Theorem 1. *With ℓ and s in $O\left(polylog(|V|)\right)$,* csav *algorithm computes a k-node placement $M \subseteq V$ in time $\tilde{O}\left(k^2|E|\right)$.*

Proof. With polylogarithmically many pivots, the complexity of rand algorithm is $\tilde{O}\left(|E|\right)$; this is the time used to approximate penalty and generate the initial candidate S_1. The score procedure takes $O(|E| + |V|\log|V|)$ and since the ℓ-confined celf repeats for k iterations each running score once, the total time of performing candidate selection in celf-r is $O(k(|E| + |V|\log|V|))$. Due to the limited search space, the add-candidate procedure evaluates marginal reward at most $k^2\ell$ times, each of which takes time $O(|E|)$. Thus we spend $O(k^2\ell \cdot |E|)$ time on updating rewards. In conclusion, the total complexity of the first phase is $\tilde{O}\left(k^2\ell|E|+k|V|\log|V| + |E|\right) = \tilde{O}\left(k^2|E|\right)$.

In the iterative improvement algorithm, each round takes $O(k(|E|+|V|\log|V|))$ to generate candidates and $O(k\ell \cdot |E|)$ to evaluate marginal reward. Thus in s rounds, it takes $O(sk(|V|\log|V|+\ell|E|)) = \tilde{O}\left(k|E|\right)$. Therefore the overall time of the algorithm is $\tilde{O}\left(k^2|E|\right)$.