SOLUTION OF FLOW INSIDE ISOLATOR

EQUATIONS

These equations are taken from M. Smart's research paper.

$$\frac{d(M^2)}{M^2} = -\left(1+\frac{\gamma-1}{2}M^2\right)\left(\frac{\frac{dP}{P}}{\frac{\gamma M^2}{2}\frac{A_c}{A}} + \frac{4C_f\frac{dx}{D}}{\frac{A_c}{A}} + \frac{dT_t}{T_t}\right) \quad (1)$$

$$\frac{d\left(\frac{A_c}{A}\right)}{\frac{A_c}{A}} = \left\{\frac{1-M^2\left[1-\gamma\left(1-\frac{A_c}{A}\right)\right]}{\gamma M^2\frac{A_c}{A}}\right\}\frac{dP}{P} + \left[\frac{1+(\gamma-1)M^2}{2\frac{A_c}{A}}\right]4C_f\frac{dx}{D} + \left(1+\frac{\gamma-1}{2}M^2\right)\frac{dT_t}{T_t} \quad (2)$$

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{89}{\mathrm{D}} \cdot \frac{C_f \cdot \gamma \cdot M^2}{2} \cdot p \qquad (3)$$

Now, by substituting (3) into (1) and (2) and taking dT_t/T_t = 0 *(adiabatic)*, and rewriting A_c/A = r(x) *(ratio),* we get

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} + \frac{M(x)}{2} \cdot \left(1 + \frac{(\gamma - 1)}{2} \cdot M(x)^2\right) \cdot \left(\frac{93 \cdot C_f}{\mathrm{D} \cdot r(x)}\right) = 0 \qquad (4)$$

$$\frac{\mathrm{d}r(x)}{\mathrm{d}x} - \frac{C_f}{2\mathrm{D}} \cdot \left(93 + M(x)^2 \cdot \left(93 \cdot (\gamma - 1) - 89 \cdot \gamma \cdot r(x)\right)\right) = 0 \qquad (5)$$

- Using (3), (1), (2) and isentropic relation between p_t and p, we can derive:

$$\frac{\mathrm{d}p\_t(x)}{\mathrm{d}x} + \frac{C_f}{\mathrm{D}} \cdot \frac{\gamma \cdot M(x)^2}{2} \cdot \left(\frac{93}{r(x)} - 89 \right) \cdot p\_t(x) = 0 \qquad (6)$$

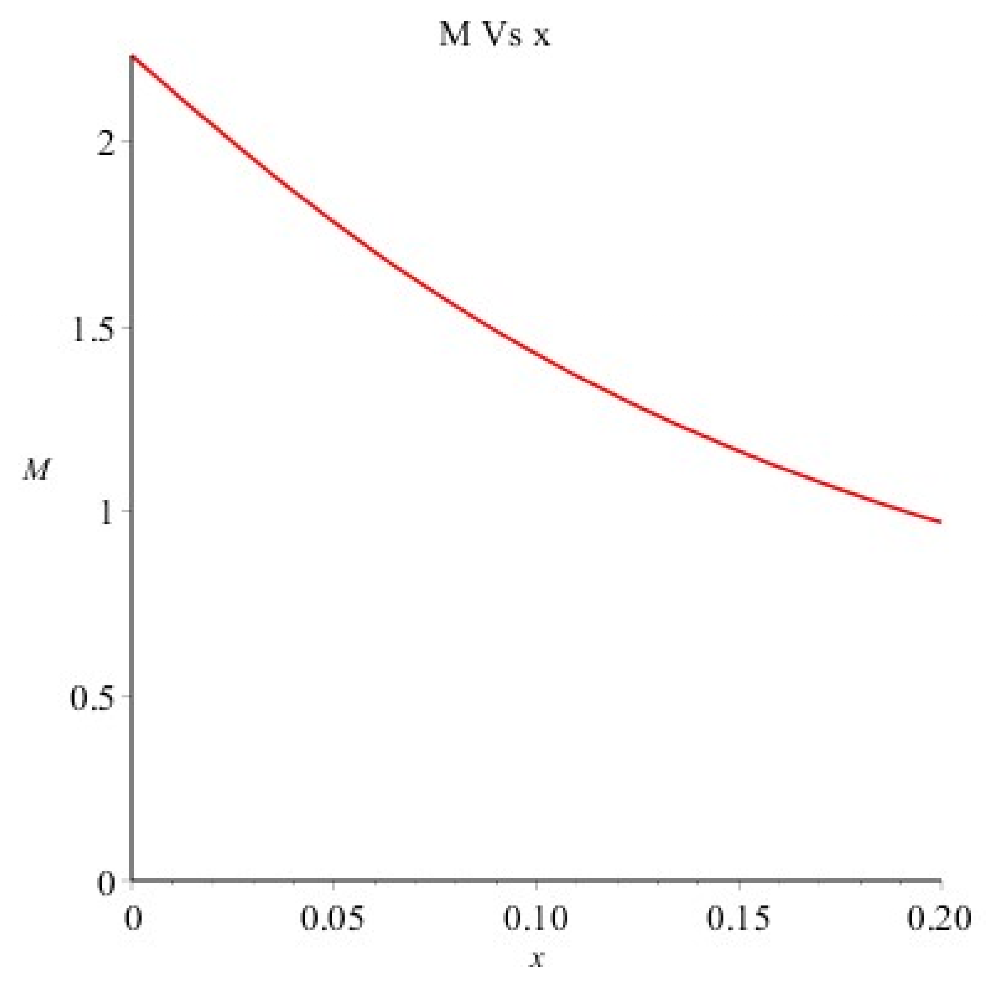
- The equations (3), (4), (5) and (6) are given to Maple (a software like Mathematica) with the following initial conditions:

