An incomplete bibliography of network geometry

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Compiled on 2022-04-22

The $\mathbb{S}^1$ and $\mathbb{H}^2$ models, their applications, and their variants

Models of undirected graphs

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Models of multiplex graphs

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Models of undirected graphs in higher dimensions

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