On Likelihood Ratio Tests in dimensionality-restricted models

Mingyue Gao

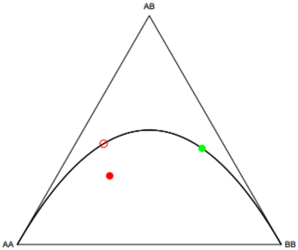
This is joint work With Michael Trosset, Carey Priebe

February 24, 2016

Conjecture

"The long time conjecture of the power superiority of the restricted LRT to its unrestricted version in the entire parameter space of alternatives for the general setting is of considerably analytic difficulty and lack of the definitive results."
(Mingtan Tsai, On the power superiority of likelihood ratio tests for restricted alternatives, JMVA 1992)

Hardy-Weinberg Submanifold of a Trinomial Parameter Space



The green dot is hypothesized; the red dot is observed (n=10); the red circle is estimated. Plausible test statistics are D(●,●) and D_{HW} (○,●). Presumably, restricted inference is more powerful.

Problem of Interest

Consider $X \sim Multinomial(n, p)$ on $K + 1$ categories. Let $\Theta \subset R^d$ with $d < K$, and consider a smooth parametrization $h : \Theta \to H = h(\Theta) \subset \Delta^K$. Let $p_0 = h(\theta_0)$, and consider simple null with composite alternative likelihood ratio test statistics

1. $T_{super} = LRTS$ for $H_0 : p = p_0$ vs $H_A : p \in \Delta^K \setminus \{p_0\}$ and
2. $T_{sub} = LRTS$ for $H_0 : p = p_0$ vs $H_A : p \in H \setminus \{p_0\}$.

Testing $H_0 : \theta = 0.3$

However......

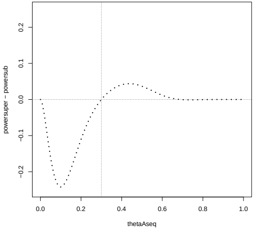


Figure: $\beta_{super} - \beta_{sub}$ when $n = 10, \theta_0 = 0.3$

The horizontal axis parametrizes the Hardy-Weinberg submanifold. The vertical axis measures power of unrestricted test minus that of the restricted test.

Conclusions

In fact, for Hardy-Weinberg submodel of Trinomial supmodel......

- If $\theta_0 = 0.5$, then for each n, α, θ_A, $\beta_{super}(n, \theta_0, \theta_A, \alpha) \leq \beta_{sub}(n, \theta_0, \theta_A, \alpha)$.
- For each n, there exists $\theta_0, \theta_A, \alpha$ such that $\beta_{super}(n, \theta_0, \theta_A, \alpha) > \beta_{sub}(n, \theta_0, \theta_A, \alpha)$.

Moreover, this phenomenon exists for general multinomial models...