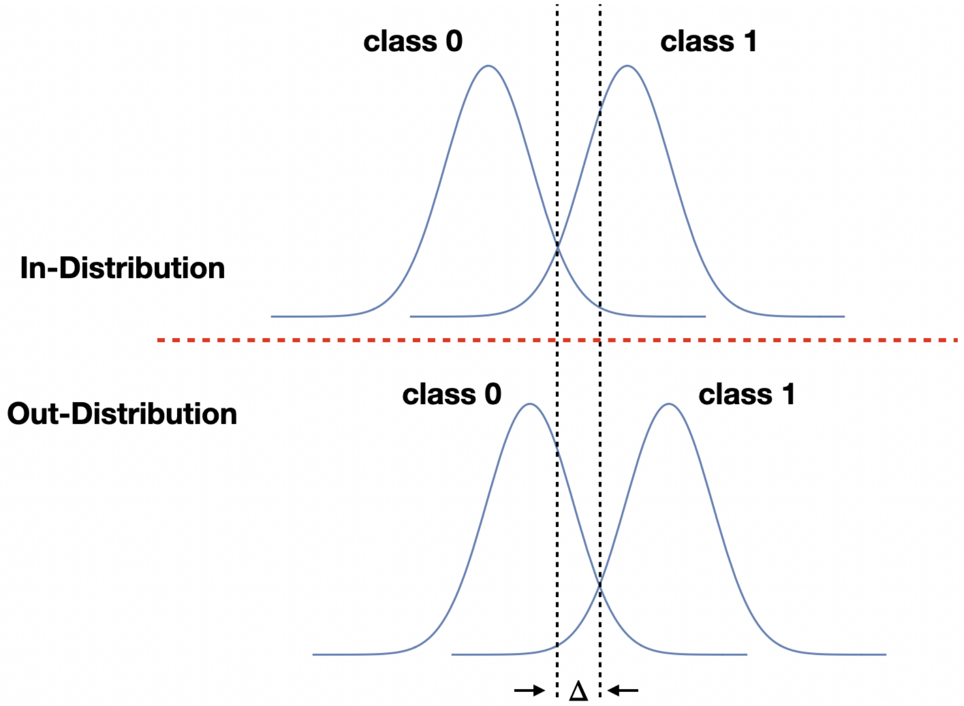
Out-of-Distribution Learning

Gaussian Tasks Experiment

- Consider an in-distribution task that consists of two class conditional gaussians.
- Now, consider an out-of-distribution task similar to the above task, but whose center is displaced by an amount Δ.
- The amount Δ reflects the "similarity" between the two tasks.



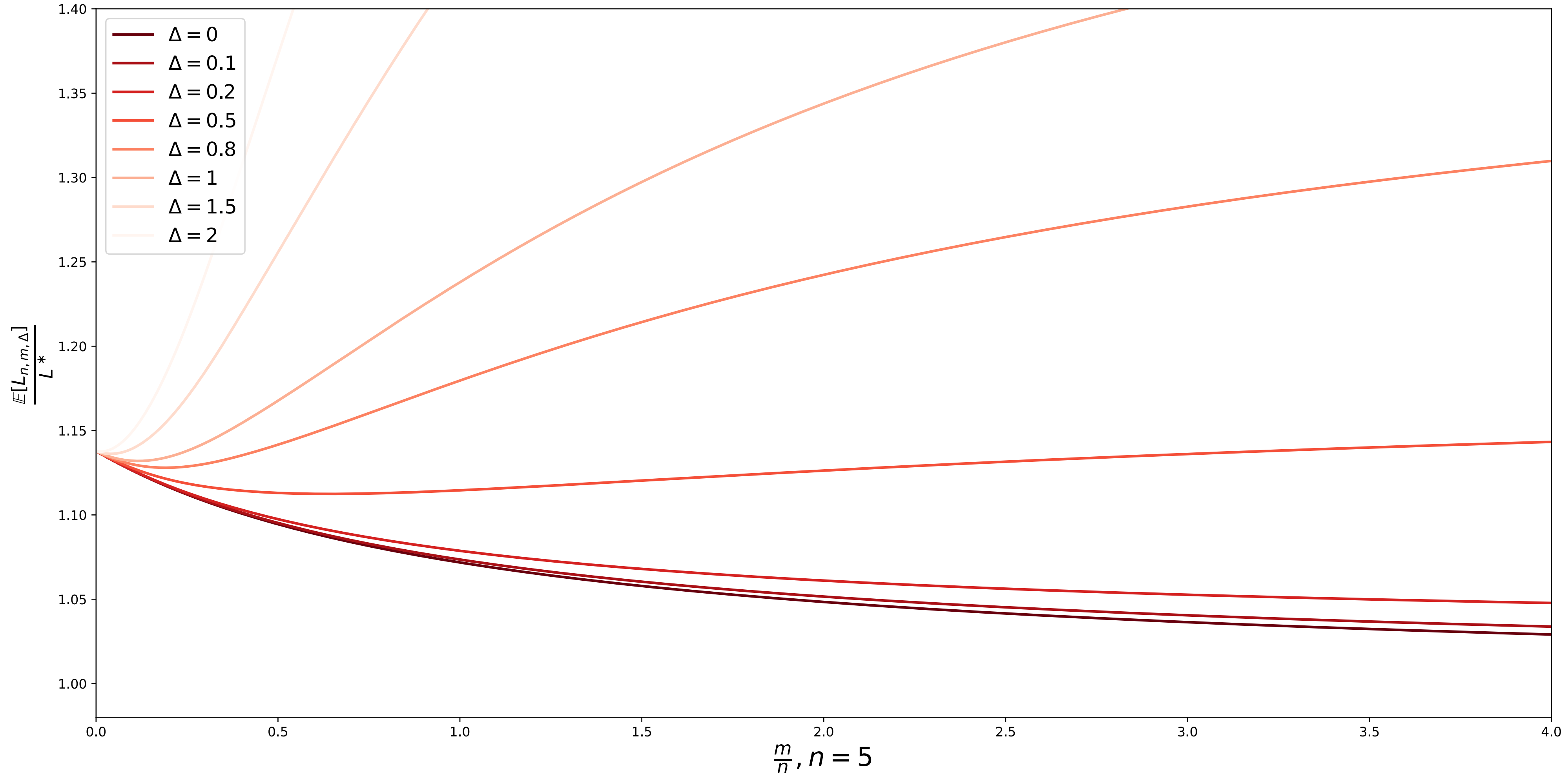
Gaussian Tasks Experiment

- We have access to n samples from the in-distribution task, and m samples from the out-of-distribution task.
- Using both the in-distribution and out-of-distribution samples, we train a classifier h aimed at the in-distribution classification task.
- Let's denote the classification error of h by $\mathbb{E}[L_{n,m,\Delta}]$.

