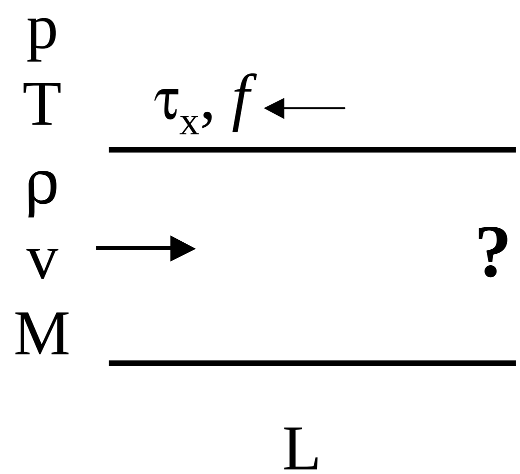
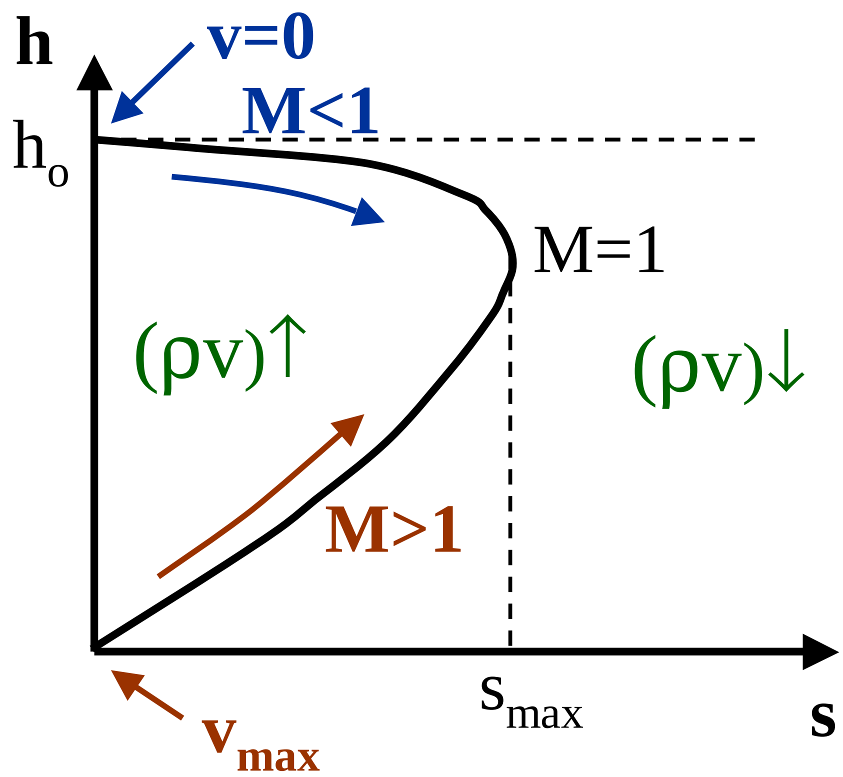
Fanno Flow - Thermodynamics



- Steady, 1-d, constant area, adiabatic flow with no external work but *with friction*
- Conserved quantities
  - since adiabatic, no work: **h_o=constant**
  - since A=const: mass flux=ρ**v=constant**
  - combining: $h_o = h + (\rho v)^2/2\rho = \text{constant}$
- On h-s diagram, can draw **Fanno Line**
  - line connecting points with same h_o and ρv

