# INTRODUCTION

## A. SCRAMJETS:

# INLET

## B. CODE TO COMPUTE FLOW PROPERTIES AFTER EACH SHOCK OF A 3-SHOCK INLET

### I. DESCRIPTION:

The design of the 3-shock inlet used as reference is shown in figure 1. The user can input the range over which the ramp angles theta 1 and theta 2 vary, along with the other necessary values of inlet flow properties (M, P, T) and gas properties R and $\gamma$.

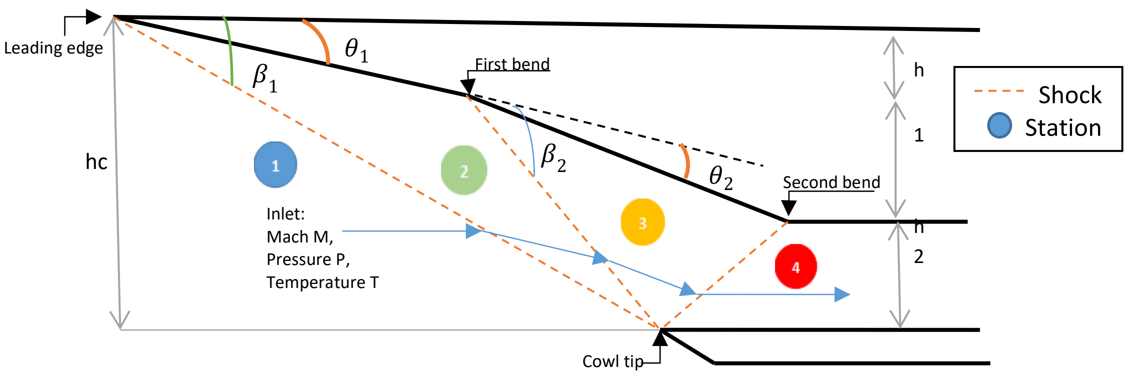



Figure 1- Scramjet Inlet

We assume that the geometry is such that all shocks meet at the cowl tip. The oblique shocks generated create a rise in temperature and pressure after each station (labelled 1 to 4). Consequently, the Mach number decreases.

### II. EQUATIONS USED:

We know the relation between Mach number before a shock M1, ramp angle ϴ and shock angle β is:

$$\frac{\tan(\beta)}{\tan(\beta-\theta)} = \frac{(\gamma+1)M_1^2\sin^2(\beta)}{2+(\gamma-1)M_1^2\sin^2(\beta)}$$

This equation was solved using the fzero function is Matlab to evaluate the value of beta. Once this was obtained, the downstream conditions (subscript 2) were simply calculated using the following shock relations with upstream values (subscript 1):

$$M_2\sin(\beta-\theta) = \left\{\frac{1+\frac{\gamma-1}{2}(M_1\sin\beta)^2}{\gamma(M_1\sin\beta)^2-\frac{\gamma-1}{2}}\right\}^{1/2}$$

