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Question :-

Problem 2.13

$$\lambda(\alpha_i|\omega_j) = \begin{cases} 0 & i=j \qquad i,j=1,\cdots c \\ \lambda_r & i=c+1 \\ \lambda_s & \text{otherwise} \end{cases}$$

Solution :-

~~we can assume risk at a point x to minimize~~

~~Based upon the textbook formula~~

Posterior probability is computed using

$$P(\omega_i|x) = \frac{P(x|\omega_i)\,P(\omega_i)}{P_x} \rightarrow ①$$

and evidence is

$$p(x) = \sum_{j=1}^{c} P(x/\omega_j)\, p(\omega_j)$$

The loss from above equation is given as

$$\lambda(\alpha_i|\omega_j)$$

and risk is $R(\alpha_i|x) = \sum_{j=1}^{c} \lambda(\alpha_i|\omega_j)\, p(\omega_j|x)$ ②

and we minimize error rate by maximizing posterior probability.

Now at a point where ω_{max} is maximum probability point, based upon equation above we have risk

$$= \lambda_s \sum_{j \neq max} (\omega_j/x) = \lambda_s(1 - P(\omega_{max}/x))$$

if we consider decision being rejected, then

$\omega_{point} \neq$ maximum posterior probability

so at ~~risk~~ risk λ_r for rejection, plugging back into above equation; risk is

$$\lambda_s \sum_{j \neq Point_r} = \lambda_s [1 - P(\omega_p | x)] \geq 1 - P[\omega_{max} | x]$$

Since we should not reject selection with $P(\omega_{max})$

so we should reject if

$$\lambda_r < \lambda_s [1 - P(\omega_{max} | x)] \text{ and}$$

this happens when

$$\boxed{P(\omega_i | x) \geq 1 - \frac{\lambda_r}{\lambda_s}}$$

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what happens if $\lambda_r = 0$

if $\lambda r = 0$ then we have from Equation above.

$P(wi/x) \geq 1$ so ~~state~~ in this case we should definitely reject

→ what happens if $\lambda r > \lambda_s$

if $\lambda_r > \lambda s$ then plugging back based upon textbook we have to decide

ω_i ~~λ_s State~~ if $P(wi|x) > P(\omega_j|x)$ for all $i \neq j$

in other words decide

$$\lambda_s[1-P(\omega_{max}|x)] < \lambda_r[1-P(\omega_{max}|x)]$$

$$\boxed{\lambda_s[1-P(\omega_{max}/x)] < \lambda_r[1-P(\omega_{max}/x)]}$$