Network

CASUAL CHALLENGES



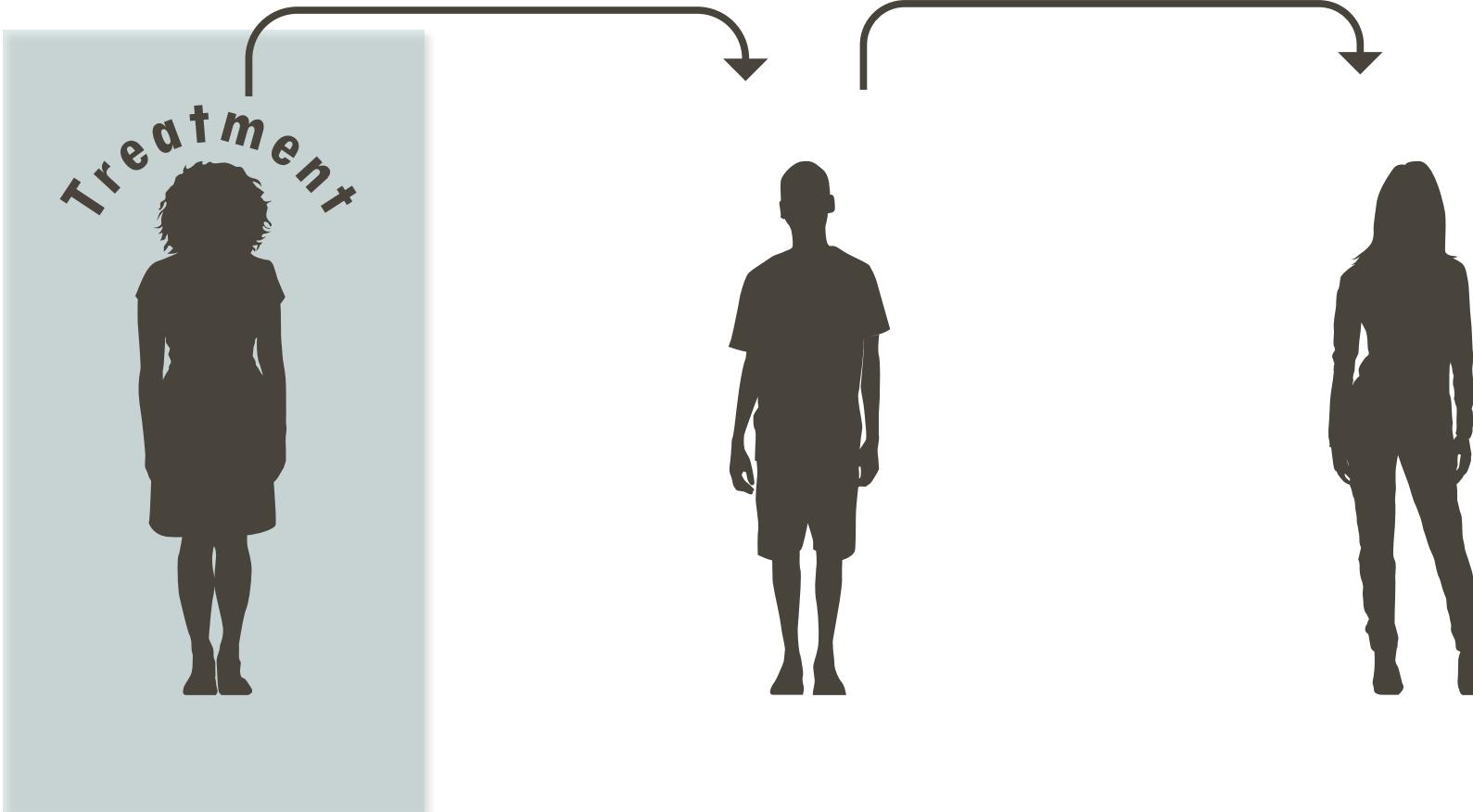
1.

Feedback

2.

Set Valued
Counterfactuals

FEEDBACK



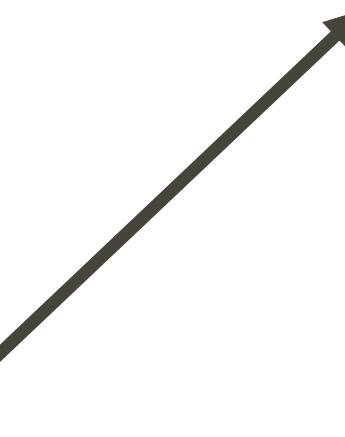
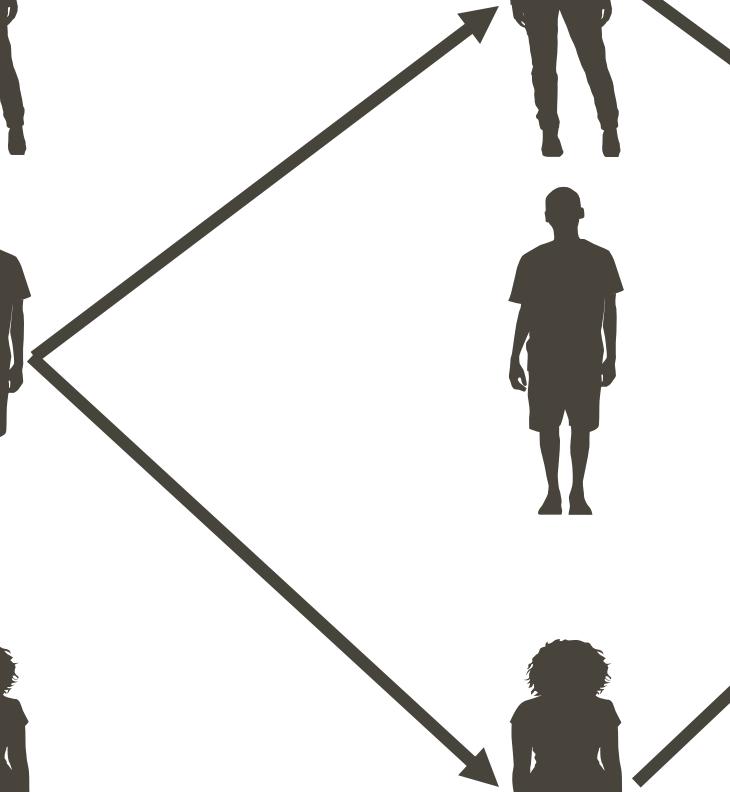


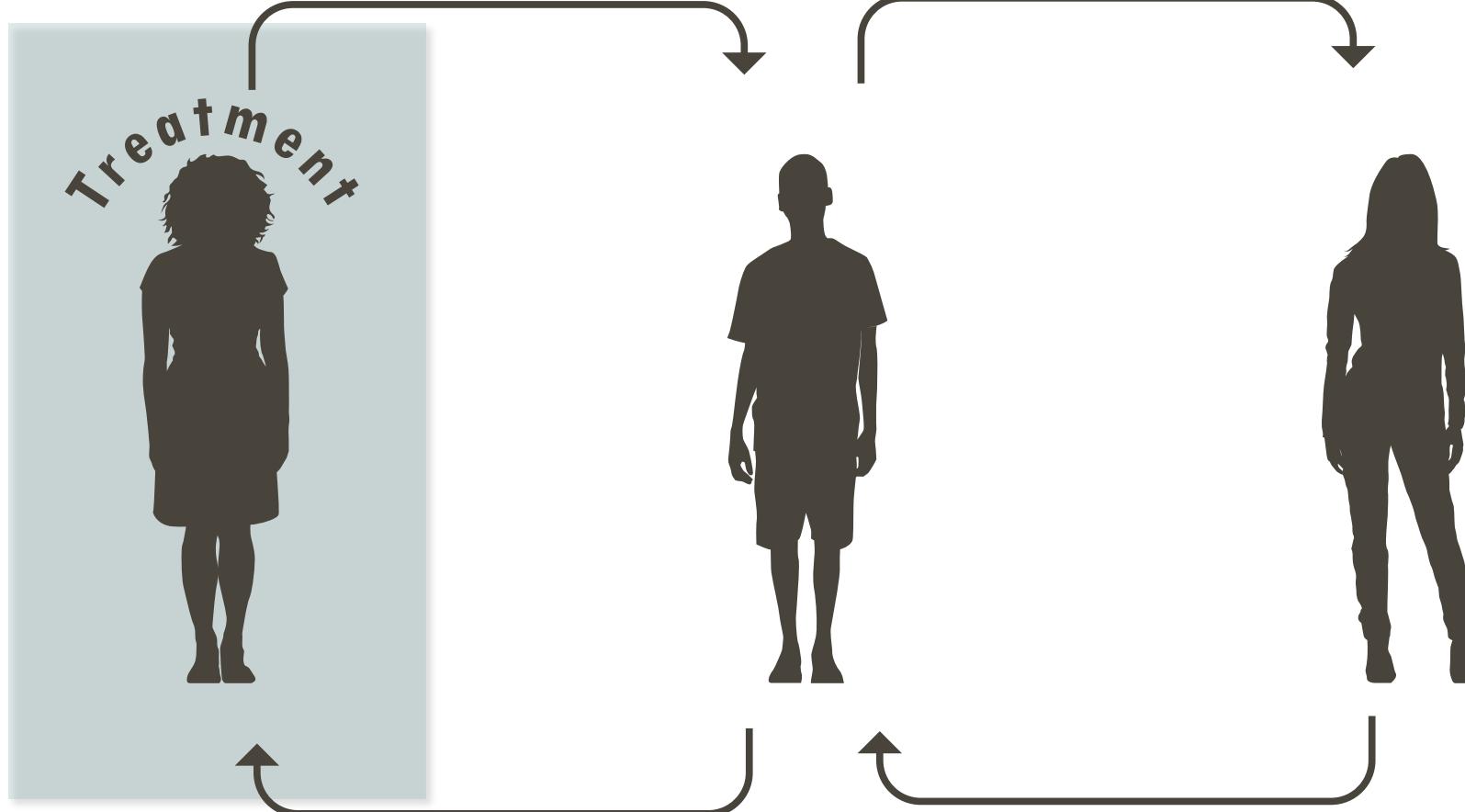
Week 1

Week 2

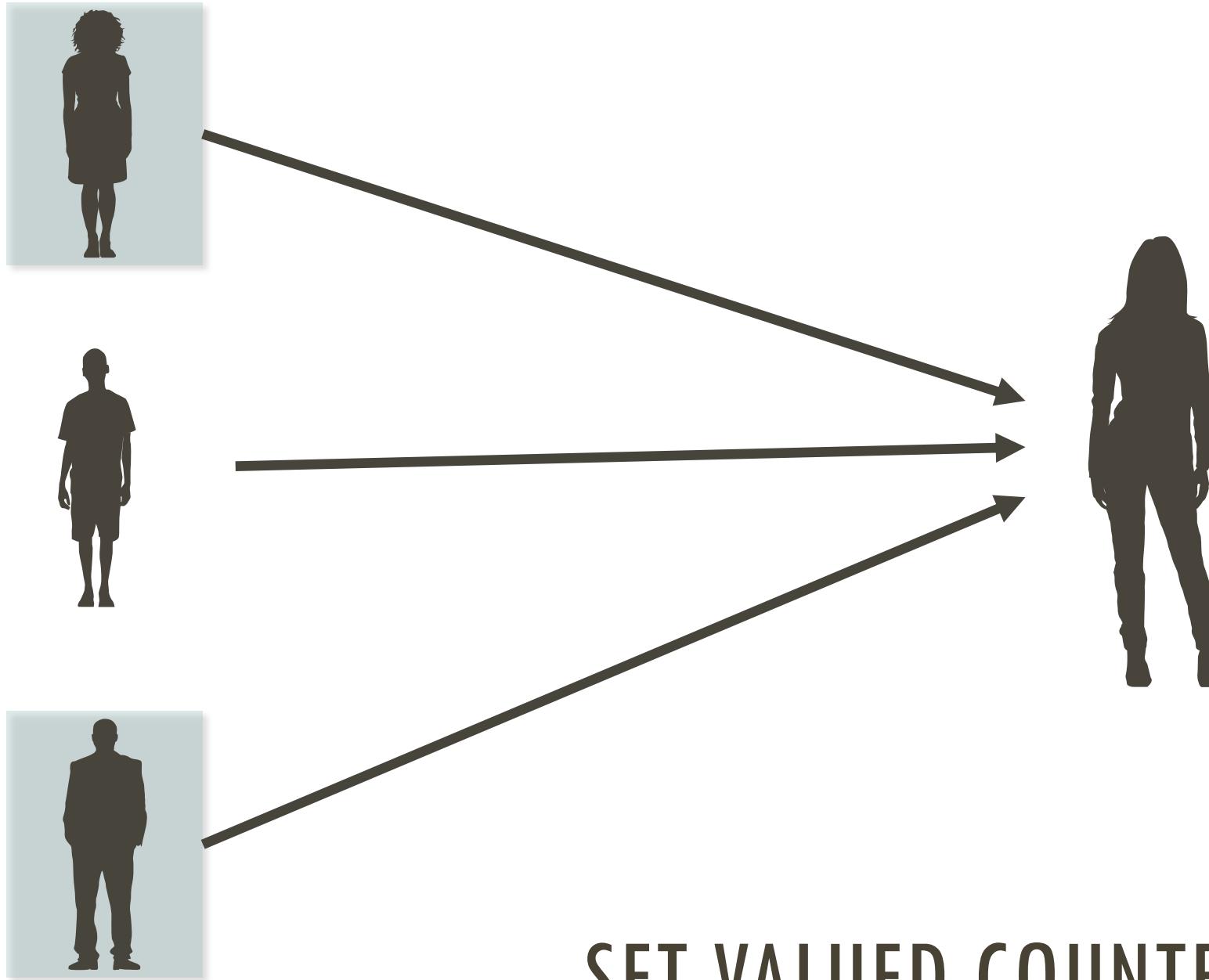
Week 3

Week 4

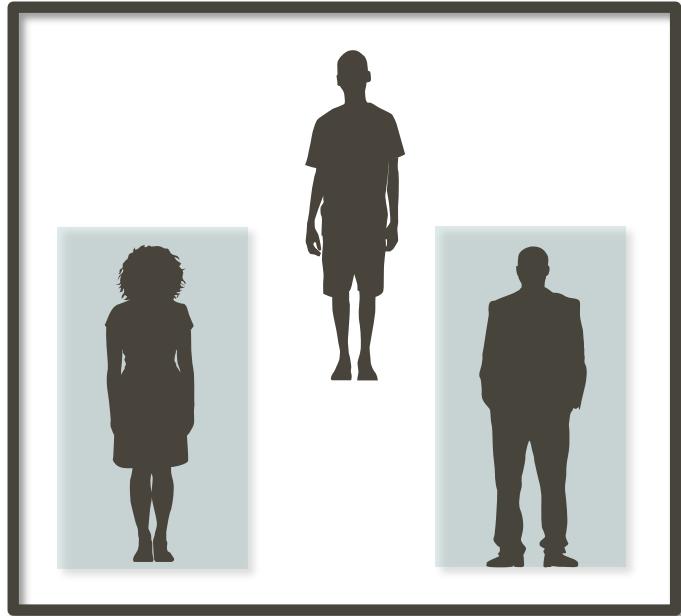




Pooled Data

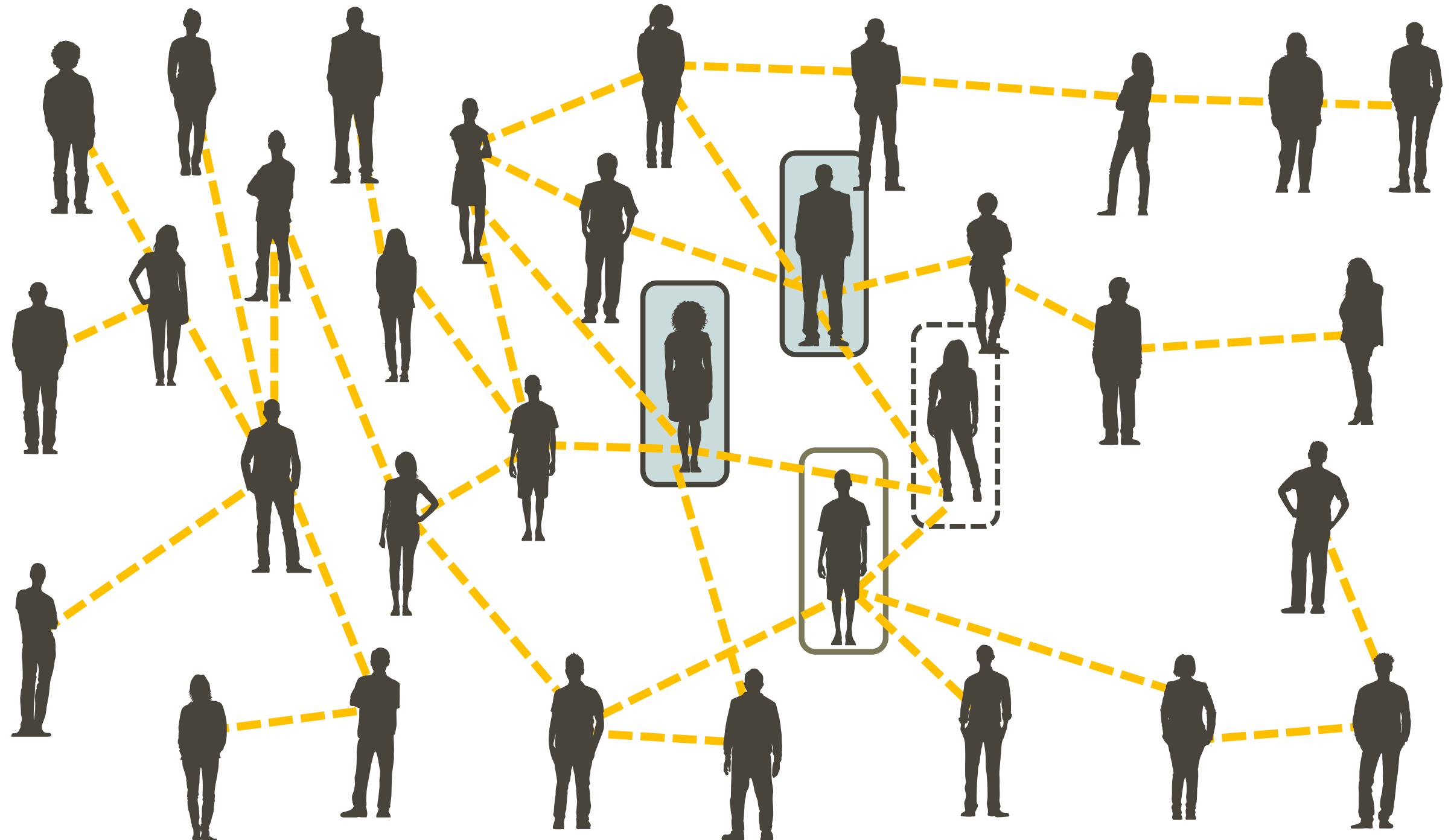


SET VALUED COUNTERFACTUALS

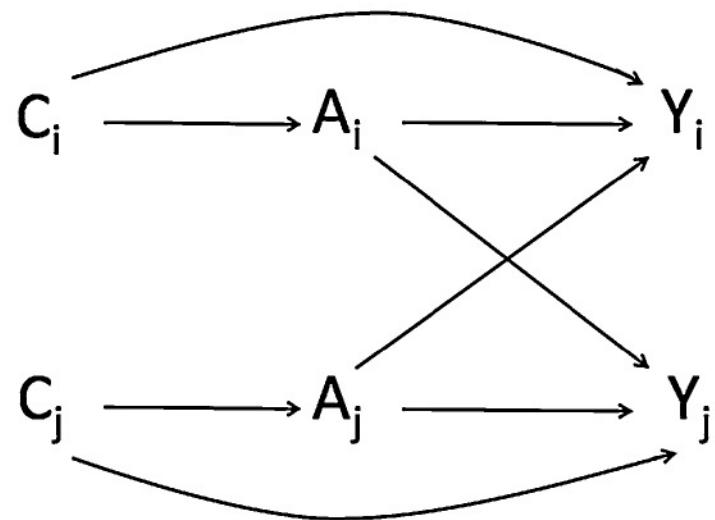


2/3 Treated

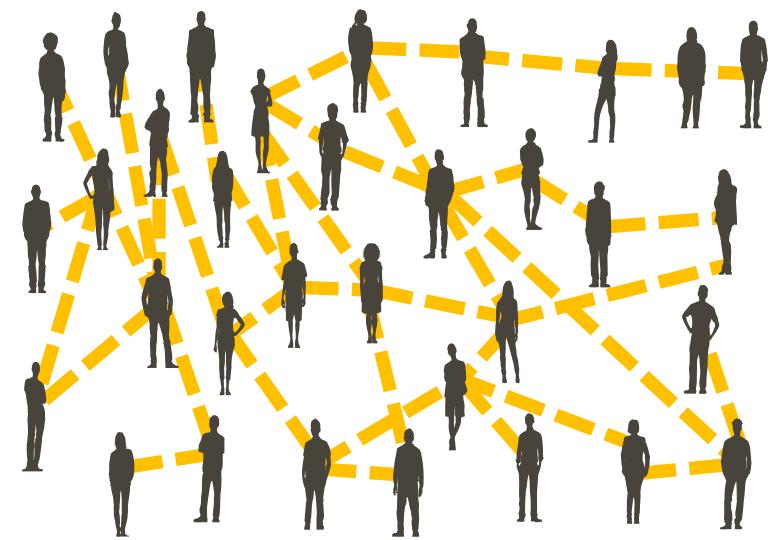
SET VALUED COUNTERFACTUALS



CHALLENGES



Causal



Network

NETWORK CHALLENGES

1.

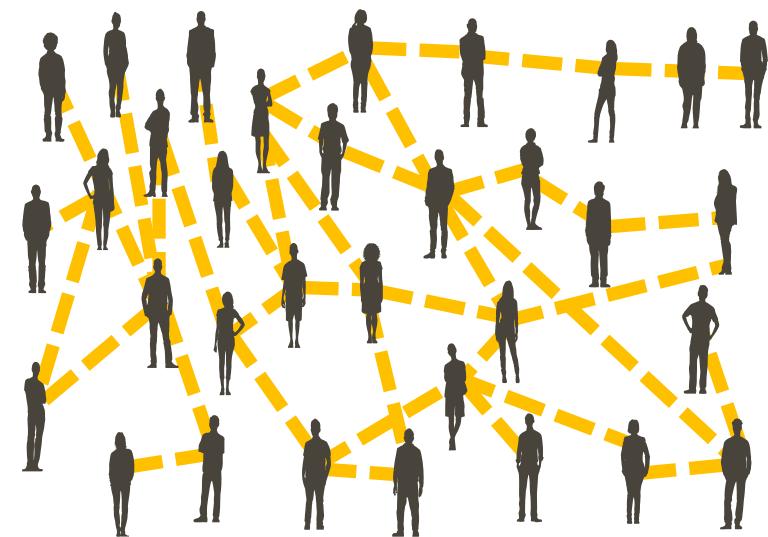
Directed /
Undirected Edges

2.

Multiple Entities &
Relationships

3.

