Network Connectivity With Inhomogeneous Correlated Mobility

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***Abstract*—In this paper, we derive the critical transmission range, i.e., the smallest transmission distance of nodes such that wireless network can be connected, in large-scale clustered wireless networks. Contrary to most previous literature on independent and homogeneous mobility of nodes, we consider general settings with inhomogeneous node distribution and correlated mobility. In particular, we consider three network states based on the degree of correlation among nodes, i.e., cluster-sparse state (strong correlations), cluster-dense state (weak correlations), and cluster-transitional state (medium correlations). Under each state, we focus on the following problems: 1) how to place cluster-head nodes to minimize the critical transmission range and 2) what is the corresponding minimum critical transmission range. We derive the optimal distribution of cluster-head nodes that minimizes the critical transmission range, and show that the inhomogeneous distribution of mobile nodes leads to a smaller critical transmission range.**

***Index Terms*—Network connectivity, optimal distribution, critical transmission range.**

I. Introduction

THE fundamental problem of determining the asymptotic connectivity of wireless networks has received significant interest in recent years. In real world, base stations adjust their transmission power so as to provide a proper service region for mobile terminals. An important problem is to comprehend the relationship between transmission power and network connectivity, based on which we can use the minimal transmission power to guarantee connectivity.

A plenty of works [1]–[4] use a distance-based connection establishment policy to analyze network connectivity: given graph $G(V, E)$, where V is the set of nodes and E is the set of indirected edges between nodes. For any two nodes $i, j \in V$, an edge e_{ij} exists between i and j, if and only if (iff) the Euclidean distance between them is less than transmission range $r(n)$. The key issue is to derive a critical value of $r(n)$ for network connectivity when the number of nodes n tends to infinity.

Many previous studies consider connectivity in stationary networks and non-clustered networks [1], [5]. Gupta and Kumar [1] investigated the critical power for asymptotic connectivity in non-clustered stationary networks, and showed that with $r(n) = \sqrt{(\log n + c(n))/\pi n}$, connectivity can be established with probability 1 as $c(n)$ goes to infinity. In [5], Xue and Kumar indicated that $\Theta(\log n)$[1] nearest neighbors are needed to achieve connectivity with n randomly and independently distributed mobile terminals. However, non-clustered networks have poor scalability [6] and energy efficiency [7], and clustering is incorporated to improve various aspects of network performance, such as efficient transmission [8], secure communication [9] and higher connectivity [10]. In [10], Kumar and Lobiyal demonstrated that higher connectivity is achieved with a limited number of sensors employing the cooperative cluster transmission algorithm. Hence, clustering is a vital element for network connectivity.

Researchers adopt "k-connected" to characterize the strength of network connectivity. Specifically, a network is said to be k-connected iff any two nodes in the network have k paths between them; or a network is said to be k-connected iff it remains connected despite the failure of any $(k-1)$ nodes [11]. Wan and Yi [12] obtained the critical transmission range for k-connectivity in ad hoc networks where nodes are uniformly and independently distributed. Then, Zhao *et al.* [13] showed that k-connectivity is particularly important in secure sensor networks, as it provides reliable communication against disconnection. Furthermore, k-connectivity is adopted to attain certain properties, e.g. unique localizability [14], robust routing [15], fault tolerance [16], etc. Therefore, many works [17]–[19] have focused on k-connectivity of wireless networks. Thereinto, Yavuz *et al.* [18] studied the secure and reliable connectivity of wireless sensor networks, and presented scaling conditions such that the resulting graph contains no node with degree less than k with high probability.

Mobility has also been incorporated to increase connectivity performance of wireless networks [20]–[22]. In particular, Santi [20] explored the impact of bounded and obstacle free mobility on the critical transmission range for connectivity in mobile

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[1]The following asymptotic notations are used throughout this paper. Given functions $f(n) > 0$ and $g(n) > 0$:
(1) $f(n) = o(g(n))$ means $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.
(2) $f(n) = \omega(g(n))$ is equivalent to $g(n) = o(f(n))$.
(3) $f(n) = O(g(n))$ means $\lim_{n\to\infty} \sup \frac{f(n)}{g(n)} < \infty$.
(4) $f(n) = \Theta(g(n))$ means $f(n) = O(g(n))$, $g(n) = O(f(n))$.
(5) $f(n) = \Omega(g(n))$ is equivalent to $g(n) = O(f(n))$.

ad hoc networks. Wang *et al.* [21] investigated k-connectivity in mobile clustered networks and studied the critical transmission range for various mobility models. However, [20] and [21] adopted uncorrelated mobility, which violates the fact that mobility exhibits significant correlation degree [23]–[25]. Hitherto [26], [27] studied the influence of correlated mobility on network connectivity, however, the assumption of uniform node distribution limits the application of results to more general scenarios. Thus, we are curious about how the distributional inhomogeneity of nodes impacts network connectivity under correlated mobility settings.

In this paper, we consider a model of inhomogeneous correlated mobility based on the Reference Point Group Mobility (RPGM) model introduced in [23]. The network contains two kinds of nodes: cluster-member nodes and cluster-head nodes. n cluster-member nodes are partitioned into clusters, and each cluster has a home point which moves over the whole network. The positions of nodes belonging to the same cluster are restricted within its cluster region (centered at the home point with radius R). n^α cluster-head nodes are fixed over the network. We would like to emphasize that the cluster-head nodes can be manually determined as a priori. Different from the previous work [26] assuming uniform distribution of cluster-head nodes and home points, we extend our design approach by considering the placement of cluster-head nodes with the computation of critical transmission range. Correspondingly, we address the following problems:

- What is the optimal placement of cluster-head nodes such that the critical transmission range can be minimized?
- What is the corresponding critical transmission range?

Based on the degree of correlation among nodes, we consider three network states: cluster-sparse state, cluster-dense state and cluster-transitional state. Accordingly, we derive the critical transmission range for each state. The contributions of this paper include the following aspects:

- We propose an inhomogeneous correlated mobility model, where we consider both distributional inhomogeneity and correlated mobility.
- We derive the optimal distribution of cluster-head nodes to improve network connectivity, and compute the corresponding critical transmission range for each state: cluster-sparse state, cluster-dense state, cluster-transitional state.

The rest of this paper is organized as follows. Section II states the network model. Section II presents main results and intuitions. We derive the exact values of the critical transmission range for the cluster-sparse state, cluster-dense state and cluster-transitional state in Section IV, Section V, and Section VI, respectively. Furthermore, Section VII derives the optimal probability density function (pdf) of cluster-head nodes. Section VIII presents some discussions. Section IX concludes this paper and outlines future works.

II. Network Model

A. Network Architecture

We consider a clustered network consisting of n cluster-member nodes (clients) and n^α ($\alpha \in (0, 1]$) cluster-head nodes

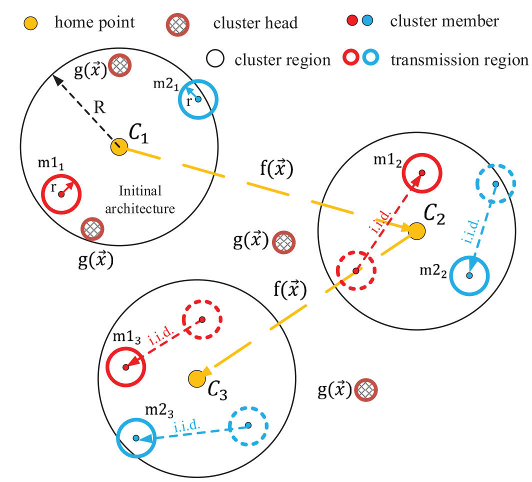

Fig. 1. The inhomogeneous correlated mobility illustration. m1 and m2 represent two cluster-member nodes respectively, and the subscripts in this picture denote three time slots.

distributed in a unit square $\mathcal{O}$ with wraparound conditions to avoid border effects. The role of a cluster-head node in the clustered network is similar to an access point which serves all the cluster-member nodes. We assume that cluster-member nodes are partitioned into m clusters, where $m = n^\gamma$ ($\gamma \in (0, 1]$), and each cluster-member node belongs to only one cluster. Each cluster has a home point and covers a circular area centering in the home point with radius $R = \Theta(n^\beta)$ ($\beta \in (-\infty, 0]$). In addition, we assume that cluster-head nodes are independently distributed according to pdf $g(\overrightarrow{x})$ over $\mathcal{O}$, home points are independently distributed according to pdf $f(\overrightarrow{x})$, and cluster-member nodes are uniformly and independent distributed within their corresponding cluster regions. We consider the scenario that $g(\overrightarrow{x})$ and $f(\overrightarrow{x})$ are continuous, and $f(\overrightarrow{x})$ does not vanish over $\mathcal{O}$. For simplicity, we assume each cluster is comprised of an integer number $\varpi = n/m = n^{1-\gamma}$ of cluster-member nodes, and the same applies to n^α and n^γ. The initial architecture is depicted in the first part of Fig. 1. The important notations are presented in Table I, and we explain the notation settings in the following remark.

Remark 1:

- As the number of cluster-head nodes is expected to increases with n, we have $\alpha > 0$; however, it should not exceed the number of cluster-member nodes, so $\alpha \leq 1$.
- Since n cluster-member nodes are uniformly allocated into n^γ clusters and each cluster consists of at least one cluster-member node, we have $\gamma \leq 1$. Since $\gamma \leq 0$ is a trivial case, we exclude it from our analysis.
- We have $\beta \leq 0$ due to the reason that the area of a cluster should not exceed that of $\mathcal{O}$.

B. Inhomogeneous Correlated Mobility

Time is divided into k slots with unit length. The cluster-head nodes keep static all the time, and the cluster-member nodes belonging to the same cluster move over the network area in a correlated fashion. Based on the Reference Point Group

TABLE I
IMPORTANT NOTATIONS

Notation	Definition
α	The cluster head exponent, $0 < \alpha \leq 1$.
β	The cluster radius exponent, $\beta \leq 0$.
γ	The cluster exponent, $0 < \gamma \leq 1$.
ϖ	The number of cluster members per cluster.
R	The radius of each cluster, $R = \Theta(n^{\beta})$.
$\mathcal{C}_j$	The jth cluster $(j = 1, 2, \ldots, m)$.
$\mathcal{F}_j$	Event that $\mathcal{C}_j$ is disconnected in k slots.
$g(\overrightarrow{x})$	The pdf of cluster-head nodes.
$f(\overrightarrow{x})$	The pdf of home points.
$\mathcal{C}_r(\overrightarrow{x})$	Circular area centering in $\overrightarrow{x}$ with radius r.

Mobility (RPGM) model, we consider a model of inhomogeneous correlated mobility. The movement of a cluster-member node consists of two parts: the home point movement and the cluster-member node movement within its cluster region. Specifically, at the beginning of each time slot, each home point independently chooses a position within $\mathcal{O}$ according to $f(\overrightarrow{x})$ and moves to the position immediately; then each cluster-member node uniformly and independently chooses a location within its corresponding cluster region and moves to the location immediately. In the rest of the time slot, the home points and cluster-member nodes remain static. In addition, we assume that all the cluster-member nodes do not record their past movement trajectory and that of the cluster-head nodes they have connected.

