University of Vavuniya

First Examination in Information Technology -2021

First Semester - March 2023

IT1122 Foundation of Mathematics

Answer Four Questions Only

Time Allowed : Two Hours

---

1. (a) Write down the elements in each of the following sets:

   i. $A = \{x \in \mathbb{N} \mid x \text{ is even and } x < 11\}$

   ii. $B = \{x \in \mathbb{N} \mid 3 < x < 19\}$

   iii. $C = \{x \in \mathbb{N} \mid 4 + x = 3\}$

   iv. $D = \{x \in \mathbb{N} \mid x^2 = 2 \text{ or } x^2 = 9\}$ [20%]

   (b) In a town of 10000 families it was found that 40% of families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy newspaper A and B, 3% families buy newspaper B and C and 4% families buy newspaper A and C. If 2% families buy all the newspaper. Find each of the following:

   i. Number of families which buy all three newspapers.

   ii. Number of families which buy newspaper C only.

   iii. Number of families which buy none of A, B or C.

   iv. Number of families which buy exactly only one newspaper. [25%]

(c) Let $\bigcup = \{1,2,3,4,5,6,7\}$, $A = \{1,2,3,4,5\}$ and $B = \{2,5,7\}$. Prove each of the following using the sets:

i. $(A \cup B)' = A' \cap B'$

ii. $(A \cap B)' = A' \cup B'$

[20%]

(d) Draw the Venn diagrams for each of the following expressions of sets A, B and C.

i. $A \cup B \cup C^c$

ii. $(A \cap B^c) \cap C$

iii. $A - (B \cap C)$

iv. $A \cup (B \cap C)^c$

[20%]

(e) Let A and B be sets; show that $A \cup (B - A) = A \cup B$. [15%]

2. (a) Construct a truth table for each of the following compound propositions:

i. $(p \rightarrow q) \wedge (\neg p \rightarrow q)$

ii. $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

[20%]

(b) Let p and q be the propositions.

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot.

Express each of the following propositions as an English sentence.

i. $\neg p \rightarrow \neg q$

ii. $\neg p \wedge q$

[20%]

[question 02 is continued on page 03]

(c) State the *converse*, *contrapositive* and *inverse* of the conditional statement: When I stay up late, it is necessary that I sleep until noon. [15%]

(d) Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology. [20%]

(e) Represent the following statements in mathematical logic forms and prove whether the conclusion is valid:

" If the lab is not available or there is a workshop, then the assessment exam will be postponed. If the assessment exam gets postponed, then new date will be announced. No new date has been announced. Therefore, Lab is available" [25%]

3. (a) Consider each of the following relations on the set of integers $\{1, 2, 3, 4, 5\}$:

$R_1 = \{(a, b) |\ a \leq b\}$

$R_2 = \{(a, b) |\ a = b \text{ or } a = -b\}$

$R_3 = \{(a, b) |\ a = b + 1\}$

$R_4 = \{(a, b) |\ a + b \leq 3\}$

Identify the ordered pairs for each of the above relations R_1 to R_4 and draw graph representation. [20%]

(b) Consider each of the following relations on {1,2,3,4}:

$R_1 = \{(1,3), (1,4), (2,2), (3,1), (3,3), (4,1), (4,4)\}$

$R_2 = \{(1,1), (2,2), (3,3), (4,4)\}$

$R_3 = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,3), (1,3), (3,2)\}$

$R_4 = \{(2,2), (3,3), (4,4), (1,4), (2,4), (3,4), (4,2), (3,2)\}$

$R_5 = \{(1,4), (4,2)\}$

Identify the *reflexive*, *symmetric* and *transitive* relations from the above set of relations. [25%]

[question 03 is continued on page 04]

(c) Let f and g be two functions from $\mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2+1$ and $g(x) = x+2$ respectively for all $x \in \mathbb{R}$.

Determine the value for each of the following functions:

i. $f(-5)$

ii. $g(0)$

iii. $fog(3)$

iv. $gof(2)$ [25%]

(d) Determine whether each of the following functions from $\{a, b, c, d\}$ to itself is one-to-one, onto or both.

i. $f(a) = b, f(b) = a, f(c) = c, f(d) = d$

ii. $f(a) = b, f(b) = b, f(c) = d, f(d) = c$

iii. $f(a) = d, f(b) = b, f(c) = c, f(d) = d$ [30%]

4. (a) Simplify the boolean expression $F(x, y, z) = xy\bar{z}+x\bar{y}\bar{z}+\bar{x}yz+\bar{x}\bar{y}\bar{z}$ using Karnaugh Map. [20%]

(b) Find the dual of each of the following expressions:

i. $\bar{x}.1 + (\bar{y} + z)$

ii. $(x + \bar{y})(y(\bar{x} + 1)) + y + (0 + z)$ [20%]

(c) Verify whether $\neg p \vee (\neg p \wedge q)$ and $(\neg p \wedge \neg q)$ are logically equivalent. [20%]

(d) Prove that $yz + x(\bar{x}\bar{z}) + y(\bar{z} + 1) + \bar{z}x$ can be simplified as $y + \bar{z}x$ using laws of Boolean algebra. [20%]

(e) Draw a logical circuit for the expression $F(x, y, z) = xyz + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$. [20%]

5. (a) Define a Finite Automaton. [10%]

(b) Construct a Finite Automaton to check whether a given binary number is divisible by three. [20%]

(c) Design a Turing Machine to accept the language $L = \{0^n 1^n 2^n | n \geq 1\}$. [20%]

(d) Consider the following Graph in Figure 1:

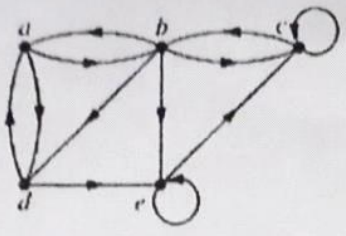

Figure 1:

i. Write the adjacency list and adjacency matrix for the Graph.

ii. Find the in-degree and out-degree of each vertex in the Graph. [20%]

(e) Draw all possible tree structures to evaluate each of the following in-order expressions:

i. $x + (y/(x + 2))$

ii. $((2 * 3) - 5) + (2^3/4)$ [20%]

(f) Represent the following graph in Figure 2 using an incident matrix:

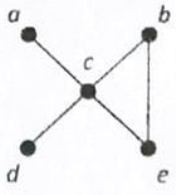

Figure 2:

[10%]