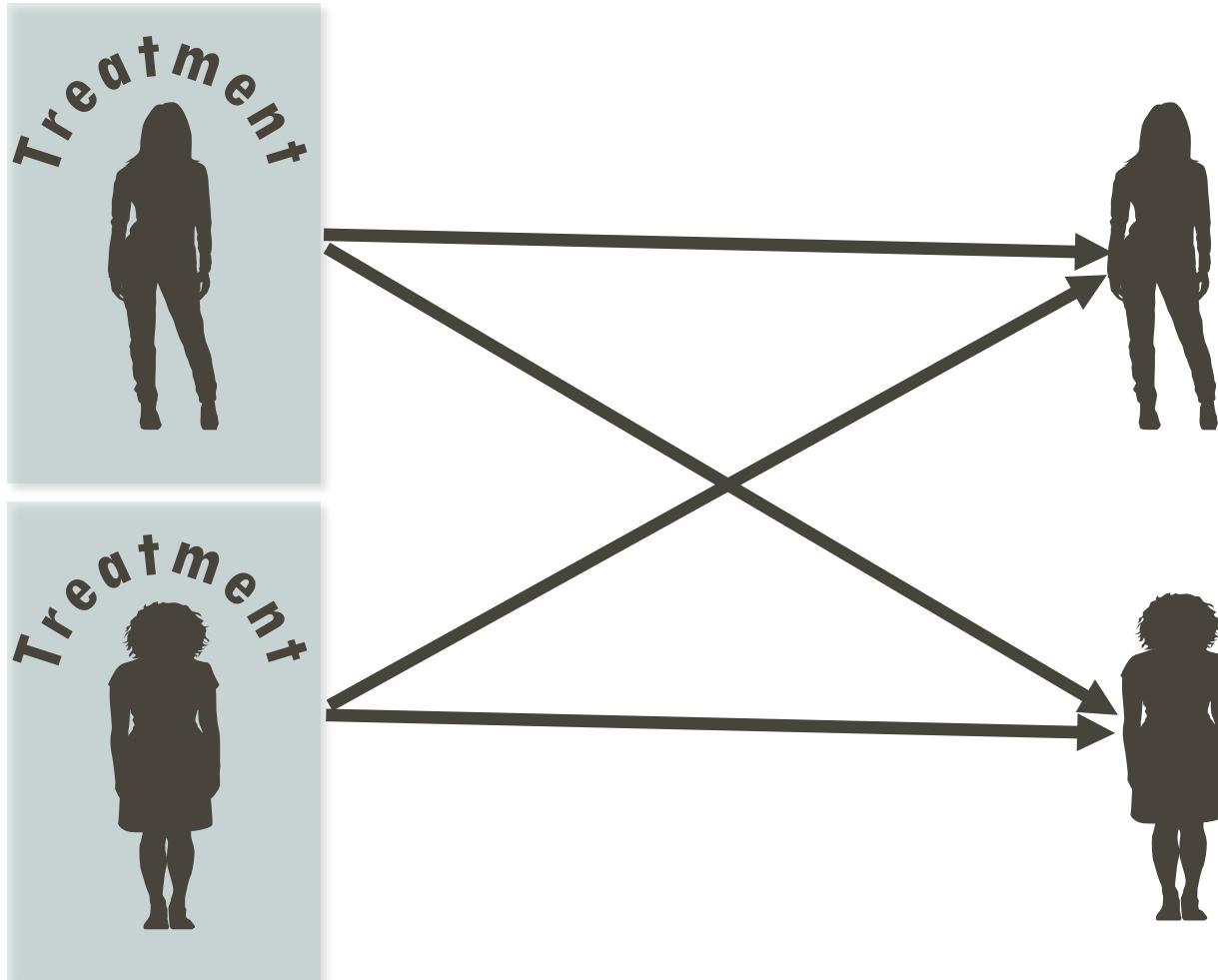
Unobserved /
Partially Observed



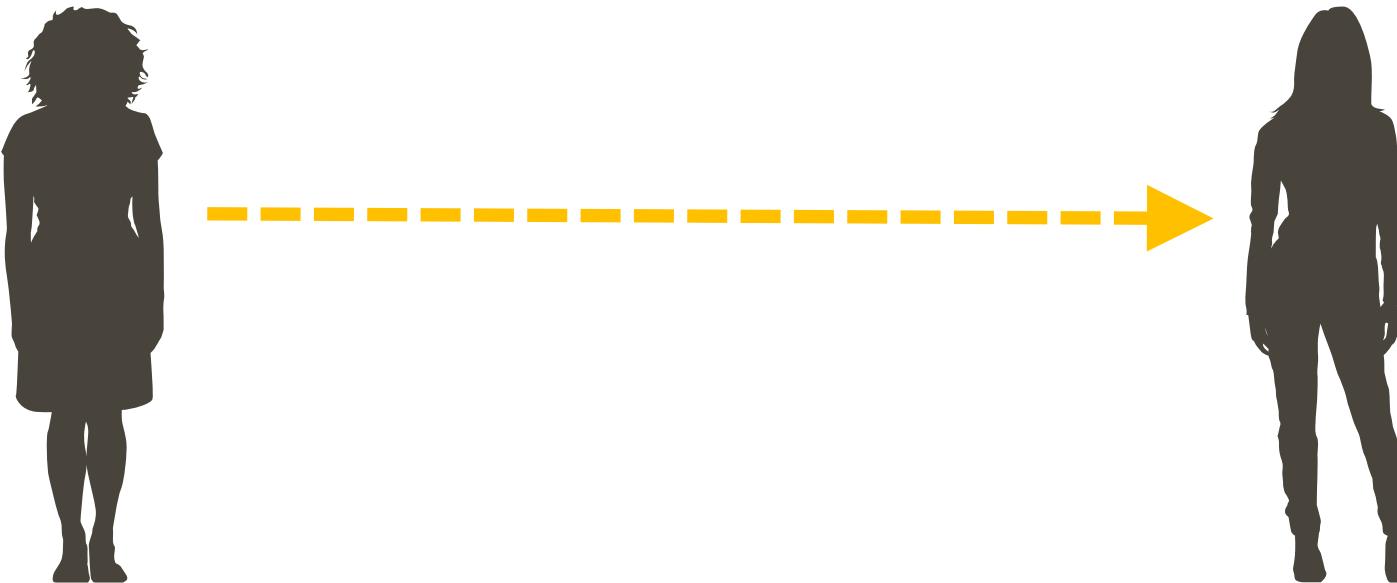
UNDIRECTED RELATIONSHIPS



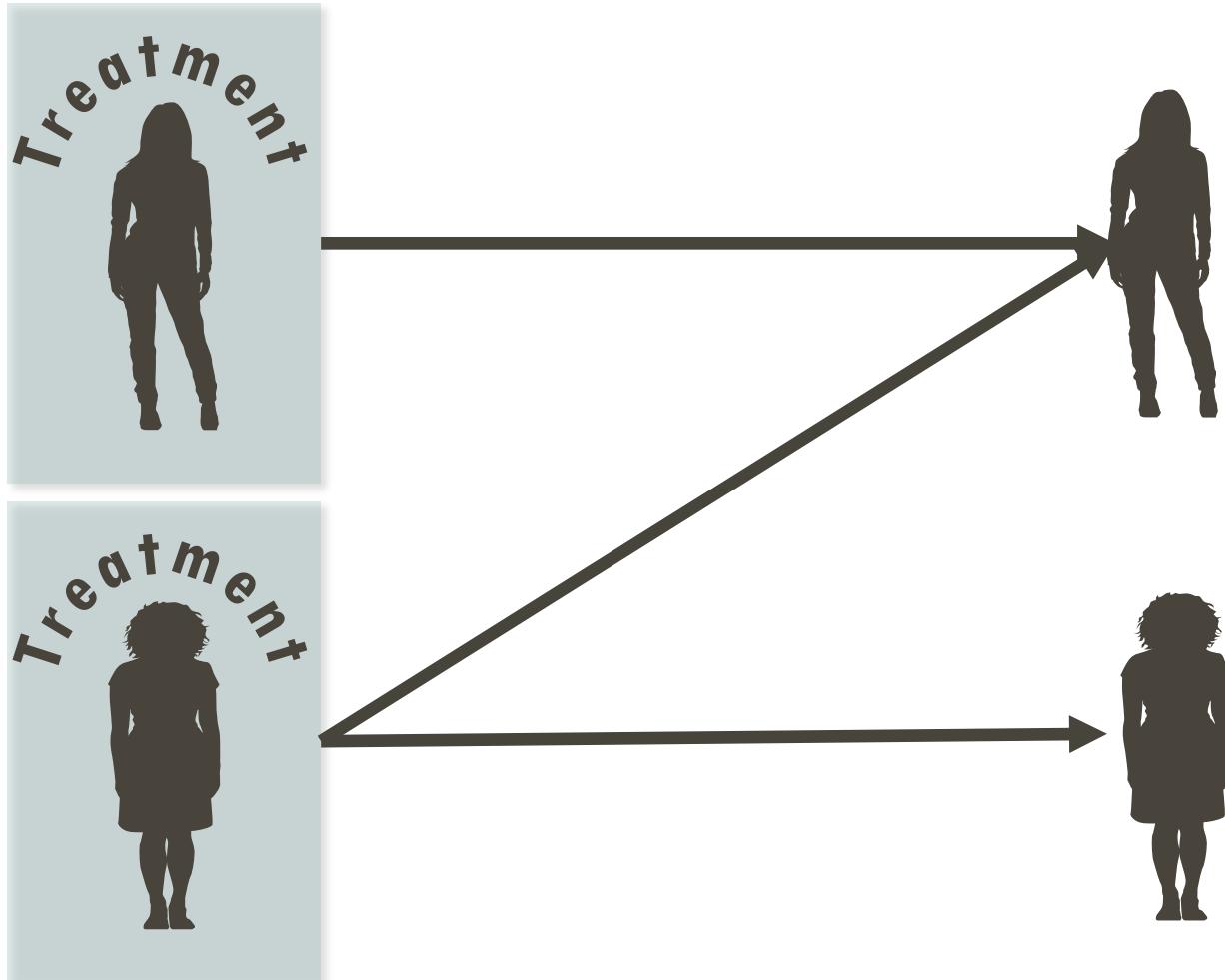
UNDIRECTED RELATIONSHIPS



DIRECTED RELATIONSHIPS



DIRECTED RELATIONSHIPS



NETWORK CHALLENGES

1.

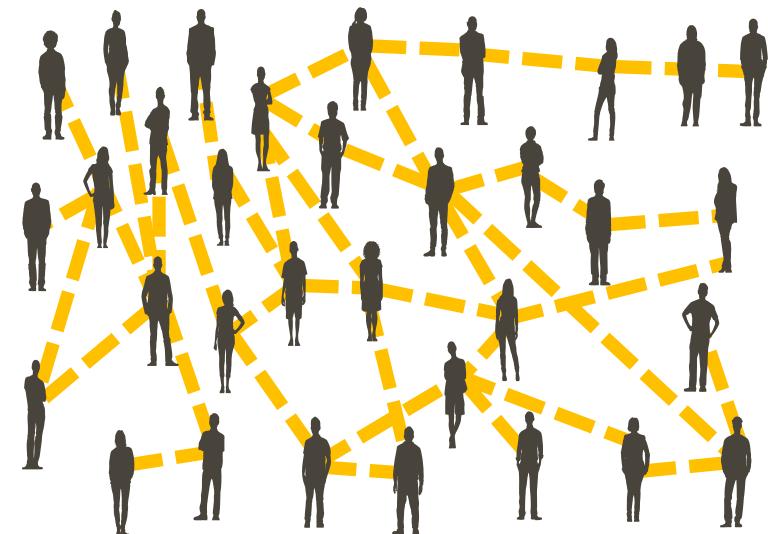
Directed /
Undirected Edges

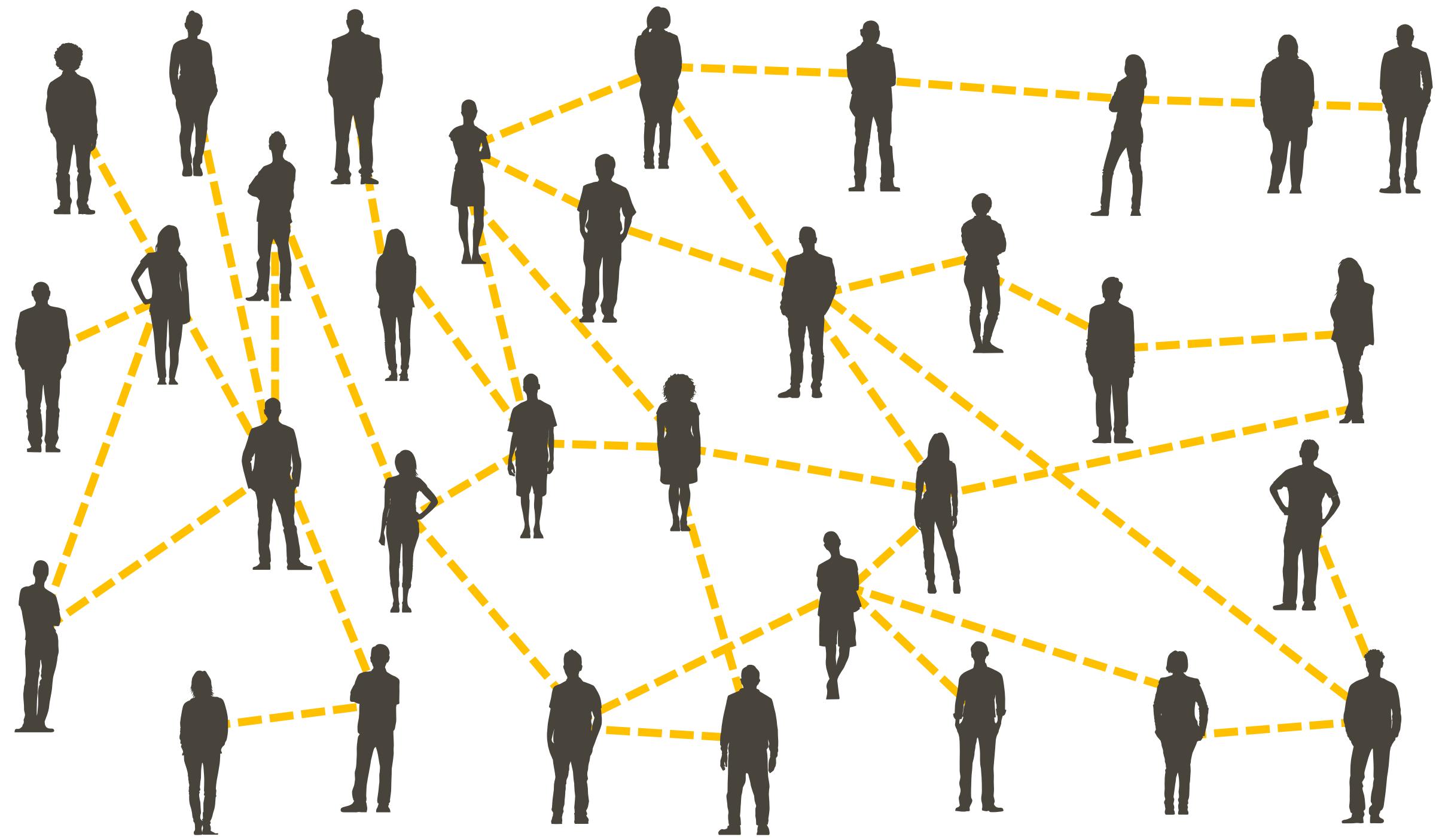
2.

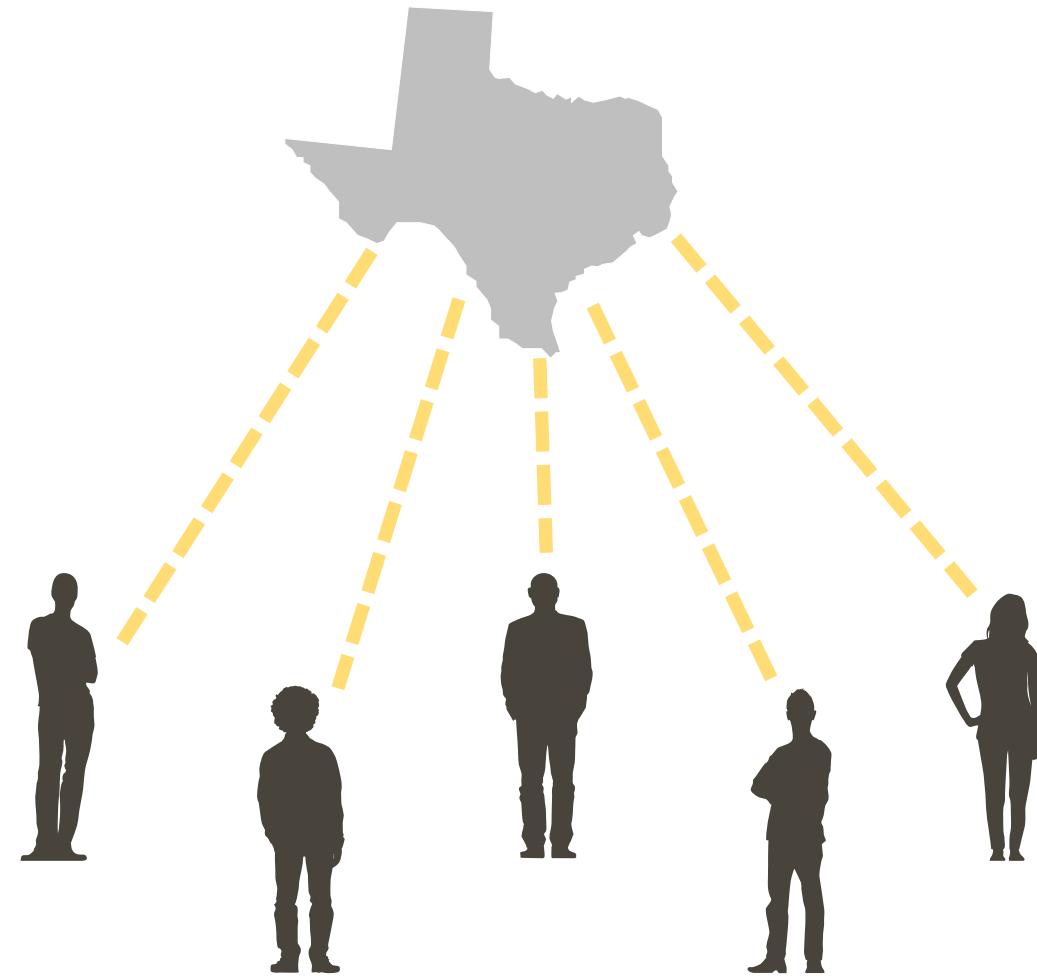
Multiple Entities &
Relationships

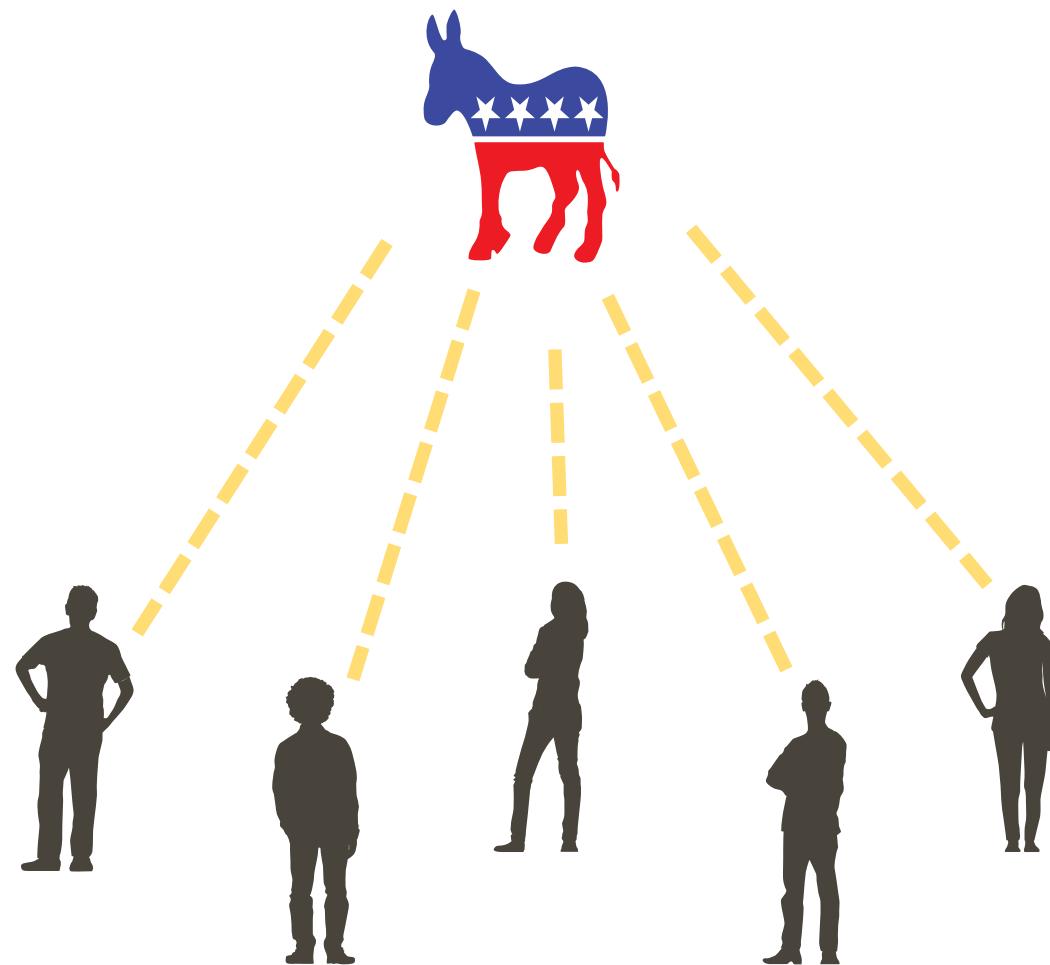
3.

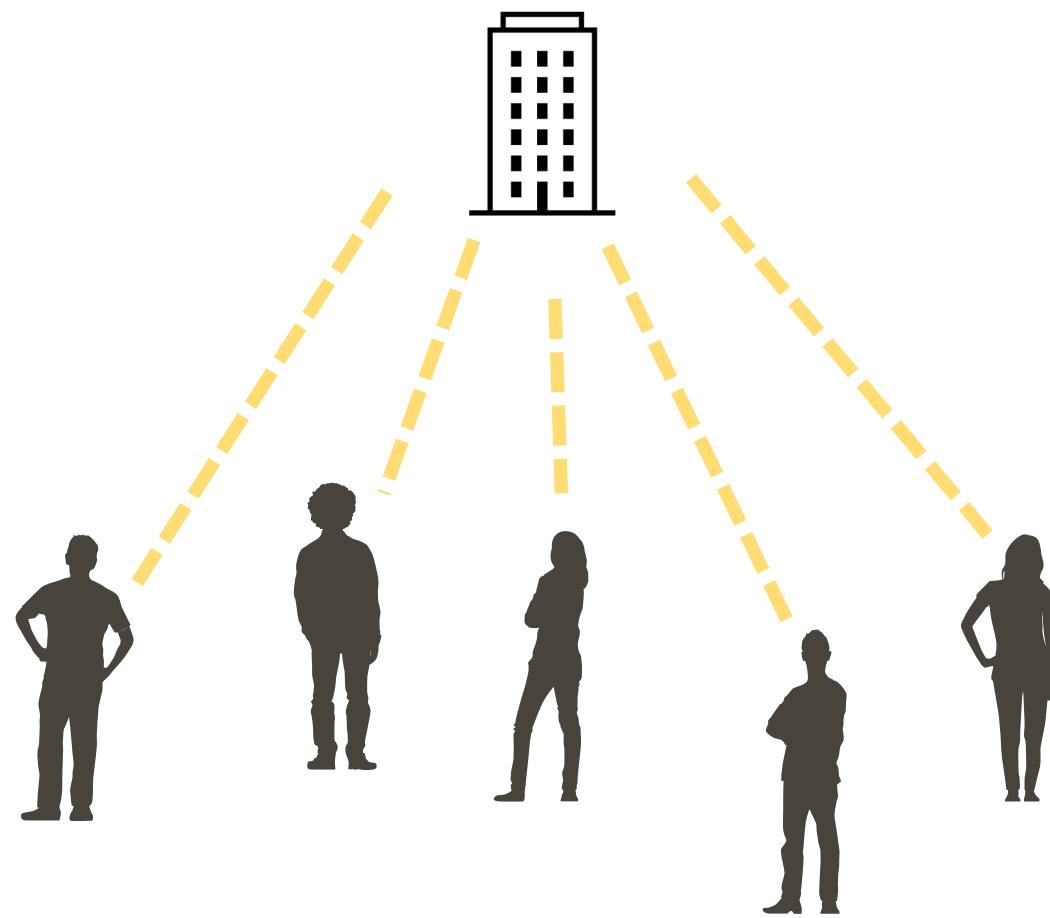
Unobserved /
Partially Observed

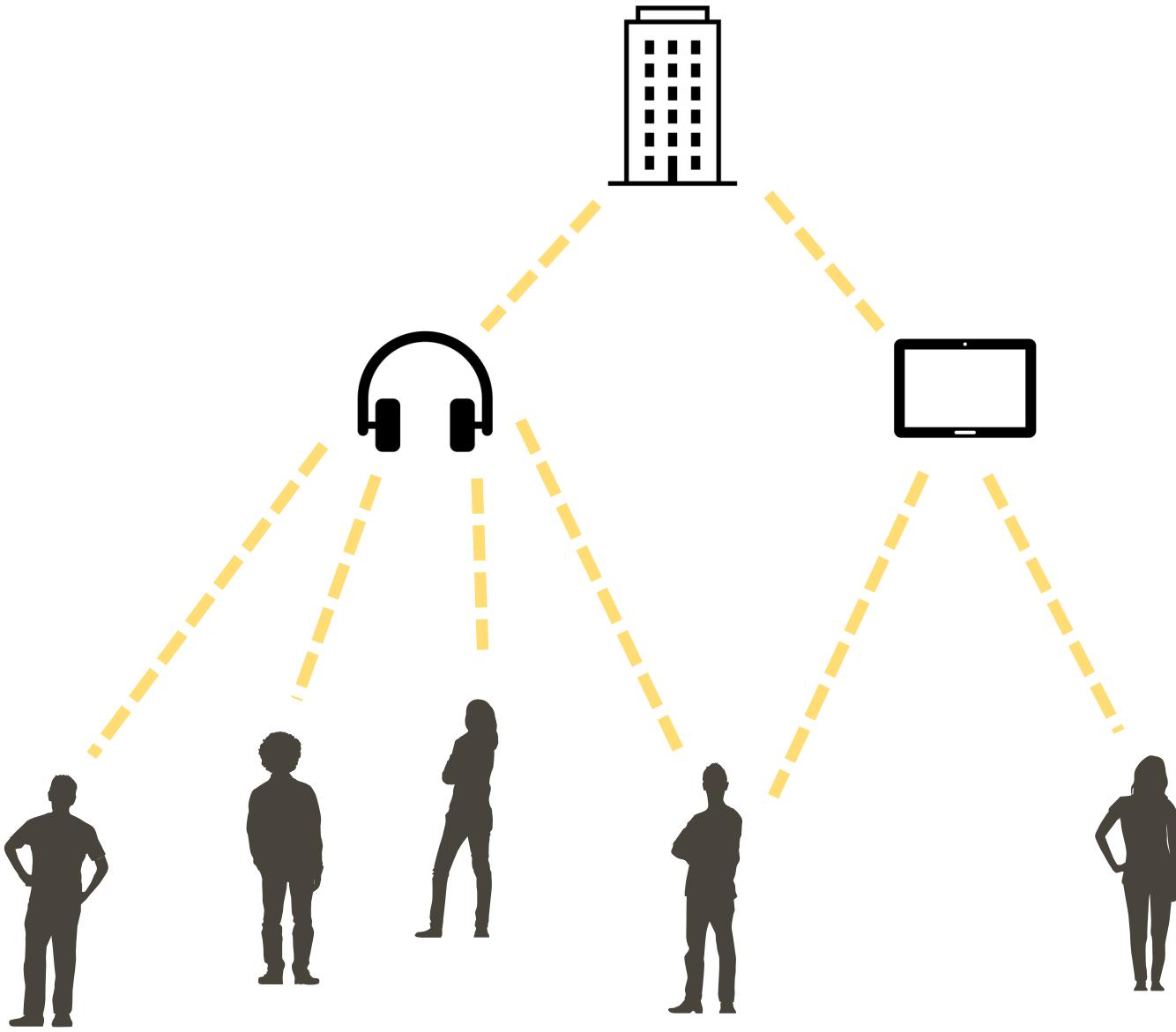












NETWORK CHALLENGES

1.