According to the proposed mobility model, the position of a cluster-member node is determined by both the movement of its corresponding cluster home point and its relative movement within the cluster region. Home points are distributed according to $f(\overrightarrow{x})$ in each slot, which stands for the distributional inhomogeneity. To sum up, our model can capture *inhomogeneous* and *correlated* mobility of cluster-member nodes.

C. Routing Protocol

Time is divided into slots with unit length. We assume that a packet can be forwarded one hop in each time slot, and the maximum transmission delay of one packet is k time slots[2]. We also assume that a period consists of k time slots, and there is a timer to record the number of hops that a packet has been forwarded. Then, we specify how a packet is forwarded from cluster-member nodes to one of cluster-head nodes. For a given cluster-member node, the initial value of timer is 1 and timer increases by 1 if no cluster-head node is located in its transmission range in each slot. We paraphrase this as follows: The cluster-member node can act as the relay of itself if it can not connect to a cluster-head node during its movements, and it sends the packet to a cluster-head node once it meets. When the timer is greater than k, the packet is discarded and we regard this period as *failed*. We do not utilize multi-hop strategy since establishing a path from a given cluster-member node to any cluster-head node is an extremely energy-consuming process [21].

[2]On the one hand, we do not investigate the influence of k on network connectivity, hence there is no problem if we set $k = \Theta(1)$; On the other hand, since a large k decrease greatly transmit efficiency, it is reasonable to regard k as a constant value.

D. Network State

In this paper, network can be divided into three states according to the correlation degree among nodes. By comparing the average coverage $(1/n^{\alpha})$ of each cluster-head node to the cluster region $\pi R^2 = \Theta(n^{2\beta})$, we obtain the following three states [26].

(**C1**). *Cluster-sparse state*

When $\pi R^2 = o(1/n^{\alpha})$, we have $\alpha + 2\beta < 0$. cluster region is sufficiently small compared to the average coverage of each cluster-head node. Clusters are sparsely distributed over the whole network. The density of each cluster $\varpi/\pi R^2 = \Theta(n^{1-2\beta-\gamma})$ is large, which leads to strong correlations among nodes in the same cluster. We can regard each cluster as an entirety.

(**C2**). *Cluster-dense State*

When $\pi R^2 = \omega(1/n^{\alpha})$, we have $\alpha + 2\beta \geq (1 - \gamma)/k$. In this state, cluster region is relatively large, and clusters are densely distributed in $\mathcal{O}$. Since the density of each cluster in this state is relatively small compared to that in cluster-sparse state, nodes show weak correlations. We can regard each cluster-member node as an independent node.

(**C3**). *Cluster-transitional State*

This state occurs when $0 \leq \alpha + 2\beta < (1 - \gamma)/k$, and $\pi R^2 = \omega(1/n^{\alpha})$ holds. It is the transitional phase between cluster-sparse state and cluster-dense state. Nodes show medium correlations based on the density of cluster-member nodes.

E. Definition of Connectivity

Now we give a definition of connectivity in inhomogeneous correlated network, then we introduce the definition of critical transmission range.

A cluster-member node is connected if it can reach a cluster-head node within k time slots. A cluster-member node is disconnected if it can not reach any cluster-head node in k time slots. If all the cluster-member nodes are connected, the clustered network has full connectivity. We denote $\mathcal{G}$ as the initial network where a path connects all the cluster-head nodes. During each time slot λ $(\lambda = 1, 2, \ldots, k)$, an edge e_{ij} is added between a cluster-member node i and a cluster-head node j into $\mathcal{G}$ if the Euclidean distance between i and j is less than $r(n)$. Recall that for mobile clustered network $\mathcal{G}$, we say the network $\mathcal{G}$ is connected iff all the cluster-member nodes can reach at least one cluster-head node in k time slots. We focus on the problem of determining $r(n)$, which guarantees $\mathcal{G}$ is asymptotically connected with probability one, i.e., the probability that $\mathcal{G}$ is connected goes to one as $n \to \infty$. The determined $r(n)$ is critical transmission range $r_c(n)$, which will be formally defined subsequently. Let $\mathcal{M}_n$ denote the event that all the cluster-member nodes are connected and $\mathbb{P}(\mathcal{M}_n)$ denote the probability that the network is connected. Then we define the critical transmission range.

Definition 1: In an inhomogeneous correlated network $\mathcal{G}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r_c(n))$, $r_c(n)$ is the critical transmission range if for constants c' and c, it satisfies

$$\lim_{n\to\infty} \mathbb{P}(\mathcal{M}_n) = 1, \text{ if } r(n) \geq c r_c(n), c > 1;$$
$$\lim_{n\to\infty} \mathbb{P}(\mathcal{M}_n) < 1, \text{ if } r(n) \leq c' r_c(n), 1 > c' > 0.$$

III. Main Results and Intuitions

In this section, we summarize our main results and the result intuitions. Let ρ be equivalent to $\iint_{\mathcal{O}} \log f(\overrightarrow{x})\, d\overrightarrow{x}$, which represents the "degree of indeterminancy" of the distribution of home points.

A. Main Results

General conditions **(C0)**, as the prerequisites for our results and subsequent proofs, are presented as follows.

(**C0**). *General conditions.*

(0.1). $|\log f(\overrightarrow{x})| = O(\log n)$, and $f(\overrightarrow{x}) = \omega(1/n^{\phi})$, where ϕ equals (γ/k), $(1/k)$, $((\alpha+2\beta)+\gamma/k)$ in three states respectively. (0.2). $\forall \overrightarrow{x_1}, \overrightarrow{x_2}, (|\overrightarrow{x_1} - \overrightarrow{x_2}| \leq \tau) \in \mathcal{O}$: $|\log f(\overrightarrow{x_1}) - \log f(\overrightarrow{x_2})| = o(1)$, where τ is defined later. (0.3). $\iint_{\mathcal{C}_{r(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} = \pi r^2(n) g(\overrightarrow{x_\lambda}) + \sigma$, where $\sigma \in [-o(1/n^{\alpha}), o(1/n^{\alpha})]$ and $r(n) = \Theta(r_c(n))$. (0.4). $\iint_{\mathcal{C}_{r(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} = o(1)$, where $r(n) = \Theta(r_c(n))$. Condition $|\log f(\overrightarrow{x})| = O(\log n)$ in (0.1) holds due to the probability that nodes congregate in one position converges to zero. Note that if x_1 locates far from x_2, the difference between $f(\overrightarrow{x_1})$ and $f(\overrightarrow{x_2})$ can be large. Conditions (0.3) and (0.4) indicate that the fluctuation of $g(\overrightarrow{x})$ is relatively gently over the whole network.

(**C1**). *Cluster-sparse state* ($\alpha + 2\beta < 0$).

The pdf of cluster-head nodes that minimizes the critical transmission range is $g(\overrightarrow{x}) = \frac{\gamma \log n + k \log f(\overrightarrow{x})}{\gamma \log n + k\rho}$. The corresponding critical transmission range is $r_{c1}(n) = \sqrt{\frac{\gamma \log n + k\rho}{k\pi n^{\alpha}}}$.

(**C2**). *Cluster-dense state* ($\alpha + 2\beta \geq \frac{1-\gamma}{k}$).

The pdf of cluster-head nodes that minimizes the critical transmission range is $g(\overrightarrow{x}) = \frac{\log n + k \log f(\overrightarrow{x})}{\log n + k\rho}$. The corresponding critical transmission range is $r_{c2}(n) = \sqrt{\frac{\log n + k\rho}{k\pi n^{\alpha}}}$.

(**C3**). *Cluster-transitional state* ($0 \leq \alpha + 2\beta < \frac{1-\gamma}{k}$).

The pdf of cluster-head nodes that minimizes the critical transmission range is $g(\overrightarrow{x}) = \frac{[k(\alpha+2\beta)+\gamma]\log n + k \log f(\overrightarrow{x})}{[k(\alpha+2\beta)+\gamma]\log n + k\rho}$. The corresponding critical transmission range is $r_{c3}(n) = \sqrt{\frac{[k(\alpha+2\beta)+\gamma]\log n + k\rho}{k\pi n^{\alpha}}}$.

B. Results Intuitions

(**C1**). *Cluster-sparse state* ($\alpha + 2\beta < 0$).

In this state, cluster region is relatively small owing to $\pi R^2 = o(1/n^{\alpha})$, and cluster-member nodes show strong correlations. Clusters are sparsely distributed over the whole network, thus we can regard each cluster as an entirety. $g(\overrightarrow{x}) = \frac{\gamma \log n + k \log f(\overrightarrow{x})}{\gamma \log n + k\rho}$ suggests that the initial locations of cluster-head nodes can be adjusted by $f(\overrightarrow{x})$ to improve network connectivity. Under this initial distribution of home points, taking the inhomogeneous distribution of nodes into results of [26], we have $r_{c1}(n) = \sqrt{\frac{\log m + k\rho}{k\pi n^{\alpha}}} = \sqrt{\frac{\gamma \log n + k\rho}{k\pi n^{\alpha}}}$.

(**C2**). *Cluster-dense state* ($\alpha + 2\beta \geq \frac{1-\gamma}{k}$).

Contrary to the cluster-sparse state, cluster region is relatively large due to $\pi R^2 = \omega(1/n^{\alpha})$, and cluster-member nodes show weak correlations. Clusters are densely distributed over the whole network, hence each cluster-member node can be seen as an individual. When $g(\overrightarrow{x})$ satisfies the optimal distribution $\frac{\gamma \log n + k \log f(\overrightarrow{x})}{\gamma \log n + k\rho}$, each cluster-member node has a critical transmission range $r_{c2}(n) = \sqrt{\frac{\log n + k\rho}{k\pi n^{\alpha}}}$.

(**C3**). *Cluster-transitional state* ($0 \leq \alpha + 2\beta < \frac{1-\gamma}{k}$).

This is the transitional state between cluster-sparse state and cluster-dense state. In this state, we have $r_{c3}(n) = o(R)$ and $\pi R^2 = \omega(1/n^{\alpha})$. We cannot regard cluster-member nodes in a same cluster as an entirety, neither can we take each cluster-member node as an individual. Each cluster is divided into $n^{k(\alpha+2\beta)}$ sub-clusters, and nodes in the same sub-cluster can be regarded as an entirety. Thus, we have $mn^{k(\alpha+2\beta)}$ alternative nodes, and the critical transmission range under the optimal distribution $g(\overrightarrow{x}) = \frac{[k(\alpha+2\beta)+\gamma]\log n + k \log f(\overrightarrow{x})}{[k(\alpha+2\beta)+\gamma]\log n + k\rho}$ is $r_{c3}(n) = \sqrt{\frac{[k(\alpha+2\beta)+\gamma]\log n + k\rho}{k\pi n^{\alpha}}}$.

Remark 2: In the three states, we present the optimal $g(\overrightarrow{x})$ of cluster-head nodes, which is optimized by $f(\overrightarrow{x})$ to minimize the $r_c(n)$. We discuss it in Section VII. Based on the optimal $g(\overrightarrow{x})$, we derive the corresponding $r_c(n)$ for each state.

