An incomplete bibliography of network geometry

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The $\mathbb{S}^1$ and $\mathbb{H}^2$ models, their applications, and their variants

Models of undirected graphs

[1] D. Krioukov et al., *Curvature and temperature of complex networks*, Phys. Rev. E **80**, 035101 (2009).
[2] D. Krioukov et al., *Hyperbolic geometry of complex networks*, Phys. Rev. E **82**, 036106 (2010).
[3] M. Á. Serrano et al., *Self-Similarity of Complex Networks and Hidden Metric Spaces*, Phys. Rev. Lett. **100**, 078701 (2008).

Models of bipartite graphs

[4] M. Kitsak et al., *Latent geometry of bipartite networks*, Phys. Rev. E **95**, 032309 (2017).
[5] M. Á. Serrano et al., *Uncovering the hidden geometry behind metabolic networks*, Mol. Biosyst. **8**, 843–850 (2012).

Models of directed graphs

[6] Z. Wu et al., *An Asymmetric Popularity-Similarity Optimization Method for Embedding Directed Networks into Hyperbolic Space*, Complexity **2020**, 8372928 (2020).

Models of multiplex graphs

[7] G. Bertagnolli and M. De Domenico, *Diffusion geometry of multiplex and interdependent systems*, Phys. Rev. E **103**, 042301 (2021).
[8] K.-K. Kleineberg et al., *Hidden geometric correlations in real multiplex networks*, Nat. Phys. **12**, 1076–1081 (2016).
[9] K.-K. Kleineberg et al., *Geometric Correlations Mitigate the Extreme Vulnerability of Multiplex Networks against Targeted Attacks*, Phys. Rev. Lett. **118**, 218301 (2017).

Models of weighted graphs

[10] A. Allard et al., *The geometric nature of weights in real complex networks*, Nat. Commun. **8**, 14103 (2017).

Models of undirected graphs in higher dimensions

[11] A. Cacciola et al., *Coalescent embedding in the hyperbolic space unsupervisedly discloses the hidden geometry of the brain*, (2017), arXiv:1705.04192.
[12] W. Yang and D. Rideout, *High Dimensional Hyperbolic Geometry of Complex Networks*, Mathematics **8**, 1861 (2020).

Models of growing graphs

[13] F. Papadopoulos et al., *Popularity versus similarity in growing networks*, Nature **489**, 537–540 (2012).
[14] M. Zheng et al., *Scaling up real networks by geometric branching growth*, Proc. Natl. Acad. Sci. U.S.A. **118**, e2018994118 (2021).

Small world, clustering, renormalization and navigability

[15] M. Boguñá and D. Krioukov, *Navigating Ultrasmall Worlds in Ultrashort Time*, Phys. Rev. Lett. **102**, 058701 (2009).
[16] M. Boguñá et al., *Navigability of complex networks*, Nat. Phys. **5**, 74–80 (2009).
[17] G. García-Pérez et al., *Multiscale unfolding of real networks by geometric renormalization*, Nat. Phys. **14**, 583–589 (2018).
[18] E. Garuccio et al., *Multiscale network renormalization: scale-invariance without geometry*, arXiv, 2009.11024 (2020).
[19] A. Gulyás et al., *Navigable networks as Nash equilibria of navigation games*, Nat. Commun. **6**, 7651 (2015).
[20] E. Ortiz et al., *Navigability of temporal networks in hyperbolic space*, Sci. Rep. **7**, 15054 (2017).
[21] M. Zheng et al., *Geometric renormalization unravels self-similarity of the multiscale human connectome*, Proc. Natl. Acad. Sci. U.S.A. **117**, 20244–20253 (2020).

Embedding algorithms

[22] G. Alanis-Lobato et al., *Efficient embedding of complex networks to hyperbolic space via their Laplacian*, Sci. Rep. **6**, 30108 (2016).

[23] G. Alanis-Lobato et al., *Manifold learning and maximum likelihood estimation for hyperbolic network embedding*, Appl. Netw. Sci. **1**, 10 (2016).

