

# A perspective on Neuro-symbolic Approaches for Knowledge Graph Refinement and Explanation

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1

## Applications

- e-Commerce
- Semantic Search
- Fact Checking
- Personalization
- Recommendation
- Medical decision support system
- Question Answering
- Machine Translation
- ...

## Research Fields

- Information Extraction
- Natural Language Processing
- Machine Learning (ML)
- Knowledge Representation
- Web
- Robotics
- ...

<sup>1</sup> picture from <https://www.csee.umbc.edu/courses/graduate/691/fall19/07/>

# ML and KGs

Two perspectives:

## KG as input to ML

**Goal:** improving the performance in many learning tasks, e.g.

- Question Answering (QA)
- image classification
- instance disambiguation
- text summarization
- ....

## ML as input to KG

**Goal:** improving the KG itself

- creating new facts
- creating generalizations
- prototyping
- improving the size, coverage, depth and accuracy of KGs → reducing their production costs

# ML as input to KG

(KG Refinement: Link Prediction)

(Explanation of Link Predictions on KGs)

(KG Refinement at Schema Level)

ML as input to KG

**(KG Refinement: Link Prediction)**

(Explanation of Link Predictions on KGs)

(KG Refinement at Schema Level)

## Incompleteness and noise



### Knowledge Graph Refinement

- **Link Prediction**: predicts missing links between entities
  - regarded as a **learning to rank** problem
- **Triple Classification**: assesses correctness of a statement wrt a KG
  - regarded as a **binary classification** problem

## Very Large Data Collections



### New scalable Machine Learning methods

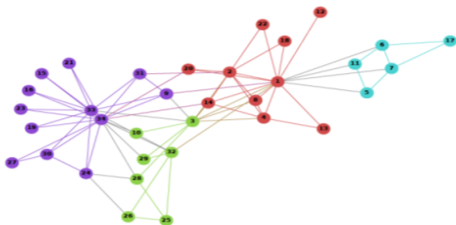
- grounded on **numeric-based approaches**
  - **KG vector embedding models** (KGE) largely investigated [Cai et al., 2018]

### ML/KGE for KGs: Issues

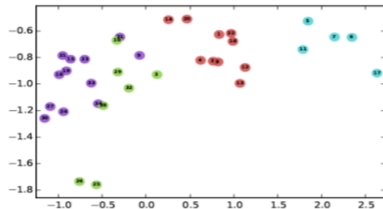
- CWA (or LCWA) mostly adopted vs. OWA
- schema level information and reasoning capabilities almost disregarded
- no interpretable models  $\Rightarrow$  hard to motivate results

# KG Embedding Models...

KGE models convert data graph into an optimal low-dimensional space [Cai *et al.*, 2018]



Input



Output

2

Graph structural information and properties preserved as much as possible

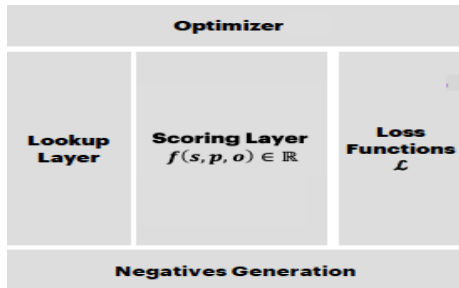
<sup>2</sup>Picture from <https://laptrinhx.com/node2vec-graph-embedding-method-2620064815/>

# ...KG Embedding Models

## Goal

Learning embeddings s.t.

- score of a valid (positive) triple is higher than
- the score of an invalid (negative) triple



Negative examples generated by **random corruption of triples**

- **false negatives** may be generated
- only triple directly observable are considered

<sup>3</sup>Picture from "ECAI-20 Tutorial: Knowledge Graph Embeddings: From Theory to Practice"



# KG Refinement by KG Embedding Methods: Injecting Semantics

# Enhancing KGE by Injecting Background Knowledge

[d'Amato *et al.*, 2021c,b]

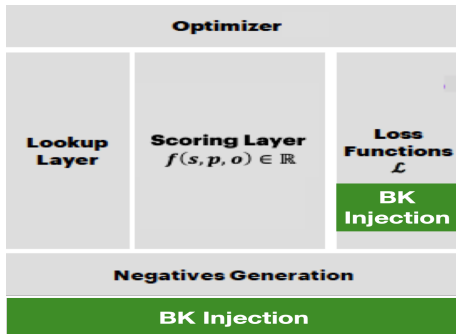
By two components:

**Reasoning:** used for generating negative triples

**Axioms:** domain, range, disjointWith, functionalProperty;

**BK Injection:** defines constraints on functions, corresponding to the considered axioms, guiding the way embedding are learned

**Axioms:** equivClass, equivProperty, inverseOf and subClassOf.



# A First Approach: TransOWL, TransROWL

[d'Amato et al., 2021c]

- Derive further triples to be considered for training via schema axioms
  - equivClass, equivProperty, inverseOf and subClassOf
- More complex loss function
  - adding a number of terms consistently with the constraints

$$\begin{aligned}
 L = & \overbrace{\sum_{\substack{\langle h,r,t \rangle \in \Delta \\ \langle h',r,t' \rangle \in \Delta'}} [\gamma + f_r(h,t) - f_r(h',t')]}_{\text{TransE loss function}} + \sum_{\substack{\langle t,q,h \rangle \in \Delta_{\text{inverseOf}} \\ \langle t',q,h' \rangle \in \Delta'_{\text{inverseOf}}}} [\gamma + f_q(t,h) - f_q(t',h')]_+ \\
 & + \sum_{\substack{\langle h,s,t \rangle \in \Delta_{\text{equivProperty}} \\ \langle h',s,t' \rangle \in \Delta'_{\text{equivProperty}}}} [\gamma + f_s(h,t) - f_s(h',t')]_+ + \sum_{\substack{\langle h,\text{typeOf},l \rangle \in \Delta \cup \Delta_{\text{equivClass}} \\ \langle h',\text{typeOf},l' \rangle \in \Delta' \cup \Delta'_{\text{equivClass}}}} [\gamma + f_{\text{typeOf}}(h,l) - f_{\text{typeOf}}(h',l')]_+ \\
 & + \sum_{\substack{\langle h,\text{subClassOf},p \rangle \in \Delta_{\text{subClass}} \\ \langle h',\text{subClassOf},p' \rangle \in \Delta'_{\text{subClass}}}} [(\gamma - \beta) + f(h,p) - f(h',p')]_+
 \end{aligned}$$

where  $q \equiv r^-$ ,  $s \equiv r$  (properties),  $l \equiv t$  and  $t \sqsubseteq p$  (classes) and  $f(h,p) = \|e_h - e_p\|$

# An Alternative Approach: TransROWL<sup>R</sup>

[d'Amato et al., 2021c]

Adopting an **axiom-based regularization** of the loss function  
as for TransE<sup>R</sup> [Minervini et al., 2017b]

- by adding specific constraints to the loss function rather than
- explicitly derive additional triples during training

## Loss function

$$\begin{aligned}
 L = & \sum_{\substack{\langle h, r, t \rangle \in \Delta \\ \langle h', r', t' \rangle \in \Delta'}} [\gamma + f'_r(h, t) - f'_r(h', t')]_+ \\
 & + \lambda_1 \sum_{r \equiv q^- \in \mathcal{T}_{\text{inverseOf}}} \|r + q\| + \lambda_2 \sum_{r \equiv q^- \in \mathcal{T}_{\text{inverseOf}}} \|M_r - M_q\| \\
 & + \lambda_3 \sum_{r \equiv p \in \mathcal{T}_{\text{equivProp}}} \|r - p\| + \lambda_4 \sum_{r \equiv p \in \mathcal{T}_{\text{equivProp}}} \|M_r - M_p\| \\
 & + \lambda_5 \sum_{e' \equiv e'' \in \mathcal{T}_{\text{equivClass}}} \|e' - e''\| + \lambda_6 \sum_{s' \subseteq s'' \in \mathcal{T}_{\text{subClass}}} \|1 - \beta - (s' - s'')\|
 \end{aligned}$$

Main Outcome: TransROWL **slightly outperforming** TransROWL<sup>R</sup>

# Other KG Embedding Methods Leveraging BK

- Jointly embedding KGs and logical rules [Guo et al., 2016]
  - triples represented as atomic formulae
  - rules represented as complex formulae modeled by t-norm fuzzy logics
- Adversarial training exploiting Datalog clauses encoding assumptions to regularize neural link predictors [Minervini et al., 2017a]

A specific form of BK required, not directly applicable to KGs

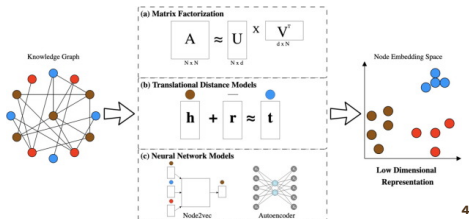
# ML as input to KG

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(KG Refinement at Schema Level)