In the following, we derive the $r_c(n)$ under the optimal $g(\overrightarrow{x})$ in each state rigorously. We consider the unique property in each state, then investigate the probability that the network is disconnected.

IV. Critical Transmission Range for Cluster-Sparse State

In this section, we consider the cluster-sparse state. We denote $\mathbb{P}_{c1}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$ as the probability that network $\mathcal{G}_{c1}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$[3] is unconnected. The main results in this state are as follows.

Theorem 1: Under the optimal $g(\overrightarrow{x}) = \frac{\gamma \log n + k \log f(\overrightarrow{x})}{\gamma \log n + k\rho}$ in network $\mathcal{G}_{c1}$ and $\tau = (2r_{c1}(n) + R)$ in condition (C0) , the critical transmission range is $r_{c1}(n) = \sqrt{\frac{\gamma \log n + k\rho}{k\pi n^{\alpha}}}$.

A. Necessary Condition of Theorem 1

If we prove $\liminf_{n\to\infty} \mathbb{P}_{c1}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n)) > 0$ when $\pi r^2(n) = \frac{\gamma \log n + k\rho + \xi(n)}{kn^{\alpha}}$, where $0 < \lim_{n\to\infty} \xi(n) = \xi < +\infty$, we can demonstrate that $\pi r^2(n) \geq \frac{\gamma \log n + k\rho}{kn^{\alpha}}$ is necessary for the connectivity of network $\mathcal{G}_{c1}$.

[3]In this paper, we use $\mathbb{P}_{c1}$ and $\mathbb{P}_{c1}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$ interchangeably, so does $\mathbb{G}_{c1}$ and $\mathbb{G}_{c1}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$. The same applies to similar situations in later sections.

We prove the necessary condition of Theorem 1 by the classical methodology introduced in [1]. In this state, cluster-member nodes show strong correlations ($\pi R^2 = o(1/n^\alpha)$). Therefore, for the estimation of disconnection probability between cluster-member nodes in two clusters, regarding a cluster as an entirety is more accurate than analyzing them separately.

To estimate $\mathbb{P}_{c1}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$, we study the probability that a cluster is disconnected in k time slots, then prove the necessary condition. We first give some notations.

- $\mathcal{C}_j$: the jth cluster ($j = 1, 2, \ldots, m$).
- $\mathcal{C}_{r(n)}(\overrightarrow{x})$: circular area centering in $\overrightarrow{x}$ with radius $r(n)$.
- $\mathcal{F}_j$: the event that $\mathcal{C}_j$ is disconnected in k time slots.

Then, we present the following lemmas to prove Theorem 1. Note that given $\alpha + 2\beta < 0$ in this state, we have $\pi R^2 = o(1/n^\alpha)$ and $r_{c1}(n) = \omega(R)$.

Lemma 1: $\forall j = 1, 2, \ldots, m.$, $\lambda = 1, 2, \ldots, k.$, $\mathbb{P}(\mathcal{F}_j)$ is bounded by

$$\mathbb{P}(\mathcal{F}_j) \le \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda}) \left(1 - \iint_{\mathcal{C}_{r(n)-R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)^{n^\alpha} d\overrightarrow{x_\lambda} \right),$$

$$\mathbb{P}(\mathcal{F}_j) \ge \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda}) \left(1 - \iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)^{n^\alpha} d\overrightarrow{x_\lambda} \right),$$

where $r(n) = \sqrt{(\gamma \log n + k\rho + \xi)/k\pi n^\alpha}$ and $R = \Theta(n^\beta)$.

Proof: Let O_h and H_j denote the positions of the hth cluster-head node and the jth home point, respectively. In time slot λ, the region centering in H_j with radius $(r(n) - R)$ is $\mathcal{C}_{r(n)-R}(H_j)$, which is shown in Fig. 2(a). If $O_h \in \mathcal{C}_{r(n)-R}(H_j)$, H_j can connect to O_h, which has done with the exact positions of cluster-member nodes. Otherwise, we cannot ensure cluster connectivity because cluster-member nodes may be aggregated in the blind spot shown in Fig. 2(a). Thus $\mathcal{C}_{r(n)-R}(H_j)$ is the minimum cluster communication region of C_j in time slot λ. Besides, the region centering in H_j with radius $(r(n) + R)$ is $\mathcal{C}_{r(n)+R}(H_j)$, which is shown in Fig. 2(b). In time slot λ, $O_h \in \mathcal{C}_{r(n)+R}(H_j)$ leads to that O_h connects to some cluster-member nodes. However, if $O_h \notin \mathcal{C}_{r(n)+R}(H_j)$, no cluster-member node can reach O_h in one time slot because the minimum distance between any point outside $\mathcal{C}_{r(n)+R}(H_j)$ and inside cluster is longer than $r(n)$. Therefore, $\mathcal{C}_{r(n)+R}(H_j)$ represents the maximum cluster communication region of C_j.

The upper bound of $\mathbb{P}(\mathcal{F}_j)$ is obtained when all the cluster-head nodes are outside the region $\mathcal{C}_{r(n)-R}(H_j)$ in k time slots. If a cluster-head node is located in this region in any time slot, it will fully cover the cluster region, which makes the cluster connected. The lower bound of $\mathbb{P}(\mathcal{F}_j)$ is derived under the circumstance that all the cluster-head nodes locate outside region $\mathcal{C}_{r(n)+R}(H_j)$ during k time slots. In other words, all the cluster-head nodes cannot reach cluster region in any time slot, which makes cluster H_j unconnected. ∎

Lemma 2: If $r(n) = \sqrt{\frac{\gamma \log n + k\rho + \xi}{k\pi n^\alpha}}$, $\alpha + 2\beta < 0$, for fixed $\theta < 1$ and sufficiently large n, we have

$$m \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda}) (1 - \iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x})^{n^\alpha} d\overrightarrow{x_\lambda} \right) \ge \theta e^{-a\xi},$$

where a is a finite positive number and $R = \Theta(n^\beta)$.

Proof: Since $r(n) = \Theta(r_{c1}(n))$, $r(n) = \omega(R)$ holds. We then have

$$\begin{aligned}
&\log \left(f(\overrightarrow{x_\lambda}) \left(1 - \iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)^{n^\alpha} \right) \\
&= \log f(\overrightarrow{x_\lambda}) + n^\alpha \log \left(1 - \iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right) \\
&= \log f(\overrightarrow{x_\lambda}) - n^\alpha \sum_{i=1}^{\infty} \frac{\left(\iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)^i}{i} \\
&= \log f(\overrightarrow{x_\lambda}) - n^\alpha \left(\sum_{i=1}^{2} \frac{\left(\iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)^i}{i} + \Delta(n) \right).
\end{aligned} \tag{1}$$

The second step of (1) follows from the Taylor Expansion.

For sufficiently large n, we have:

$$\begin{aligned}
&\iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \\
&= \iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} \frac{\gamma \log n + k \log f(\overrightarrow{x})}{\gamma \log n + k \iint_{\mathcal{O}} \log f(\overrightarrow{x})\, d\overrightarrow{x}} d\overrightarrow{x} \\
&= \frac{\pi (r(n) + R)^2 \gamma \log n + k \iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} \log f(\overrightarrow{x})\, d\overrightarrow{x}}{\gamma \log n + k \iint_{\mathcal{O}} \log f(\overrightarrow{x})\, d\overrightarrow{x}} \\
&= \frac{\pi (r(n) + R)^2 \gamma \log n + k\pi (r(n) + R)^2 \left(\log f(\overrightarrow{x_\lambda}) + o(1) \right)}{\gamma \log n + k \iint_{\mathcal{O}} \log f(\overrightarrow{x})\, d\overrightarrow{x}} \\
&= \pi (r(n) + R)^2 g(\overrightarrow{x_\lambda}) + o(\frac{1}{n^\alpha}).
\end{aligned} \tag{2}$$

The third step of (2) holds since condition $\forall \overrightarrow{x_1}, \overrightarrow{x_2}, (|\overrightarrow{x_1} - \overrightarrow{x_2}| \le 2r_c(n) + R) \in \mathcal{O}, \quad |\log f(\overrightarrow{x_1}) - \log f(\overrightarrow{x_2}|) = o(1)$ is satisfied.

$\Delta(n)$ can be upper bounded as

$$\begin{aligned}
\Delta(n) &= \sum_{i=3}^{\infty} \frac{\left(\iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)^i}{i} \\
&\le \frac{1}{3} \sum_{i=3}^{\infty} \left(\iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)^i \\
&= \frac{\left(\iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)^3}{3 \left(1 - \iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)} \\
&\le \frac{\left(\iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \right)^2}{3}.
\end{aligned} \tag{3}$$

Since inequality $\iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \le 1/2$ holds for sufficiently large n, we have $1 - \iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x} \ge \iint_{\mathcal{C}_{r(n)+R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\, d\overrightarrow{x}$. Consequently, the last step of (3) is satisfied. Applying $r(n)$, (2), (3) to (1), the left part of the final result in (1) is lower bounded as follows.

$$
\begin{aligned}
&\log\left(f(\vec{x_\lambda})\left(1-\iint_{\mathcal{C}_{r(n)+R}(\vec{x_\lambda})} g(\vec{x})\,d\vec{x}\right)^{n^\alpha}\right)\\
&\geq \log f(\vec{x_\lambda}) - n^\alpha\left(\left(\iint_{\mathcal{C}_{r(n)+R}(\vec{x_\lambda})} g(\vec{x})\,d\vec{x}\right.\right.\\
&\quad\left.\left.+\frac{5}{6}\left(\iint_{\mathcal{C}_{r(n)+R}(\vec{x_\lambda})} g(\vec{x})\,d\vec{x}\right)^2\right)\right) \qquad (4)\\
&\geq -\frac{g(x_\lambda)\xi+\gamma\log n}{k} - \Theta(n^{\frac{\alpha+2\beta}{2}}\sqrt{\log n})\\
&\quad -\Theta(n^{\alpha+2\beta}) - \Theta\left(\frac{\log^2 n}{k^2 n^\alpha}\right) - \Theta\left(\frac{\sqrt{\log n}}{kn^{\frac{\alpha}{2}-\beta}}\right)\\
&\geq -\frac{g(\vec{x_\lambda})\xi+\gamma\log n}{k} - \epsilon_\lambda.
\end{aligned}
$$

By taking the exponent of both sides in (4) and using $\theta = e^{-\sum_{\lambda=1}^{k}\epsilon_\lambda} < 1$, we have

$$
\begin{aligned}
&m\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(e^{-\frac{g(\vec{x_\lambda})\xi+\gamma\log n}{k}}e^{-\epsilon_\lambda}\,d\vec{x_\lambda}\right)\\
&=\theta\iint_{\mathcal{O}^k} e^{-\frac{\xi\sum_{\lambda=1}^{k} g(\vec{x_\lambda})}{k}}\,d\vec{x_1}\ldots d\vec{x_k} = \theta e^{-a\xi}. \qquad (5)
\end{aligned}
$$

The last step is obtained due to the mean value theorem of integrals, and hence a is a finite positive number. Until now, we complete the proof of Lemma 2. ■

Lemma 3: If $r(n) = \sqrt{\frac{\gamma\log n+k\rho+\xi}{k\pi n^\alpha}}$, $\alpha+2\beta<0$, for fixed $\eta>1$ and sufficiently large n, we have

$$
m\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\vec{x_\lambda})(1-\iint_{\mathcal{C}_{r(n)-R}(\vec{x_\lambda})} g(\vec{x})\,d\vec{x})^{n^\alpha}\,d\vec{x_\lambda}\right) \leq \eta e^{-a\xi},
$$

where a is a finite positive number and $R=\Theta(n^\beta)$.