[24] M. Boguñá et al., *Sustaining the Internet with hyperbolic mapping*. Nat. Commun. **1**, 62 (2010).

[11] A. Cacciola et al., *Coalescent embedding in the hyperbolic space unsupervisedly discloses the hidden geometry of the brain*, (2017), arXiv:1705.04192.

[25] G. García-Pérez et al., *Mercator: uncovering faithful hyperbolic embeddings of complex networks*, New J. Phys. **21**, 123033 (2019).

[26] A. Muscoloni et al., *Machine learning meets complex networks via coalescent embedding in the hyperbolic space*, Nat. Commun. **8**, 1615 (2017).

[27] F. Papadopoulos et al., *Network geometry inference using common neighbors*, Phys. Rev. E **92**, 022807 (2015).

Community structure

[28] M. Bruno et al., *Community Detection in the Hyperbolic Space*, (2019), arXiv:1906.09082.

[29] A. Faqeeh et al., *Characterizing the Analogy Between Hyperbolic Embedding and Community Structure of Complex Networks*, Phys. Rev. Lett. **121**, 098301 (2018).

[30] S. Galhotra et al., *The Geometric Block Model*, (2018), arXiv:1709.05510.

[31] G. García-Pérez et al., *Soft Communities in Similarity Space*, J. Stat. Phys. **173**, 775–782 (2018).

[32] S. Metzler and P. Miettinen, *HyGen: generating random graphs with hyperbolic communities*, Appl. Netw. Sci. **4**, 53 (2019).

[33] A. Muscoloni and C. V. Cannistraci, *A nonuniform popularity-similarity optimization (nPSO) model to efficiently generate realistic complex networks with communities*, New J. Phys. **20**, 052002 (2018).

[34] A. Muscoloni and C. V. Cannistraci, *Leveraging the nonuniform PSO network model as a benchmark for performance evaluation in community detection and link prediction*, New J. Phys. **20**, 063022 (2018).

[35] K. Zuev et al., *Emergence of Soft Communities from Geometric Preferential Attachment*, Sci. Rep. **5**, 9421 (2015).

Temporal graphs

[20] E. Ortiz et al., *Navigability of temporal networks in hyperbolic space*, Sci. Rep. **7**, 15054 (2017).

Applications to real networks

[36] G. Alanis-Lobato et al., *The latent geometry of the human protein interaction network*, Bioinformatics **34**, 2826–2834 (2018).

[37] A. Allard and M. Á. Serrano, *Navigable maps of structural brain networks across species*, PLOS Comput. Biol. **16**, e1007584 (2020).

[24] M. Boguñá et al., *Sustaining the Internet with hyperbolic mapping*. Nat. Commun. **1**, 62 (2010).

[38] G. García-Pérez et al., *The hidden hyperbolic geometry of international trade: World Trade Atlas 1870-2013*, Sci. Rep. **6**, 33441 (2016).

[5] M. Á. Serrano et al., *Uncovering the hidden geometry behind metabolic networks*, Mol. Biosyst. **8**, 843–850 (2012).

[21] M. Zheng et al., *Geometric renormalization unravels self-similarity of the multiscale human connectome*, Proc. Natl. Acad. Sci. U.S.A. **117**, 20244–20253 (2020).

Applications to connectomics

[37] A. Allard and M. Á. Serrano, *Navigable maps of structural brain networks across species*, PLOS Comput. Biol. **16**, e1007584 (2020).

[11] A. Cacciola et al., *Coalescent embedding in the hyperbolic space unsupervisedly discloses the hidden geometry of the brain*, (2017), arXiv:1705.04192.

[39] W. Whi et al., *Hyperbolic disc embedding of functional human brain connectomes using resting state fMRI*, bioRxiv, 2021.03.25.436730 (2021).

[21] M. Zheng et al., *Geometric renormalization unravels self-similarity of the multiscale human connectome*, Proc. Natl. Acad. Sci. U.S.A. **117**, 20244–20253 (2020).

Cosmology

[40] M. Boguñá et al., *Cosmological networks*, New J. Phys. **16**, 093031 (2014).

