

Assignment 10: Boolean Expressions, Normal Forms and Landau Sets

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1 Problem 3

1.1 CNF

$$\begin{aligned} & (x_1 + \overline{x_3}) * (\overline{x_2 * x_1 + x_3}) \\ & (x_1 + \overline{x_3}) * ((\overline{x_2 * x_1}) * (\overline{x_3})) \\ & (x_1 + \overline{x_3}) * ((\overline{x_2} + \overline{x_1}) * (\overline{x_3})) \\ & (x_1 + \overline{x_3}) * (\overline{x_2} + \overline{x_1}) * (\overline{x_3}) \end{aligned}$$

1.2 DNF

$$\begin{aligned} & (x_1 + \overline{x_3}) * (\overline{x_2 * x_1 + x_3}) \\ & (x_1 + \overline{x_3}) * ((\overline{x_2 * x_1}) * (\overline{x_3})) \\ & (x_1 + \overline{x_3}) * ((\overline{x_2} + \overline{x_1}) * (\overline{x_3})) \\ & (x_1 + \overline{x_3}) * ((\overline{x_2} * \overline{x_3}) + (\overline{x_1} * \overline{x_3})) \\ & (x_1 + \overline{x_3}) * (\overline{x_2} * \overline{x_3}) + (x_1 + \overline{x_3}) * (\overline{x_1} * \overline{x_3}) \\ & (x_1 * \overline{x_2} * \overline{x_3}) + (\overline{x_2} * \overline{x_3} * \overline{x_3}) + (x_1 * \overline{x_1} * \overline{x_3}) + (\overline{x_1} * \overline{x_3} * \overline{x_3}) \\ & (x_1 * \overline{x_2} * \overline{x_3}) + (\overline{x_2} * \overline{x_3}) + (\overline{x_1} * \overline{x_3}) \end{aligned}$$

1.3 Depth and Cost

We must perform six operations, namely: $op_1 = (x_2 * x_1)$, $op_2 = op_1 + x_3$, $op_3 = \overline{op_2}$, $op_4 = \overline{x_3}$, $op_5 = x_1 + op_4$ and $op_6 = op_3 * op_5$, so the cost is 5. Similarly we can see that the depth is 4.