Proof: For sufficiently large n, we obtain that

$$
\begin{aligned}
&f(\vec{x_\lambda})\left(1-\iint_{\mathcal{C}_{r(n)-R}(\vec{x_\lambda})} g(\vec{x})\,d\vec{x}\right)^{n^\alpha}\\
&\leq f(\vec{x_\lambda})\exp\left\{-n^\alpha\iint_{\mathcal{C}_{r(n)-R}(\vec{x_\lambda})} g(\vec{x})\,d\vec{x}\right\}\\
&\leq f(\vec{x_\lambda})\exp\{-\pi n^\alpha(r(n)-R)^2 g(\vec{x_\lambda})\} \qquad (6)\\
&= e^{-\frac{\gamma\log n+g(\vec{x_\lambda})\xi}{k}+\Theta(n^{\frac{\alpha+2\beta}{2}}\sqrt{\frac{\log n}{k}})-\Theta(n^{\alpha+2\beta})}\\
&\leq \eta_\lambda n^{-\frac{\gamma}{k}}e^{-\frac{g(\vec{x_\lambda})\xi}{k}}.
\end{aligned}
$$

Substituting $(r(n)-R)$ for $(r(n)+R)$, (1) holds, hence the second step of (6) is established. Let $\eta=\prod_{\lambda=1}^{k}\eta_\lambda$, and we have that η is greater than 1 but close to 1. Applying (6) to Lemma 3, we complete the proof. ■

Proposition 1: If $r(n) = \sqrt{\frac{\gamma\log n+k\rho+\xi(n)}{k\pi n^\alpha}}$, $\alpha+2\beta<0$, and $0<\lim_{n\to\infty}\xi(n)=\xi<+\infty$, we have

$$
\liminf_{n\to\infty}\mathbb{P}_{c1}(n,\alpha,\beta,\gamma,f(\vec{x}),g(\vec{x}),r(n)) \geq e^{-a\xi}(1-e^{-a\xi}),
$$

where a is a finite positive number.

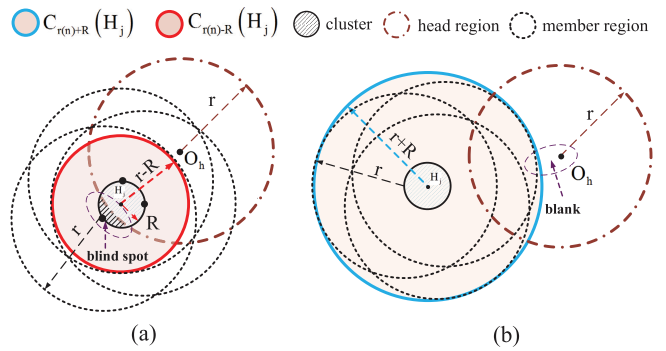

Fig. 2. (a). The overlapping portion of all the cluster-member nodes' coverage ares in cluster H_j is $C_{r(n)-R}(H_j)$, which is the minimum cluster connectivity region. $\mathcal{C}_{r(n)-R}(H_j)$: If the cluster-head node lies outside the circle centering in H_j with radius $(r(n)-R)$, a blind spot exists. Cluster-member nodes located in the blind spot can not communicate with the cluster-head node. Otherwise all the cluster-member nodes can exchange packets with the cluster-head node. (b). While the coverage by all the cluster-member nodes in cluster H_j is $C_{r(n)+R}(H_j)$, which is the maximum cluster communication region. $\mathcal{C}_{r(n)+R}(H_j)$: If the cluster-head node lies outside the circle centering in H_j with radius $(r(n)+R)$, the blank, existing between the maximum cluster communication region and the cluster-head node O_h, leads to a result that none of cluster-member nodes can communicate with O_h. Note that r is short for $r(n)$ in this figure.

Proof: We first study the case where $r(n) = \sqrt{\frac{\gamma\log n+k\rho+\xi}{k\pi n^\alpha}}$ with a fixed ξ. Let $\mathcal{K}_i^c$ denote the event that $\mathcal{C}_i$ is the only disconnected cluster during k time slots, then we bound the disconnection probability between two clusters as follows.

$$
\begin{aligned}
&\mathbb{P}_{c1}(n,\alpha,\beta,\gamma,f(\vec{x}),g(\vec{x}),r(n)) \geq \sum_{i=1}^{m}\mathbb{P}(\mathcal{K}_i^c)\\
&\geq \sum_{i=1}^{m}\left(\mathbb{P}(\mathcal{F}_i)-\sum_{j\neq i}\mathbb{P}(\mathcal{F}_i\cap\mathcal{F}_j)\right) \geq \sum_{i=1}^{m}\mathbb{P}(\mathcal{F}_i) - \left(\sum_{i=1}^{m}\mathbb{P}(\mathcal{F}_i)\right)^2\\
&\geq \theta e^{-a\xi} - (\eta e^{-a\xi})^2 \geq \theta e^{-a\xi} - (1+\mu)e^{-2a\xi} \qquad (7)
\end{aligned}
$$

is established for any $\mu>0$ and all the $n>N(\mu,\theta,\xi)$.

The third step of (7) is obtained since $\mathcal{F}_i$ and $\mathcal{F}_j$ are independent of each other. The fourth step of (7) follows from Lemmas 1-3. Now, considering the case where $\lim_{n\to\infty}\xi(n)=\xi$, for all the $\mu>0$ and all the $n>N'(\mu)$, we have $\xi(n)<\xi+\mu$. As $\mathbb{P}_{c1}$ is monotone decreasing with $\xi(n)$, we have

$$
\begin{aligned}
&\mathbb{P}_{c1}(n,\alpha,\beta,\gamma,f(\vec{x}),g(\vec{x}),r(n)) \geq \theta e^{-a(\xi+\mu)}\\
&\quad -(1+\mu)e^{-2a(\xi+\mu)},
\end{aligned}
$$

for all the $n>\max\{N(\mu,\theta,\xi),N'(\mu)\}$. Taking the limits of both sides in the above inequality, we have

$$
\begin{aligned}
&\liminf_{n\to\infty}\mathbb{P}_{c1}(n,\alpha,\beta,\gamma,f(\vec{x}),g(\vec{x}),r(n))\\
&\qquad \geq \theta e^{-a(\xi+\mu)} - (1+\mu)e^{-2a(\xi+\mu)}. \qquad (8)
\end{aligned}
$$

Since (8) holds for all the $\mu>0$ and $\theta<1$, Proposition 1 is proved. Note that $0<\xi<+\infty$, thus the right part of (8) is larger than zero. ■

Consequently, we give a summary of the necessity of $r_{c1}(n)$ for network connectivity: In the cluster-sparse state, network is to have failed periods with positive probability bounded away from zero if $\pi r^2(n) = \frac{\gamma \log n + k\rho + \xi(n)}{k\pi n^\alpha}$ and $\lim_{n\to\infty} \xi(n) = \xi < +\infty$, which means $\pi r^2(n) \geq \frac{\gamma \log n + k\rho}{k\pi n^\alpha}$ is necessary for the connectivity of inhomogeneous correlated network. We complete the proof of necessity.

B. Sufficient Condition of Theorem 1

To prove the sufficiency of $r_{c1}(n)$ for given $g(\overrightarrow{x})$, we need to show that the probability that at least one cluster is unconnected equals zero when the transmission range of each node is $cr_{c1}(n)(c > 1)$, which is equivalent to prove

$$\lim_{n\to\infty} \mathbb{P}(\bigcup_{j=1}^{m} \mathcal{F}_j) = 0. \tag{9}$$

Proof: Using the union bound, we have

$$\begin{aligned}
&\mathbb{P}(\bigcup_{j=1}^{m} \mathcal{F}_j) \leq \sum_{j=1}^{m} \mathbb{P}(\mathcal{F}_j)\\
&\leq m \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda}) \left(1 - \iint_{\mathcal{C}_{r(n)-R}(\overrightarrow{x_\lambda})} g(\overrightarrow{x}) d\overrightarrow{x}\right)^{n^\alpha} d\overrightarrow{x_\lambda}\right)\\
&\leq m \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda}) e^{-n^\alpha \pi (cr_{c1}(n)-R)^2 g(\overrightarrow{x_\lambda}) - o(1)} d\overrightarrow{x_\lambda}\right)\\
&\leq \frac{1}{n^{\gamma(c^2-1)}} \frac{e^{\Theta\left(n^{\frac{\alpha+2\beta}{2}}\sqrt{\frac{\log n}{k}}\right)}}{e^{\Theta(n^{\alpha+2\beta})+o(1)}} \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda})^{1-c^2} d\overrightarrow{x_\lambda}\right)\\
&= \frac{b}{n^{\gamma(c^2-1)}} \frac{e^{\Theta\left(n^{\frac{\alpha+2\beta}{2}}\sqrt{\frac{\log n}{k}}\right)}}{e^{\Theta(n^{\alpha+2\beta})+o(1)}},
\end{aligned} \tag{10}$$

where $b = \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda})^{1-c^2} d\overrightarrow{x_\lambda}\right)$. Based on Jensen's Inequality, we have

$$\iint_{\mathcal{O}} \left(f(\overrightarrow{x_\lambda})^{1-c^2} d\overrightarrow{x_\lambda}\right) \geq (\iint_{\mathcal{O}} f(\overrightarrow{x_\lambda}) d\overrightarrow{x_\lambda})^{1-c^2} = 1. \tag{11}$$

Since $f(\overrightarrow{x}) = \omega(1/n^{\gamma/k})$, we obtain $b = o(n^{\gamma(c^2-1)})$.

Taking the limits of both sides in (10), (9) holds since $c > 1, \gamma > 0$ and $\alpha + 2\beta < 0$. ■

From the other perspective, (9) also embodies that network $\mathcal{G}_{c1}$ is asymptotically connected with probability one, conforming to Definition 1. Therefore, the proof of sufficiency is completed.

V. Critical Transmission Range for Cluster-Dense State

In this state, we consider the cluster-dense state. We denote $\mathbb{P}_{c2}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$ as the probability that network $\mathcal{G}_{c2}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$ is disconnected. The main results are given as follows.

Theorem 2: Under the optimal $g(\overrightarrow{x}) = \frac{\log n + k \log f(\overrightarrow{x})}{\log n + k\rho}$ in network $\mathcal{G}_{c2}$ and $\tau = 2r_{c2}(n)$ in condition (C0) , the critical transmission range is $r_{c2}(n) = \sqrt{\frac{\log n + k\rho}{k\pi n^\alpha}}$.