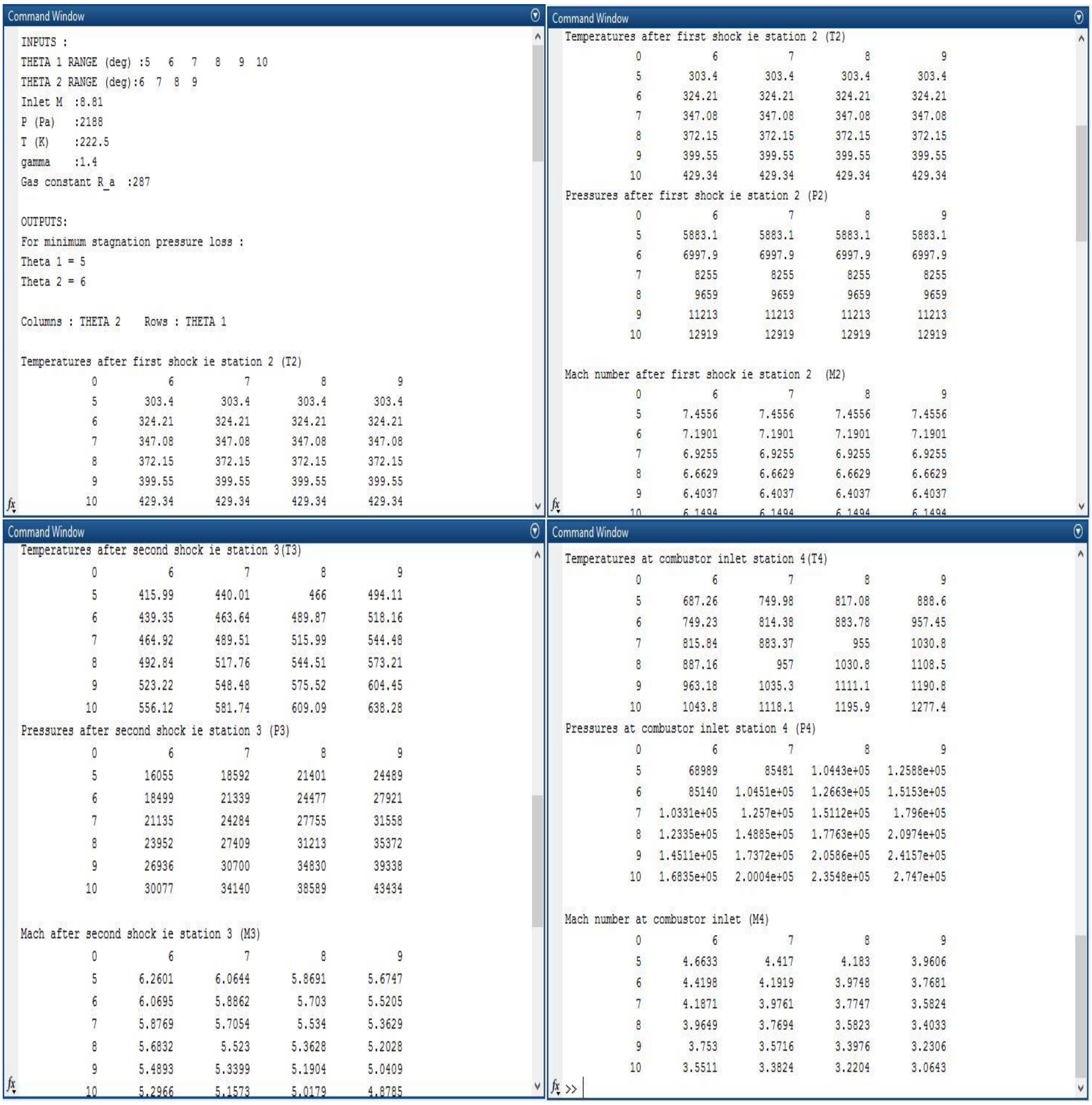
$$\frac{P_2}{P_1} = \frac{2\gamma\,(M_1\sin\beta)^2-\gamma+1}{\gamma+1}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)(M_1 \sin\beta)^2}{2+(\gamma-1)(M_1 \sin\beta)^2}$$

$$\frac{T_2}{T_1} = \frac{\rho_1}{\rho_2} * \frac{P_2}{P_1}$$

The code stores the value of flow properties (M, P, T, ρ, v, To, Po) and shock angles (β1, β2, β3) at each station, for all combinations of $\Theta_1$ and $\Theta_2$ in the given range. Mach number, Pressure and Temperature values at each station are displayed.

## III. OUTPUT:



```
Command Window
INPUTS :
THETA 1 RANGE (deg) :5   6   7   8   9  10
THETA 2 RANGE (deg):6  7  8  9
Inlet M  :8.81
P (Pa)   :2188
T (K)    :222.5
gamma    :1.4
Gas constant R_a  :287

OUTPUTS:
For minimum stagnation pressure loss :
Theta 1 = 5
Theta 2 = 6

Columns : THETA 2    Rows : THETA 1

Temperatures after first shock ie station 2 (T2)
         0         6         7         8         9
         5     303.4     303.4     303.4     303.4
         6    324.21    324.21    324.21    324.21
         7    347.08    347.08    347.08    347.08
         8    372.15    372.15    372.15    372.15
         9    399.55    399.55    399.55    399.55
        10    429.34    429.34    429.34    429.34
```

```
Command Window
Temperatures after first shock ie station 2 (T2)
         0         6         7         8         9
         5     303.4     303.4     303.4     303.4
         6    324.21    324.21    324.21    324.21
         7    347.08    347.08    347.08    347.08
         8    372.15    372.15    372.15    372.15
         9    399.55    399.55    399.55    399.55
        10    429.34    429.34    429.34    429.34
Pressures after first shock ie station 2 (P2)
         0         6         7         8         9
         5    5883.1    5883.1    5883.1    5883.1
         6    6997.9    6997.9    6997.9    6997.9
         7      8255      8255      8255      8255
         8      9659      9659      9659      9659
         9     11213     11213     11213     11213
        10     12919     12919     12919     12919

Mach number after first shock ie station 2  (M2)
         0         6         7         8         9
         5    7.4556    7.4556    7.4556    7.4556
         6    7.1901    7.1901    7.1901    7.1901
         7    6.9255    6.9255    6.9255    6.9255
         8    6.6629    6.6629    6.6629    6.6629
         9    6.4037    6.4037    6.4037    6.4037
        10    6.1494    6.1494    6.1494    6.1494
```

