Assignment 5: ADT's, Inductive Functions and SML

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1 Problem 7

1.1 Part 1

The functions written in a more 'readable' look something like this:

$$f(x,y) = \begin{cases} 0 & \text{if } x,y = 0 \\ g(0,y) & \text{if } x = 0 \\ 3 + g(x-1, y+1) & \text{otherwise} \end{cases} \quad g(x,y) = \begin{cases} 0 & \text{if } x,y = 0 \\ f(x,0) & \text{if } y = 0 \\ 2 + g(x, y-1) & \text{otherwise} \end{cases}$$

We can see that it first loops through x until it gose to zero and then through y. For every 'loop' it dose in x, x is reduced by one and 5 units are added (or the function returns five times the succesor). Then after x gose to zero it start iterating over y wheras 2 is added per y iteration. So the 'final' version would look something like this:

$$f \circ g(x,y) = 5x + 2y$$

1.2 Part 2

- **Substraction:** $sub(0,0) = 0$ and $sub(0,y) = 0$ and $sub(x,0) = x$ and $sub(s(x), s(y)) = sub(s,y)$
- **Factorial:** Assuming multiplication by n μ is defined as in the homework: $fact(0) = 0$ and $fact(s(0)) = s(0)$ and $fact(s(x)) = \mu(s(x), fact(x))$
- **Modulo:** We need to define some helper functions:
  $ge(0,0) = 1$ and $ge(0,n) = 0$ and $ge(n,0) = 1$ and $ge(s(n), s(m)) = ge(n,m)$
  $myIf(0,a,b) = b$ and $myIf(s(n),a,b) = b$
  Now we can define the modulo:
  $mod(0,0) = 0$ and $mod(n,0) = 0$ and $mod(0,n) = 0$ and $mod(a,b) = myIf(ge(a,b), mod(sub(a,b)), a)$

- **Division by two:** $\gamma(0) = 0$ and $\gamma(n) = myIf(ge(n, 2), s(\gamma(sub(n, 2))), 0)$