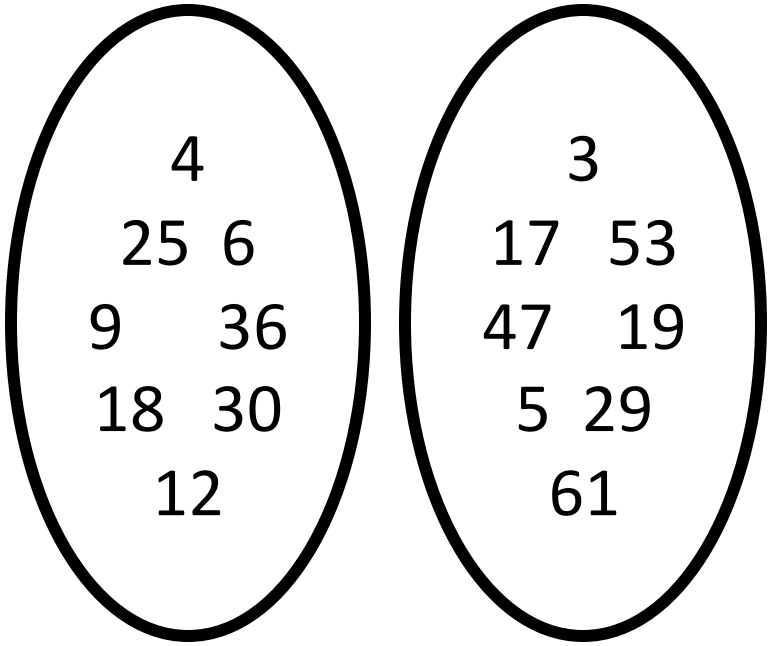
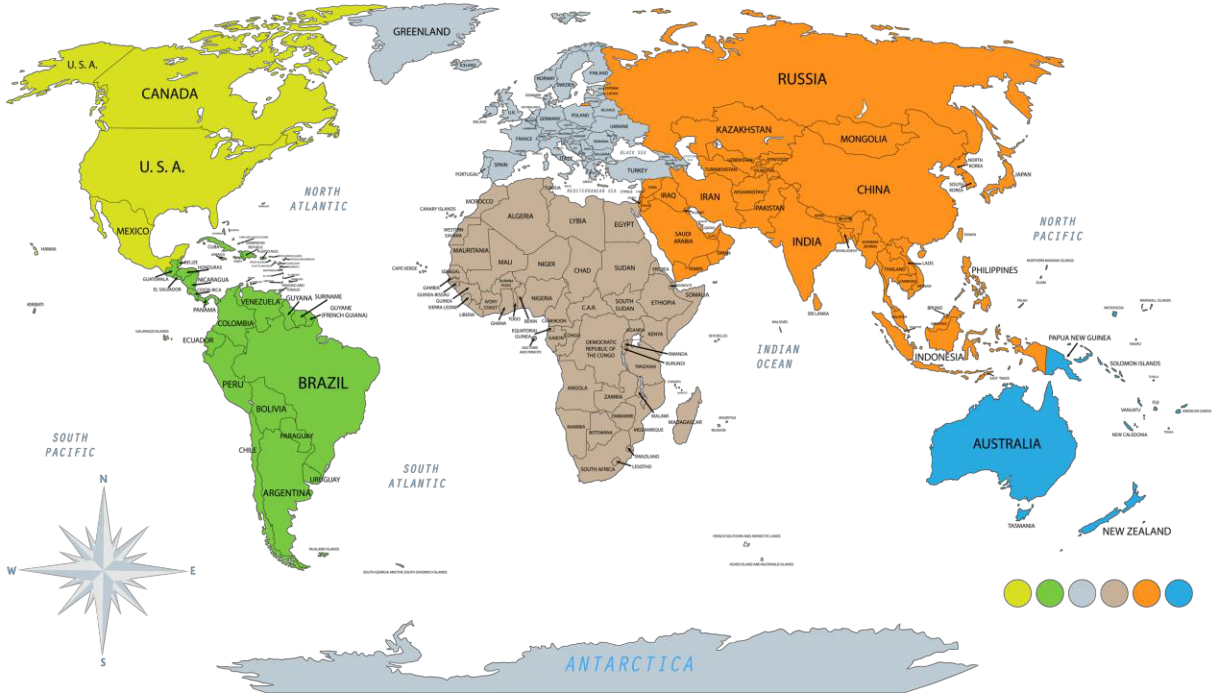
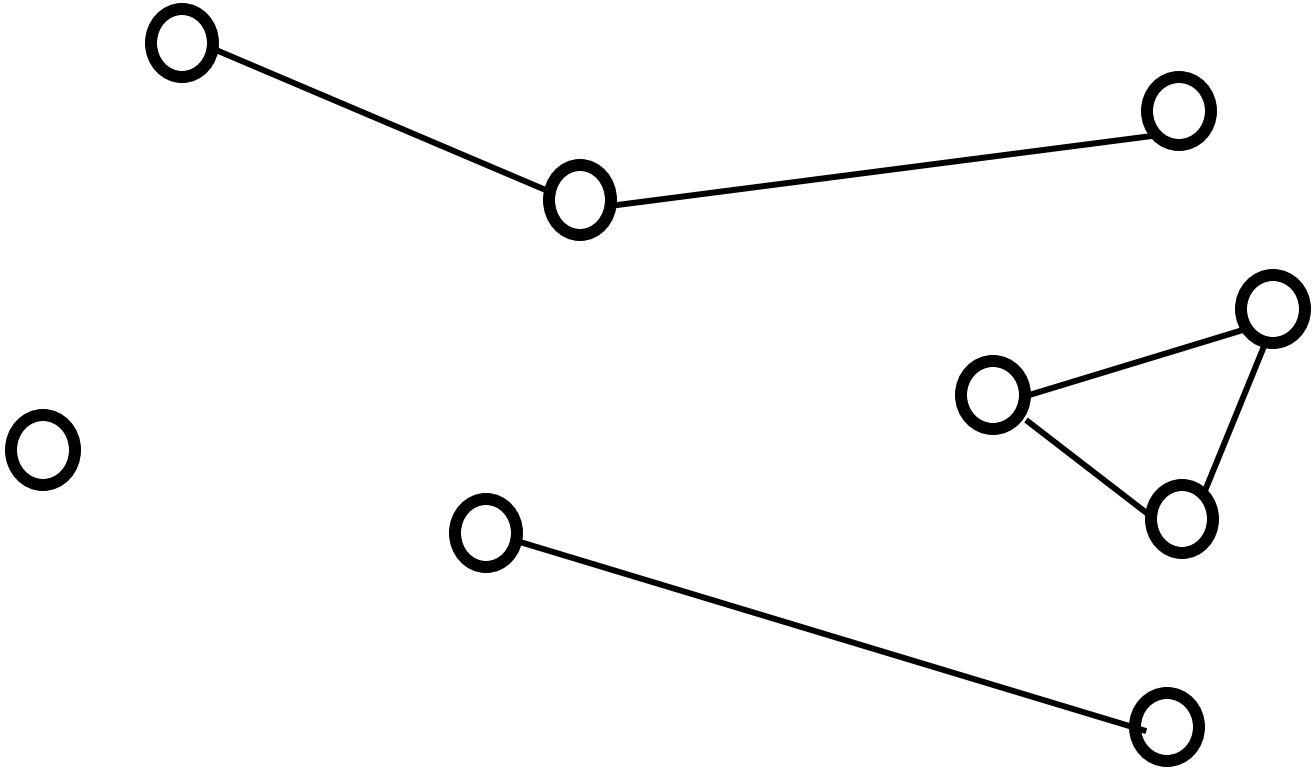
The Union-Find algorithm finds disjoint sets.

