

# Trees

**Welcome back to CS 2100!**

**Prof. Rasika Bhalerao**

## Poll: What's wrong with this recursive function?

```
def find_max(nums: list[int], index: int = 0) -> int:
    """Returns the maximum of the numbers in nums, starting from the given index"""
    if index == len(nums) - 1:
        return nums[index]
    else:
        max_of_rest: int = find_max(nums, index)
        if nums[index] < max_of_rest:
            return max_of_rest
        else:
            return nums[index]
```

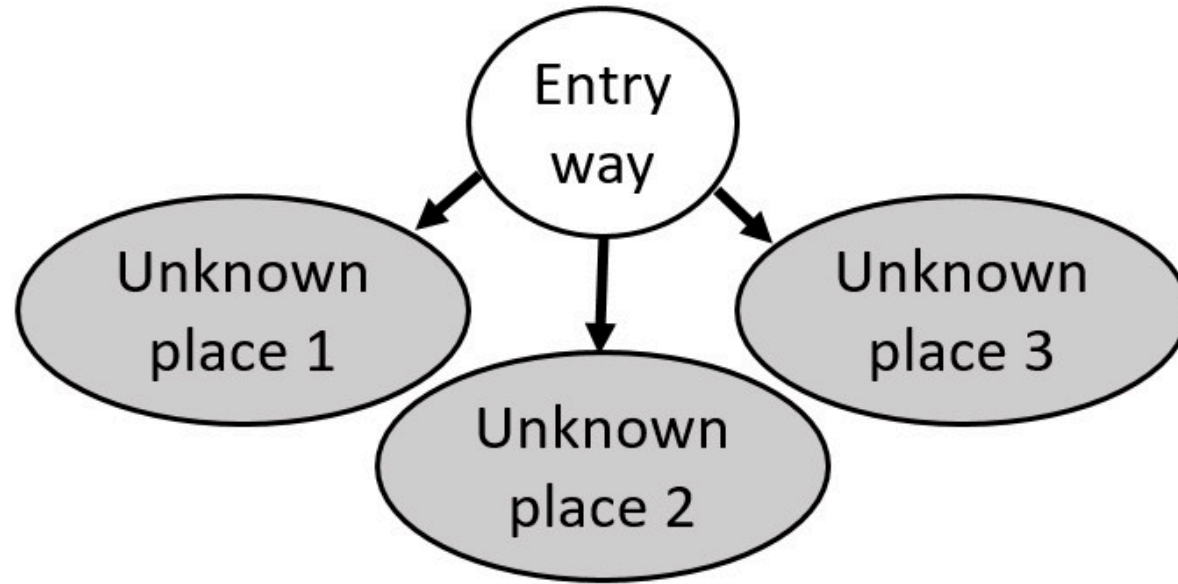
1. It's missing a base case
2. It's missing a recursive case
3. The recursive case doesn't progress towards the base case
4. It has an "off-by-one bug"

## Poll: What's wrong with this recursive function?

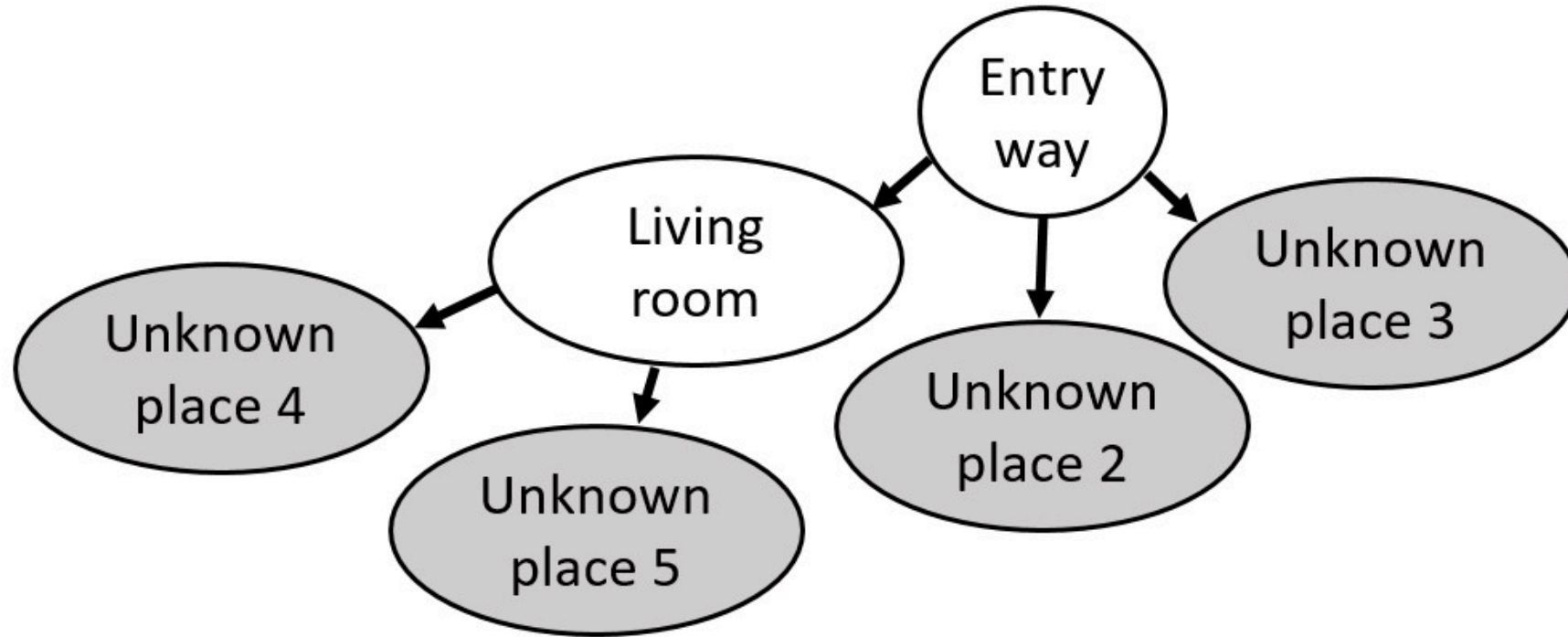
```
def countdown(n: int) -> None:
    """Prints numbers from n down to 0, one on each line"""
    if n > 0:
        print(n)
        countdown(n - 1)
```