  M(0)=2.231, p(0)=36515, p_t(0)=409384.4, r(0)=1

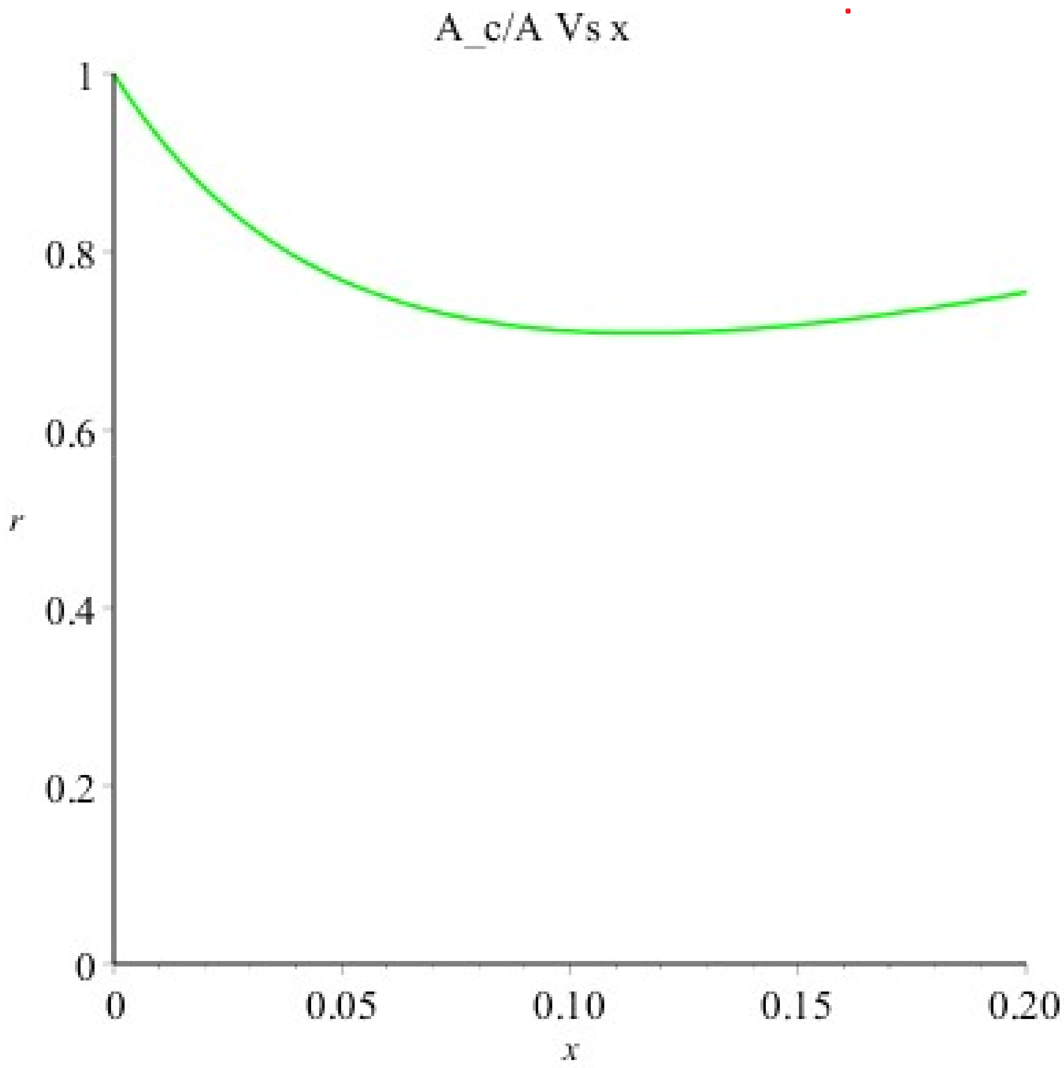
- The values of gamma, C_f and diameter are taken to be: $\gamma = 1.37,\ C_f = 0.002,\ \mathrm{D} = 0.042$