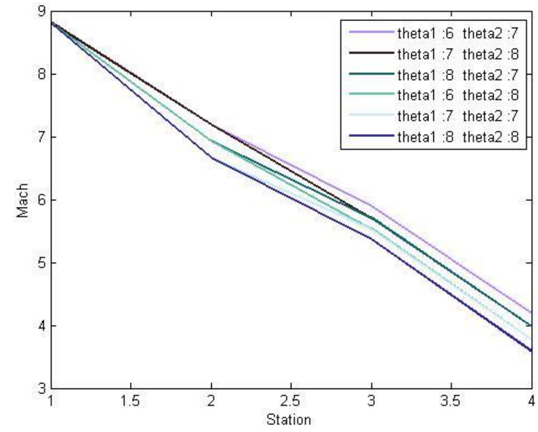
```
Command Window
Temperatures after second shock ie station 3(T3)
         0         6         7         8         9
         5    415.99    440.01       466    494.11
         6    439.35    463.64    489.87    518.16
         7    464.92    489.51    515.99    544.48
         8    492.84    517.76    544.51    573.21
         9    523.22    548.48    575.52    604.45
        10    556.12    581.74    609.09    638.28
Pressures after second shock ie station 3 (P3)
         0         6         7         8         9
         5     16055     18592     21401     24489
         6     18499     21339     24477     27921
         7     21135     24284     27755     31558
         8     23952     27409     31213     35372
         9     26936     30700     34830     39338
        10     30077     34140     38589     43434

Mach after second shock ie station 3 (M3)
         0         6         7         8         9
         5    6.2601    6.0644    5.8691    5.6747
         6    6.0695    5.8862     5.703    5.5205
         7    5.8769    5.7054     5.534    5.3629
         8    5.6832     5.523    5.3628    5.2028
         9    5.4893    5.3399    5.1904    5.0409
        10    5.2966    5.1573    5.0179    4.8785
```

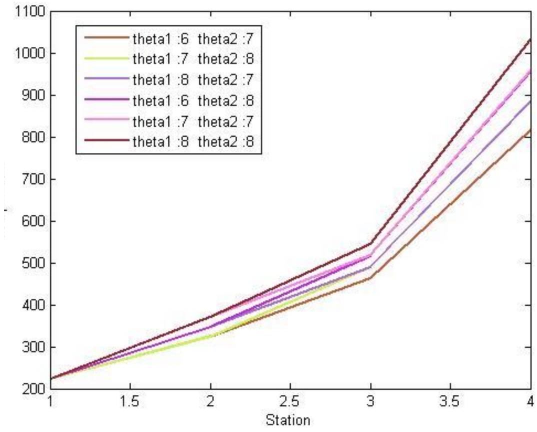
```
Command Window
Temperatures at combustor inlet station 4(T4)
         0         6         7         8         9
         5    687.26    749.98    817.08     888.6
         6    749.23    814.38    883.78    957.45
         7    815.84    883.37       955    1030.8
         8    887.16       957    1030.8    1108.5
         9    963.18    1035.3    1111.1    1190.8
        10    1043.8    1118.1    1195.9    1277.4
Pressures at combustor inlet station 4 (P4)
         0         6         7         8         9
         5     68989     85481 1.0443e+05 1.2588e+05
         6     85140 1.0451e+05 1.2663e+05 1.5153e+05
         7 1.0331e+05  1.257e+05 1.5112e+05  1.796e+05
         8 1.2335e+05 1.4885e+05 1.7763e+05 2.0974e+05
         9 1.4511e+05 1.7372e+05 2.0586e+05 2.4157e+05
        10 1.6835e+05 2.0004e+05 2.3548e+05  2.747e+05

Mach number at combustor inlet (M4)
         0         6         7         8         9
         5    4.6633     4.417     4.183    3.9606
         6    4.4198    4.1919    3.9748    3.7681
         7    4.1871    3.9761    3.7747    3.5824
         8    3.9649    3.7694    3.5823    3.4033
         9     3.753    3.5716    3.3976    3.2306
        10    3.5511    3.3824    3.2204    3.0643
>>
```