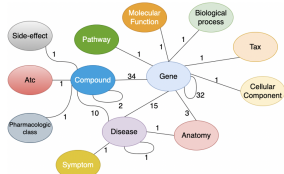
Numeric-based methods consist of series of numbers without any obvious human interpretation



This may affect:

- the **interpretability** of the results
- the **explainability**
- and thus also somehow the **trustworthiness** of results

DRKG – Drug Repurposing Knowledge Graph



5

<sup>4</sup> Picture from D. N. Nicholson et al. Constructing knowledge graphs and their biomedical applications, Computational and Structural Biotechnology Journal, Vol. 18, pp. 1414–1428, (2020) ISSN 2001-0370

<sup>5</sup> Picture from <https://github.com/topics/knowledge-graph-embeddings>

# A-Posteriori Explanations of Link Predictions

**A-posteriori methods:** find suitable explanation(s) based on the observed output and the model input, **independently on the KGE adopted** - Developed solutions

**KELPIE** [Rossi *et al.*, 2022]: generates necessary and sufficient (path) conditions and an articulated new evaluation protocol

**CrossE** [Zhang *et al.*, 2019]: embedding model for link predictions providing explanations

- the **search for a path linking the subject  $h$  and object  $t$  of a predicted triple  $\langle h, r, t \rangle$** 
  - Max lenght 2**  $\rightarrow$  six types of paths possible:  
 Length 1:  $P_1 = \{\langle h, r_s, t \rangle\}$ ,  $P_2 = \{\langle t, r_s, h \rangle\}$   
 Length 2:  $P_3 = \{\langle e', r_s, h \rangle, \langle e', r', t \rangle\}$ ,  $P_4 = \{\langle e', r_s, h \rangle, \langle t, r', e' \rangle\}$ ,  
 $P_5 = \{\langle h, r_s, e' \rangle, \langle e', r', t \rangle\}$ ,  $P_6 = \{\langle h, r_s, e' \rangle, \langle t, r', e' \rangle\}$ ,  
 where  $r_s$  similar to  $r$ ,  $r'$  any other relationship,  $e'$  any other entity;
- search driven by similarities between relation/entity embeddings** by **Euclidean distance**
- structural comparisons** with other paths in the KG to reinforce the reliability of the explanation found (referred to as **support**)



# Explaining Link Predictions with CrossE

Given the predicted triple:  $\langle \text{NickMason}, \text{recordLabel}, \text{CapitolRecords} \rangle$   
 why is it provided?

User is able to understand motivations, and trust (or not) the prediction

## Example of explanation

$\langle \text{NickMason}, \text{associatedBand}, \text{PinkFloyd} \rangle,$   
 $\langle \text{PinkFloyd}, \text{recordLabel}, \text{CapitolRecords} \rangle$

Ideally supported by analogous situations to be found in the KG e.g.

$\langle \text{RingoStarr}, \text{recordLabel}, \text{Parlophone} \rangle$

for which the computed explanation is:

$\langle \text{RingoStarr}, \text{associatedBand}, \text{TheBeatles} \rangle,$   
 $\langle \text{TheBeatles}, \text{recordLabel}, \text{Parlophone} \rangle.$

# Exploiting Semantics for Providing Explanations to Link Predictions on KGs

**Idea:** SemanticCrossE provide semantic-based explanations for link predictions on KGs

[d'Amato et al., 2021a]

Extends CrossE by adopting a Semantic Cosine similarity that leads the explanation process

- exploits the underlying KG semantics  $\rightarrow$  Domain, Range and Classes considered
- increases the cosine similarity of two (entities / relationships) vector embeddings on the ground of available additional semantic information which is captured by a semantic Score function

### Definition (semantic Cosine)

Given KG  $\mathcal{K}(\mathcal{E}, \mathbb{R})$ , the semantic Cosine measure for two entities  $e, e' \in \mathcal{E}$  is defined by:

$$\text{semCos}_{\alpha, \beta}(e, e') = \alpha \cdot \text{sScore}(e, e') + \beta \cdot \text{sim}_{\text{cos}}(e, e')$$

where  $e$  the respective entity embedding vector;  $\alpha, \beta \in [0, 1]$  chosen s.t.  $\alpha + \beta = 1$ .

In the case of relations  $r, r' \in \mathbb{R}$  the measure is defined analogously.