A. Necessary Condition of Theorem 2

We prove the necessity of $r_{c2}(n)$ by showing that $\liminf_{n\to\infty} \mathbb{P}_{c2}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n)) > 0$ if $\pi r^2(n) = \frac{\log n + k\rho + \xi(n)}{kn^\alpha}$, where $0 < \lim_{n\to\infty} \xi(n) = \xi < +\infty$. However, the analytic methodologies in this state are different from those in cluster-sparse state, because each cluster-member node moves as an individual due to weak correlations among nodes in this state. Some notations are listed as follows.

- $\mathcal{N}_{j\kappa}$: the κth cluster-member node in the jth cluster ($\kappa = 1, 2, \ldots, \varpi$).
- $f_{j\kappa}^{\lambda}$: the event that $\mathcal{N}_{j\kappa}$ is disconnected in the λth slot.
- $f_{j\kappa}$: the event that $\mathcal{N}_{j\kappa}$ is disconnected in k time slots.

Lemma 4: $\forall j = 1, 2, \ldots, m, \kappa = 1, 2, \ldots, \varpi, \mathbb{P}(f_{j\kappa})$ has

$$\mathbb{P}(f_{j\kappa}) = \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda})(1 - \iint_{\mathcal{C}_{r(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x}) d\overrightarrow{x})^{n^\alpha} d\overrightarrow{x_\lambda}\right),$$

where $r(n) = \sqrt{(\log n + k\rho + \xi)/k\pi n^\alpha}$ and $R = \Theta(n^\beta)$.

Proof: Lemma 4 can be regarded as a special case of Lemma 1, in which R approaches 0, and we neglect the proof here for simplicity. ■

Lemma 5: If $r(n) = \sqrt{\frac{\log n + k\rho + \xi}{k\pi n^\alpha}}$, $\alpha + 2\beta \geq \frac{1-\gamma}{k}$, for fixed $\theta < 1$ and sufficiently large n, we have

$$n \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda})(1 - \iint_{\mathcal{C}_{r(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x}) d\overrightarrow{x})^{n^\alpha} d\overrightarrow{x_\lambda}\right) \geq \theta e^{-a\xi},$$

where a is a finite positive number.

Proof: The proof is similar to that of Lemma 2, and we neglect it here for simplicity. ■

Lemma 6: If $r(n) = \sqrt{\frac{\log n + k\rho + \xi}{k\pi n^\alpha}}$, $\alpha + 2\beta \geq \frac{1-\gamma}{k}$, for fixed $\eta > 1$ and sufficiently large n, we have

$$n \iint_{\mathcal{O}^k} \prod_{\lambda=1}^{k} \left(f(\overrightarrow{x_\lambda})(1 - \iint_{\mathcal{C}_{r(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x}) d\overrightarrow{x})^{n^\alpha} d\overrightarrow{x_\lambda}\right) \leq \eta e^{-a\xi},$$

where a is a finite positive number.

Proof: The proof of Lemma 6 is similar to that of Lemma 3, and the difference is substituting $(r(n) - R)$ in (6) to $r(n)$ in this lemma. Notice that (1) also holds although the probability distribution function $g(\overrightarrow{x})$ and the integrand region are different from that of Section IV. We neglect the proof here for simplicity. ■

Proposition 2: If $r(n) = \sqrt{\frac{\log n + k\rho + \xi(n)}{k\pi n^\alpha}}$, $\alpha + 2\beta \geq \frac{1-\gamma}{k}$, and $0 < \lim_{n\to\infty} \xi(n) = \xi < +\infty$, we have

$$\liminf_{n\to\infty} \mathbb{P}_{c2}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n)) \geq e^{-a\xi}(1 - e^{-a\xi}).$$

Proof: Let $\mathcal{K}_{j\kappa}$ denote the event that $\mathcal{N}_{j\kappa}$ is the only disconnected cluster-member node during k time slots. Owing to the independence between cluster-member nodes in two different clusters, we get

$$\begin{aligned}
&\mathbb{P}_{c2}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n)) \\
&\geq \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(\mathcal{K}_{j\kappa}) \\
&\geq \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa}) - \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{\kappa'=1,\kappa'\neq\kappa}^{\varpi}\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'}) \\
&\quad -\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\sum_{\kappa=1}^{\varpi}\sum_{\kappa'=1}^{\varpi}\mathbb{P}(f_{i\kappa}\cap f_{j\kappa'}) \\
&\geq \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa}) - \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{\kappa'\neq\kappa}\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'}) \\
&\quad -\left(\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa})\right)^2. \qquad (12)
\end{aligned}$$

Since the analytic methodology of (12) is similar to that of (7), it is unnecessary to present the details of (12). Next, we evaluate the three terms on the right part of the final result in (12). In fact, we have already obtained the lower bound of the first term by Lemma 5 and the upper bound of the third term by Lemma 6, respectively. Hence, we only need to derive an upper bound of the second term, which is equivalent to the probability that two cluster-member nodes in the same cluster are disconnected. We give an upper bound of the second term as (13).

$$\begin{aligned}
&\mathbb{P}(f_{j\kappa}^{\lambda}\cap f_{j\kappa'}^{\lambda}) \\
&=\mathbb{P}(f_{j\kappa}^{\lambda}\cap f_{j\kappa'}^{\lambda}|d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})<2r(n))\mathbb{P}(d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})<2r(n)) \\
&+\mathbb{P}(f_{j\kappa}^{\lambda}\cap f_{j\kappa'}^{\lambda}|d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})\geq 2r(n))\mathbb{P}(d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})\geq 2r(n)) \\
&\leq\mathbb{P}(f_{j\kappa}^{\lambda}\cap f_{j\kappa'}^{\lambda}|d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})<2r(n))\mathbb{P}(d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})<2r(n)) \\
&\quad+\mathbb{P}(f_{j\kappa}^{\lambda}\cap f_{j\kappa'}^{\lambda}|d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})\geq 2r(n)) \\
&\leq\mathbb{P}(f_{j\kappa})\cdot\mathbb{P}(d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})<2r(n))+ \\
&\quad\mathbb{P}(f_{j\kappa}^{\lambda}\cap f_{j\kappa'}^{\lambda}|d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})\geq 2r(n)). \qquad (13)
\end{aligned}$$

We obtain (13) by using the total probability formula and inequality zoom. The terms $\mathbb{P}(f_{j\kappa})$, $\mathbb{P}(d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})<2r(n))$ and $\mathbb{P}(f_{j\kappa}^{\lambda}\cap f_{j\kappa'}^{\lambda}|d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})\geq 2r(n))$ in the right part of the final result in (13) are shown in Lemma 4, (14) and (15), respectively.

$$\mathbb{P}(d(\mathcal{N}_{j\kappa},\mathcal{N}_{j\kappa'})<2r)=4r^2(n)/R^2. \qquad (14)$$

$$\iint_{\mathcal{O}} f(\overrightarrow{x_\lambda})\iint_{\mathcal{C}_R(\overrightarrow{x_\lambda})} f(\overrightarrow{x_\lambda}')\cdot\Phi d\overrightarrow{x_\lambda}' d\overrightarrow{x_\lambda}, \qquad (15)$$

where $\Phi=\left(1-\iint_{\mathcal{C}_{r(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})d\overrightarrow{x}-\iint_{\mathcal{C}_{r(n)}(\overrightarrow{x_\lambda}')} g(\overrightarrow{x})d\overrightarrow{x}\right)^{n^\alpha}$.

Applying the three terms in (13), we have

$$\mathbb{P}(f_{j\kappa}^{\lambda}\cap f_{j\kappa'}^{\lambda})\leq \eta_\lambda n^{-\frac{1}{k}}e^{-\frac{a\xi}{k}}\Theta(\frac{r^2(n)}{R^2})+\eta_\lambda^2 n^{-\frac{2}{k}}e^{-\frac{2a\xi}{k}}. \quad (16)$$

Resulting from the independence of each time slot, we have

$$\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'})=(\mathbb{P}(f_{j\kappa}^{\lambda}\cap f_{j\kappa'}^{\lambda}))^k. \qquad (17)$$

Then, combining (16) and (17), we can obtain that

$$\begin{aligned}
&\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{\kappa'\neq\kappa}\mathbb{P}\left(f_{j\kappa}\cap f_{j\kappa'}\right)\leq \\
&e^{-2a\xi}\left(e^{\frac{a\xi}{k}}\Theta\left(\frac{\log n}{n^{\alpha+2\beta-\frac{1-\gamma}{k}}}\right)+\eta_\lambda^2 n^{-\frac{\gamma}{k}}\right)^k. \qquad (18)
\end{aligned}$$

Applying (18), Lemma 5 and Lemma 6 in (12), we can derive the following result.

$$\begin{aligned}
&\mathbb{P}_{c2}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r_{c2}) \\
&\geq\theta e^{-a\xi}-\eta^2 e^{-2a\xi}-e^{-2a\xi}\left(e^{\frac{a\xi}{k}}\Theta\left(\frac{\log n}{n^{\alpha+2\beta-\frac{1-\gamma}{k}}}\right)+\eta_\lambda^2 n^{-\frac{\gamma}{k}}\right)^k \\
&\geq\theta e^{-a\xi}-(1+\mu)e^{-2a\xi}, \qquad (19)
\end{aligned}$$

is established for any $\mu>0$ and all the $n>N(\mu,\theta,\xi)$. the remaining part of proof is resemblant of that of cluster-sparse state and we omit it here for simplicity. ■

Until now, we complete the proof of necessary condition of Theorem 2.

B. Sufficient Condition of Theorem 2

To prove the sufficient condition of Theorem 2, it suffices to show the probability that at least one cluster-member node is disconnected equals 0 when the transmission range is $cr_{c2}(n)$ $(c>1)$. In other words, we need to prove

$$\lim_{n\to\infty}\mathbb{P}\left(\bigcup_{j=1}^{m}\left(\bigcup_{\kappa=1}^{\varpi}f_{j\kappa}\right)\right)=0. \qquad (20)$$

Proof: By exploiting the same techniques in proving the sufficient condition of Theorem 1, we have

$$\begin{aligned}
&\mathbb{P}\left(\bigcup_{j=1}^{m}\left(\bigcup_{\kappa=1}^{\varpi}f_{j\kappa}\right)\right)\leq\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa}) \\
&\leq n\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})\left(1-\iint_{\mathcal{C}_{r(n)}(\overrightarrow{x_\lambda})}g(\overrightarrow{x})d\overrightarrow{x}\right)^{n^\alpha}d\overrightarrow{x_\lambda}\right) \\
&\leq n\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})e^{-n^\alpha\pi c^2 r_{c2}^2(n)g(\overrightarrow{x_\lambda})-o(1)}d\overrightarrow{x_\lambda}\right) \\
&\leq\frac{e^{-o(1)}}{n^{(c^2-1)}}\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})^{1-c^2}d\overrightarrow{x_\lambda}\right)=\frac{be^{-o(1)}}{n^{(c^2-1)}}, \qquad (21)
\end{aligned}$$

where $b=o(n^{(c^2-1)})$ in this section[4].

Due to $c>1$ and taking the limits of both sides in (21), we complete the proof of (20). ■

[4]The upper bound and lower bound are similar to that of cluster-sparse state, so does the following cluster-transitional state.