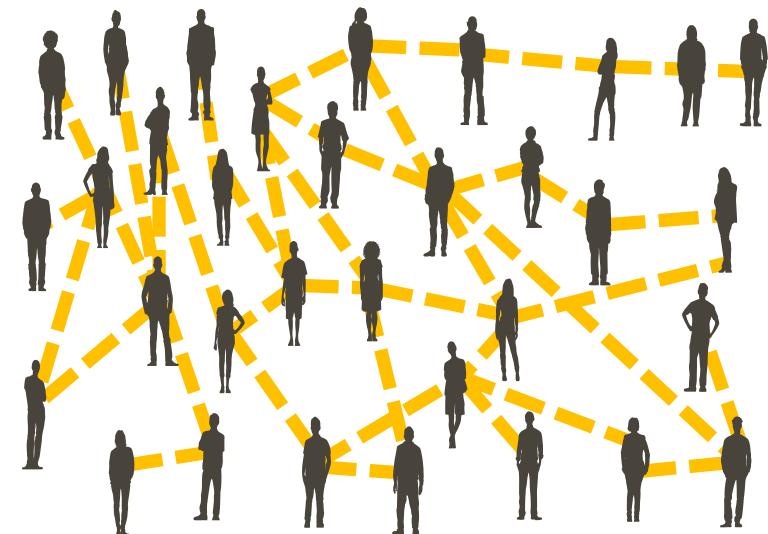
Directed /
Undirected Edges

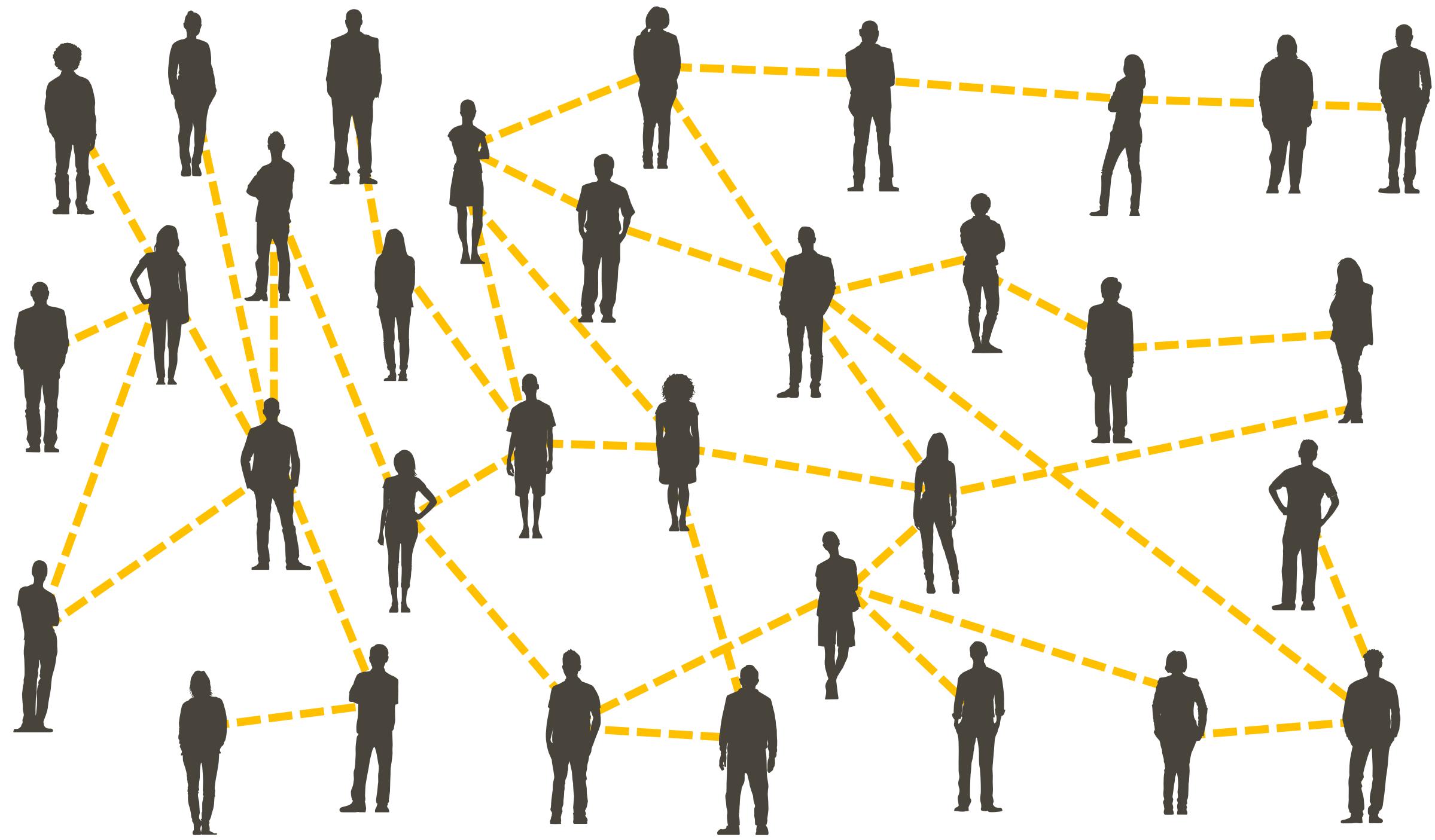
2.

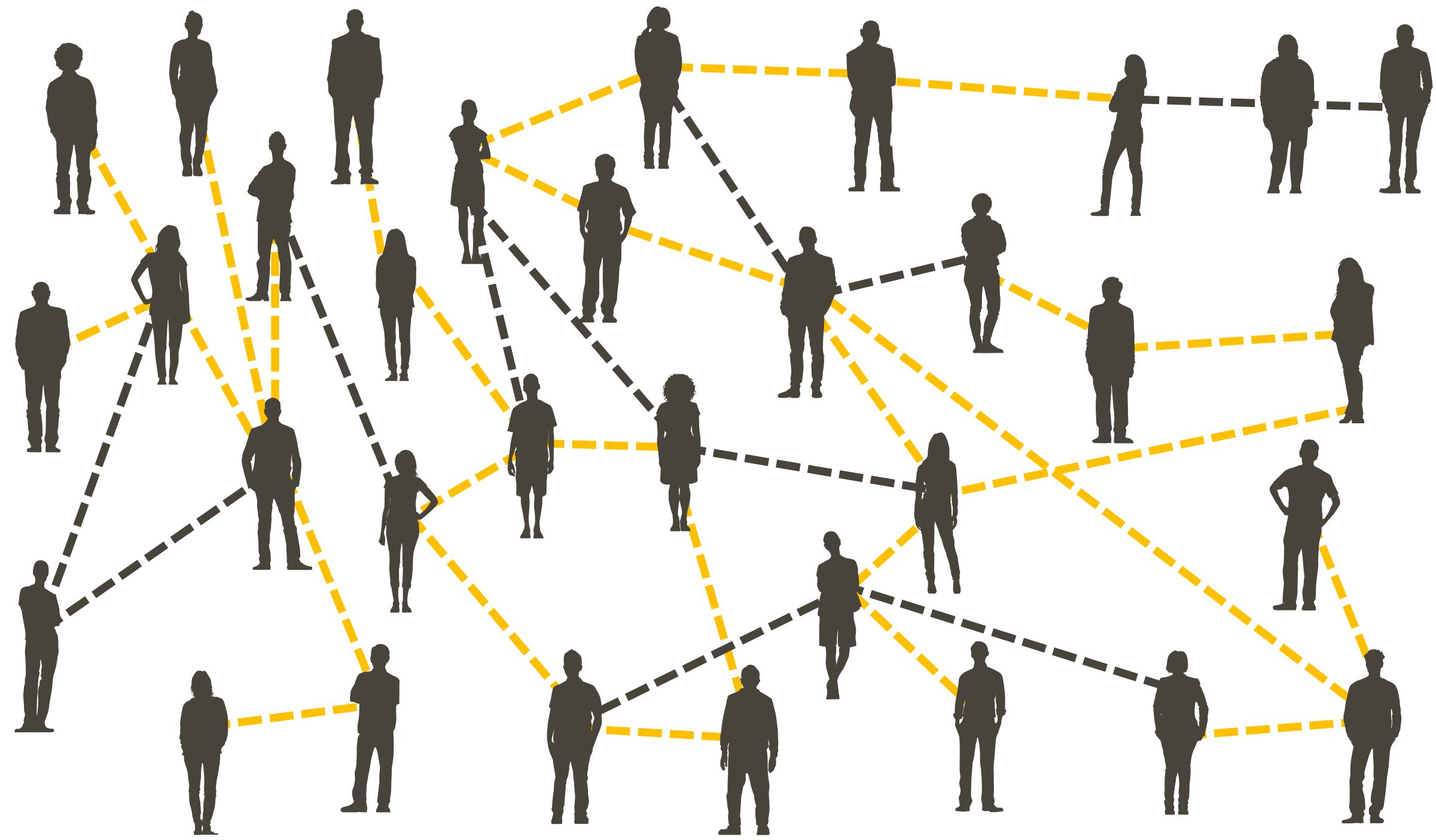
Multiple Entities &
Relationships

3.

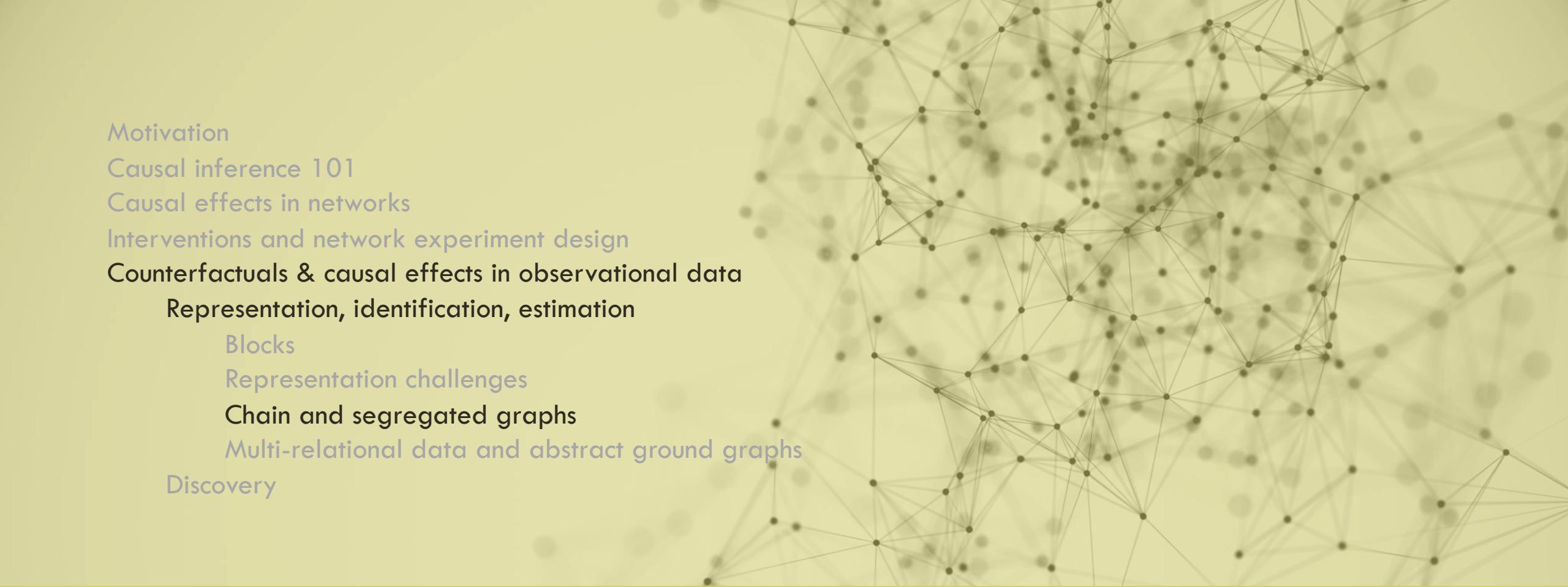
Unobserved /
Partially Observed







| | Directed & Undirected Edges | Multiple Entities and Relationships | Partially Observed Networks |
|-------------------------|-----------------------------|-------------------------------------|-----------------------------|
| Chain Graphs | ✓ | | ✓ in discovery |
| Aggregate Ground Graphs | ✓ | ✓ | |

A large, abstract network graph is positioned at the top right of the slide. It consists of numerous small, dark grey dots representing nodes, connected by thin, light grey lines representing edges. The graph is dense and organic, with no single central node but many local clusters of connections.

Motivation

Causal inference 101

Causal effects in networks

Interventions and network experiment design

Counterfactuals & causal effects in observational data

Representation, identification, estimation

Blocks

Representation challenges

Chain and segregated graphs

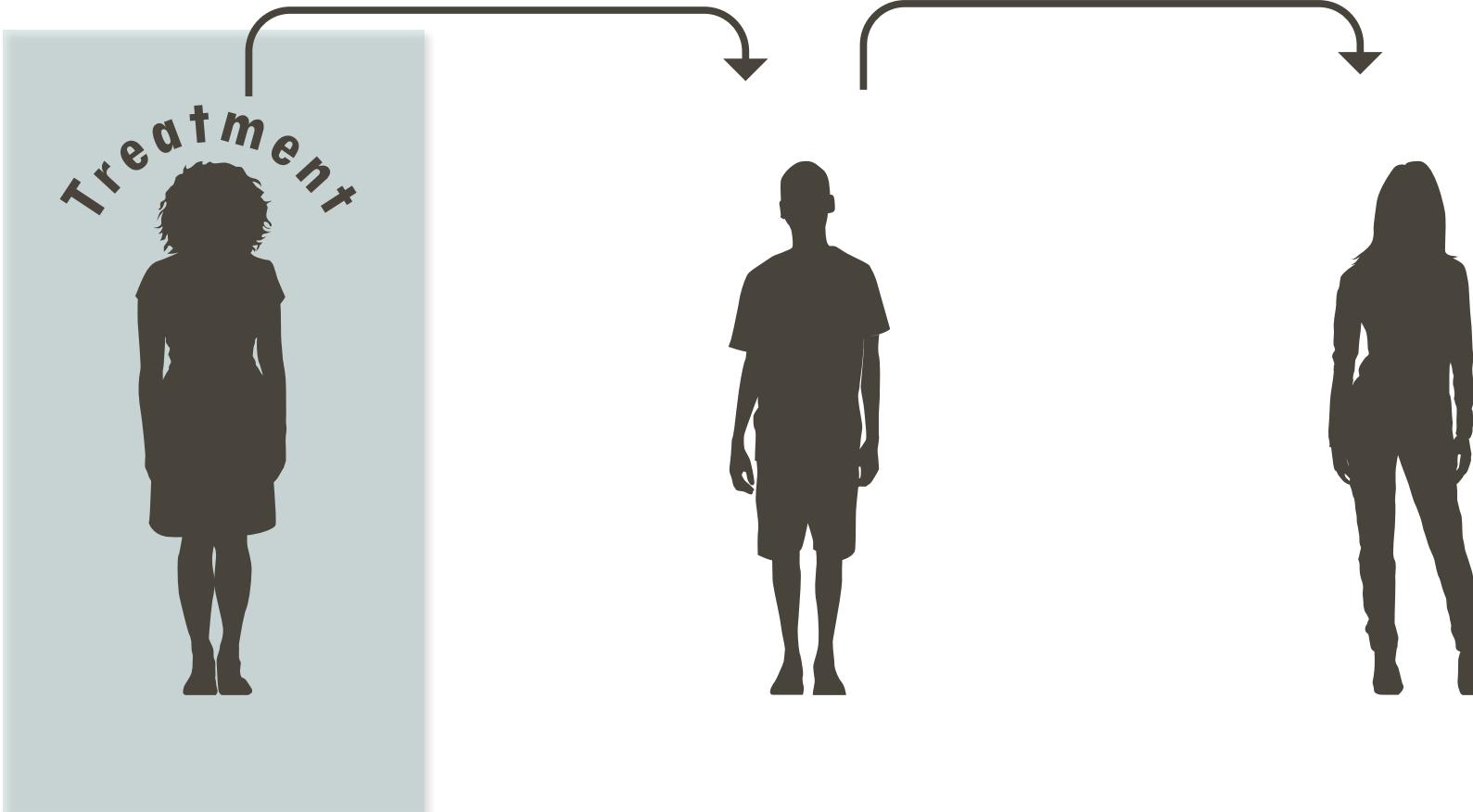
Multi-relational data and abstract ground graphs

Discovery

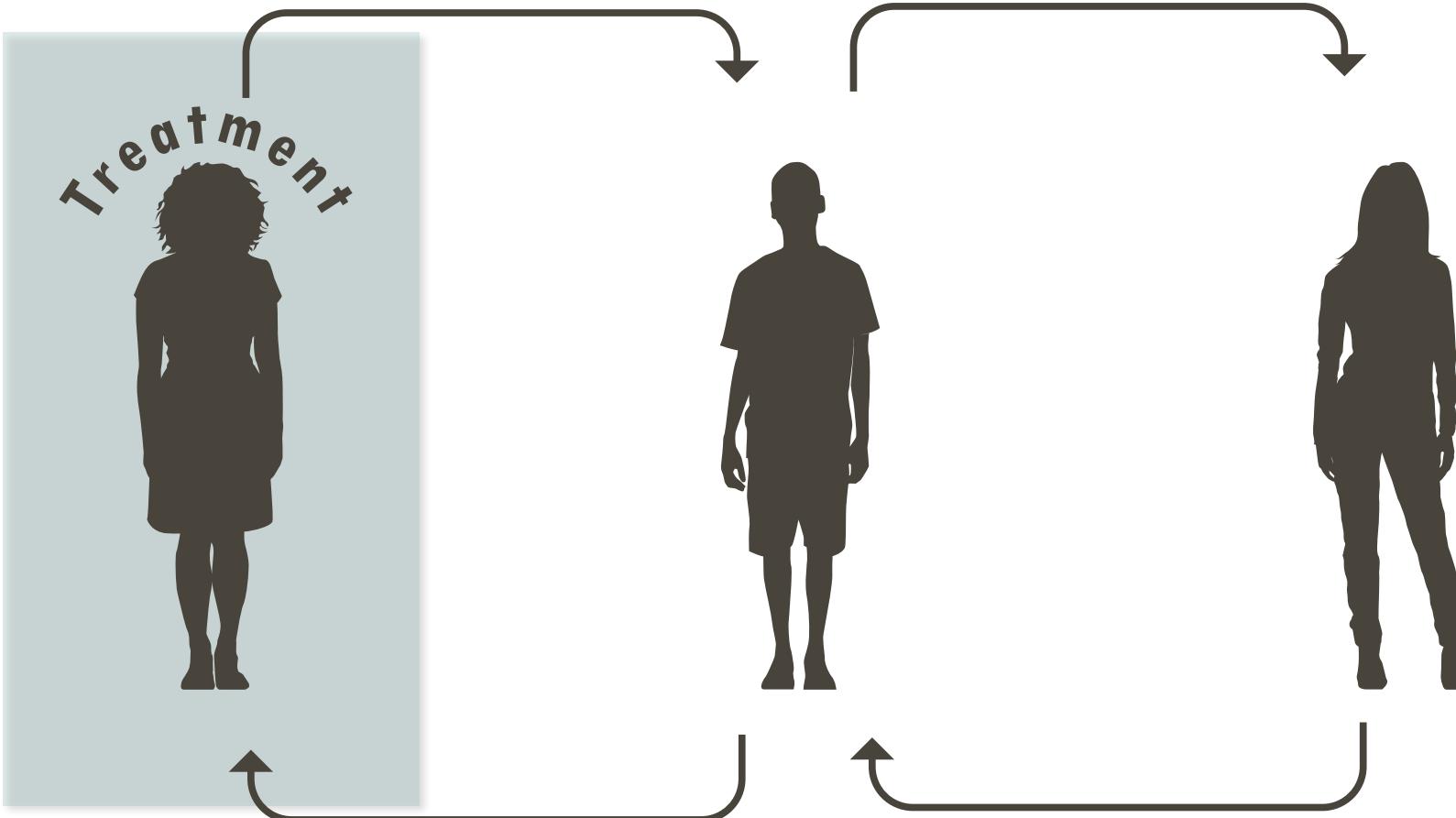
COUNTERFACTUALS & CAUSAL EFFECTS IN OBSERVATIONAL NETWORK DATA

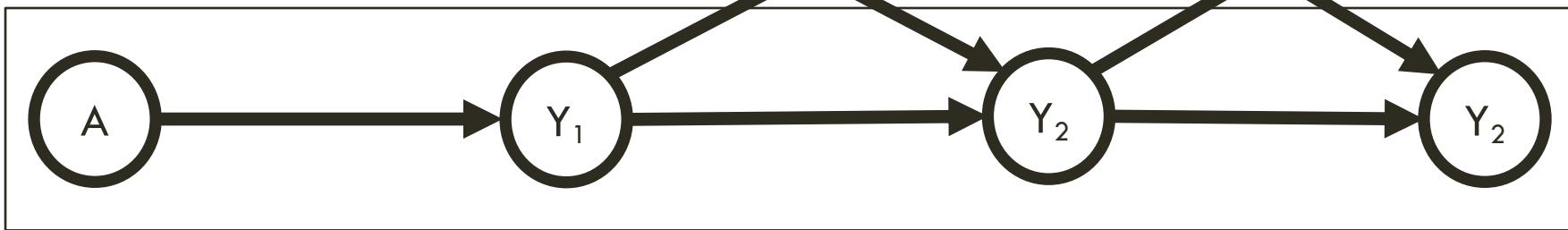
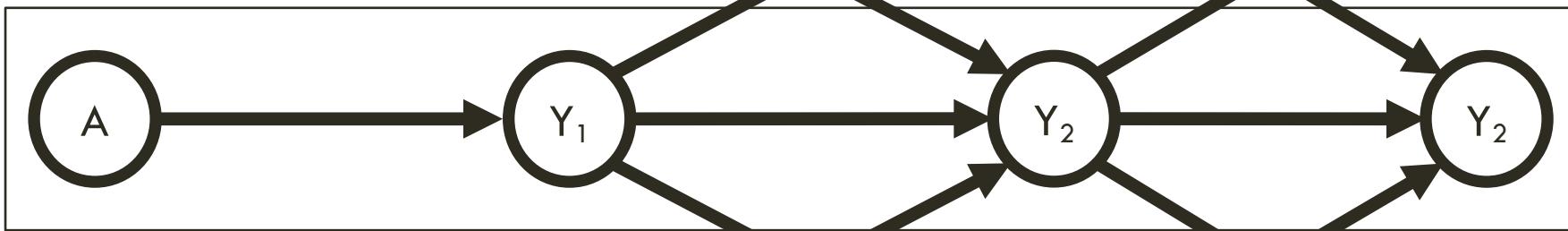
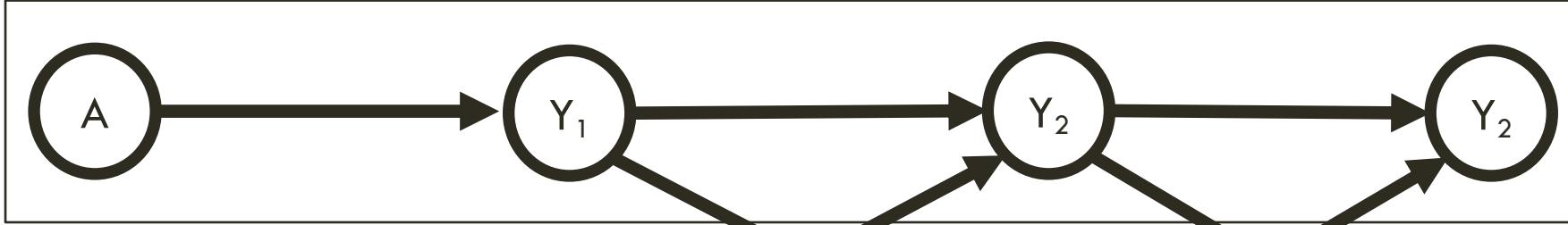
Chain and
Segregated
Graphs

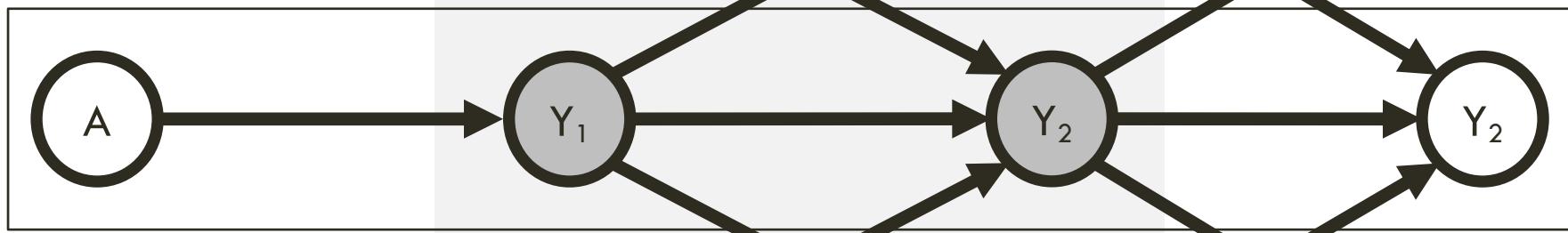
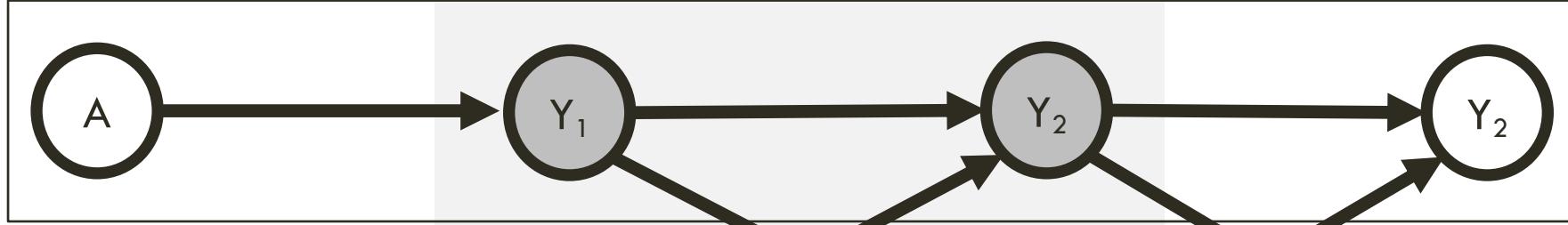
ACYCLICITY