PLOTS

Numerical solutions are obtained for x from 0 to 0.2 m.

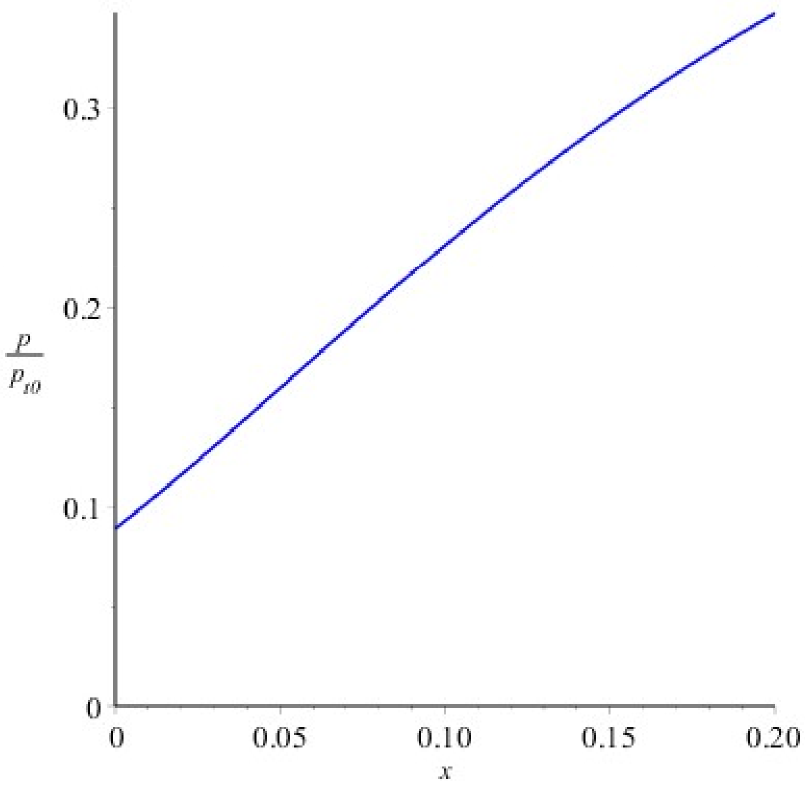


(m)



(m)

Comparison with Billig's plot (p/p_t0 Vs x)



