Solar Energy Vol. 49, No. 3, pp. 175–179, 1992
Printed in the U.S.A.

0038-092X/92 $5.00 + .00

ANALYSIS OF THE INCIDENCE ANGLE OF THE BEAM RADIATION ON CPC

J. M. PINAZO, J. CAÑADA,[†] and F. ARAGO
Department of Applied Thermodynamics, Universidad Politecnica de Valencia,
P. O. Box 22012, 46071 Valencia, Spain

Abstract—Analytic expressions have been derived for the projected incidence angles ϑ_l and ϑ_t from a two-dimensional compound parabolic concentrator solar collector. For a CPC the fraction of the incident rays on the aperture at angle ϑ, which reaches the absorber, depends only on the ϑ_l angle. In this paper, a mathematical expression for ϑ_l and ϑ_t has been calculated to determine the times at which acceptance of the sun's beam radiation begins and ceases for a CPC consisting of arbitrary orientation.

1. INTRODUCTION

The compound parabolic concentrator (CPC) is a nonimaging concentrator design that requires minimum tracking[1,2,3,4]. The CPC is known for its ability to collect and concentrate solar radiation within a given acceptance angle, that is, the angular range over which radiation is accepted without moving all or part of the collectors. Because of the CPC's acceptance angle for radiation, minimal tracking is necessary. The tilt frequency and number of hours of collection of a single curvature CPC has been calculated[3,5,6,7].

To estimate the radiation absorbed by the receiver of a CPC, it is necessary to determine whether the angle of incidence of the beam radiation is within the acceptance angle.

In the following paragraphs, a mathematical expression has been derived in order to determine the times at which acceptance of the sun's beam radiation begins and ceases, for a CPC consisting of arbitrary orientation.

2. SUN-EARTH GEOMETRY: LOCAL REFERENCE FRAME

The relative position of the sun to a local reference frame (fixed at the earth's surface) has been studied in other papers[7].

Figure 1 shows this local reference frame with the S-axis pointed south, the E-axis pointed east and the V-axis pointed in the vertical direction.

The direction cosines of a unit vector locating the sun with respect to a local frame on the surface of the earth is given by

$$\overline{\mathbf{SOL}} = (\cos \sigma \bar{\mathrm{S}}, \cos \alpha \bar{\mathrm{V}}, \cos \pi \bar{\mathrm{E}}) \tag{1}$$

where:

$$\begin{aligned}\cos \sigma &= -\sin \delta \cos \lambda + \cos \delta \sin \lambda \cos w\\ \cos \alpha &= \sin \delta \sin \lambda + \cos \delta \cos \lambda \cos w = \sin h\\ \cos \pi &= \cos \delta \sin w\end{aligned} \tag{2}$$

where λ—is the geographic latitude, δ—is the declination, w—is the hour angle, and h—is the solar altitude.

3. CPC COLLECTOR'S SOLAR INCIDENCE ANGLES

Figure 2 shows a typical CPC. It is convenient to define a collector reference frame that rotates with the collector, by three unit vectors:

$\bar{\mathbf{V}}_c$—defines the direction normal to the aperture of the collector.
$\bar{\mathbf{E}}_c$—defines the direction of the rotational axis of the collector.
$\bar{\mathbf{S}}_c$—defines the direction of the aperture of the collector (normal to the $\bar{\mathbf{V}}_c$–$\bar{\mathbf{E}}_c$ plane).

In this collector reference system, the three solar incidence angles ϑ, ϑ_l and ϑ_t are as follows:

ϑ—angle of incidence, that is, the angle between the beam radiation on a surface and the one normal to that surface.
ϑ_l—angle between the one normal to the aperture plane (unit vector $\bar{\mathbf{V}}_c$) and the projection of $\overline{\mathbf{SOL}}$ into a plane normal to the collector and to the rotational axis of the collector ($\bar{\mathbf{E}}_c$).
ϑ_t—angle between the rotational axis ($\bar{\mathbf{E}}_c$) and the projection of $\overline{\mathbf{SOL}}$ into a plane normal to collector and parallel to the rotational axis of the collector.