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2. It's missing a recursive case
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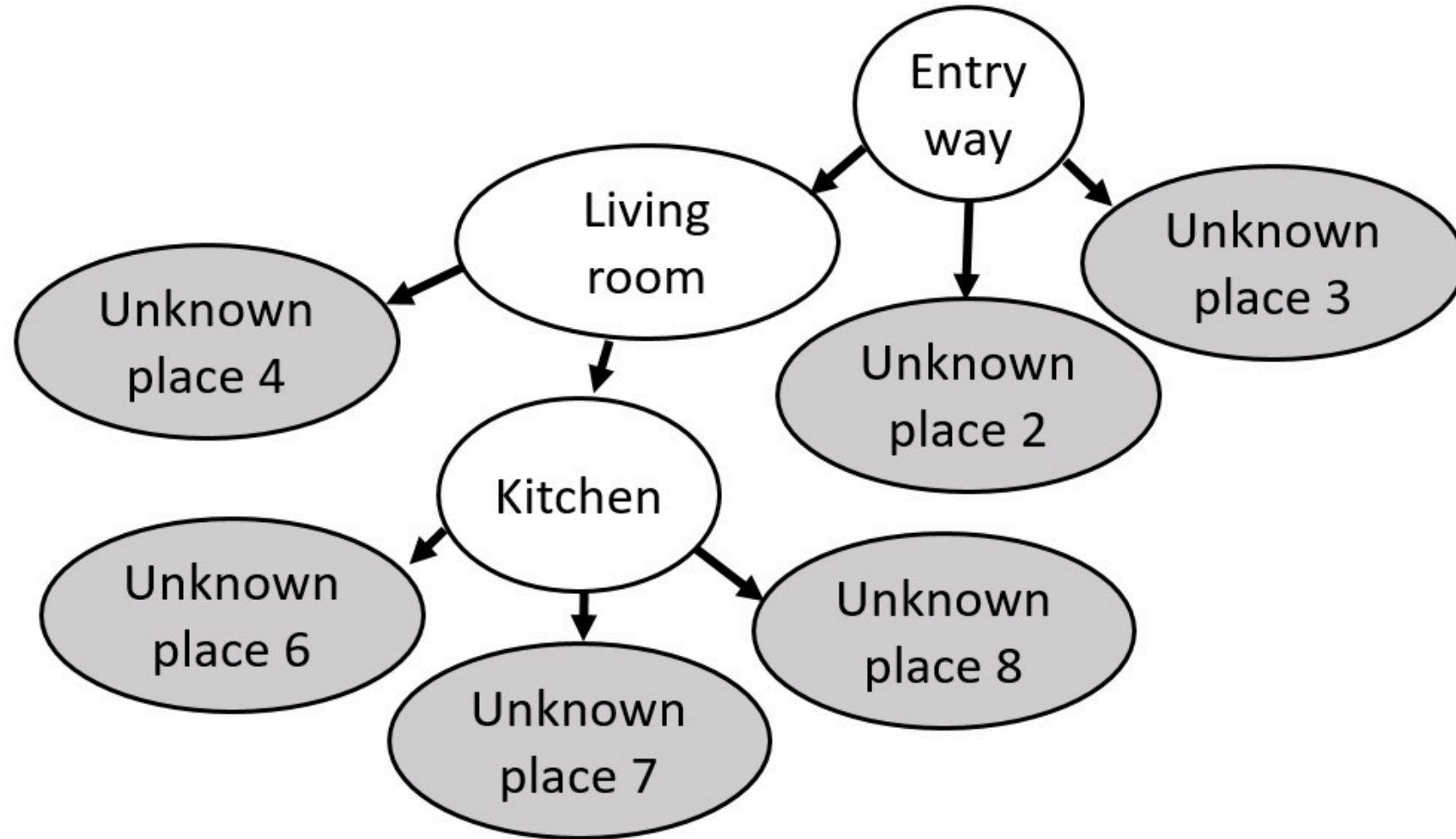
Let's say you're exploring a place...



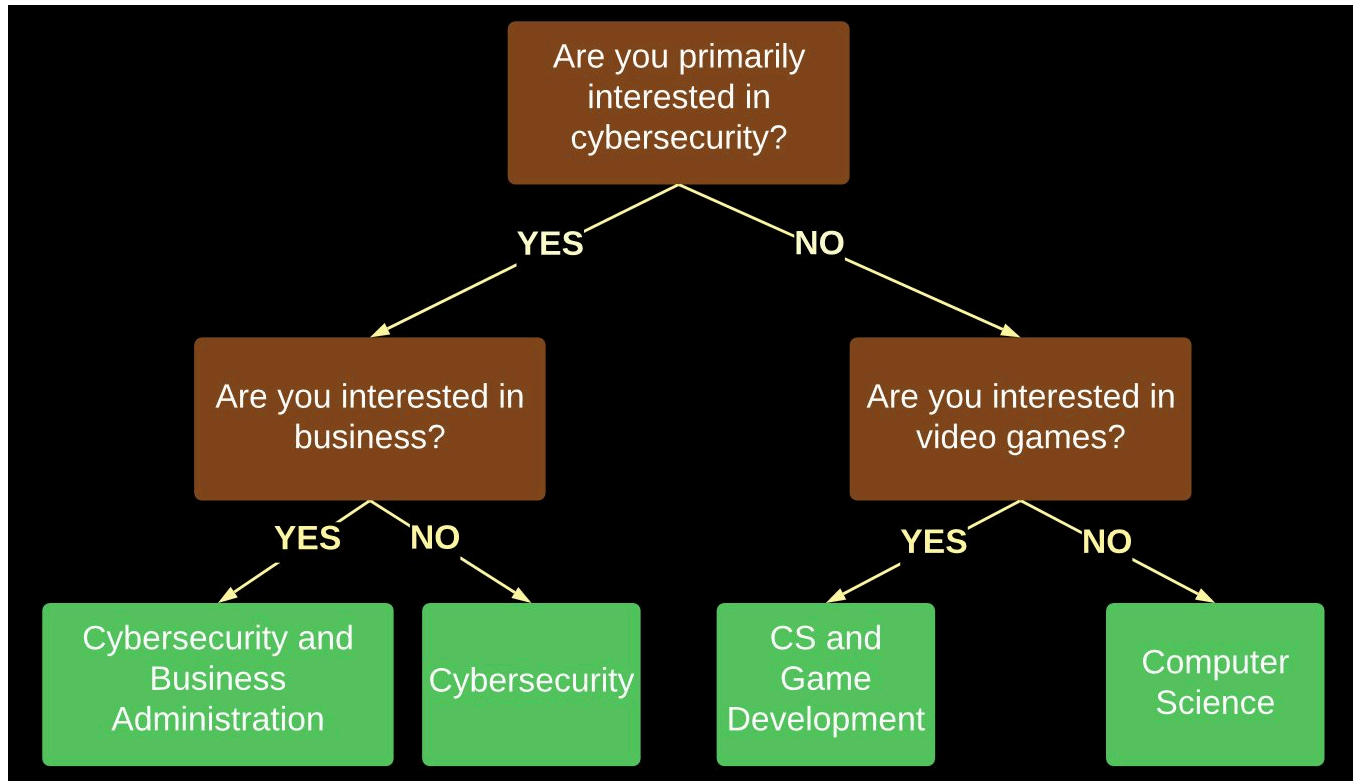
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Let's say you're exploring a place...



You may have seen trees like this decision tree:



## Some tree terminology / rules

- Each *node* may have data
- There is a *root* node (the start)
- Each node points to any number of *child* nodes
- Cycles are not permitted
- There are any number of *leaf* nodes (node with no children)



Source: [Reddit](#)

Note: In computer science, we draw our trees with the root at the top and the leaves at the bottom.