Gaussian Tasks Experiment

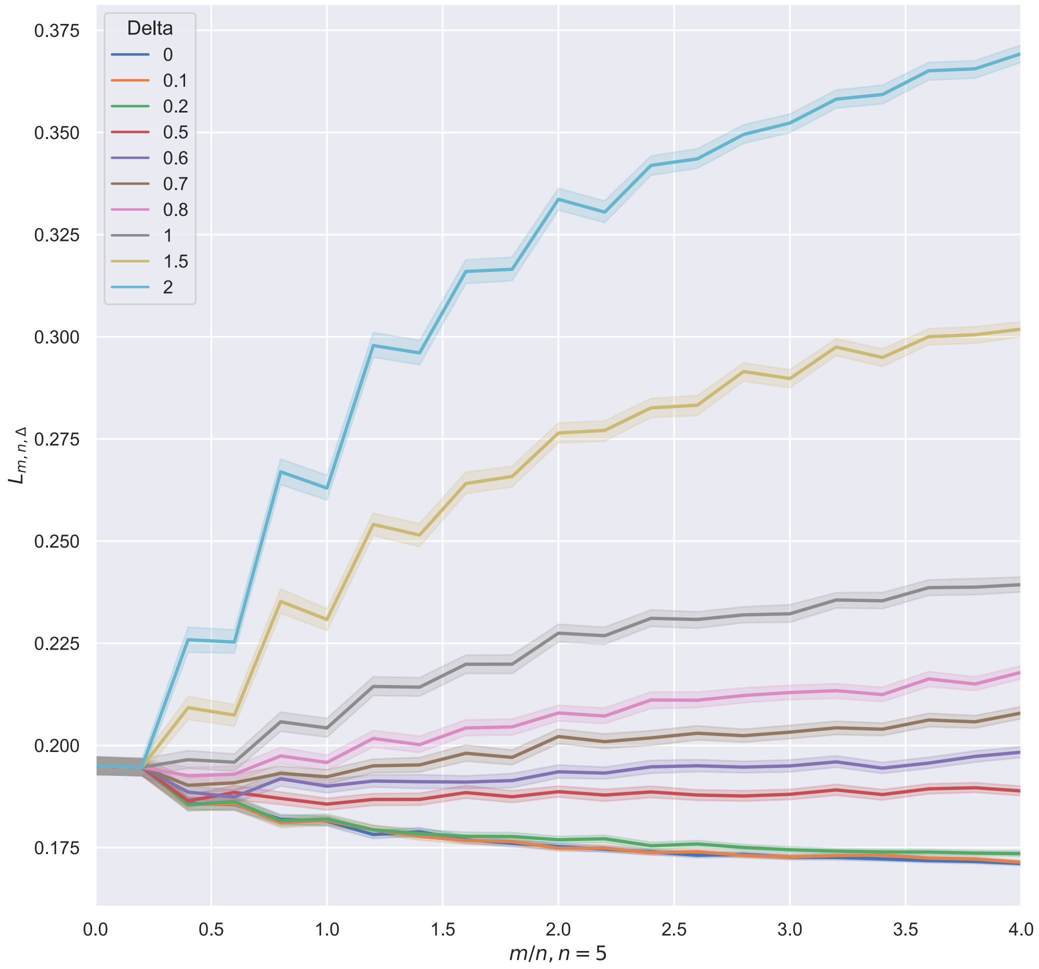
- Let n be a small fixed constant. We hypothesize that,
  - For very small Δ, as we add more out-of-distribution data (as m increases) the $\mathbb{E}[L_{n,m,\Delta}]$ would decrease.
  - For moderately large Δ, as we add more out-of-distribution data (as m increases) the $\mathbb{E}[L_{n,m,\Delta}]$ would initially decrease and start increasing later. The initial decrease is due to the reduction in the variance of h. The later increase is due to the increase in bias of h caused by the out-of-distribution samples.
  - For very large Δ, as we add more out-of-distribution data (as m increases) the $\mathbb{E}[L_{n,m,\Delta}]$ would keep increasing.

