Linear Algebra Self-Study

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Chapter 1

Vector and Matrices

1.1 My bad guys

So, the issue, right? I picked up LaTeX after I already finished chapter 1 , and nearly finished chapter 2. So to future me and any poor sods reading this, good luck lol.

Chapter 2

Solving Linear Equation $Ax = b$

2.1 Elimination and Back Substitution

I fucked up

2.2 Elimination Matrices and Inverse Matrices

This too.

2.3 Matrix Computation and $A = LU$

This one too.

2.4 Permutation and Tranposes

Also this shit.

2.5 Derivatives and Finite Difference Matrices

Second difference matrices includes K, T, B. They all have the $-1, 2, -1$ pattern. Now we can approximate $-\frac{d^2u}{dx^2} = f = (x)$

$$K_4 = \frac{1}{h^2}\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} f(h) \\ f(2h) \\ f(3h) \\ f(4h) \end{bmatrix}$$

So, we want to compute $-\frac{d^2u}{dx^2}$ with a computer, but the computer can't understand derivative. So what we do is we turn $\frac{d^2u}{dx^2}$ into the matrix $\frac{K^2}{h}$, function $u(x)$ into vector u, and function $f(x)$ into F. We also need the boundary conditions, which are given where $u(0) = 0$ and $u(1) = 0$. We can't pick out the infinite space between 0 and 1, so we pick N equally spaced points at a regular interval. The space between each points (and the first and the last point) becomes meshwidth (h). If we have N interal points $u_0, u_1, u_2, \ldots$ plus two boundary points u_0 and u_{N+1}, we divide the total length into N+1 segments. Therefore the spacing is $h = \frac{1}{N+1}$. If we have 4 N, then the spacing is h = $\frac{1}{5}$. So instead of finding the continuous function $u(x)$, we will find the value at each internal points, and they becomes the unknown vector $U = [u_1, u_2, u_3, u_4]^T$.

$-\frac{d^2u}{dx^2} \approx \frac{-u(x+h)+2u(x)-u(x-h)}{h^2}$ which means that it will do THAT. Test. Boo. Hoo. Too.