

# General Computer Science I (320101) Fall 2011

## Assignment 10: Boolean expressions, normal forms, and Landau notation

(Given Nov.24., Due Dec. 1.)

25pt

### Problem 10.1 (Properties of the domination relation)

Given the domination relation on monomials in normal form (i.e. where each variable appears at most once), prove or refute whether it possesses the following properties:

- reflexivity
- transitivity
- symmetry
- antisymmetry

For each of them either give a formal proof that the property holds, or refute via a counter-example.

30pt

### Problem 10.2 (Landau Sets)

Determine for the following functions  $f$  and  $g$  whether  $f \in O(g)$ , or  $f \in \Omega(g)$ , or  $f \in \Theta(g)$ , explain your answers.

$f$	$g$	$f$	$g$
$n^4$	$n^3 \log n$	$\frac{n^2}{\log n}$	$n^4 + \log n$
$n + n^2$	$n + 3n^3$	$\ln n$	$\log_{10}(n)$
$e^{(n^2)}$	$2^n$	$5^n$	$n!$
$n$	$\sin n$	$9^n$	$n^{\log_n(8^n)}$
12	42	$((n^3) + 3)^2$	$(3(n^3) + 2)^2$

20pt

### Problem 10.3 (CNF and DNF)

You are given the following boolean expression:

$$e := (x_1 + \overline{x_3}) * \overline{x_2 * x_1 + x_3}$$

1. Find the CNF and DNF of the boolean function that corresponds to the expression  $e$ .
2. What are the depth and the cost of the expression ?

25pt

### Problem 10.4 (Algorithm complexities)

Find the time complexity in big-O notation of the following operations:

- addition  $\pi$  of two unary natural numbers , where  $\pi$  is defined by the equations  $\pi(n, o) = o$  and  $\pi(n, s(m)) = s(\pi(n, m))$ .
- multiplication  $\mu$  of two unary natural numbers, where  $\mu$  is defined by the equations  $\mu(n, o) = o$  and  $\mu(n, s(m)) = \pi(n, \mu(n, m))$ .
- exponentiation  $\epsilon$  of unary natural numbers, where  $\epsilon(n, o) = s(o)$  and  $\epsilon(n, s(m)) = \mu(n, \epsilon(n, m))$ .
- the follwing SML function (Take one evaluation step to be a creation of a head in function `unwork` and disregard other operations.)

```

fun gigatwist lst = let
    fun unwork nil = nil |
      unwork(hd::tl) = hd::unwork(tl)

    fun nextwork(nil, _) = nil |
      nextwork(hd::tl, fnc) = fnc(lst)@nextwork(tl, fnc)

    fun nthwork 1 = unwork |
      nthwork n = let
        fun work arg = nextwork(arg, nthwork(n-1))
        in
          work
        end
  in
    nthwork(length lst) lst
  end

```