In-memory & Semi-External Memory Large-Scale Clustering



K-means Motivation

- One of the most widely used & important clustering techniques.
- Problems with k-means:
 - Historically does not scale well computation complexity: O(kdn), Space complexity: O(nd + kd)
 - Others did not fully utilize CPU resources (toward computation efficiently)

Others have not take advantage of NUMA architecture nor new vectorization enabled

CPUs

Our approach:

Scale-up not scale-out

• **The how** — Packages:

• k||means : Shared memory

 Core 0
 Thread 0
 data[0] ... data[PAGE_SZ-1]

 Core 1
 Thread 1
 data[PAGE_SZ] ... data[2*PAGE_SZ-1]

 NUMA Node 0
 :
 :

 Core T-1
 Thread T-1
 data[(T-1)*PAGE_SZ] ... data[(T*PAGE_SZ)-1]

 NUMA Node N
 Core (2NT-1)
 Thread NT
 data[(NT)*PAGE_SZ] ... data[(NT*PAGE_SZ)-1]

 Core N(T+1)
 Thread T+2
 data[(N(T+1))*PAGE_SZ] ... data[(N(T+2))*PAGE_SZ-1]

 ...
 ...
 ...

 Core (2NT-1)
 Thread (2NT-1)
 data[((2NT-1)*PAGE_SZ] ... data[(2NT)*PAGE_SZ)-1]

• SEM-kmeans, Min-Triangle-SEM-kmeans: Semi-external memory



k||means

- Rethink Lloyd's EM steps. Why not (mostly) merge the two EM-steps into a super-step?
 - How? Share no data!
 - Per-thread data structures; combined recursively in || at end of super-step.
 - Result: Embarrassing ||ism
- Developed ||ized initializations via || kmeans++or kmeans|| — they matter

```
Algorithm 1 k||means algorithm
```

```
1: procedure K||MEANS(V, C, K)|
       ptCentroids
                                    > Per-thread centroids
       clusterAssignment
                                      ⊳ Shared, no conflict
      parfor \vec{v}_i \in V do
         for \vec{c_i} \in C do
            [dist_{min}, ci\vec{d}_{min}] = min(\mathbf{d}(\vec{v}_i, \vec{c}_i))
         end for
         ptCentroids[CURR\_THREAD][cid_{min}] += \vec{v_i}
       end parfor
       clusterMeans = mergePtStructs(ptClusters)
11: end procedure
12: procedure MERGEPTSTRUCTS(vectors)
       while |vectors| > 1 do
         PAR_MERGE(vectors)
                                               \triangleright O(T \log n)
14:
       end while
15:
       return vectors[0]
17: end procedure
```

Old memory bound: O(nd + kd)

New memory bound: O(nd + Tkd), where T = #threads

k||means Performance on Friendster eigs (66 Mil x 8)

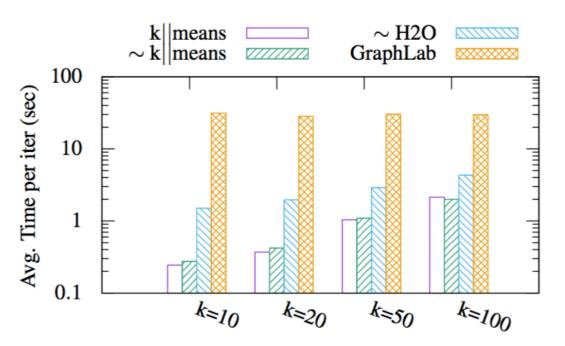


Fig 1: Log scale average time per iteration

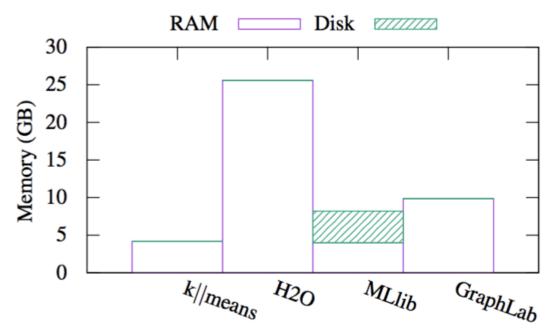


Fig 2: Log scale average time per iteration

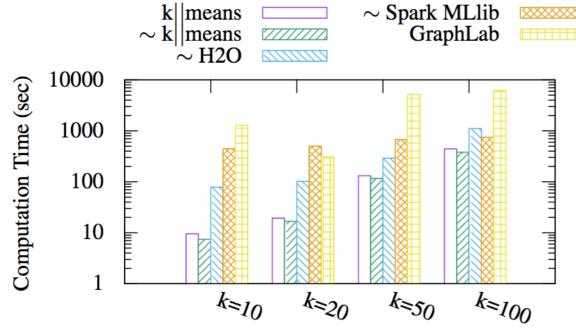


Fig 3: Log scale average computation time.
Important because we see **greater** improvement than the per cost per iteration (Fig. 1) due to || initialization modules



