Welcome to the Physics Olympiad Club! This is a designed as a physics-inspired scavenger hunt, where you'll run around in groups of 3-4 to try and solve the problems. Each problem has a certain number of points, for a total score of **215** points. Every problem can be solved with zero background knowledge.

1 Mechanics

[9] **Problem 1.** Jon runs 1 mile at a pace of 200 meters per minute. Note that 1 kilometer = 1000 meters and 1 mile = 1600 meters.

(a) Convert Jon's speed to kilometers/second.

(b) Find how long it takes, in minutes, for Jon to run a mile.

(c) Measure how fast you can run 100 meters and convert it to meters per minute.

[9] **Problem 2.** Suppose you are at the 41st floor of the Empire State Building. You take an elevator to the 67th floor. Circle each correct option.

(a) At some point in the journey, you will feel lighter. (True/False)

(b) At some point in the journey, you will feel heavier. (True/False)

(c) A bowl of water sits on the floor of the elevator, and a mass floats in it. In an ideal scenario, the mass will stay in place relative to the bowl for the entire journey. (True/False)

[9] **Problem 3.** There are 4 kinematics equations:

$$v_f = v_i + at, \qquad \Delta x = \left(\frac{v_f + v_i}{2}\right) t, \qquad \Delta x = v_i t + \frac{1}{2}at^2, \qquad v_f^2 = v_i^2 + 2a\Delta x$$

Where Δx represents an object's displacement, a represents acceleration, t represents time, and v_i and v_f represent the initial and final velocities, respectively.

A toy ball is thrown from the ground directly upwards with an initial velocity of 10 m/s. Find

(a) The time until the ball reaches its maximum height.

(b) The maximum height of the ball.

(c) The total time the ball is in the air.

Hint: Try plugging in $a = -10$ (gravity), $v_i = 10$, $v_f = 0$ into each equation to solve for Δx and t.

[27] **Problem 4.** Appoint someone in your group to run around the center quad as fast as possible (along the square perimeter which is 100 meters long) starting at one of the vertices, appoint another person to record the time for first person to finish the whole loop.

Assume uniform acceleration for each linear sprint along the 4 edges, estimate the final velocity.

2 Electricity&Magnetism

[3] **Problem 5.** A magnet consists of a North Pole and a South Pole where the magnetic field are the strongest. Suppose you have two magnets. Circle each correct option.

(a) Opposite charges (attract/repel) each other.

(b) The further the distance, the (stronger/weaker) the force.

(c) The same magnets on the moon would act (same/different) than on Earth.

3 Thermodynamics

[45] **Problem 6. (Gary Sat, Vivek Ran)**

It is a cold winter morning. Gary and Vivek are two spherical creatures sitting outside. Gary has a thick layer of fat that reduces the rate of heat loss to the environment, allowing him to stay warm even while sitting still. Vivek, however, has almost no fat. To maintain his body temperature, he must **run** to generate heat through metabolism.

We will model this situation using basic heat transfer and energy balance principles.

Let both creatures:

- Have the same internal body temperature T_b (in kelvins).
- Be surrounded by an ambient temperature T_a ($T_a < T_b$).
- Have spherical shapes with radius R.

Heat is transferred from the body to the surroundings mainly by **conduction through fat** and then **convection** from the outer surface.

We will model the fat layer as a spherical shell with:

- Inner radius r_i
- Outer radius $r_o = R$
- Thermal conductivity k

Part (a): Gary's Heat Loss [36 points]

The rate of heat transfer from the inner surface of the fat shell (where blood and muscles connect to the fat) to outer surface (where fat and skin touches air) is given by

$$\dot{Q}_{\text{cond}} = \frac{4\pi k(T_b - T_s)}{\frac{1}{r_i} - \frac{1}{r_o}}$$

At the outer surface, the heat leaving the fat layer is lost to air by convection:

$$\dot{Q}_{\text{conv}} = hA(T_s - T_a)$$

where

$$A = 4\pi r_o^2$$

and h is the convective heat transfer coefficient (depends on air speed and conditions).