[41] D. Krioukov et al., *Network Cosmology*, Sci. Rep. **2**, 793 (2012).

Theses

[42] G. García-Pérez, *A geometric approach to the structure of complex networks*, Ph.D. Thesis (Universitat de Barcelona, 2018).

[43] E. Ortiz Castillo, *Mapping complexity through network geometry: From structure to dynamics of real-world networks*, Ph.D. Thesis (Universitat de Barcelona, 2021).

All references

[22] G. Alanis-Lobato et al., *Efficient embedding of complex networks to hyperbolic space via their Laplacian*, Sci. Rep. **6**, 30108 (2016).

[23] G. Alanis-Lobato et al., *Manifold learning and maximum likelihood estimation for hyperbolic network embedding*, Appl. Netw. Sci. **1**, 10 (2016).

[36] G. Alanis-Lobato et al., *The latent geometry of the human protein interaction network*, Bioinformatics **34**, 2826–2834 (2018).

[44] R. Aldecoa et al., *Hyperbolic graph generator*, Comput. Phys. Commun. **196**, 492–496 (2015).

[45] L. Alemany-Puig et al., *Reappraising the distribution of the number of edge crossings of graphs on a sphere*, J. Stat. Mech. **2020**, 083401 (2020).

[37] A. Allard and M. Á. Serrano, *Navigable maps of structural brain networks across species*, PLOS Comput. Biol. **16**, e1007584 (2020).

[10] A. Allard et al., *The geometric nature of weights in real complex networks*, Nat. Commun. **8**, 14103 (2017).

[46] T. Aste et al., *Complex networks on hyperbolic surfaces*, Physica A **346**, 20–26 (2005).

[47] K. E. Avrachenkov and A. V. Bobu, *Cliques in high-dimensional random geometric graphs*, Appl. Netw. Sci. **5**, 92 (2020).

[48] I. Benjamini and O. Schramm, *Percolation in the hyperbolic plane*, Journal of the American Mathematical Society **14**, 487–507 (2001).

[7] G. Bertagnolli and M. De Domenico, *Diffusion geometry of multiplex and interdependent systems*, Phys. Rev. E **103**, 042301 (2021).

[49] G. Bianconi and C. Rahmede, *Emergent Hyperbolic Network Geometry*, Sci. Rep. **7**, 41974 (2017).

[50] G. Bianconi and R. M. Ziff, *Topological percolation on hyperbolic simplicial complexes*, Phys. Rev. E **98**, 052308 (2018).

[51] G. Bianconi et al., *Percolation on branching simplicial and cell complexes and its relation to interdependent percolation*, Phys. Rev. E **100**, 062311 (2019).

[52] M. Bode et al., *On the Largest Component of a Hyperbolic Model of Complex Networks*, The Electronic Journal of Combinatorics, P3.24–P3.24 (2015).

[53] M. Boguna et al., *Network Geometry*, (2020), arXiv:2001.03241.

[15] M. Boguñá and D. Krioukov, *Navigating Ultrasmall Worlds in Ultrashort Time*, Phys. Rev. Lett. **102**, 058701 (2009).

[16] M. Boguñá et al., *Navigability of complex networks*, Nat. Phys. **5**, 74–80 (2009).

[24] M. Boguñá et al., *Sustaining the Internet with hyperbolic mapping*. Nat. Commun. **1**, 62 (2010).

[40] M. Boguñá et al., *Cosmological networks*, New J. Phys. **16**, 093031 (2014).

[54] M. Boguñá et al., *Small worlds and clustering in spatial networks*, Phys. Rev. Research **2**, 023040 (2020).

[55] M. Boguñá et al., *Network geometry*, Nat. Rev. Phys. **3**, 114–135 (2021).

[56] M. Borassi et al., *Hyperbolicity measures democracy in real-world networks*, Phys. Rev. E **92**, 032812 (2015).