# Poll: Is this a tree?

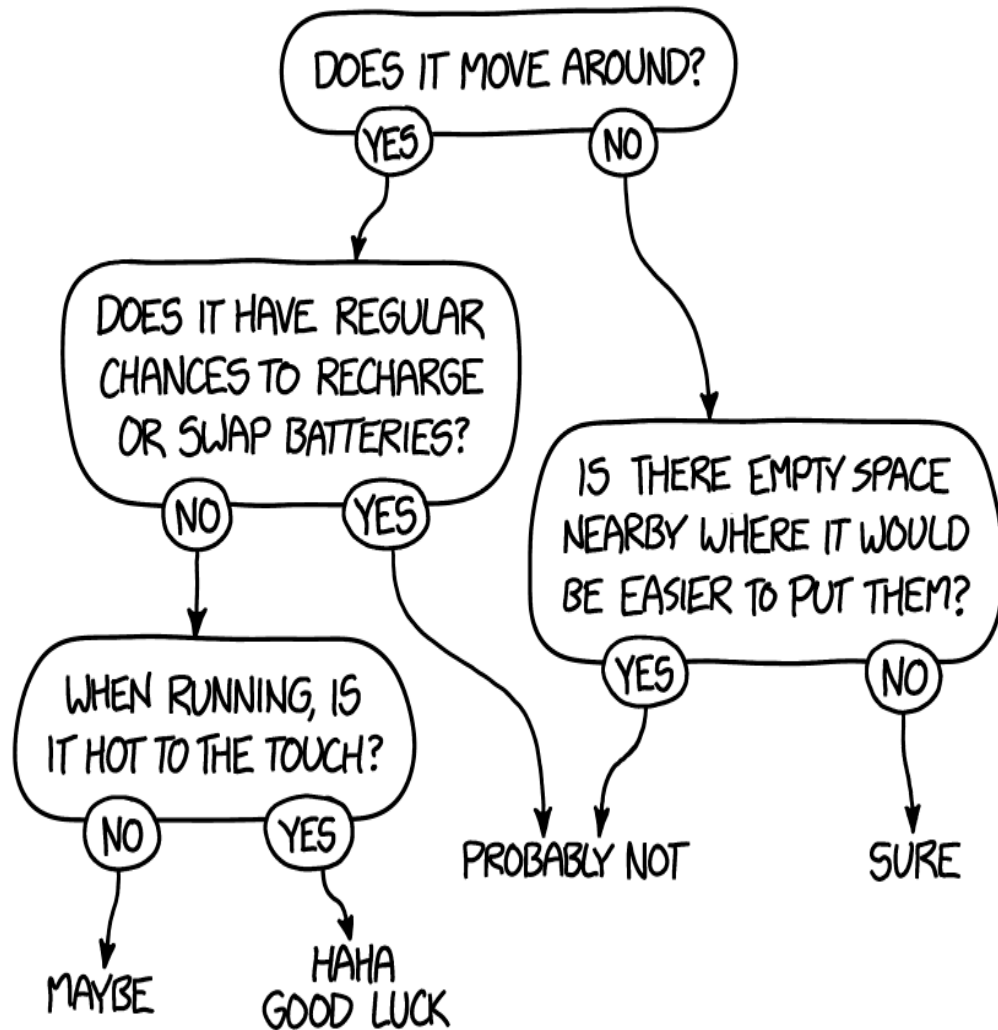


Source: <https://www.canva.com/graphs/decision-trees/>

1. Yes

2. No

SHOULD I PUT SOLAR PANELS ON IT?



Source: <https://xkcd.com/1924/>

**Poll: Is this a tree?**

1. Yes
2. No

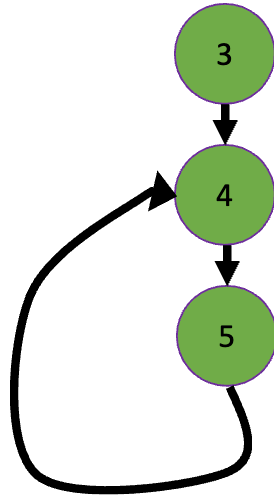
## Poll: Which of these are trees?



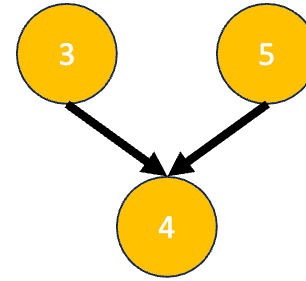
A



B



C



D

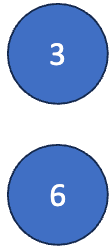
1. A

2. B

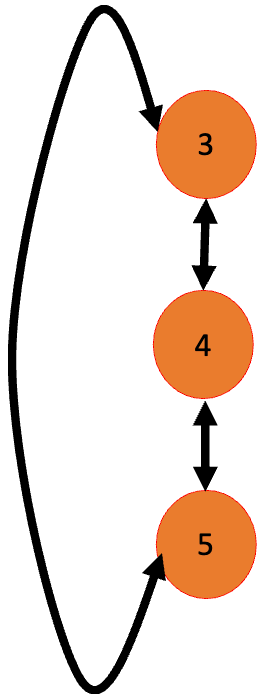
3. C

4. D

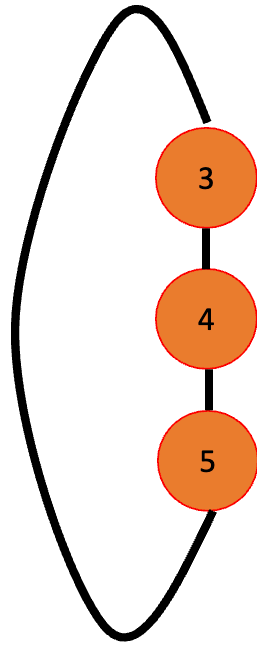
**Poll: Which of these are trees? (B and B' are equivalent.)**



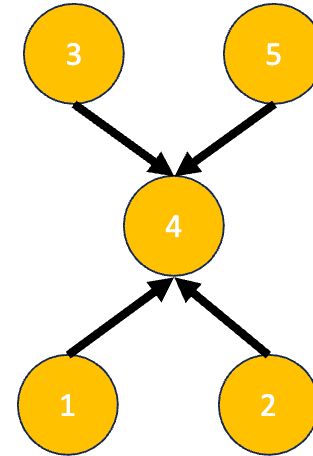
A



B



B'



C

1. A

2. B and B'

3. C

```

T = TypeVar('T')

class Node(Generic[T]):
    def __init__(self, data: T):
        self.data = data
        self.left: Optional[Node[T]] = None
        self.right: Optional[Node[T]] = None

    def __eq__(self, other: object) -> bool:
        if not isinstance(other, Node):
            raise NotImplementedError
        return self.data.__eq__(other.data)

    def __str__(self) -> str:
        value: str = f'{self.data}'
        if self.left is not None:
            value += f' {self.left}'
        else:
            value += ' *'
        if self.right is not None:
            value += f' {self.right}'
        else:
            value += ' *'
        return f'({value})'

```

```

class Tree(Generic[T]):
    def __init__(
        self,
        root_data: Optional[T] = None
    ) -> None:
        if root_data is None:
            self.root: Optional[Node[T]] = None
        else:
            self.root = Node[T](root_data)

    def __str__(self) -> str:
        return self.root.__str__()

tree: Tree[str] = Tree[str]('Entry way')

assert tree.root is not None
tree.root.left = Node[str]('Living room')

tree.root.left.right = Node[str]('Kitchen')

print(tree)

```

```

(Entry way (Living room * (Kitchen * *) *)

```

```
(Entry way (Living room * (Kitchen * *))) *)
```

Split that printed output into multiple lines:

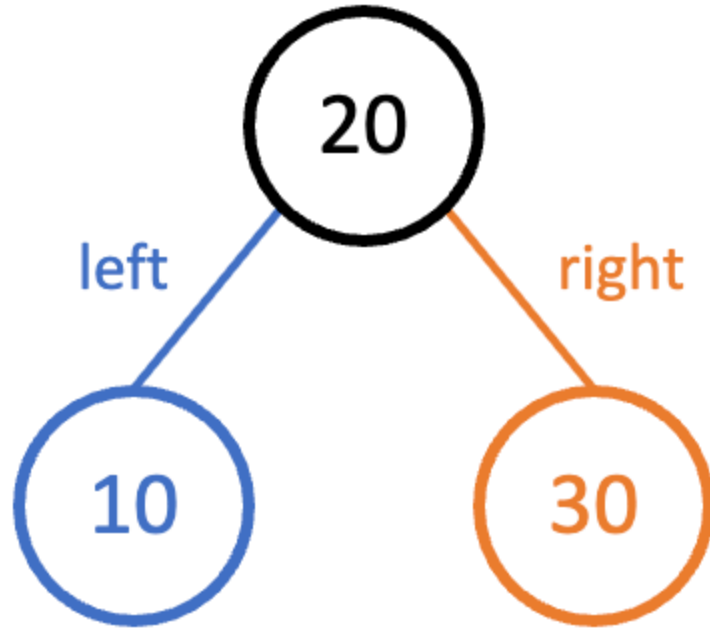
```
(Entry way  
    (Living room  
        *  
        (Kitchen * *)))  
    *  
)
```

# Binary Search Trees

**Binary Tree:** A tree in which each node has at most 2 children

The first and second child of a node are called the left and right child, respectively.