Gaussian Tasks Experiment



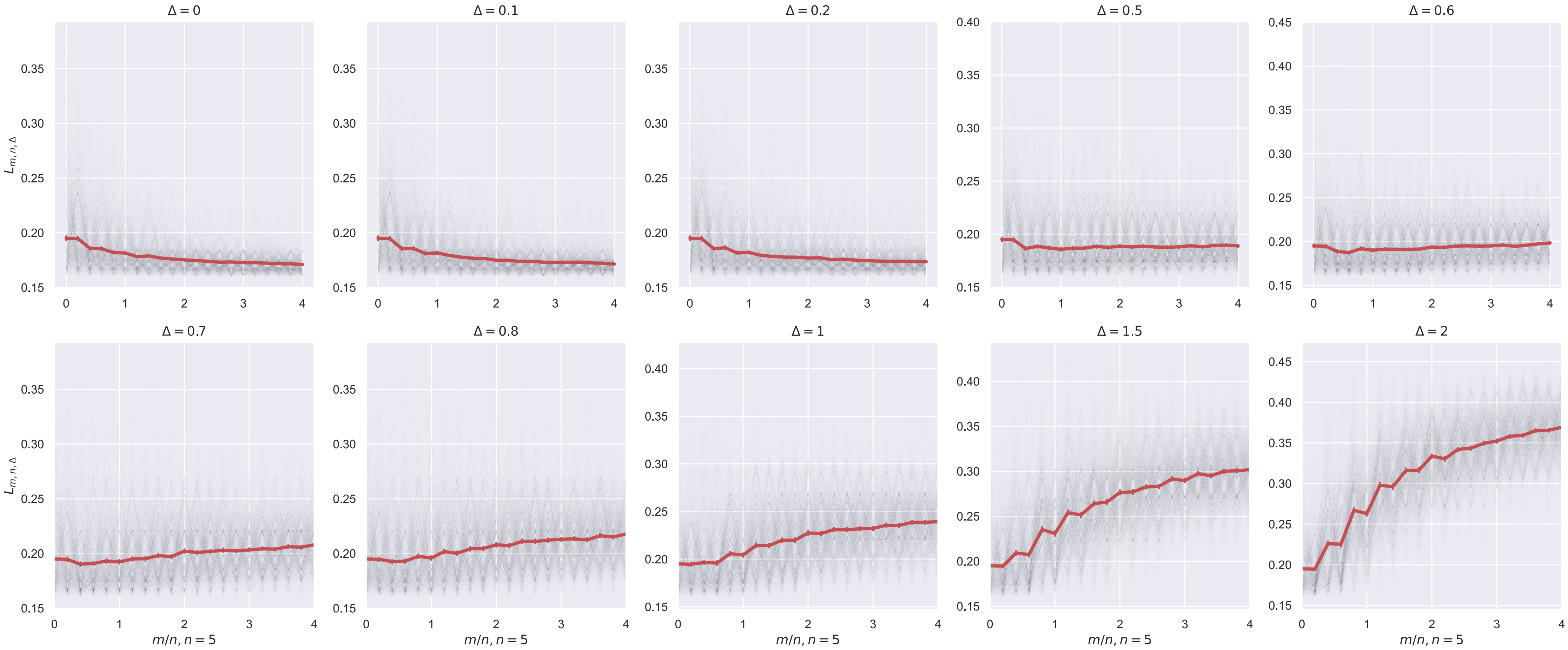
Gaussian Tasks Experiment

- Number of replicates: 1000



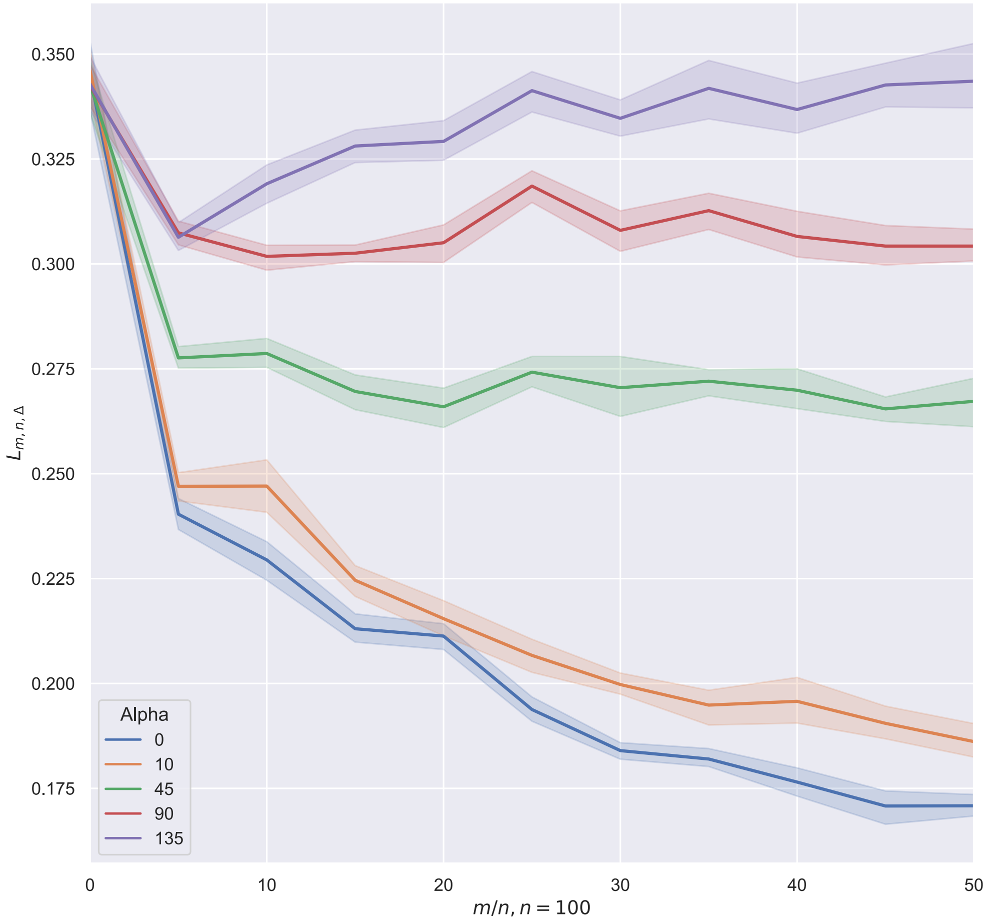
Gaussian Tasks Experiment

- Number of replicates: 1000



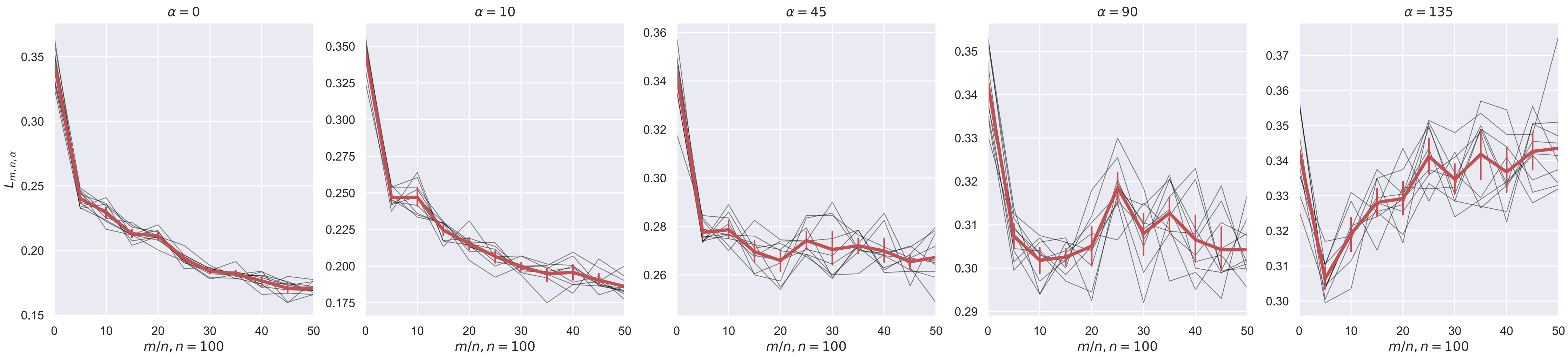
Bird vs. Cat & α-Rotated Bird vs. Cat (Single-Head Network)

- Number of replicates: 10, Network: SmallConv



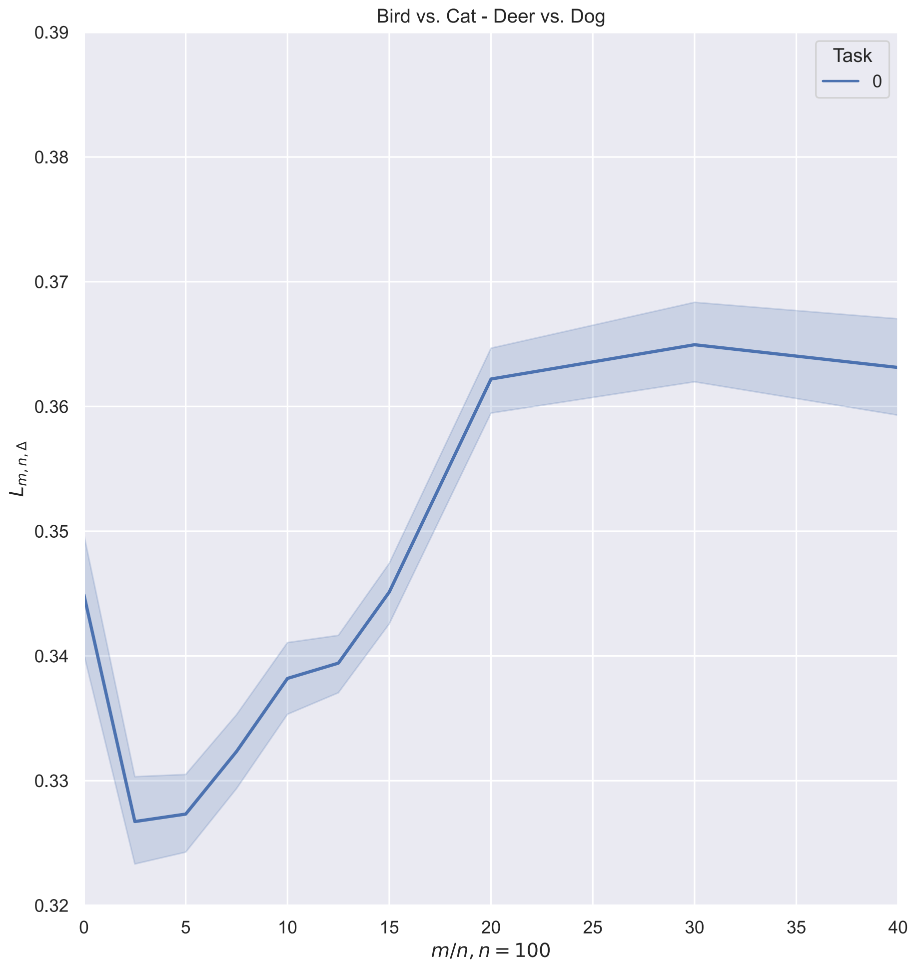
Bird vs. Cat & α-Rotated Bird vs. Cat (Single-Head Network)

