Assignment 7: SML Playground

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1 Problem 1

1.1 Substitution Application

- $\omega(s)$:

$$\begin{aligned}&\omega(h(g(x,y,c),x))\\&h(\omega(\langle g(x,y,c),x\rangle))\\&h(\omega(g(x,y,c)),\omega(x))\\&h(g(\omega(\langle x,y,c\rangle)),h(a,b))\\&h(g(\omega(x),\omega(y),\omega(c)),h(a,b))\\&h(g(h(a,b),g(c,f(c),b),c),h(a,b))\end{aligned}$$

- $\omega(t)$:

$$\begin{aligned}&\omega(g(y,h(y,x),a))\\&g(\omega(\langle x,h(y,x),a\rangle))\\&g(\omega(x),\omega(h(y,x)),\omega(a))\\&g(h(a,b),h(\omega(\langle y,x\rangle)),a)\\&g(h(a,b),h(\omega(y),\omega(x)),a)\\&g(h(a,b),h(g(c,f(c),b),h(a,b)),a)\end{aligned}$$

- $\tau(s)$:

$$\begin{aligned}&\tau(h(g(x,y,c),x))\\&h(\tau(\langle g(x,y,c),x\rangle))\\&h(\tau(g(x,y,c)),\tau(x))\\&h(g(\tau(\langle x,y,c\rangle)),a)\\&h(g(\tau(x),\tau(y),\tau(c)),a)\\&h(g(a,h(b,c),b),a)\end{aligned}$$

- $\tau(t)$:

$$\begin{aligned}&\tau(g(y,h(y,x),a))\\&g(\tau(\langle y,h(y,x),a\rangle))\\&g(\tau(y),\tau(h(y,x)),\tau(a))\\&g(h(b,c),h(\tau(\langle y,x\rangle)),a)\\&g(h(b,c),h(\tau(y),\tau(x)),a)\\&g(h(b,c),h(h(b,c),a),a)\end{aligned}$$

1.2 Commutativity Proof

No, substitution isn't commutative. The coutner-example is the following:

We have $\omega(t) = g(h(a,b), h(g(c, f(c), b), h(a,b)), a)$ and $\tau(t) = g(h(b,c), h(h(b,c), a), a)$ so $\tau(\omega(t))$:

$$
\begin{array}{l}
\tau(g(h(a,b), h(g(c, f(c), b), h(a,b)), a)) \\
g(h(\tau(a), \tau(b)), h(g(\tau(c), f(\tau(c))), \tau(b)), h(\tau(a), \tau(b)), \tau(a)) \\
g(h(a,b), h(g(b, f(b)), a), a)
\end{array}
$$

Now $\omega\tau(t)$:

$$
\begin{array}{l}
\omega(g(h(b,c), h(h(b,c), a), a)) \\
g(h(\omega(b), \omega(c)), h(h(\omega(b), \omega(c)), \omega(a)), \omega(a)) \\
g(h(b,c), h(h(b,c), a), a)
\end{array}
$$

We can see that $g(h(b,c), h(h(b,c), a), a) \neq g(h(a,b), h(g(b, f(b)), a), a)$ thus $\tau(\omega(t)) \neq \omega(\tau(t))$.