Fanno Line

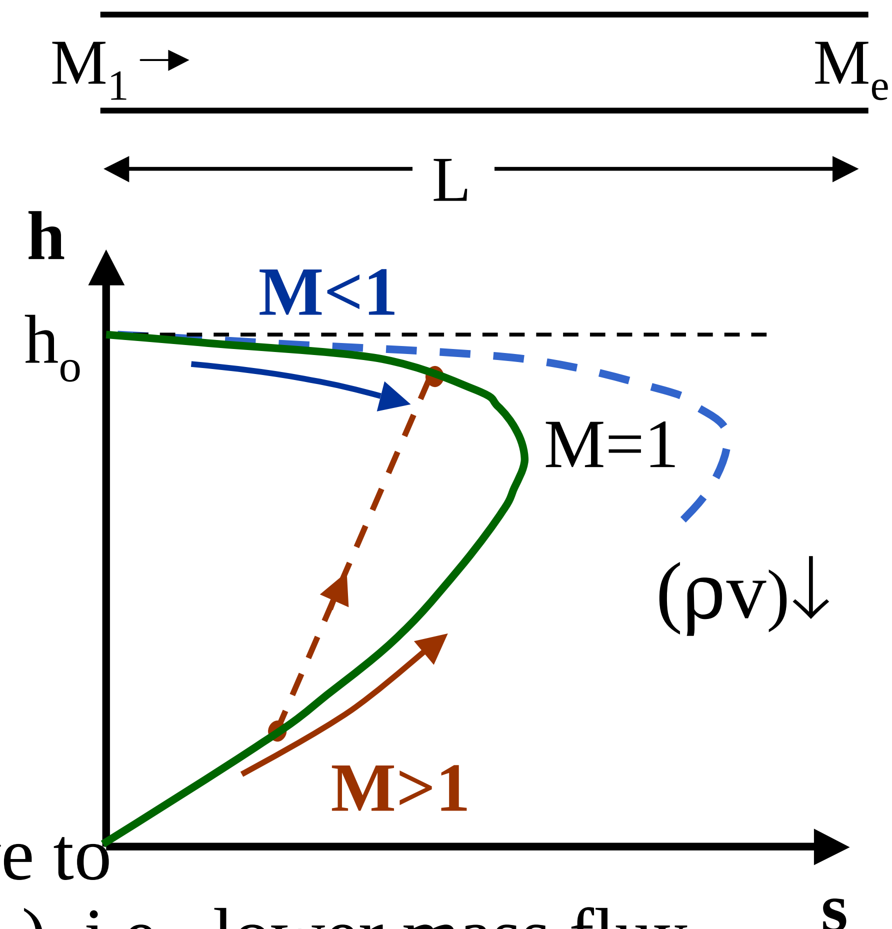
- Velocity change (due to friction) associated with entropy change
- Friction can only increase entropy
  - can only approach M=1
  - friction alone can not allow flow to transition between sub/supersonic



- Two solutions given (ρv,h_o,s): subsonic & supersonic
  - change mass flux: new Fanno line

Fanno Line - Choking

- Total friction experienced by flow increases with length of "flow", e.g., duct length, L
- For long enough duct, M_e=1 ($L=L_{max}$)
- What happens if $L>L_{max}$
  - flow already "choked"
  - **subsonic flow**: must move to different Fanno line (– – –), i.e., lower mass flux
  - **supersonic flow**: get a shock (– – –)



Fanno Line – Mach Equations

- Simplify (X.4-5) for $\delta q = dA = 0$

$$\frac{dM^2}{M^2} = \frac{\gamma M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2}\frac{f dx}{D} \qquad \textbf{(X.6)}$$

$$\frac{dp}{p} = \frac{-\gamma M^2\left[1+(\gamma-1)M^2\right]}{2\left(1-M^2\right)}\frac{f dx}{D} \qquad \textbf{(X.7)}$$

$$\frac{dT}{T} = \frac{dh}{h} = \frac{-\gamma(\gamma-1)M^4}{2\left(1-M^2\right)}\frac{f dx}{D} \qquad \textbf{(X.8)}$$

$$\frac{d\rho}{\rho} = -\frac{dv}{v} = \frac{-\gamma M^2}{2\left(1-M^2\right)}\frac{f dx}{D} \qquad \textbf{(X.9)}$$

$$\frac{ds}{R} = -\frac{dp_o}{p_o} = \frac{\gamma M^2}{2}\frac{f dx}{D} \qquad \textbf{(X.10)}$$

- can write each as only f(M)
- p_o loss due to entropy rise

Property Variations

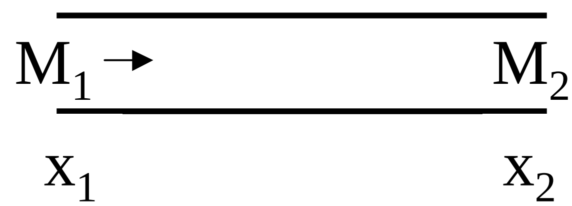
- Look at signs of previous equations to see how properties changed by friction as we move along flow
  - $(1-M^2)$ term makes $M<1$ different than $M>1$