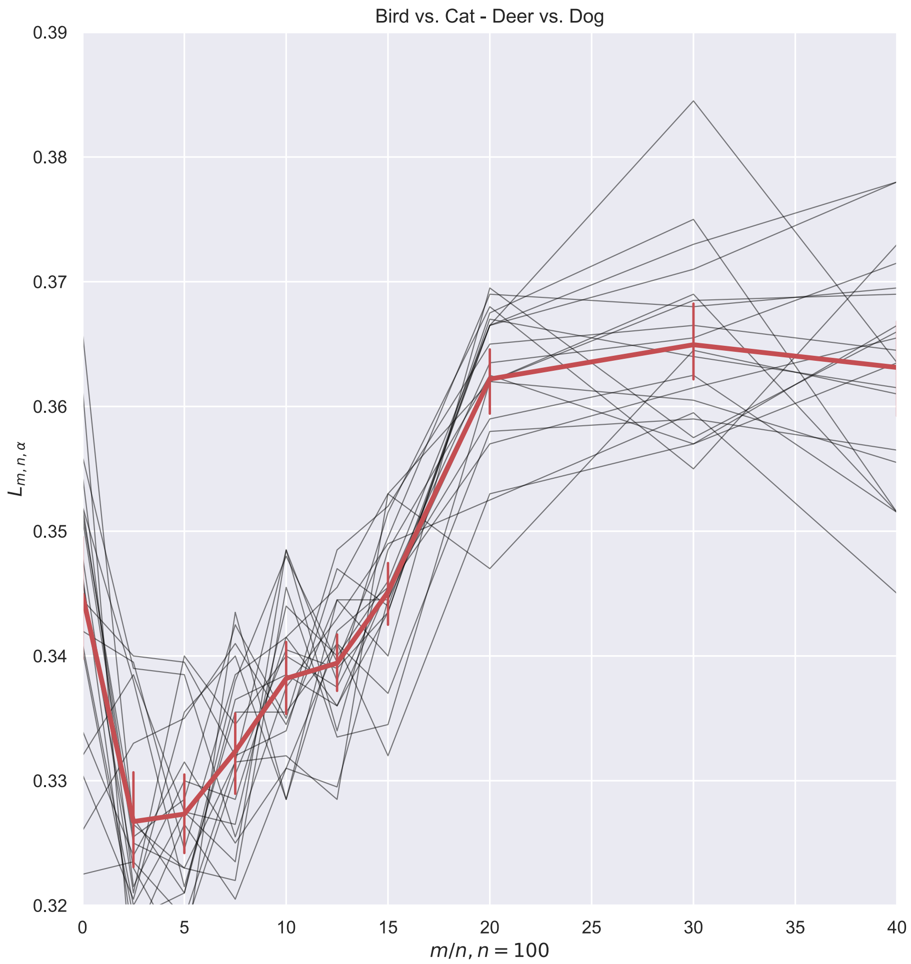
- Number of replicates: 10, Network: SmallConv



Task 2: Bird vs. Cat & Task 3: Deer vs. Dog (Single-Head Network)

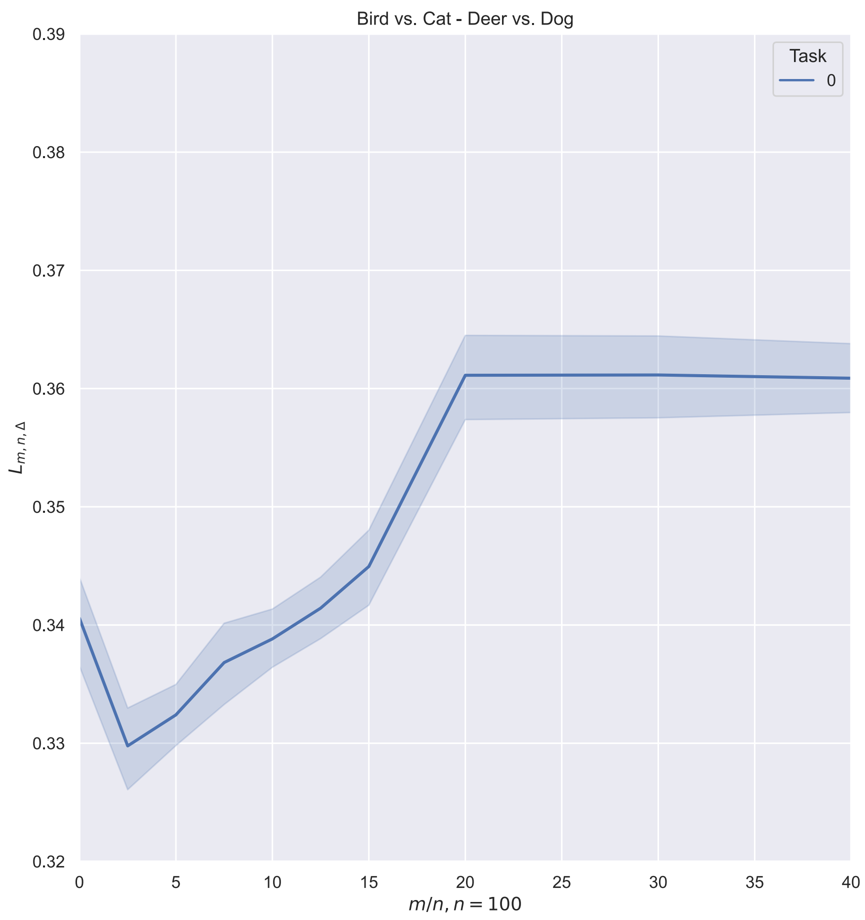
- Number of replicates: 20, Network: SmallConv

