Kinematic modeling of (1,2) robots*

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Parameterization

Figure 1 shows the schematic representation of the robot, with wheel 1 being arbitrarily considered as the front wheel. Angles β are the usual angles used in the parameterization. Angles δ are "user-friendly" wheel orientation angles, defined such that:

- Both δ angles are positive counter-clockwise.
- When both angles are zero, the robot translates along x_m.
- When the robot moves in the direction of $+x_m$, the rotation speeds $\dot{\varphi}$ of both wheels are positive.

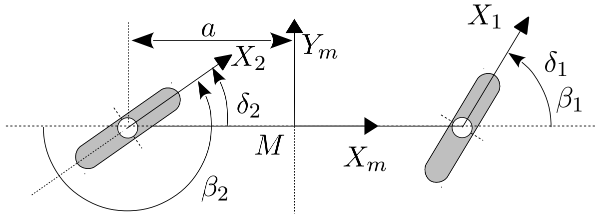

Figure 1: Sketch of the $(1, 2)$ robot.

Table 1 gives the parameters of the robot, where the β angles are replaced by their expressions as a funciton of the δ angles.

Table 1: Table of parameters of the $(1, 2)$ robot.

W	L	α	d	β	γ	φ	ψ
$1s$	a	0	0	δ_1	0	φ_{1s}	δ_1
$2s$	a	π	0	$\delta_2 + \pi$	0	φ_{2s}	δ_2

The configuration vector is:

$$q = [x, y, \theta, \delta_1, \delta_2, \varphi_{1s}, \varphi_{2s}]^T$$

*This document is not self-contained. It is does not introduce the theory and notations. You need the correponding course documents to read it.

Where x, y and θ are the coordinates of origin M of the robot frame in the absolute reference frame.

Configuration kinematic model

$$J_1 = \begin{pmatrix} \cos\delta_1 & \sin\delta_1 & a\,\sin\delta_1 \\ \cos\delta_2 & \sin\delta_2 & -a\,\sin\delta_2 \end{pmatrix} \tag{1}$$

$$C_1 = \begin{pmatrix} -\sin\delta_1 & \cos\delta_1 & a\,\cos\delta_1 \\ -\sin\delta_2 & \cos\delta_2 & -a\,\cos\delta_2 \end{pmatrix} \tag{2}$$

$J_2 = -r * I_{2*2}$ and $C_2 = \begin{pmatrix} 0 & 0 \end{pmatrix}$

$$C_1^* = \begin{bmatrix} C_{1f} \\ C_{1s} \end{bmatrix} = C_{1s} \tag{3}$$

So:

$$C_1^* = \begin{pmatrix} -\sin\delta_1 & \cos\delta_1 & a\,\cos\delta_1 \\ -\sin\delta_2 & \cos\delta_2 & -a\,\cos\delta_2 \end{pmatrix} \tag{4}$$

The determinants of the three minors corresponding to columns (1,2), (1,3) and (2,3) are:

$$\begin{cases} d_{12} = -\sin(\delta_1 - \delta_2) \\ d_{13} = a\sin(\delta_1 + \delta_2) \\ d_{23} = -2a\cos\delta_1\cos\delta_2 \end{cases}$$

The rank of C_1^* is strictly less than two if and only if $d_{12} = d_{13} = d_{23} = 0$. The only solution to this system of equations is $\delta_1 = \pm\pi/2$ and $\delta_2 = \pm\pi/2$. This is logical because, in the latter configuration the robot is equivalent to a $(2, 0)$ robot. Otherwise, $rank(C_1^*) = 2$ and $\delta_m = 1$. So we have a type $(1, 2)$ robot.

A base vector of C_1^* can be taken as:

$$\Sigma = \begin{pmatrix} 2\,\cos\delta_1\,\cos\delta_2 \\ \sin(\delta_1 + \delta_2) \\ \frac{1}{a}\sin(\delta_1 - \delta_2) \end{pmatrix} \tag{5}$$

So we get the configuration kinematic model matrix as:

$$S(q) = \begin{pmatrix} 2\,\cos\delta_1\,\cos\delta_2\,\cos\theta - \sin(\delta_1{+}\delta_2)\,\sin\theta & 0 & 0 \\ \sin(\delta_1{+}\delta_2)\,\cos\theta + 2\,\cos\delta_1\,\cos\delta_2\,\sin\theta & 0 & 0 \\ \sin(\delta_1{-}\delta_2)\,/\,a & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2\,\cos\delta_2\,/\,R & 0 & 0 \\ 2\,\cos\delta_1\,/\,R & 0 & 0 \end{pmatrix} \tag{6}$$

And the configuration kinematic model is expressed as:

$$\dot{q} = S(q)u \tag{7}$$

Where $u = [u_m, u_s]^T$ and $u_s = [\dot{\delta}_1, \dot{\delta}_2]^T$