Figure 3 shows these angles within the planes $\bar{\mathbf{V}}_c$–$\bar{\mathbf{S}}_c$ and $\bar{\mathbf{V}}_c$–$\bar{\mathbf{E}}_c$.

The solar incidence angle ϑ, from[7] is given by:

$$\begin{aligned}\cos \vartheta = {} & \sin \delta \sin \lambda \cos s - \sin \delta \cos \lambda \sin s \cos \gamma\\ & + \cos \delta \cos \lambda \cos s \cos w + \cos \delta \sin \lambda \sin s\\ & \times \cos \gamma \cos w + \cos \delta \sin s \sin \gamma \sin w\end{aligned} \tag{3}$$

where s is the surface tilt angle from the horizonal and γ is the surface azimuth angle, that is, the deviation

---

[†] ISES member.

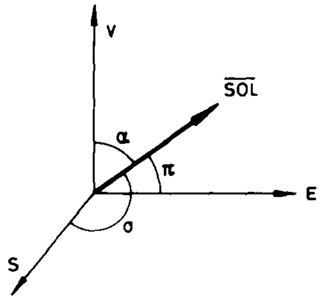

Fig. 1. Direction cosines of the unit vector $\overline{SOL}$.

of the normal to the surface from the local meridian with east positive and south zero.

For CPC collectors, the angular acceptance function $F(\vartheta)$ is defined as that fraction of the rays incident on the aperture at angle ϑ which reaches the absorber, dependent only on the ϑ_l angle. For a CPC the angular acceptance function is given by

$$F(\vartheta) = 1 \quad |\vartheta_l| \le \vartheta_a \tag{4a}$$

$$F(\vartheta) = \text{Function of CPC geometry} \quad \vartheta_a < |\vartheta_l| \le \vartheta_D \tag{4b}$$

$$F(\vartheta) = 0 \quad |\vartheta_l| > \vartheta_a \tag{4c}$$

where ϑ_a is the acceptance half-angle of untruncated CPC and ϑ_D is the acceptance half-angle of truncated CPC. On the other hand, the average number of reflections of solar radiation depends on ϑ_l[8,9].

The incidence angle projected along the absorber, ϑ_t, is important in order to determine the optical performance of the concentrator for the effects of end-mirrors (see Fig. 4).

McIntire[10] calculated the end-mirror reflection factor for several absorbers. The end-mirror reflection factor must be determined for each individual ray traced, since it depends upon the depth to which the ray travels in the trough before striking the absorber. Therefore, it is dependent upon ϑ_l, as well as upon ϑ_t, because ϑ_l determines the path of the rays. However, the average over all the rays does not vary significantly with ϑ_l.

For CPC collectors, the incidence angles ϑ, ϑ_l and ϑ_t must be known with respect to the collector orientation in order to determine thermal study. Miller[11] derived a mathematical condition which determines the times at which acceptance of the sun's beam radiation begins and ceases for a trough-type concentrator consisting of arbitrary orientation. In this paper, mathematical expressions are presented for the angles ϑ_l and ϑ_t.

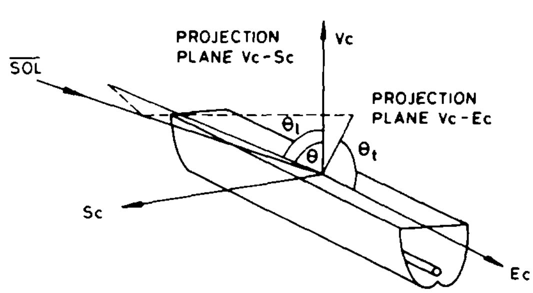

Fig. 2. Definition of ϑ, ϑ_l, and ϑ_t.

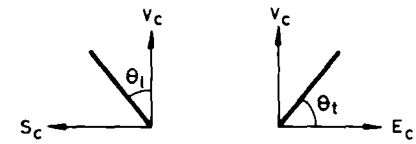

Fig. 3. Sun projection angles ϑ_l and ϑ_t.