**Binary Search Tree:** A binary tree in which each node's data is greater than everything in its left subtree and less than everything in its right subtree



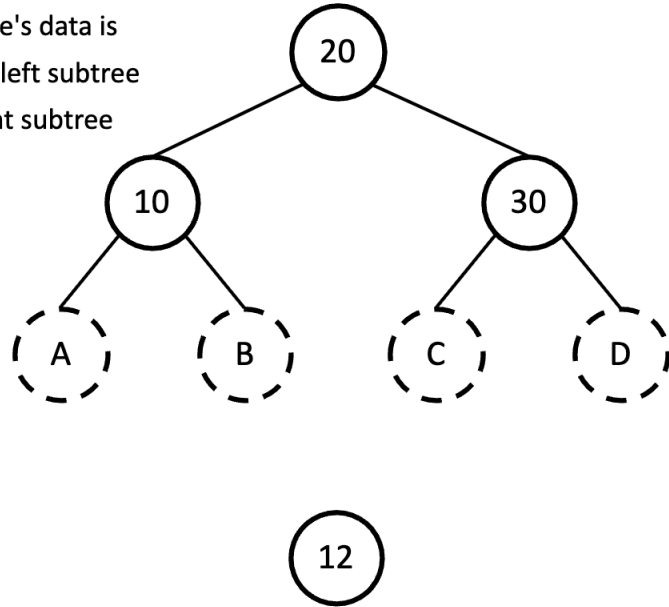
A binary tree in which each node's data is

- greater than everything in its left subtree
- less than everything in its right subtree

# Poll: This is a Binary Search Tree. Where should the 12 go?

A binary tree in which each node's data is

- greater than everything in its left subtree
- less than everything in its right subtree



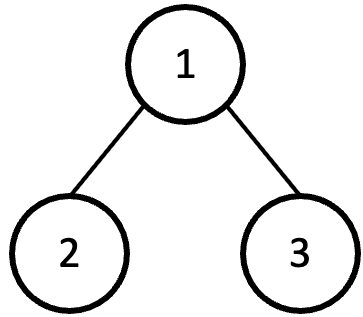
1. A

2. B

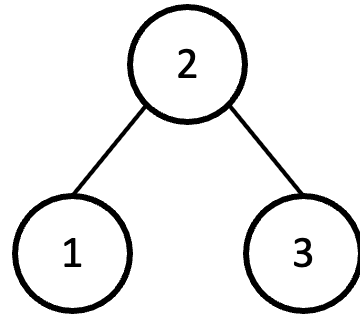
3. C

4. D

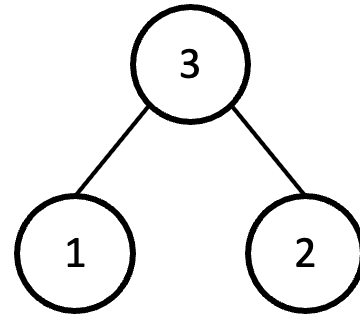
## Poll: Which of these are Binary Search Trees?



A



B



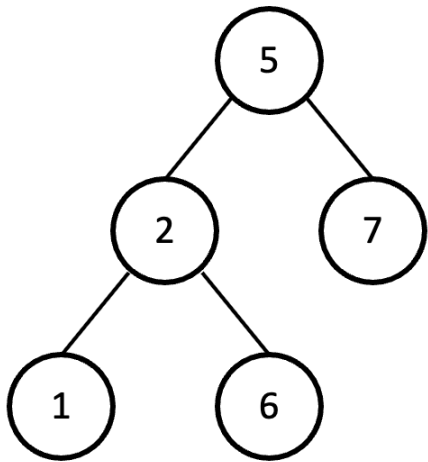
C

1. A

2. B

3. C

**Poll: Is this a BST?**

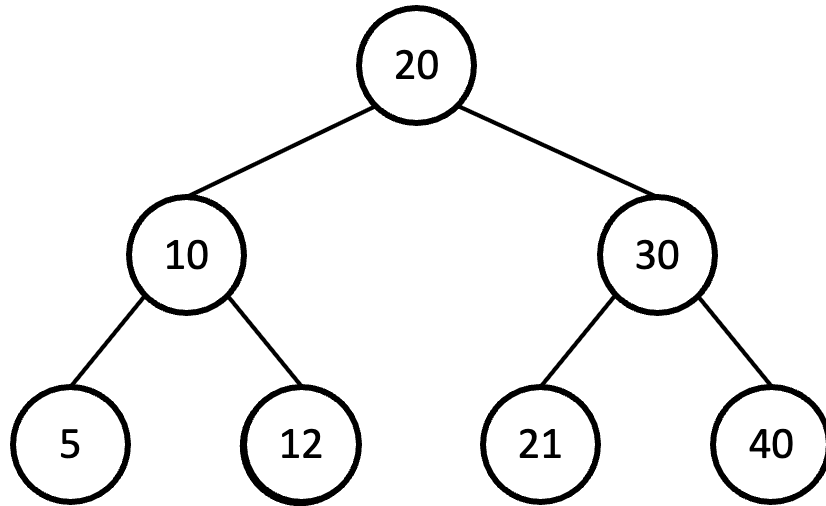


1. Yes

2. No

## What's a Binary Search Tree good for?

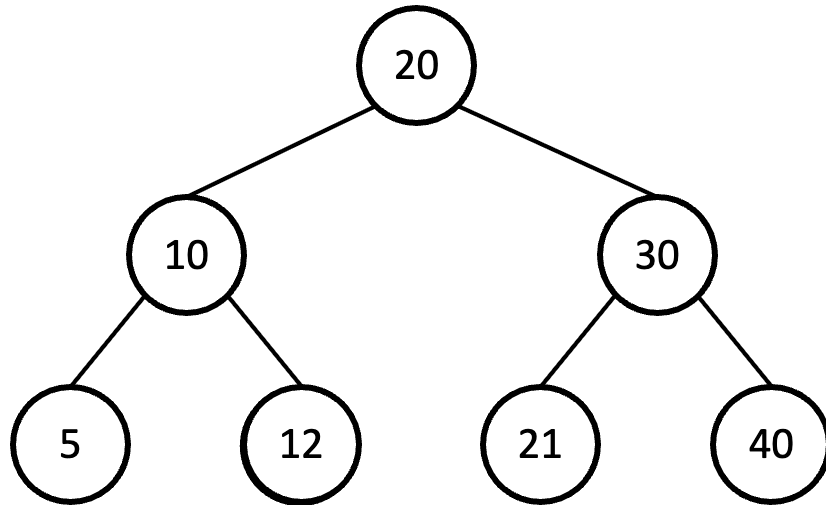
Let's say I want to check for whether the tree contains 12. How do we do that?



What about searching for 11 (with the same BST)?

# What's a Binary Search Tree good for?

Let's say I want to check for whether the tree contains 12. How do we do that?



What about searching for 11 (with the same BST)?

Hey, this is more efficient than searching a list!

It's not as efficient as hashing, but it does make a pretty good set.

And it's more useful if you care about the order of things.

