Homework 1

Sample Solution

January 24, 2013

1. See repository on `github.com`.
2. See code in `sample-solution.rkt`.
3. (a) Demonstrate that $f(n) = 2n^2 - 3n + 4 \in O(n^2)$. That is, choose a specific $c > 0$ and a specific $n_0 > 0$. Then, show that for any possible $n > n_0$, it must be true that $f(n) \le cn^2$.

   Solution: Solving for c and n_0 as we go.

$$\begin{array}{rcll} 2n^2 - 3n + 4 & \le & cn^2 & \\ 2n^2 - 3n + 4n & \le & cn^2 & \text{because } 4 \le 4n \text{ if we choose } n_0 \ge 1 \\ 2n^2 + n & \le & cn^2 & \\ 2n^2 + n^2 & \le & cn^2 & \text{because } n \le n^2 \text{ given } n_0 \ge 1 \\ 3n^2 & \le & cn^2 & \text{which is true if we choose } c \ge 3 \end{array}$$

   Therefore, the equation holds for $c = 3$ and $n_0 = 1$.

   (b) Demonstrate that $f(n) = 3\sqrt{n} \in O(n)$. That is, choose a specific $c > 0$ and a specific $n_0 > 0$. Then, show that for any possible $n > n_0$, it must be true that $f(n) \le cn$.

   Solution: Solving for c and n_0 as we go.

$$\begin{array}{rcll} 3\sqrt{n} & \le & cn & \\ 3n & \le & cn & \text{because } \sqrt{n} \le n \text{ if we choose } n_0 \ge 1 \\ & & & \text{and this is true if we choose } c \ge 3 \end{array}$$

   Therefore, the equation holds for $c = 3$ and $n_0 = 1$.

   (c) Demonstrate that $f(n) = \Sigma_{i=1}^{n}(2i^2 + 3i + 5) \in O(n^3)$. That is, choose a specific $c > 0$ and a specific $n_0 > 0$. Then, show that for any possible $n > n_0$, it must be true that $f(n) \le cn^3$.

   (d) Demonstrate that $f(n) = \sqrt{n} \notin O(\log_2 n)$. That is, for any possible $c > 0$ and for any possible $n_0 > 0$, give a formula for some $n > n_0$ and show that n always satisfies $f(n) > c \log_2 n$.