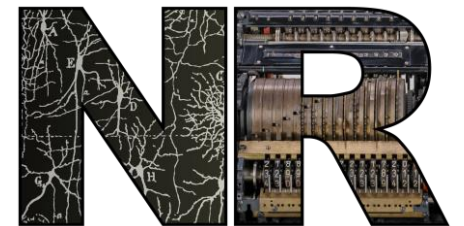
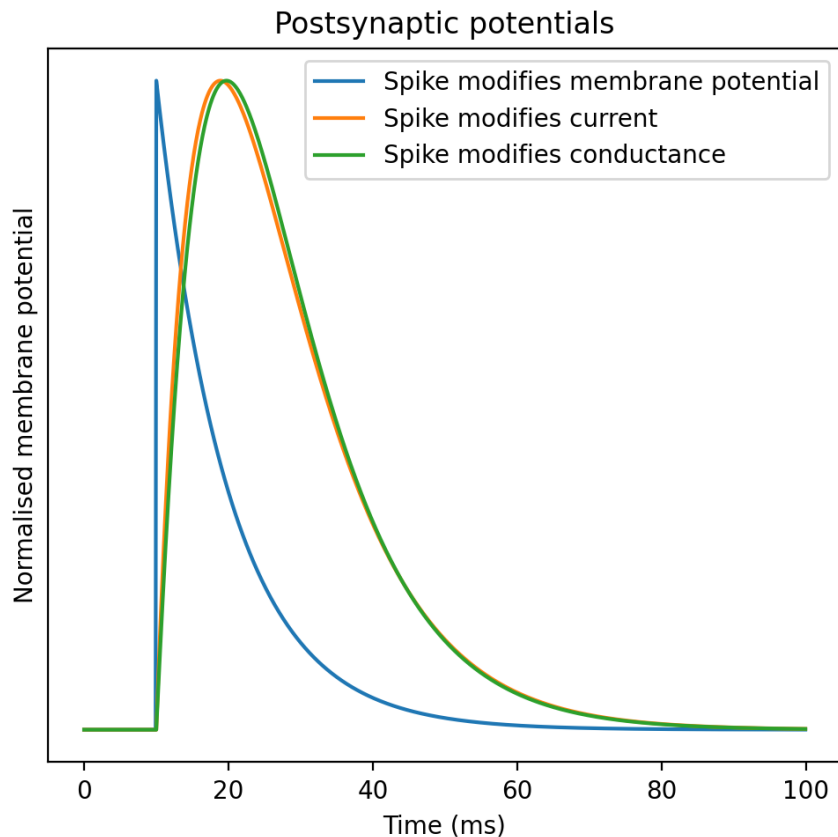


# Synapse models



# Level of abstraction

Q: Are these changes fast enough to ignore?



Neurotransmitter  
release and binding

changes

Conductance

$g_{syn}(t)$

changes

Current

$I_{syn}(t)$

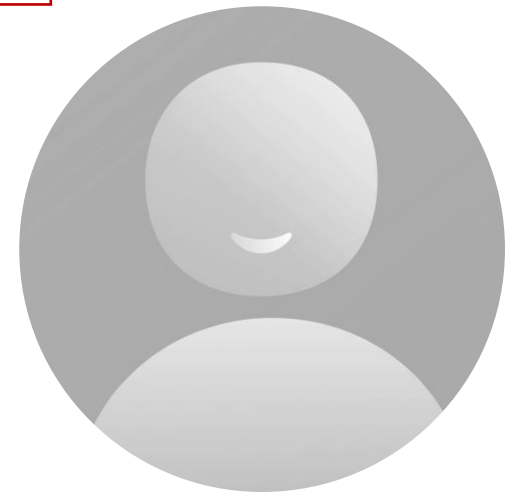
changes

Membrane potential  
 $V(t)$

Q: Are we modelling this level of detail?

$$I_{syn}(t) = g_{syn}(t)(V(t) - E_{syn})$$

$$CV'(t) = I_{syn}(t) - V(t) + \dots$$



# Synapse time course

Postsynaptic potential form

Differential equation form  
(ignoring constants)