VI. Critical Transmission Range for Cluster-Transitional State

In this section, we investigate the transitional state between cluster-sparse state and cluster-dense state $(0 < \alpha + 2\beta < (1-\gamma)/k)$. We denote $\mathbb{P}_{c3}$ $(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$ as the probability that network $\mathcal{G}_{c3}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$ is disconnected. The following theorem illustrates our main results.

Theorem 3: Under the optimal probability distribution function $g(\overrightarrow{x}) = \frac{[k(\alpha+2\beta)+\gamma]\log n + k\log f(\overrightarrow{x})}{[k(\alpha+2\beta)+\gamma]\log n + k\rho}$ in network $\mathcal{G}_{c3}$ and $\tau = (2r_{c3}(n) + 1/(n^{\frac{\alpha}{2}}\log n))$ in condition C(0) , we have $r_{c3}(n) = \sqrt{\frac{[k(\alpha+2\beta)+\gamma]\log n + k\rho}{k\pi n^{\alpha}}}$.

In the cluster-transitional state, $r_{c3}(n) = o(R)$ and $\pi R^2 = \omega(1/n^{\alpha})$ hold. Accordingly, we can not regard cluster-member nodes in the same cluster as an entirety, neither can we take each cluster-member node as an individual because the correlation among nodes belonging to the same cluster is greater than that of cluster-dense state but less than that of cluster-sparse state. We divide each cluster into sub-clusters, which is introduced at the beginning of Section VI-A.

A. Basis of Sub-Cluster

We perform segmentation on clusters by the same approach as [27]. In particular, each cluster is divided into $n^{\alpha+2\beta}$ non-overlapping circular areas, whose radii are the same with $\bar{r}(n) = 1/(n^{\frac{\alpha}{2}}\log n)$.

After segmentation, each cluster possesses $n^{\alpha+2\beta}$ sub-areas, and here we use a sequence $(a_1, a_2, \ldots, a_k)$, where $1 \le a_i \le n^{\alpha+2\beta} (i = 1, 2, \ldots, k)$, to interpret the concept of sub-cluster. A node x satisfies $x \in (a_1, a_2, \ldots, a_k)$ iff x is in sub-area a_i during time slot i $(i = 1, 2, \ldots, k)$. Thus with $n^{\alpha+2\beta}$ sub-areas, we have $n^{k(\alpha+2\beta)}$ sub-clusters in each individual cluster, and the average number of cluster-member nodes in each sub-cluster is $\varpi \cdot (\frac{\bar{r}^2(n)}{R^2})^k = \Theta(n^{k[\frac{1-\gamma}{k}-(\alpha+2\beta)]}/\log^{2k} n)$. Due to $\alpha + 2\beta < (1-\gamma)/k$, each sub-cluster is not empty. Furthermore, from the perspective of inter-sub-clusters, sub-clusters have the features of clusters in cluster-sparse state on account of $\bar{r}(n) = o(r_{c3}(n))$, and we can regard each sub-cluster as an independent node. However, from the perspective of intra-sub-clusters, sub-clusters have the characteristics of clusters in cluster-dense state since the radius of intra-nodes in sub-clusters satisfies $r_{c3}(n) = \omega(R)$, hence we can treat each sub-cluster as a cluster of cluster-dense state.

Notice that we number the sub-clusters in each cluster from 1 to $n^{k(\alpha+2\beta)}$. The number is the decimal representation of $a_1 a_2 \ldots a_k$, which is under the base of $n^{k(\alpha+2\beta)}$. Then we present the necessary condition of Theorem 3.

B. Necessary Condition of Theorem 3

We prove the necessity of $r_{c3}(n)$ by showing that $\liminf_{n\to\infty} \mathbb{P}_{c3}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n)) > 0$ if $\pi r^2(n) = \frac{[k(\alpha+2\beta)+\gamma]\log n + k\rho + \xi(n)}{kn^{\alpha}}$, where $0 < \lim_{n\to\infty} \xi(n) = \xi < +\infty$ holds.

To begin with, we give some notations.

- $\mathcal{C}_{j\kappa}$: the κth sub-cluster in the jth cluster.
- $\mathcal{S}_{j\kappa}$: the event that $\mathcal{C}_{j\kappa}$ is disconnected.
- $\mathcal{K}_{j\kappa}$: the event that $\mathcal{C}_{j\kappa}$ is the only disconnected cluster in k time slots.

Then, two lemmas are presented.

Lemma 7: If $r(n) = \sqrt{\frac{[k(\alpha+2\beta)+\gamma]\log n + k\rho + \xi}{k\pi n^{\alpha}}}$, $0 < \alpha + 2\beta < \frac{1-\gamma}{k}$, for fixed $\theta < 1$ and sufficiently large n, we have

$$m\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})n^{\alpha+2\beta}(1-\iint_{\mathcal{C}_{r(n)+\bar{r}(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})d\overrightarrow{x})^{n^{\alpha}}d\overrightarrow{x_\lambda}\right)$$
$$\ge \theta e^{-a\xi},$$

where $\bar{r}(n) = 1/(n^{\frac{\alpha}{2}}\log n)$.

Proof: The proof is similar to that of Lemma 2, and we neglect it here for simplicity. ■

Lemma 8: If $r(n) = \sqrt{\frac{[k(\alpha+2\beta)+\gamma]\log n + k\rho + \xi}{k\pi n^{\alpha}}}$, $0 < \alpha + 2\beta < \frac{1-\gamma}{k}$, for fixed $\eta > 1$ and sufficiently large n, we have

$$m\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})n^{\alpha+2\beta}(1-\iint_{\mathcal{C}_{r(n)-\bar{r}(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})d\overrightarrow{x})^{n^{\alpha}}d\overrightarrow{x_\lambda}\right)$$
$$\le \eta e^{-a\xi},$$

where $\bar{r}(n) = 1/(n^{\frac{\alpha}{2}}\log n)$.

Proof: The proof is similar to that of Lemma 3, and the difference is substituting $(r(n) - \bar{r}(n))$ in this section for $(r(n) - R)$ in (6). We neglect it here for simplicity. ■

Proposition 3: If $r(n) = \sqrt{\frac{[k(\alpha+2\beta)+\gamma]\log n + k\rho + \xi}{k\pi n^{\alpha}}}$, $0 < \alpha + 2\beta < \frac{1-\gamma}{k}$ and $0 < \lim_{n\to\infty} \xi(n) = \xi < +\infty$, we have

$$\liminf_{n\to\infty} \mathbb{P}_{c3}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n)) \ge e^{-a\xi}(1 - e^{-a\xi}).$$

Proof: We use the approaches similar to cluster-dense state to bound $\mathbb{P}_{c3}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n))$.

$$\begin{aligned}
&\mathbb{P}_{c3}(n, \alpha, \beta, \gamma, f(\overrightarrow{x}), g(\overrightarrow{x}), r(n)) \\
&\ge \sum_{j=1}^{m}\sum_{\kappa=1}^{n^{k(\alpha+2\beta)}} \mathbb{P}(\mathcal{K}_{j\kappa}) \\
&\ge \sum_{j=1}^{m}\sum_{\kappa=1}^{n^{k(\alpha+2\beta)}} \mathbb{P}(\mathcal{S}_{j\kappa}) - \sum_{j=1}^{m}\sum_{\kappa\neq\kappa'} \mathbb{P}(\mathcal{S}_{j\kappa}\cap\mathcal{S}_{j\kappa'}) \\
&\quad - \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{\kappa=1}^{n^{k(\alpha+2\beta)}}\sum_{\kappa'=1}^{n^{k(\alpha+2\beta)}} \mathbb{P}(\mathcal{S}_{i\kappa}\cap\mathcal{S}_{j\kappa'}) \\
&= \sum_{j=1}^{m}\sum_{\kappa=1}^{n^{k(\alpha+2\beta)}} \mathbb{P}(\mathcal{S}_{j\kappa}) - \sum_{j=1}^{m}\sum_{\kappa\neq\kappa'} \mathbb{P}(\mathcal{S}_{j\kappa}\cap\mathcal{S}_{j\kappa'}) \\
&\quad - \left(\sum_{j=1}^{m}\sum_{\kappa=1}^{n^{k(\alpha+2\beta)}} \mathbb{P}(\mathcal{S}_{j\kappa})\right)^2.
\end{aligned} \tag{22}$$

As the inter-sub-clusters have the feature of clusters in cluster-sparse state, we can bound $\mathbb{P}(\mathcal{S}_{j\kappa})$ as the inequalities of Lemma 1:

$$\mathbb{P}(\mathcal{S}_{j\kappa})\geq\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})(1-\iint_{\mathcal{C}_{r(n)+\bar{r}(n)}(\overrightarrow{x_\lambda})}g(\overrightarrow{x})\,d\overrightarrow{x})^{n^\alpha}d\overrightarrow{x_\lambda}\right);$$

$$\mathbb{P}(\mathcal{S}_{j\kappa})\leq\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})(1-\iint_{\mathcal{C}_{r(n)-\bar{r}(n)}(\overrightarrow{x_\lambda})}g(\overrightarrow{x})\,d\overrightarrow{x})^{n^\alpha}d\overrightarrow{x_\lambda}\right).$$

Combining lemma 7 and lemma 8, we have

$$\theta e^{-a\xi}\leq\sum_{j=1}^{m}\sum_{\kappa=1}^{n^{k(\alpha+2\beta)}}\mathbb{P}(\mathcal{S}_{j\kappa})\leq\eta e^{-a\xi}. \tag{23}$$

Then we evaluate the term $\mathbb{P}(\mathcal{S}_{j\kappa}\cap\mathcal{S}_{j\kappa'})$. Let $\kappa=(a_1,a_2,\ldots,a_k)$ and $\kappa'=(b_1,b_2,\ldots,b_k)$ and suppose there are exactly t $(0\leq t\leq k-1)$ identical elements between the sequences κ and κ', i.e., $\exists\{s(1),s(2),\ldots,s(t)\}\subseteq\{1,2,\ldots,k\}$ that $a_{s(i)}=b_{s(i)},\ i\leq t$. Since sub-areas do not overlap each other, we have

$$\begin{aligned}&\mathbb{P}(\mathcal{S}_{j\kappa}\cap\mathcal{S}_{j\kappa'})\\&\leq C_k^t(C^1_{n^{\alpha+2\beta}})^t(C^2_{n^{\alpha+2\beta}})^{k-t}\cdot\\&\quad\iint_{\mathcal{O}^t}\prod_{\lambda=1}^{t}\left(f(\overrightarrow{x_\lambda})\left(1-\iint_{\mathcal{C}_{r(n)-\bar{r}(n)}(\overrightarrow{x_\lambda})}g(\overrightarrow{x})d\overrightarrow{x}\right)^{n^\alpha}d\overrightarrow{x_\lambda}\right)\\&\quad\cdot\iint_{\mathcal{O}^{2(k-t)}}\prod_{\lambda=t+1}^{k}\left(f(\overrightarrow{x_\lambda})f(\overrightarrow{x_\lambda}')\left(1-\iint_{\mathcal{C}_{r(n)-\bar{r}(n)}(\overrightarrow{x_\lambda})}g(\overrightarrow{x})d\overrightarrow{x}\right.\right.\\&\quad\left.\left.-\iint_{\mathcal{C}_{r(n)-\bar{r}(n)}(\overrightarrow{x_\lambda}')}g(\overrightarrow{x})d\overrightarrow{x}\right)^{n^\alpha}d\overrightarrow{x_\lambda}\,d\overrightarrow{x_\lambda}'\right)\\&\leq C_k^t n^{\frac{t-2k}{k}\gamma}e^{\frac{(t-2k)a\xi}{k}+\Theta(\frac{1}{\sqrt{\log n}}-\frac{1}{\log n^2})}\leq C_k^t n^{\frac{t-2k}{k}\gamma}.\end{aligned} \tag{24}$$