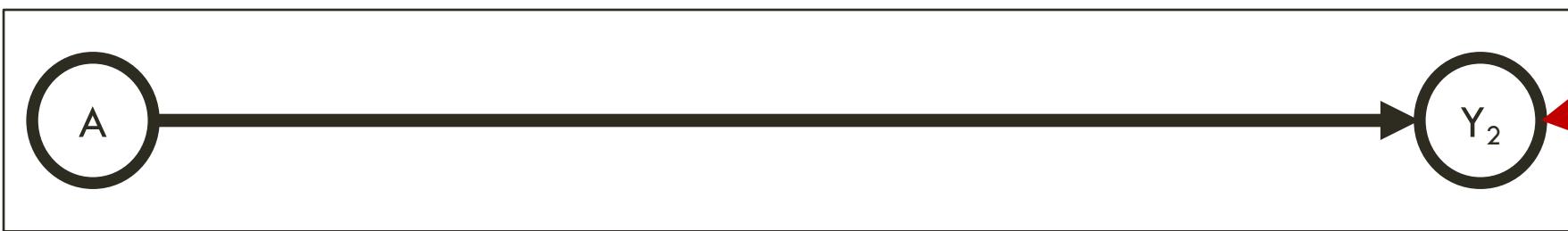
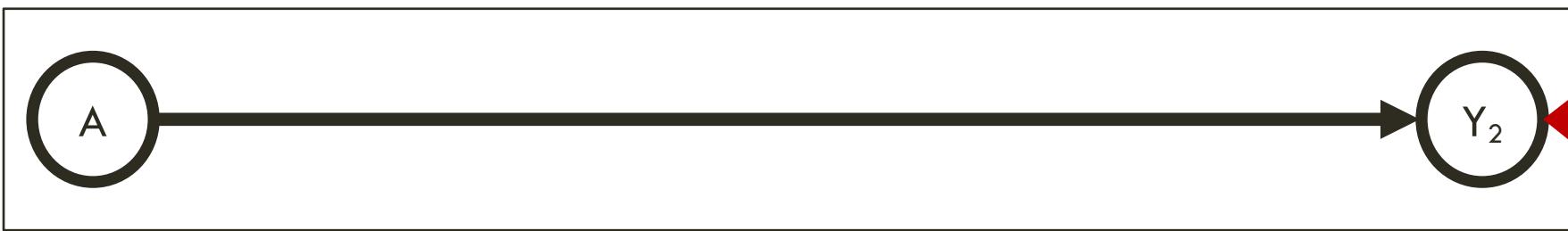
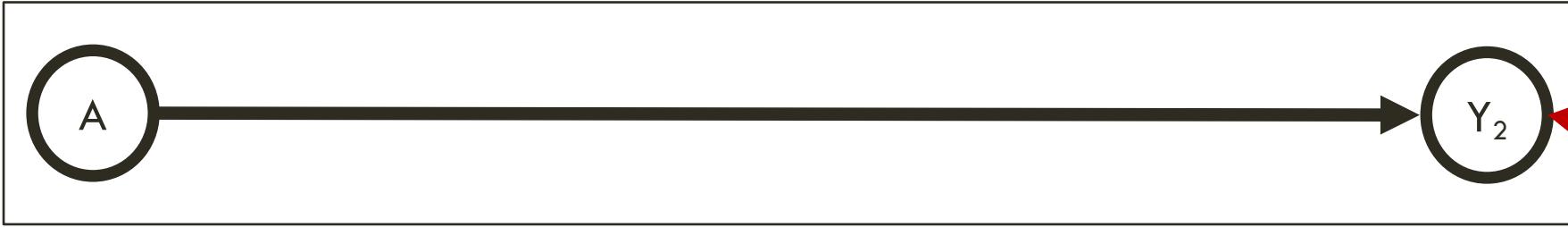


FEEDBACK

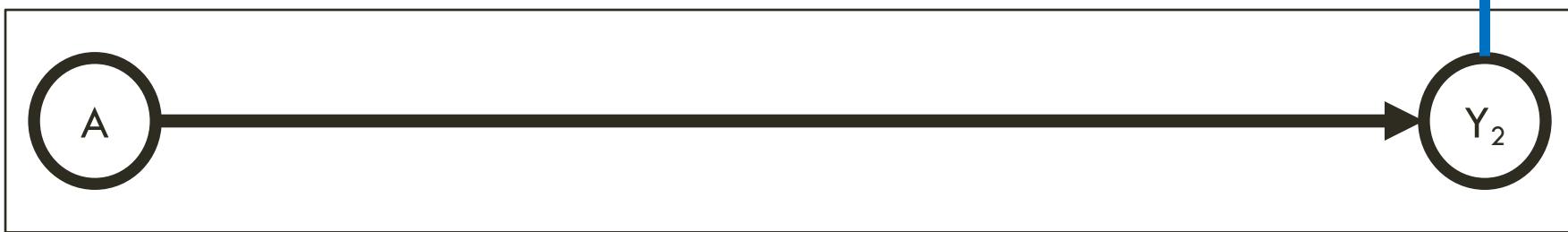
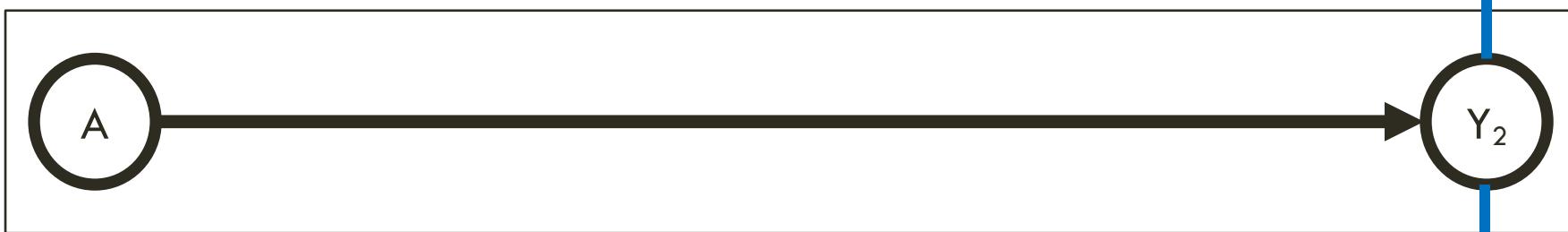
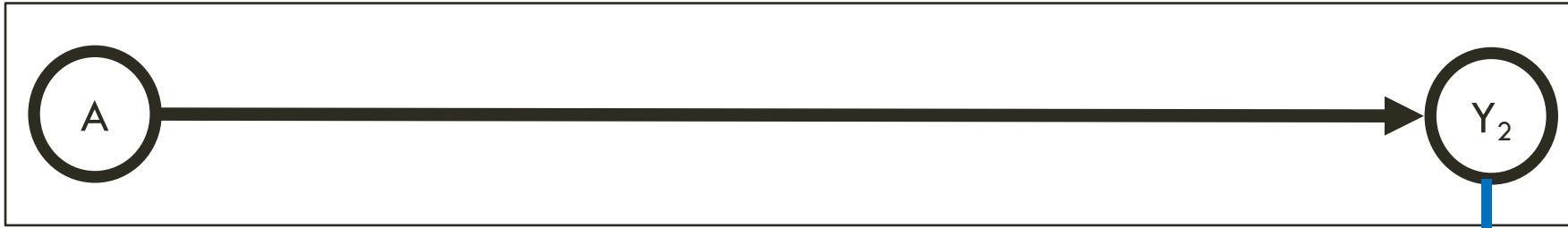








**Latent
edges**



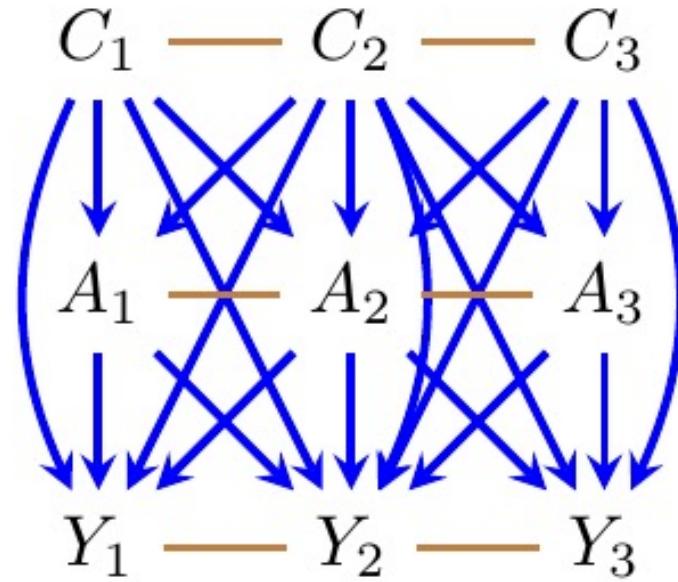
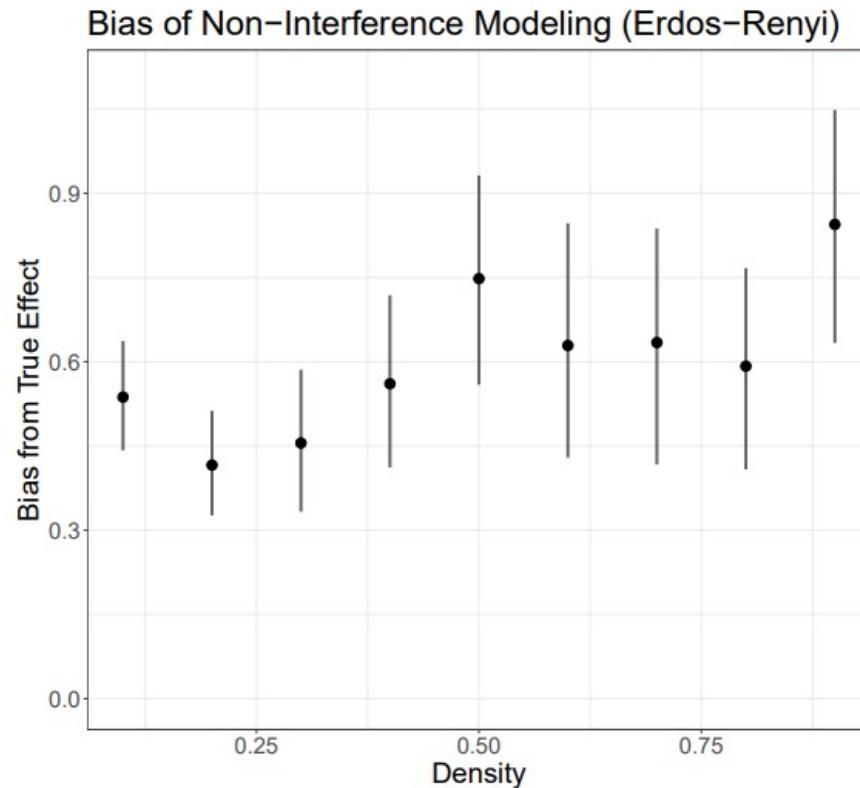
**non causal
undirected
edges**

CHAIN GRAPHS

Ogburn, Shpitser and Lee. *Causal inference, social networks and chain graphs*. JRSSB 2020.

Lauritzen & Richardson. *Chain Graph Models and Their Causal Interpretation*. JRSSB. 2002.

WHY DEPENDENCE-AWARE MODELING?¹



Lee & Ogburn. Network Dependence Can Lead to Spurious Associations and Invalid Inference. Journal of American Statistical Association. 2020.

Sherman, Arbour, and Shpitser. General Identification of Dynamic Treatment Regimes Under Interference. AISTATS. 2020.

CHAIN GRAPHS

Undirected edges represent stable equilibrium
between 2+ edges

‘DAG of blocks’ with 2-level factorization

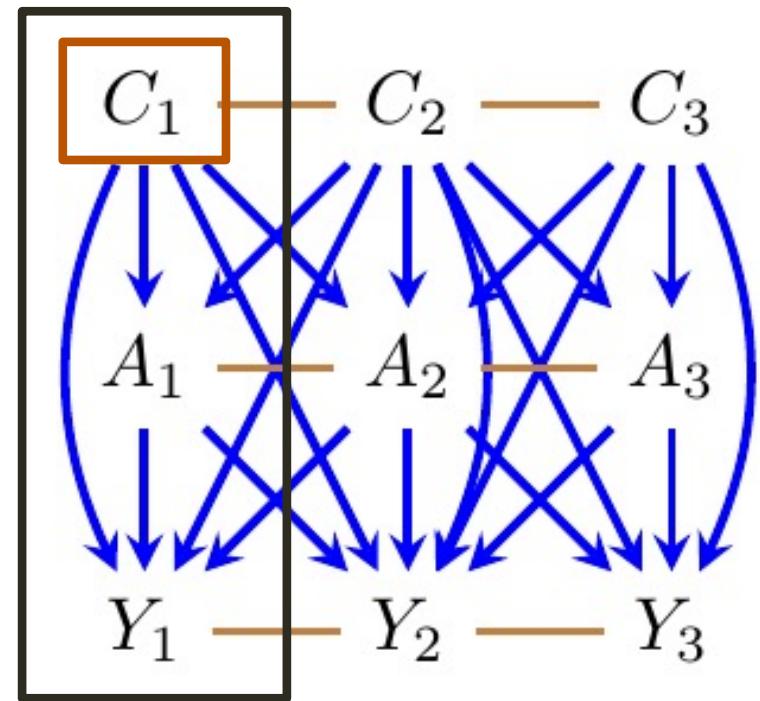
$$V \leftarrow f_V(\mathcal{B}(V), \text{pa}_{\mathcal{G}}(\mathcal{B}(V)), \epsilon_V)$$

$$p(\mathbf{V}) = \prod_{\mathbf{B} \in \mathcal{B}(\mathcal{G})} p(\mathbf{B} | \text{pa}_{\mathcal{G}}(\mathbf{B})) = \prod_{\mathbf{B} \in \mathcal{B}(\mathcal{G})} \frac{1}{Z(\text{pa}_{\mathcal{G}}(\mathbf{B}))} \prod_{\mathbf{C} \in \mathcal{C}^*} \phi_{\mathbf{C}}(\mathbf{C}),$$

DATA GENERATING PROCESS

Procedure 1 CG Data Generating Process

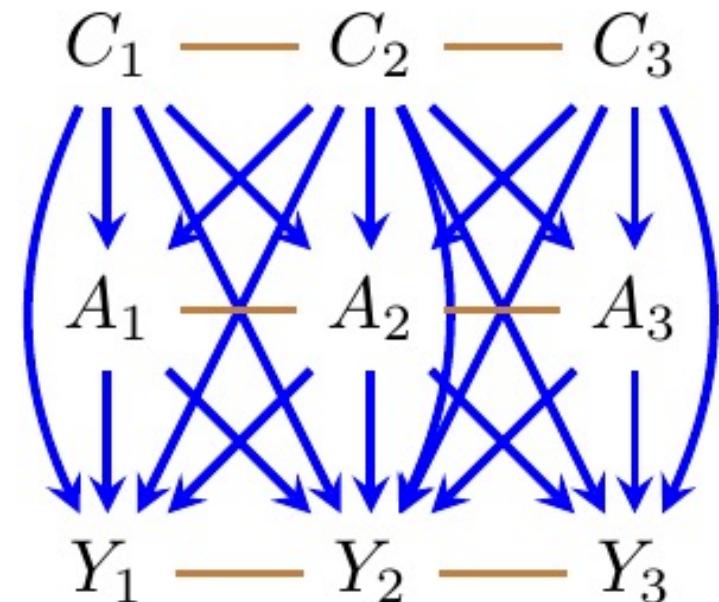
```
1: procedure CG-DGP( $\mathcal{G}, \{f_B : B \in \mathbf{V}\}$ )
2:   for each block  $\mathbf{B}_i \in \mathcal{B}(\mathcal{G})$  do
3:     repeat
4:       for each variable  $B_j \in \mathbf{B}_i$  do
5:          $B_j \leftarrow f_{B_j}(\mathbf{B}_i \setminus B_j, \text{pa}_{\mathcal{G}}(\mathbf{B}_i), \epsilon_{B_j})$ 
6:     until equilibrium
    return  $\mathbf{V}$ 
```



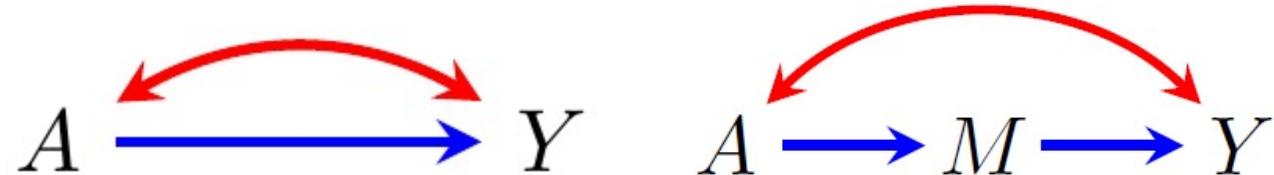
IDENTIFICATION

$$p(\mathbf{V}_C(\mathbf{a})) = \prod_{\mathbf{B} \in \mathcal{B}(\mathcal{G})} p(\mathbf{B} \setminus \mathbf{A} | \text{pa}_{\mathcal{G}}(\mathbf{B}), \mathbf{B} \cap \mathbf{A})|_{\mathbf{A}=\mathbf{a}}$$

$$p(\mathbf{V}_D(\mathbf{a})) = \prod_{V \in \mathbf{V}_D \setminus \mathbf{A}} p(V | \text{pa}_{\mathcal{G}}(V))|_{\mathbf{A}=\mathbf{a}}$$



HANDLING LATENT VARIABLES



Acyclic Directed Mixed Graphs (ADMGs) – latent projection DAGs

- A B means A and B share a common cause

Markov Kernels

- ADMGs factorize as product of densities that relate *distinct* variables¹

$$p(V) = \prod_{D \in \mathcal{D}(\mathcal{G})} q_D(D \mid \text{pa}_{\mathcal{G}}(D)),$$

THE ID ALGORITHM

Fixing

- Truncated factorization provided notion of ‘fixing’ a variable in a DAG
- Corresponding notion in ADMGs – yields conditional ADMG (CADMG)
- Reframe Pearl’s ‘graph surgery’ via fixing operator



HANDLING LATENTS IN CHAIN GRAPHS

Segregation Property

- Do not permit  and – edge at the same node
 - No known likelihood to support violations

Block-safeness

- Enforces segregation property in underlying chain graph
- Block-safe CGs can undergo latent projection operation to yield segregated graph

HANDLING LATENTS IN CHAIN GRAPHS

Factorization-Blocks and districts

Conditional Chain Graph

$$q(\mathbf{B}^* | \text{pa}_{\mathcal{G}}^s(\mathbf{B}^*)) = \prod_{\mathbf{B} \in \mathcal{B}^{nt}(\mathcal{G})} p(\mathbf{B} | \text{pa}_{\mathcal{G}}(\mathbf{B}))$$

CADMG

$$q(\mathbf{D}^* | \text{pa}_{\mathcal{G}}^s(\mathbf{D}^*)) = \frac{p(\mathbf{V})}{q(\mathbf{B}^* | \text{pa}_{\mathcal{G}}^s(\mathbf{B}^*))}$$

THE SEGREGATED GRAPH ID ALGORITHM

Theorem 2 Assume $\mathcal{G}(\mathbf{V} \cup \mathbf{H})$ is a causal CG, where \mathbf{H} is block-safe. Fix disjoint subsets \mathbf{Y}, \mathbf{A} of \mathbf{V} . Let $\mathbf{Y}^* = \text{ant}_{\mathcal{G}(\mathbf{V})_{\mathbf{V} \setminus \mathbf{A}}} \mathbf{Y}$. Then $p(\mathbf{Y}|do(\mathbf{a}))$ is identified from $p(\mathbf{V})$ if and only if every element in $\mathcal{D}(\tilde{\mathcal{G}}^d)$ is reachable in \mathcal{G}^d , where $\tilde{\mathcal{G}}^d$ is the induced CADMG of $\mathcal{G}(\mathbf{V})_{\mathbf{Y}^*}$.

Moreover, if $p(\mathbf{Y}|do(\mathbf{a}))$ is identified, it is equal to

$$\sum_{\mathbf{Y}^* \setminus \mathbf{Y}} \left[\prod_{\mathbf{D} \in \mathcal{D}(\tilde{\mathcal{G}}^d)} \phi_{\mathbf{D}^* \setminus \mathbf{D}}(q(\mathbf{D}^* | \text{pa}_{\mathcal{G}(\mathbf{V})}(\mathbf{D}^*)); \mathcal{G}^d) \right] \left[\prod_{\mathbf{B} \in \mathcal{B}(\tilde{\mathcal{G}}^b)} p(\mathbf{B} \setminus \mathbf{A} | \text{pa}_{\mathcal{G}(\mathbf{V})_{\mathbf{Y}^*}}(\mathbf{B}), \mathbf{B} \cap \mathbf{A}) \right]_{\mathbf{A}=\mathbf{a}}$$

where $q(\mathbf{D}^* | \text{pa}_{\mathcal{G}(\mathbf{V})}(\mathbf{D}^*)) = p(\mathbf{V}) / (\prod_{\mathbf{B} \in \mathcal{B}^{nt}(\mathcal{G}(\mathbf{V}))} p(\mathbf{B} | \text{pa}_{\mathcal{G}(\mathbf{V})}(\mathbf{B})))$, and $\tilde{\mathcal{G}}^b$ is the induced CCG of $\mathcal{G}(\mathbf{V})_{\mathbf{Y}^*}$.

$$p(\mathbf{Y}|do(\mathbf{a})) = \sum_{\mathbf{Y}^* \setminus \mathbf{Y}} \prod_{\mathbf{D} \in \mathcal{D}(\mathcal{G}(\mathbf{V})_{\mathbf{Y}^*})} \phi_{\mathbf{V} \setminus \mathbf{D}}(p(\mathbf{V}); \mathcal{G}(\mathbf{V}))|_{\mathbf{A}=\mathbf{a}}.$$

$$p(\mathbf{V}_C(\mathbf{a})) = \prod_{\mathbf{B} \in \mathcal{B}(\mathcal{G})} p(\mathbf{B} \setminus \mathbf{A} | \text{pa}_{\mathcal{G}}(\mathbf{B}), \mathbf{B} \cap \mathbf{A})|_{\mathbf{A}=\mathbf{a}}$$

EASY

Modeling feedback

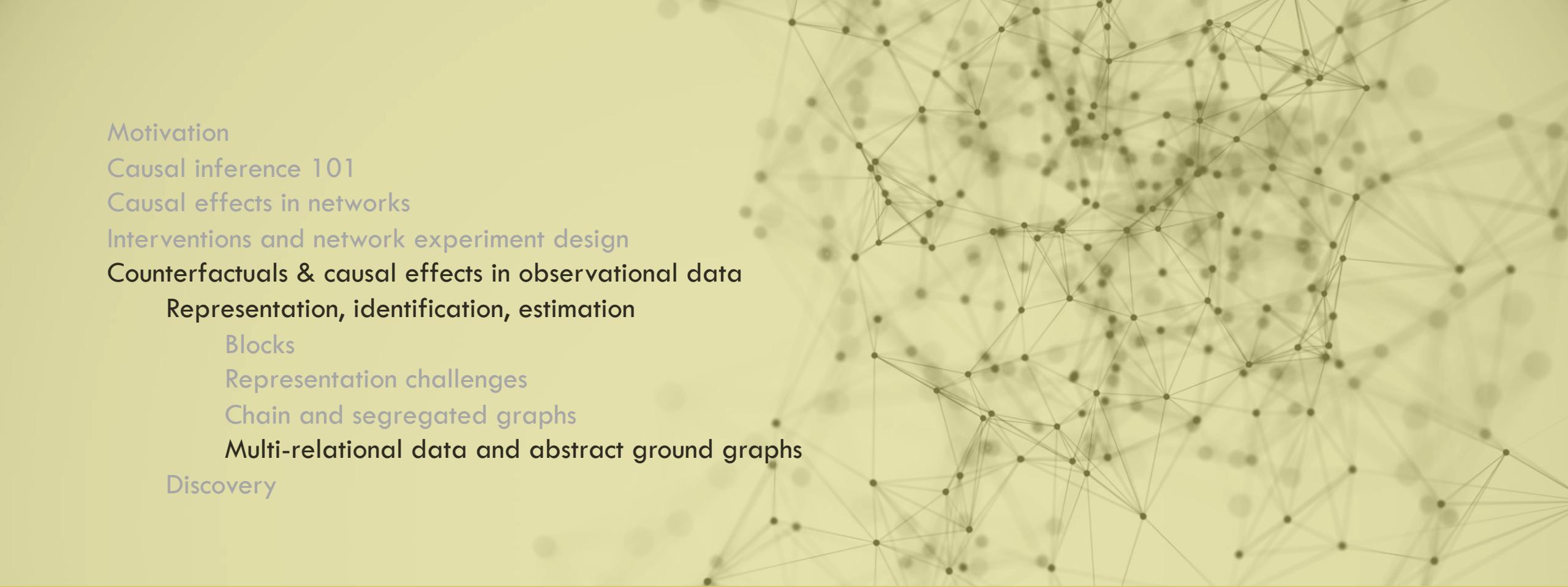
Modeling latent
variables

Identification

HARD

Expensive—Gibbs sampling
is required for inference

Difficult to represent
interventions on distributions

A complex network graph composed of numerous small, semi-transparent nodes and a dense web of thin, light-colored lines representing connections between them, set against a yellow gradient background.