Matrix form  
 $\mathbf{y}' = M\mathbf{y}$

## Exponential

$$v(t) = H(t) e^{-t/\tau}$$

$$\tau v' = -v$$

$$v(0) = 1$$

$$\mathbf{y} = (v)$$

$$M = (-1/\tau)$$

Single eigenvalue  $-1/\tau$

## Alpha

$$v(t) = H(t) e^{-t/\tau} t/\tau$$

$$\tau v' = x - v$$

$$\tau x' = -x$$

$$x(0) = 1$$

$$\mathbf{y} = (v, x)$$

$$M = \begin{pmatrix} -1/\tau & 1 \\ 0 & -1/\tau \end{pmatrix}$$

Repeated eigenvalue  $-1/\tau$

## Biexponential

$$v(t) = H(t)(e^{-t/\tau} - e^{-t/\tau_x})$$

$$\tau_x \neq \tau$$

$$\tau v' = x - v$$

$$\tau_x x' = -x$$

$$x(0) = 1$$

$$\mathbf{y} = (v, x)$$

$$M = \begin{pmatrix} -1/\tau & 1 \\ 0 & -1/\tau_x \end{pmatrix}$$

Two eigenvalues  $-1/\tau, -1/\tau_x$

Differential equation

$$\mathbf{y}' = M\mathbf{y}$$

Has solutions

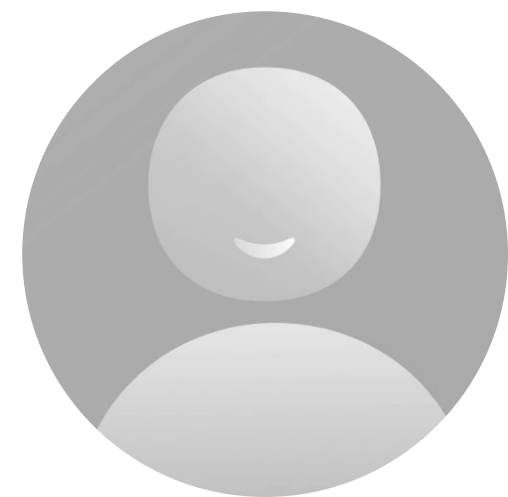
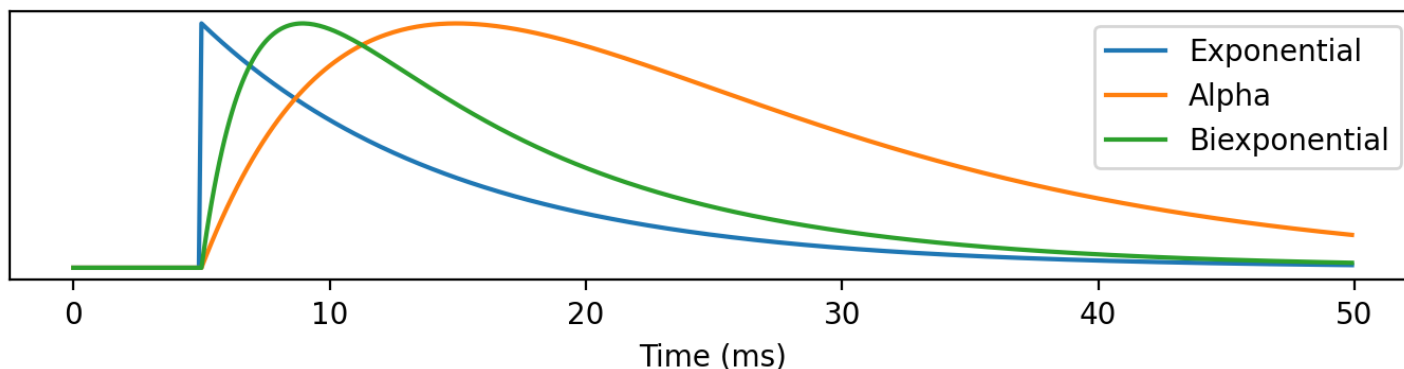
$$\mathbf{y}(t) = \mathbf{y}^* e^{\lambda t}$$