	$M<1$	$M>1$
s	↑	↑
p_o	↓	↓
M	↑	↓
h,T	↓	↑
p	↓	↑
ρ	↓	↑
v	↑	↓

- Friction increases s, $\Rightarrow p_o$ drop
- Friction drives $M \rightarrow 1$
- h_o, T_o const: h,T opposite to M
- p, ρ same as T (like isen. flow)
- ρv=const: v opposite of ρ

A Solution Method

- Need to integrate (X.6-10) to find how properties change along length of flow ($f$dx/D)
  - can integrate or use tables of integrated values
- Mach number variation



$$\int_{M_1^2}^{M_2^2} \frac{\left(1-M^2\right)dM^2}{\gamma M^4\left(1+\frac{\gamma-1}{2}M^2\right)} = \int_0^L \frac{f\left(\mathrm{Re},\mathrm{surface}\right)dx}{D}$$

function of Reynolds number (ν) and surface roughness

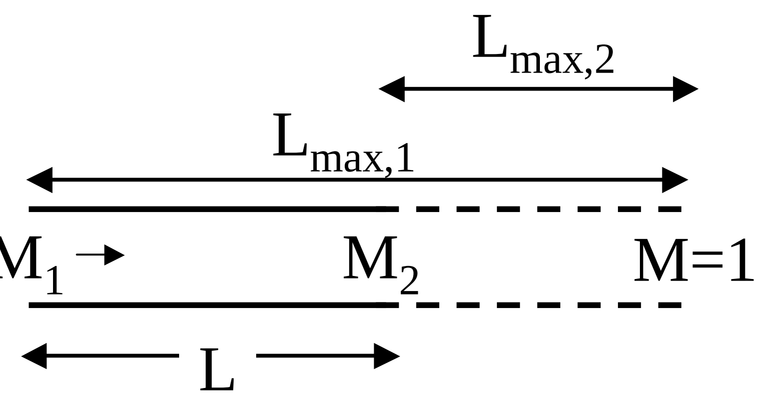
$$\int_{M_1^2}^{1} \frac{\left(1-M^2\right)dM^2}{\gamma M^4\left(1+\frac{\gamma-1}{2}M^2\right)} = \frac{\bar{f}\,L_{max}}{D}$$

1) use avg. f
2) to **tabularize solution**, use reference condition: $M_2=1$, $L_2=L_{max}$

$$\Rightarrow \frac{\bar{f}\,L_{max}}{D} = f(M)\ \text{only}$$

Use of Tables

- To get change in M, use change in $f\,L_{max}/D$ (like using A/A*)



$$\frac{fL}{D}=\int_{M_1^2}^{M_2^2}\left\{\frac{\left(1-M^2\right)}{\gamma M^4\left(1+\frac{\gamma-1}{2}M^2\right)}\right\}dM^2$$

$$=\int_{M_1^2}^{1}\left\{-\right\}dM^2-\int_{M_2^2}^{1}\left\{-\right\}dM^2$$

(X.11)

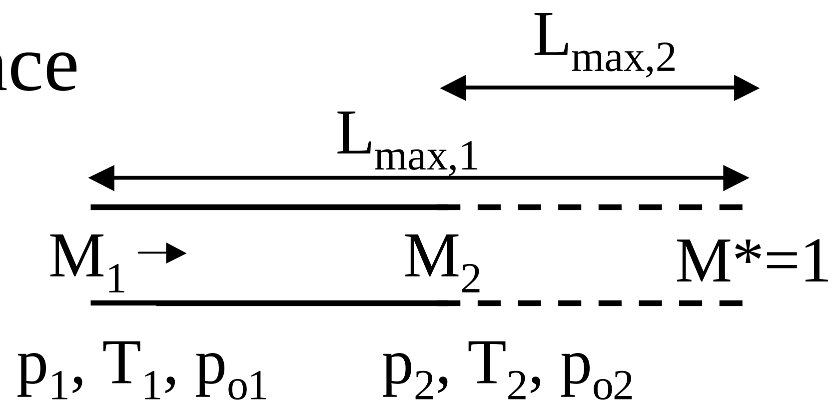
$$\frac{fL}{D}=\left.\frac{fL_{max}}{D}\right)_{M_1}-\left.\frac{fL_{max}}{D}\right)_{M_2}$$