[57] M. Borassi et al., *On Computing the Hyperbolicity of Real-World Graphs*, in *Algorithms - ESA 2015*, Vol. 9294, edited by N. Bansal and I. Finocchi (Springer Berlin Heidelberg, Berlin, Heidelberg, 2015), pp. 215–226.

[58] C. Bordenave, *Eigenvalues of Euclidean random matrices*, Random Structures & Algorithms **33**, 515–532 (2008).

[28] M. Bruno et al., *Community Detection in the Hyperbolic Space*, (2019), arXiv:1906.09082.

[11] A. Cacciola et al., *Coalescent embedding in the hyperbolic space unsupervisedly discloses the hidden geometry of the brain*, (2017), arXiv:1705.04192.

[59] E. Candellero and N. Fountoulakis, *Clustering and the Hyperbolic Geometry of Complex Networks*, in *Algorithms and Models for the Web Graph*, Vol. 8882, edited by A. Bonato et al., Lecture Notes in Computer Science (Springer International Publishing, 2014), pp. 1–12.

[60] E. Candellero and N. Fountoulakis, *Bootstrap percolation and the geometry of complex networks*, Stochastic Processes and their Applications **126**, 234–264 (2016).

[61] E. Candellero and N. Fountoulakis, *Clustering and the Hyperbolic Geometry of Complex Networks*, Internet Mathematics **12**, 2–53 (2016).

[62] C. V. Cannistraci and A. Muscoloni, *Geometrical congruence and efficient greedy navigability of complex networks*, arXiv, 2005.13255 (2020).

[63] J. Carifio et al., *Vacuum Selection from Cosmology on Networks of String Geometries*, Phys. Rev. Lett. **121**, 101602 (2018).

[64] S. Chanpuriya et al., *Node Embeddings and Exact Low-Rank Representations of Complex Networks*, (2020), arXiv:2006.05592.

[65] W. Chen et al., *On the Hyperbolicity of Small-World and Tree-Like Random Graphs*, in *Algorithms and Computation*, Vol. 7676, edited by D. Hutchison et al. (Springer Berlin Heidelberg, 2012), pp. 278–288.

[66] E. Chien et al., *Active learning in the geometric block model*, (2019), arXiv:1912.06570.

[67] P. Cui et al., *A Survey on Network Embedding*, IEEE Transactions on Knowledge and Data Engineering **31**, 833–852 (2019).

[68] J. Dall and M. Christensen, *Random geometric graphs*, Phys. Rev. E **66**, 016121 (2002).

[69] M. De Domenico, *Diffusion Geometry Unravels the Emergence of Functional Clusters in Collective Phenomena*, Phys. Rev. Lett. **118**, 168301 (2017).

[70] C. P. Dettmann and O. Georgiou, *Random geometric graphs with general connection functions*, Phys. Rev. E **93**, 032313 (2016).

[71] C. P. Dettmann et al., *Spectral statistics of random geometric graphs*, EPL **118**, 18003 (2017).

[72] W. Fang, *On Hyperbolic Geometry Structure of Complex Networks*, Technical Report (Microsoft Research Asia, 2011).

[29] A. Faqeeh et al., *Characterizing the Analogy Between Hyperbolic Embedding and Community Structure of Complex Networks*, Phys. Rev. Lett. **121**, 098301 (2018).

[73] N. Fountoulakis and T. Müller, *Law of large numbers for the largest component in a hyperbolic model of complex networks*, Annals of Applied Probability **28**, 607–650 (2018).

[74] T. Friedrich and A. Krohmer, *On the Diameter of Hyperbolic Random Graphs*, in *Automata, Languages, and Programming*, Vol. 9135, edited by M. M. Halldórsson et al., Lecture Notes in Computer Science (Springer Berlin Heidelberg, 2015), pp. 614–625.

[30] S. Galhotra et al., *The Geometric Block Model*, (2018), arXiv:1709.05510.

[42] G. García-Pérez, *A geometric approach to the structure of complex networks*, Ph.D. Thesis (Universitat de Barcelona, 2018).

[38] G. García-Pérez et al., *The hidden hyperbolic geometry of international trade: World Trade Atlas 1870-2013*, Sci. Rep. **6**, 33441 (2016).