Motivation

Causal inference 101

Causal effects in networks

Interventions and network experiment design

Counterfactuals & causal effects in observational data

Representation, identification, estimation

Blocks

Representation challenges

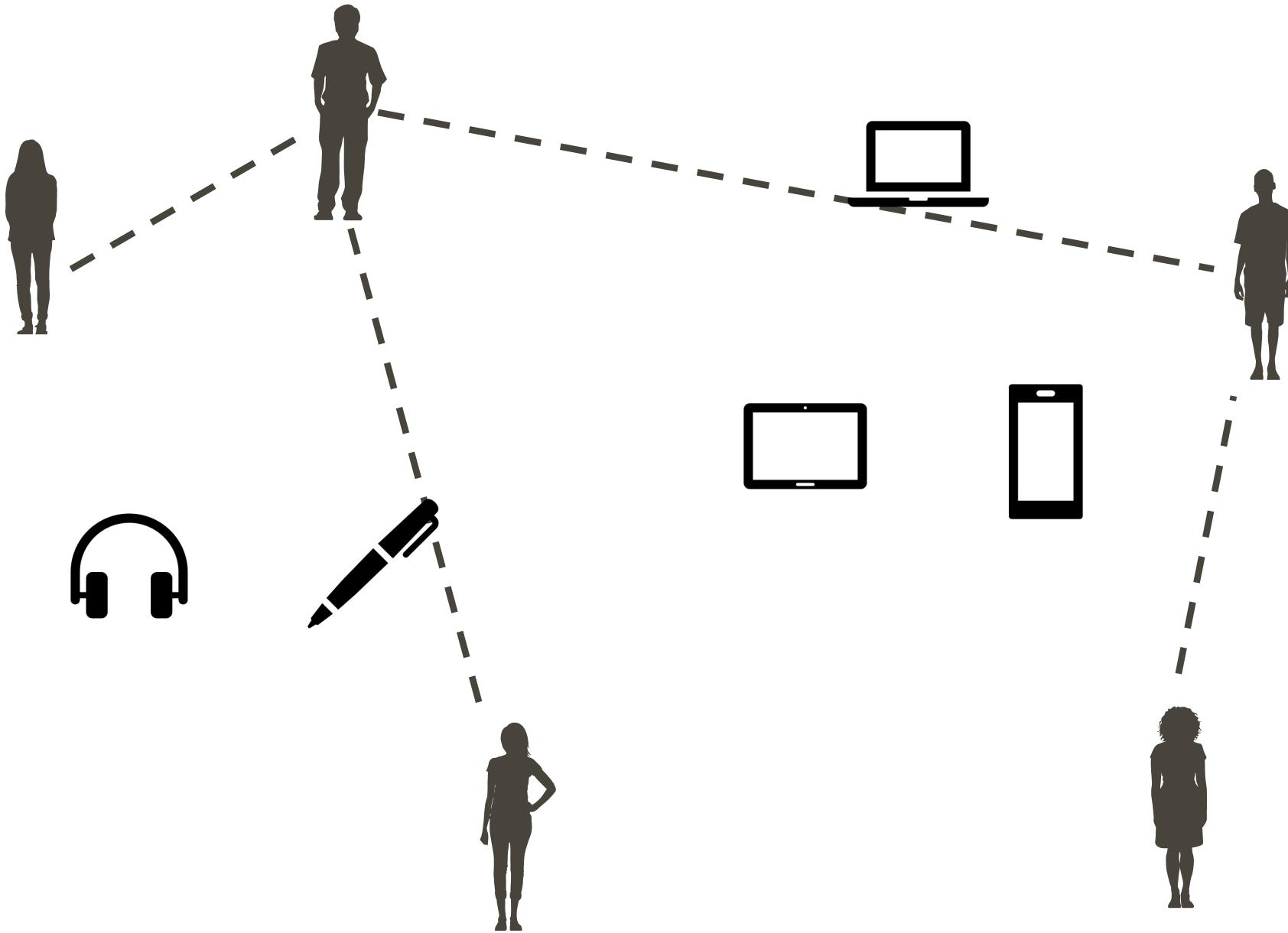
Chain and segregated graphs

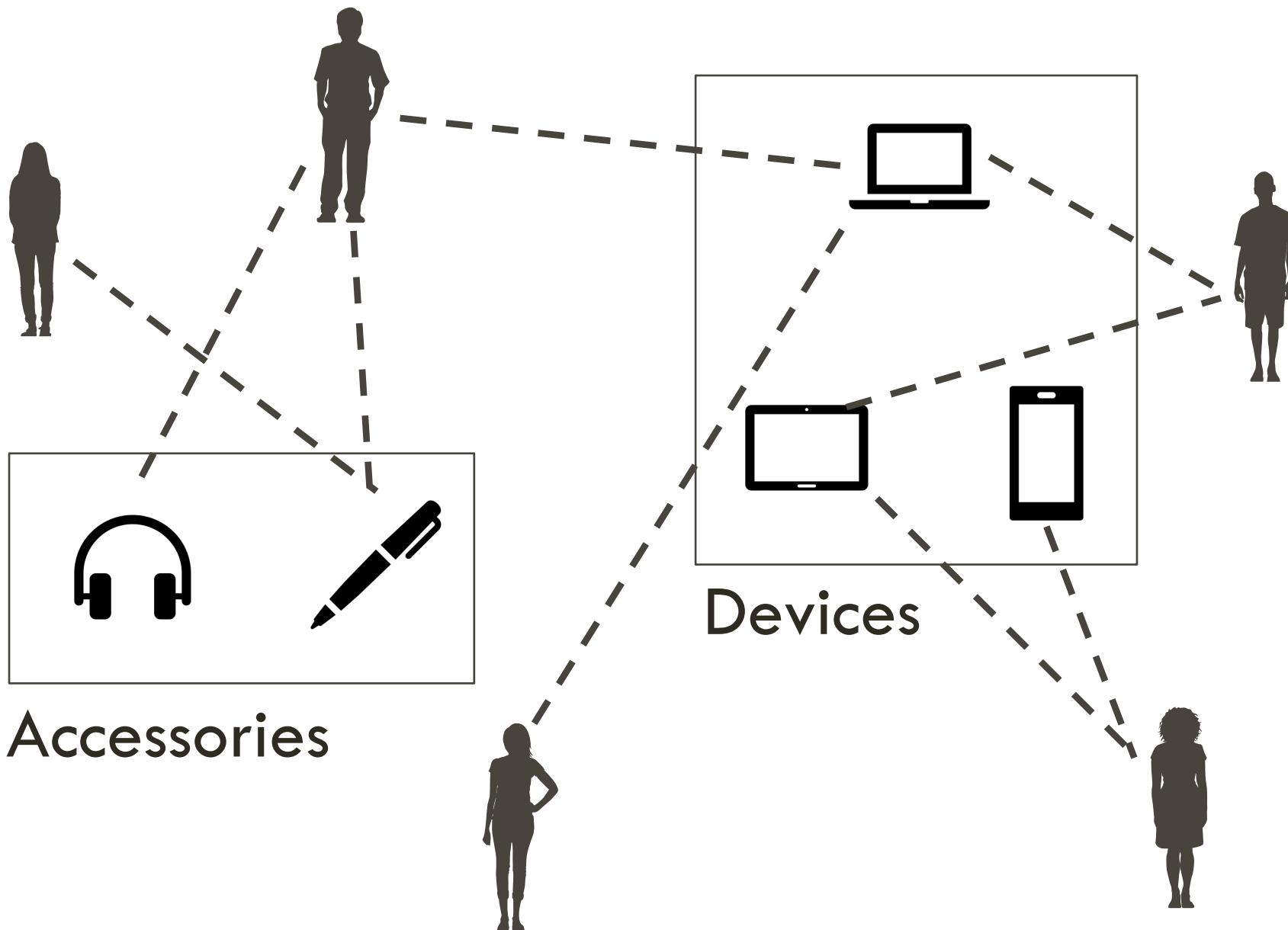
Multi-relational data and abstract ground graphs

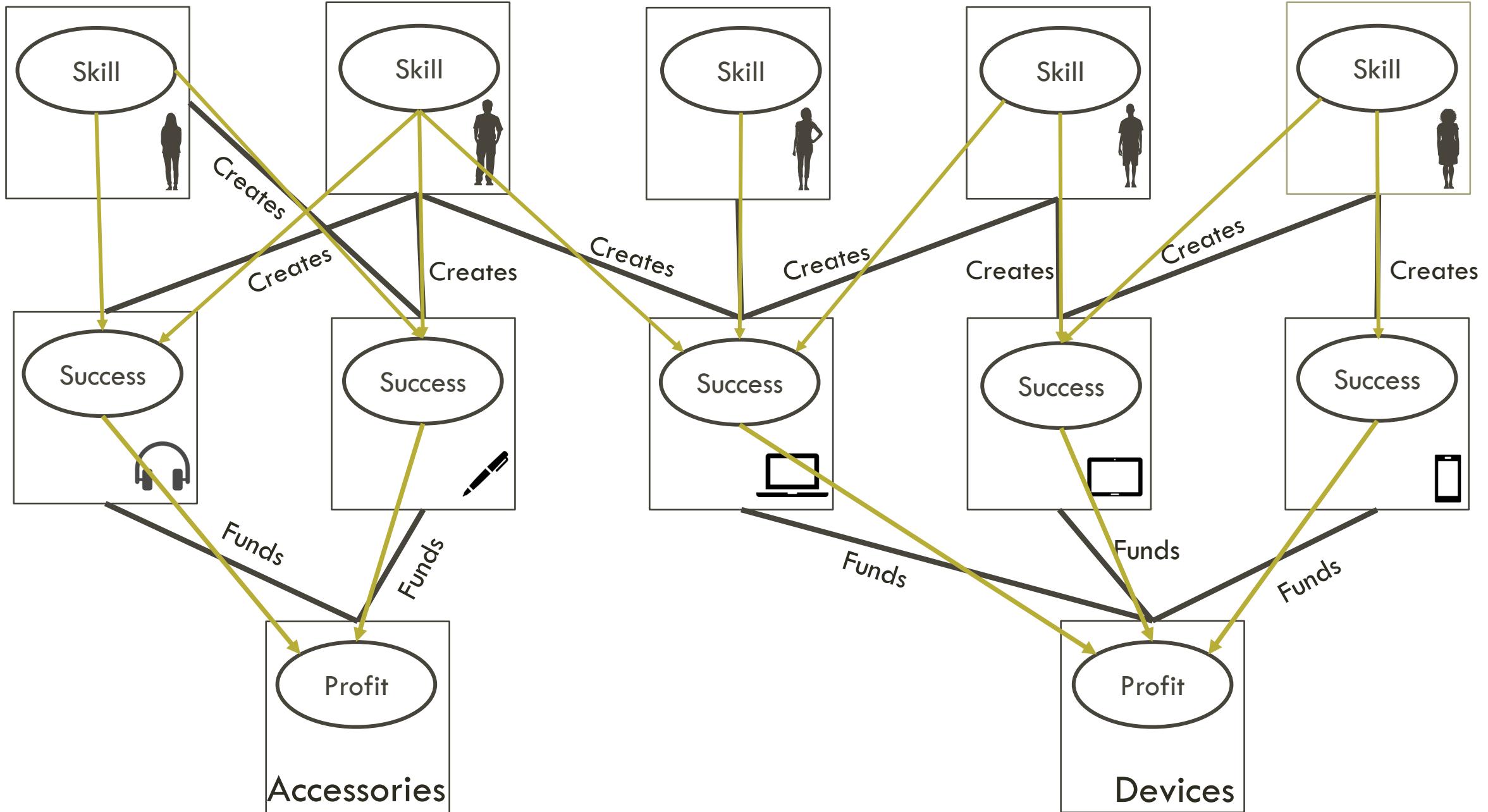
Discovery

COUNTERFACTUALS & CAUSAL EFFECTS IN OBSERVATIONAL NETWORK DATA

Multi-relational
data and abstract
ground graphs



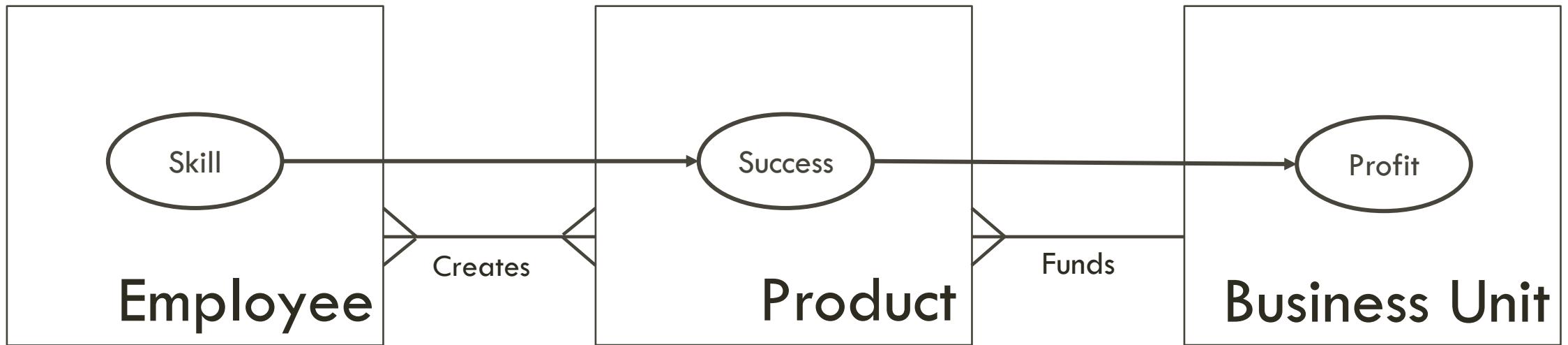




TEMPLATES

Assume shared marginal and conditional distributions

Allows a general model which represents relationships and dependencies more abstractly



OVERVIEW OF TEMPLATE MODELS



OVERVIEW OF TEMPLATE MODELS

Schema



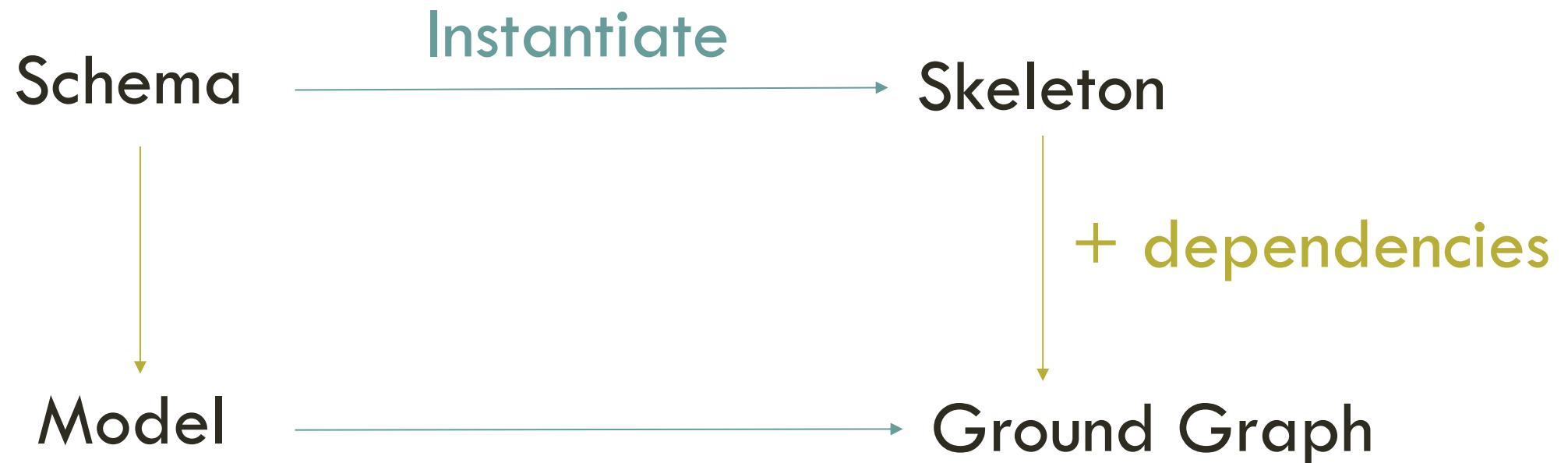
Model

Skeleton

+ dependencies

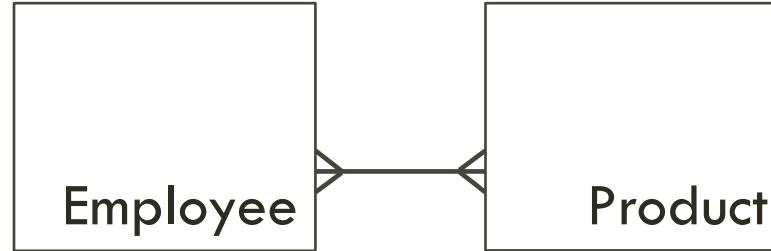
Ground Graph

OVERVIEW OF TEMPLATE MODELS



Products an Employee
works on

[Employee, Product]



Business units an
Employee works in

[Employee, Product, Business Unit]



An employee's
coworkers

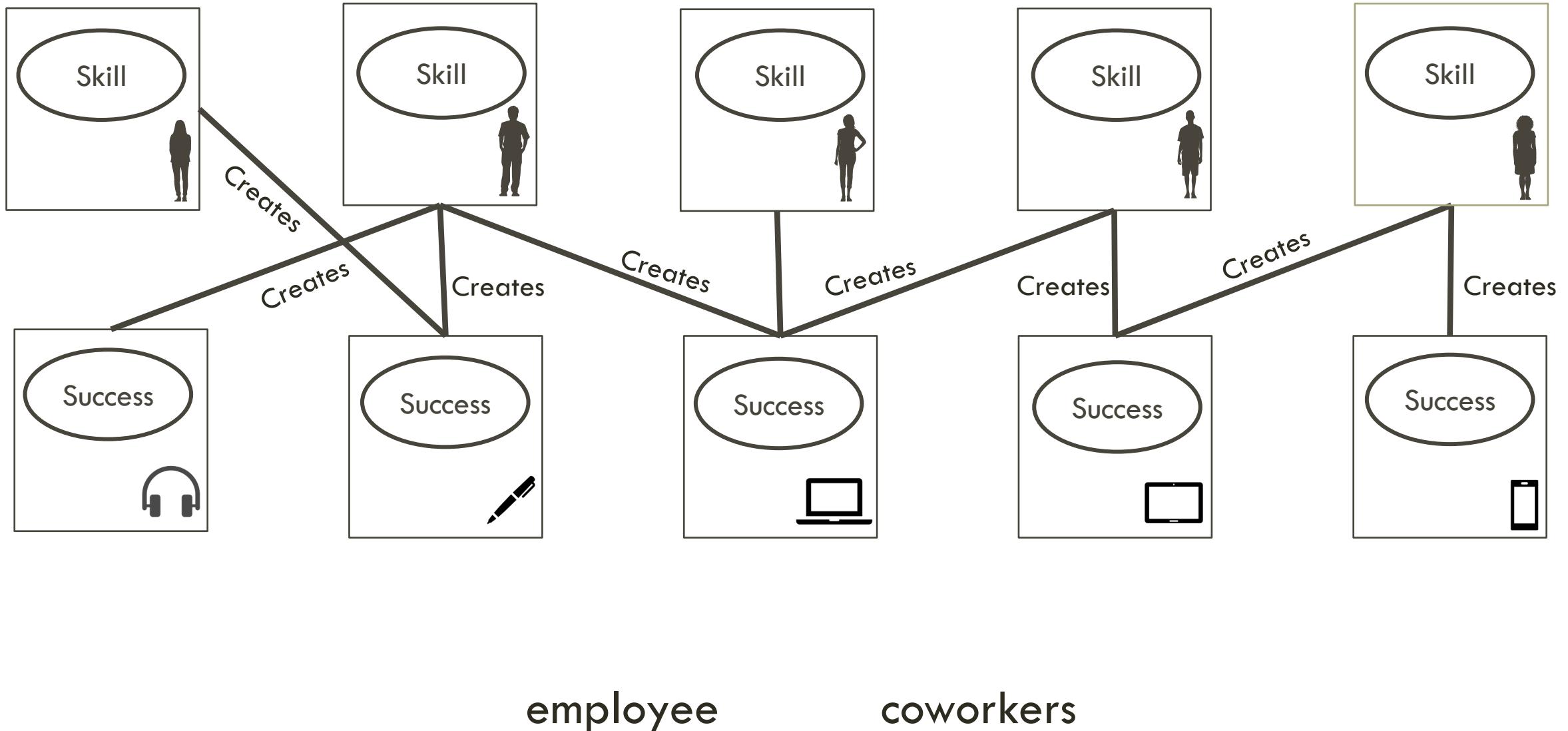
[Employee, Product, Employee]



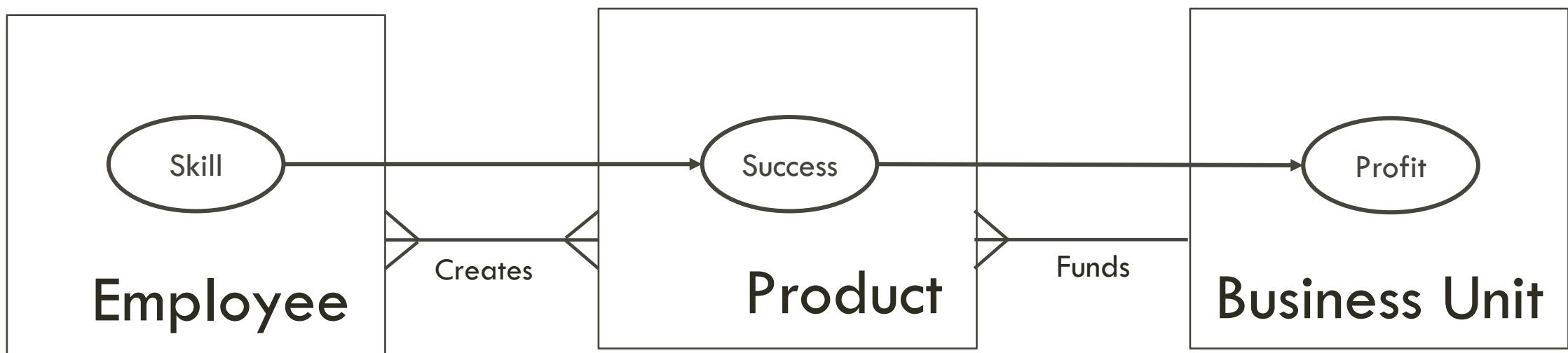
RELATIONAL PATHS

An employee's coworkers

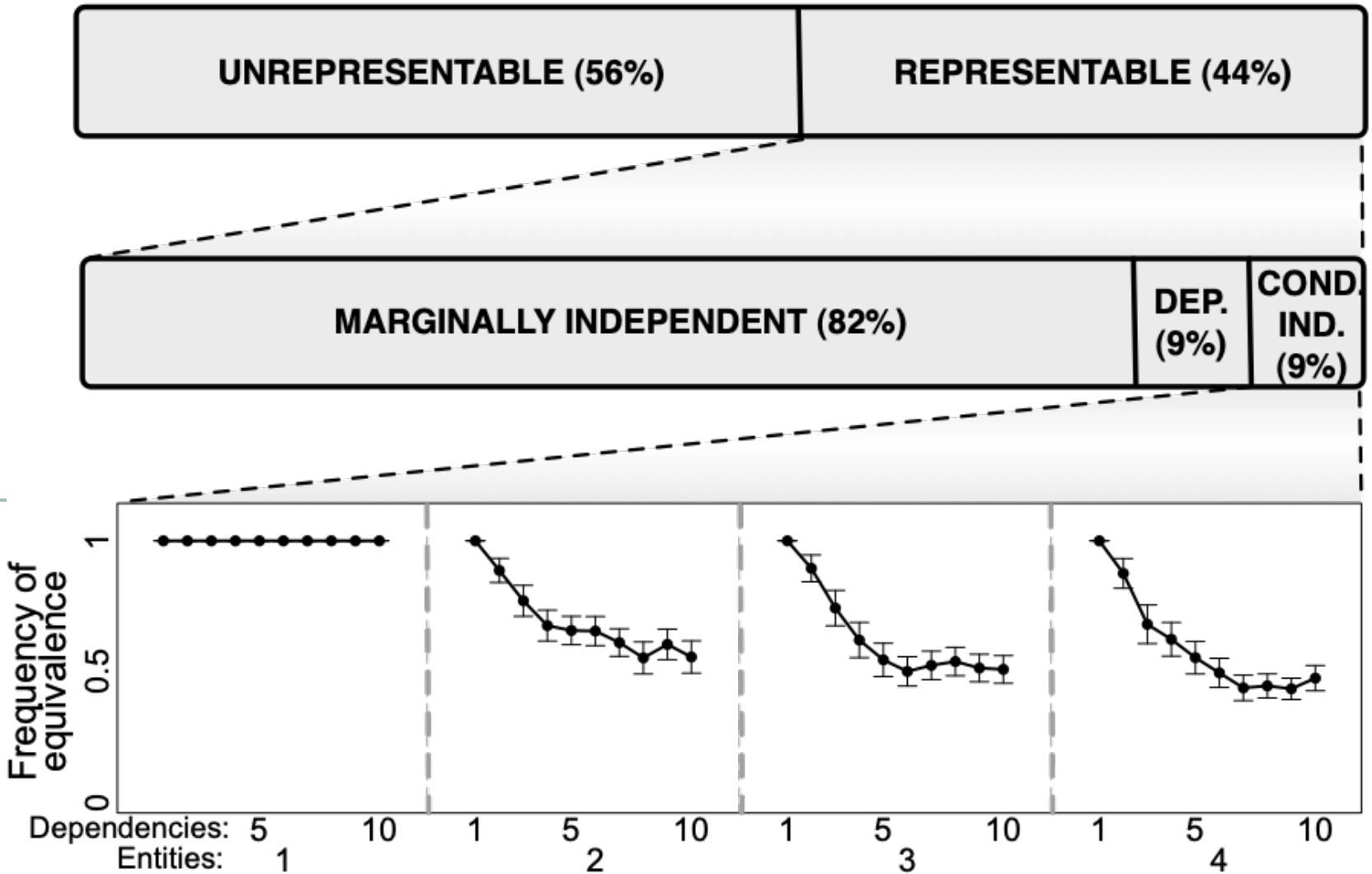
[Employee, Product, Employee]

