

# Simultaneously Learning Contact and Continuous Dynamics

Bibit Bianchini and Michael Posa

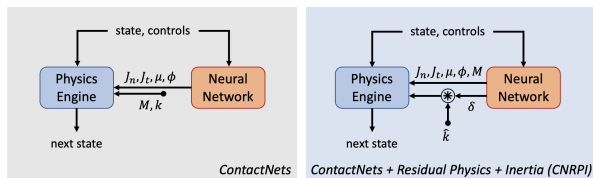


Fig. 1. ContactNets [4] (left) and our proposed amendment (right) for performing dynamics predictions of an object/system whose parameters are learned implicitly.

## I. INTRODUCTION

Model-predictive control (MPC) is increasingly used for controlling complex robotic systems in contact-rich scenarios [1], since models generalize across tasks and can be significantly more data efficient than their model-free alternatives [2]. Constructing models for novel objects on the fly can enable robots to gain the benefits of MPC. The model construction process can be challenging, particularly for contact-rich tasks where dynamics are discontinuous [3]. It is the aim of this work to build off recent work demonstrating success at model building through frictional contact [4], eliminating limiting assumptions preventing its use with entirely novel objects.

This goal requires simultaneously learning the contact dynamics and continuous dynamics of an unknown system. While the smoothness of continuous dynamics simplifies relevant parameter estimation [5], inherently stiff or discontinuous contact dynamics are difficult to model explicitly [6]. Thus implicit approaches are increasingly common to represent these sharp functions [7], [8], [9]. Aiming to accomplish useful tasks with novel objects, we push the bounds on how much we can simultaneously learn.

## II. MATHEMATICAL FORMULATION

The discretized linear complementarity problem (LCP) formulation of a rigid body undergoing frictional contact

Bibit Bianchini and Michael Posa are with the Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, PA 19104, {bibit, posa}@seas.upenn.edu

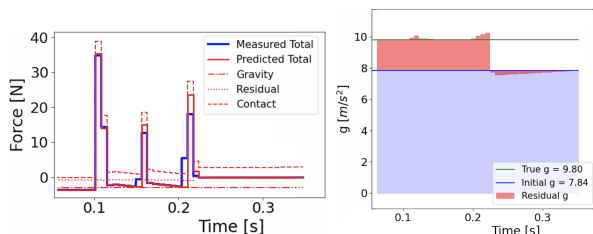


Fig. 2. Force breakdown and estimation of gravity throughout a cube toss trajectory as it falls then strikes, slides against, and comes to rest on a flat ground. The system begins with an incomplete continuous dynamics model amended by a learned residual. The residual accurately fills the model-to-reality gap during free-fall and tumbling. At rest, the residual goes to zero, as the continuous dynamics and contact dynamics are not independently observable, and the residual is encouraged to zero by regularization.



Fig. 3. Left: the articulated object in MuJoCo simulation environment. Middle: the fabricated articulated object. Right: an example of an automated toss conducted by a Franka Panda robotic arm.

[10] inspires our setup.<sup>1</sup> With this model, a system’s inertia, geometry, frictional behaviors, and continuous dynamics are parameterized by  $\{M, J_n, J_t, \phi, \mu, k\}$ . Given initial conditions  $\{q, v\}$ , this LCP formulation determines the contact forces  $\{\lambda_n, \lambda_t\}$  such that contact assumptions like rigidity, inelasticity, and Coulomb friction hold. In this way, the dynamics of a system are implicitly determined via its physical properties.

Through observations of a novel object’s dynamics through a contact-rich trajectory, our methods can implicitly learn these parameters. Prior work demonstrates a physics-inspired learning pipeline with a novel loss that implicitly determines geometry and friction ( $J_n, J_t, \phi, \mu$ ) through contact-rich trajectories, outperforming end-to-end methods in dynamics predictions [4]. Our work explores removing the remaining assumptions ( $M, k$ ) while still distinguishing continuous dynamics from contact-related effects. While all the parameters together are impossible to entirely disambiguate without active exploration (Fig. 2), these conflations can be inconsequential if predictive ability remains strong. Thus, we analyze individual dynamics parameters (Figs. 2 and 5) purely for development insight, emphasizing our end goal is high performance on dynamics prediction metrics (Fig. 4).