so if you know $f\,L/D$ and M_1,
1) look up $f\,L_{max}/D$ at M_1
2) calculate $f\,L_{max}/D$ at M_2
3) look up corresponding M_2

- Find values in Appendix E in John

TD Property Changes

- To get changes in T, p, p_o, ... can also use M=1 condition as reference condition (*)



- Integrate (X.7-10), e.g.,

$$\int_{p_1}^{p_2} \frac{dp}{p} = \int_{M_1}^{M_2} -\frac{1}{2} \frac{1+(\gamma-1)M^2}{1+\frac{\gamma-1}{2}M^2} \frac{dM^2}{M^2}$$

$$\frac{p_2}{p_1} = \left[\frac{M_1^2\left(1+\frac{\gamma-1}{2}M_1^2\right)}{M_2^2\left(1+\frac{\gamma-1}{2}M_2^2\right)} \right]^{1/2} \Rightarrow \frac{p}{p^*} = \frac{1}{M}\sqrt{\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M^2}}$$

Fanno Flow Property Changes

- Summarize results in terms of reference conditions

$$\frac{T}{T^*} = \frac{(\gamma+1)/2}{1+\frac{\gamma-1}{2}M^2} \quad \textbf{(X.12)}$$

$$\frac{p_o}{p_o^*} = \frac{1}{M}\left(\frac{T}{T^*}\right)^{\frac{\gamma+1}{2(1-\gamma)}} \quad \textbf{(X.14)}$$

$$\frac{p}{p^*} = \frac{1}{M}\sqrt{\frac{T}{T^*}} \quad \textbf{(X.13)}$$

$$\frac{v}{v^*} = \frac{\rho^*}{\rho} = M\sqrt{\frac{T}{T^*}} \quad \textbf{(X.15)}$$

- In terms of initial and final properties

$$\frac{T_2}{T_1} = \frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)}{\left(1+\frac{\gamma-1}{2}M_2^2\right)}$$

(X.16) (T_o=const)

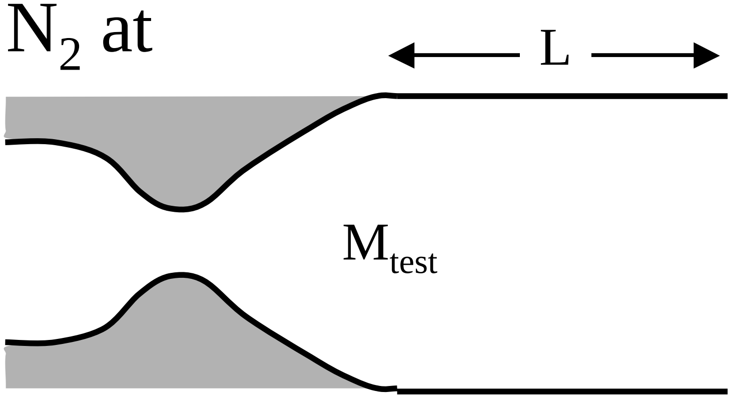
$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}} \quad \textbf{(X.17)}$$

$$\frac{p_{o2}}{p_{o1}} = \frac{M_1}{M_2}\left(\frac{T_2}{T_1}\right)^{\frac{\gamma+1}{2(1-\gamma)}} \quad \textbf{(X.18)}$$

$$\frac{v_2}{v_1} = \frac{\rho_1}{\rho_2} = \frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}} \quad \textbf{(X.19)}$$