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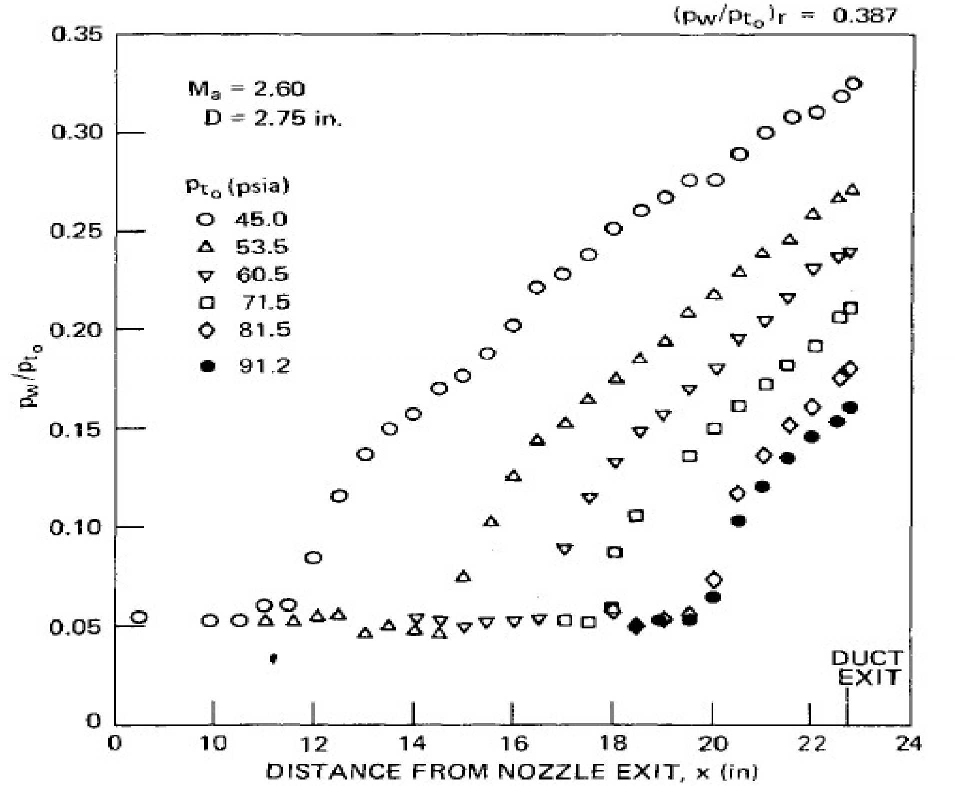
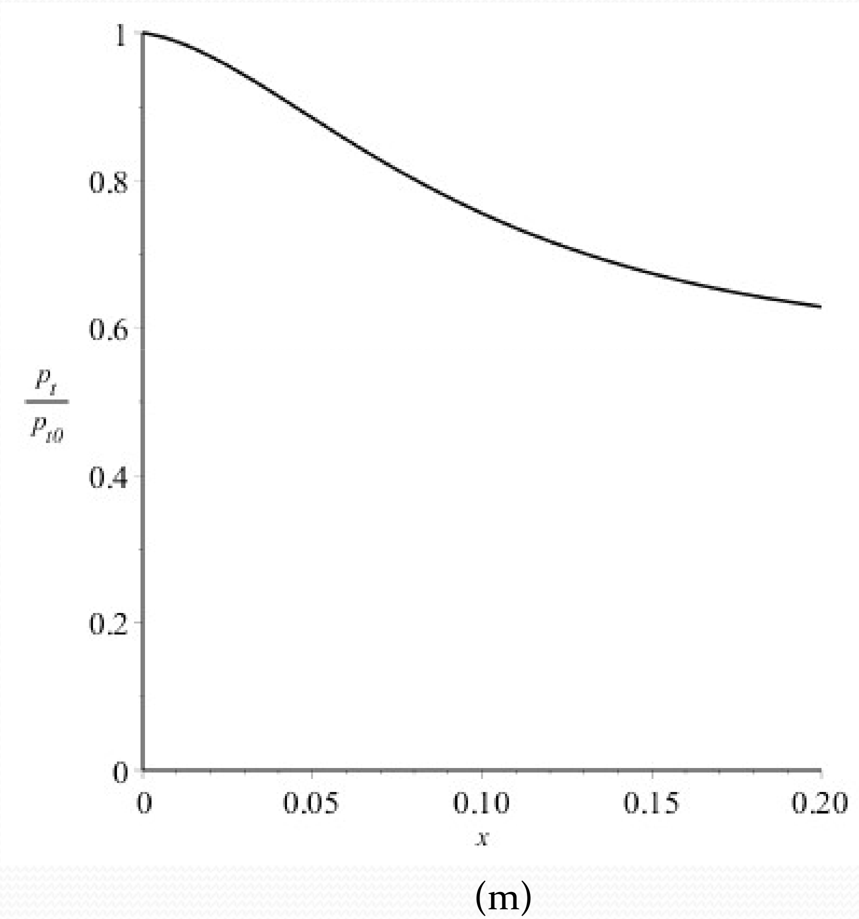


Fig. 2 Wall pressure distribution for case 3.