Therefore, we can obtain

$$\sum_{j=1}^{m}\sum_{\kappa=1}^{n^{k(\alpha+2\beta)}}\sum_{\kappa'\neq\kappa}\mathbb{P}(\mathcal{S}_{j\kappa}\cap\mathcal{S}_{j\kappa'})\leq m\sum_{t=0}^{k-1}C_k^t n^{\frac{t-2k}{k}\gamma}=\Theta(n^{-\frac{\gamma}{k}}). \tag{25}$$

Applying (23) and (25) in (22), we have

$$\begin{aligned}&\mathbb{P}_{c3}(n,\alpha,\beta,\gamma,f(\overrightarrow{x}),g(\overrightarrow{x}),r_{c3})\\&\geq\theta e^{-a\xi}-\eta^2e^{-2a\xi}-\Theta(n^{-\frac{\gamma}{k}})\\&\geq\theta e^{-a\xi}-(1+\mu)e^{-2a\xi}\end{aligned} \tag{26}$$

is established for all the $\mu>0$ and $n>N(\mu,\theta,\xi)$.

The rest is similar to that in cluster-sparse state, and we neglect it here for simplicity. ■

C. Sufficient Condition of Theorem 3

In this part, we segment each cluster in another way [27]. This method of segmentation has two requisites: (1) each cluster region is covered by the sub-areas without overlapping each other; (2) we can use circles with sufficiently small radius (compared with $r_c(n)$) to cover each sub-area. After this segmentation, all the cluster-member nodes are grouped into sub-clusters during k time slots, and nodes in the same sub-cluster can be regarded as a whole. The steps of segmentation are listed as follows:

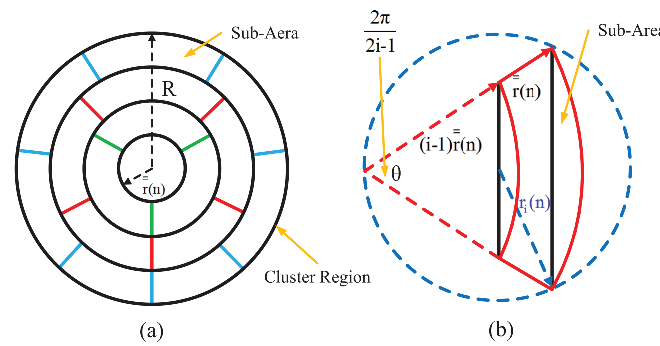

Fig. 3. (a) Segmentation illustration: Network is divided into $n^{\alpha+2\beta}$ sub-areas and the area of each sub-area is equal. (b) Radius proof: This picture demonstrates that each sub-area can be covered by a circle with radius $r_i(n)$.

Step.1: Divide each cluster region into a series of circular rings. The ith ring has an inner radius of $i\bar{\bar{r}}(n)$ and an outer radius of $(i+1)\bar{\bar{r}}(n)$ $(i=1,2,\cdots,n^{\frac{\alpha+2\beta}{2}})$, where $\bar{\bar{r}}(n)=n^{-\frac{\alpha+2\beta}{2}}R$, and the 0th ring is a circle centering at the home point with radius $\bar{\bar{r}}(n)$. We assume that $n^{\frac{\alpha+2\beta}{2}}$ is an integer.

Step.2: Cut the ith ring into $2i-1$ pieces equally.

Fig. 3(a) presents the process of segmentation. We prove that each sub-area can be covered by a circle, whose radius is less than $4\bar{\bar{r}}(n)$. As shown in Fig. 3(b) and (27), we can obtain

$$\begin{aligned}r_i(n)&=\bar{\bar{r}}(n)\sqrt{\cos^2\frac{\pi}{2i-1}+(i\sin\frac{\pi}{2i-1})^2}\\&\leq\bar{\bar{r}}(n)\sqrt{1+(\frac{i\pi}{2i-1})^2}\\&\leq\bar{\bar{r}}(n)\sqrt{1+\pi^2}<4\bar{\bar{r}}(n).\end{aligned} \tag{27}$$

After this segmentation, we get $n^{\alpha+2\beta}$ sub-areas with area $\pi\bar{\bar{r}}^2(n)$, and $n^{k(\alpha+2\beta)}$ sub-clusters in each individual cluster. These sub-clusters contain all the cluster-member nodes during k time slots. Moreover, the average number of cluster-member nodes in each sub-cluster is $\varpi\cdot\left(1/n^{(\alpha+2\beta)}\right)^k=n^{k[\frac{1-\gamma}{k}-(\alpha+2\beta)]}$, which indicates that every sub-cluster is not empty when n is sufficiently large.

We regard cluster-member nodes in the same sub-cluster as a whole and treat each sub-cluster as an independent individual. Therefore, similar to the previous sufficient condition proofs, in order to prove the sufficiency of r_{c3}, we need to show that the probability that at least one sub-cluster is unconnected equals 0 when the transmission range of each node is $cr_{c3}(n)(c>1)$, which is equivalent to prove

$$\lim_{n\to\infty}\mathbb{P}\left(\bigcup_{j=1}^{m}(\bigcup_{\kappa=1}^{n^{k(\alpha+2\beta)}}\mathcal{S}_{j\kappa})\right)=0. \tag{28}$$

Proof: By combining (23) with (27), we can obtain the following process.

$$
\begin{aligned}
&\mathbb{P}\left(\bigcup_{j=1}^{m}\left(\bigcup_{\kappa=1}^{n^{k(\alpha+2\beta)}} \mathcal{S}_{j\kappa}\right)\right) \leq \sum_{j=1}^{m}\sum_{\kappa=1}^{n^{k(\alpha+2\beta)}} \mathbb{P}(\mathcal{S}_{j\kappa})\\
&\leq n^{[k(\alpha+2\beta)+\gamma]}\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})\left(1-\iint_{\mathcal{C}_{r(n)-4\bar{r}(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})d\overrightarrow{x}\right)^{n^\alpha} d\overrightarrow{x_\lambda}\right)\\
&\leq n^{[k(\alpha+2\beta)+\gamma]}\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})e^{-\pi n^\alpha (cr_c(n)-4\bar{r}(n))^2 g(\overrightarrow{x_\lambda})}\, d\overrightarrow{x_\lambda}\right)\\
&\leq \frac{\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})^{1-c^2}d\overrightarrow{x_\lambda}\right)e^{\Theta(\sqrt{k\log n})}}{n^{[k(\alpha+2\beta)+\gamma](c^2-1)}}\\
&=\frac{be^{\Theta(\sqrt{k\log n})}}{n^{[k(\alpha+2\beta)+\gamma](c^2-1)}}, \qquad (29)
\end{aligned}
$$

where $b = o(n^{[k(\alpha+2\beta)+\gamma](c^2-1)})$.

Due to $c > 1$ and taking the limits of both sides in (29), the proof is finished. ■

Until now, the sufficiency of Theorem 3 is proved.

VII. The Optimality of Distribution Density of Cluster Head

We prove the optimality of $g(\overrightarrow{x})$ from the perspective of maximizing the expected number of isolated nodes ($\mathbb{N}_c(n)$) when network connectivity is guaranteed. First, based on the sufficient condition of Definition 1, we present a interval $\pi r_c^2(n)n^\alpha g(\overrightarrow{x})$. Second, according to the necessary condition of Definition 1, we observe that the critical transmission range is minimized when $\lim_{n\to\infty}\mathbb{P}(\mathcal{M}_n)$ is minimized; Namely the critical transmission range is minimized when $\mathbb{N}_c(n)$ is maximized. Finally, on the basis of the maximum $\mathbb{N}_c(n)$, we derive the corresponding critical transmission range.

Retrospecting to the preceding proofs of necessary condition and sufficient condition in each state, we observe that $\mathbb{N}_c(n)$ has similar formulations. To avoid repetitions, we only present the detailed proof of $g(\overrightarrow{x})$ in cluster-dense state.

In the beginning, we provide a interval of $\pi r_c^2(n)n^\alpha g(\overrightarrow{x})$ under the sufficient condition of definition 1. Similar to Section VI-B, it suffices to show (20), where $r(n) = cr_c(n)(c > 1)$. Note that $r_c(n)$ is unknown in this section. Then we have

$$
\begin{aligned}
&\mathbb{P}\left(\bigcup_{j=1}^{m}\left(\bigcup_{\kappa=1}^{\varpi} f_{j\kappa}\right)\right) \leq \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa})\\
&= n\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})\left(1-\iint_{\mathcal{C}_r(\overrightarrow{x_\lambda})} g(\overrightarrow{x})d\overrightarrow{x}\right)^{n^\alpha} d\overrightarrow{x_\lambda}\right)\\
&= n\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})\exp\{-n^\alpha\iint_{\mathcal{C}_{r(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\,d\overrightarrow{x}\}\,d\overrightarrow{x_\lambda}\right)\\
&= \left(\iint_{\mathcal{O}}\exp\left\{\frac{\log n}{k}+\log f(\overrightarrow{x})-c^2\pi n^\alpha r_c^2(n)g(\overrightarrow{x})+o(1)\right\}d\overrightarrow{x}\right)^k\\
&= \eta'\left(\iint_{\mathcal{O}}\exp\left\{\frac{\log n}{k}+\log f(\overrightarrow{x})-c^2\pi n^\alpha r_c^2(n)g(\overrightarrow{x})\right\}d\overrightarrow{x}\right)^k, \qquad (30)
\end{aligned}
$$

where $\eta' \to 1$. The second step of (30) follows from condition (0.4) and the equivalent infinitesimal replacement. The third step is obtained due to condition (0.3).

According to the addition formula of Probability, we obtain

$$
\mathbb{P}\left(\bigcup_{j=1}^{m}\left(\bigcup_{\kappa=1}^{\varpi} f_{j\kappa}\right)\right)=\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa})-\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{\kappa'\geq\kappa}\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'})+\ldots \qquad (31)
$$

Since the second term of the right part in (31) is less than the first term, the right part in the last step of (30) must approach 0 when n goes to ∞ to ensure the sufficiency of $r_c(n)$. Therefore, we obtain $\pi n^\alpha r_c^2(n)g(\overrightarrow{x}) = \Omega(\log n)$. Actually, $\pi n^\alpha r_c^2(n)g(\overrightarrow{x}) \geq (\log n)/k + \log f(\overrightarrow{x})$.