At steady state:

$$\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}}$$

Combine both expressions to eliminate T_s and find the net rate of heat loss $\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}}$ in terms of T_b, T_a, and the parameters (k, r_i, r_o, h). (Good luck, it's extremely bashy, but this is worth 36 points!)

Gary would need to generate heat energy at this rate to maintain body temperature while sitting.

Part (b): Vivek's Metabolic Heat Generation and Heat Loss [9 points]

Vivek compensates for his higher heat loss by **running**, which generates additional internal heat. Assume that the rate of heat generation $\dot{Q}_{\text{gen}}$ is proportional to his running speed v:

$$\dot{Q}_{\text{gen}} = cv$$

where c is a proportionality constant depending on body mass and metabolism.

At steady state (constant body temperature):

$$\dot{Q}_{\text{gen}} = \dot{Q}_{\text{loss}}$$

Thus:

$$cv = \dot{Q}_{\text{conv}}$$

Step 1: Substitute $\dot{Q}_{conv}$ from the heat balance equation similar to Part (a).

Note that Vivek does not have a heat transfer process through fat, heat directly leaves his body by convection.

Therefore you can directly apply this formula

$$\dot{Q}_{\text{conv}} = hA(T_b - T_a)$$

Step 2: Solve for the minimum speed required:

$$v_{\text{min}} = \frac{\dot{Q}_{\text{conv}}}{c}$$

This gives Vivek's minimum running speed to maintain body temperature.

Now it is for you to express v_{min} in terms of T_b, T_a, and the parameters (c, R, h).

4 Special Relativity

Context (given formulas). For this problem you do *not* need any prior knowledge of special relativity beyond the two short formulas given below. Treat them as definitions you may use directly.

$$\gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{(Lorentz factor)} \tag{1}$$

$$K(v) = (\gamma(v)-1)\,mc^2 \quad \text{(relativistic kinetic energy of a particle of rest mass } m \text{ moving at speed } v\text{)} \tag{2}$$

[56] **Problem 7.** A spacecraft of rest mass $m = 2.00 \times 10^3$ kg travels at speed $v = 0.400\,c$ relative to an observer (where $c = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}$ is the speed of light).

(a) Compute the Lorentz factor $\gamma(v)$ to five significant figures.

(b) Compute the relativistic kinetic energy $K(v)$ in joules. Give your answer in scientific notation with three significant figures.

(c) Convert the kinetic energy you found in part (b) into an equivalent mass Δm using $\Delta m = K/c^2$. Interpret Δm briefly (one sentence).

(d) Suppose instead the spacecraft had speed $0.100\,c$. Without re-deriving formulas, compute the new K and Δm and report the ratio $\frac{\Delta m_{0.400c}}{\Delta m_{0.100c}}$.

Hints.

- Evaluate v^2/c^2 carefully as a decimal, then compute γ using the square root.
- Use $c^2 = (3.00 \times 10^8\ \mathrm{m\,s^{-1}})^2 = 9.00 \times 10^{16}\ \mathrm{m^2\,s^{-2}}$ when converting energies to mass.

Bonus: Einstein's derivation of $E = mc^2$ (nothing to do here, but very fascinating background information)

Intuition. Starting from elementary assumptions about energy and the transformation of electromagnetic energy between inertial frames, derive that if a body loses energy L as radiation, its mass must decrease by L/c^2. From this, infer the general relation $E = mc^2$.

Setup

Consider a body at rest in an inertial frame S (the rest frame of the body). The body emits two short flashes of electromagnetic radiation (light) simultaneously in opposite directions along the x–axis. Each flash carries energy $L/2$ in frame S, so the total energy radiated is L. By symmetry the body remains at rest in S after emission (no net momentum emitted).

Now observe the same process from another inertial frame S' moving at speed v relative to S along the x–axis. Quantities measured in S' will be primed.

Energy of a light pulse under a boost

The energy E of a light pulse measured in two frames related by a Lorentz boost along x transforms according to

$$E' = E\,\gamma(1 - \beta\cos\theta), \tag{3}$$

where $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$, and θ is the angle between the light's direction and the x–axis in the unprimed frame S. (For light moving exactly along $+x$, $\cos\theta = 1$; for light moving along $-x$, $\cos\theta = -1$.)