4. CONCENTRATOR ORIENTATION

In order to determine the position of the CPC, it is necessary to prescribe the slope of the surface with respect to the horizontal (s), its orientation in relation to the local meridian (γ), and (β), the angle between the longitudinal axis of the CPC and a horizontal line lying on the plane of the CPC. The angle β is measured in a plane which contains the collector (see Fig. 5).

In order to calculate the angles ϑ_l and ϑ_t, it is necessary to describe the unit vector $\overline{\mathbf{SOL}}$ in the collector reference system, defined by $\bar{\mathbf{V}}_c$, $\bar{\mathbf{E}}_c$ and $\bar{\mathbf{S}}_c$.

The local reference frame will here be defined in terms of the collector reference frame. The two reference frames can be related to each other through three successive rotations[12]. The three rotation angles are as follows, γ, s, and β, defined in the previous section.

The first local reference frame transformation accounts for rotation around the normal vector $\bar{\mathrm{V}}$ away from south by the γ angle

$$\begin{bmatrix} S_1 \\ V_1 \\ E_1 \end{bmatrix} = \begin{bmatrix} \cos\gamma & 0 & \sin\gamma \\ 0 & 1 & 0 \\ -\sin\gamma & 0 & \cos\gamma \end{bmatrix} \begin{bmatrix} S \\ V \\ E \end{bmatrix} \tag{5}$$

The second rotation applied to the local reference frame is a rotation around $\mathbf{E}_1$ by an s angle

$$\begin{bmatrix} S_2 \\ V_2 \\ E_2 \end{bmatrix} = \begin{bmatrix} \cos s & -\sin s & 0 \\ \sin s & \cos s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_1 \\ V_1 \\ E_1 \end{bmatrix} \tag{6}$$

Finally, the rotation around the $\mathbf{V}_2$ vector by the β angle

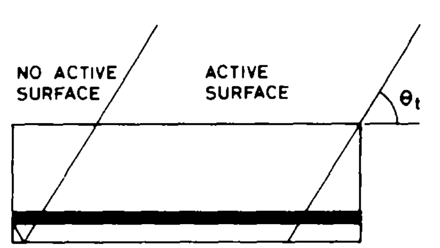

Fig. 4. End-mirror reflection factor for a CPC.

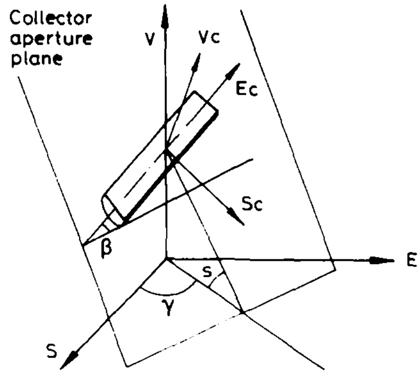

Fig. 5. Position of the CPC relative to a local reference frame.

$$\begin{bmatrix} S_3 \\ V_3 \\ E_3 \end{bmatrix} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} S_2 \\ V_2 \\ E_2 \end{bmatrix} \quad (7)$$

The collector reference frame can here be related to the local reference frame by

$$\begin{bmatrix} S_c \\ V_c \\ E_c \end{bmatrix} = [\beta][s][\gamma] \begin{bmatrix} S \\ V \\ E \end{bmatrix} \quad (8)$$

Multiplying all the terms

$$S_c = [\cos\beta \cos s \cos\gamma - \sin\beta \sin\gamma]\cos\sigma - [\cos\beta \sin s]\cos\alpha + [\cos\beta \cos s \sin\gamma + \sin\beta \cos\gamma]\cos\pi \quad (9)$$

$$V_c = [\sin s \cos\gamma]\cos\sigma + [\cos s]\cos\alpha + [\sin s \sin\gamma]\cos\pi \quad (10)$$

$$E_c = -[\sin\beta \cos s \cos\gamma + \cos\beta \sin\gamma]\cos\sigma + [\sin\beta \sin s]\cos\alpha + [-\sin\beta \cos s \sin\gamma + \cos\beta \cos\gamma]\cos\pi \quad (11)$$

where $\cos\sigma$, $\cos\alpha$, and $\cos\pi$ are calculated in eqn (2).