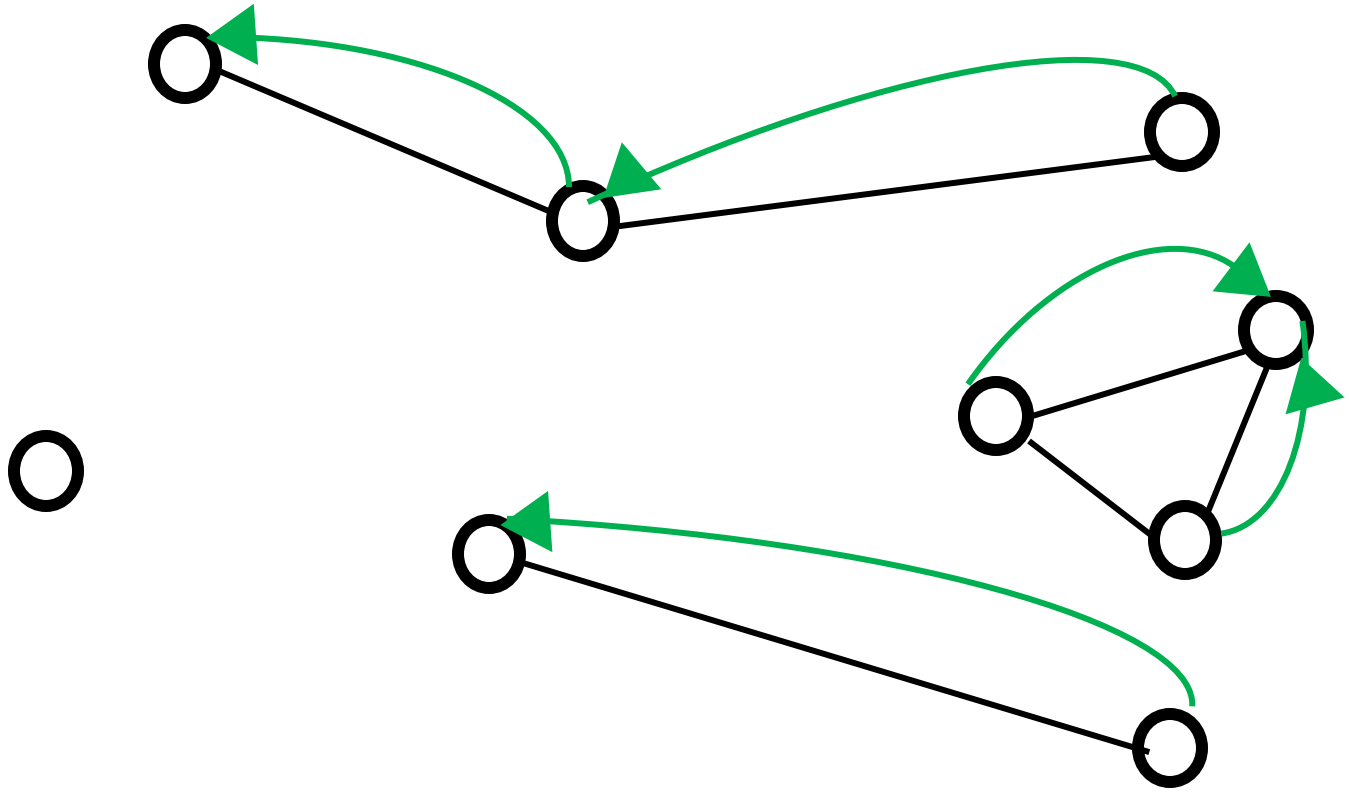
Disjoint sets are sets that have no elements in common.