Example

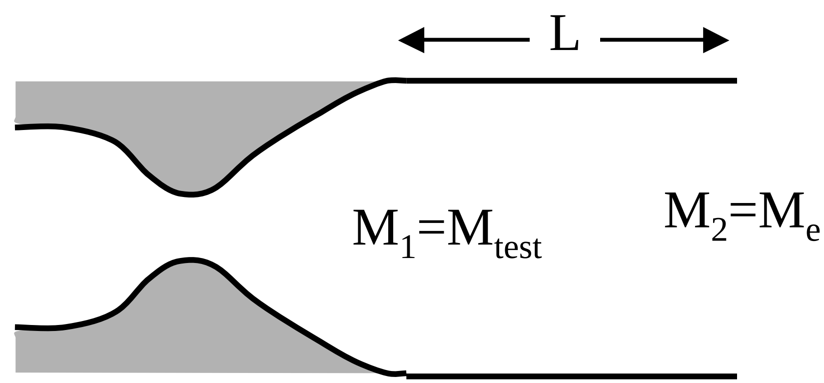
- **Given:** Exit of supersonic nozzle connected to straight walled test section. Test section flows N_2 at M_{test}=3.0, T_o=290. K, p_o=500. kPa, L=1m, D=10 cm, f=0.005



- **Find:**
  - M, T, p at end of test section
  - $p_{o,exit}/p_{o,inlet}$
  - L_{max} for test section

- **Assume:** N_2 is tpg/cpg, γ=1.4, steady, adiabatic, no work

Solution



- **Analysis:**
  - **M_e** (X.11)

$$\frac{fL}{D} = \left.\frac{fL_{max}}{D}\right)_{3.0} - \left.\frac{fL_{max}}{D}\right)_{M_e}$$

$$\left.\frac{fL_{max}}{D}\right)_{M_e} = 0.5222 - \frac{0.005(100)}{10} = 0.4722$$

(Appendix E) $\rightarrow$ $M_e = 2.70$

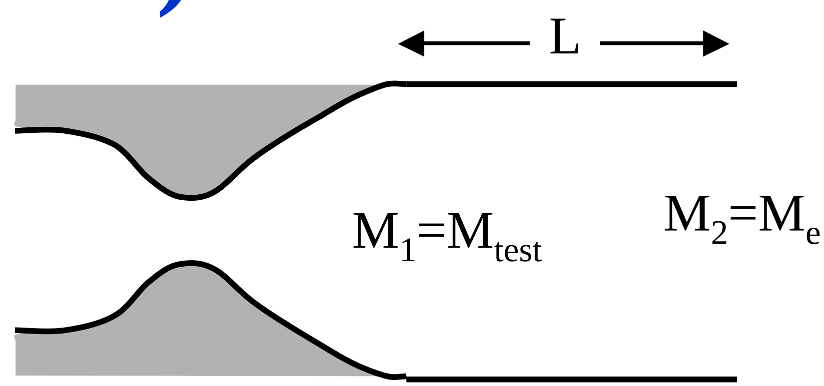
another solution is M=0.605, but since started M>1, can't be subsonic

  - **T**

(T$_o$ const)

$$T_2 = T_1 \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} = \frac{T_o}{1 + \frac{\gamma-1}{2} M_2^2} = 118\,\text{K}$$

Solution (con't)



– **p**

$$p_2 = p_1 \frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}} \quad \text{(X.17)}$$

$$p_1 = p_{o1}\left(1+\frac{\gamma-1}{2}M_1^2\right)^{-\gamma/\gamma-1}$$

$$= \frac{500\,\text{kPa}}{2.8^{3.5}} = 13.6\text{kPa} \qquad \frac{T_2}{T_1} = \frac{1+((\gamma-1)/2)M_1^2}{1+((\gamma-1)/2)M_2^2} = 1.14$$