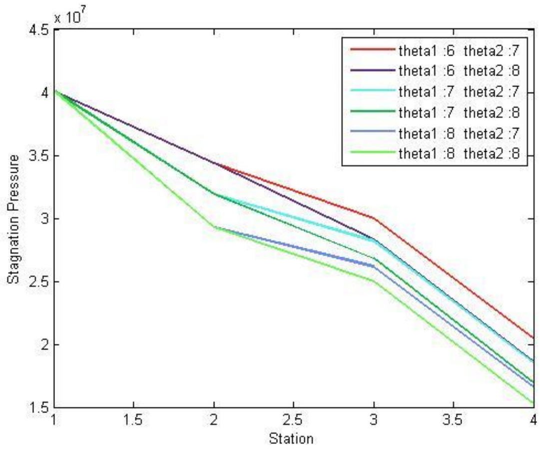
Output 1



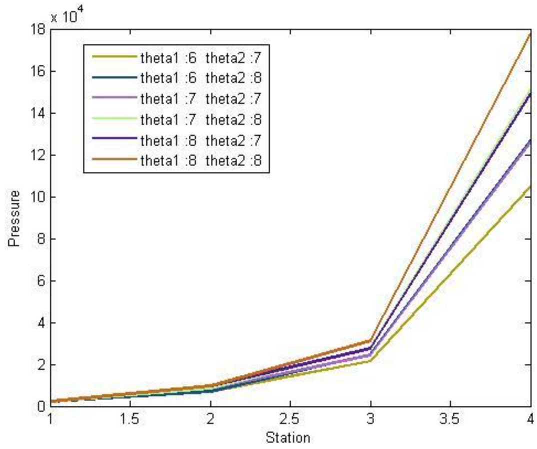

Plot 1




Plot 1




Plot 2




Plot 4

*Note: The variation is not linear in plots 1 to 4, only take the abscissa values 1,2,3,4.*

## IV. ANALYSIS AND CONCLUSION:

i. The inlet shocks lead to slowing down of the flow (plot 1) with consequent rise in temperature (plot 3) and pressure (plot 4).

ii. Higher the angle of deflection, stronger is the shock; leading to lower Mach numbers and greater P and T rise.

iii. The effect of variation of the first angle of deflection is more than the second one.

iv. The stagnation pressure loss increases with deflection angles theta.(plot 2)

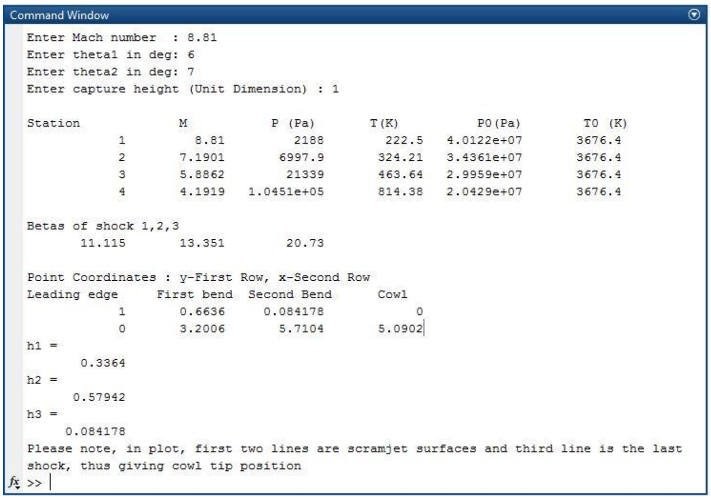
## C. CODE TO EVALUATE INLET GEOMETRY FOR CERTAIN $\Theta_1$, $\Theta_2$

### I. DESCRIPTION:

This code gives you the geometry of a scramjet inlet for a particular design point (Mach, $\Theta_1$, $\Theta_2$) so that all shocks will meet at the cowl tip.

The capture height at inlet is denoted by hc. See figure 1 for reference.
The output of this code is a plot whose first two lines define the inlet boundary and the third line denotes the third shock. The flow properties at each station are also displayed.

### II. OUTPUT:



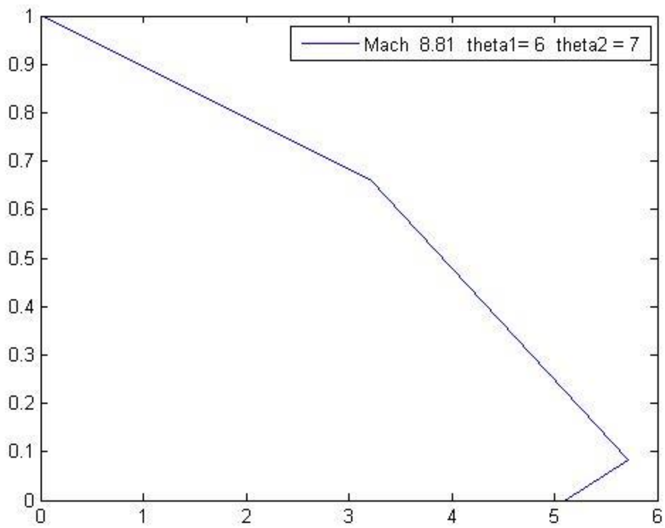
```
Command Window
Enter Mach number  : 8.81
Enter theta1 in deg: 6
Enter theta2 in deg: 7
Enter capture height (Unit Dimension) : 1

Station           M          P (Pa)       T(K)        P0(Pa)        T0 (K)
           1        8.81        2188        222.5   4.0122e+07      3676.4
           2      7.1901      6997.9       324.21   3.4361e+07      3676.4
           3      5.8862       21339       463.64   2.9959e+07      3676.4
           4      4.1919  1.0451e+05       814.38   2.0429e+07      3676.4

Betas of shock 1,2,3
       11.115       13.351        20.73

Point Coordinates : y-First Row, x-Second Row
Leading edge     First bend  Second Bend      Cowl
           1       0.6636     0.084178           0
           0       3.2006       5.7104      5.0902
h1 =
       0.3364
h2 =
      0.57942
h3 =
     0.084178
Please note, in plot, first two lines are scramjet surfaces and third line is the last
shock, thus giving cowl tip position
>>
```