[75] G. García-Pérez et al., *A new approach to international trade from Network Geometry. The World Trade Atlas 1870-2013*, in *Networks of International Trade and Investment: Understanding globalisation through the lens of network analysis*, edited by S. Gorgoni et al. (Vernon Press, 2018), p. 344.

[17] G. García-Pérez et al., *Multiscale unfolding of real networks by geometric renormalization*, Nat. Phys. **14**, 583–589 (2018).

[31] G. García-Pérez et al., *Soft Communities in Similarity Space*, J. Stat. Phys. **173**, 775–782 (2018).

[25] G. García-Pérez et al., *Mercator: uncovering faithful hyperbolic embeddings of complex networks*, New J. Phys. **21**, 123033 (2019).

[18] E. Garuccio et al., *Multiscale network renormalization: scale-invariance without geometry*, arXiv, 2009.11024 (2020).

[76] A. J. Gates et al., *Nature's reach: narrow work has broad impact*, Nature **575**, 32–34 (2019).

[77] W. Gu et al., *Principled approach to the selection of the embedding dimension of networks*, Nat. Commun. **12**, 3772 (2021).

[78] L. Gugelmann et al., *Random Hyperbolic Graphs: Degree Sequence and Clustering*, in *Automata, Languages, and Programming*, Vol. 7392, edited by A. Czumaj et al., Lecture Notes in Computer Science (Springer Berlin Heidelberg, 2012), pp. 573–585.

[79] L. Gugelmann et al., *Random Hyperbolic Graphs: Degree Sequence and Clustering*, (2012), arXiv:1205.1470.

[80] P. Guha et al., *Gravity and depth of social media networks*, J. Complex Netw. **9**, cnab016 (2021).

[19] A. Gulyás et al., *Navigable networks as Nash equilibria of navigation games*, Nat. Commun. **6**, 7651 (2015).

[81] H. Hartle et al., *Dynamic hidden-variable network models*, Phys. Rev. E **103**, 052307 (2021).

[82] T. Hutchcroft, *Percolation on Hyperbolic Graphs*, Geometric and Functional Analysis **29**, 766–810 (2019).

[83] M. Keller-Ressel and S. Nargang, *Hydra: a method for strain-minimizing hyperbolic embedding of network- and distance-based data*, J. Complex Netw. **8**, 10.1093/comnet/cnaa002 (2020).

[84] S. Kerrache et al., *A Scalable Similarity-Popularity Link Prediction Method*, Sci. Rep. **10**, 6394 (2020).

[4] M. Kitsak et al., *Latent geometry of bipartite networks*, Phys. Rev. E **95**, 032309 (2017).

[85] M. Kitsak et al., *Link prediction with hyperbolic geometry*, Phys. Rev. Research **2**, 043113 (2020).

[86] M. Kiwi and D. Mitsche, *A Bound for the Diameter of Random Hyperbolic Graphs*, in *2015 Proceedings of the Meeting on Analytic Algorithmics and Combinatorics (ANALCO)* (Society for Industrial and Applied Mathematics, 2014), pp. 26–39.

[8] K.-K. Kleineberg et al., *Hidden geometric correlations in real multiplex networks*, Nat. Phys. **12**, 1076–1081 (2016).

[9] K.-K. Kleineberg et al., *Geometric Correlations Mitigate the Extreme Vulnerability of Multiplex Networks against Targeted Attacks*, Phys. Rev. Lett. **118**, 218301 (2017).

[87] M. Knor et al., *Graphs with the edge metric dimension smaller than the metric dimension*, Appl. Math. Comput. **401**, 126076 (2021).

[88] B. Kovács and G. Palla, *Optimisation of the coalescent hyperbolic embedding of complex networks*, (2020), arXiv:2009.04702.

[89] D. Krioukov, *Clustering Implies Geometry in Networks*, Phys. Rev. Lett. **116**, 208302 (2016).

[1] D. Krioukov et al., *Curvature and temperature of complex networks*, Phys. Rev. E **80**, 035101 (2009).