https://www.worldatlas.com/geography/continents-by-number-of-countries.html

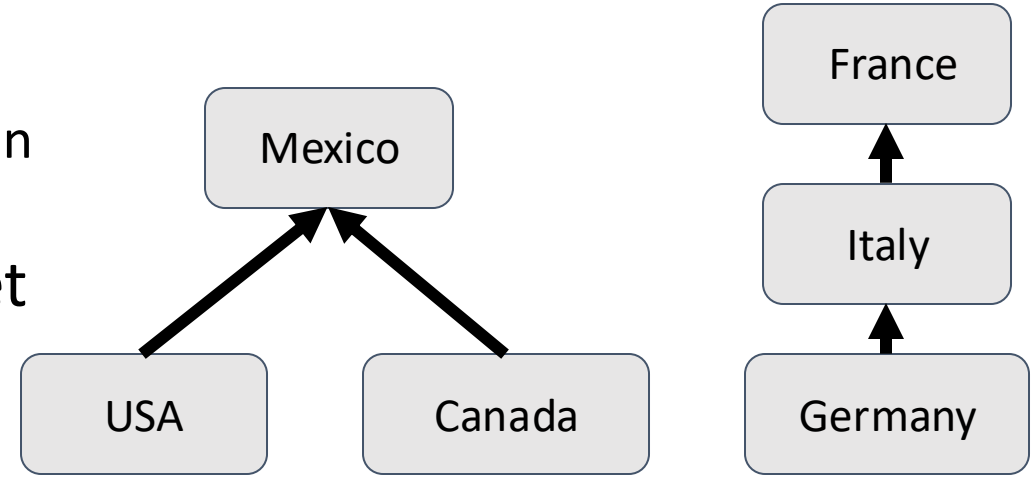




Representing disjoint sets

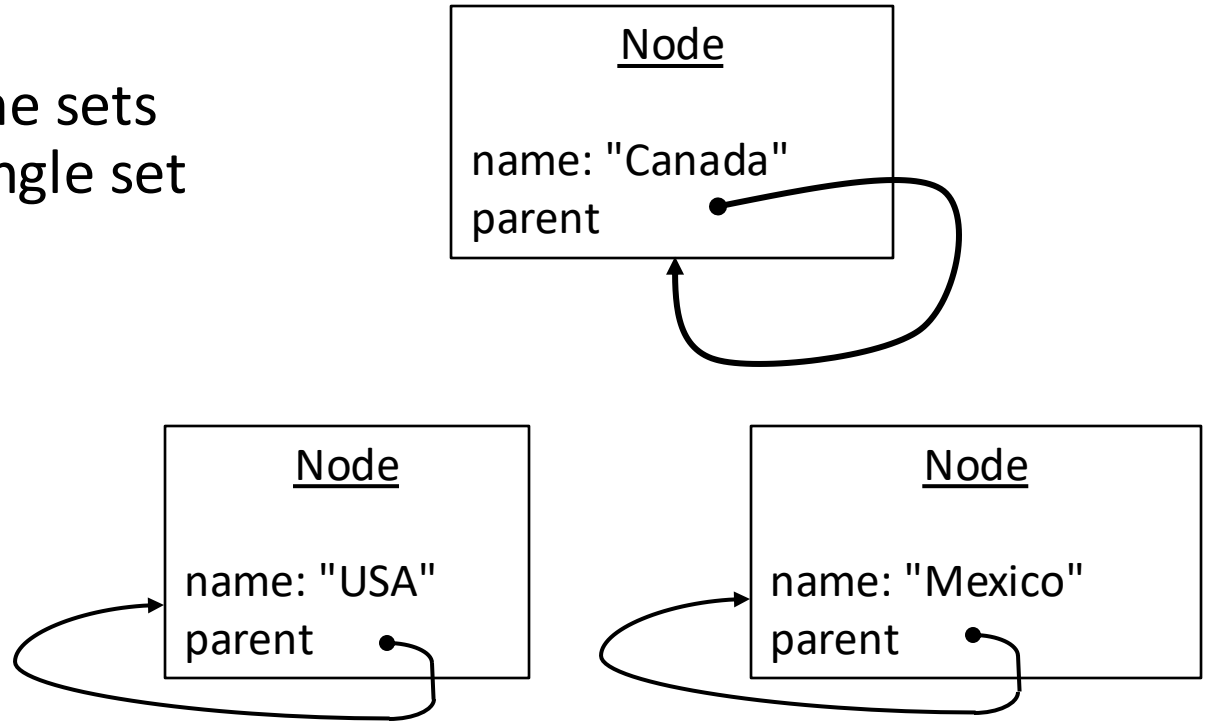
- The goal of Union-Find is to result in a representation of disjoint sets:
  - Each item is a node in a graph
  - Each disjoint set is a tree within a *forest*
  - Each disjoint set has one element which is the *representative element*
    - That item is the root of the tree.

- The algorithm for finding / managing disjoint sets is called Union-Find
  - union: combine two sets
    - Do this when we find that those two sets have an element in common
  - find: find the representative element of a set
    - Given an element, find the root of the tree that it belongs to



Union-Find overview

- Each node keeps track of:
  - Its data (the element)
  - Its parent (another node)
    - `node.parent == node` if node is a root
    - `node.parent == node2` if node2 is its parent
- Operations
  - `union(node1, node2)` combines the sets containing node1 and node2 into a single set
  - `find(node)` returns node's root