**Output 2**




**Plot 5- Inlet geometry for given M, $\Theta_1$, $\Theta_2$**

# COMBUSTOR

## D. DERIVATION OF COMBINED RAYLEIGH-FANNO (HEAT INPUT+FRICTION) FLOW

### I. ASSUMPTIONS:

i. The flow is one dimensional, compressible

ii. Steady state has been reached

iii. Mass of fuel injected is very small compared to mass of flowing air and is hence neglected.

iv. Air and its products of combustion remain an ideal gas throughout.

v. Specific heats are constant.

vi. Friction coefficient does not vary.

vii. Angle of divergence/convergence is small; friction force is entirely along flow direction.

viii. Heat input and cross-section area vary as some function of distance x from combustor start.

### II. EQUATIONS:

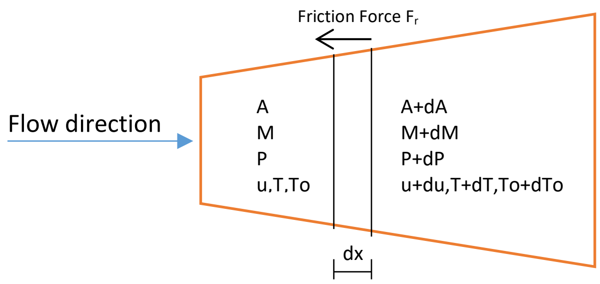



**Figure 2- Combustor**

#### 1. MOMENTUM EQUATION:

$$(\rho uA)du = PA - (P + dP)(A + dA) + \left(P + \frac{dP}{2}\right)dA - F_r$$

$$F_r = \frac{\rho u^2}{2} * f * P_{ermtr} * dx$$

*On manipulating the momentum equation and substituting Fr we get -*

$$\gamma M^2 \frac{du}{u} = -\frac{dP}{P} - \frac{\gamma M^2 f}{2} * \frac{P_{ermtr}}{A} * dx$$

### 2. DEFINITION OF MACH NUMBER:

$$\frac{dM^2}{M^2} = \frac{2du}{u} - \frac{dT}{T}$$

### 3. IDEAL GAS EQUATION:

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

### 4. CONTINUITY EQUATION:

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

### 5. USING EQUATIONS 2,3 AND 4 AND SUBSTITUTING IN 1

$$\frac{\gamma M^2 - 1}{2} \cdot \frac{dM^2}{M^2} + \frac{\gamma M^2 + 1}{2} \cdot \frac{dT}{T} = \frac{dA}{A} - \frac{\gamma M^2 f}{2} * \frac{P_{ermtr}}{A} dx$$

### 6. T-To-M RELATION:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} * M^2 = X$$

$$\frac{dT}{T} = \frac{dT_0}{T_0} - \frac{dX}{X} = \frac{dT_0}{T_0} - \frac{\gamma - 1}{2} \cdot \frac{M^2}{X} \cdot \frac{dM^2}{M^2}$$

### 7. ESUBSTITUTE DT/T IN EQUATION 5:

$$\left(\frac{1 - M^2}{X}\right) * \frac{dM^2}{M^2} = (\gamma M^2 + 1) * \frac{dT_o}{T_o} + \gamma M^2 f * \frac{P_{ermtr}}{A} * dx - 2 * \frac{dA}{A}$$

*This is the final equation consisting of three terms on the R.H.S*

## III. ANALYSING THE FINAL EQUATION:

i. The 3 terms on the R.H.S denote the effect of heat addition, friction and area variation on Mach number respectively.

ii. Consider a diverging duct (dA>0) with a supersonic flow(M>1).

- The first and second term, being positive, tend to reduce the Mach number [$(1-M^2)dM^2$ tends to be positive].
- On the other hand, the effect of the divergence term (which has a negative sign)will be to accelerate the flow.

- This analysis is in agreement with our knowledge that friction and heat input decelerate a supersonic flow and a diverging section behaves as a nozzle and will thus accelerate the flow.

iii. Similar analysis can be made for a supersonic flow with a converging cross-section.

iv. The first term on R.H.S can be expressed as a function of distance x from the start of the combustor using the definition of total temperature:

If heat added-

$$\mathrm{Q = f(x)\ Joule/kgair}$$

Then-

$$C_P dT_O = dQ = f'(x).dx$$

$$T_{o,x} = T_{o,4} + \frac{Q(x)}{C_P}$$

Where $T_{o,4}$ is the stagnation temperature at the beginning of the combustor.