`sortedcontainers.SortedSet` stores elements using a Binary Search Tree

( `pip install sortedcontainers` )

```
ss = SortedSet([3, 1, 4, 1, 5])
print(ss)  # SortedSet([1, 3, 4, 5])
ss.add(2)
print(ss)  # SortedSet([1, 2, 3, 4, 5])
```

Corresponding `SortedDict` :

```
sm = SortedDict({2: [1, 2, 3], 1: [0, 0, 0]})
print(sm)  # SortedDict({1: [0, 0, 0], 2: [1, 2, 3]})
```

<b>set</b>	<b>sortedcontainers.SortedSet</b>
Stored as a hash table (list of lists) Index of each element is calculated using <code>__hash__()</code>	Stored as a Binary Search Tree Elements must implement Comparable protocol
Constant time to look up / add / remove	Logarithmic time to look up / add / remove
Use when care more about speed than order	Use when care more about order than speed

## Poll: Which are true?

1. For a `SortedDict`, it is constant time to check whether it contains a key
2. For a `SortedDict`, it is constant time to check whether it contains a value
3. `set` s can store things which don't implement the `Comparable` protocol
4. `set` s can store things which aren't hashable
5. When we iterate over a `set`, the elements will be increasing in size
6. When we iterate over a `SortedSet` the elements will be increasing in size

Let's use recursion to search for an element in a Tree :

```
class Tree(Generic[T]):
    def __init__(self, root_data: Optional[T] = None) -> None:
        if root_data is None:
            self.root: Optional[Node[T]] = None
        else:
            self.root = Node[T](root_data)

    def __str__(self) -> str:
        return self.root.__str__()

    def __contains__(self, item: T) -> bool:
        return self.contains(item, self.root)

    def contains(self, item: T, node: Optional[Node[T]]) -> bool:
        if node is None:
            return False
        elif node.data == item:
            return True
        else:
            return self.contains(item, node.left) or self.contains(item, node.right)

tree: Tree[str] = Tree[str]('Entry way')

assert tree.root is not None
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tree.root.left.right = Node[str]('Kitchen')

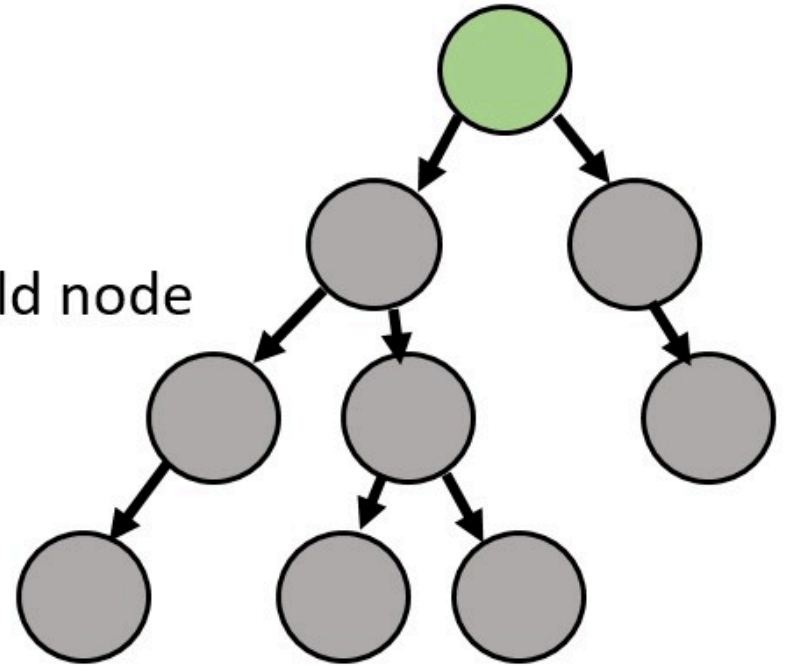
print('Kitchen' in tree) # True
print('Bathroom' in tree) # False
```

# Recursive backtracking

Recursive backtracking is a strategy to search every node of a tree.

Strategy: given a node, for each of its children:

1. Choose a child to explore
2. Recursively perform this strategy for that child node
3. Un-choose that child

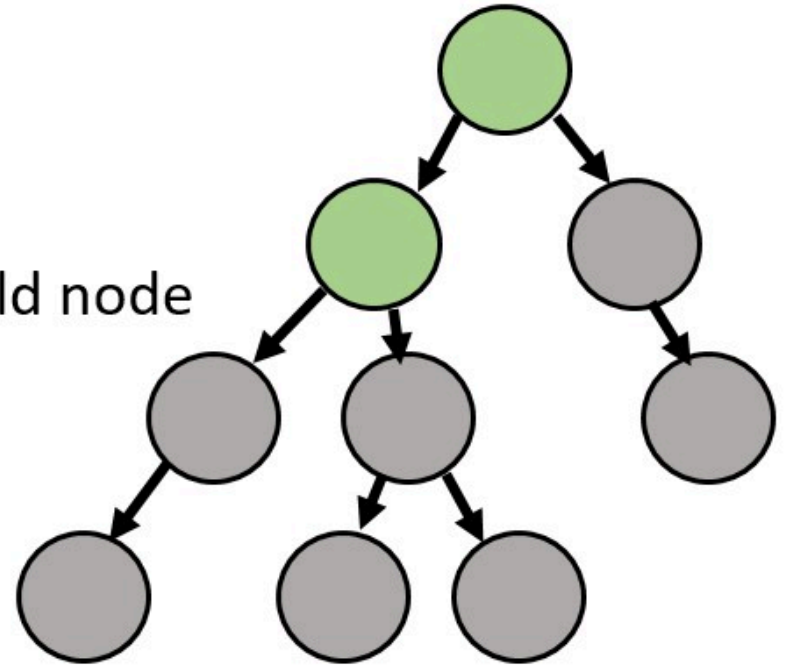


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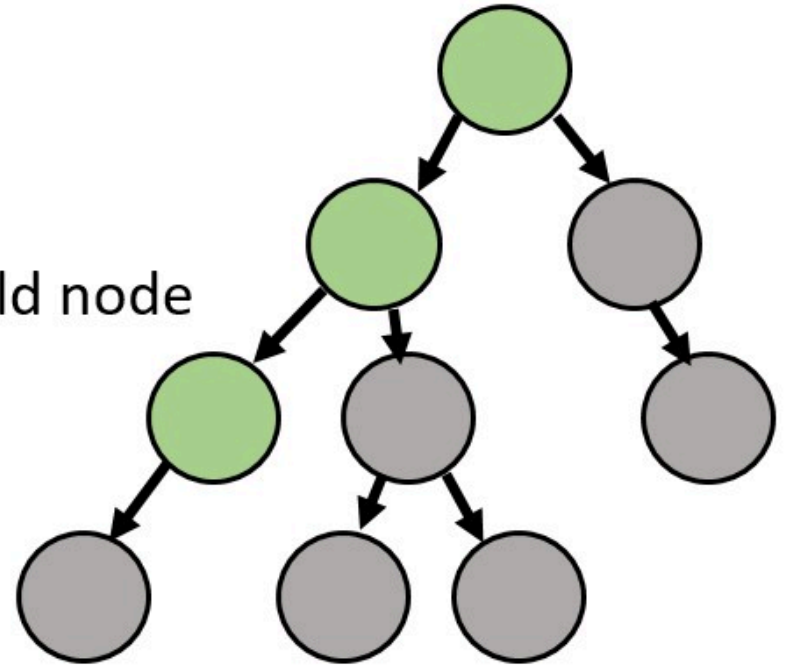


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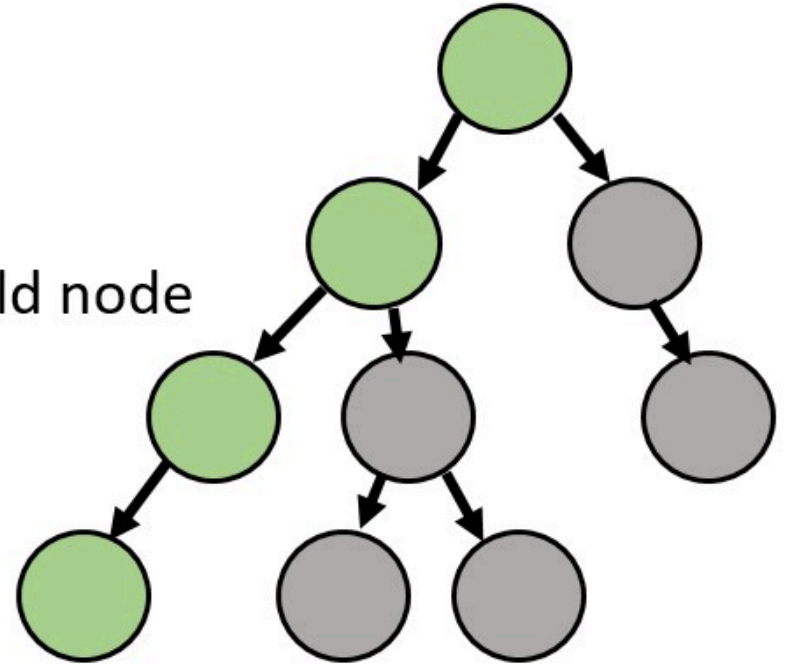


