Homework 1

Sample Solution

January 24, 2013

1. See repository on `github.com`.
2. See code in `sample-solution.rkt`.
3. (a) Demonstrate that $f(n) = 2n^2 - 3n + 4 \in O(n^2)$. That is, choose a specific $c > 0$ and a specific $n_0 > 0$. Then, show that for any possible $n > n_0$, it must be true that $f(n) \leq cn^2$.

   Solution: Solving for c and n_0 as we go.

$$
\begin{array}{rcll}
2n^2 - 3n + 4 & \leq & cn^2 & \\
4 & \leq & (c-2)n^2 + 3n & \text{by subtracting } 2n^2 - 3n \text{ from both sides} \\
4 & \leq & (c-2)n^2 + 3 & \text{because } 3 < 3n \text{ if we choose } 1 \leq n_0 < n \\
1 & \leq & (c-2)n^2 & \text{by subtracting 3 from both sides} \\
1 & \leq & n^2 & \text{if we choose } c = 3
\end{array}
$$

   Therefore, the equation holds for $c = 3$ and $n_0 = 1$.

   (b) Demonstrate that $f(n) = 3\sqrt{n} \in O(n)$. That is, choose a specific $c > 0$ and a specific $n_0 > 0$. Then, show that for any possible $n > n_0$, it must be true that $f(n) \leq cn$.

   (c) Demonstrate that $f(n) = \Sigma_{i=1}^{n}(2i^2 + 3i + 5) \in O(n^3)$. That is, choose a specific $c > 0$ and a specific $n_0 > 0$. Then, show that for any possible $n > n_0$, it must be true that $f(n) \leq cn^3$.

   (d) Demonstrate that $f(n) = \sqrt{n} \notin O(\log_2 n)$. That is, for any possible $c > 0$ and for any possible $n_0 > 0$, give a formula for some $n > n_0$ and show that n always satisfies $f(n) > c \log_2 n$.