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    `union(USA, Canada)`

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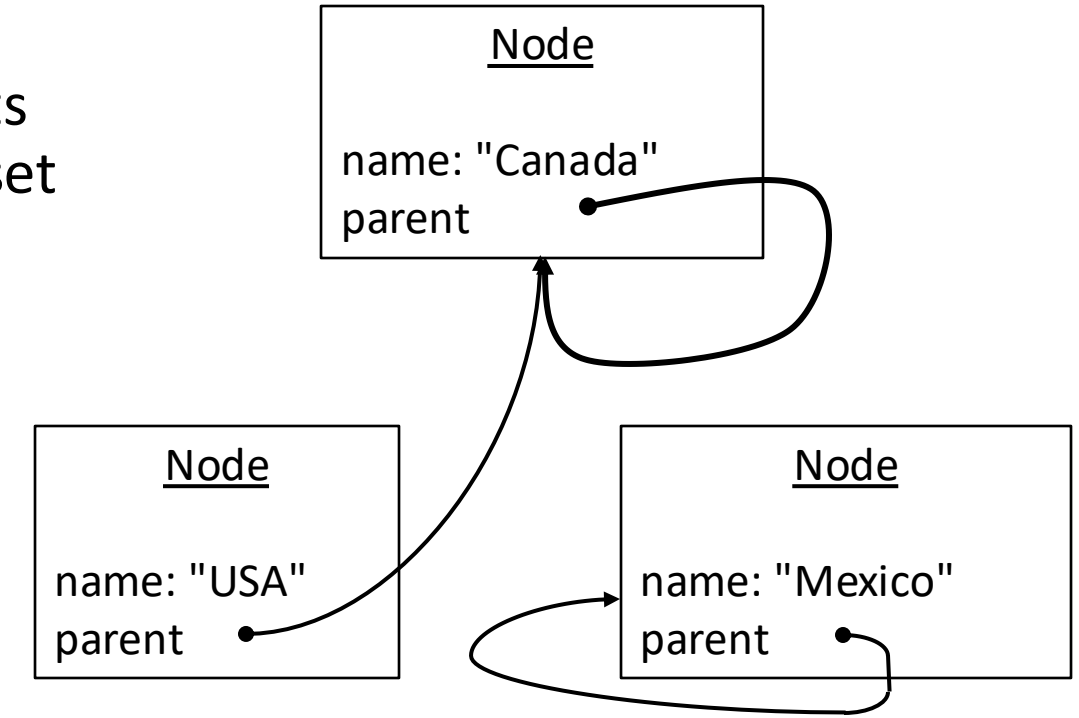

Poll: How might we implement `find(node)`?

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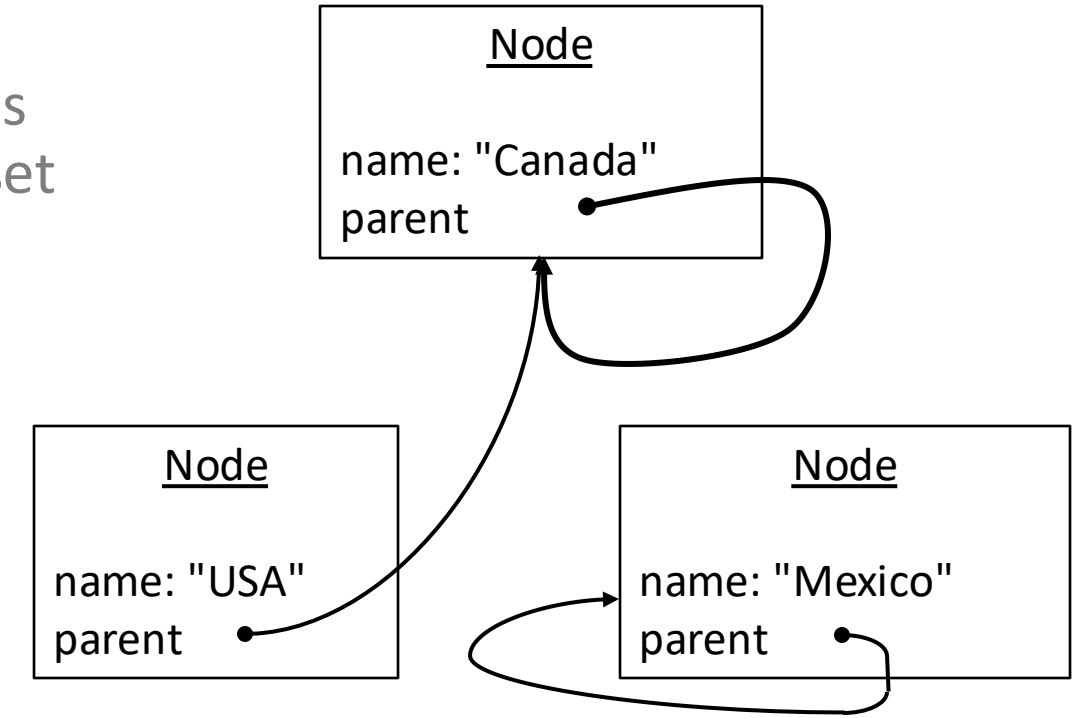
  - **`find(node)` returns node's root**

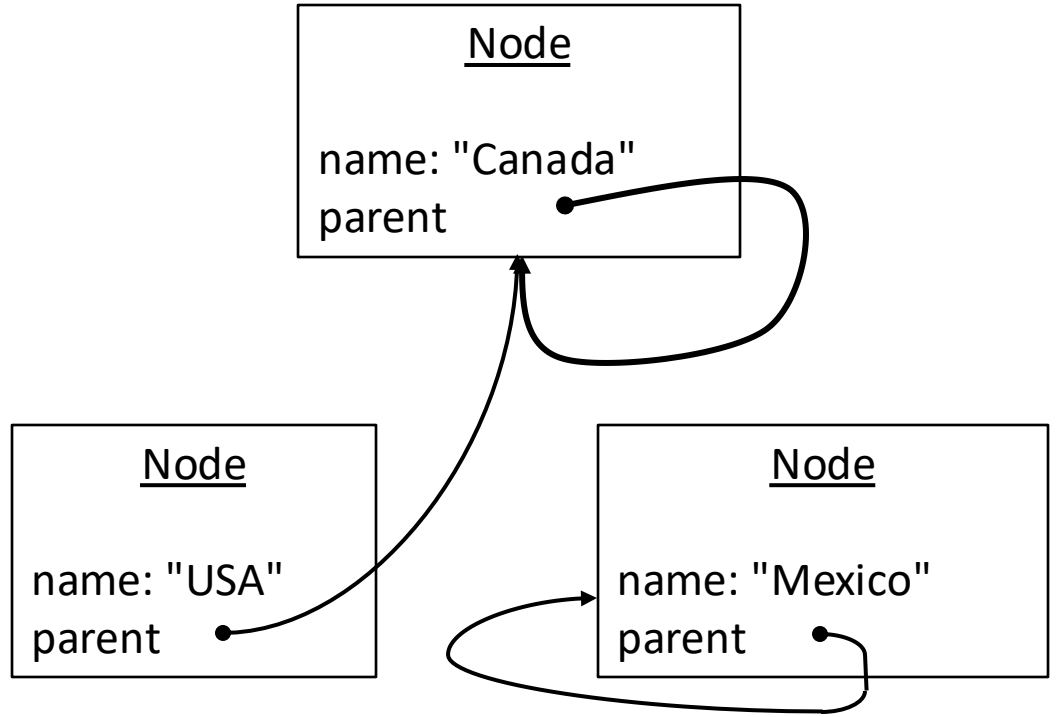


Implementing `find(node)`

- Each node keeps track of:
  - Its data (the element)
  - Its parent (another node)
    - `node.parent == node` if node is a root
    - `node.parent == node2` if node2 is its parent