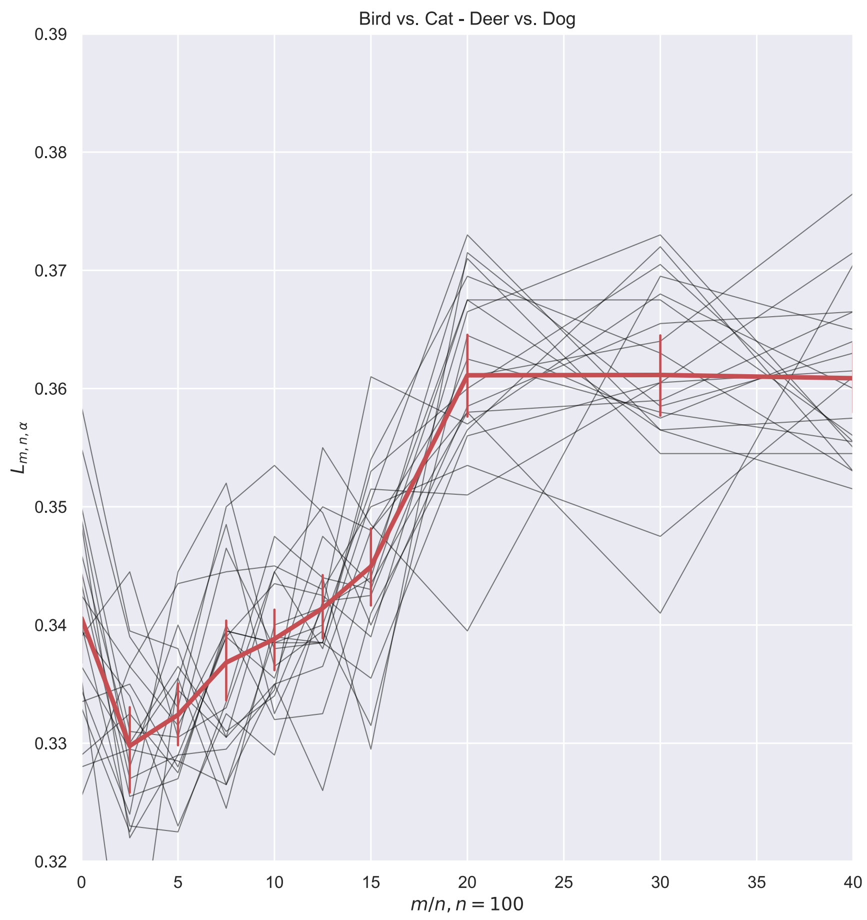




Task 2: Bird vs. Cat & Task 3: Deer vs. Dog (Single-Head Network)

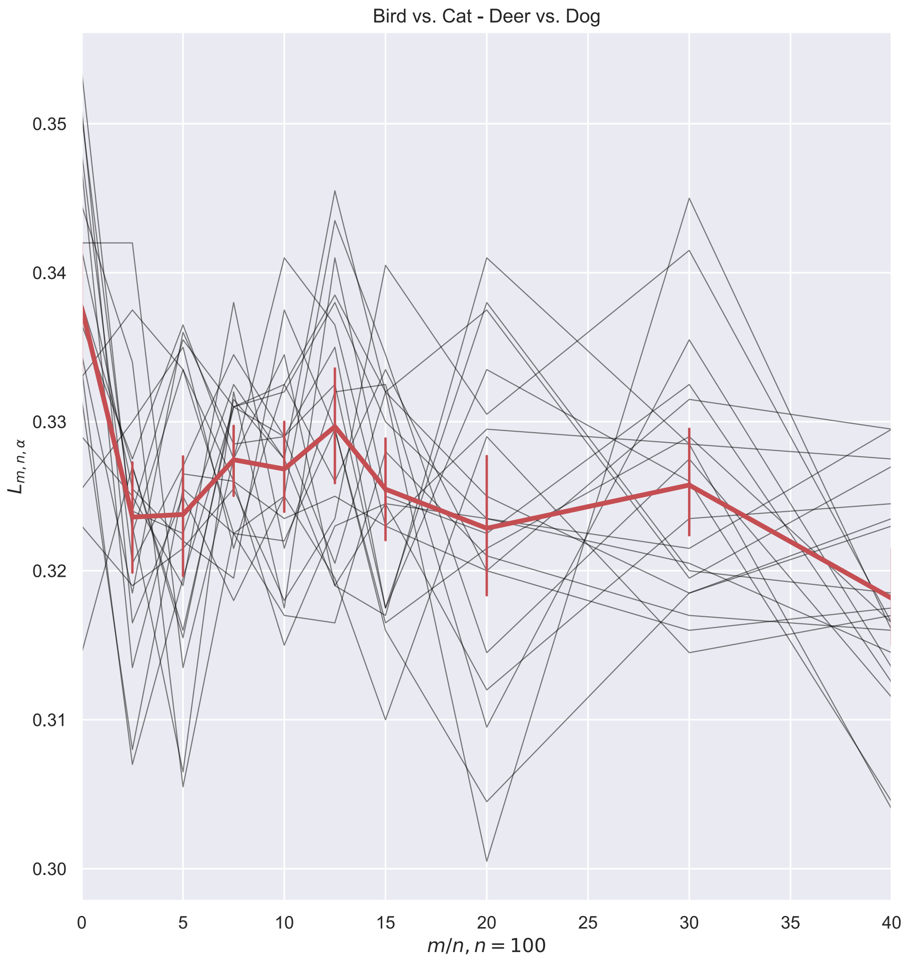
- Number of replicates: 20, Network: SmallConv, each model was trained for 100 epochs





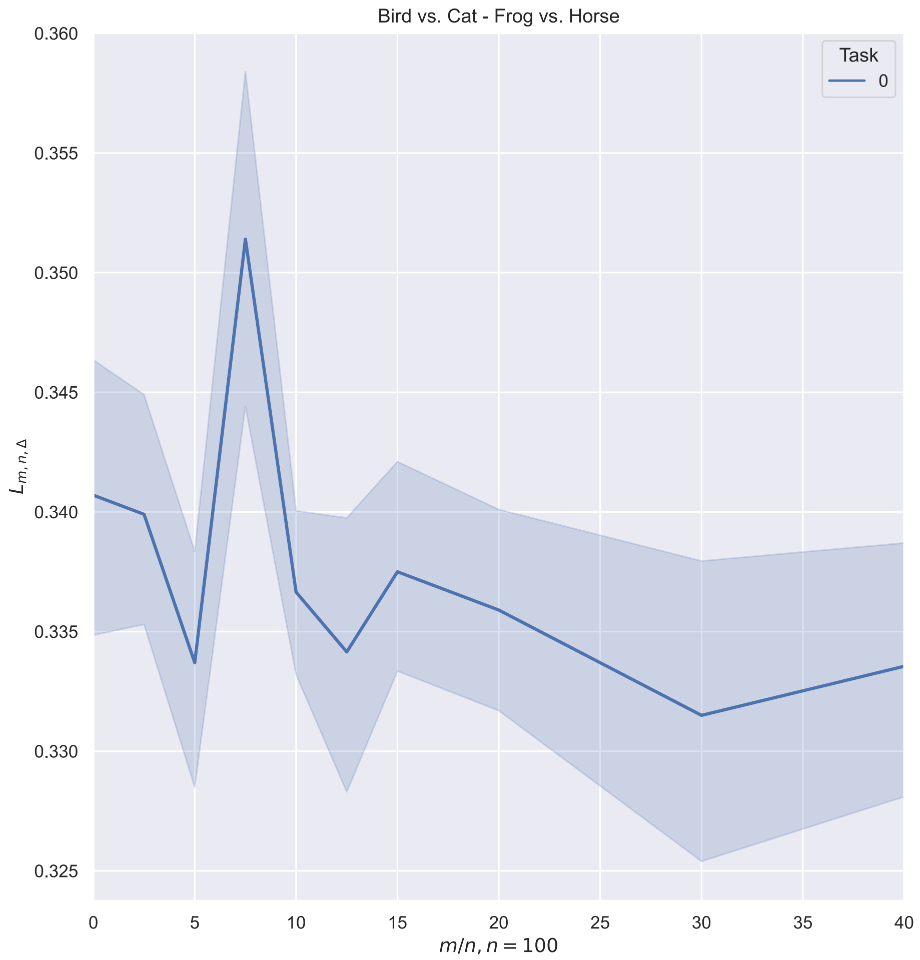
Task 2: Bird vs. Cat & Task 3: Deer vs. Dog (Multi-Head Network)

