Assignment 1: Problem 1

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1 Notation

Zero or first unary natural number: 0 Successor of n: s(n) Multiple successor operatios: sss(n), equals to the successor of the succesor of the successor of n.

2 Solution

1. **Base Case:**
   $a + b = b + a, b = 0$
   $a + 0 = 0 + a$
   $a = 0 + a$
   $a + 0 = 0 + a + 0$
   $a = 0 + a$
   Step Case:
   $a + b = b + a$
   $s(a + b) = s(b + a)$
   $a + s(b) = b + s(a)$
   $a + b + s(0) = b + a + s(0)$, a+b=b+a So we have:
   $s(0) = s(0)$
2. **Base Case:**
   $(a + b) * c = a * c + b * c$,c=0
   $(a + b) * 0 = a * 0 + b * 0$
   $(a + b) * 0 = 0$
   Step Case:
   $(a + b) * c = a * c + b * c$,c=s(c)
   $(a + b) * s(c) = a * s(c) + b * s(c)$
   $(a + b) + (a + b) * s(c) = a + a * c + b + b * c$
   $a + b + a * c + b + b * c = a + a * c + b + b * c$

3. **Base Case:**

   $a * 0 = 0 * a$

   $a * s(0) = s(0) * a$

   $a + a * 0 = s(0) * a$

   $a = s(0) * a$

   Step Case:

   $a * s(b) = s(b) * a$

   $a * (b + s(0)) = (b + s(0)) * a$

   $a * b + a * s(0) = b * a + s(0) * a$

   $a * b = b * a$ and $a * s(0) = s(0) * a$