Given a node, we want to find its root

```
find (node):
    while (node.parent != node):
        node = node.parent
    return node
```



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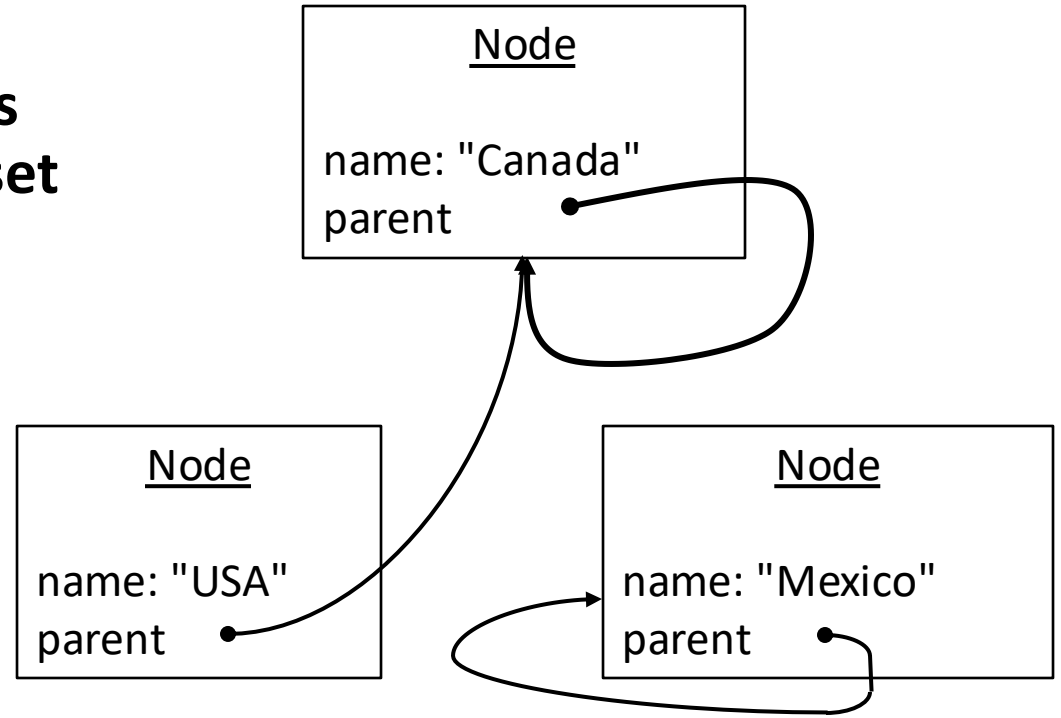