[2] D. Krioukov et al., *Hyperbolic geometry of complex networks*, Phys. Rev. E **82**, 036106 (2010).

[41] D. Krioukov et al., *Network Cosmology*, Sci. Rep. **2**, 793 (2012).

[90] I. Kryven et al., *Renormalization group for link percolation on planar hyperbolic manifolds*, Phys. Rev. E **100**, 022306 (2019).

[91] S. P. Lalley, *Percolation clusters in hyperbolic tessellations*, Geometric & Functional Analysis GAFA **11**, 971–1030 (2001).

[92] M. Ławniczak et al., *Hearing Euler characteristic of graphs*, Phys. Rev. E **101**, 052320 (2020).

[93] W. Leal et al., *Ricci curvature of random and empirical directed hypernetworks*, Appl. Netw. Sci. **5**, 65 (2020).

[94] A. Liu and M. A. Porter, *Spatial strength centrality and the effect of spatial embeddings on network architecture*, Phys. Rev. E **101**, 062305 (2020).

[95] B. I. Mahler, *Analysis of Contagion Maps on a Class of Networks That Are Spatially Embedded in a Torus*, SIAM J. Appl. Math. **81**, 1416–1440 (2021).

[32] S. Metzler and P. Miettinen, *HyGen: generating random graphs with hyperbolic communities*, Appl. Netw. Sci. **4**, 53 (2019).

[96] M. Mézard et al., *Spectra of euclidean random matrices*, Nuclear Physics B **559**, 689–701 (1999).

[33] A. Muscoloni and C. V. Cannistraci, *A nonuniform popularity-similarity optimization (nPSO) model to efficiently generate realistic complex networks with communities*, New J. Phys. **20**, 052002 (2018).

[34] A. Muscoloni and C. V. Cannistraci, *Leveraging the nonuniform PSO network model as a benchmark for performance evaluation in community detection and link prediction*, New J. Phys. **20**, 063022 (2018).

[26] A. Muscoloni et al., *Machine learning meets complex networks via coalescent embedding in the hyperbolic space*, Nat. Commun. **8**, 1615 (2017).

[97] A. I. Nesterov and P. H. Mata Villafuerte, *Complex networks in the framework of nonassociative geometry*, Phys. Rev. E **101**, 032302 (2020).

[98] C. Norrenbrock et al., *Fragmentation properties of two-dimensional proximity graphs considering random failures and targeted attacks*, Phys. Rev. E **94**, 062125 (2016).

[99] E. Ortiz et al., *Geometric detection of hierarchical backbones in real networks*, arXiv, 2006.03207 (2020).

[100] E. Ortiz et al., *Geometric detection of hierarchical backbones in real networks*, Phys. Rev. Research **2**, 033519 (2020).

[101] E. Ortiz and M. Á. Serrano, *Multiscale opinion dynamics on real networks*, (2021), arXiv:2107.06656.

[20] E. Ortiz et al., *Navigability of temporal networks in hyperbolic space*, Sci. Rep. **7**, 15054 (2017).

[43] E. Ortiz Castillo, *Mapping complexity through network geometry: From structure to dynamics of real-world networks*, Ph.D. Thesis (Universitat de Barcelona, 2021).

[102] S. Osat et al., *k-core structure of real multiplex networks*, Phys. Rev. Research **2**, 023176 (2020).

[103] F. Papadopoulos and M. A. Rodríguez Flores, *Latent geometry and dynamics of proximity networks*, Phys. Rev. E **100**, 052313 (2019).

[13] F. Papadopoulos et al., *Popularity versus similarity in growing networks*, Nature **489**, 537–540 (2012).

[27] F. Papadopoulos et al., *Network geometry inference using common neighbors*, Phys. Rev. E **92**, 022807 (2015).

[104] M. A. Rodríguez Flores and F. Papadopoulos, *Similarity Forces and Recurrent Components in Human Face-to-Face Interaction Networks*, Phys. Rev. Lett. **121**, 258301 (2018).

