Welcome! Today, we have a variety of questions from QOTW for you guys to solve. There is a total of **41** points. Points indicate relative difficulty. You may find the following formulas helpful:

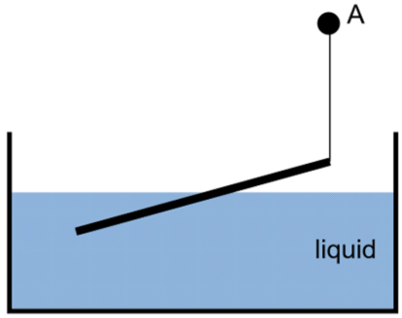
$$F_{net} = ma \text{ (Newton's 2nd law)} \qquad U = \frac{1}{2}kx^2 \text{ (Spring potential energy)}$$
$$F_b = \rho_w V g \text{ (Bouyancy Force)} \qquad (1+x)^n \approx 1 + nx \text{ (Binomial Approximation)}$$

1 Week 1

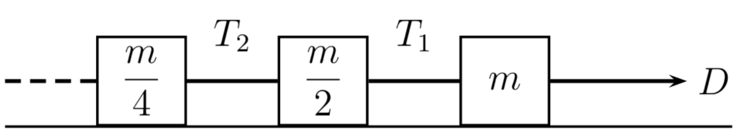
[8] **Problem 1.** A uniform rod, of length $2a$, floats partially submerged in a liquid, being supported by a string fastened to one of its ends, the other end of the string being attached to a fixed point A. The density of the liquid is a factor 4/3 times that of the rod. Determine

(a) the fraction of the rod's length that will be submerged.

(b) the tension, T in the string.



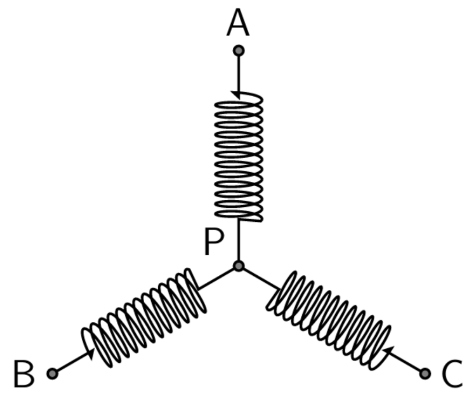
[6] **Problem 2.** A long chain of blocks, connected via light extensible strings, travels without friction in a straight line on horizontal ground. The front block is being pulled with a force D, causing the entire chain to accelerate at a rate a. The block at the front has mass m, and each other block has half the mass of the block directly in front of it. The tensions in the strings are labeled $T_1, T_2, \cdots$ from the front.



Given that $T_n = D - f(n)ma$, find $f(n)$.

(A) 2^{-n} **(B)** 2^{1-n} **(C)** $2 - 2^{-n}$ **(D)** $2 - 2^{1-n}$ **(E)** $1 - 2^{-n}$

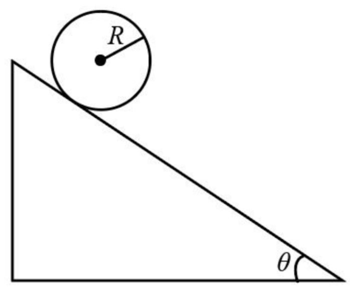
[7] **Problem 3.** Three ideal springs of spring constant k and natural length a are arranged equidistant from a point P, as shown below. The points A, B and C are fixed in the plane of the paper, and are initially each a distance a away from P. The point P is then moved a small distance d perpendicular to the plane of the paper. Which of these options gives the best approximation for the total energy stored in the system?



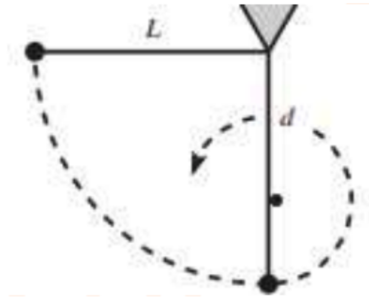
(A) $\frac{kd^4}{2a^2}$ **(B)** $\frac{3kd^4}{a^2}$ **(C)** $\frac{3kd^4}{2a^2}$ **(D)** $\frac{3kd^4}{4a^2}$ **(E)** $\frac{3kd^4}{8a^2}$

2 Week 2

[6] **Problem 4.** A wheel with mass m, radius R, and moment of inertia $\beta m R^2$ (where β is a numerical factor) rolls without slipping down a plane inclined at angle θ. Find the linear acceleration of the wheel.



[7] **Problem 5.** A pendulum of length L is held with its string horizontal, and then released. The string runs into a peg a distance d below the pivot, as shown. In terms of L, find the smallest value of d for which the string remains taut at all times.



3 Week 3

[7] **Problem 6. Adiabatic Compression in a Gas Cylinder**

An *adiabatic process* is a thermodynamic process in which no heat is transferred into or out of the system. Interestingly, even without heat exchange, the temperature of the gas can still change due to *adiabatic heating* or *adiabatic cooling*.

Consider a closed gas cylinder with a movable piston. The gas is compressed from an initial volume V_0 and temperature T_0 to a final volume V_f and temperature T_f. Throughout the process, no heat is exchanged with the surroundings ($Q = 0$).

(a) Derivation of Work Expression

Starting from the general definition of mechanical work,

$$W = \int F\,dx,$$

where F is the force applied by the gas on the piston and dx is an infinitesimal piston displacement, **show that** the work done by the gas during compression or expansion can be written as

$$W = \int P\,dV,$$

where P is the pressure and V is the volume of the gas.

(b) Relation Between Temperature and Volume in an Adiabatic Process

Using the result from part (a) and the following relations:

1. **First Law of Thermodynamics (differential form):**

$$dE = \delta Q - \delta W,$$

   or equivalently, for an ideal gas,

$$mC_v\,dT = -P\,dV$$

   (since $\delta Q = 0$ for adiabatic processes),

2. **Ideal Gas Law:**

$$PV = mRT,$$

derive an expression that relates the **temperature ratio** $\frac{T_f}{T_0}$ to the **volume ratio** $\frac{V_f}{V_0}$.

Your final relation should involve the adiabatic index $\gamma = \frac{C_p}{C_v}$, and demonstrate how compression (decreasing V) leads to heating (increasing T) of the gas.