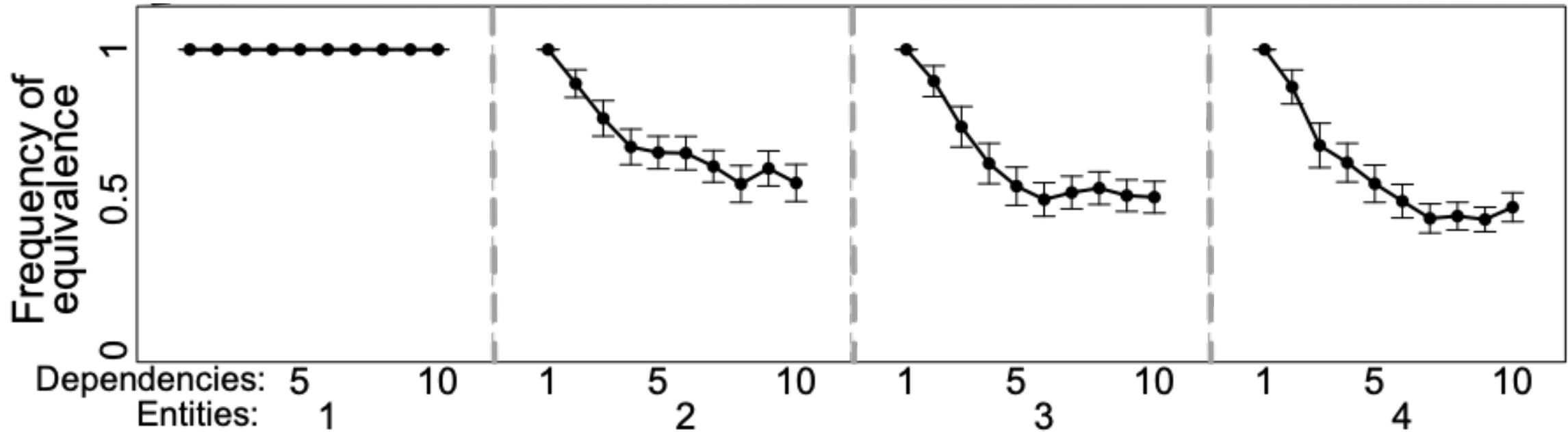


D-SEPARATION ON TEMPLATES

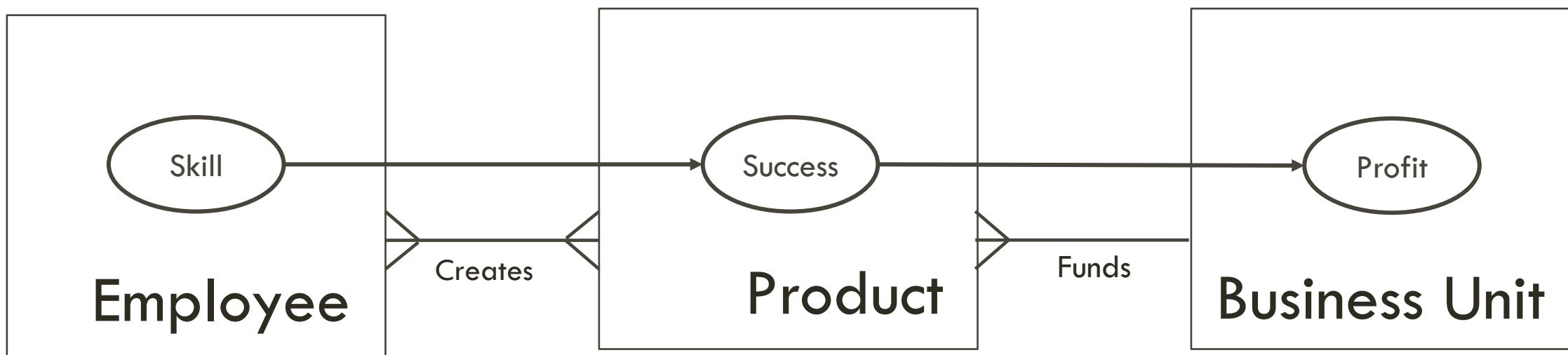


D-separation on templates often fails

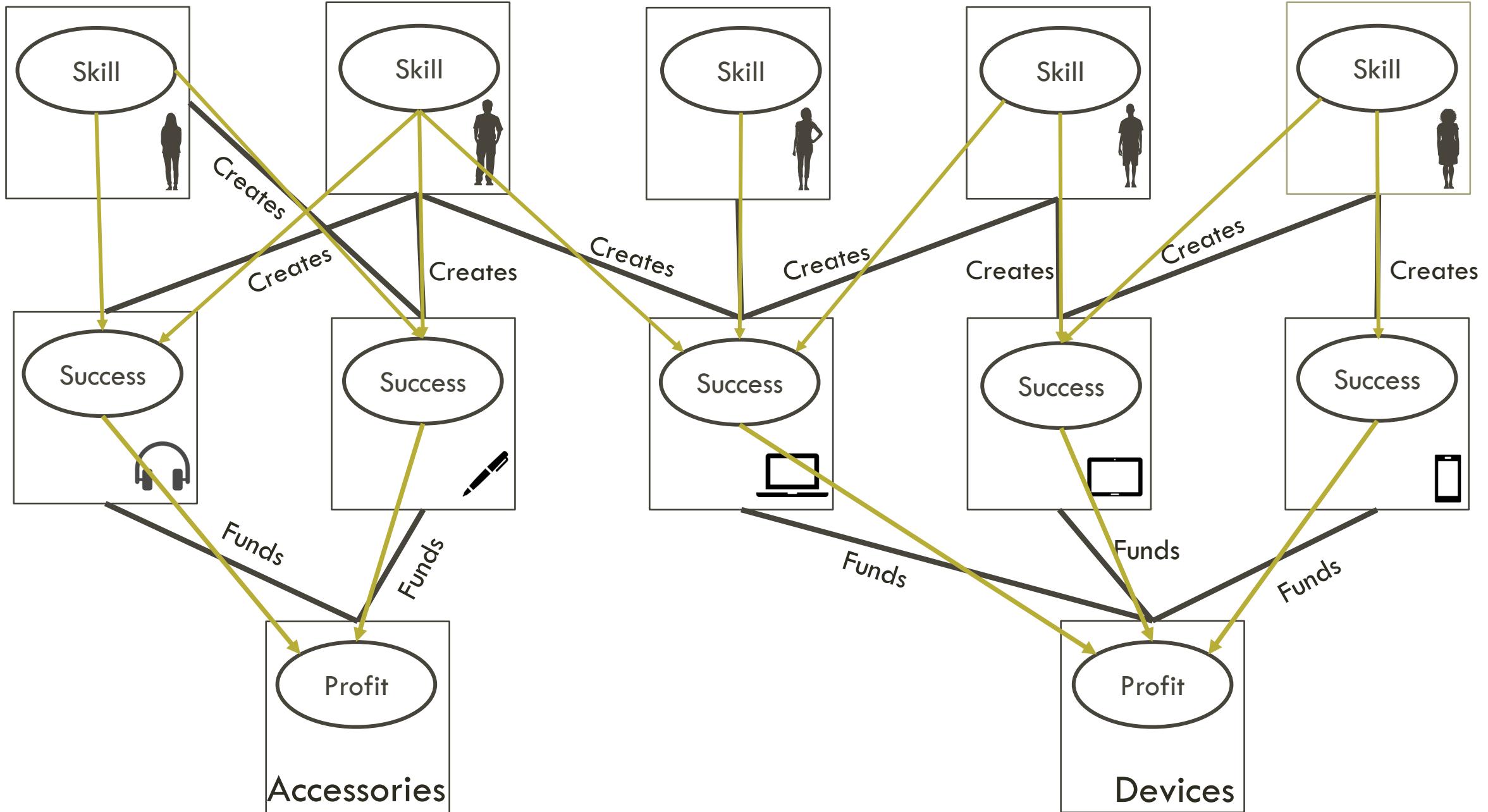




D-SEPARATION ON TEMPLATES



$[\text{Employee}, \text{Product}, \text{Employee}].\text{Skill} \amalg [\text{Employee}].\text{Skill}$

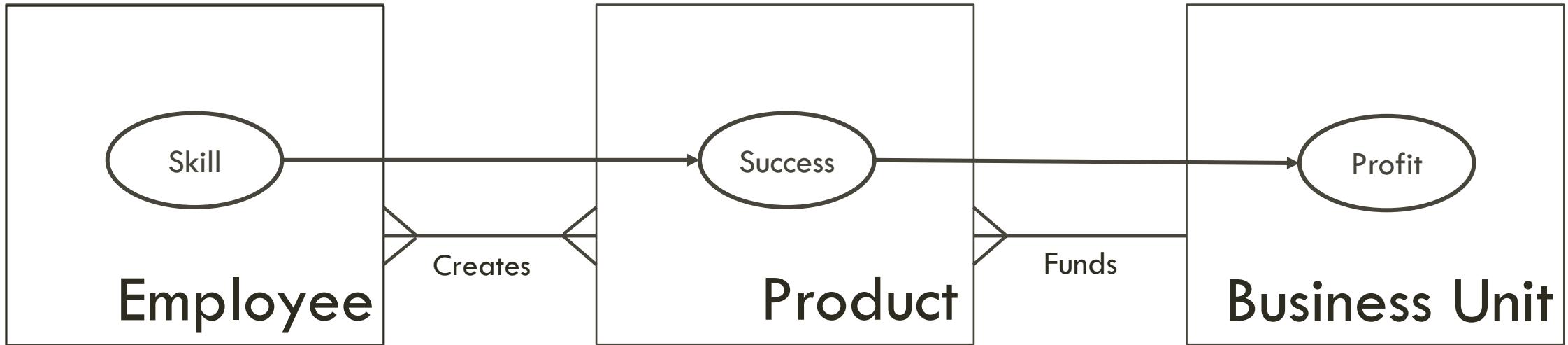


HOW DO WE FIND AN
INTERMEDIATE
REPRESENTATION THAT
ALLOWS FOR D-SEPARATION?

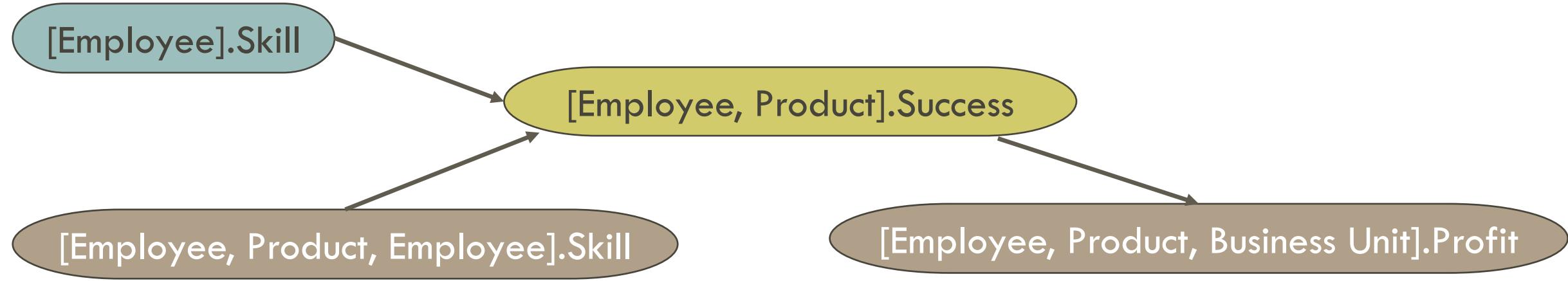
ABSTRACT GROUND GRAPHS

Lifted representation
with **d-separation** semantics

EMPLOYEE PERSPECTIVE | Hop threshold = 2



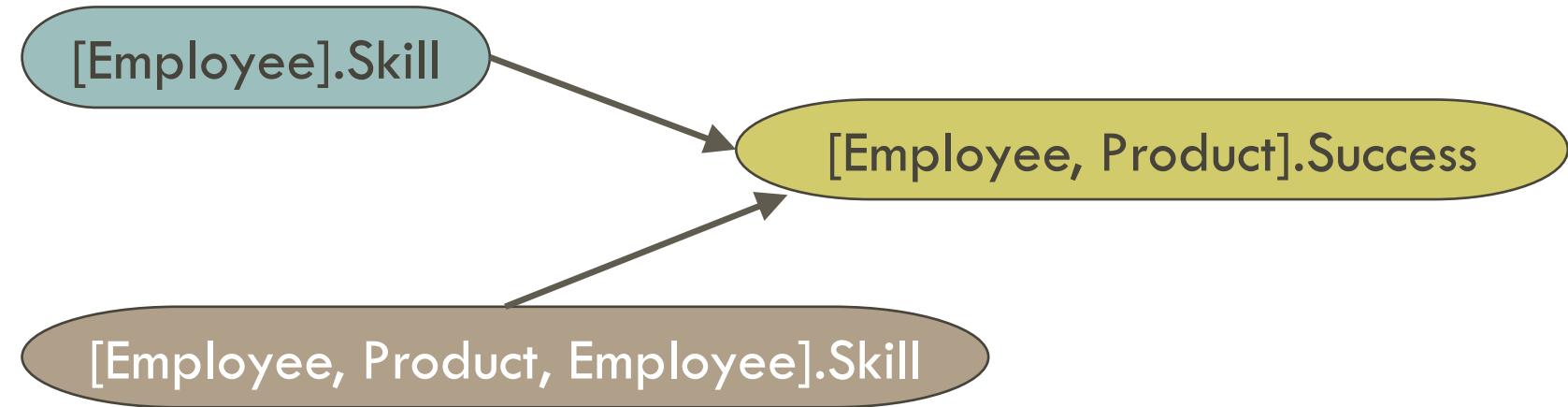
AGGS INHERIT THE PROPERTIES OF BAYES NETS



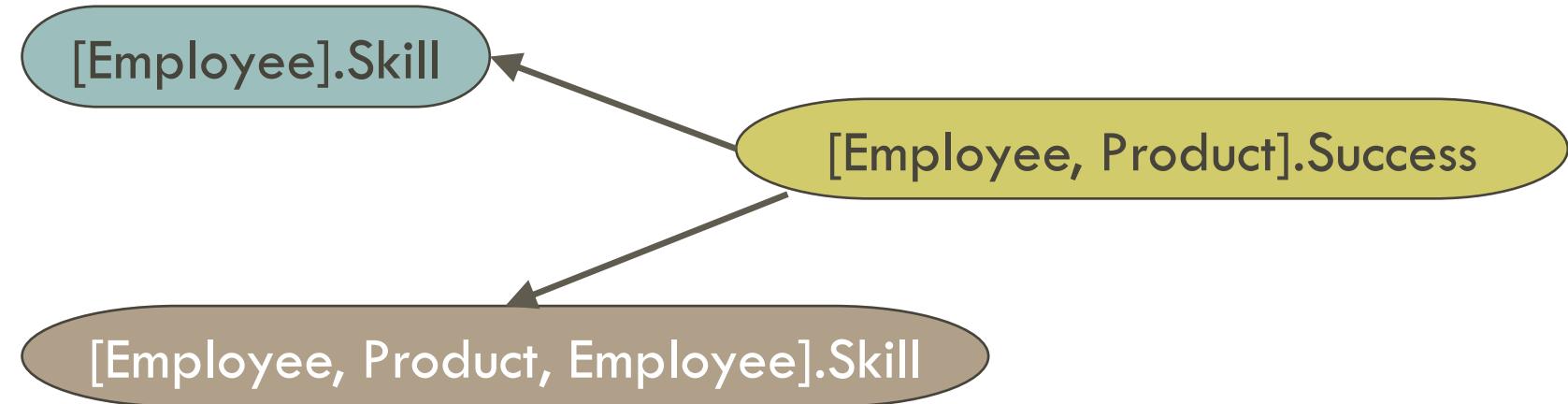
d-separation and identification theory from Bayesian networks can be applied directly.

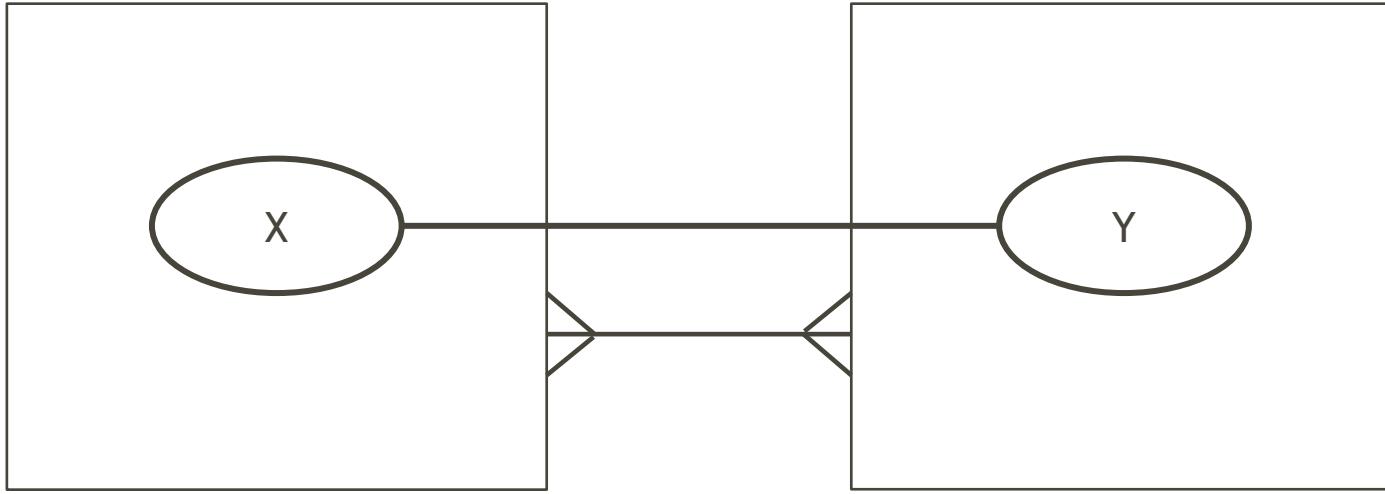
RELATIONAL BIVARIATE ORIENTATION

Unshielded collider



Non-collider

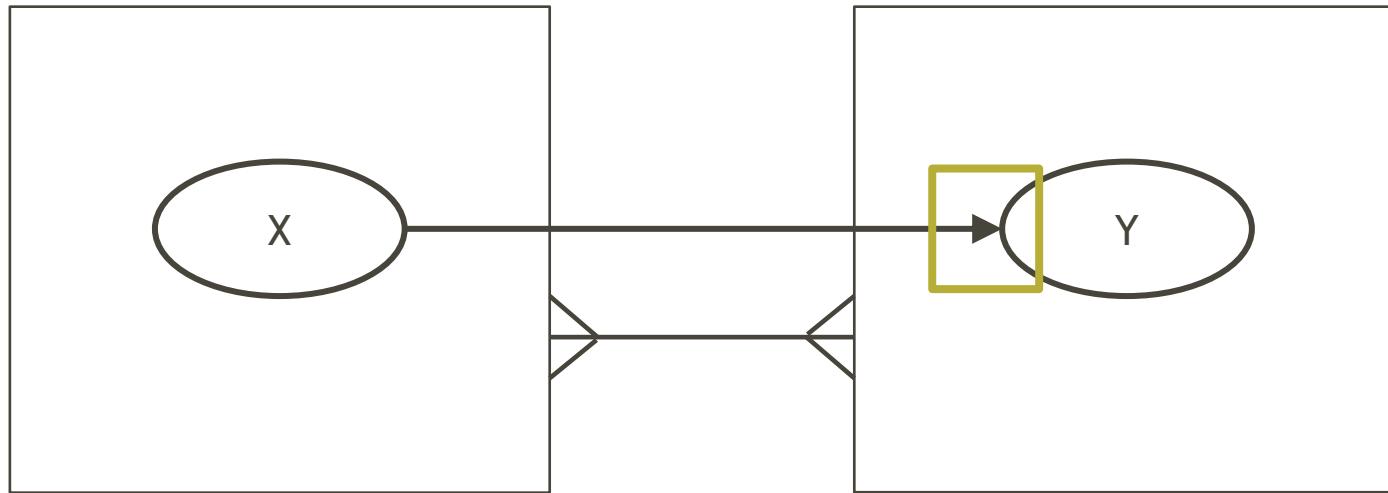




Compare:

$$\begin{aligned} &\text{cov}([A].X, [A, B].Y) \\ &\text{cov}([B].Y, [B, A].X) \end{aligned}$$

INFERRING DIRECTION OF RELATIONAL DEPENDENCIES DIRECTLY

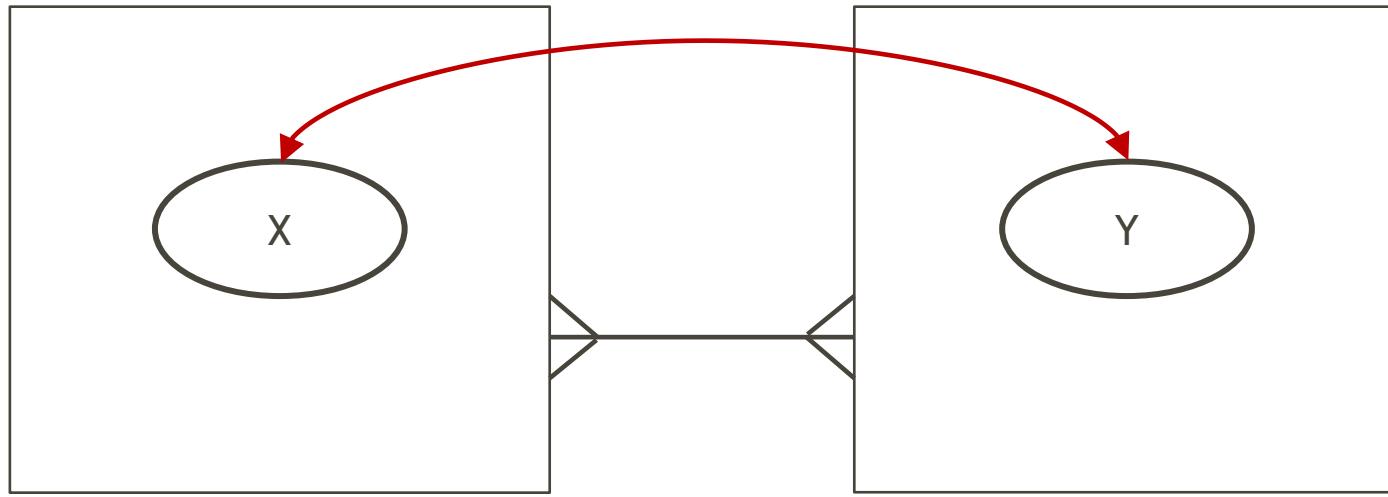


Larger covariance is true direction

Compare:

$$\begin{aligned} & \text{cov}([A].X, [A, B].Y) \\ & \text{cov}([B].Y, [B, A].X) \end{aligned}$$

INFERRING DIRECTION OF RELATIONAL DEPENDENCIES DIRECTLY



Equal covariances implies a latent confounder

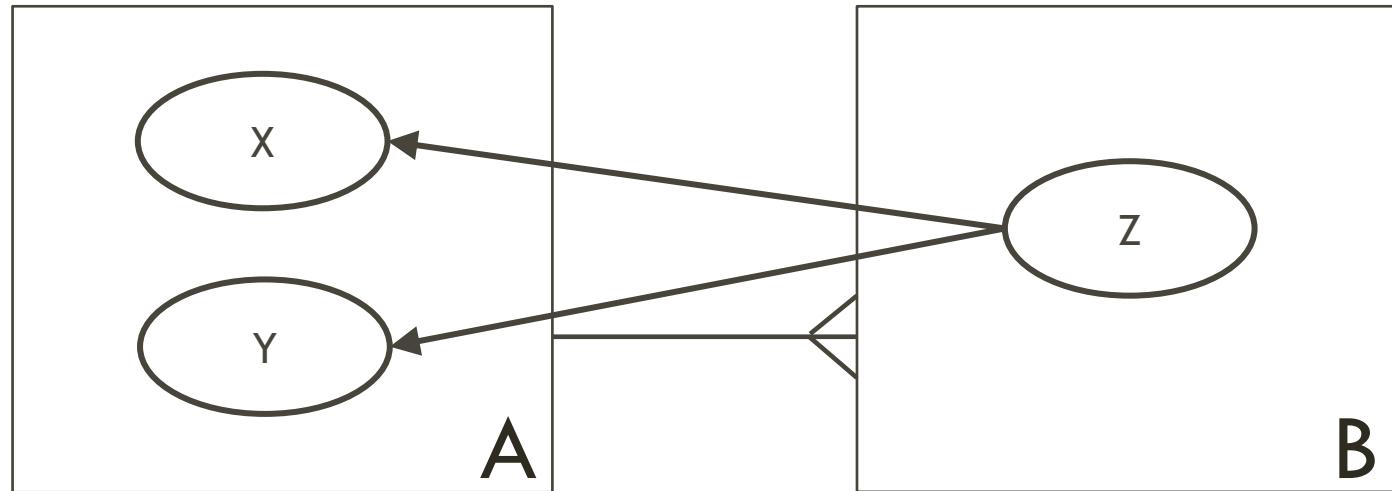
Compare:

$$\begin{aligned} \text{cov}([A].X, [A, B].Y) \\ \text{cov}([B].Y, [B, A].X) \end{aligned}$$