For eigenvalues  $\lambda$

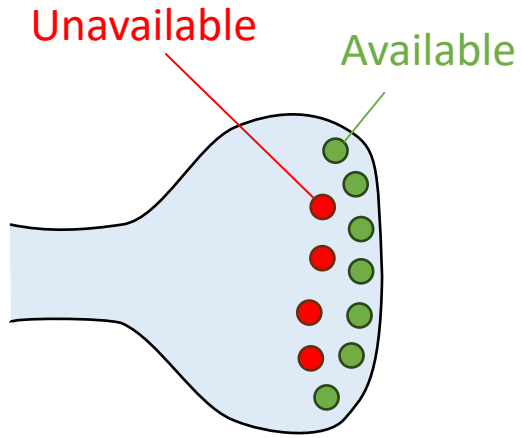
Eigenvectors  $\mathbf{y}^*$

Repeated eigenvalues give

$$\mathbf{y}(t) = \mathbf{y}^* t e^{\lambda t}, t^2 e^{\lambda t}, \dots$$



# Short term plasticity



Fraction available  $x$

↘ Decreases after a spike



Probability of release  $u$

↗ Increases after a spike

**Continuous evolution:**

$$\tau_f u' = -u$$

$$\tau_d x' = 1 - x$$

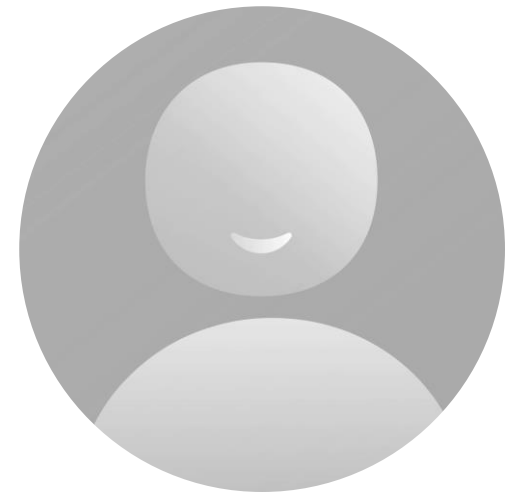
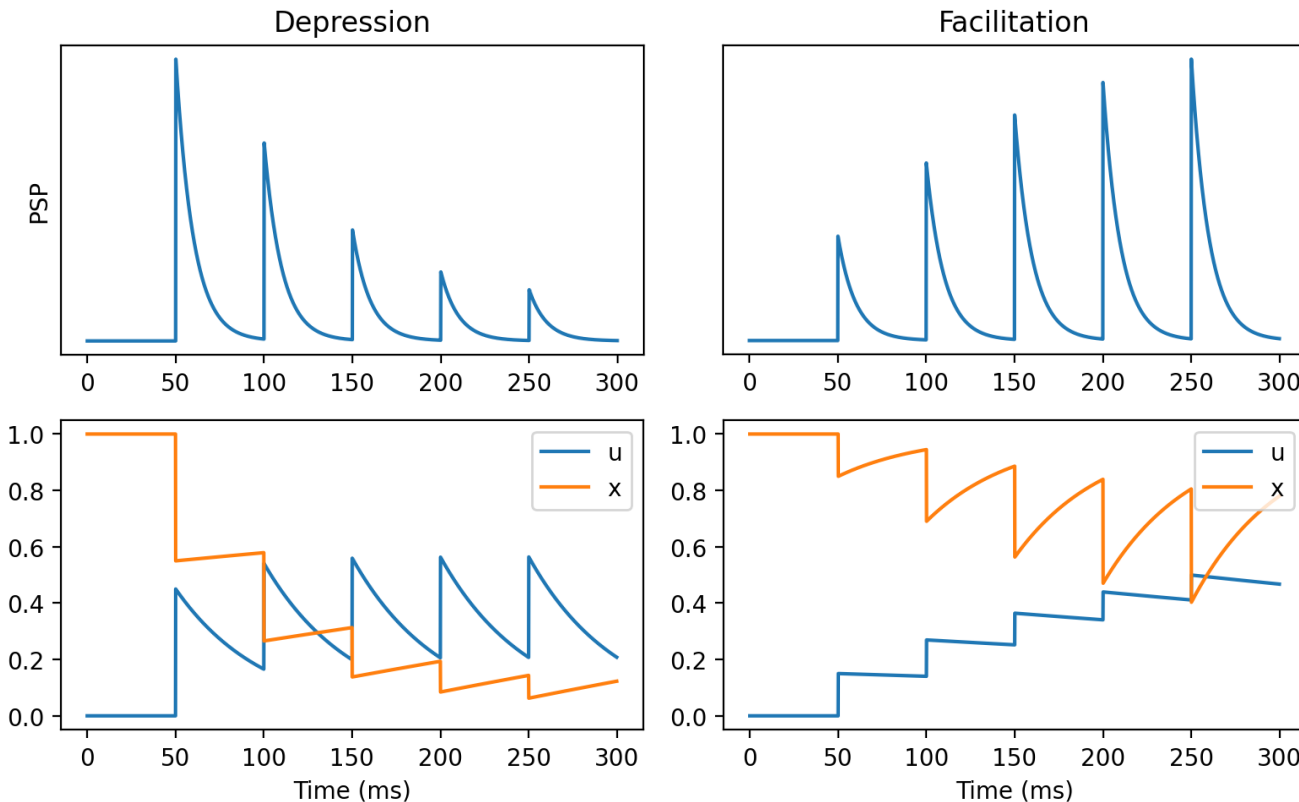
**When a spike arrives:**