### A. Learning Continuous Dynamics

Prior works demonstrate high sample efficiency and performance by combining analytical and learned approaches, via differentiable physics [11], [12], [13], embedding neural networks in dynamics prediction blocks [14], [15], or residual physics [16], [17], [18], [19]. We take a residual physics approach for the continuous dynamics, illustrated in Fig. 1.

### B. Learning Inertia

We learn inertia as a collection of parameters associated with each rigid body. There are methods to learn these smoothly [20]. We also attempted other approaches such as learning an upper-triangular matrix  $A$  such that  $M = A^T A$ , or learning directly the mass, center of mass location, and moments of inertia.

<sup>1</sup>Notation used throughout this document:  $M$ : inertia matrix;  $q$ : configuration;  $v$ : velocity;  $J_n$  and  $J_t$ : normal and tangential contact Jacobians;  $\lambda_n$  and  $\lambda_t$ : normal and tangential contact impulses;  $k$ : continuous forces;  $\phi$ : signed distances;  $\mu$ : friction coefficients for each contact.

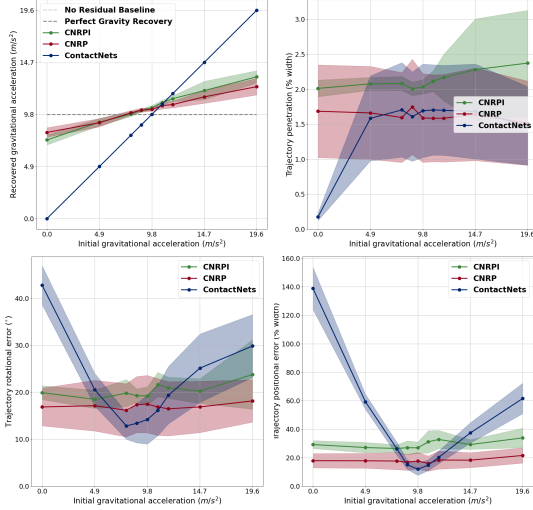


Fig. 4. Top left: the different models’ abilities to recover gravity as the main continuous dynamics contributor (the ContactNets data is superimposed on the No Residual Baseline curve). Top right: predicted penetration over a toss rollout prediction after the models were trained. Bottom left and right: rotational and positional errors, respectively, of the predicted toss rollout prediction after training. All results are with real data; shaded regions indicate 95% confidence intervals.

### III. EXPERIMENTS AND RESULTS

We begin with results on the cube toss dataset from ContactNets [4]: a cube with AprilTags is tossed manually and tracked by three cameras using TagSLAM [21]. Since the cube is a single rigid body with symmetry, its inertia matrix is constant, and the continuous force dynamics  $k(q, v)$  are dominated by gravity, also constant in the system’s state space. As comparison to the real data, we additionally generated simulation data via the LCP formulation [10].

We continue with a 2-link object with a rotary joint and no joint limits. The articulation introduces state dependency for both the inertia and continuous dynamics. The results in this report use MuJoCo simulation data [22], though we automated real data collection using a Franka Panda 7-DOF robotic arm for future analysis (see Fig. 3). While we assume the kinematics of this articulation are known, recent works demonstrate the ability to recognize and accurately model unknown articulation modes [23], [24].

**Cube:** We compare the performance of (C)ontact(N)ets, which cannot amend the inertia and continuous dynamics models it is given, against versions that add a (R)esidual (P)hysics (CNRP) contribution to the continuous dynamics, and additionally learning the (I)ntertia (CNRPI). To test the residual’s ability to correct for a poor initial continuous dynamics model,  $\hat{k}(q, v)$ , we provided varying contributions of gravity (varying over the  $x$ -axis in Fig. 4) and evaluated the system performance after training. The ContactNets curves provide a direct comparison when all methods start with a reasonable continuous dynamics model ( $x = 9.8$  in the plots in Fig. 4) and a reference for lost performance when beginning with faulty initial gravity models.