[105] M. A. Rodríguez-Flores and F. Papadopoulos, *Hyperbolic Mapping of Human Proximity Networks*, arXiv, 2005.13216 (2020).

[106] A. Samal et al., *Comparative analysis of two discretizations of Ricci curvature for complex networks*, Sci. Rep. **8**, 8650 (2018).

[107] A. Samal et al., *Network geometry and market instability*, R. Soc. Open Sci. **8**, 201734.

[3] M. Á. Serrano et al., *Self-Similarity of Complex Networks and Hidden Metric Spaces*, Phys. Rev. Lett. **100**, 078701 (2008).

[108] M. Á. Serrano et al., *Percolation in Self-Similar Networks*, Phys. Rev. Lett. **106**, 048701 (2011).

[5] M. Á. Serrano et al., *Uncovering the hidden geometry behind metabolic networks*, Mol. Biosyst. **8**, 843–850 (2012).

[109] A. L. Smith et al., *The Geometry of Continuous Latent Space Models for Network Data*, Stat. Sci. **34**, 428–453 (2019).

[110] M. Starnini et al., *Geometric randomization of real networks with prescribed degree sequence*, New J. Phys. **21**, 053039 (2019).

[111] B. Tadić et al., *Large-scale influence of defect bonds in geometrically constrained self-assembly*, Phys. Rev. E **102**, 032307 (2020).

[112] B. Tadić and N. Gupte, *Hidden geometry and dynamics of complex networks: Spin reversal in nanoassemblies with pairwise and triangle-based interactions (a)*, EPL **132**, 60008 (2021).

[113] B. Tadić et al., *Functional Geometry of Human Connectomes*, Sci. Rep. **9**, 12060 (2019).

[114] A. Tandon et al., *Community detection in networks using graph embeddings*, Phys. Rev. E **103**, 022316 (2021).

[115] P. Tee and C. A. Trugenberger, *Enhanced Forman curvature and its relation to Ollivier curvature*, EPL **133**, 60006 (2021).

[116] N. G. Trillos et al., *Geometric structure of graph Laplacian embeddings*, J. Mach. Learn. Res. **22**, 63 (2021).

[117] P. van der Hoorn et al., *Ollivier-Ricci curvature convergence in random geometric graphs*, Phys. Rev. Research **3**, 013211 (2021).

[118] J. van der Kolk et al., *A geometry-induced topological phase transition in random graphs*, (2021), arXiv:2106.08030.

[39] W. Whi et al., *Hyperbolic disc embedding of functional human brain connectomes using resting state fMRI*, bioRxiv, 2021.03.25.436730 (2021).

[119] M. Wilsher et al., *Connectivity in one-dimensional soft random geometric graphs*, Phys. Rev. E **102**, 062312 (2020).

[120] F. Wolf et al., *Edge directionality properties in complex spherical networks*, Phys. Rev. E **99**, 012301 (2019).

[6] Z. Wu et al., *An Asymmetric Popularity-Similarity Optimization Method for Embedding Directed Networks into Hyperbolic Space*, Complexity **2020**, 8372928 (2020).

[121] Z. Xie and T. Rogers, *Scale-invariant geometric random graphs*, Phys. Rev. E **93**, 032310 (2016).

[12] W. Yang and D. Rideout, *High Dimensional Hyperbolic Geometry of Complex Networks*, Mathematics **8**, 1861 (2020).

[122] Y.-J. Zhang et al., *Model-free hidden geometry of complex networks*, Phys. Rev. E **103**, 012305 (2021).

[21] M. Zheng et al., *Geometric renormalization unravels self-similarity of the multiscale human connectome*, Proc. Natl. Acad. Sci. U.S.A. **117**, 20244–20253 (2020).

[14] M. Zheng et al., *Scaling up real networks by geometric branching growth*, Proc. Natl. Acad. Sci. U.S.A. **118**, e2018994118 (2021).

[35] K. Zuev et al., *Emergence of Soft Communities from Geometric Preferential Attachment*, Sci. Rep. **5**, 9421 (2015).