University of Vavuniya

First Examination in Information Technology - 2022

First Semester- January / February 2024

IT1122 - Foundation of Mathematics

Answer any Four Questions Only

Time Allowed: Two Hours

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1. (a) In a group of students, 25 study Computer Science, 28 study Health Science, 20 study Mathematics, 9 study Computer Science only, 12 study Health Science only, 8 study Computer Science and Health Science only, and 5 students study Health Science and Mathematics only.

   i. Draw a Venn diagram to illustrate the above information.

   ii. Find how many students study all the subjects.

   iii. Find how many students are in there group.

[30%]

   (b) Let A and B be two non-empty sets, then draw the Venn diagrams for each of the following;

   i. $(A \cup B) \setminus (A \cap B)$;

   ii. $(A \cup B) \cap (A \cup B^c)$;

   iii. $(A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)$.

[30%]

[Question 1 continues on page 2]

(c) Prove that if A, B, C and D are non-empty sets, then

i. $(A \times B) \cap (B \times D) = (A \cap B) \times (B \cap D)$;

ii. $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$.

[20%]

(d) Let A, B and C be subsets of a non-empty set X. Prove each of the following:

i. $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$;

ii. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

[20%]

2. (a) Construct the truth table for each of the following compound proposition:

i. $(p \to q) \wedge (p \to r)$;

ii. $(q \wedge \neg r) \to \neg p$.

[20%]

(b) Determine whether the two compound propositions in each of the following are logically equivalent:

i. $p \leftrightarrow (q \leftrightarrow r)$ and $(p \leftrightarrow q) \leftrightarrow r$;

ii. $(p \to q) \to r$ and $(p \wedge \neg r) \to \neg q$. [20%]

(c) Show that each of the following compound proposition is a tautology by using truth table:

i. $(p \wedge (q \wedge r)) \to [((r \wedge p) \wedge q) \vee q]$;

ii. $[p \leftrightarrow (q \wedge r)] \to ((q \wedge r) \to p)$. [30%]

(d) Determine the validity of the following argument:

If students play cricket they study well. Either students do not study well or they pass annual examinations. But students are failing the annual examination. Therefore, students do not play cricket. [30%]

(a) Determine whether each of the following relations is reflexive, symmetric or transitive: 4.

i. A relation R in a set $A = \{1, 2, 3, \ldots\ldots, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$;

ii. A relation R in a set $\mathbf{N}$ of natural numbers defined as $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$;

iii. A relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y) : y \text{ is divisible by } x\}$;

iv. A relation R in the set $\mathbf{Z}$ of all integers defined as $R = \{(x, y) : x - y \text{ is an integer}\}$.

[20%]

(b) Let R is a relation defined in A. Prove each of the following:

i. If R is an anti-symmetric relation then R^{-1} is also an anti-symmetric relation;

ii. If R is a transitive relation then R^{-1} is also a transitive relation. [20%]

(c) Consider the relation $R = \{(1,1), (2,1), (1,2), (2,2), (3,2), (3,4), (4,3), (4,4)\}$ on the set $A = \{1, 2, 3, 4\}$.

i. Draw the directed graph of the above relation R;

ii. Find the relation matrix of the above relation R.

[20%]

(d) Let $f : A \to B$ and $g : B \to C$ be two functions. Prove each of the following:

i. If g and f are invertible functions then $g \circ f$ is invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$;

ii. If f is a bijection then f^{-1} is also a bijection with $(f^{-1})^{-1} = f$.

[20%]

(e) Consider the function $f : \mathbf{R} \to \mathbf{R}$ defined by,

$$f(x) = 5x - 7.$$

Show that f is bijective and find the inverse of f. [20%]

4. (a) Construct the truth table for the following Boolean expression:

$$F(x,y,z) = x\bar{y} + (\overline{xy\bar{z}}).$$

[20%]

(b) Prove the following Boolean expression using Boolean Identities:

$$(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C).$$

[20%]

(c) Construct circuits using the basic logic gates that produce each of the following outputs:

i. $\bar{x}\,(\overline{y+\bar{z}})$;

ii. $(x+y+z)(\bar{x}\bar{y}\bar{z})$.

[20%]

(d) Find the Sum of Product (SOP) and the Product of Sum (POS) of the following non-canonical Boolean expression:

$$F(x,y,z) = (x+y)(x+\bar{z}) + x\bar{y}.$$

[20%]

(e) Simplify the Boolean expression $F(x,y,z) = x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$ using Karnaugh Map. [20%]

5. (a) Draw the directed graph for the following adjacency matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

[10%]

(b) Write down the *prefix* and *postfix* expressions of each of the following infix arithmetic expressions:

i. $((H \times ((((A + ((B + C)) \times D)) \times F) \times G) \times E)) + J$;

ii. $((A \times X) + (B \times (C \times Y)) / (D - E))$.

[20%]

[Question 5 continues on page 5]

(c) Consider the following graph;

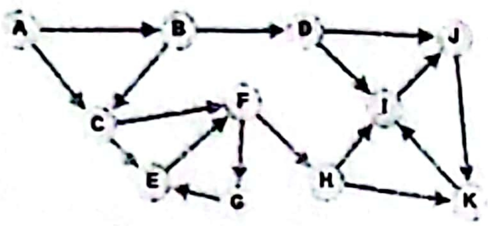

Figure 1: G1

i. Write down the adjacency matrix for the above graph G1 shown in the Figure 1.

ii. Find the in-degree and out-degree of each of the vertices in the graph. [20%]

(d) Determine whether the given pair of graphs G2 and G3 shown in Figure 2 and Figure 3 are isomorphic.

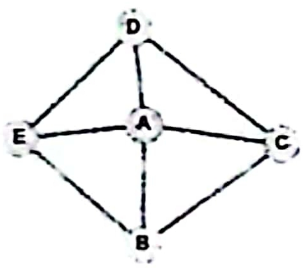

Figure 2: G2

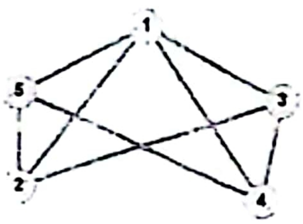

Figure 3: G3

[20%]

(e) Construct a Finite Automaton to check whether a given binary number is divisible by two. [15%]

(f) Design a Turing Machine to accept the language $L = \{0^n1^n2|n \geq 1\}$.

[15%]