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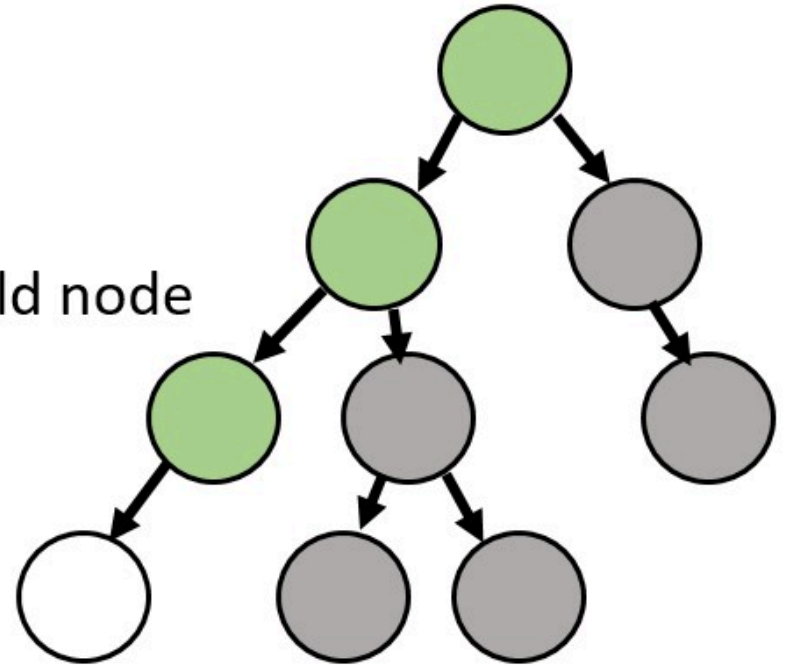


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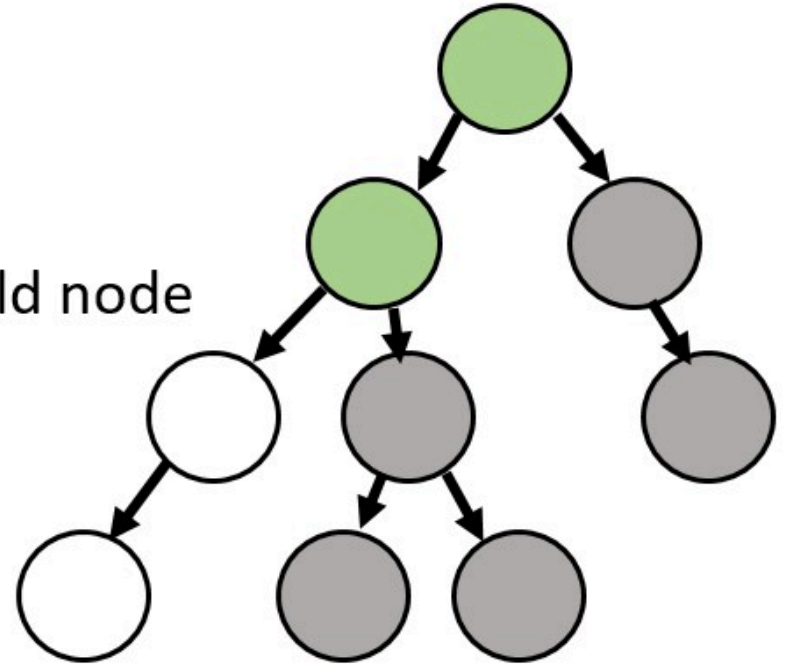


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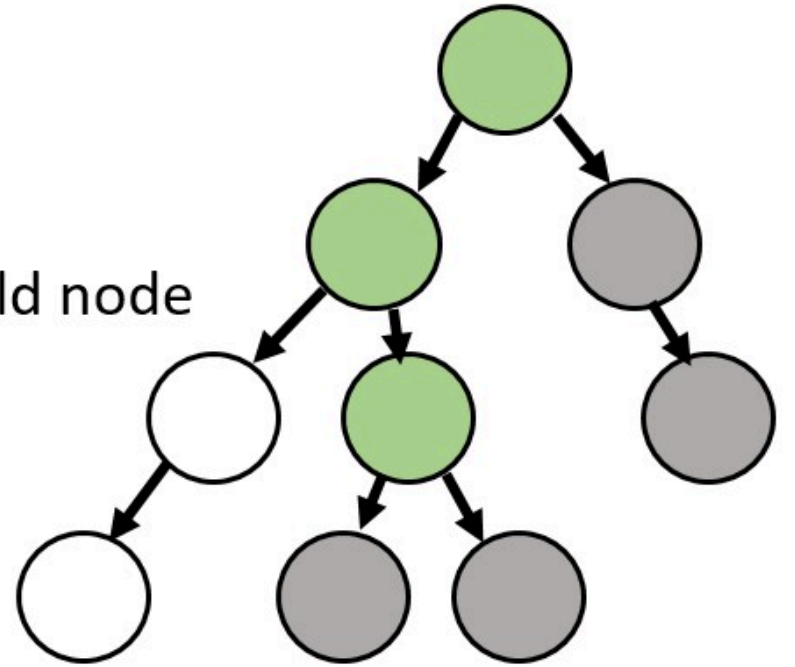


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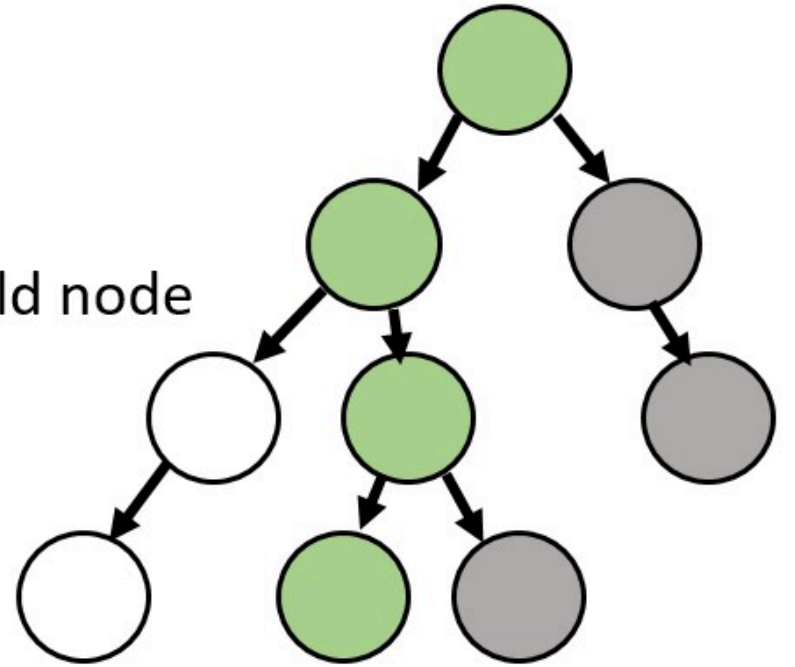