Implementing `union(node1, node2)`

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Given two nodes, we want to make them have the same root

```
union (node1, node2):
    root1 = find(node1)
    root2 = find(node2)
    if (root1 != root2):
        root1.parent = root2
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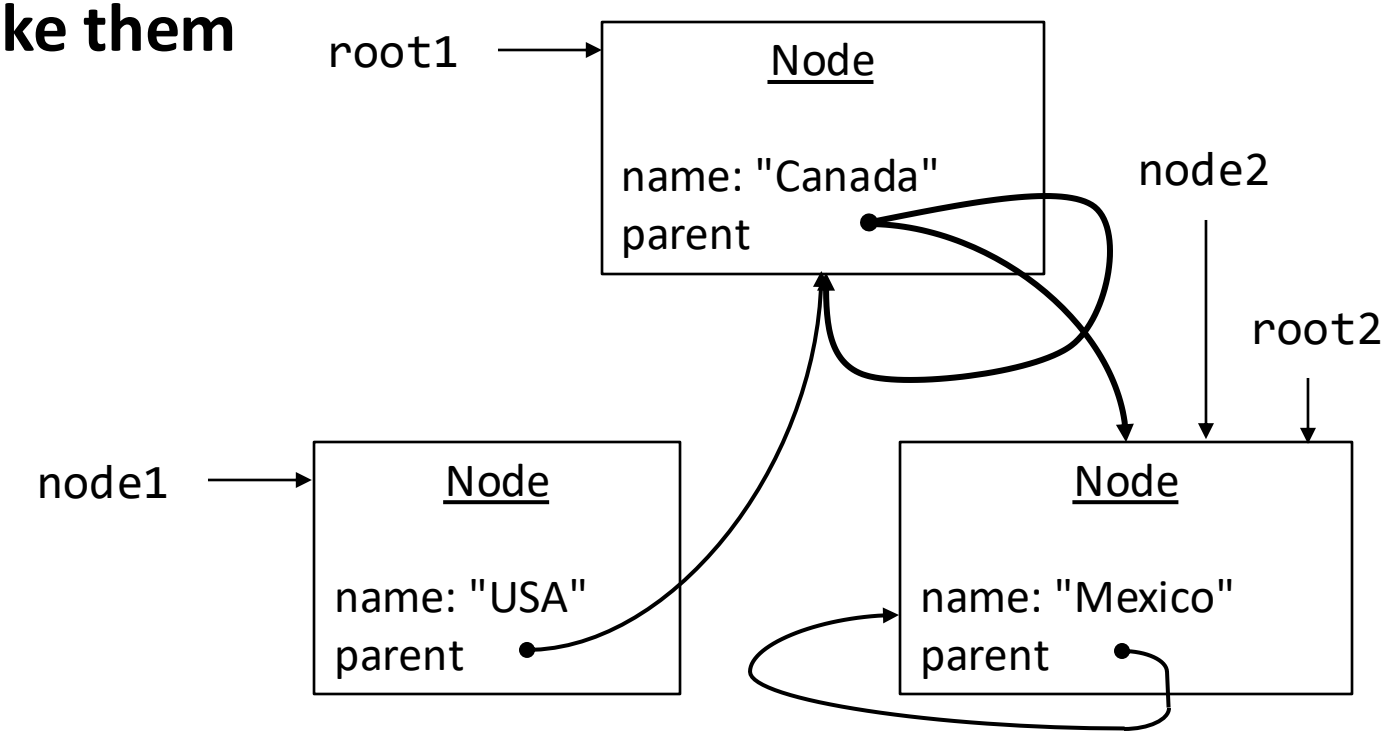
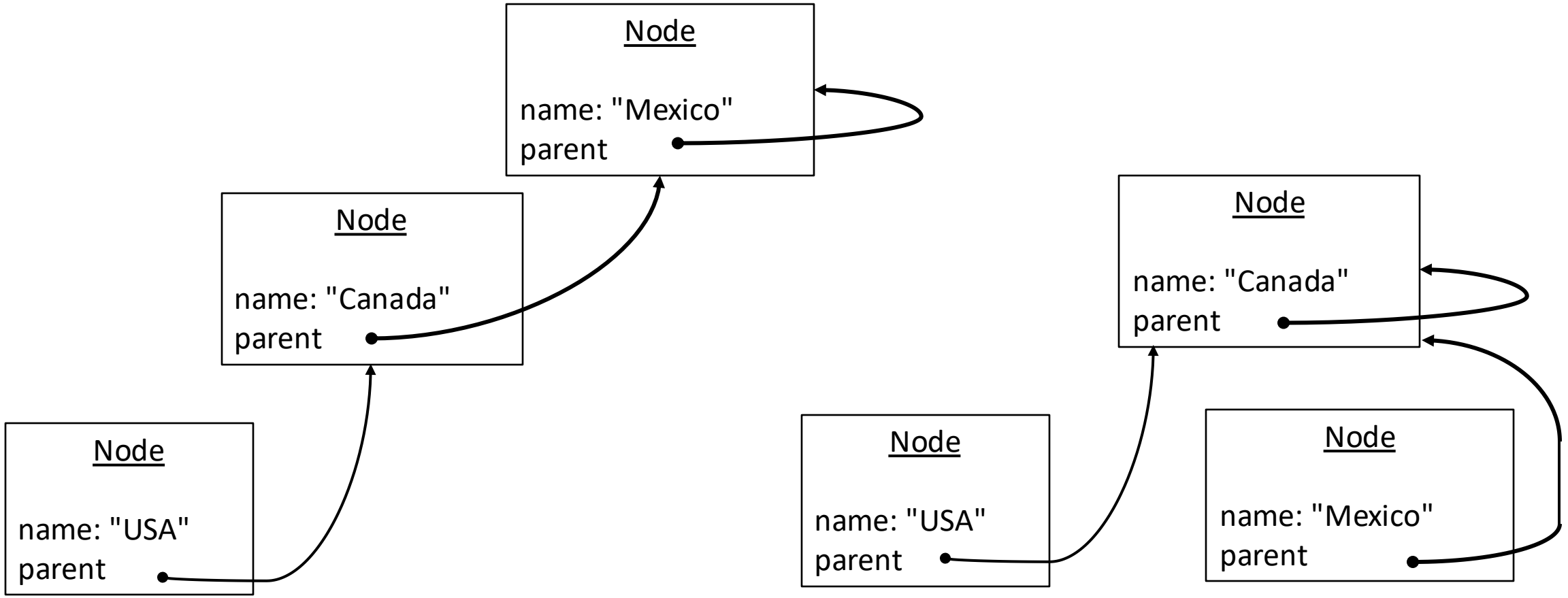


Problem with that union() implementation

Algorithm can lead to long paths (trees of great height)

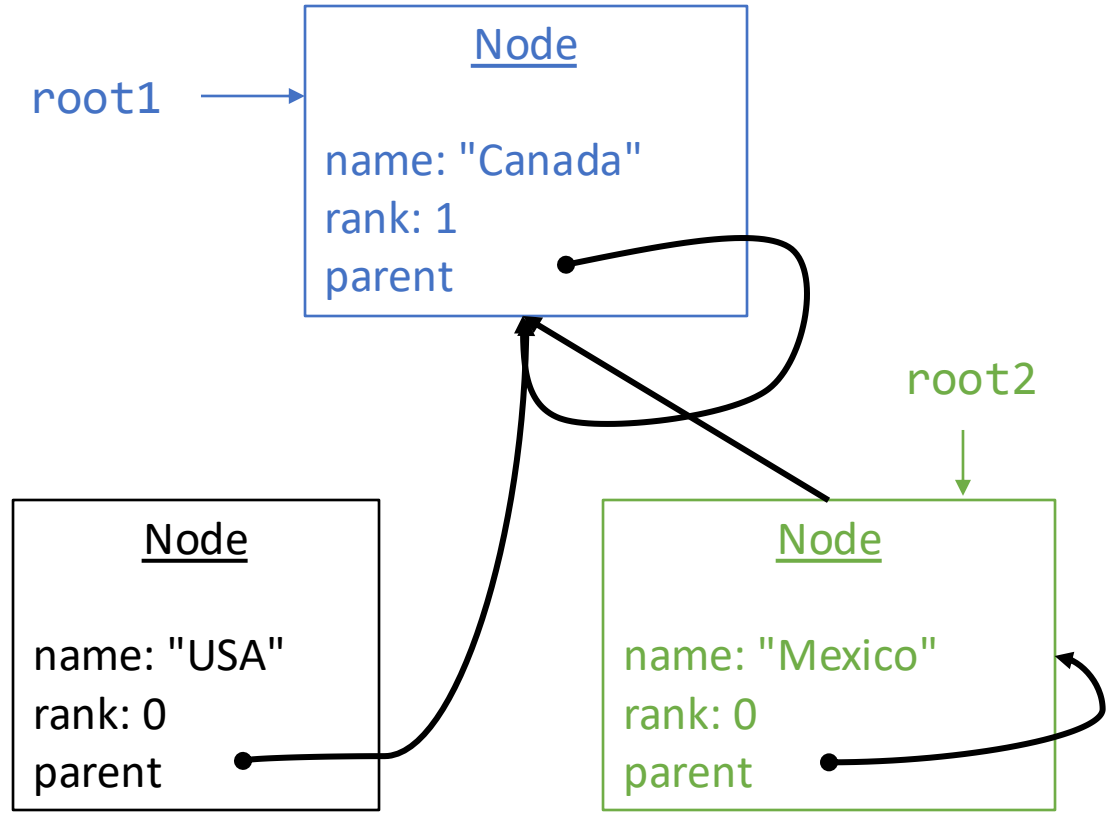


A better implementation of `union(node1, node2)`

Rank: how "high up" that node is in the tree
Aim is to keep ranks low