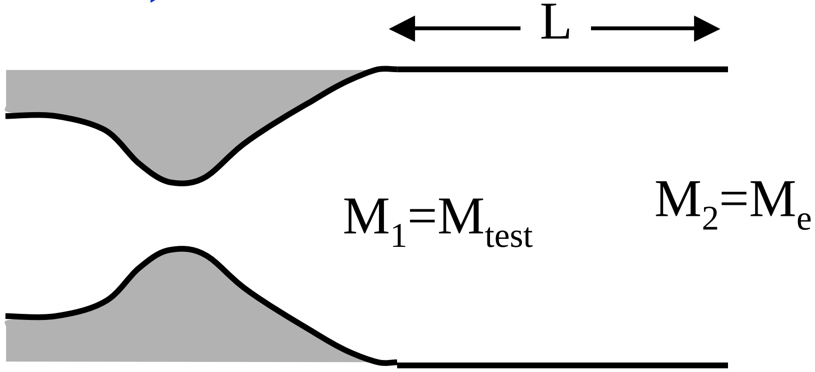
$$p_2 = 13.6\text{kPa}\,\frac{3.0}{2.7}\sqrt{1.14} = 16.1\text{kPa}$$

– **$p_{o,e}/p_{o,test}$**

$$\text{(X.18)} \qquad \frac{p_{o2}}{p_{o1}} = \frac{M_1}{M_2}\left(\frac{T_2}{T_1}\right)^{\frac{\gamma+1}{2(1-\gamma)}} = \frac{3.0}{2.7}(1.14)^{-3} = 0.75$$

25% loss in stagnation pressure due to friction

Solution (con't)