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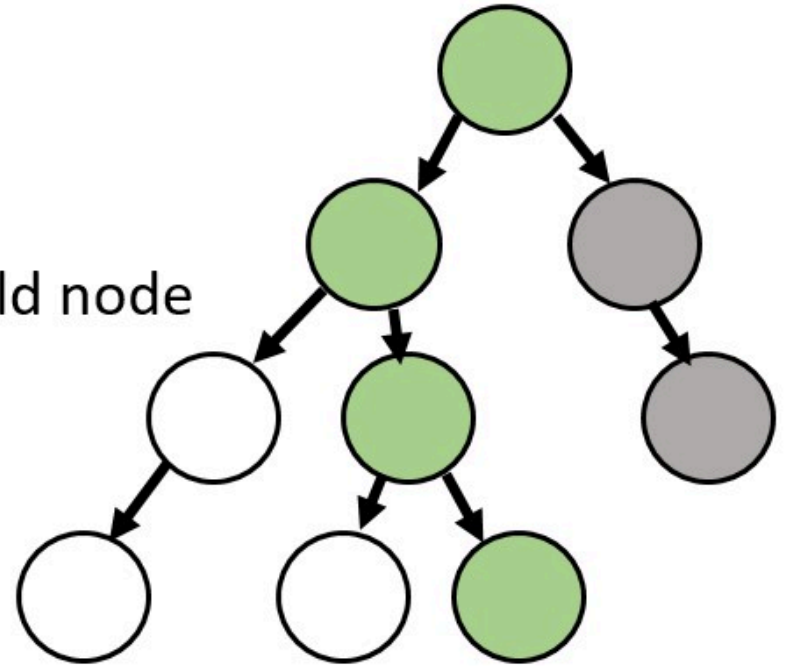


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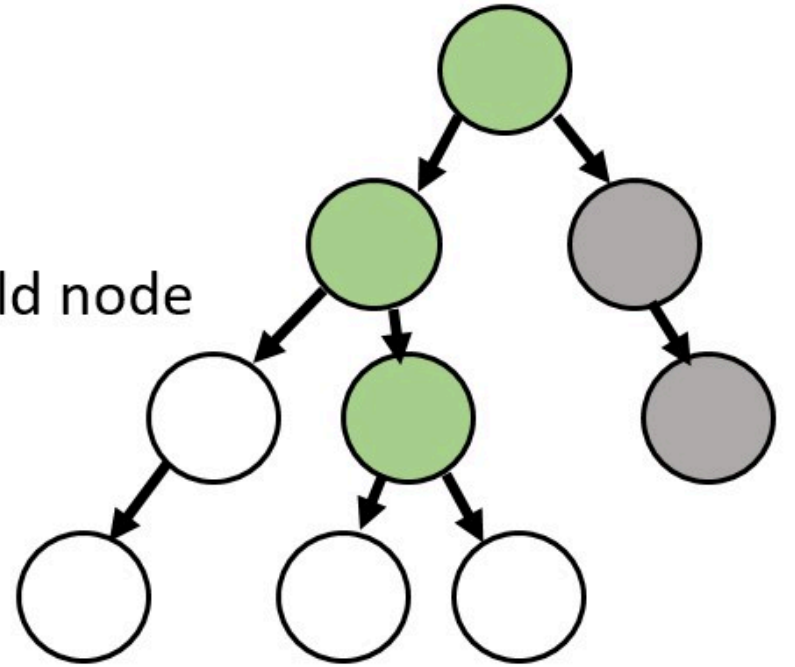


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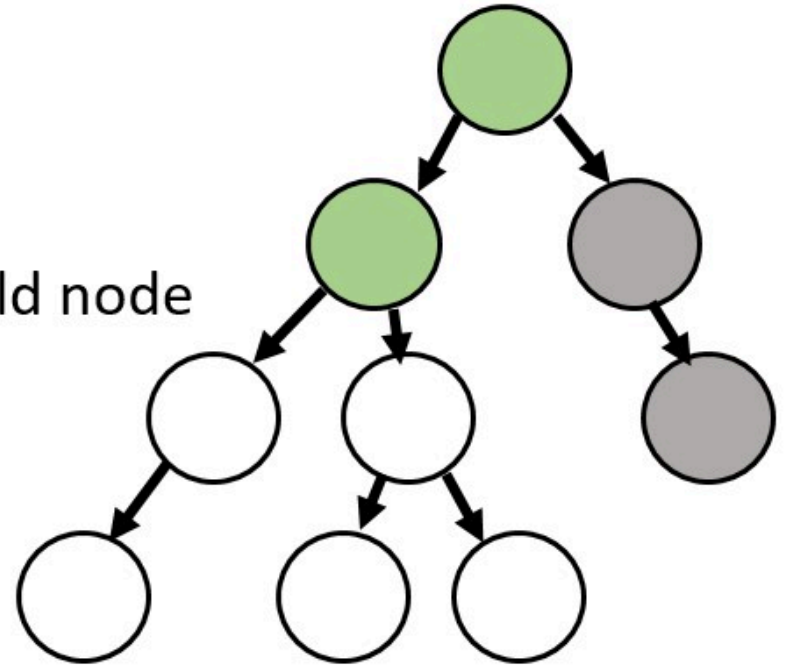


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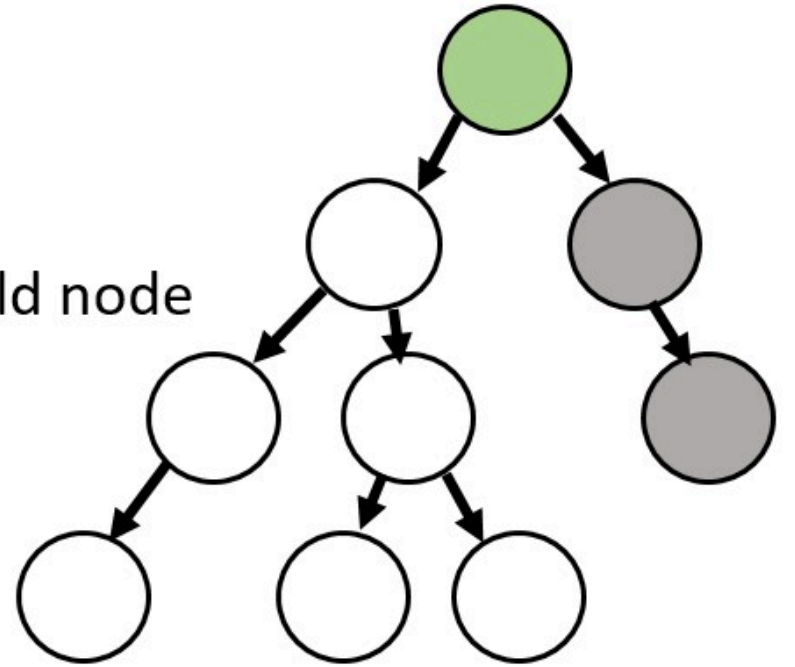


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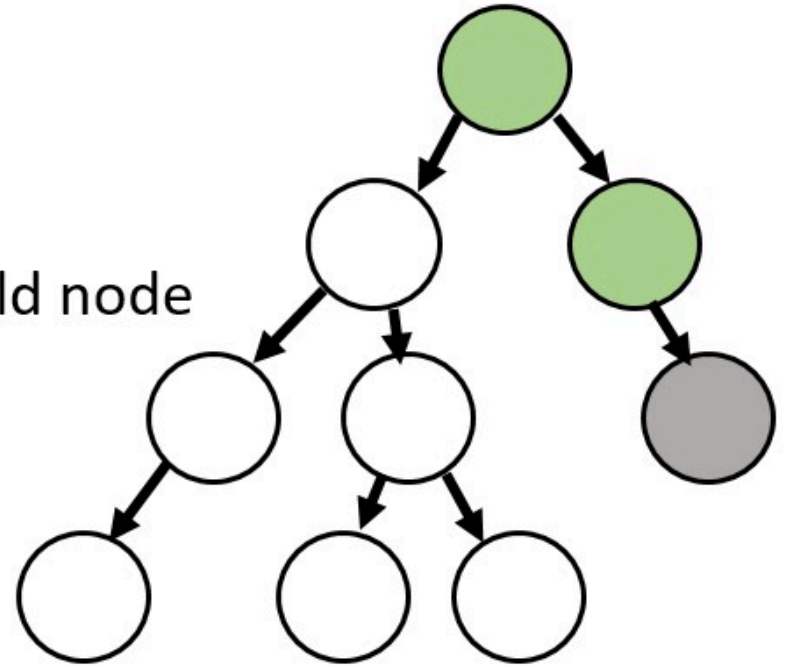