$$u \leftarrow u + U \cdot (1 - u)$$

$$v \leftarrow v + A \cdot u \cdot x$$

$$x \leftarrow x - u \cdot x$$

Order of operations matters!



# Channel types: excitation and inhibition

## Channel types include:

### Inhibitory:

GABA<sub>A</sub> fast

GABA<sub>B</sub> slow

### Excitatory:

AMPA fast

NMDA slow

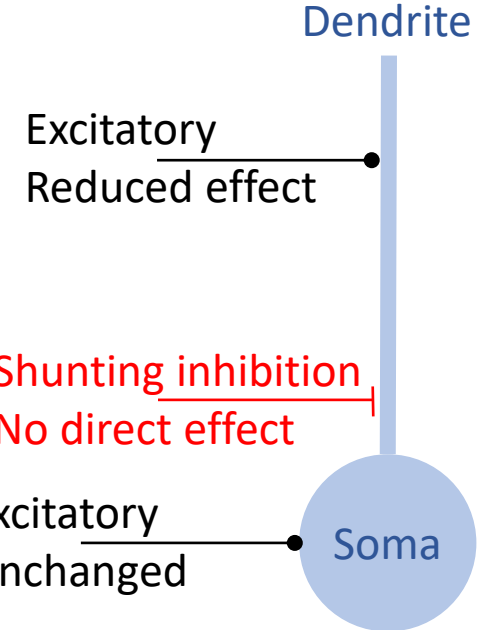
## Shunting inhibition

Conductance increases locally

Reversal potential near resting potential

No effect on its own

Reduces effect of passing excitatory currents



## Simplest model

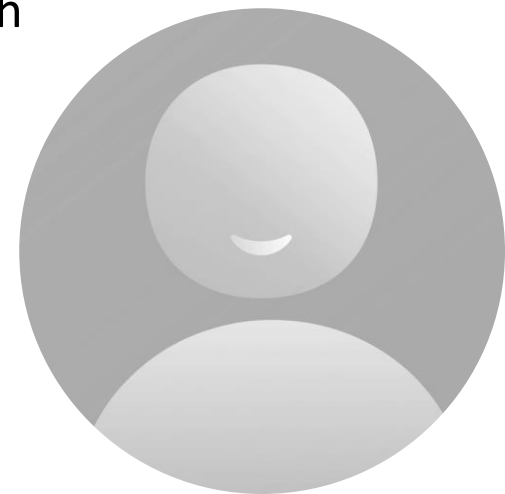
Excitation  $v \leftarrow v + w$

Inhibition  $v \leftarrow v - w$

## NMDA

Highly nonlinear

