Assignment 5: ADT's, Inductive Functions and SML

Ernesto Rodriguez

October 27, 2011

1 Problem 1

The statement says that if x,y are coprime we can find an a,b such that $a < y$ and $b < x$ and:

$$a * x - b * y = 1 \tag{1}$$

Now assume $a > y$ and $b > x$ now we have:

$$a * x - b * y = 1$$
$$a * x - 1 = b * y$$
$$\frac{a * x}{b} - \frac{1}{b} = y \tag{2}$$

Since we assume $b > x$ from equation 2 we can deduce that:

$$\frac{a}{b} - \frac{1}{b} \leq \frac{a * x}{b} - \frac{1}{b} = y$$
$$\frac{a}{b} - \frac{1}{b} \leq y$$
$$\frac{a}{b} \leq y + \frac{1}{b}$$

Since $b \geq 1$ we have:

$$y \leq y + \frac{1}{b}$$

This implies that:

$$\frac{a}{b} \leq y$$
$$a \leq b * y \tag{3}$$

We know $b \geq 1$ so we also know that:

$$y \leq y * b$$

Thus we can rewrite equation 3 as:

$$a \leq y \tag{4}$$

Which is a contradiction to the original assumption that $y > a$.