Next, we focus on how to minimize $\lim_{n\to\infty}\mathbb{P}(\mathcal{M}_n)$ under the premise of connectivity. (32) is established in accordance with probability theory.

$$
\lim_{n\to\infty}\mathbb{P}(\mathcal{M}_n) = 1 - \lim_{n\to\infty}\mathbb{P}\left(\bigcup_{j=1}^{m}\left(\bigcup_{\kappa=1}^{\varpi} f_{j\kappa}\right)\right). \qquad (32)
$$

Thus, minimizing $\lim_{n\to\infty}\mathbb{P}(\mathcal{M}_n)$ is equivalent to maximizing $\lim_{n\to\infty}\mathbb{P}\left(\bigcup_{j=1}^{m}\left(\bigcup_{\kappa=1}^{\varpi} f_{j\kappa}\right)\right)$. In the following, we estimate the second term of the right part in (31). Utilizing (13)–(18), we have

$$
\begin{aligned}
&\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{\kappa'\geq\kappa}\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'})\\
&\qquad\leq H^2(n^{2\beta-\frac{\gamma}{k}}+\frac{1}{H}\Theta(n^{-(1-\gamma)(1-\frac{1}{k})}))^k, \qquad (33)
\end{aligned}
$$

where

$$
H = \left(\iint_{\mathcal{O}}\exp\left\{\frac{\log n}{k}+\log f(\overrightarrow{x})-\pi n^\alpha r_c^2(n)g(\overrightarrow{x})\right\}d\overrightarrow{x}\right)^k.
$$

H is a finite positive number on account of $\pi n^\alpha r_c^2(n)g(\overrightarrow{x}) \geq (\log n)/k + \log f(\overrightarrow{x})$. Since $\beta \leq 0$ and $0 < \gamma \leq 1$, the right part of (33) approaches 0 when n goes to ∞. As a result, we obtain

$$
\lim_{n\to\infty}\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{\kappa'\geq\kappa}\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'}) = 0. \qquad (34)
$$

Hence, taking the limits of both sides of (31), we have

$$
\lim_{n\to\infty}\mathbb{P}\left(\bigcup_{j=1}^{m}\left(\bigcup_{\kappa=1}^{\varpi} f_{j\kappa}\right)\right) = \lim_{n\to\infty}\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa}). \qquad (35)
$$

Combining (32) with (35), the problem becomes maximizing $\lim_{n\to\infty}\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa})$, which actually is $\mathbb{N}_c(n)$. Accordingly, the expression of $\mathbb{N}_c(n)$ is presented in the following.

$$\begin{aligned}\mathbb{N}_c(n) &= n\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})\left(1-\iint_{\mathcal{C}_{r_c(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\,d\overrightarrow{x}\right)^{n^\alpha} d\overrightarrow{x_\lambda}\right)\\ &= n\iint_{\mathcal{O}^k}\prod_{\lambda=1}^{k}\left(f(\overrightarrow{x_\lambda})\exp\{-n^\alpha\iint_{\mathcal{C}_{r_c(n)}(\overrightarrow{x_\lambda})} g(\overrightarrow{x})\,d\overrightarrow{x}\}\,d\overrightarrow{x_\lambda}\right)\\ &= \nu\left(\iint_{\mathcal{O}}\exp\left\{\frac{\log n}{k}+\log f(\overrightarrow{x})-\pi n^\alpha r_c^2(n)g(\overrightarrow{x})\right\}d\overrightarrow{x}\right)^k,\end{aligned} \tag{36}$$

where $\nu \to 1$. The second step of (36) follows from the equivalent infinitesimal replacement, and the last step is obtained on account of condition (0.3) . Based on (35), we have $\lim_{n\to\infty}\mathbb{N}_c(n) \le 1$, which means $\max\{\lim_{n\to\infty}\mathbb{N}_c(n)\} = 1$.

Finally, we discuss whether $\lim_{n\to\infty}\mathbb{N}_c(n)$ can reach 1 or not; if yes, we derive the corresponding minimum critical transmission range.

From the first step, we have the following inequality.

$$\frac{\log n}{k} + \log f(\overrightarrow{x}) - \pi n^\alpha r_c^2(n)g(\overrightarrow{x}) \le 0. \tag{37}$$

Combining (36) with (37), $\lim_{n\to\infty}\mathbb{N}_c = 1$ iff (38) holds.

$$\frac{\log n}{k} + \log f(\overrightarrow{x}) = \pi n^\alpha r_c^2(n)g(\overrightarrow{x}). \tag{38}$$

Since $\iint_{\mathcal{O}} g(\overrightarrow{x})\,d\overrightarrow{x} = 1$ and integrating two sides of (38) over $\mathcal{O}$, we obtain $r_c(n) = \sqrt{\frac{\log n + k\rho}{k\pi n^\alpha}}$ and the corresponding $g(\overrightarrow{x}) = \frac{\log n + k\log f(\overrightarrow{x})}{\log n + k\rho}$. Consequently, we complete the proof of the optimality of $g(\overrightarrow{x})$.

Remark 3: The minimum critical transmission range prompts Definition 1 to satisfy the zero-one law, which is listed as follows.

$$\lim_{n\to\infty}\mathbb{P}(\mathcal{M}_n) = \begin{cases}1, & r(n) \ge c\min\{r_c(n)\} \text{ for any } c > 1;\\ 0, & r(n) \le c'\min\{r_c(n)\} \text{ for any } 1 > c' > 0,\end{cases}$$

where c and c' are constants.

Based on the results, we observe that the critical $\mathbb{N}_c(n)$ is 1. If $\mathbb{N}_c(n) > 1$, the network would be disconnected; otherwise the critical transmission range increases with the reduction of $\mathbb{N}_c(n)$. Namely, the minimum critical transmission range is derived when the definition of connectivity satisfies zero-one law.

VIII. Discussion

In this section, we mainly discuss the distribution of cluster-head nodes for better connectivity and the impact of distributional inhomogeneity on the critical transmission range. We interpret the results in Fig. 4.

A. Distribution of Cluster-Head Nodes

According to the mathematical expression of $g(\overrightarrow{x})$, we observe that for given location, the probability of cluster-head nodes increases with respect to that of home points. From the aspect of network design, once the mobility of home points is determined, we can adjust the initial distribution of cluster-head nodes to achieve better connectivity so as to reduce the critical transmission range, which can economize energy cost and abate the interference pollution in turn. Taking the large-scale clustered sensor networks for example, if cluster-head nodes are selected according to the mobility of home points, energy will be saved.

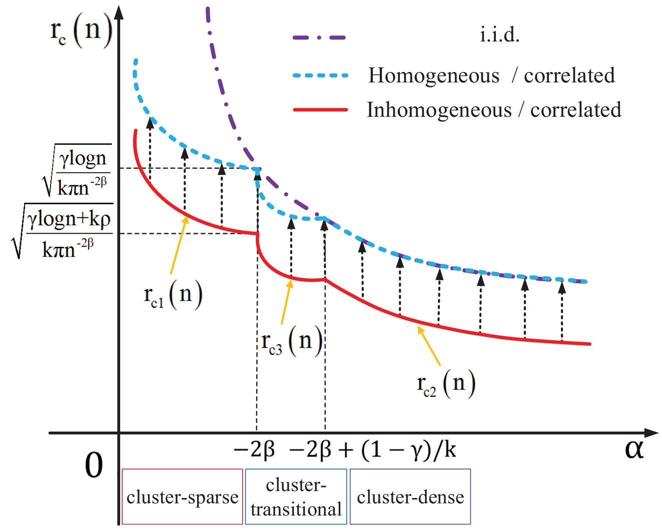

Fig. 4. The network state is determined by α, β and γ. Correlated mobility improve the performance of network connectivity compared with i.i.d. model, and inhomogeneity makes further improvement on the connectivity performance.

B. Distributional Inhomogeneity

ρ is the “analogous differential entropy” of $f(\overrightarrow{x})$, which represents the “degree of indeterminancy” of home points. Fig. 4 illustrates the impact of distributional inhomogeneity on the critical transmission range by comparison. As Fig. 4 shows, in the same state, $r_c(n)$ of the inhomogeneous correlated mobility is smaller than that of the correlated mobility. Moreover, the critical transmission range decreases with ρ, which indicates that the more inhomogeneously the mobile nodes are distributed, the smaller the critical transmission range is. Specifically, according to the Jensen's inequality, we have $\iint_{\mathcal{O}}\log f(\overrightarrow{x})\,d\overrightarrow{x} \le \log\iint_{\mathcal{O}} f(\overrightarrow{x})\,d\overrightarrow{x} = 0$. Hence the maximum value of ρ is 0, and it can be achieved when home points are uniformly and independently distributed ($f(\overrightarrow{x}) = 1$). Theoretically, the minimum value $-\infty$ of ρ is obtained when the pdf of home points converges Dirac function ($f(\overrightarrow{x}) \to \delta(\overrightarrow{x})$). However, based on condition (0.1) , we obtain that ρ is greater than $-(\gamma\log n)/k$, $-(\log n)/k$, $-((\alpha+2\beta)+\gamma/k)\log n$ in three states respectively. The above analysis shows that $f(\overrightarrow{x})$ is a transitional function between the uniform distribution function and the Dirac function, which coincides with real world. Therefore, under the practical scenario, ρ varies from $-(\gamma\log n)/k$, $-(\log n)/k$, $-((\alpha+2\beta)+\gamma/k)\log n$ to 0 in three states respectively, and the trend of $r_c(n)$ is shown as the black dotted arrow in Fig. 4.

IX. Conclusion and Future Work

In this paper, we studied the connectivity of large-scale clustered wireless ad hoc networks under inhomogeneous correlated mobility model. We provided the design principle for the placement of cluster-head nodes such that a minimum critical transmission range can be obtained to guarantee network connectivity. We also derived the corresponding critical transmission ranges for cluster-sparse state, cluster-dense state and cluster-transitional state, respectively. These results help us understand the impact of distributional inhomogeneity on network connectivity. It is also shown that the critical transmission range decreases with respect to the extent of inhomogeneity of distribution of mobile nodes. Our results give important insights to design energy-efficient wireless ad hoc networks. For example, during disaster reliefs where various rescue crews (e.g., firemen, policemen and medical assistants) form groups and work cooperatively, sometimes base stations are damaged and temporary base stations are needed to guarantee successful communications.

There are several directions for future work. First, in our current model, each cluster has identical radius and the same number of nodes. An extension is to study coexistence different kinds of clusters. Another interesting problem is to consider the application of zero-one law on network connectivity.

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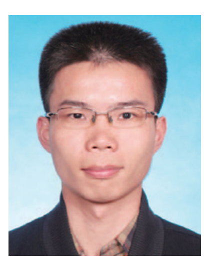

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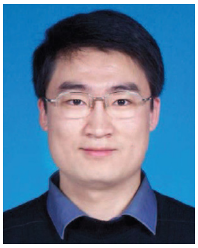

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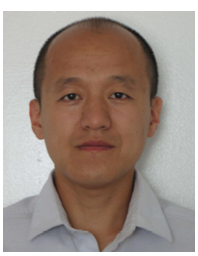

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