Remark. Equation (3) follows from combining the Lorentz transformation of the energy–momentum four-vector with the fact that for light the magnitude of the spatial momentum equals E/c and its direction is given by θ. The derivation is standard: if a photon has energy E and momentum $\mathbf{p}$ in S, then in S' the energy is $E' = \gamma(E - vp_x)$ and $p_x = (E/c)\cos\theta$, hence (3).

Considering the two opposite pulses

In S the two pulses have energies $E_1 = E_2 = L/2$, traveling in directions $\theta_1 = 0$ (to the right) and $\theta_2 = \pi$ (to the left). Using (3):

$$\begin{aligned} E_1' &= \frac{L}{2}\gamma(1 - \beta \cdot 1) = \frac{L}{2}\gamma(1-\beta), \\ E_2' &= \frac{L}{2}\gamma(1 - \beta \cdot (-1)) = \frac{L}{2}\gamma(1+\beta). \end{aligned}$$

The total energy of the two pulses in S' is

$$E'_{\text{rad}} = E_1' + E_2' = \frac{L}{2}\gamma(1-\beta) + \frac{L}{2}\gamma(1+\beta) = L\gamma. \tag{4}$$

Energy conservation before and after emission in both frames

In frame S (rest frame of the original body):

- Before emission, let the body's rest energy be E_0 (we will interpret this soon). The total energy of the isolated system (body + radiation) before emission is E_0.
- After emission, the body (by symmetry) remains at rest but has lost energy L carried away as radiation, so the body's new energy is $E_0 - L$. The radiation energy (in S) is L.

Thus energy conservation in S is satisfied: before $= E_0$, after $= E_0 - L + L = E_0$.

In frame S' (moving at speed v relative to S):

- Before emission, the whole isolated system has energy E'_{before} which equals the energy of the body as seen in S' (call it $E'_{\text{body, before}}$); there is no radiation yet.
- After emission, the radiation energy measured in S' is $E'_{\text{rad}} = L\gamma$ by (4). The body recoils negligibly (to first approximation for our symmetric emission) and moves with some speed u' in S'; denote its energy after emission by $E'_{\text{body, after}}$. Energy conservation in S' demands

$$E'_{\text{body, before}} = E'_{\text{body, after}} + E'_{\text{rad}}. \tag{5}$$

Relate the body energies to kinetic energy

Assume that the body's total energy in frame S' can be written as the sum of a rest contribution (its rest energy) and a kinetic contribution due to motion in S'. Specifically, before emission the body (which is at rest in S) has total energy in S' equal to

$$E'_{\text{body, before}} = \gamma M c^2, \tag{6}$$

where M denotes the body's mass before emission and we have written γ for the Lorentz factor of the boost between S and S'. After emission the body's mass is $M - \Delta M$ (we allow its mass to change) and its total energy in S' is, to leading order consistent with relativity,

$$E'_{\text{body, after}} = \gamma (M - \Delta M) c^2. \tag{7}$$

Equation (7) assumes the body remains (in S) moving slowly so that in S' its motion is entirely accounted for by the same Lorentz factor; the symmetric emission causes only negligible recoil in S and hence small changes in the body's velocity in S' that can be ignored to first order. (Einstein's original derivation linearizes in v; the algebra above captures the transformation exactly for the radiation energy and assumes negligible recoil so mass change is the main effect.)

Combine energy conservation in S'

Plug (6), (7), and (4) into (5):

$$\gamma M c^2 = \gamma (M - \Delta M) c^2 + L\gamma.$$

Divide both sides by γ (nonzero):

$$M c^2 = (M - \Delta M) c^2 + L.$$

Cancel the Mc^2 terms on left and right, leaving

$$0 = -\Delta M\, c^2 + L,$$

so

$$\Delta M = \frac{L}{c^2}. \tag{8}$$

This is the striking conclusion: when the body loses energy L (radiated away as light), its mass decreases by L/c^2.

5 Nuclear Physics

[57] **Problem 8.** *(56 or 57 points total)*

We've all heard the saying, "What's heavier: one kilogram of lead or one kilogram of feathers?"

