Assignment 10: Boolean Expressions, Normal Forms and Landau Sets

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December 1, 2011

1 Problem 1

1.1 Reflexivity

Let $f(x) = f(x)$ for all $x \in \mathbb{B}^n$. Assume that $f(x) \nleq f(x)$, then there exists $x \in \mathbb{B}^n$ such that $f(x) \neq f(x)$. This is a contradiction to our original assumption.

1.2 Transitivity

Let $f(x) \leq g(x)$ and $g(x) \leq h(x)$ for all $x \in \mathbb{B}^n$. Assume there exists $x^* \in \mathbb{B}^n$ such that $h(x^*) \leq f(x^*)$. By our original assumption we know that $g(x) \leq h(x)$ for all $x \in \mathbb{B}^n$. Then we can say that $g(x^*) \leq h(x^*) \leq f(x^*)$ which implies $g(x^*) \leq f(x^*)$ but $\leq$ is not symmetric (see below) so $f(x) \leq g(x)$ and $g(x^*) \leq f(x^*)$ is not true for all $x \in \mathbb{B}^n$ which is a contradiction.

1.3 Symetry

Let $f(x) := x + \overline{x}$ and $g(x) := x * \overline{x}$. Clearly, when $x = T$, $f(x) = T + \overline{T} = T$ and $g(x) = T * \overline{T} = F$. So $g(x) \leq f(x)$ but $f(x) \nleq g(x)$.

1.4 Antisymmetry

Let $f(x) \leq g(x)$ for all $x \in \mathbb{B}^n$ and $f(x) \neq g(x)$. Assume that $g(x) \leq f(x)$ for all $x \in \mathbb{B}^n$ then the relation is symmetric, which is a contradiction (proved above). So $g(x) \nleq f(x)$ for all $x \in \mathbb{B}^n$.