5. THE INCIDENT ANGLES PROJECTED ϑ_l AND ϑ_t

The angle between the collector normal $\bar{V}_c$, and the sun position vector $\overline{SOL}$, projected onto the $\bar{V}_c$-$\bar{S}_c$ plane, named ϑ_l, is given by (see Fig. 3).

$$\operatorname{tg}\vartheta_l = S_c/V_c \quad (12)$$

The angle between the collector vector $\bar{E}_c$ along the absorber tube and the sun position vector, $\overline{SOL}$, projected onto the $\bar{V}_c$-$\bar{E}_c$ plane, named ϑ_t, is given by (see Fig. 3)

$$\operatorname{tg}\vartheta_t = V_c/E_c \quad (13)$$

where S_c, V_c, and E_c are defined in eqns (9), (10), and (11).

For the case of a typical CPC: horizontal ($\beta = 0°$), vertical ($\beta = 90°$), east-west collector ($\gamma = 0°$) and north-south collector ($\gamma = 90°$), the eqns (12) and (13) reduce to:

(a) CPC collector, horizontal ($\beta = 0°$) and absorber tube aligned east-west ($\gamma = 0°$).

$$\operatorname{tg}\vartheta_l = \frac{\cos\delta \sin\lambda \cos w - \sin\delta \cos\lambda - \operatorname{tg} s \sin\delta \sin\lambda - \operatorname{tg} s \cos\delta \cos\lambda \cos w}{\sin\delta \sin\lambda + \cos\delta \cos\lambda \cos w + \operatorname{tg} s \cos\delta \sin\lambda \cos w - \operatorname{tg} s \sin\delta \cos\lambda} \quad (14)$$

$$\operatorname{tg}\vartheta_t = \frac{\sin s \cos\delta \sin\lambda \cos w - \sin s \sin\delta \cos\lambda + \cos s \sin\delta \sin\lambda + \cos s \cos\delta \cos\lambda \cos w}{\cos\delta \sin w} \quad (15)$$

(b) CPC with tilt equal to latitude ($s = \lambda$), horizontal ($\beta = 0°$), and aligned east-west ($\gamma = 0°$).

$$\operatorname{tg}\vartheta_l = -\frac{\operatorname{tg}\delta}{\cos w} \quad (16)$$

$$\operatorname{tg}\vartheta_t = \frac{1}{\operatorname{tg} w} \quad (17)$$

(c) Vertical CPC collector ($\beta = 90°$) and absorber tube aligned north-south ($\gamma = 90°$).

$$\operatorname{tg}\vartheta_l = \frac{-\cos\delta \sin\lambda \cos w - \sin\delta \cos\lambda}{\sin s \cos\delta \sin w + \cos s \sin\delta \sin\lambda + \cos s \cos\delta \cos\lambda \cos w} \quad (18)$$

$$\operatorname{tg}\vartheta_t = \frac{\sin\delta \sin\lambda + \cos\delta \cos\lambda \cos w + \operatorname{tg} s \cos\delta \sin w}{\sin s \sin\delta \sin\lambda + \sin s \cos\delta \cos\lambda \cos w - \cos s \cos\delta \sin w} \quad (19)$$

6. SAMPLE GRAPHICS CALCULATIONS

Figures 6 and 7 show the angles ϑ, ϑ_t, and ϑ_l versus solar time for a CPC consisting of an acceptance half-angle $\vartheta_a = 30°$, tilt = 50° ($s = 50°$), $\beta = 10°$ and $\gamma = 15°$, located in Valencia ($\lambda = 40°$N).

The hours of operation of the collector in the winter solstice vary from 8 h solar time to sunset; at the vernal equinox the solar radiation was accepted from 7 to 14.5 h of solar time.

Figure 8 shows the angle ϑ_l versus solar time for the same CPC, horizontal ($\beta = 0°$) aligned east-west ($\gamma = 0°$) and tilt = latitude ($s = 40°$N), for various times of the year. Each curve is similar to the path the sun takes across the sky on that particular day.

