Example:

Biggyan in this class knows oop. Everyone who knows oop can easily get hired. Hence, some are in this class can get hired easily.

Soln:

x represent member of class.

$oop(x)$: x knows oop

Then,

OOP (Biggyan) : Biggyan knows OOP.

Hired (x) : x get hired easily

Now, In Symbolic form:

i) $\forall x \; OOP(x) \rightarrow Hired(x)$

ii) Hired (Biggyan)

We have to prove:

Someone in this class get hired easily.

i.e. $\exists x \; Hired(x)$

Now,

1. $\forall x \; OOP(x) \longrightarrow H(x)$ — from argument
2. $OOP(Biggyan) \longrightarrow H(Biggyan)$ using universal instantiation in 1
3. $OOP(Biggyan)$ — from argument
4. $H(Biggyan)$ — using Modus ponens on 2, 3

Hence, we can use existential generalization

Hired (Bigyan),

for some c (constant) Hired (c).

i.e. $\exists x$ Hired (x)

Proof Methods

- often used to establish the truth of statement or proposition.

(i) Direct proof

(ii) Indirect proof

Direct proof $(p \to q)$

- If p is true then q must also be true.

- We assume a hypothesis 'p' is true

Example

if n is even integer then n^2 is also even.

Sol^n: let $p \to q$: if n is even, then n^2 is also even.

let, n be even integer,

we can write, $\boxed{n = 2K}$, K is integer

Then, $n^2 = (2K)^2 = 4K^2$

Since, $4K^2$ is multiple of 2

$\therefore$ if n is even integer then n^2 is even.

Example: if n is odd integer, then n^2 is also odd integer.

Indirect proof (proof by contradiction)

eg:- The square root of 2 is irrational.

Soln:-

Assume, square root of 2 is rational.

i.e. it can be expressed in fraction

a/b

- Here a & b are integers with no common factor other than 1.

So,

$$\sqrt{2} = a/b$$

Squaring both side

$$2 = a^2/b^2$$

or, $2b^2 = a^2$

i.e. a^2 is even [it is 2 times some integer)

Thus, a is even

Let, $a = 2k$, k is integer.

Substitute, $2b^2 = (2k)^2 = 4k^2$

Then, dividing by 2

$$\frac{2b^2}{2} = \frac{4k^2}{2} \Rightarrow b^2 = 2k^2$$

i.e. b^2 is also even.

Hence, both a & b are even, this contradicts our assumptions.

Thus, square root of 2 must be irrational.

1. $\forall x \; Plant(x) \vee Animal(x)$
2. $alive(dog) \wedge \neg plant(dog)$
3. $\forall x \; Animal(x) \rightarrow Heart(x)$
4. $\cancel{plant(dog)} \vee animal(dog)$ (UI : 1)
5. $\neg \cancel{plant(dog)}$ (Simplification 2)
6. $animal(dog)$ Resolution 4, 5)
7. $animal(dog) \rightarrow Heart(dog)$ (UI:3)
8. $Heart(dog)$ (Modus ponens 6, 7)

Indirect proof (proof by contradiction

If a^2 is even then prove a is even.

Assume:

If a^2 is even then a is odd.

$a = 2k+1$ where k is integer

$a^2 = 4k^2 + 4k + 1$

$= 4\underbrace{(k^2+k)}_{\text{is odd number}} + 1$ is $4 + 1$

$k^2 + k \simeq 1$