

Linear Algebra Self-Study

Kansapat Udomchokpaiboon (Hu Jinlin)

June 14, 2025

Contents

1	Vector and Matrices	3
1.1	My bad guys	3
2	Solving Linear Equation $Ax = b$	4
2.1	Elimination and Back Substitution	4
2.2	Elimination Matrices and Inverse Matrices	4
2.3	Matrix Computation and $A = LU$	4
2.4	Permutation and Tranposes	4
2.5	Derivatives and Finite Difference Matrices	4

Chapter 1

Vector and Matrices

1.1 My bad guys

So, the issue, right? I picked up LaTeX after I already finished chapter 1 , and nearly finished chapter 2. So to future me and any poor sods reading this, good luck lol.

Chapter 2

Solving Linear Equation $Ax = b$

2.1 Elimination and Back Substitution

I fucked up

2.2 Elimination Matrices and Inverse Matrices

This too.

2.3 Matrix Computation and $A = LU$

This one too.

2.4 Permutation and Transposes

Also this shit.

2.5 Derivatives and Finite Difference Matrices

Second difference matrices includes K, T, B . They all have the $-1, 2, -1$ pattern. Now we can approximate $-\frac{d^2u}{dx^2} = f(x)$

$$K_4 = \frac{1}{h^2} \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} f(h) \\ f(2h) \\ f(3h) \\ f(4h) \end{bmatrix}$$

So, we want to compute $-\frac{d^2u}{dx^2}$ with a computer, but the computer can't understand derivative. So what we do is we turn $\frac{d^2u}{dx^2}$ into the matrix $\frac{K^2}{h}$, function $u(x)$ into vector u , and function $f(x)$ into F . We also need the boundary conditions, which are given where $u(0) = 0$ and $u(1) = 0$. We can't pick out the infinite space between 0 and 1, so we pick N equally spaced points at a regular interval. The space between each points (and the first and the last point) becomes meshwidth (h). If we have N internal points u_0, u_1, u_2, \dots plus two boundary points u_0 and u_{N+1} , we divide the total length into $N+1$ segments. Therefore the spacing is $h = \frac{1}{N+1}$. If we have 4 N , then the spacing is $h = \frac{1}{5}$. So instead of finding the continuous function $u(x)$, we will find the value at each internal points, and they becomes the unknown vector $U = [u_1, u_2, u_3, u_4]^T$.

$$-\frac{d^2u}{dx^2} \approx \frac{-u(x+h) + 2u(x) - u(x-h)}{h^2}$$

$$A_u =: \begin{bmatrix} 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 \end{bmatrix}$$