Depends on postsynaptic membrane potential being high



# Long term plasticity

Will come back to this in more detail in week 4

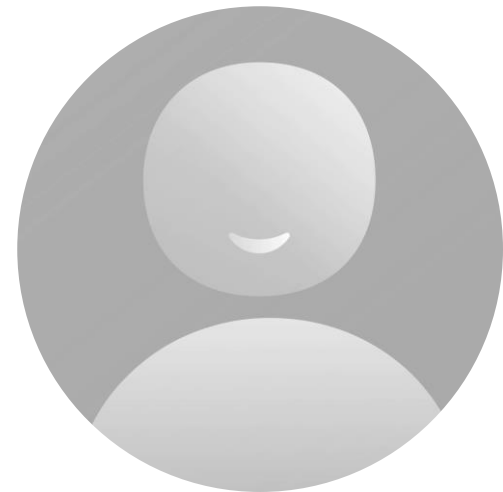
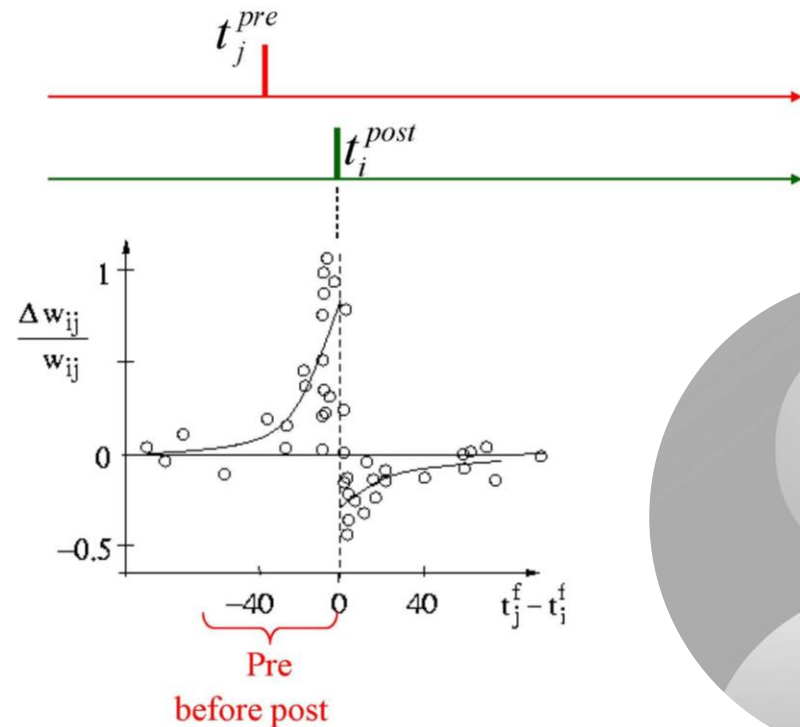
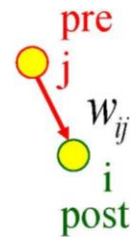
Hebbian rules can be modelled as:

$$\frac{dw}{dt} = \text{some function of } w \text{ and pre- and post-synaptic activity}$$

Spike timing-dependent plasticity (STDP):

$$\Delta w = \sum_{\text{presynaptic spikes } t_j} \sum_{\text{postsynaptic spikes } t_i} W(t_i - t_j)$$

$$W(\delta t) = \begin{cases} A_+ \exp(-\delta t / \tau_+) & \text{for } \delta t > 0 \\ A_- \exp(-\delta t / \tau_-) & \text{for } \delta t < 0 \end{cases}$$



