Homework 3: Sample Solution

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1. (a) The running time for `difference/recursive` is $T(n)$, where n is the sum of the lengths of the two input strings. For `difference-between/recursive` and its helpers, the length includes only characters from indices `i` and `j` in the respective strings.

   The running time for `difference-between/recursive` can be stated as this recurrence relation:

   $$T(n) = T(n-2) + T(n-1) + T(n-1) + T(n-2) + T(n-4) + 1$$

   Here, the first four recursive references to T correspond to the replace, insert, delete, and swap operations, and the fifth corresponds to the case where we "copy" a character by performing no operations. We can find a lower bound on $T(n)$ by reducing all the $T(n-1)$ and $T(n-2)$ terms with $T(n-4)$ and combining them. This gives us:

   $$T(n) = 5T(n-4) + 1$$

   This is in $Theta(5^{n/4})$, or $\Theta((\sqrt[4]{5})^n)$.

   (b) The memoization table for `difference` has $(m+1) \times (n+1)$ entries, one for every pair of locations in the input strings of lengths m and n respectively. It takes $\Theta(1)$ time to compute the solution at each entry in terms of others, so `difference` runs in $O(mn)$ time.

2. (a) The running time for `shared/recursive` is $T(n)$, again using n to mean the sum of the lengths of the two input strings. Again, we consider the indices passed to helper functions to effectively mark the new start of the string.

   The running time for `shared-at/recursive` is $O(n)$; it simply traverses both strings as far as both have the same characters.

   The running time for `max-shared-starting-at/recursive` can be stated as this recurrence relation:

   $$T(n) = 2T(n-1) + O(n)$$

   In both cases where it recurs, it drops a single character off of one of its inputs. A lower bound on this recurrence is $T(n) = 2T(n-1) + \Theta(1)$, which is in $\Theta(2^n)$.

   (b) The two memoization tables for `shared` each have $(m+1) \times (n+1)$ entries, one for every pair of locations in the input strings of lengths m and n respectively. Each step of both `shared-at` and `max-shared-starting-at` takes $\Theta(1)$ time to compute a solution based on other entries in the table. The running time of `shared` is therefore $O(mn)$.