Thus, the first term can be expressed in terms of length x as –

$$(\gamma M^2 + 1) * \frac{dT_o}{T_o} = (\gamma M^2 + 1) * \frac{f'(x).dx}{C_P T_{o,4} + Q(x)}$$

v. The third term on R.H.S is converted in terms of x since area variation is known.

Let Area-

$$A = g(x)$$

Then-

$$dA = g'(x).dx$$

Thus third term-

$$-2 * \frac{dA}{A} = -2 * \frac{g'(x).dx}{g(x)}$$

vi. Final equation is now written as-

$$\left(\frac{1 - M^2}{X}\right) * \frac{dM^2}{M^2} = \left((\gamma M^2 + 1) * \frac{f'(x)}{C_P T_{o,4} + Q(x)} + \gamma M^2 f * \frac{P_{ermtr}}{A} - 2 * \frac{g'(x)}{g(x)}\right) dx$$

This differential equation was solved using ode45 on MATLAB and the results were obtained using the following code.

## E. CODE TO SOLVE COMBUSTOR FLOW DIFFERENTIAL EQUATION:

### I. DESCRIPTION:

The following equation derived earlier was solved using ode45 on MATLAB

$$(\gamma M^2+1)*\frac{f'(x)}{C_P T_{o,4}+Q(x)}+\gamma M^2 f*\frac{P_{ermtr}}{A}-2*\frac{g'(x)}{g(x)}$$

We took heat addition to be a linear function of length x so that

$$Q=f(x)=k.x$$

(Where k is a constant)

The cross-section area is assumed rectangular with initial dimensions $d1\ and\ d2$.

The divergence angle of the four combustor boundary walls are taken as $\theta_1$ ,$\theta_2$ ,$\theta_3$ ,$\theta_4$ such that they affect the dimensions at a distance x in the following manner –

$$d1,x=d1+x*tan(\theta 1)+x*tan(\theta 2)$$

$$d2,x=d2+x*tan(\theta 3)+x*tan(\theta 4)$$

Hence at 'x' length from start of combustor -

$$P_{ermtr}=2(d1,x+d2,x)$$

$$A=g(x)=d1,x*d2,x$$

### WE CALCULATED THE TOTAL HEAT ADDED AS FOLLOWS:

- Fuel used = $C_xH_y$
- Combustion equation

$$C_xH_y+\left(x+\frac{y}{4}\right)(O_2+3.76N_2)\longrightarrow xCO_2+yH_2O+3.76(x+y/4)N_2$$

- Stoichiometric Air/Fuel ratio by mass:

$$af_{stoich}=4.76*28.96*(x+y/4)/(12*x+2*y)$$

- Equivalence ratio phi (ϕ)

$$af_{actual}=\frac{af_{stoich}}{\varphi}$$

- Combustion Efficiency eff_comb (η)
- Calorific value of fuel cv_fuel
- Total heat released per unit mass flow rate of air is hence:

$$Q=\frac{cv\_fuel*\eta}{af_{actual}} \qquad \therefore k=\frac{Q}{combustor\ length\ l}$$

## II. OUTPUT:

Command Window

```
Flow properties :
g =
          1.4
R =
   287
friction =
        0.001

Combustor inlet conditions
M4 =
       4.1919
T04 =
       3676.4
P4 =
      104150

Initial geometry :
d1 =
          0.5
d2 =
     0.084178
A0 =
     0.042089
combustor_length =
          0.8
```

Command Window

```
combustor_length =
          0.8

Properties of fuel CxHy :
x =
     0
y =
     2
phi =
          0.4
cv_fuel =
   120000000
af_actual =
       43.078
eff_comb =
          0.7
T4 =
       814.38
rho4 =
      0.44561
m_dotair =
       44.973
m_dotfuel =
        1.044
k =
   2.4374e+06
```

Command Window

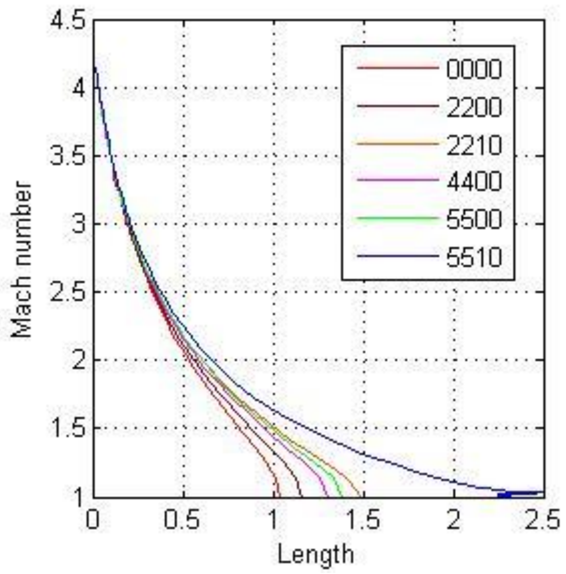
Conditions at combustor exit :

