

# Assignment 10: Boolean Expressions, Normal Forms and Landau Sets

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## 1 Problem 2

- $n^3 \log(n) \in O(n^4)$ : We know that  $n^4 = n^3 * n$  and  $\log(n) \leq n$  for all  $n \in \mathbb{N}/0$ .
- $n + 3n^3 \in \Omega(n + n^2)$ :  $n^2 \leq n^3$  for all  $n \in \mathbb{N}$ .
- $2^n \in O(e^{n^2})$ : We know that  $2^n \in O(e^n)$  and that  $n \in O(n^2)$ .
- $\sin(n) \in O(n)$ :  $\sin(n) \leq 1$  for all  $n \in \mathbb{N}$  so there is no  $k \in \mathbb{R}$  such that  $1 * k \geq n$  for all  $n \in \mathbb{N}$ .
- $42 \in \Theta(12)$ :  $42 > 12$  and if we define  $k = 100$ ,  $12 * k > 42$ .
- $n^4 \in \Omega(\frac{n^2}{\log(n)})$ : Since  $n^2 \in \Omega(n^4)$ .
- $\log_{10}(n) \in \Omega(\ln(n))$ :  $\ln(n)$  always grows faster than  $\log_{10}(n)$  so for every  $k$  we can always choose some  $n$  large enough such that  $\ln(n) - k * \log_{10}(n) > 0$ .
- $n! \in \Omega(5^n)$ : Since  $5^n = 5 * 5 * 5 * \dots * 5$  and  $n! = 1 * 2 * 3 * 4 \dots$  so as  $n$  grows it gets multiplied by bigger numbers so we can't find a  $k$  to make them equivalent.
- $n^{\log_n(8^n)} \in O(9^n)$ : Taking the  $\log_n$  of both functions we get  $\log_n(8^n) * \log_n(n) = \log_n(8^n)$  and  $n * \log_n(9) \approx n$  since  $\log_n(9) \rightarrow 1$  as  $n \rightarrow \infty$ . Therefore we have  $\log_n(8^n)$  and  $n$  and  $\log_n(8^n) \in O(n)$ .
- $(3(n^3) + 2)^2 \in \Theta((n^3 + 3)^2)$ : Since  $9n^6 \in \Theta(n^6)$ .