Lemma 1. *Take any $v \in V$. If there exists a node s that lies on a shortest path between u and v and $s \notin D$. Then $v \notin D$.*

Proof. Suppose s lies on a shortest path between u and v and $s \notin D$. Then $d(u,v) = d(u,s) + d(s,v)$. Since $d(v,s) + d(s,M) \geq d(v,M)$, we have $d(u,v) \geq d(u,s) + d(v,M) - d(s,M)$. Since $s \notin D$, $d(u,s) \geq d(s,M)$. Thus $d(u,v) \geq d(v,M)$ and $v \notin D$.