Without a residual, ContactNets cannot recover from a poor initial model. CNRP and CNRPI recover at least 75% of the true gravitational acceleration, even at the extremes of required residual size. CNRP slightly lags ContactNets

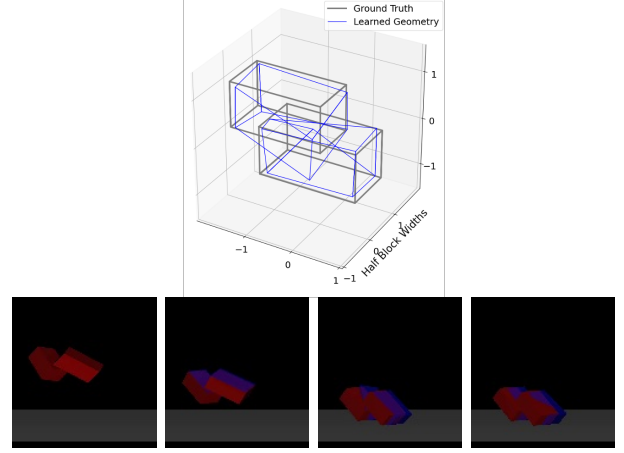


Fig. 5. Top: the converged geometric representation of the articulated system after training. Bottom left to right: the true simulated (red) articulated object trajectory overlaid with the predicted (blue) trajectory after learning the full continuous dynamics.

with the perfect continuous dynamics assumption ( $x = 9.8$  in Fig. 4) in performance, with a difference of just  $3^\circ$  and 6% in rotational and positional error, respectively (bottom row in Fig. 4). CNRPI nearly matches the CNRP performance with only slight degradation in its predictive ability. Unlike ContactNets, both maintain performance with little variation across all tested  $\hat{k}(q, v)$ . Interpenetration (top right in Fig. 4) is within the sensing noise floor of our TagSLAM tracking for all models with real data. ContactNets sees notably low penetration for  $\hat{k}(q, v) = 0$ , a result of the block predictions drifting away into freespace without gravity.

**Articulated Object:** Compared to the cube, all of the articulated object’s physical properties are more complicated. Geometrically, the two rectangular bodies contain some vertices which rarely contact the table, due to the other link getting in the way. While we see reasonable recovery of the convex hull of the object across articulation angles (top in Fig. 5), those interior corners are not observable in most of the collected data and thus lie randomly within this hull with insignificant effect on the predicted rollouts after training.

We observe in many cases that the continuous dynamics can be learned qualitatively well, at the same time as the contact dynamics. An example toss in Fig. 5 (bottom) shows a trajectory rollout prediction after training, when the model began with an initial belief of  $\hat{k}(q, v) = 0$ . Preliminary observations on learning the inertial parameters show a small basin of attraction, and thus we intend to explore what may assist this challenging learning problem.

### IV. CONCLUSION

With promising results on our single rigid body experiments and with an articulated object dataset ready for use, we are confident we can simultaneously learn contact dynamics and continuous dynamics for an unknown object of interest. Building such a model will enable us to perform tasks using MPC for future demonstrations.

### ACKNOWLEDGMENTS

This work was supported by a National Defense Science and Engineering Graduate Fellowship. Toyota Research Institute also provided funds to support this work.