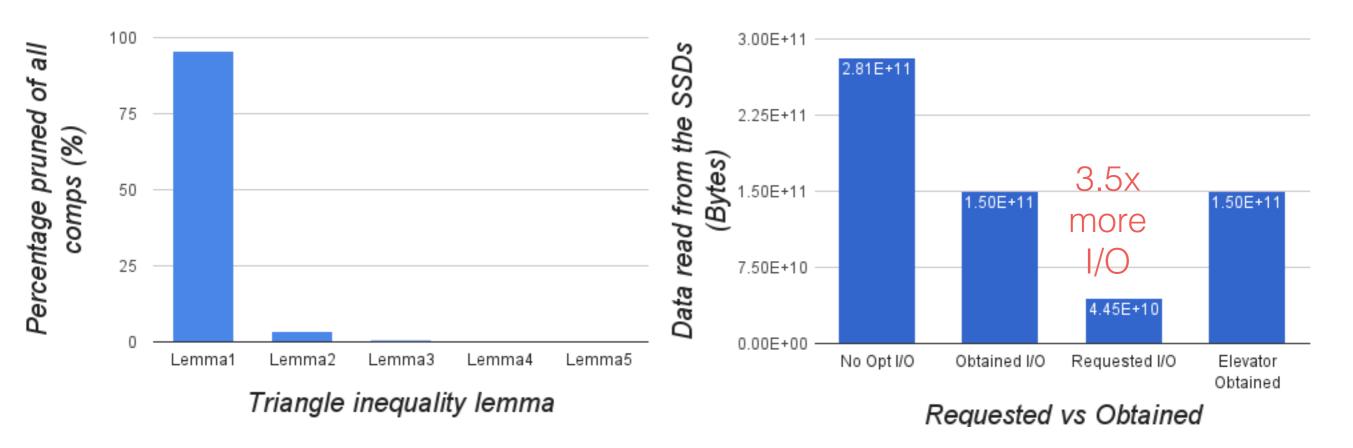
SEM-kmeans

- Can we be comparable with less resources?
 - Semi-external:
 - Memory now O(n + Tkd) < O(nd + kd) < O(nd + Tkd)
 - T = #threads/processes and Tkd << O(nd)
- Further improve speed using a modified triangle inequality pruning algorithm with same memory bound: Min-Triangle-SEM-Kmeans
 - Performance improvement from 2 factors:
 - Reduction in # of distance computations
 - Reduction in I/O complexity



Min-Triangle-SEM-kmeans on Friendster

Micro Data: 66Mil x 8, Dense, Size: 4 GB, K=10

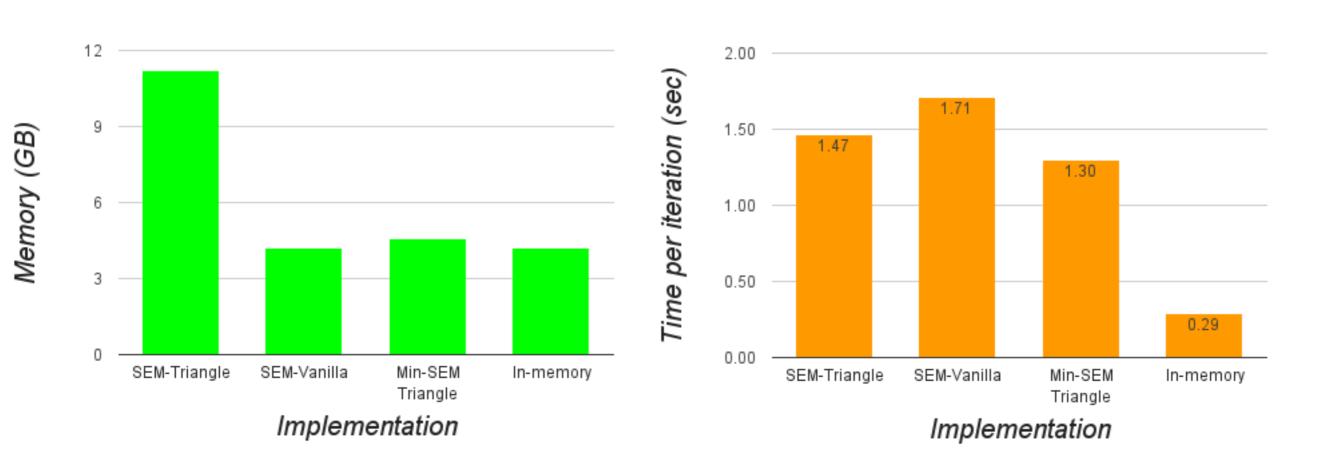


Insight: The effect of upper bound matrix, O(nk) is minimal. It contributes only < 2% (Lemma 4 & Lemma 5)

Both depend a lot on the data ... But still very I/O bound

Only 40% CPU utilization ...

Variant comparison on Friendster



*More optimizations to follow, 1GB cache for all SEM



Applications

- Together with the FlashEigen-solver we can perform extremely fast and scalable spectral embedding!
 - Applies to:
 - Connectomics
 - Anomaly detection
 - Machine-Learning
 - NLP
 - A host of other interesting problems:
 - https://sites.google.com/site/dataclusteringalgorithms/ clustering-algorithm-applications