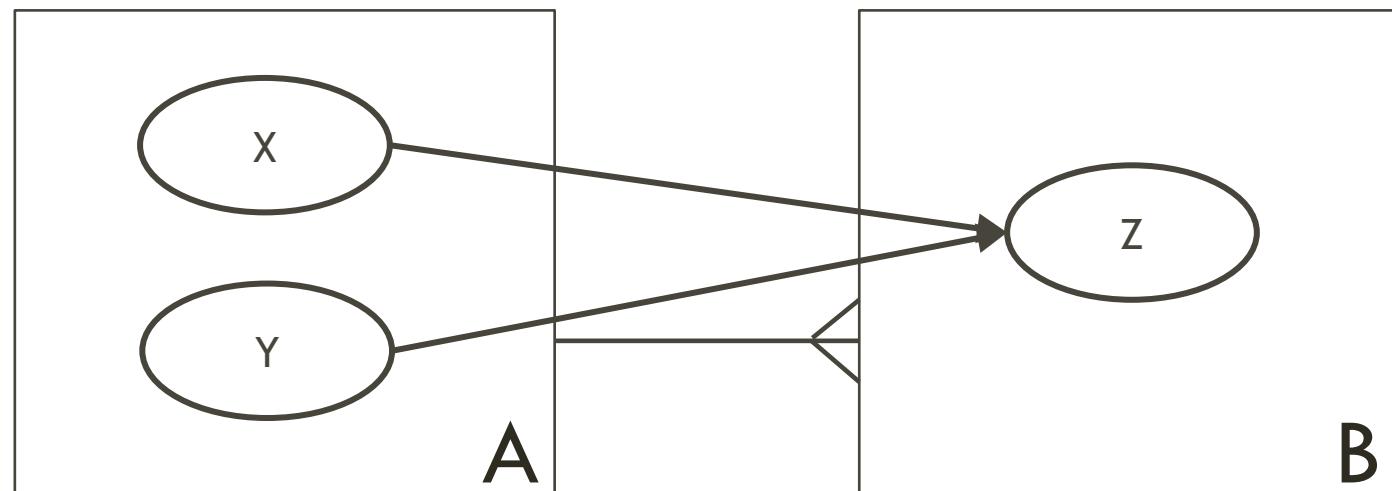
INFERRING DIRECTION OF RELATIONAL DEPENDENCIES DIRECTLY

OBJECT CONDITIONING

$[A].X \perp\!\!\!\perp [A].Y \mid [B].ID$



$[A].X \perp\!\!\!\perp [A].Y \mid [B].ID$



EASY

Modeling multiple entity and relationships

ID for acyclic ground graphs

HARD

Specifying the right relational path semantic

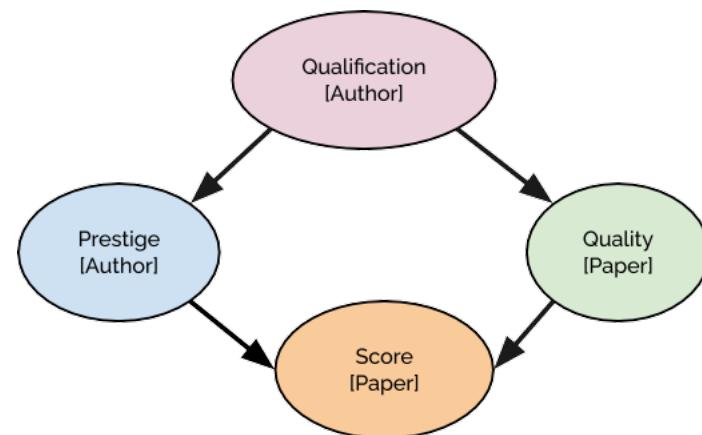
Feedback

Network uncertainty and topological features

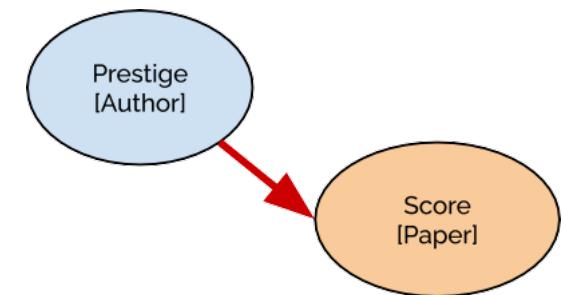
INFERENCE WITHIN THE CARL FRAMEWORK



Relational DB

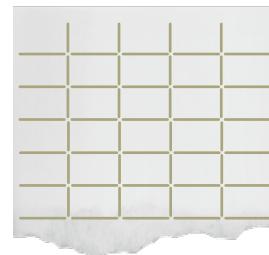


Background Knowledge



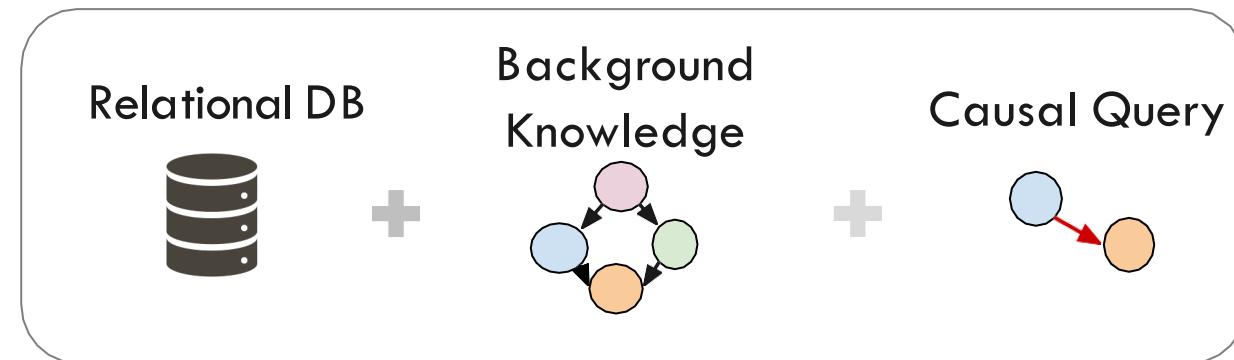
Causal Query

Single flat data-table



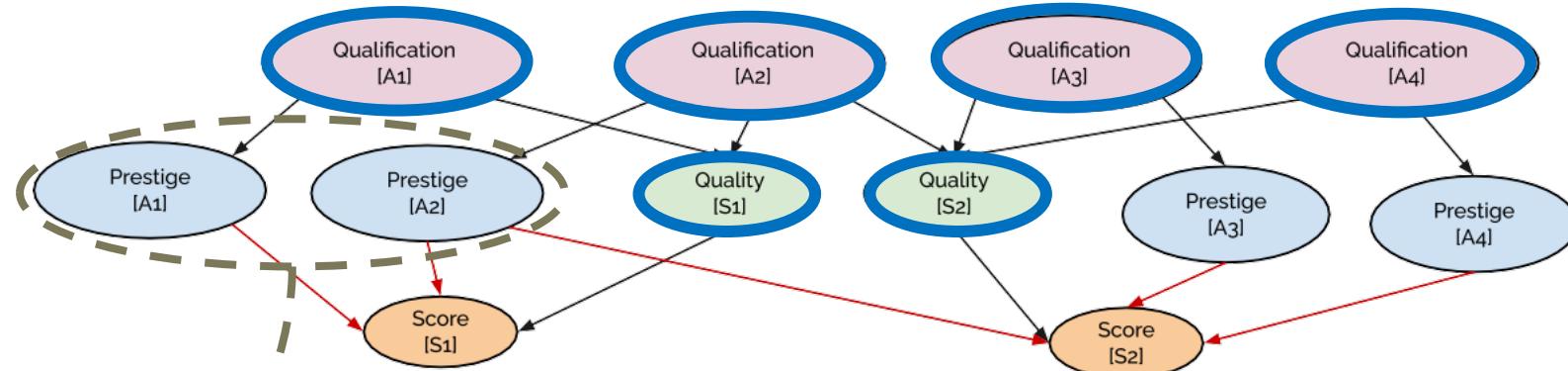
Causal Effects(s)
Estimates

Skeleton Traversal



Grounding

Grounded Causal DAG



Confounder Identification

Flat Table

| Summary-Prestige | Summary-Qualification | Score | Quantity |
|---|--|-----------|--------------|
| $g_{\text{prestige}}(\text{Prestige}[A_1, A_2])$ | $g_{\text{qual}}(\text{Qualification}[A_1, A_2])$ | Score[S1] | Quantity[S1] |
| $g_{\text{prestige}}(\text{Prestige}[A_1, A_2, A_3])$ | $g_{\text{qual}}(\text{Qualification}[A_1, A_2, A_3])$ | Score[S2] | Quantity[S2] |

Causal Inference

A large, abstract network graph is positioned at the top of the slide, consisting of numerous small, semi-transparent nodes connected by thin, light-colored lines.

Motivation

Causal inference 101

Causal effects in networks

Interventions and network experiment design

Counterfactuals & causal effects in observational data

Representation, identification, estimation

Blocks

Representation challenges

Chain and segregated graphs

Multi-relational data and abstract ground graphs

Discovery

COUNTERFACTUALS & CAUSAL EFFECTS IN OBSERVATIONAL NETWORK DATA

Discovery

DISCOVERING RELATIONAL STRUCTURE OF CHAIN GRAPHS

Assume: **Causal** structure is known a priori

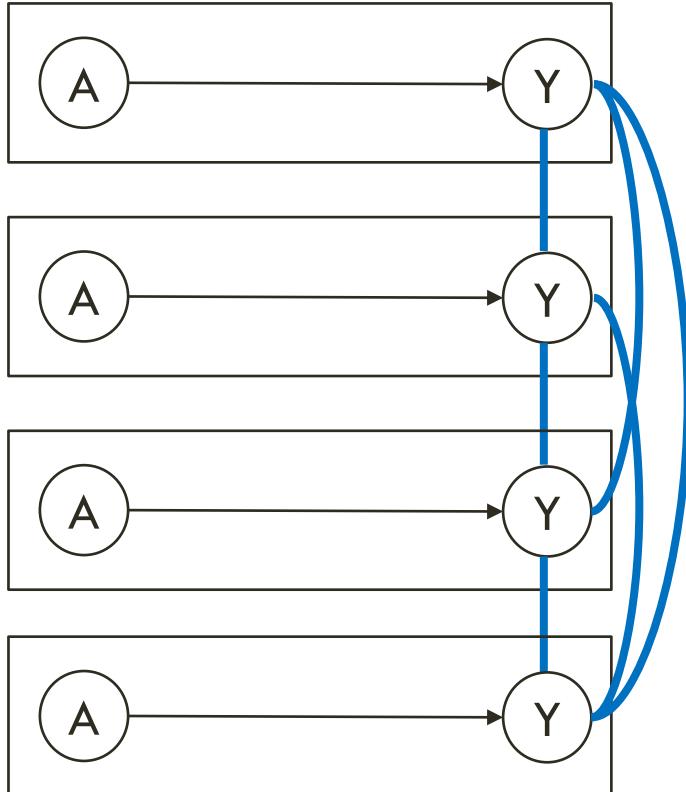
Learn: The **relational** structure

DISCOVERING RELATIONAL STRUCTURE

Assume: **Causal** graph is known

Learn: Greedily search for the relational structure that maximizes the pseudo-likelihood

$$PL(D; G) \equiv \prod_{i=1}^n \prod_{j=1}^d p(x_{j,i} \mid x_{-j,i}; G)$$

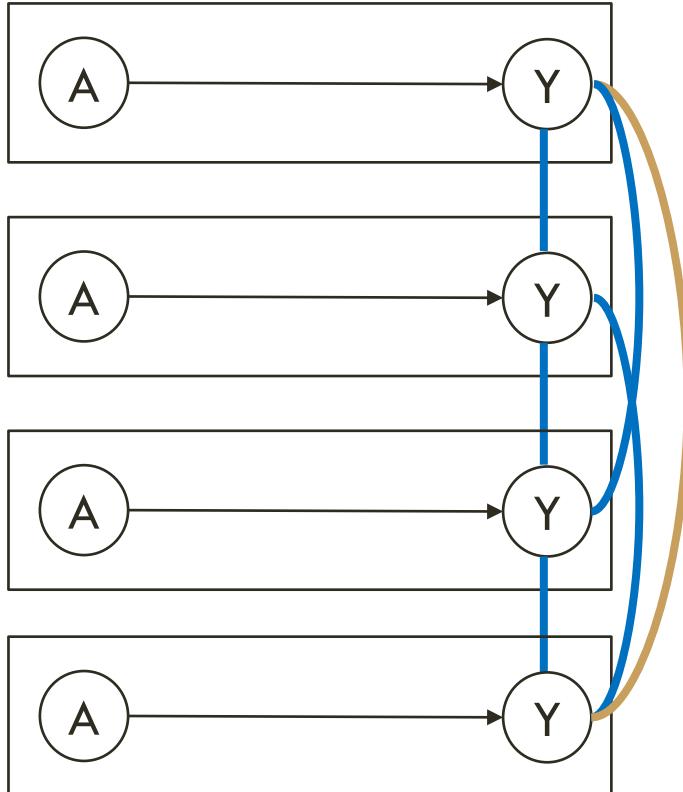


Algorithm 1 GREEDY NETWORK SEARCH($\mathcal{G}^{\text{init}}, \mathbf{D}$)

```

1:  $\mathcal{G}^* \leftarrow \mathcal{G}^{\text{init}}$ 
2: score change  $\leftarrow$  True
3: while score change do
4:   score change  $\leftarrow$  False
5:    $\mathcal{E}_N^* \leftarrow$  network ties in  $\mathcal{G}^*$ 
6:    $E_{max} \leftarrow \operatorname{argmax}_{E \in \mathcal{E}_N^*} \text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E)$ 
7:   if  $\text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E_{max}) > \text{PBIC}(\mathbf{D}; \mathcal{G}^*)$  then
8:      $\mathcal{G}^* \leftarrow \mathcal{G}^* \setminus E_{max}$   $\triangleright$  deleting edge  $E_{max}$ 
9:     score change  $\leftarrow$  True
10:  return  $\mathcal{E}_N^*$ 

```

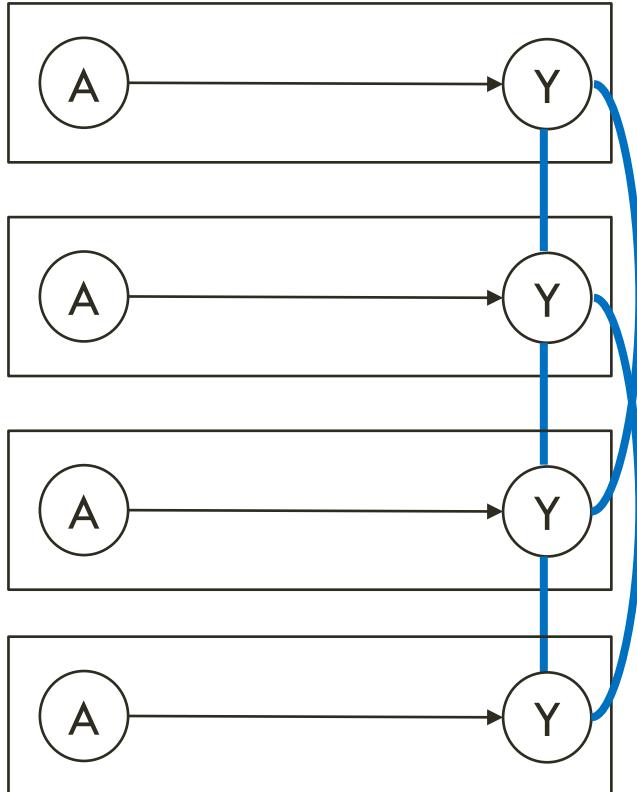


Algorithm 1 GREEDY NETWORK SEARCH($\mathcal{G}^{\text{init}}, \mathbf{D}$)

```

1:  $\mathcal{G}^* \leftarrow \mathcal{G}^{\text{init}}$ 
2: score change  $\leftarrow$  True
3: while score change do
4:   score change  $\leftarrow$  False
5:    $\mathcal{E}_{\mathcal{N}}^* \leftarrow$  network ties in  $\mathcal{G}^*$ 
6:    $E_{max} \leftarrow \operatorname{argmax}_{E \in \mathcal{E}_{\mathcal{N}}^*} \text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E)$ 
7:   if  $\text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E_{max}) > \text{PBIC}(\mathbf{D}; \mathcal{G}^*)$  then
8:      $\mathcal{G}^* \leftarrow \mathcal{G}^* \setminus E_{max}$   $\triangleright$  deleting edge  $E_{max}$ 
9:     score change  $\leftarrow$  True
10: return  $\mathcal{E}_{\mathcal{N}}^*$ 

```

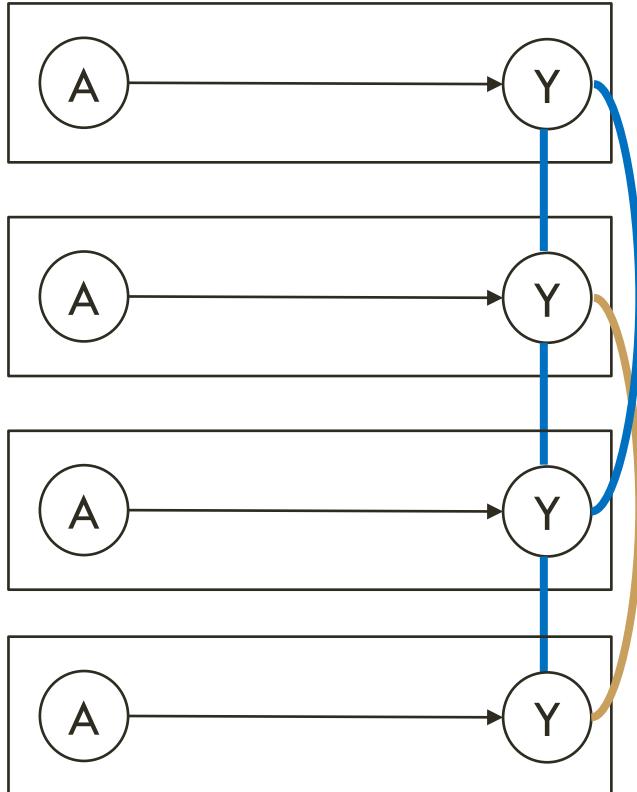


Algorithm 1 GREEDY NETWORK SEARCH($\mathcal{G}^{\text{init}}, \mathbf{D}$)

```

1:  $\mathcal{G}^* \leftarrow \mathcal{G}^{\text{init}}$ 
2: score change  $\leftarrow$  True
3: while score change do
4:   score change  $\leftarrow$  False
5:    $\mathcal{E}_{\mathcal{N}}^* \leftarrow$  network ties in  $\mathcal{G}^*$ 
6:    $E_{max} \leftarrow \operatorname{argmax}_{E \in \mathcal{E}_{\mathcal{N}}^*} \text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E)$ 
7:   if  $\text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E_{max}) > \text{PBIC}(\mathbf{D}; \mathcal{G}^*)$  then
8:      $\mathcal{G}^* \leftarrow \mathcal{G}^* \setminus E_{max}$   $\triangleright$  deleting edge  $E_{max}$ 
9:     score change  $\leftarrow$  True
10: return  $\mathcal{E}_{\mathcal{N}}^*$ 

```

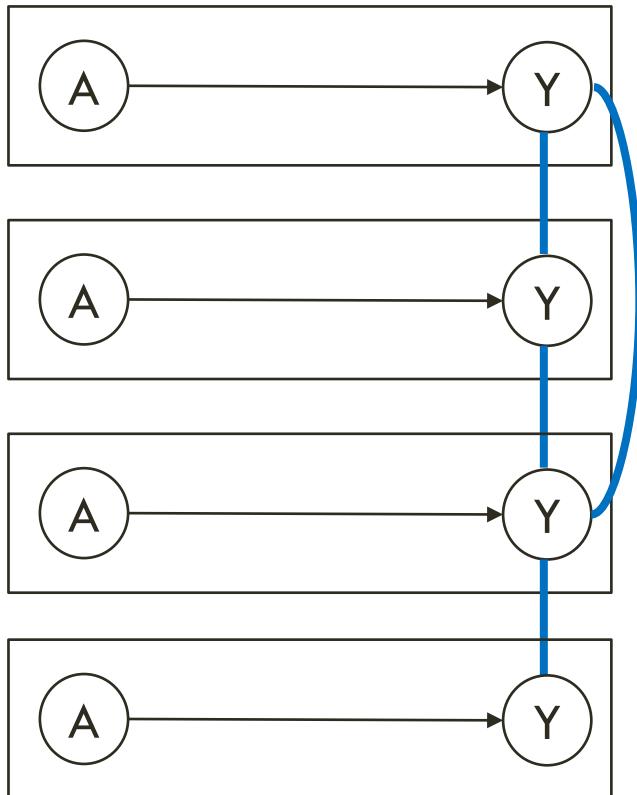


Algorithm 1 GREEDY NETWORK SEARCH($\mathcal{G}^{\text{init}}, \mathbf{D}$)

```

1:  $\mathcal{G}^* \leftarrow \mathcal{G}^{\text{init}}$ 
2: score change  $\leftarrow$  True
3: while score change do
4:   score change  $\leftarrow$  False
5:    $\mathcal{E}_{\mathcal{N}}^* \leftarrow$  network ties in  $\mathcal{G}^*$ 
6:    $E_{max} \leftarrow \operatorname{argmax}_{E \in \mathcal{E}_{\mathcal{N}}^*} \text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E)$ 
7:   if  $\text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E_{max}) > \text{PBIC}(\mathbf{D}; \mathcal{G}^*)$  then
8:      $\mathcal{G}^* \leftarrow \mathcal{G}^* \setminus E_{max}$   $\triangleright$  deleting edge  $E_{max}$ 
9:     score change  $\leftarrow$  True
10: return  $\mathcal{E}_{\mathcal{N}}^*$ 

```

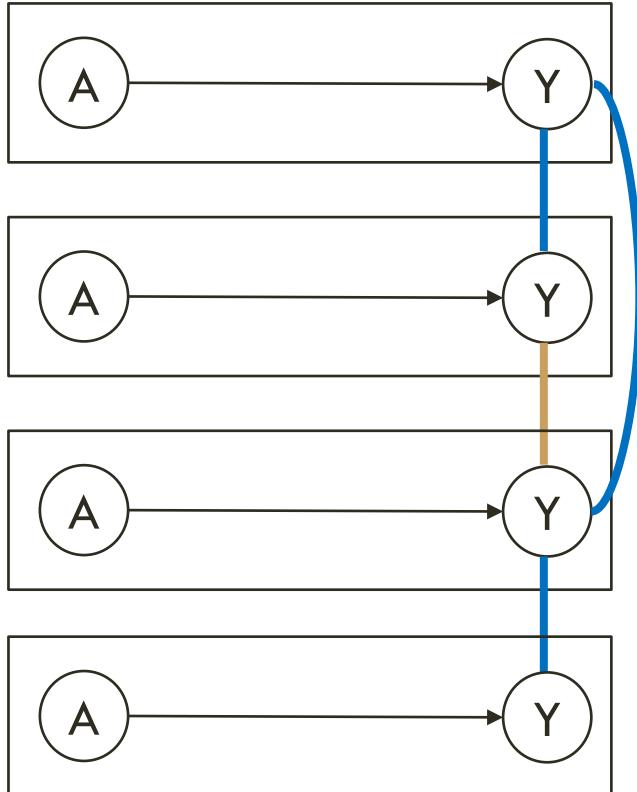


Algorithm 1 GREEDY NETWORK SEARCH($\mathcal{G}^{\text{init}}, \mathbf{D}$)

```

1:  $\mathcal{G}^* \leftarrow \mathcal{G}^{\text{init}}$ 
2: score change  $\leftarrow$  True
3: while score change do
4:   score change  $\leftarrow$  False
5:    $\mathcal{E}_{\mathcal{N}}^* \leftarrow$  network ties in  $\mathcal{G}^*$ 
6:    $E_{max} \leftarrow \operatorname{argmax}_{E \in \mathcal{E}_{\mathcal{N}}^*} \text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E)$ 
7:   if  $\text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E_{max}) > \text{PBIC}(\mathbf{D}; \mathcal{G}^*)$  then
8:      $\mathcal{G}^* \leftarrow \mathcal{G}^* \setminus E_{max}$   $\triangleright$  deleting edge  $E_{max}$ 
9:     score change  $\leftarrow$  True
10: return  $\mathcal{E}_{\mathcal{N}}^*$ 

```

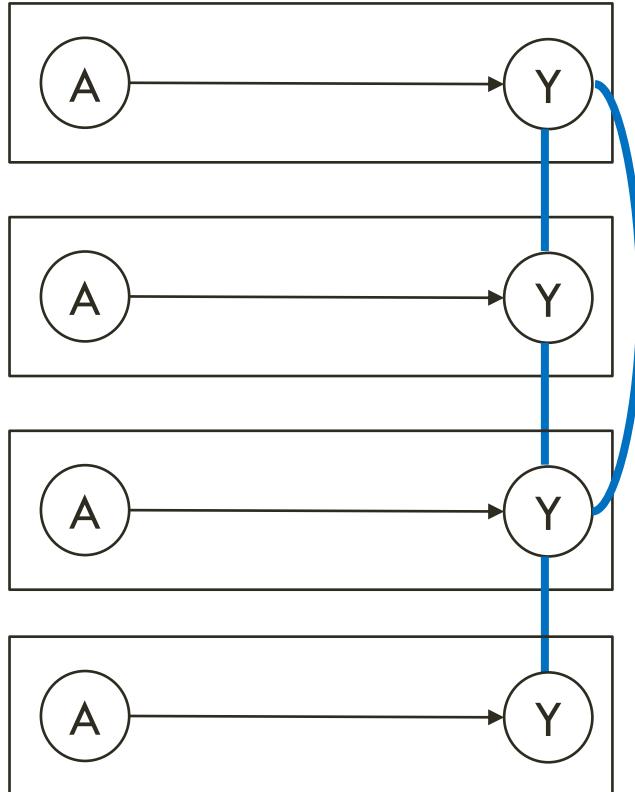


Algorithm 1 GREEDY NETWORK SEARCH($\mathcal{G}^{\text{init}}, \mathbf{D}$)

```

1:  $\mathcal{G}^* \leftarrow \mathcal{G}^{\text{init}}$ 
2: score change  $\leftarrow$  True
3: while score change do
4:   score change  $\leftarrow$  False
5:    $\mathcal{E}_{\mathcal{N}}^* \leftarrow$  network ties in  $\mathcal{G}^*$ 
6:    $E_{max} \leftarrow \operatorname{argmax}_{E \in \mathcal{E}_{\mathcal{N}}^*} \text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E)$ 
7:   if  $\text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E_{max}) > \text{PBIC}(\mathbf{D}; \mathcal{G}^*)$  then
8:      $\mathcal{G}^* \leftarrow \mathcal{G}^* \setminus E_{max}$   $\triangleright$  deleting edge  $E_{max}$ 
9:     score change  $\leftarrow$  True
10: return  $\mathcal{E}_{\mathcal{N}}^*$ 