But since this is the nuclear section, let's upgrade the comparison: one kilogram of uranium dioxide versus one kilogram of feathers.

Although their masses are identical, their potential energies couldn't be more different. In this section, you'll explore just how much energy is stored in nuclear fuel by working through the classic "one-kilogram" puzzle — but this time, we'll be measuring it in energy, not weight.

(a) *(2 points)* Estimation: Let us define a unit of energy u_E as the following, given C is carbon, H is hydrogen, and O is oxygen:

$$C + O_2 - > CO_2 + u_E \tag{9}$$

$$CH_4 + 2O_2 - > CO_2 + 2H_2O + 2u_E \tag{10}$$

Given the definition for this 'energy unit', estimate how many energy units are released in one fission reaction? *Please submit your estimation before moving on to the next part.*

(b) *(9 points)* Uranium fission involves neutrons and uranium-235. There are a lot of really interesting phenomena that occur here which if I was to add, this section would take maybe 6 or 7 chapters worth of information. Instead, I will give some specifics for a specific reaction pathway.

U represents uranium, n represents a neutron, and Kr and Ba are reaction products. The mass numbers are important for knowing fissile properties, but keep them as an identity for now.

Exact atomic masses:

$$^{235}U = 235.044amu \tag{11}$$

$$^{92}Kr = 91.926amu \tag{12}$$

$$^{141}Ba = 140.914amu \tag{13}$$

$$n = 1.009amu \tag{14}$$

The reaction is as follows:

$$^{235}U + n - >^{92} Kr +^{141} Ba + 3n \tag{15}$$

Use $E = mc^2$ to find energy, where c is the speed of light. Your unit will be in $amu * (m/s)^2$. To convert to electron volts, divide by 10^8. Assume $c = 3 * 10^8$.

(c) : *(9 points)*: Have the shortest and the tallest person in your group nearly run into each other and take a photo. This is to symbolize a neutron colliding into a uranium atom.

(d) : *(9 points)*: Find locker number 2351, as it represents fission between ^{235}U and a neutron, and take a photo of it with a group member.

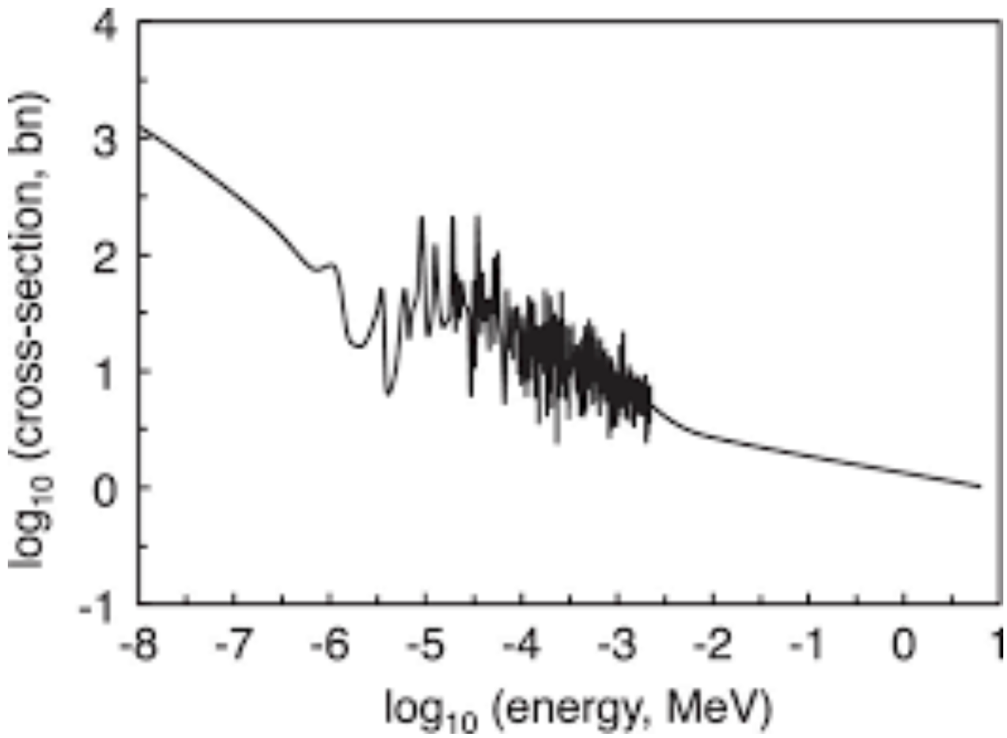

Figure 1: The energy vs cross section of fissile uranium on a log scale.

