General Computer Science I (320101) Fall 2011
Assignment 2: More Induction, Sets and Relations
(Given Sep. 22., Due Sep. 29.)

15pt

Problem 2.1 (Division by 17)
Prove by induction or refute that for all natural numbers $n \geq 1$ the following assertion holds: $3(5^{2n+1}) + (2^{3n+1})$ is divisible by 17.

Note: The statement does not require the use of Unary Natural Numbers, but rather "normal" natural numbers.

20pt

Problem 2.2 (A wrong induction proof)
Identify what is the problem in the following faulty induction proof:

> ***Theorem***: All elements of a non-empty set are equal.
>
> ***Proof***: We prove the assertion by induction over the number n, which corresponds to the size of the set ($n > 0$).
>
> **base case:** $n = 1$. Obviously the sole element of a set with size 1 is equal to itself.
>
> **step case:** $n > 1$. Let us assume that the assertion is true for all sets with size n (induction hypothesis), aiming to prove that it is also true for sets with size $n + 1$. Let us take a set S with $\#(S) = n + 1$ elements and consider some $a \in S$. Then we form two subsets of S, S_1 and S_2 with $\#(S_1) = \#(S_2) = n$, such that $S_1 \cup S_2 = S$, and $a \in S_1$ and $a \in S_2$. Hence from the induction hypothesis it follows that all elements in S_1 equal a, and also all elements in S_2 equal a. Since $S = S_1 \cup S_2$, then all elements in S equal a, i.e. they are equal to each other. Thus we have proven what we were aiming for. □

10pt

Problem 2.3 (Five Djinn Postulates)
Restate in MathTalk the following five Postulates of the Djinns:

1. Genie is a djinn.
2. Every djinn has a meta (which is also a djinn).
3. Genie is not the meta of any djinn.
4. Different djinns have different metas.
5. If Genie has X, and each djinn that has X gives X to its meta, then all djinns get X.

20pt

Problem 2.4 (Determine the Set)
Determine the sets S, such that:

1. S contains only natural numbers

2. S contains at least two elements

3. $\forall x, y \in S. x > y \Rightarrow \frac{(xy)}{((x^2)-(y^2))} \in S$

25pt

Problem 2.5 (Set Sizes)
Let S be a set with n elements. Determine the sizes of the following sets:

1. $\{\langle X, Y\rangle \mid (X, Y \subset S) \wedge (X \cup Y \equiv S)\}$

2. $\{\langle X, Y\rangle \mid (X, Y \subset S) \wedge (X \cap Y \equiv \emptyset)\}$