

Welcome! Today, we are continuing with Newtons Laws, as well as learning how to draw a free body diagram. There is a total of **54** points. Points indicate relative difficulty.

### Idea 1: Newton's Laws

As a recap, Newton's 1st Law, also known as the law of inertia, states that **A body moves with constant velocity (which may be zero) unless acted on by a force.**

Newton's 2nd Law, also known as  $F = ma$ , states that **The rate of change of the momentum of a body equals the force acting on the body.**

Newton's 3rd Law states that **Given two bodies A and B, if A exerts a force on B, then B exerts an equal and opposite force on A.**

Now, lets learn how to draw a free body diagram.

### Idea 2: Free Body Diagram (FBD)

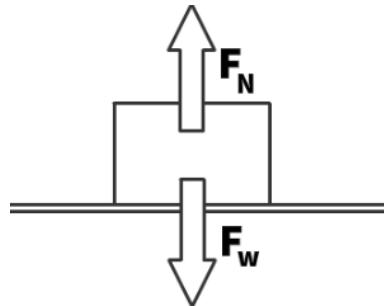
We can define a free body diagram (FBD) as **a drawing of an object with all external forces and moments acting on it, shown as vectors.** By doing this, we isolate the object to analyze the forces acting on it, making it easier to solve problems.

When drawing a FBD, the main types of forces we typically draw are:

1. The force of gravity, denoted as  $F_g$  or  $F_W$  (weight).
2. Normal force, denoted as  $F_N$ .
3. Friction, which opposes motion, denoted as  $F_f$ , or  $F_k$  for kinetic friction and  $F_s$  for static friction.
4. Tension, denoted as  $F_T$ .
5. Applied forces, if present, denoted as  $F_A$ .

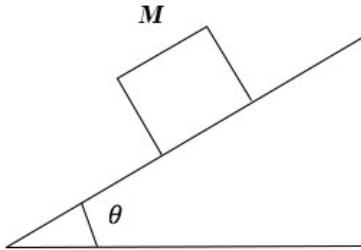
Its important to note that a free body diagram should **only include forces**, which means you should not draw things like velocity or centripetal acceleration. You should also try to draw your force vectors **to scale**, particularly when forces are equal and opposite.

Here is an example of a free body diagram of a box resting on the floor:

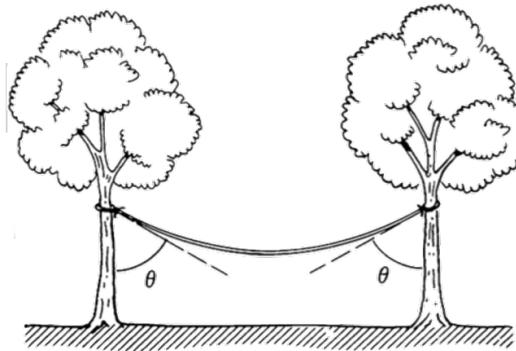


## 1 Practice Problems

- [3] **Problem 1.** Draw a corresponding free diagram for each of the scenarios below:
- An egg is free-falling from a nest in a tree. Neglect air resistance.
  - A rightward force is applied to a book, moving it across a rough desk.
  - A girl is suspended motionless from a bar which hangs from the ceiling by two ropes.
- [3] **Problem 2.** A block is placed on an inclined plane with angle  $\theta$  respect to the ground. There is static friction.



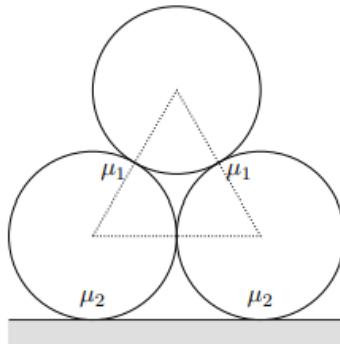
- Draw the free body diagram of the block.
  - Assume no other forces act on the block, find the least static friction coefficient required for the block to be in equilibrium, express your answer using  $\theta$  and  $g$ .
- [4] **Problem 3.** A block of mass  $M_1$  sits on a block of mass  $M_2$  on a frictionless table. The coefficient of friction between the blocks is  $\mu$ . Find the maximum horizontal force that can be applied to (a) block 1 or (b) block 2 so that the blocks will not slip on each other.
- [5] **Problem 4.** A uniform rope of weight  $W$  hangs between two trees. The ends of the rope are the same height, and they each make angle  $\theta$  with the trees. Find the tension at either end of the rope, and the tension at the middle of the rope.



