General Computer Science I (320101) Fall 2010
Assignment 1: Elementary Math
(Given Sep. 15., Due Sep. 22.)

Problem 1.1   (Bernoulli inequality)
Prove by induction the Bernoulli inequality:

$$(1 + x)^n \geq nx$$

where $n \in \mathbb{N}$, $x \in \mathbb{Q}$, and $x \geq -1$

    Hint: You can acomplish this by proving a stronger statement first, namely that the left hand side is greater or equal to $nx + 1$.

Problem 1.2   (Newton's Formula)
Prove the following formula by induction after n:

$$\forall n \geq 1.(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} (x^k)(y^{n-k})$$

Problem 1.3   (Unary Natural Numbers)
Let $\oplus$ be the addition operation and $\odot$ be the multiplication operation on unary natural numbers as defined on the slides. Prove that:

1. $a \oplus b = b \oplus a$

2. $(a \oplus b) \odot c = a \odot c \oplus b \odot c$

3. $a \odot b = b \odot a$

Problem 1.4   (Helly's Theorem)
There are $n \geq 4$ convex polygons in a plane. Each 3 of these polygons have a common point. Prove that all n figures share at least one common point.

    Hint: Use the method of mathematical induction to prove the statement. Start with a rigorous proof for the case $n = 4$ and use it as a base case for your induction proof.