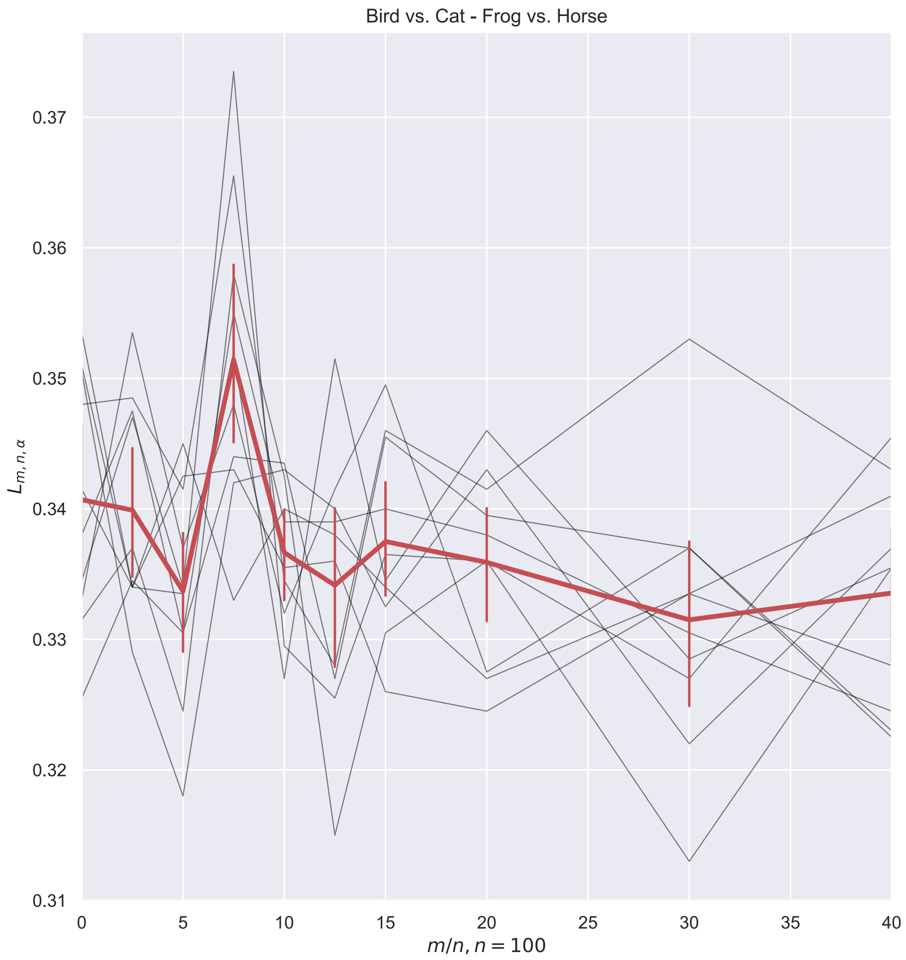
- Number of replicates: 20, Network: SmallConv



Task 2: Bird vs. Cat & Task 4: Frog vs. Horse (Multi-Head Network)

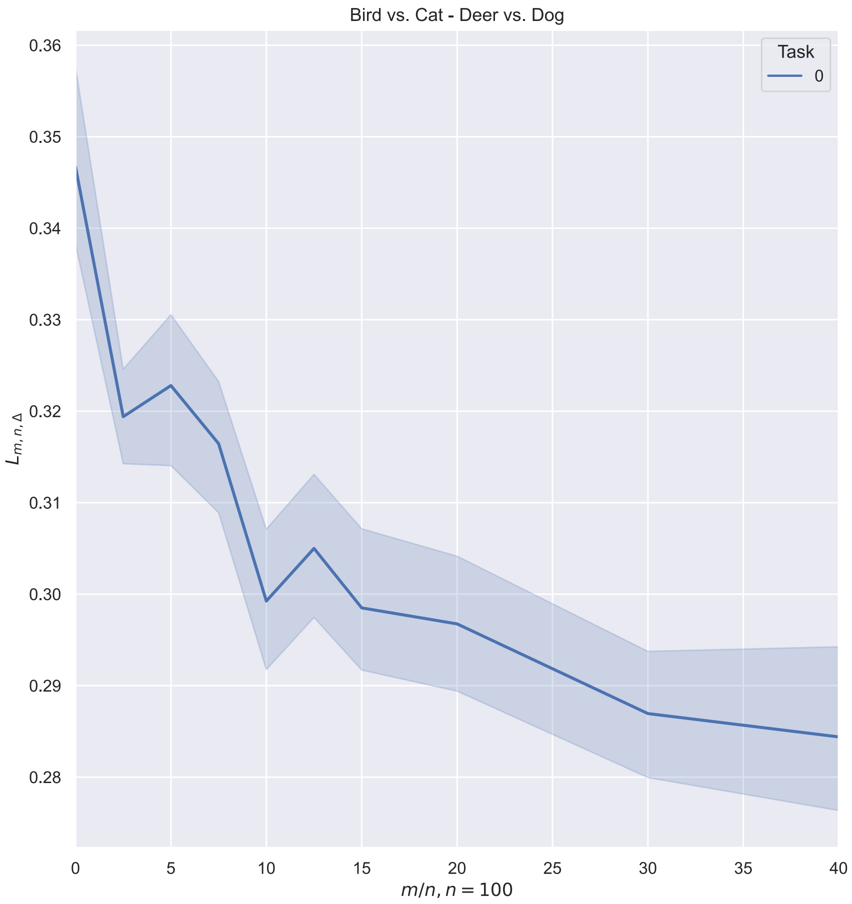
- Number of replicates: 20, Network: SmallConv