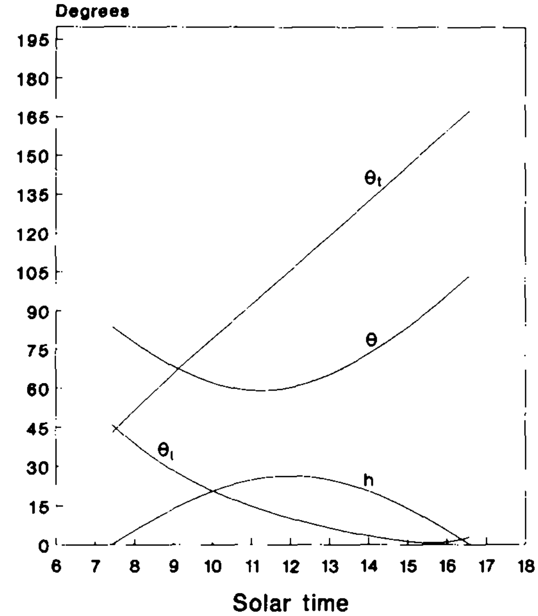

Fig. 6. ϑ, ϑ_l, and ϑ_t versus solar time for a CPC with ϑ_a = 30°, s = 50°, β = 10°, and γ = 50°, located in Valencia (λ = 40°N) at winter solstice (21 December).

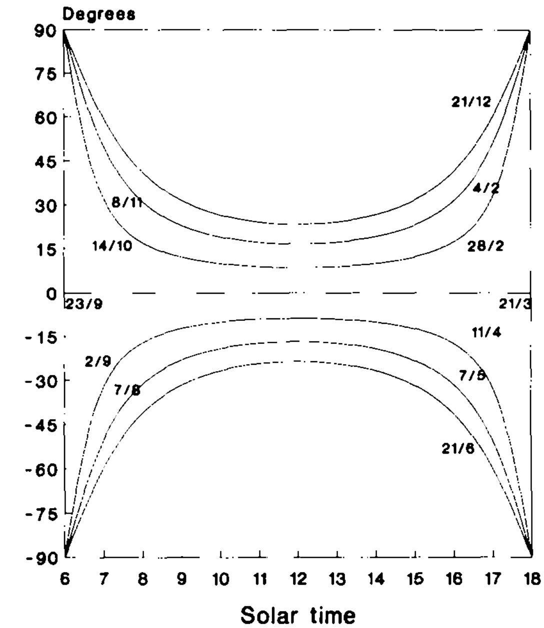

Fig. 8. Seasonal variation of ϑ_l for a CPC, horizontal (β = 0°), aligned east-west (γ = 0°) and s = 40°.

7. CONCLUSIONS

A mathematical expression for ϑ_l and ϑ_t has been calculated to determine the times at which acceptance of the sun's beam radiation begins and ceases for a CPC consisting of arbitrary orientation.

The angle between the collector normal, $\bar{V}_c$, and the sun position vector, $\overline{SOL}$, projected onto the $\bar{V}_c$–$\bar{E}_c$ plane, denoted ϑ_l, is the projected incidence angle traverse to the absorber and is the angle to be compared with the acceptance half-angle, ϑ_a, to determine turn-on and turn-off times for the collector. ϑ_l is given by

$$\tan \vartheta_l = S_c/V_c$$

The incident angle projected along the absorber is denoted ϑ_t and is given by

$$\tan \vartheta_t = V_c/E_c$$

In summary, collection times for a CPC concentrator having arbitrary orientation has been described in an analytic and graphic form. The simple analytic expressions described above will show the effect of a nonoptimal orientation of a CPC.

