**Generating Dimensional Formulae and Checking for Dimensional Consistency.**

Definition of dimensionless products (Langhaar, 1951).

A set of dimensionless products of given variables is *complete*, if each product in the set is independent of the others, and every other dimensionless product of the variables is a product of powers of dimensionless products in the set. By convention the symbol pi is used to denote a dimensionless product, it is not related to the number 3.1416.

Buckingham’s theorem (Buckingham, 1914)

If an equation is dimensionally homogeneous, it can be reduced to a relationship among a complete set of dimensionless products.

Definition of rank of matrix.

If a matrix contains a nonzero determinant of order r, and if all determinants of order greater than r that the matrix contains have the value zero, the rank of the matrix is said to be r.

Theorem for number of dimensionless products.

The number of dimensionless products in a complete set is equal to the total number of variables minus the rank of their dimensional matrix. (proof ch4).

The theorem for number of dimensionless products is also stated as (Van Driest, 1946)

The number of dimensionless products in a complete set is equal to the total number of variables minus the maximum number of these variables that will not form a dimensionless product.

Theorem of linear dependence.

The rows of a matrix are linearly dependent if, and only if, the rank of the matrix is less than the number of rows.

Theorem of linear algebra.

Disregarding the trivial solution, ki = 0, the system of homogeneous equation possesses exactly (n – r) linearly independent solutions, in which r is the rank of the matrix of the coefficients. A set of (n – r) linearly independent solutions is called a fundamental system of solutions. Any solution is a linear combination of the solutions in any fundamental system.

Rule of thumb in choosing which variables are included or excluded in pi expressions.

Since the first (n – r) variables in the dimensional matrix each occur in only one dimensionless product, in the dimensional matrix, le the first variable be the dependent variable, the second variable be that which is easiest to regulate experimentally, and the third variable be that which is next easiest to regulate experimentally, and so on.

Let us imagine we derived an expression given below.

Before we begin the dimensional analysis using diman© we must first do some preliminary setting up.

**Definitions setup.**

We set the definitions for the symbols in the expression.

|  |  |  |
| --- | --- | --- |
|  | (**def** varpars [{*:symbol* **"x"**, *:dimension* **"length"**}  {*:symbol* **"v"**, *:dimension* **"velocity"**}  {*:symbol* **"t"**, *:dimension* **"time"**}  {*:symbol* **"a"**, *:dimension* **"acceleration"**}]) |  |

**Expressions and equation setup.**

We then define the equation whose left- and right-hand sides are based on their defined expressions.

|  |  |  |
| --- | --- | --- |
|  | (**def** leftside **"x^(1)"**) (**def** rightside {*:term1* **"x^(1)"**,  *:term2* **"v^(2)"**,  *:term3* **"t^(1)"**,  *:term4* **"0.5\*a^(1)\*t^(2)"**}) (**def** equation {*:lhs* leftside, *:rhs* rightside}) |  |

**Importing functions from diman©**

To do our analysis we must import function required for it.

|  |  |  |
| --- | --- | --- |
|  | (**require** '[diman.formula *:refer* [formula-term formula-eqn-side]])  (**require** '[diman.filter *:refer* [remove-zero-powers]]) (**require** '[diman.analyze *:refer* [dimnames consistent?]]) |  |

**Getting dimensional formula.**

***Sub-formula of the dimensional formula for one side of the equation.***

A sub-formula is practically a dimensional formula for one of the terms on a chosen side (left- or right-hand sides) of the equation. This is because the sub-formula **IS** the dimensional formula for the expression if there is just one term.

Based on our definition we setup we know that the right-hand side of the given equation is

|  |
| --- |
| => (:rhs equation)  {:term1 "x^(1)", :term2 "v^(2)", :term3 "t^(1)", :term4 "0.5\*a^(1)\*t^(2)"} |

Say, we are interested in viewing the dimensional formula for the :term4 expression in the :rhs of the equation. Then using the formula-term function and passing our expression of interest as its argument we get

|  |
| --- |
| => (formula-term varpars (:term4 (:rhs equation)))  "[T^(0)\*L^(1)]" |

Notice that this is consistent with the composite unit of the dimensions in the expression.

where is the unit for acceleration.