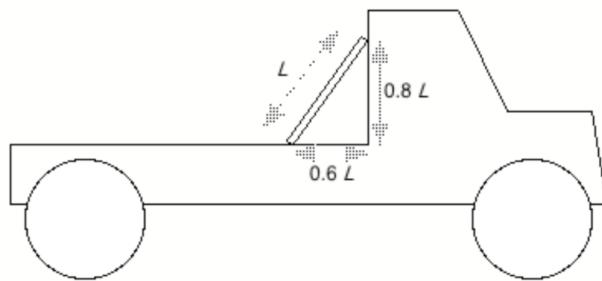
## 2 Challenge Problems

- [5] **Problem 5.** (2025 F=MA Problem 5) Three identical cylinders are used in this setup. Two of them are placed side by side on a horizontal surface, with a negligible distance between their surfaces so they do not touch. The third identical cylinder is placed on top of the first two, such that their centers form an equilateral triangle, as shown in the figure below.

Draw the FBD for each of the 3 cylinders.

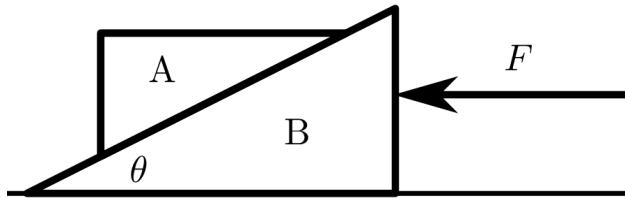


- [6] **Problem 6.** You are at the center of a disk space station of radius  $R$ , spinning about its center at angular velocity  $\omega$  that simulates gravity through the use of centripetal acceleration. There is a path that moves an object from the center of the station to the edge that is a straight tube that is frictionless. If you nudge an object down the tube starting at  $r = 0$  and going to  $r = R$  with initial velocity  $v_0$ , how long would it take to reach the edge at radius  $R$ ? Remember your reference frame.
- [8] **Problem 7.** A uniform board of length  $L$  is placed on the back of a truck. There is no friction between the top of the board and the vertical surface of the truck. The coefficient of static friction between the bottom of the board and the horizontal surface of the truck is  $\mu_s = 0.5$ . The truck always moves in the forward direction.



- What is the maximum starting acceleration the truck can have if the board is not to slip or fall over?
- What is the maximum stopping acceleration the truck can have if the board is not to slip or fall over?
- For what value of stopping acceleration is the static frictional force equal to zero?

- [7] **Problem 8.** (2017 USAPhO A1) A pair of wedges are located on a horizontal surface. The coefficient of friction (both sliding and static) between the wedges is  $\mu$ , the coefficient of friction between the bottom wedge  $B$  and the horizontal surface is  $\mu_b$ , and the angle of the wedge is  $\theta$ . The mass of the top wedge  $A$  is  $m$ , and the mass of the bottom wedge  $B$  is  $M = 2m$ . A horizontal force  $F$  directed to the left is applied to the bottom wedge as shown in the figure.



Determine the range of values for  $F$  so that the top wedge does not slip on the bottom wedge. Express your answer(s) in terms of any or all of  $m$ ,  $g$ ,  $\mu$ ,  $\mu_b$ , and  $\theta$ .

### Idea 3: Center of mass and stability

An extended object supported at a point may be static if its center of mass lies directly above or below that point. More generally, if the object is supported at a set of points, it can be static if its center of mass lies above the convex hull of the points.

- [5] **Problem 9.**  $N$  identical uniform bricks of length  $L$  are stacked, one above the other, near the edge of a table. What is the maximum possible length the top brick can protrude over the edge of the table? How does this limit grow as  $N \rightarrow \infty$ ?
- [8] **Problem 10.** A sphere is made of two homogeneous hemispheres stuck together, with different densities. Is it possible to choose the densities so that the sphere can be placed on an inclined plane with incline  $30^\circ$  and remain in equilibrium? Assume the coefficient of friction is sufficiently high so that the sphere cannot slip.