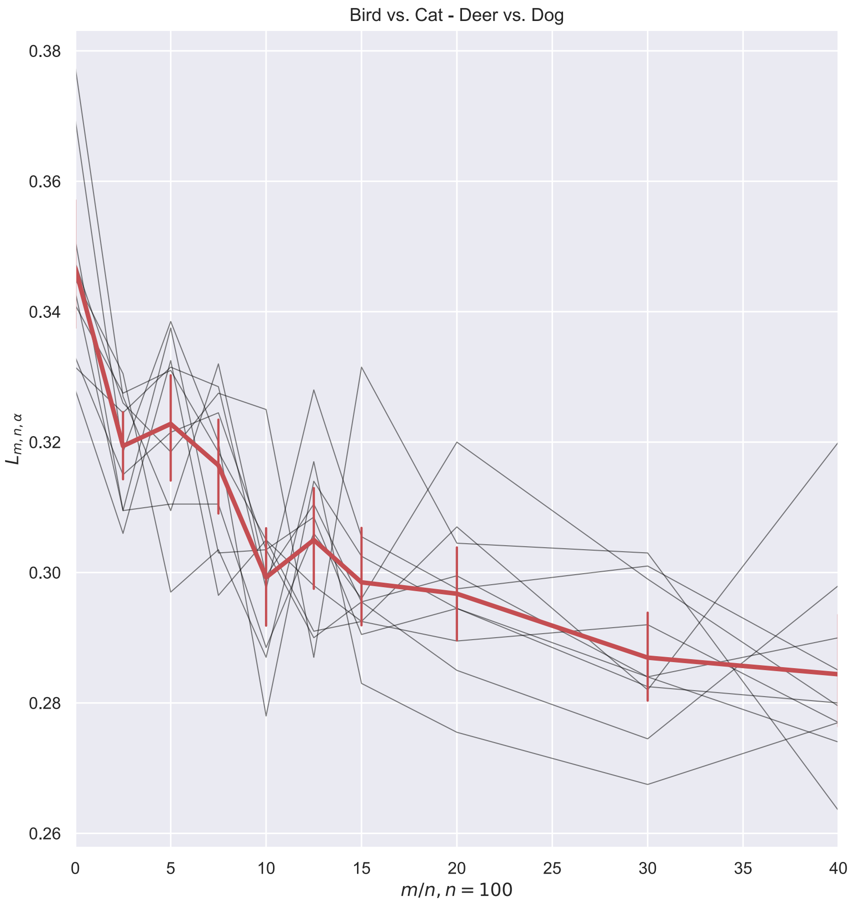




Task 2: Bird vs. Cat & Task 3: Deer vs. Dog (Multi-Head Network)

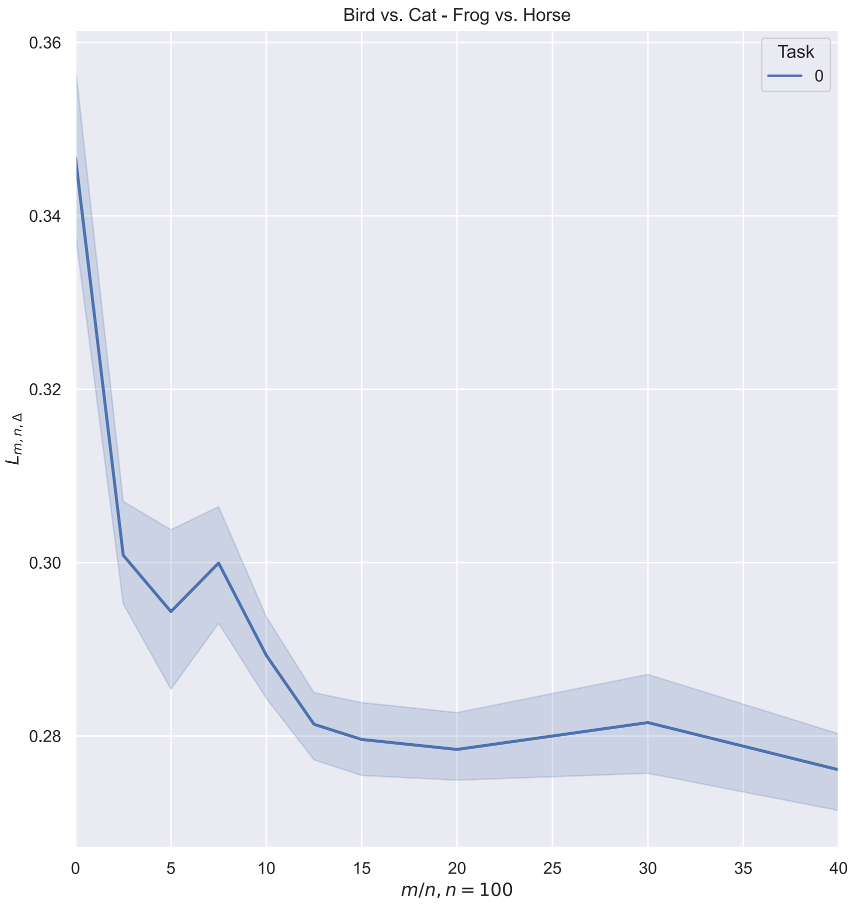
- Number of replicates: 10, Network: Wide Res-Net

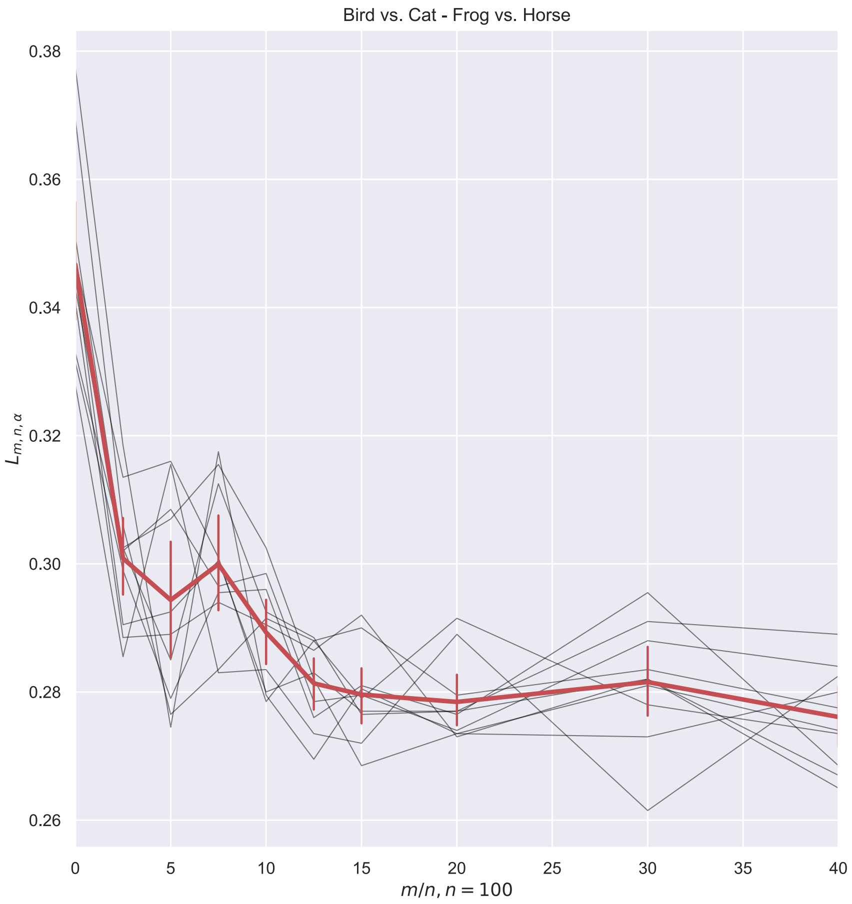




Task 2: Bird vs. Cat & Task 4: Frog vs. Horse (Multi-Head Network)