Comparison with Billig's plot (p_t/p_t0 Vs x)



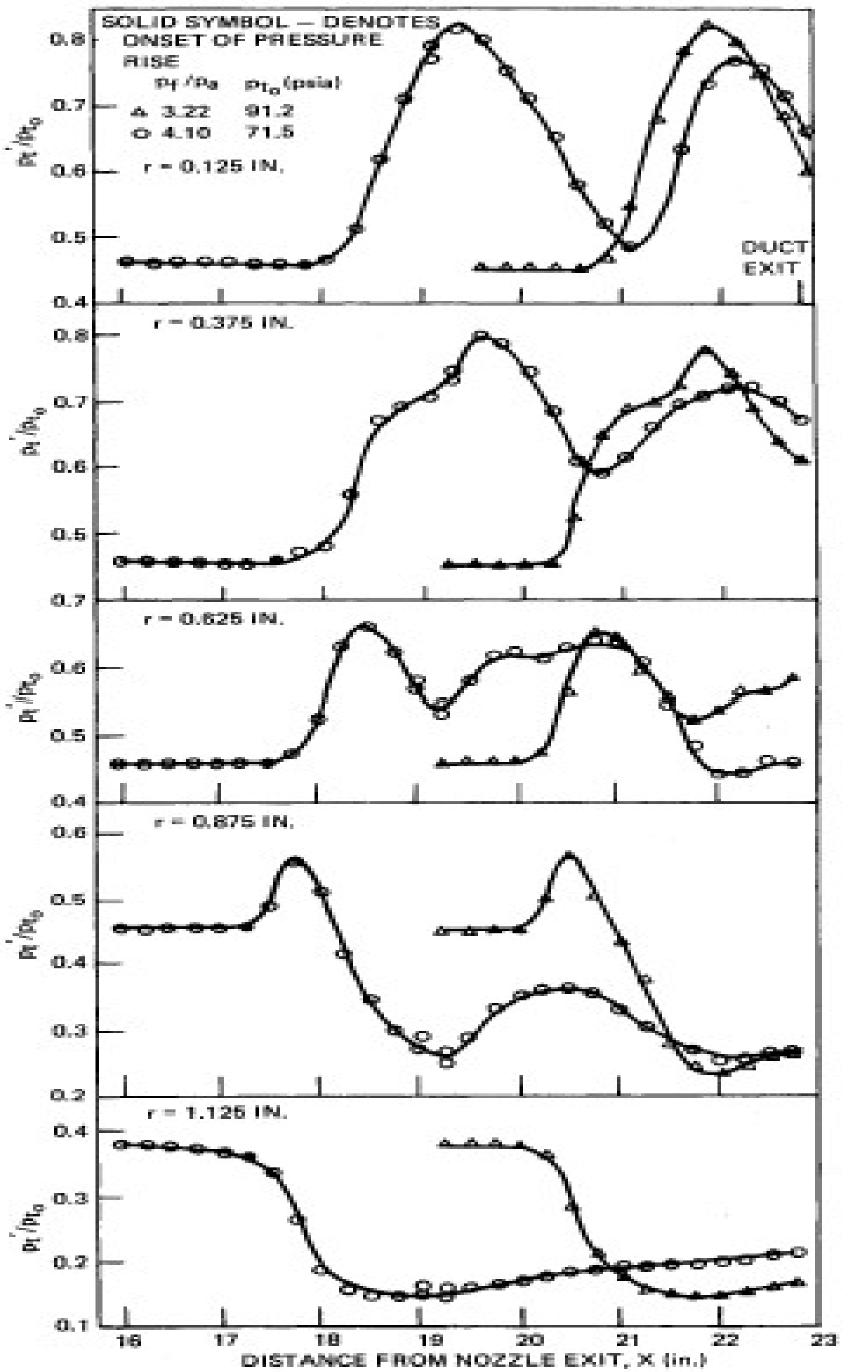


Fig. 5 Axial pitot pressure profiles for case 3.

CONCLUSION

- After comparing with Billig's experimental results (qualitatively), we can see that total pressure plot does not match with experimental plots.

- The pressure Vs x plot matches qualitatively with experimental plots except that the constant pressure plateau in the beginning is missing.

Thank You

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& DEEPENDRA