# More synaptic fun

## Synaptic failure

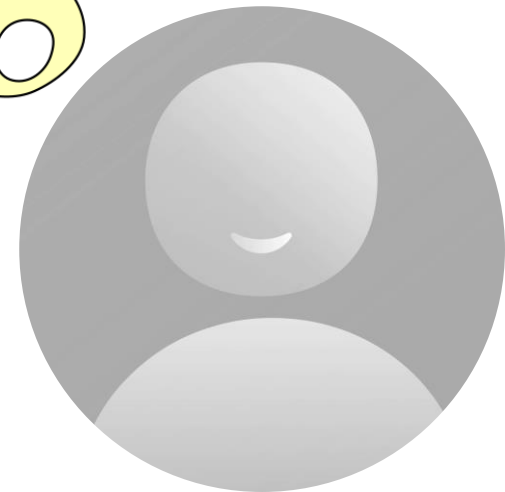
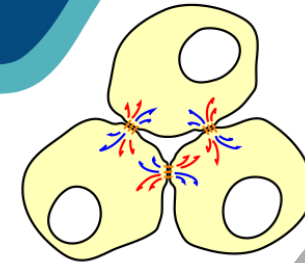
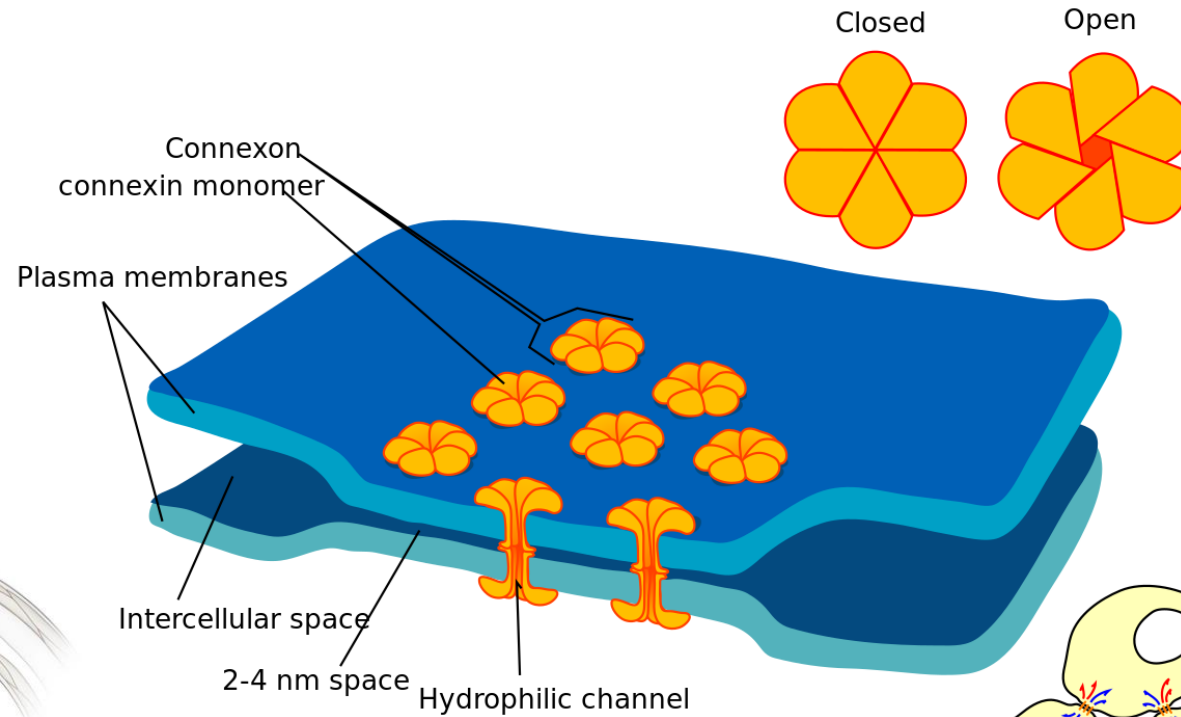
$$v \leftarrow v + w \quad (\text{with probability } p)$$

## Gap junctions

For example...

$$\tau v' = I_{gap} - v$$

$$I_{gap} = w(v_{pre} - v_{post})$$



## C. Elegans

Precisely 302 neurons

Always the same pattern

Mostly non-spiking, using gap junctions instead