## Definition (semantic Score)

Given  $\mathcal{C}$  set of the classes occurring in  $\mathcal{K}(\mathcal{E}, \mathbb{R})$ , and the functions  $CI: \mathcal{E} \rightarrow \mathcal{C}$ ,  $Do: \mathbb{R} \rightarrow \mathcal{C}$ , and  $Ra: \mathbb{R} \rightarrow \mathcal{C}$  that return, resp., the conjunction of the classes an entity belongs to, and the domain and range of a relation, the **semantic Score** function for pairs of entities  $e, e' \in \mathcal{E}$  is defined by:

$$\text{sScore}(e, e') = \frac{|\text{ret}[CI(e) \sqcap CI(e')]|}{|\text{ret}[CI(e) \sqcup CI(e')]|}$$

where  $\text{ret}_{\mathcal{K}}(\mathcal{C})$  returns the entities that can be proven to belong to a given class  $\mathcal{C}$

Analogously, given any two relationships  $r, r' \in \mathbb{R}$ , it is defined:

$$\text{sScore}(r, r') = \frac{|\text{ret}[Do(r) \sqcap Do(r')]|}{|\text{ret}[Do(r) \sqcup Do(r')]|} + \frac{|\text{ret}[Ra(r) \sqcap Ra(r')]|}{|\text{ret}[Ra(r) \sqcup Ra(r')]|}$$

- **Approximated** form of **semantic Cosine measure** (specifically of the semantic Score function) employed [d'Amato *et al.*, 2021a]
- **Efficient computation** obtained by a **preliminary clustering phase** [d'Amato *et al.*, 2023]

## Main Outcome:

- ApproxSemanticCrossE showed improved results both in terms of recall and support
- ApproxSemanticCrossE not affected by noisy (irrelevant) explanations as for CrossE and partially cosineCrossE → qualitative evaluation conducted

A more efficient solution developed with **KELPIE++** [Barile *et al.*, 2024]

# ML as input to KG

(KG Refinement: Link Prediction)

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**(KG Refinement at Schema Level)**

## Learning Disjointness Axioms

# Missing Disjointness Axioms: Issues

Disjointness axioms often missing [Wang et al., 2006]

Problems:

- introduction of noise

$\mathcal{K} = \{ \text{JournalPaper} \sqsubseteq \text{Paper}, \text{ConferencePaper} \sqsubseteq \text{Paper}, \text{ConferencePaper}(a), \text{Author}(a) \}$

$\mathcal{K}$  is Consistent !!!

**Cause Axiom:**  $\text{Author} \equiv \neg \text{ConferencePaper}$  missing

- counterintuitive inferences

$\mathcal{K} = \{ \text{JournalPaper} \sqsubseteq \text{Paper}, \text{ConferencePaper} \sqsubseteq \text{Paper}, \text{ConferencePaper}(a) \}$

$\mathcal{K} \models \text{JournalPaper}(a)$ ?

**Answer:** Unknown

**Cause Axiom:**  $\text{JournalPaper} \equiv \neg \text{ConferencePaper}$  missing

- hard collecting negative examples when adopting numeric approaches



**Observation:** extensions of disjoint concepts do not overlap

**Question:** would it be possible to **automatically capture** disjointness axioms by analyzing the data configuration/distribution?

**Idea:** Exploiting **(Conceptual) clustering methods** for mining disjointness axioms

[Rizzo *et al.*, 2021]

### Definition (Problem Definition)

Given

- an ontological knowledge base  $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$
- a set of individuals (aka entities)  $I \subseteq \text{Ind}(\mathcal{A})$

Find

- $n$  pairwise disjoint clusters  $\{C_1, \dots, C_n\}$
- for each  $i = 1, \dots, n$ , a concept description  $D_i$  that describes  $C_i$ , such that:
  - $\forall a \in C_i : \mathcal{K} \models D_i(a)$
  - $\forall b \in C_j, j \neq i : \mathcal{K} \models \neg D_i(b).$
- Hence  $\forall D_i, D_j, i \neq j : \mathcal{K} \models D_j \sqsubseteq \neg D_i.$

# Learning Disjointness Axioms: Developed Methods

## Statistical-based approach

- NAR - exploiting negative association rules [Fleischhacker and Völker, 2011]
- PCC - exploiting Pearson's correlation coeff. [Völker *et al.*, 2015]

do not exploit any background knowledge and reasoning capabilities

Disjointness axioms learning/discovery can be hardly performed without symbol-based methods

# Terminological Cluster Tree

Defined a method <sup>6</sup> for eliciting disjointness axioms [Rizzo et al., 2021]

- solving a clustering problem via learning Terminological Cluster Trees
- providing a concept description for each cluster