The base quantities with zero exponents can be removed by using the remove-zero-powers function.

|  |
| --- |
| => (remove-zero-powers (formula-term varpars (:term4 (:rhs equation))))  "[L^(1)]" |

***Dimensional formula for one side of the equation.***

Similarly, the dimensional formula for a side of the equation can be derived. However, for this we use the formula-eqn-side function.

|  |
| --- |
| => (formula-eqn-side varpars (:rhs equation))  "[L^(1)] + [T^(-2)\*L^(2)] + [T^(1)] + [T^(0)\*L^(1)]" |

As was in the case shown above the base quantities with zero exponents can be removed using the remove-zero-powers function.

|  |
| --- |
| => (remove-zero-powers (formula-eqn-side varpars (:rhs equation)))  "[L^(1)] + [T^(-2)\*L^(2)] + [T^(1)] + [L^(1)]" |

***View dimensional names in the derived formula.***

Using the dimnames function the notations for the base quantities in the formula can be reflected in terms of their dimensional names.

For the sub-formula of the fourth term in the right-hand side of the equation this is

|  |
| --- |
| => (dimnames (formula-term varpars (:term4 (:rhs equation))))  "length^(1)" |

For the formula of the right-hand side is

|  |
| --- |
| => (dimnames (formula-eqn-side varpars (:rhs equation)))  "length^(1) + time^(-2)\*length^(2) + time^(1) + length^(1)" |

**Analyze.**

***Consistency check.***

If the correctness of an equations is in doubt checking for dimensional consistency is a useful preliminary step. In diman© this is done using the consistent? function.

Thus,

|  |
| --- |
| => (consistent? varpars equation)  false |

Notice that this is consistent when we view the composite unit of the dimensions in the

That is, the equation *fails* the consistency check. Thus,

An equation that is not dimensionally consistent **must be wrong**.

What about the equation ?

We define this second equation as

|  |  |  |
| --- | --- | --- |
|  | (**def** equation2 {*:lhs* **"x^(1)"**, *:rhs* {*:term1* **"x^(1)"**,  *:term2* **"v^(1)\*t^(1)"**,  *:term3* **"0.5\*a^(1)\*t^(2)"**}}) |  |

And, consistent? function on this equation returns

=> (consistent? varpars equation2)

true

This agrees with

Thus, equation *passes* the consistency check.

Consistency checking in dimensional analysis although useful it is still a preliminary step in the analysis because

A dimensionally consistent equation **does not guarantee** correct equation.

Let us illustrate this with multiple equations.

|  |  |
| --- | --- |
| **Equation** | **Definition setup** |
|  | (**def** eqn1 {*:lhs* **"e^(1)"**,  *:rhs* **"m^(2)\*v^(2)"**}) |
|  | (**def** eqn2 {*:lhs* **"e^(1)"**,  *:rhs* **"0.5\*m^(1)\*v^(2)"**}) |
|  | (**def** eqn3 {*:lhs* **"e^(1)"**,  *:rhs* **"m^(1)\*a^(1)"**}) |
|  | (**def** eqn4 {*:lhs* **"e^(1)"**,  *:rhs* **"0.1875\*m^(1)\*v^(2)"**}) |
|  | (**def** eqn5 {*:lhs* **"e^(1)"**,  *:rhs* {*:term1* **"0.5\*m^(1)\*v^(2)"**,  *:term2* **"m^(1)\*a^(1)"**}}) |

