General Computer Science I (320101) Fall 2011
Assignment 6: Abstract procedures
(Given Oct. 28., Due Nov. 3.)

25pt

Problem 6.1 (Abstract Procedures on Trains)

A train can have locomotives, passenger cars, cargo cars or mail cars. You are required to define abstract procedures using the ADT for trains defined below (in the ADT there is also defined a sort for boolean values true and false).

$$\langle \{\mathbb{B}, \mathbb{T}\}, \{[nil\colon \mathbb{T}], [m\colon \mathbb{T} \to \mathbb{T}], [c\colon \mathbb{T} \to \mathbb{T}], [p\colon \mathbb{T} \to \mathbb{T}], [l\colon \mathbb{T} \to \mathbb{T}], [T\colon \mathbb{B}], [F\colon \mathbb{B}]\}\rangle$$

1. given two trains, swap their cars:

$$swap(l(p(c(m(nil)))), l(c(m(m(nil))))) = \langle l(c(m(m(nil)))), l(p(c(m(nil))))\rangle$$

2. given two trains, find out if the first is "included" in the second:

$$include(m(p(nil)), l(c(m(p(c(nil)))))) = T$$

3. given two trains, find out if the two trains are equal:

$$equal(l(p(c(m(nil)))), l(m(p(nil)))) = F$$

4. given two trains, find out if the two trains have the same number of cars (length):

$$samelen(l(l(c(nil))), l(m(p(nil)))) = T$$

5. given one train, return a train with the locomotives first and all the cars in reverse order:

$$\rho(l(c(m(l(p(nil)))))) = l(l(p(m(c(nil)))))$$

Hint: Define additional abstract procedures (for example a procedure for logical AND) if needed.

25pt

Problem 6.2 (Another abstract procesure)

Consider the following abstract procedures on the abstract data type of natural numbers:

$$\mathcal{P} := \langle f{::}\mathbb{N} \to \mathbb{N}\,;\, \{f(o) \rightsquigarrow o, f(s(o)) \rightsquigarrow o, f(s(s(x_{\mathbb{N}}))) \rightsquigarrow s(f(x_{\mathbb{N}}))\}\rangle$$

$$\mathcal{P} := \langle g{::}\mathbb{N} \times \mathbb{N} \to \mathbb{N}\,;\, \{g(o,o) \rightsquigarrow s(g(s(o),o)), g(o,s(y_{\mathbb{N}})) \rightsquigarrow f(y_{\mathbb{N}}), g(s(x_{\mathbb{N}}),y_{\mathbb{N}}) \rightsquigarrow s(s(g(y_{\mathbb{N}},x_{\mathbb{N}})))\}\rangle$$

1. Show the computation process for g on the arguments $\langle s(s(o)), o\rangle$.

2. Give the recursion relation of f g.

3. Do f and g terminate on all inputs? Justify your answer.

4. What function is computed by g and f?

10pt

Problem 6.3 (Substitutions)

You are given the ADT

$$\langle\{\mathbb{A}, \mathbb{B}, \mathbb{C}\}, \{[a\colon \mathbb{A}], [b\colon \mathbb{B}], [c\colon \mathbb{A}], [f\colon \mathbb{A} \to \mathbb{A}], [g\colon \mathbb{A} \times \mathbb{B} \to \mathbb{A}], [h\colon \mathbb{B} \times \mathbb{B} \to \mathbb{A}]\}\rangle$$

Which of the following mappings are valid substitutions?

$\sigma_1 := [(f(x_\mathbb{A}))/x_\mathbb{A}], [b/y_\mathbb{B}]$
$\sigma_2 := [(h(a, b))/x_\mathbb{A}], [(g(a, b))/y_\mathbb{A}]$
$\sigma_3 := [(f(a, c))/x_\mathbb{A}], [(g(a, b))/y_\mathbb{A}]$
$\sigma_4 := [f^{i+1}(x_\mathbb{A})/f^i(x_\mathbb{A})]$, $i \in \mathbb{N}$ with $f^0(x_\mathbb{A}) = x_\mathbb{A}$ and $f^{i+1}(x_\mathbb{A}) = f(f^i(x_\mathbb{A}))$