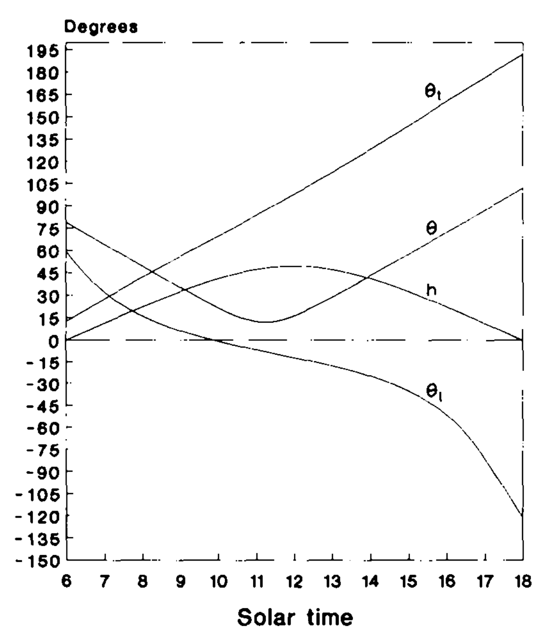

Fig. 7. ϑ, ϑ_l, and ϑ_t versus solar time for a CPC with ϑ_a = 30°, s = 50°, β = 10°, and γ = 50°, located in Valencia (λ = 40°N) at equinox (21 March).

NOMENCLATURE

- $\bar{E}$ unit vector pointing east
- $\bar{E}_c$ unit vector along the rotational axis of the collector
- h solar altitude
- $\bar{S}$ unit vector pointing south
- s surface tilt angle from the horizontal
- $\bar{S}_c$ unit vector traverse to rotational axis and perpendicular to the collector normal, $\bar{V}_c$
- $\overline{SOL}$ sun position unit vector
- $\bar{V}$ unit vector to the zenith
- $\bar{V}_c$ unit vector normal to the collector aperture
- w hour angle (zero at noon, mornings positive)

Greek

- β angle between the longitudinal axis of the CPC and a horizontal line lying on the plane of the CPC
- δ declination (zero on equinox, positive north)
- λ latitude (positive north)

γ surface azimuth angle (east positive, south zero)
ϑ angle of incidence on collector
ϑ_l traverse projected component of the angle ϑ between the collector normal and the sun position vector (angle in the $\bar{V}_c$–$\bar{S}_c$ plane)
ϑ_t longitudinal projected component of the angle ϑ between the collector normal and the sun position vector (angle in the $\bar{V}_c$–$\bar{E}_c$ plane)
ϑ_a acceptance half-angle of CPC
$\cos\sigma$ direction cosines of the unit vector $\overline{\mathbf{SOL}}$
$\cos\alpha$ direction cosines of the unit vector $\overline{\mathbf{SOL}}$
$\cos\pi$ direction cosines of the unit vector $\overline{\mathbf{SOL}}$

REFERENCES

1. R. Winston, Principles of solar concentrators of a novel design, *Solar Energy* **16**, 89 (1974).
2. R. Winston and H. Hinterberger, Principles of cylindrical concentrators for solar energy, *Solar Energy* **17**, 255 (1975).
3. A. Rabl, Comparison of solar concentrators, *Solar Energy* **18**, 93 (1976).
4. R. Winston and W. Welford, The optics of nonimaging concentrators, Academic Press, New York (1978).
5. M. Collares and A. Rabl, Simple procedure for predicting long-term average performance of nonconcentrating and of concentrating solar collectors, *Solar Energy* **23**, 235 (1979).
6. F. Grosso, F. Musumeci, and A. Triglia, Relation between concentration and acceptance in solar collectors, *Solar Energy* **22**, 521 (1979).
7. J. A. Duffie and W. A. Beckman, Solar Engineering of thermal processes, Wiley, New York (1980).
8. M. J. Carvalho, M. Collares, and A. Rabl, Truncation of CPC solar collectors and its effect on energy collection, *Solar Energy* **35**, 393 (1985).
9. A. Rabl, Radiation transfer through specular passages. A simple approximation, *Int. J. Heat Mass Transfer* **20**, 323 (1977).
10. W. R. McIntire, Factored approximations for biaxial incident angle modifiers, *Solar Energy* **29**, 315 (1982).
11. C. W. Miller, Collection times for trough-type concentrators having arbitrary orientation, *Solar Energy* **20**, 399 (1978).
12. W. R. McIntire and K. A. Reed, Orientational relationships for optically nonsymmetric solar collectors, *Solar Energy* **31**, 405 (1983).