```
union(node1, node2):
  root1 = find(node1)
  root2 = find(node2)
  if root1 != root2:
    if root1.rank > root2.rank:
      root2.parent = root1
    else:
      root1.parent = root2
      if root1.rank == root2.rank:
        root2.rank++

union(USA, Mexico)
```



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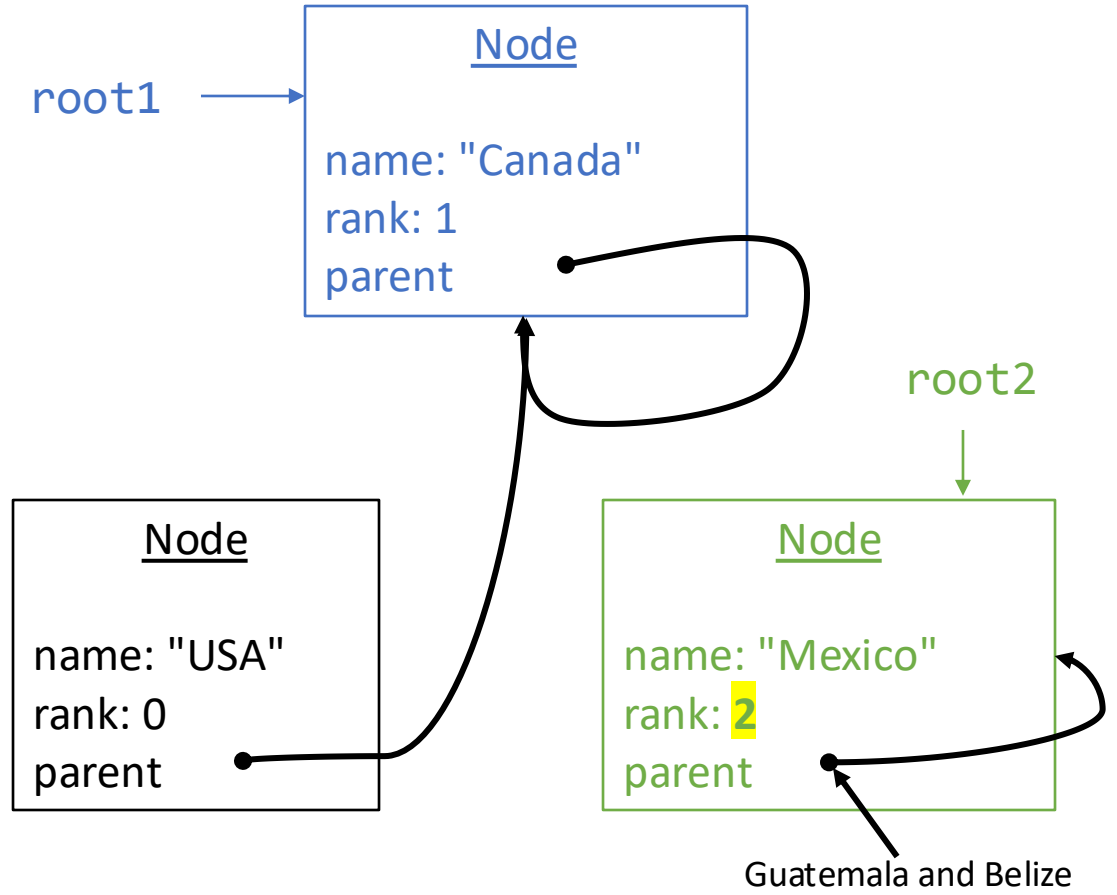

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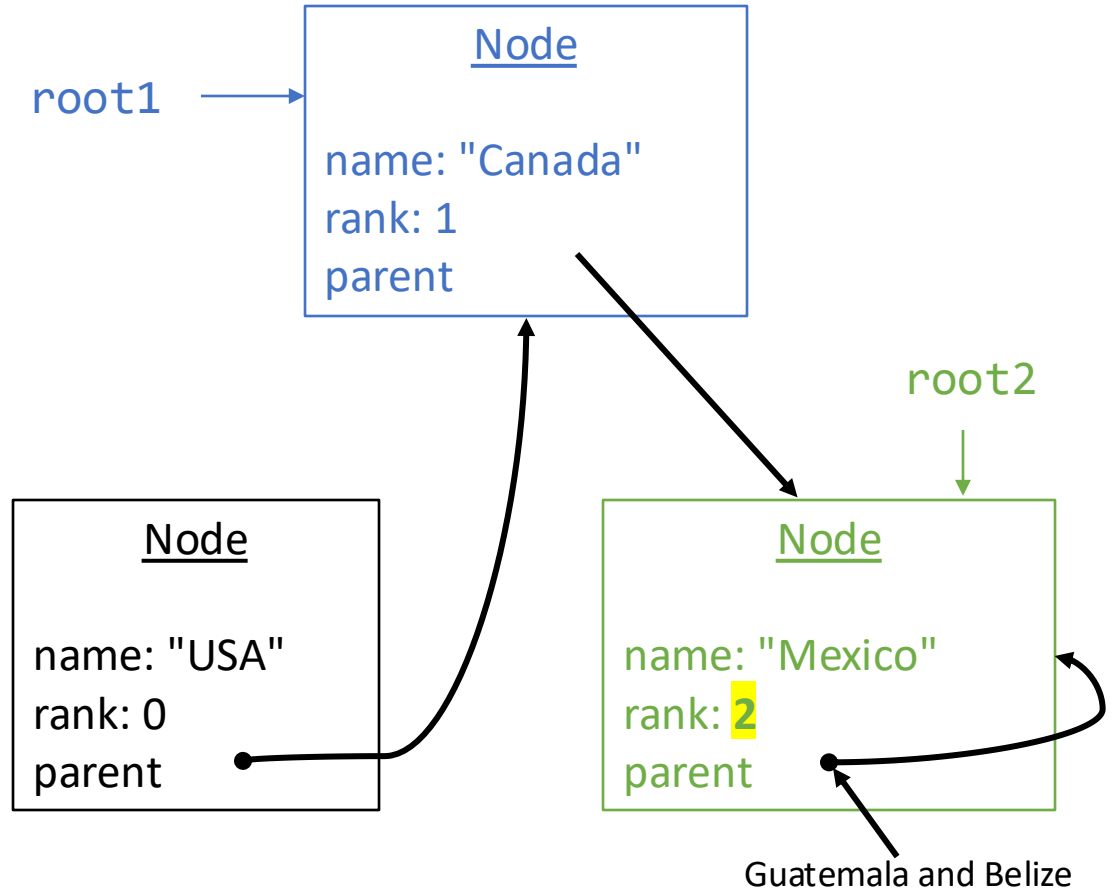

Poll: Why do we check if `root1.rank == root2.rank` before incrementing `root2.rank`?