– L_{max}

$$L_{max} = \left.\frac{fL_{max}}{D}\right)_{M_{test}} \frac{D}{f}$$

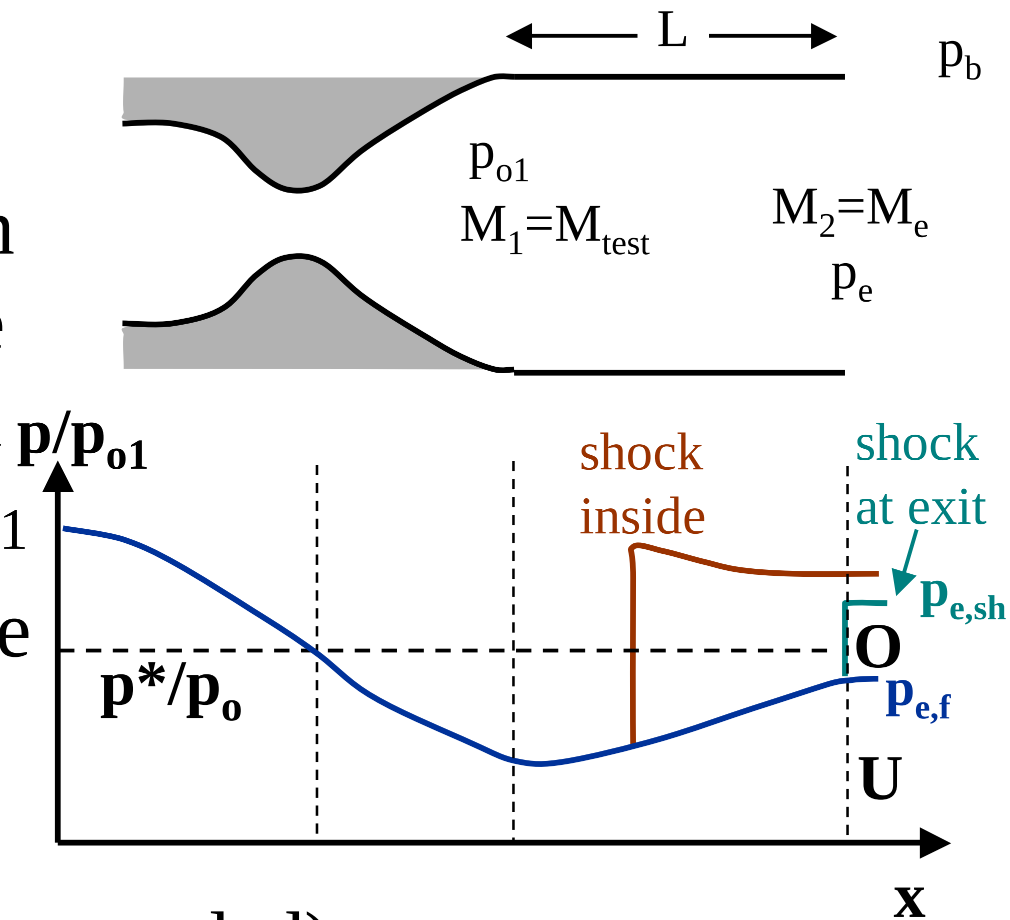
$$= 0.5222 \frac{0.1\text{m}}{0.005}$$

$$= 10.4\text{m}$$

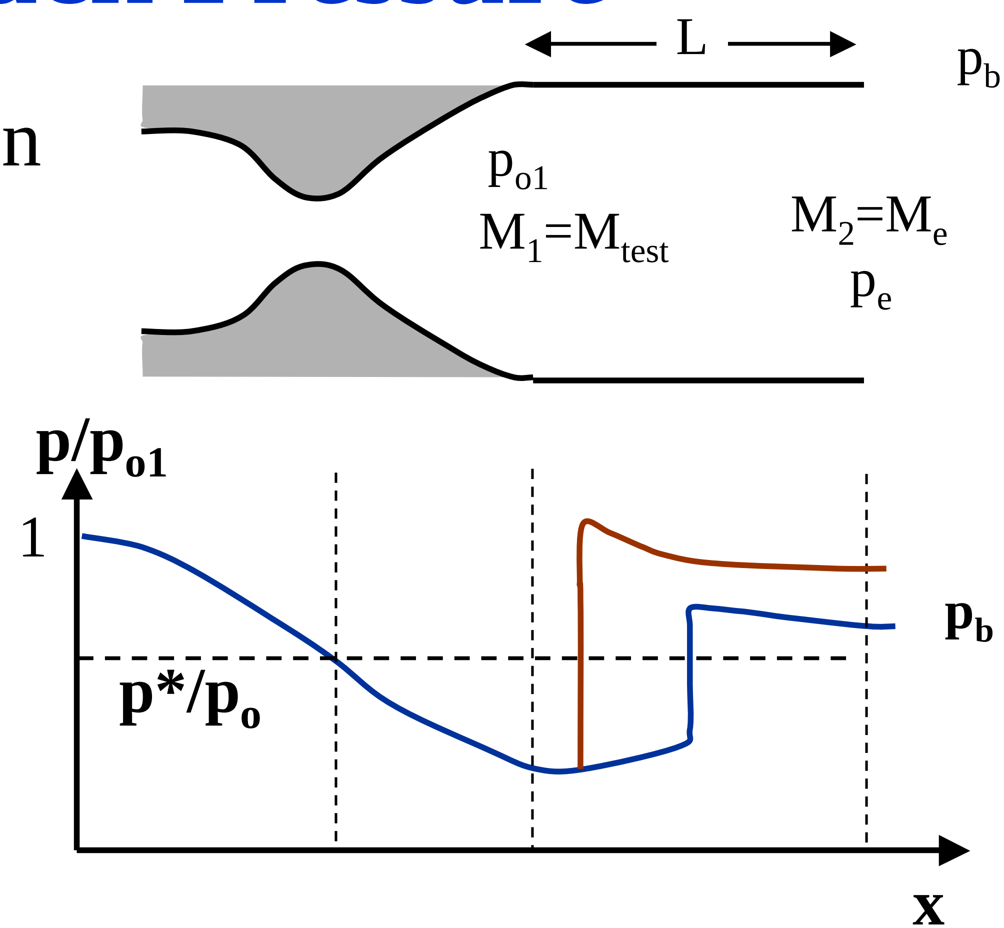
10 m long section would have M=1 at exit

$L<L_{max}$, Back Pressure

- Last problem (supersonic duct), what would happen if calculated exit pressure ($p_{e,f}$) did not match actual back pressure (p_b)
- $p_b < p_{e,f}$: expansion outside duct (underexpanded)
- $p_{e,f} < p_b < p_{e,sh}$: oblique shocks outside duct (overexpanded)
- $p_{e,sh} < p_b$: shocks inside duct (until shock reaches ~throat)



$L>L_{max}$, Back Pressure



- Can't have flow transition to subsonic with pure Fanno flow
  $\Rightarrow$shock in duct
- Shock location determined by back pressure
  – raise p_b
  – shock moves upstream until shock reaches M=1 location in nozzle