| Angle | Length | Mach | T05 | T5 | v5 | RHo5 | P5 | P05 | CS-Area |
|---|---|---|---|---|---|---|---|---|---|
| 0000 | 0.83430640328 | 1.4604726014 | 5700.9021877 | 3996.157295 | 1850.6302952 | 0.57737774554 | 662192.88718 | 2296349.9107 | 0.042089 |
| 2200 | 0.80556016622 | 1.5986279686 | 5631.1488765 | 3726.4680459 | 1956.1451348 | 0.49098654121 | 525108.3035 | 2227396.399 | 0.046824985745 |
| 2210 | 0.79997438626 | 1.7269275174 | 5617.5948712 | 3518.7915113 | 2053.4108089 | 0.40146247075 | 405434.20471 | 2084344.331 | 0.054554115438 |
| 4400 | 0.76477369592 | 1.7257644355 | 5532.1797018 | 3467.0327239 | 2036.8800354 | 0.43214230589 | 429998.18507 | 2206738.1629 | 0.051092373552 |
| 5500 | 0.78775227222 | 1.7269778092 | 5587.9376704 | 3500.138443 | 2048.0206659 | 0.40898224292 | 410838.91316 | 2112290.9038 | 0.05369199341 |
| 5510 | 0.77923332012 | 1.8463801401 | 5567.2662651 | 3310.255125 | 2129.3978915 | 0.33942871919 | 322471.95364 | 1989407.6801 | 0.06222184865 |

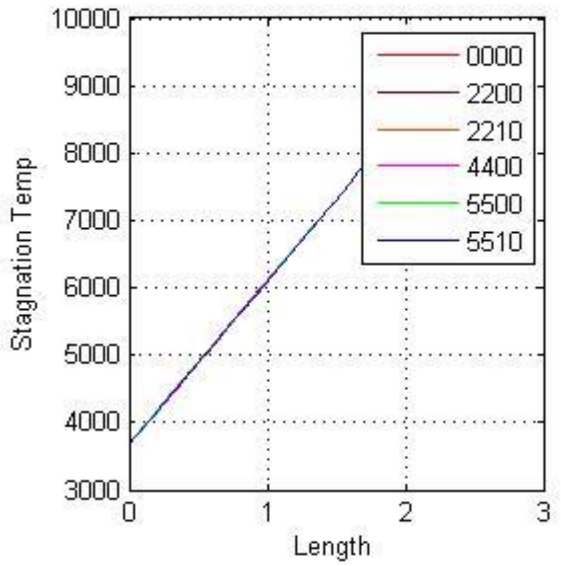
Note: Angles are displayed as theta1 theta2 theta3 theta4

```
>>
```