```



Algorithm 1 GREEDY NETWORK SEARCH($\mathcal{G}^{\text{init}}, \mathbf{D}$)

```

1:  $\mathcal{G}^* \leftarrow \mathcal{G}^{\text{init}}$ 
2: score change  $\leftarrow$  True
3: while score change do
4:   score change  $\leftarrow$  False
5:    $\mathcal{E}_{\mathcal{N}}^* \leftarrow$  network ties in  $\mathcal{G}^*$ 
6:    $E_{max} \leftarrow \operatorname{argmax}_{E \in \mathcal{E}_{\mathcal{N}}^*} \text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E)$ 
7:   if  $\text{PBIC}(\mathbf{D}; \mathcal{G}^* \setminus E_{max}) > \text{PBIC}(\mathbf{D}; \mathcal{G}^*)$  then
8:      $\mathcal{G}^* \leftarrow \mathcal{G}^* \setminus E_{max}$   $\triangleright$  deleting edge  $E_{max}$ 
9:     score change  $\leftarrow$  True
10: return  $\mathcal{E}_{\mathcal{N}}^*$ 

```

DISCOVERING RELATIONAL STRUCTURE

1

Can additionally search over
heterogenous relationship
types

2

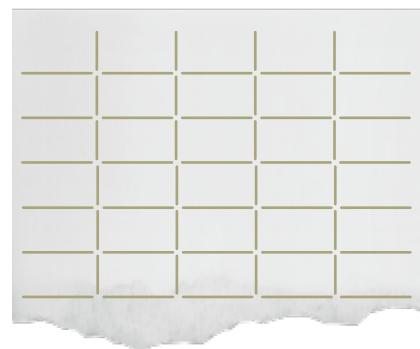
Consistent assuming true
distribution is in the curved
exponential family

DISCOVERING THE CAUSAL STRUCTURE OF MULTI- RELATIONAL DATA

Assume: **Relational** structure is known a priori

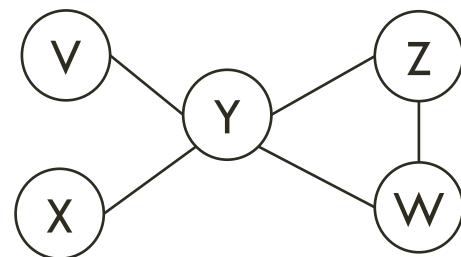
Learn: The **causal** structure

PC ALGORITHM



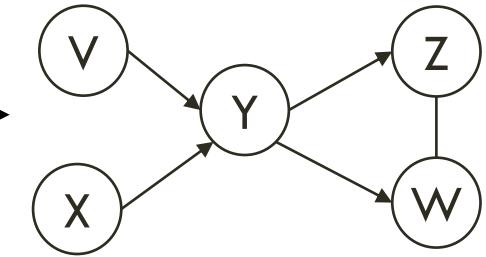
DATA

conditional
independencies



SKELETON

orientation
rules



MARKOV EQUIVALENCE
CLASS

ORIENTATION RULES

Collider Detection (CD)



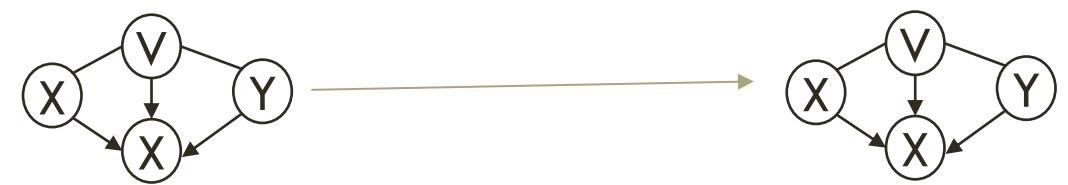
Cycle Avoidance (CA)



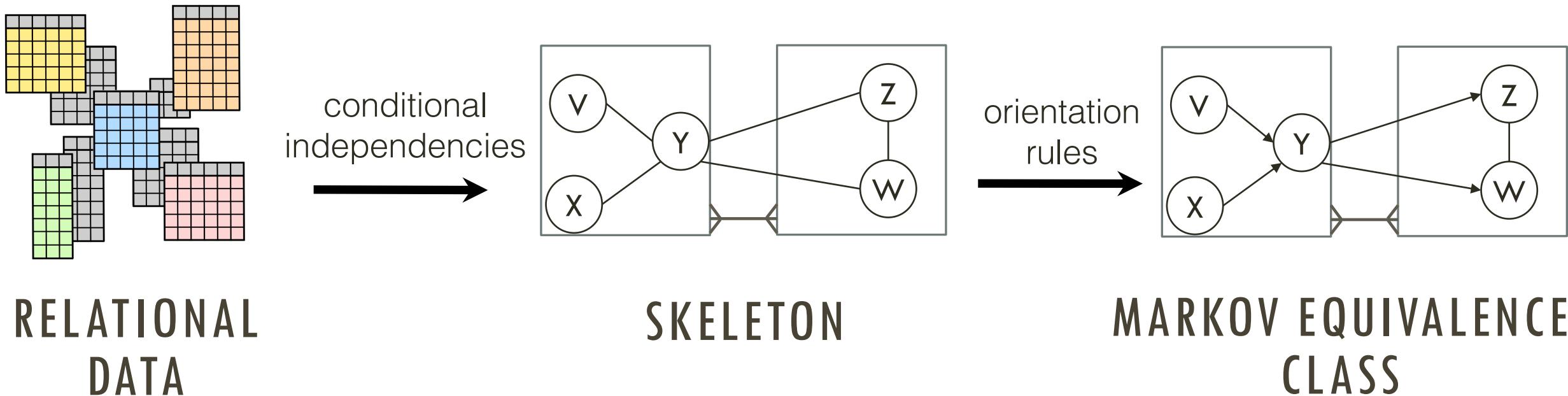
Known Non-Colliders (KNC)



Meek Rule 3 (MR3)

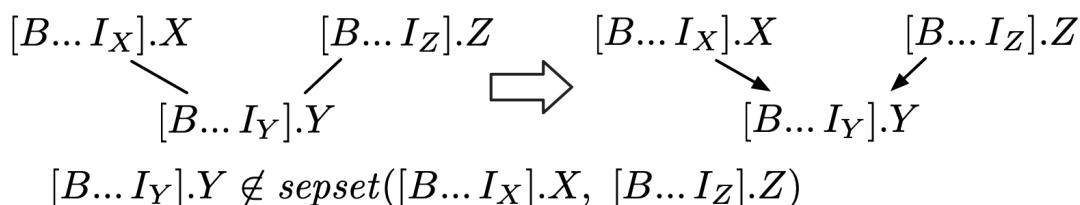


RELATIONAL CAUSAL DISCOVERY (RCD)

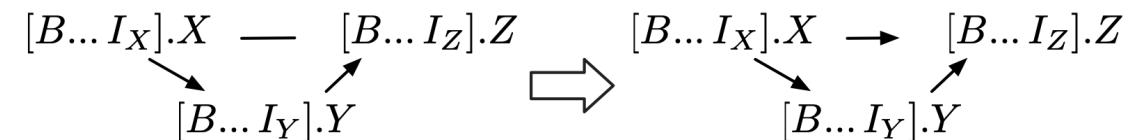


RELATIONAL CAUSAL DISCOVERY (RCD)

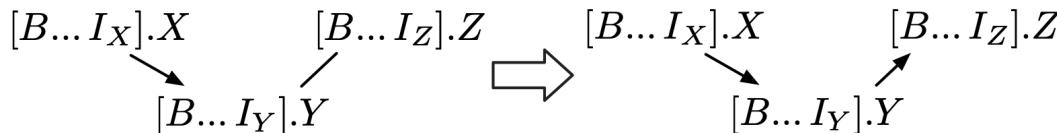
Collider Detection (CD)



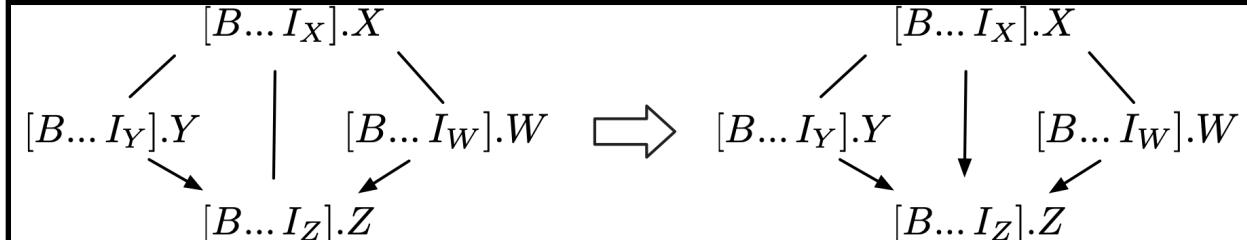
Cycle Avoidance (CA)



Known Non-Colliders (KNC)

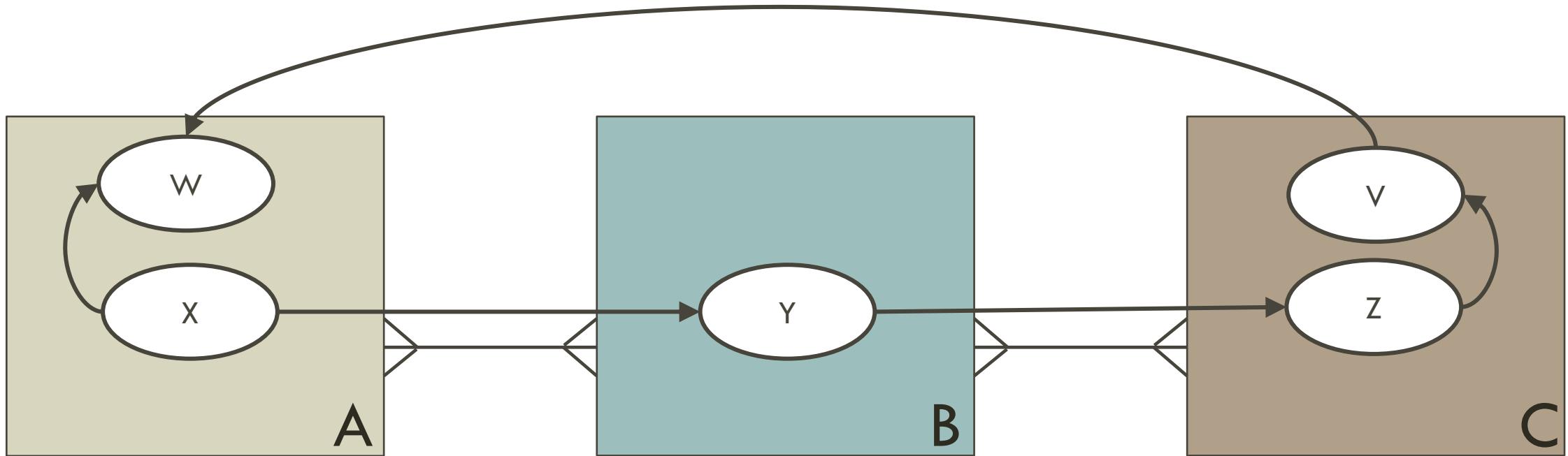


Meek Rule 3 (MR3)

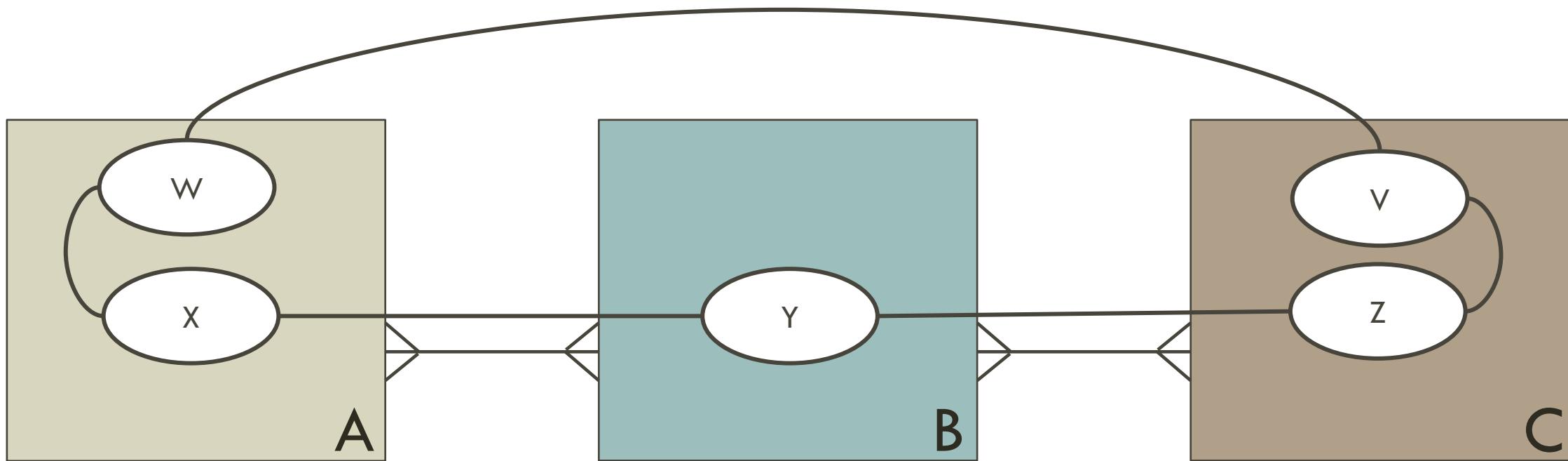


Orientations are propagated across perspectives

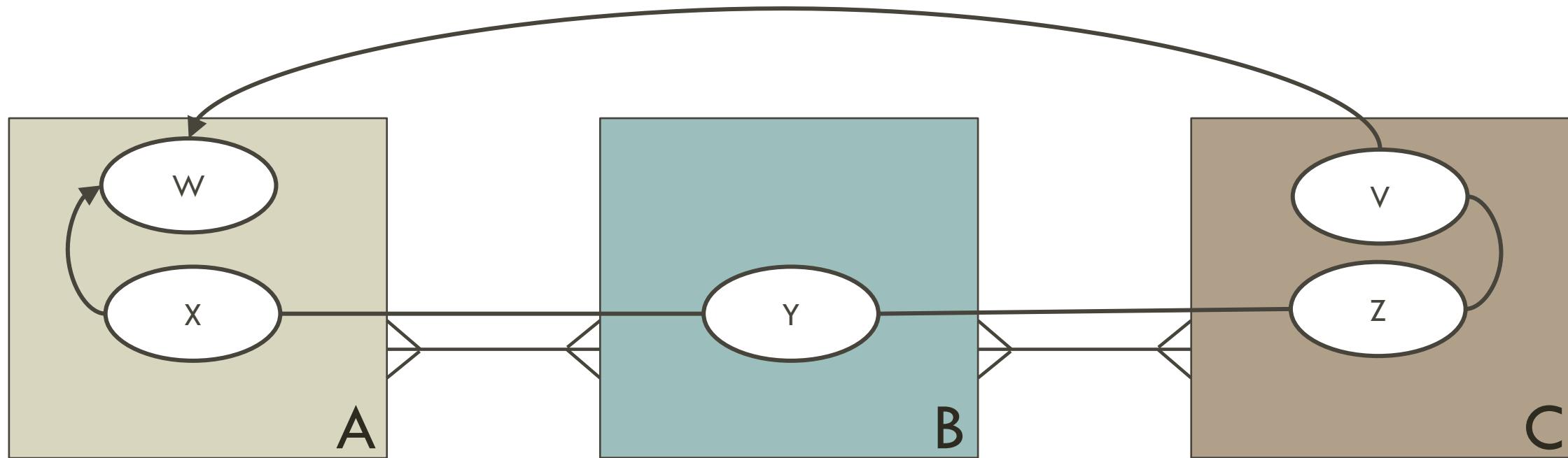
TRACING THE EXECUTION OF RCD



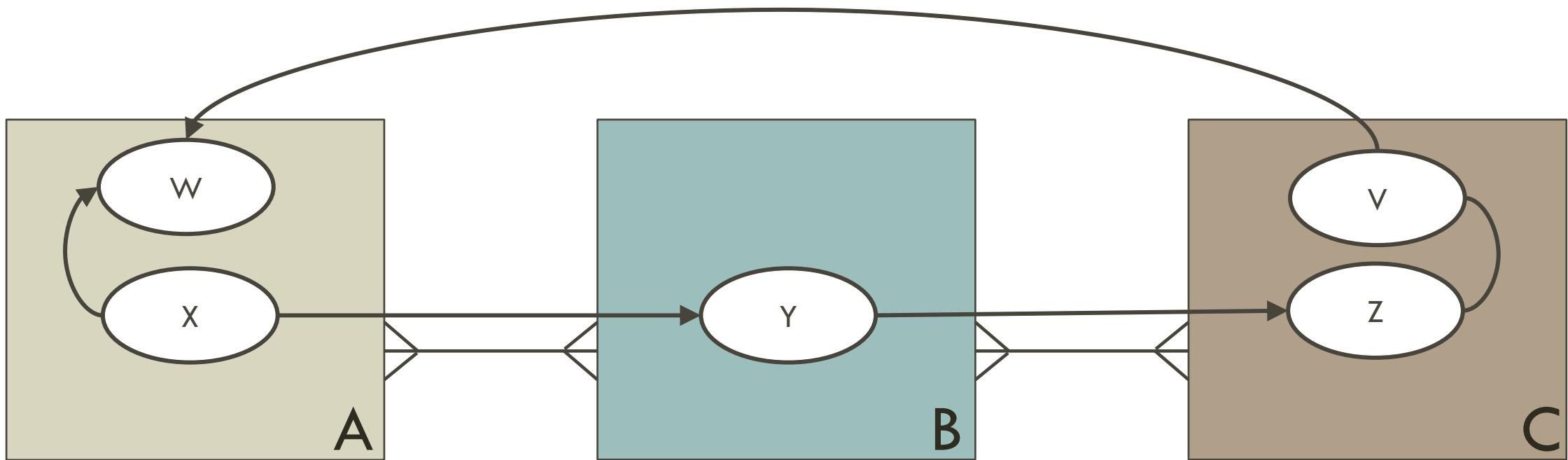
IDENTIFY UNDIRECTED EDGES



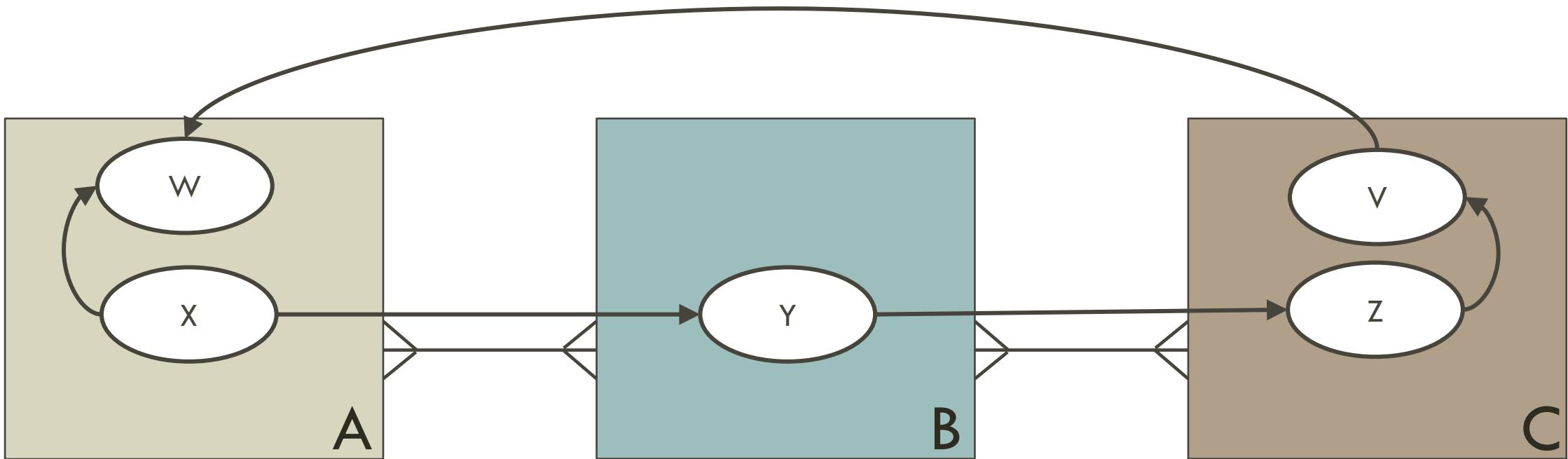
APPLY COLLIDER DETECTION



ORIENT RELATIONAL DEPENDENCIES



APPLY KNOWN NON-COLLIDERS



Relational domains hold considerable promise and unique challenges to causal inference

There is a growing literature with many open research problem in:

- Experimental design
- Graphical representations
- Observational causal inference
- Discovery

SUMMARY

THANK YOU!

David Arbour, Adobe Research

[@darbour26](#)

Elena Zheleva, University of Illinois at Chicago

[@elenadata](#)

Website: <https://netcause.github.io>

- All materials, slides & references
- Our contact information

REFERENCES

Introduction

- Christakis & Fowler. The Spread of Obesity in a Large Social Network Over 32 Years. *New England Journal of Medicine*. 2007.
- Lee & Ogburn. Network Dependence Can Lead to Spurious Associations and Invalid Inference. *Journal of American Statistical Association*. 2020.
- Bail, Argyle, Brown, Bumpus, Chen, Hunzaker, Lee, Mann, Merhout, Volkovskiy. Exposure to opposing views on social media can increase political polarization. *PNAS* 2018.
- T. Sun, S. Viswanathan, E. Zheleva. Creating Social Contagion through Firm-mediated Message Design: Evidence from A Randomized Field Experiment. *Management Science* 2021.
- J. Pearl. The seven tools of causal inference, with reflections on machine learning. *Communications of the ACM* 2019.
- Pearl. *Causality: Models, Reasoning and Inference*. 2009.

Causal Effects in Networks

- Halloran, Struchiner. Causal inference in infectious diseases. *Epidemiology* 1995.
- Hudgens, Halloran. Toward causal inference with interference. *JASA* 2008.
- Ugander, Karrer, Backstrom, Kleinberg. Graph cluster randomization: Network exposure to multiple universes. *KDD* 2013.

Interventions and Network Experiments

- H. Leavitt. Some effects of certain communication patterns on group performance. *The Journal of Abnormal and Social Psychology*, 46(1): 38. 1951.
- S. Aral. Networked Experiments. *The Oxford Handbook of the Economics of Networks*. 2016.
- Manski. Economic analysis of social interactions. *Journal of Economic Perspectives*. 2000
- P. Toulis, E. Kao. Estimation of Causal Peer Influence Effects. *ICML* 2013.
- D. Eckles, R. Kizilcec, E. Bakshy. Estimating peer effects in networks with peer encouragement designs. *PNAS* 2016
- Z. Fatemi, E. Zheleva. Minimizing interference and selection bias in network experiment design. *ICWSM* 2020.
- Saveski, Pouget-Abadie, Saint-Jacques, Duan, Ghosh, Xu, Airolid. Detecting network effects: Randomizing over randomized experiments. *KDD* 2017.
- Stuart. Matching methods for causal inference: a review and look forward. *Stat. Science* 2010.
- R. Johari, H. Li, I. Liskovic, G. Weintraub. Experimental design in two-sided platforms: An analysis of bias. *Arxiv* 2020.
- P. Bajari, B. Burdick, G. Imbens, J. McQueen, T. Richardson, I. Rosen. Multiple randomization designs for interference. *ASSA Annual Meeting* 2020.

Counterfactuals: Blocks

- Ogburn, VanderWeele. Causal Diagrams for Interference. *Statistical Science* 2014.
- Shalizi & Thomas. Homophily and Contagion Are Generically Confounded in Observational Social Network Studies. *Sociological Methods & Research* 2011.

REFERENCES

Representation: Chain and Segregated Graphs

- Ogburn, et al. Causal Inference, Social Networks and Chain Graphs. JRSSB 2020.
- Sherman, Arbour, and Shpitser. General Identification of Dynamic Treatment Regimes Under Interference. AISTATS. 2020.
- Lauritzen & Richardson. Chain Graph Models and Their Causal Interpretation. JRSSB. 2002.
- Richardson. Markov Properties for Acyclic Directed Mixed Graphs. Scandinavian Journal of Statistics. 2002.
- Richardson, Robins and Shpitser. Nested Markov Properties for Acyclic Directed Mixed Graphs. UAI. 2012.
- Tian and Pearl. A General Identification Condition for Causal Effects. AAAI. 2002.
- Shpitser and Pearl. Identification of Joint Interventional Distributions in Recursive Semi-Markovian Causal Models. AAAI. 2006.
- Huang and Voltorta. Pearl's Calculus of Intervention is Complete. UAI. 2006.
- Shpitser. Segregated Graphs and Marginals of Chain Graph Models. NeurIPS. 2015.
- Sherman & Shpitser. Identification of Causal Effects from Dependent Data. NeurIPS. 2018.

Representation: Multi-Relational Data

- Getoor, Friedman, Koller & Pfeffer. Learning Probabilistic Relational Models. IJCAI. 1999.
- Heckerman, Meek, and Killer. Probabilistic Models for Relational Data. MSR Tech Report. 2004.
- Maier, Marazopoulou, and Jensen. Reasoning about Independence in Probabilistic Models of Relational Data. Arxiv. 2013.
- Lee and Hanovar. A Characterization of Markov Equivalence Classes of Relational Causal Models under Path Semantics. UAI. 2016
- Arbour, Garant, and Jensen. Inferring Network Effects from Observational Data. KDD. 2016.
- Maier, Marazopoulou, Arbour, and Jensen. A Sound and Complete Algorithm for Learning Causal Models from Relational Data. UAI. 2013.
- Arbour, Marazopoulou, and Jensen. Inferring Causal Direction from Relational Data. UAI. 2016.
- Jensen, Burroni, and Rattigan. Object Conditioning for Causal Inference. UAI. 2020.
- Salimi, Parikh, Kayali, Getoor, Roy, and Suciu. Causal Relational Learning. SIGMOD. 2020.

Discovery

- Bhattacharya, Malinksy, and Shpitser. Causal inference under interference and network uncertainty. UAI, 2019.
- Spirites, Glymour, Scheines. Causation, Prediction, and Search. MIT Press, 1993.
- Lee and Hanovar. On Learning Causal Models from Relational Data. AAAI. 2016.