

# Assignment 4: Relations, Functions and Introduction to SML

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## 1 Problem 3

First let's get the set  $C$  of all elements paired with themselves, namely  $\{a, a\}$ . These elements, the pair of an element with itself must be included in the relation in order for it to be reflexive.

$$C = \{ \langle a, a \rangle \mid a \in \{a, b, c, d, e\} \}$$

The power set minus the empty set contains all subsets of pairs with the same element, namely  $P(C)$ . The size is  $2^5 - 1$ .

Let's consider case by case basis:

1. **Sets with one pair:** These sets are already reflexive and symmetric relations, we have 5 different. And we can't generate any new relations.
2. **Sets with more than one pair:** They are already reflexive and symmetric. But we can generate from each one more reflexive and symmetric relations by adding to it more pairs. To be exact, we can know the amount of relations that can be generated by multiplying the amount of sets with  $n$  elements by  $n$ . So this boils to:

$$|R| = 1 * \binom{5}{1} + 2 * \binom{5}{2} + 3 * \binom{5}{3} + 4 * \binom{5}{4} + 5 * \binom{5}{5} = 80$$

The General case for  $n$  elements is:

$$|R| = \sum_{i=0}^n i * \binom{n}{i}$$

The answer to the question boils down to 80 relations.