```
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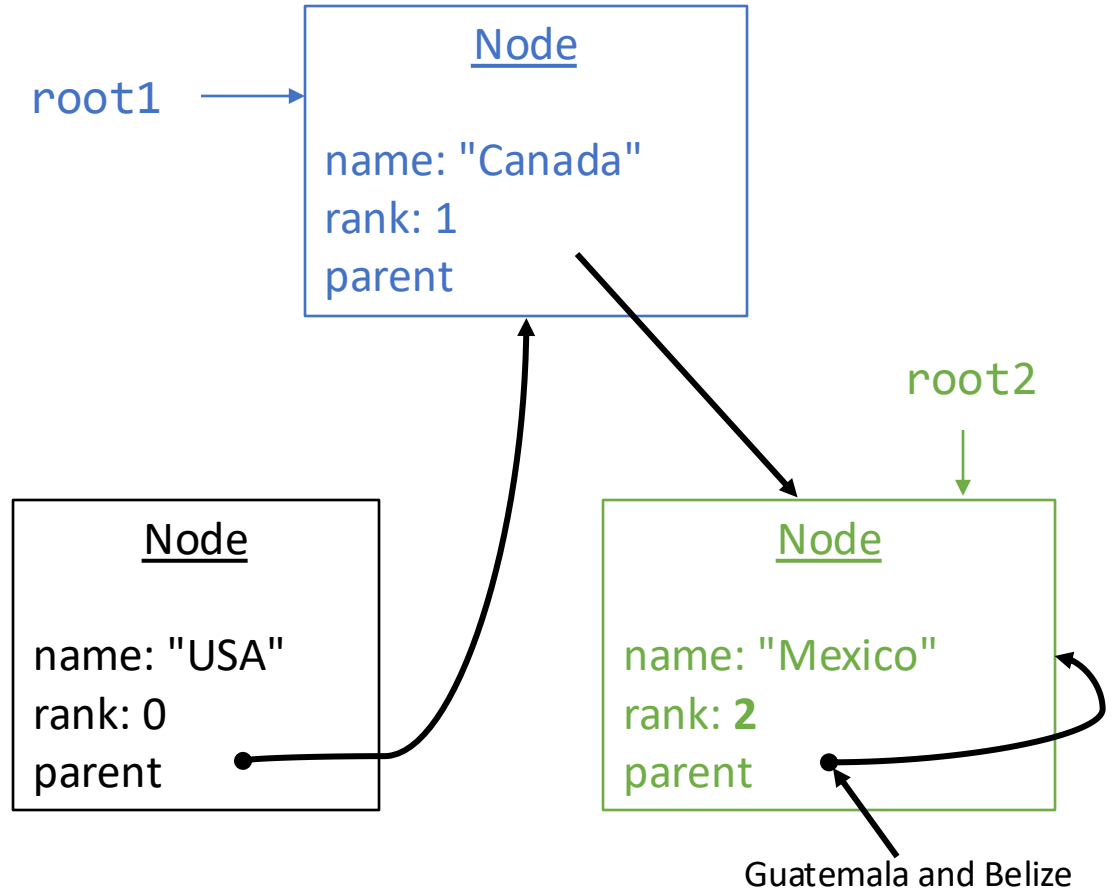

Poll: What does the Union-Find algorithm do?

A. It groups nodes into sets such that none of the nodes in each set are connected to each other, and all of the edges are between two sets
B. It groups nodes into sets such that the nodes in each set are connected to each other, and there are no edges between two sets
C. It groups edges into sets such that none of the sets of edges have any nodes in common
D. It works on undirected graphs (all edges go both ways)
E. It works on directed graphs (all edges are one-way)

Union-Find Summary

- The Union-Find algorithm finds disjoint sets
  - It groups nodes into sets such that, for each set:
    - The nodes in that set are connected to each other
    - None of the nodes in that set have an edge with a node outside that set

- Each node keeps track of:
  - Its data (the element)
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