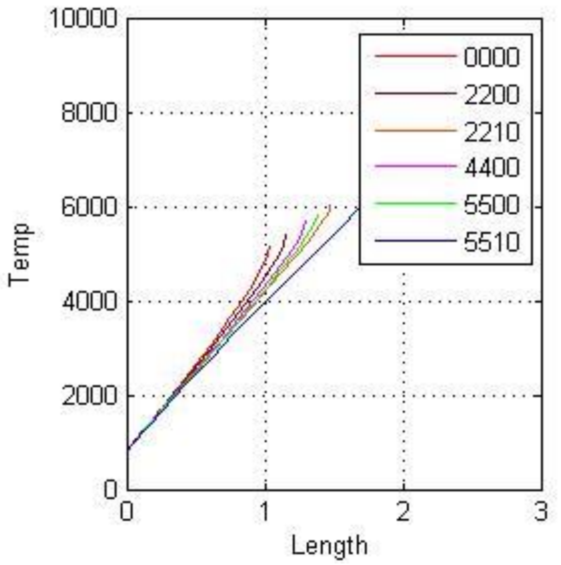
Output 3



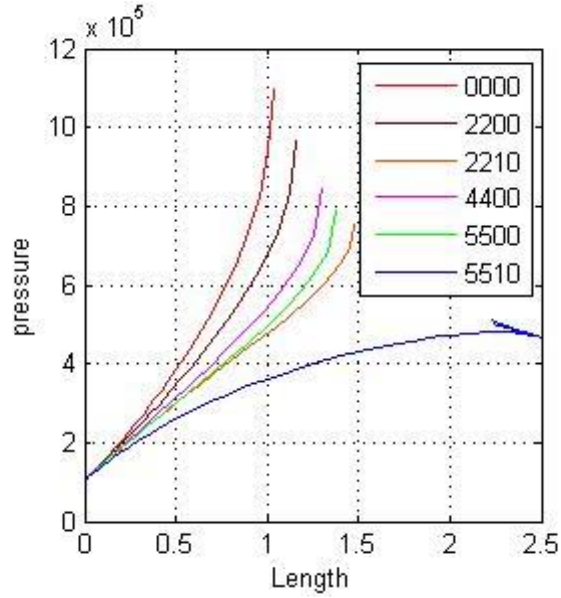

PLOT 6



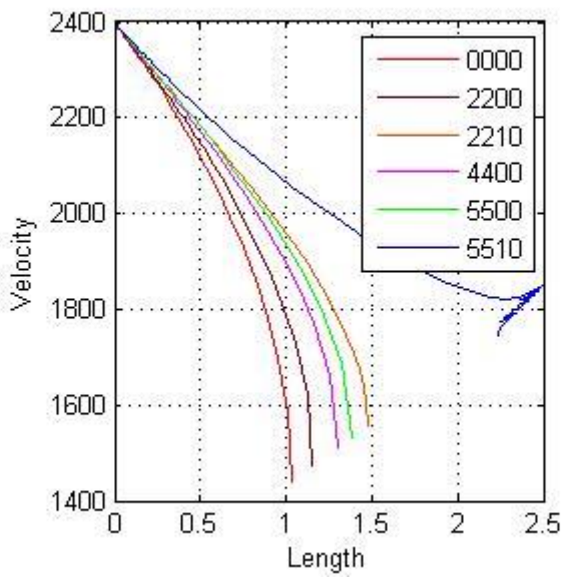

PLOT 7



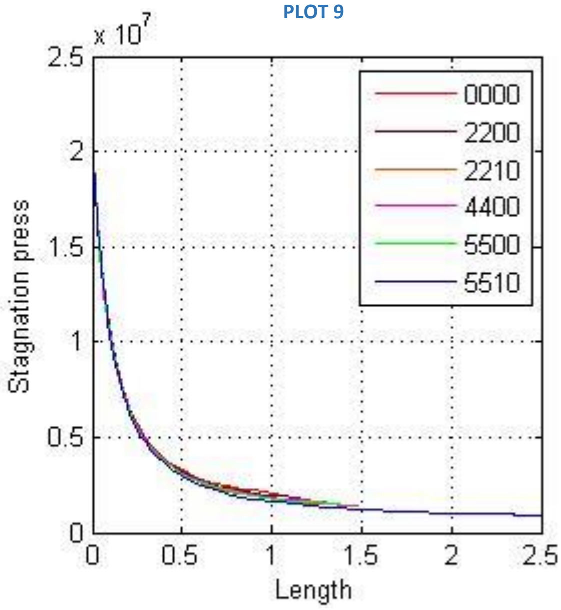

PLOT 8



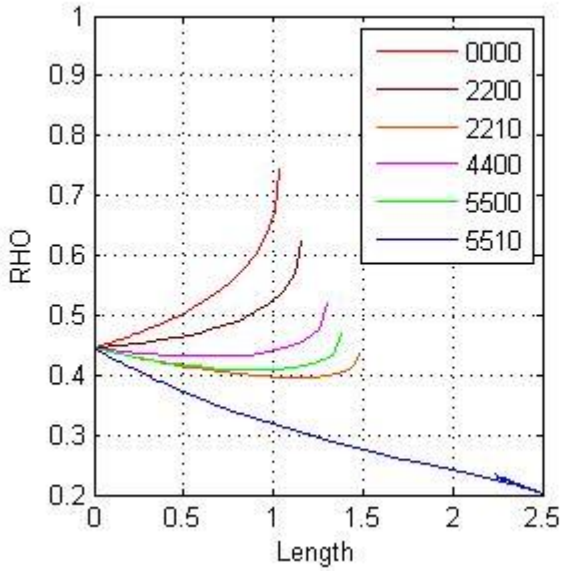

PLOT 9



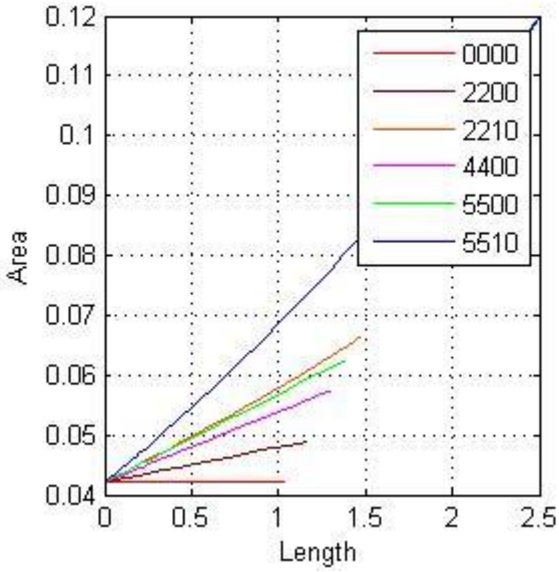

PLOT 10




PLOT 11




Plot 12




Plot 13