## Definition (Terminological cluster tree (TCT))

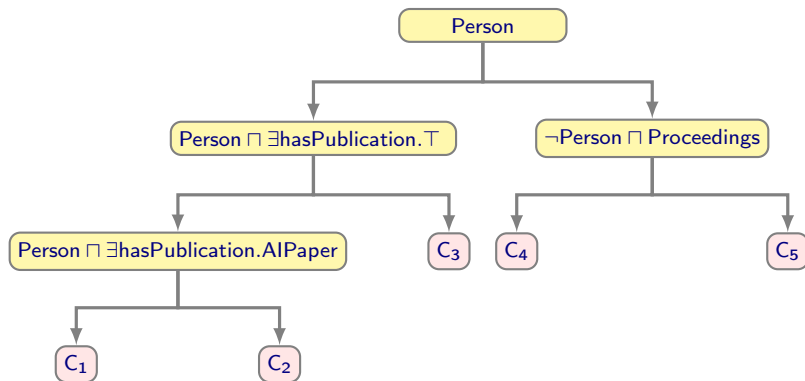
A binary logical tree where

- a leaf node stands for a cluster of individuals  $C$
- each inner node contains a description  $D$  (over the signature of  $\mathcal{K}$ )
- each departing edge corresponds to positive (left) and negative (right) examples of  $D$

<sup>6</sup> Implemented system publicly available at <https://github.com/Giuseppe-Rizzo/TCTnew>

# Example of TCT

Given  $I \subseteq \text{Ind}(\mathcal{A})$ , an example of TCT describing the AI research community



# Collecting Disjointness Axioms

Given a TCT  $T$ :

Step I:

- Traverse the  $T$  to collect the concept descriptions describing the clusters at the leaves
- A set of concepts  $CS$  is obtained

Step II:

- A set of candidate axioms  $A$  is generated from  $CS$ :
  - an axiom  $D \sqsubseteq \neg E$  ( $D, E \in CS$ ) is generated if
    - $D \not\sqsubseteq E$  (or  $D \not\sqsupseteq E$  or viceversa - **reasoner needed**)
    - $E \sqsubseteq \neg D$  has not been generated

# Conclusions

## Conclusions:

- Injecting semantics and exploiting reasoning capabilities may improve the effectiveness of ML solutions for KG
  - Framework for injecting BK into KGE models
  - Solution for injecting semantics when computing a-posteriori explanations to link predictions
- Supplementing schema level information (disjointness axioms) needed

## Next Research Challenges:

- Extend the framework for injecting BK to more complex KGE models
- Learning disjointness axioms from KGE
- Develop a standardized evaluation protocol for evaluating explanations
- New solutions required for enhancing LLMs with KG semantics

Thank you



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# Inducing a TCT

Given the set of individuals  $I$  and  $T$  concept

Divide-and-conquer approach adopted

- **Base Case:** test the **stopCondition**
  - the cohesion of the cluster  $I$  exceeds a threshold  $\nu$ 
    - distance between **medoids** below a threshold  $\nu$
- **Recursive Step** (**stopCondition** does not hold):
  - a set  $S$  of refinements of the current (parent) description  $C$  generated
  - the **bestConcept**  $E^* \in S$  is selected and installed as **current node**
    - the one showing the **best cluster separation**  $\Leftrightarrow$  with max distance between the **medoids** of its positive  $P$  and negative  $N$  individuals
  - $I$  is **split** in:
    - $I_{left} \subseteq I \Leftrightarrow$  individuals with the smallest distance wrt the **medoid** of  $P$
    - $I_{right} \subseteq I \Leftrightarrow$  individuals with the smallest distance wrt the **medoid** of  $N$
    - **reasoner employed** for collecting  $P$  and  $N$

**Note:** Number of clusters not required - obtained from data distribution

# Distance measure between individuals adopted for TCT

Distance Function (adapted from [d'Amato et al.@ESWC2008]):

$$d_n^{\mathcal{C}} : \text{Ind}(\mathcal{A}) \times \text{Ind}(\mathcal{A}) \rightarrow [0, 1]$$

$$d_n^{\mathcal{C}}(a, b) = \left[ \sum_{i=1}^m w_i [1 - \pi_i(a)\pi_i(b)]^n \right]^{1/n}$$

Context: a set of atomic concepts  $\mathcal{C} = \{B_1, B_2, \dots, B_m\}$

Projection Function:

$$\forall a \in \text{Ind}(\mathcal{A}) \quad \pi_i(a) = \begin{cases} 1 & \text{if } \mathcal{K} \models B_i(a) \\ 0 & \text{if } \mathcal{K} \models \neg B_i(a) \\ 0.5 & \text{otherwise} \end{cases}$$