Vavuniya Campus of the University of Jaffna

First Examination in Information ~~and Communication~~ Technology - 2018

First Semester - September/October 2019

IT1122 Foundation of Mathematics

Answer Four Questions Only

Time Allowed : **Two hours**

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1. (a) Write down the elements in each of the following sets:

   i. $A=\{x \in \mathbb{N} \mid x \text{ is even}, x < 11\}$;

   ii. $B=\{x \in \mathbb{N} \mid 3 < x < 9\}$;

   iii. $C=\{x \in \mathbb{N} \mid 4 + x = 3\}$;

   iv. $D=\{x \in \mathbb{R} \mid x^2 = 4 \text{ OR } x^2 = 9\}$. [20%]

   (b) Suppose A is the set of distinct letters in the word *ELEPHANT*, B is the set of distinct letters in the word *SYCOPHANT*, C is the set of distinct letters in the word *FANTASTIC*, and D is the set of distinct letters in the word *STUDENT*. The universe **U** is the set of *English alphabet*. Find:

   i. $A \cup B$;

   ii. $A \cap B$;

   iii. $A \cap (C \cup D)$;

   iv. $(A \cup B \cup C \cup D)^C$. [30%]

[This question is continued on the next page]

(c) If A={ 0, 1 } , B={ 1, 2 } and C={ 0, 1, 2 } find:

i. P(C), the power set of C;

ii. $A \times C$;

iii. $A \times B \times C$. [30%]

(d) Find A^2 if:

i. A={ 0, 1, 3 }

ii. A={ 1, 2, a, b } [20%]

2. (a) In an examination, 73% of the candidates passed in mathematics, 70% passed in physics, 64% passed in both subjects and 63 candidates are failed in both subjects. Use a Venn diagram to find the total number of candidates who appeared at the examination. [30%]

(b) Using a membership table to show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. [20%]

(c) Using the set identities prove the following:

i. $(A^C \cup B)^C \cap A^C = \phi$

ii. $(A \cup B) \cap (A^C \cap B)^C = A$

iii. $(A \cap B) \cup (A \cup B^C)^C = B$ [30%]

(d) A genetics experiments classifies fruit flies according to the following criterions: `Sex:m-male,f-female; Wing Span:s-short winged,m-medium winged,l-long winged`. List all the categories in this classification scheme. [20%]

3. (a) Define the following function:

i. *Injective* function;

ii. *Surjective* function;

iii. *Bijective*function. [30%]

[This question is continued on the next page]

(b) Determine whether each of the following functions is a **bijection** from $\mathbb{R}$ to $\mathbb{R}$. Justify your answer:

i. $f(x) = 2x + 1$

ii. $f(x) = x^2 + 1$

iii. $f(x) = x^3$

iv. $f(x) = (x^2 + 1)/(x^2 + 2)$ [40%]

(c) Let $f\colon \mathbb{R} \to \mathbb{R}$ and $g\colon \mathbb{R} \to \mathbb{R}$. Find fog and gof, if :

i. $f(x)=2x$ and $g(x)=x^2$

ii. $f(x)=4x-3$ and $g(x)=x^2+2$ [30%]

4. (a) Define the terms *arithmetic progression* and *geometric progression.* [20%]

(b) Decide whether the each of the following scenarios describes an arithmetic or geometric sequence. Then, write the formula for the sequence.

i. Round 1 of a tennis tournament starts with 128 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 64 players, in round 3 there are 32 players and so on.

ii. Paul has Rs 6800.00 in a saving account. He makes a deposit after he receives each paycheck. After one month he has Rs 7580.00 in the account. The next month the balance is Rs 8360.00. The balance after the third month is Rs 9140.00. [30%]

(c) Find the sum of the first six terms in the sequence of $\{a_n\}$, $n > 2$ where $a_n = 2a_{n-1} + a_{n-2}$, $a_1 = 1$, and $a_2=1$. [20%]

(d) Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n= 8a_{n-1} - 16a_{n-2}$, $n > 2$ if

i. $a_n = 4^n$

ii. $a_n = 2.4^n + 3n.4^n$ [30%]

5. (a) Find the product AB, where

i. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$;

ii. $A = \begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 2 & 3 \\ -1 & 0 & 3 & -1 \\ -3 & -2 & 0 & 2 \end{bmatrix}$;

iii. $A = \begin{bmatrix} 0 & -1 \\ 7 & 2 \\ -4 & -3 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 & 2 & 3 & 0 \\ -2 & 0 & 3 & 4 & 1 \end{bmatrix}$.

[30%]

(b) Let A be the 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

i. Show that if ad - bc $\neq 0$, then $A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$; [30%]

ii. if $A = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$; Find A^{-1}. [15%]

(c) Write an algorithm for swapping two numbers a and b. [25%]

6. (a) Consider the graph G1 in Figure 1:

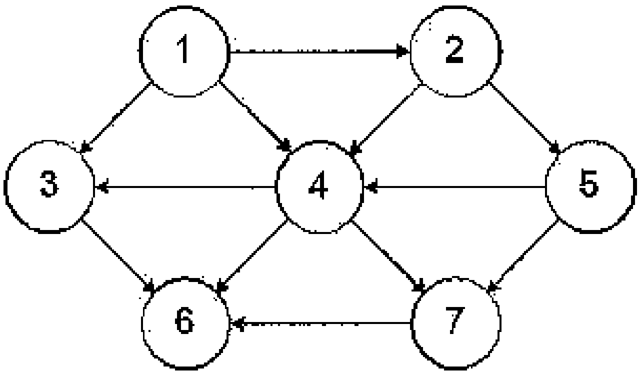

Figure 1

i. Find the in-degree and out-degree of each vertex. [20%]

ii. Represent the graph G1 in an adjacency matrix. [15%]

(b) Represent the following graph in an incidence matrix.

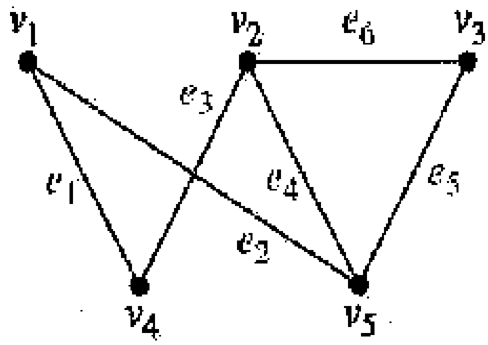

Figure 2

[20%]

(c) A couple has two children; the sample space is S = { bb, bg, gb, gg }, where b represents boy and g represents girl.In the set elements, first letter and second letter represent elder child and younger child respectively. Find the probability p:

i. at least one of the children is a boy

ii. the elder child is a boy [20%]

[This question is continued on the next page]

(d) Consider the finite-state machine M defined by the state table shown in Table 1.

State	Input	
	0	1
S_0	$S_1, 0$	$S_0, 0$
S_1	$S_2, 1$	$S_0, 1$
S_2	$S_0, 0$	$S_3, 1$
S_3	$S_1, 1$	$S_2, 0$

Table 1:

i. What are the states of M? [05%]

ii. What are the input symbols of M? [05%]

iii. Draw the state diagrams for the finite-state machine M. [15%]