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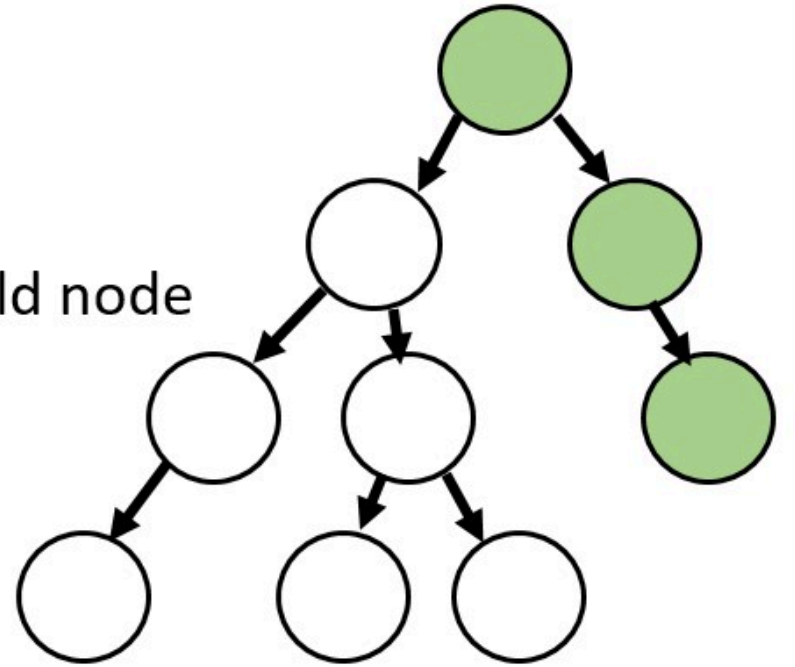


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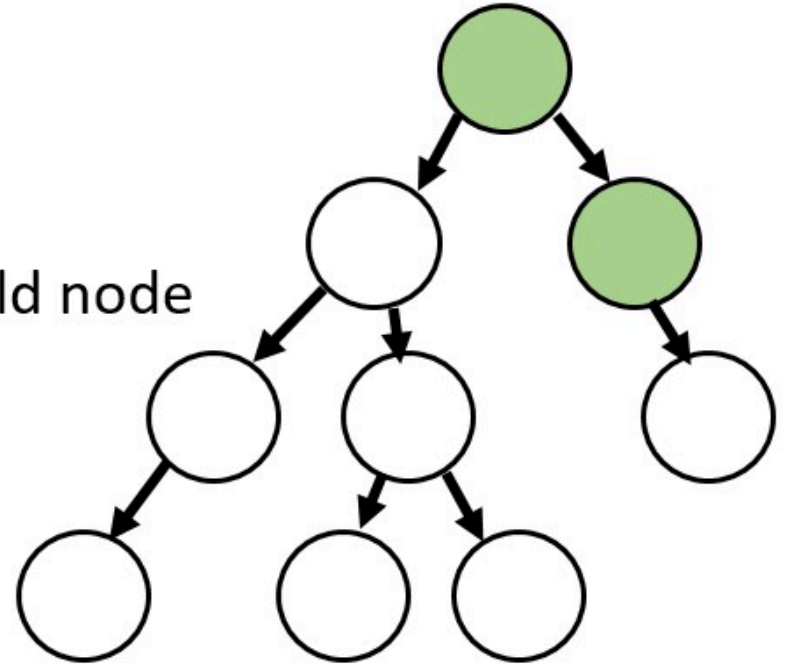


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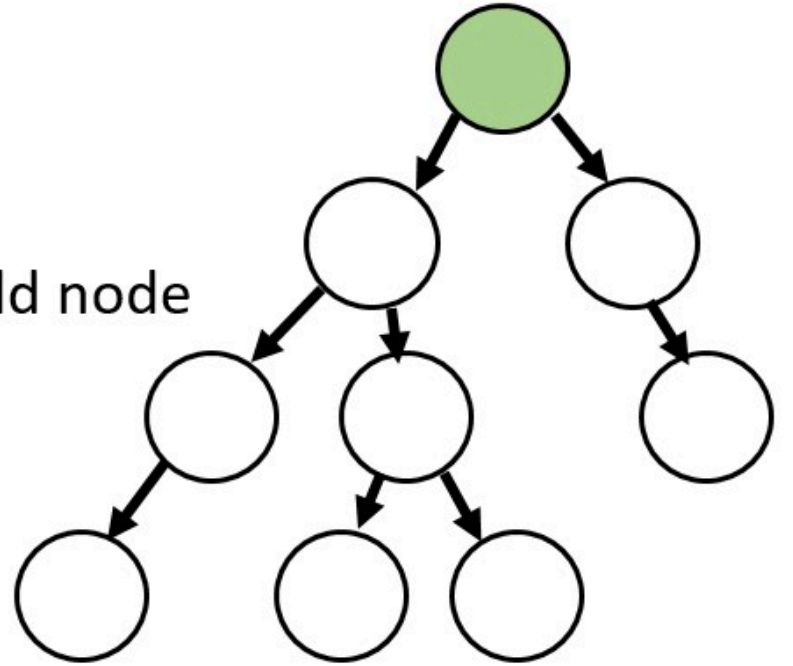


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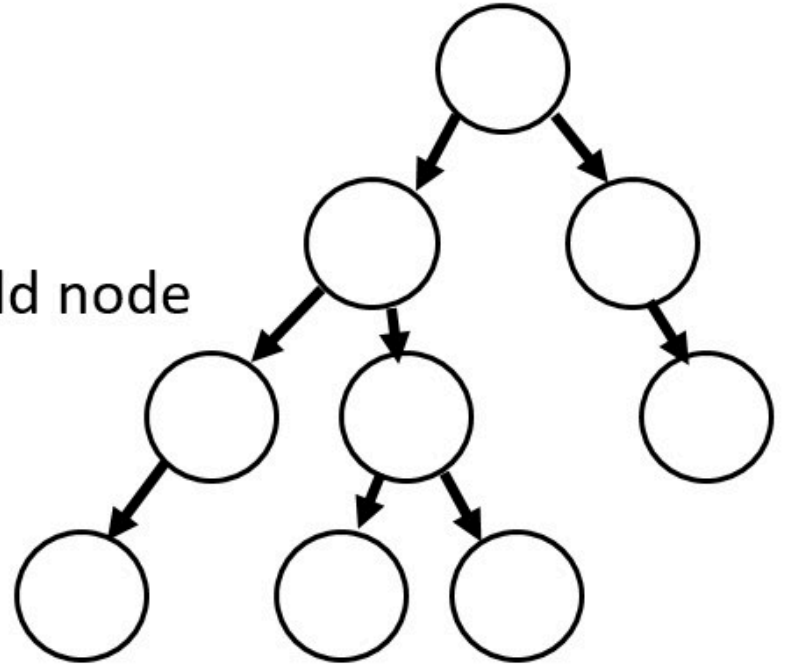


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That recursive backtracking algorithm is called **Depth-First Search**, since it explores "deep" in one area before moving on to other unexplored "shallow" places (closer to the root).

**Poll: This is pseudocode for a Depth-First Search on a graph that is not a tree (because it has cycles). What's a good base case?**

```
DFS(node):  
    Base case:  
        ???  
    Recursive case:  
        For each child:  
            Add child to explored nodes  
            DFS(child)
```

1. If the node is in the set of explored nodes, do nothing
2. If the node is a leaf, add it to the set of explored nodes
3. If the node is `None`, do nothing
4. If the node is `None`, add it to the set of explored nodes

## **Poll:**

- 1. What is your main takeaway from today?**
- 2. What would you like to revisit next time?**