The defined variables/parameters used in the equations are

|  |  |  |
| --- | --- | --- |
|  | (**def** varpars [{*:symbol* **"e"**, *:dimension* **"energy"**}  {*:symbol* **"m"**, *:dimension* **"mass"**}  {*:symbol* **"v"**, *:dimension* **" velocity "**}  {*:symbol* **"a"**, *:dimension* **"acceleration"**}]) |  |

Then, running the consistency check using the consistent? function we get

=> (consistent? varpars eqn1)

false

=> (consistent? varpars eqn2)

true

=> (consistent? varpars eqn3)

false

=> (consistent? varpars eqn4)

true

=> (consistent? varpars eqn5)

false

Thus, and are dimensionally correct equations. But which of these two is the actual “correct” equation? Consistency checking cannot answer this. For this particular example, referring to the definition of kinetic energy we know that is the correct equation.

**Standardizing a formula.**

What if the user has determined the correct equation and would like to perform dimensional analysis on another equation such that it has expressions that incorporate the correct equation? Can the user avoid again deriving the dimensional formula for the equation he/she knows is correct? diman© provides the flexibility to insert the previously determined correct equation into standard\_formula. This is a collection of predefined formulae available in diman©.

After importing this predefined collection with

|  |  |
| --- | --- |
|  | (**require** '[diman.dimensions *:refer* [standard\_formula]]) |

the predefined formulae in standard\_formula can be viewed with

|  |
| --- |
| => (pprint standard\_formula)  [{:quantity "volume", :sformula "[M^(0)\*L^(3)\*T^(0)]"}  {:quantity "velocity", :sformula "[M^(0)\*L^(1)\*T^(-1)]"}  {:quantity "acceleration", :sformula "[M^(0)\*L^(1)\*T^(-2)]"}  {:quantity "force", :sformula "[M^(1)\*L^(1)\*T^(-2)]"}  {:quantity "mass density", :sformula "[M^(1)\*L^(-3)\*T^(0)]"}] |

Let us illustrate how the user can extend this collection of predefined formulae using as our correct equation. Since we know that the correct dimensional formula for kinetic energy is

|  |
| --- |
| => (formula-eqn-side varpars (:rhs eqn2))  "[M^(1)\*T^(-2)\*L^(2)]" |

our objective is to insert [M^(1)\*T^(-2)\*L^(2)] into the standard\_formula.

To do this we first create a new collection.

|  |  |  |
| --- | --- | --- |
|  | (**def** updated\_sform  (**conj** standard\_formula  {*:quantity* **"energy"**,  *:sformula* **"[M^(1)\*T(-2)\*L^(2)]"**})) |  |

The newly defined collection however needs to replace the standard\_formula. This can be done with

|  |  |  |
| --- | --- | --- |
|  | (**intern** 'diman.dimensions 'standard\_formula updated\_sform) |  |

One can check the updated standard\_formula with

|  |
| --- |
| => (pprint standard\_formula)  [{:quantity "volume", :sformula "[M^(0)\*L^(3)\*T^(0)]"}  {:quantity "velocity", :sformula "[M^(0)\*L^(1)\*T^(-1)]"}  {:quantity "acceleration", :sformula "[M^(0)\*L^(1)\*T^(-2)]"}  {:quantity "force", :sformula "[M^(1)\*L^(1)\*T^(-2)]"}  {:quantity "mass density", :sformula "[M^(1)\*L^(-3)\*T^(0)]"}  {:quantity "energy", :sformula "[M^(1) \*T(-2)\*L^(2)]"}] |

Furthermore, let say the variables/parameters in the equation were already defined prior to the update. Then this definition can be updated with

|  |  |  |
| --- | --- | --- |
|  | (**def** varpars (**conj** varpars {*:symbol* **"e"**, *:dimension* **"energy"**})) |  |

Thus,

|  |
| --- |
| => (pprint varpars)  [{:symbol "m", :dimension "mass"}  {:symbol "v", :dimension "velocity"}  {:symbol "a", :dimension "acceleration"}  {:symbol "e", :dimension "energy"}] |

**Bibliography**

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