(e) : *(6 or 7 points, we didnt decide yet)*: The graph above shows an effective cross section of the fissile uranium atoms. This essentially means the area that neutrons have to collide with the atom. The x-axis represents the energy level of the atom, which correlates with velocity. There is a general trend downwards as velocities are higher, but there is a regime in the middle with various spikes. What might be the cause for those spikes (think oscillations/waves)?

(f) : *(9 points)*: A formula exists to find out the rate of nuclear reactions. This formula is not mathematically complex, but it involves knowing exactly what happens to start a fission reaction:

Fission requires neutrons that collide with ^{235}U atoms. This would involve a product between the neutron side (neutron flux through the reaction), and the uranium side (macroscopic 'cross section' as described above).

The neutron flux ϕ can be expressed as the product of the neutron density n and their velocity v, $\phi = nv$. Similarly, the macroscopic cross section Σ is the product of the microscopic cross section σ and the nuclide density N, $\Sigma = \sigma N$. Hence, for a constant volume,

$$R_{fission} = \Sigma\phi = \sigma N n v \tag{16}$$

In this case, Neutron flux $= 5 * 10^18$, while Nuclide density is $10^{27}\ {}^{235}U/m^3$ where ^{235}U is number of Uranium-235 atoms. If the effective cross section is 200 barns (1 barn $= 10^{-28}m^2$), how many reactions per second occur a cubic meter?

(g) : *(9 points)*: The density of UO_2 is about 11,000 kg/m^3. Find the energy released by doing nuclear fission on 1 kg of the UO_2 every second: note that In every reaction, 2 out of 3 neutrons are absorbed, so you would have to subtract $2 * 2 = 4MeV$ from your number from part A and convert to joules. $1eV = 1.6 * 10^{-19}J$. This will give you the power output of 1kg in watts.

The nuclear fuel will burn for about $15,000,000$ seconds. If we were to burn the feathers for that long, the wattage of the feathers will be **only 1 watt**.

(h) : *(3 points)*: Fission occurs at medium energy levels. Knowing the formula for reaction rates, are the spikes beneficial for fission?

Solution to SR Problem (brief)

(a) $v = 0.400c$ so $v^2/c^2 = (0.400)^2 = 0.1600$. Thus

$$\gamma = \frac{1}{\sqrt{1 - 0.1600}} = \frac{1}{\sqrt{0.8400}} \approx 1.0905.$$

(b) $K = (\gamma - 1)mc^2 = (1.0905 - 1)(2.00 \times 10^3 \text{ kg})(9.00 \times 10^{16} \text{ m}^2\,\text{s}^{-2})$

$$\begin{aligned} K &= 0.0905 \times 2.00 \times 10^3 \times 9.00 \times 10^{16} \text{ J} \\ &\approx 1.63 \times 10^{19} \text{ J.} \end{aligned}$$

(c) $\Delta m = K/c^2 = 1.63 \times 10^{19} \ / \ 9.00 \times 10^{16}$ kg ≈ 181 kg. Interpretation: the kinetic energy is equivalent to an increase in the system's inertial energy equal to the mass $\approx 181\,\text{kg}$ via $E = mc^2$.
(d) For $v = 0.100c$ one finds $\gamma = 1/\sqrt{1 - (0.1)^2} = 1/\sqrt{0.99} \approx 1.00504$, so $K = (0.00504)(2.00 \times 10^3)(9.00 \times 10^{16}) \approx 9.07 \times 10^{17}$ J and $\Delta m \approx 10.1$ kg. Ratio $\Delta m_{0.400c}/\Delta m_{0.100c} \approx 181/10.1 \approx 17.9$.

Figure 2: The roughly traced trajectory of your sprint.