## REFERENCES

- [1] A. Aydinoglu and M. Posa, “Real-Time Multi-Contact Model Predictive Control via ADMM,” *Accepted to the IEEE International Conference on Robotics and Automation (ICRA)*, Sep. 2022.
- [2] A. Gupta, J. Yu, T. Z. Zhao, V. Kumar, A. Rovinsky, K. Xu, T. Devlin, and S. Levine, “Reset-free reinforcement learning via multi-task learning: Learning dexterous manipulation behaviors without human intervention,” *arXiv preprint arXiv:2104.11203*, 2021.
- [3] M. Parmar, M. Halm, and M. Posa, “Fundamental challenges in deep learning for stiff contact dynamics,” in *2021 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2021, pp. 5181–5188.
- [4] S. Pfrommer\*, M. Halm\*, and M. Posa, “ContactNets: Learning of Discontinuous Contact Dynamics with Smooth, Implicit Representations,” in *The Conference on Robot Learning (CoRL)*, 2020. [Online]. Available: <https://proceedings.mlr.press/v155/pfrommer21a.html>
- [5] N. Fazeli, R. Kolbert, R. Tedrake, and A. Rodriguez, “Parameter and contact force estimation of planar rigid-bodies undergoing frictional contact,” *The International Journal of Robotics Research*, vol. 36, no. 13-14, pp. 1437–1454, 2017.
- [6] B. Bianchini, M. Halm, N. Matni, and M. Posa, “Generalization Bounded Implicit Learning of Nearly Discontinuous Functions,” *4th Annual Learning for Dynamics and Control Conference (LADC)*, Jun. 2022.
- [7] P. Florence, C. Lynch, A. Zeng, O. A. Ramirez, A. Wahid, L. Downs, A. Wong, J. Lee, I. Mordatch, and J. Tompson, “Implicit behavioral cloning,” in *Conference on Robot Learning*. PMLR, 2022, pp. 158–168.
- [8] N. Fazeli, S. Zapolsky, E. Drumwright, and A. Rodriguez, “Learning data-efficient rigid-body contact models: Case study of planar impact,” in *Conference on Robot Learning*. PMLR, 2017, pp. 388–397.
- [9] B. Amos and J. Z. Kolter, “Optnet: Differentiable optimization as a layer in neural networks,” in *International Conference on Machine Learning*. PMLR, 2017, pp. 136–145.
- [10] D. E. Stewart and J. C. Trinkle, “An implicit time-stepping scheme for rigid body dynamics with inelastic collisions and coulomb friction,” *International Journal for Numerical Methods in Engineering*, vol. 39, no. 15, pp. 2673–2691, 1996.
- [11] C. Song and A. Boularias, “Learning to slide unknown objects with differentiable physics simulations,” *arXiv preprint arXiv:2005.05456*, 2020.
- [12] M. Geilinger, D. Hahn, J. Zehnder, M. Bächer, B. Thomaszewski, and S. Coros, “Add: Analytically differentiable dynamics for multi-body systems with frictional contact,” *ACM Transactions on Graphics (TOG)*, vol. 39, no. 6, pp. 1–15, 2020.
- [13] F. de Avila Belbute-Peres, K. Smith, K. Allen, J. Tenenbaum, and J. Z. Kolter, “End-to-end differentiable physics for learning and control,” *Advances in neural information processing systems*, vol. 31, pp. 7178–7189, 2018.
- [14] E. Heiden, D. Millard, E. Coumans, Y. Sheng, and G. S. Sukhatme, “Neuralsim: Augmenting differentiable simulators with neural networks,” *arXiv preprint arXiv:2011.04217*, 2020.
- [15] P. W. Battaglia, R. Pascanu, M. Lai, D. Rezende, and K. Kavukcuoglu, “Interaction networks for learning about objects, relations and physics,” *arXiv preprint arXiv:1612.00222*, 2016.
- [16] A. Zeng, S. Song, J. Lee, A. Rodriguez, and T. Funkhouser, “Tossing-bot: Learning to throw arbitrary objects with residual physics,” 2019.
- [17] A. Ajay, J. Wu, N. Fazeli, M. Bauza, L. P. Kaelbling, J. B. Tenenbaum, and A. Rodriguez, “Augmenting physical simulators with stochastic neural networks: Case study of planar pushing and bouncing,” in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018, pp. 3066–3073.
- [18] C. Olah, “Understanding lstm networks,” Aug 2015. [Online]. Available: <https://colah.github.io/posts/2015-08-Understanding-LSTMs/>
- [19] P. Abbeel, M. Quigley, and A. Y. Ng, “Using inaccurate models in reinforcement learning,” in *Proceedings of the 23rd international conference on Machine learning*, 2006, pp. 1–8.
- [20] C. Rucker and P. M. Wensing, “Smooth parameterization of rigid body inertia,” *IEEE Robotics and Automation Letters*, 2022.
- [21] B. Pfrommer and K. Daniilidis, “Tagslam: Robust slam with fiducial markers,” *arXiv preprint arXiv:1910.00679*, 2019.
- [22] E. Todorov, T. Erez, and Y. Tassa, “Mujoco: A physics engine for model-based control,” in *2012 IEEE/RSJ international conference on intelligent robots and systems*. IEEE, 2012, pp. 5026–5033.
- [23] E. Heiden, Z. Liu, V. Vineet, E. Coumans, and G. S. Sukhatme, “Inferring articulated rigid body dynamics from rgbd video,” *arXiv preprint arXiv:2203.10488*, 2022.
- [24] Z. Jiang, C.-C. Hsu, and Y. Zhu, “Ditto: Building digital twins of articulated objects from interaction,” *arXiv preprint arXiv:2202.08227*, 2022.