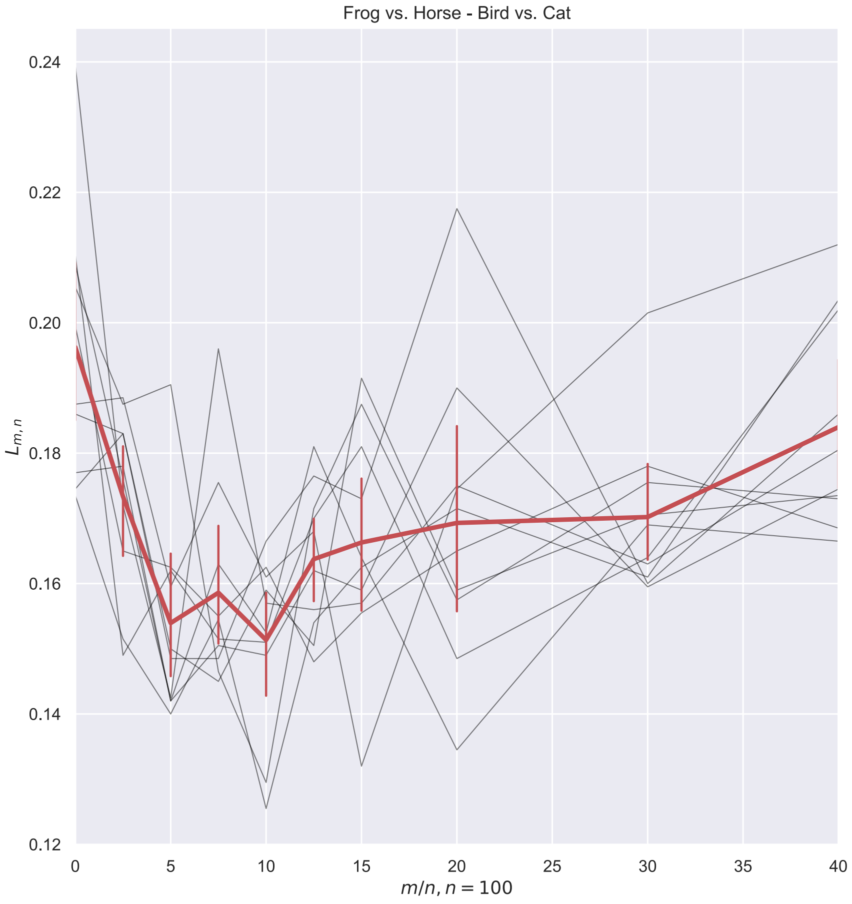
- Number of replicates: 10, Network: Wide Res-Net



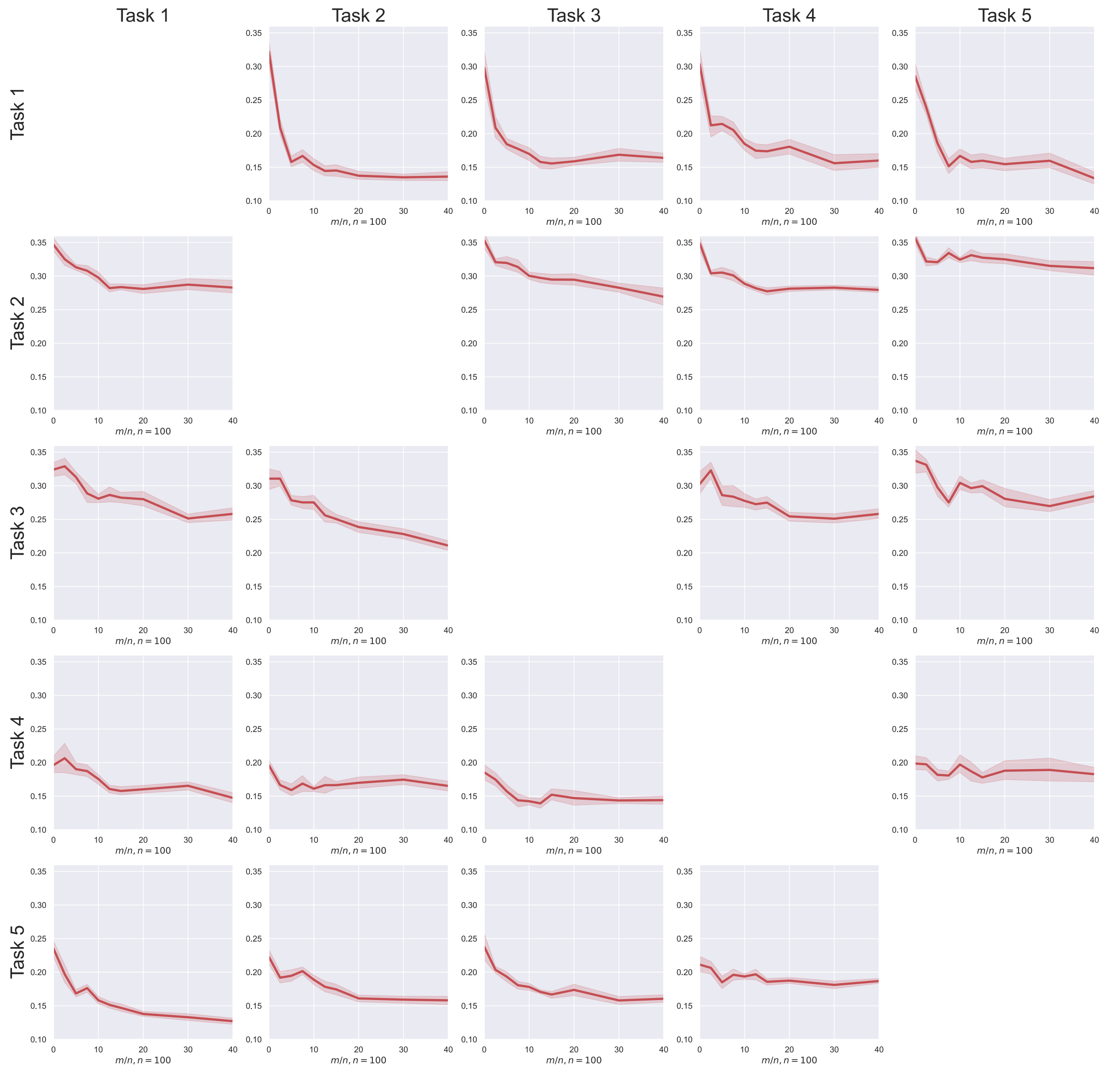


Task 4: Frog vs. Horse & Task 2: Bird vs. Cat (Multi-Head Network)

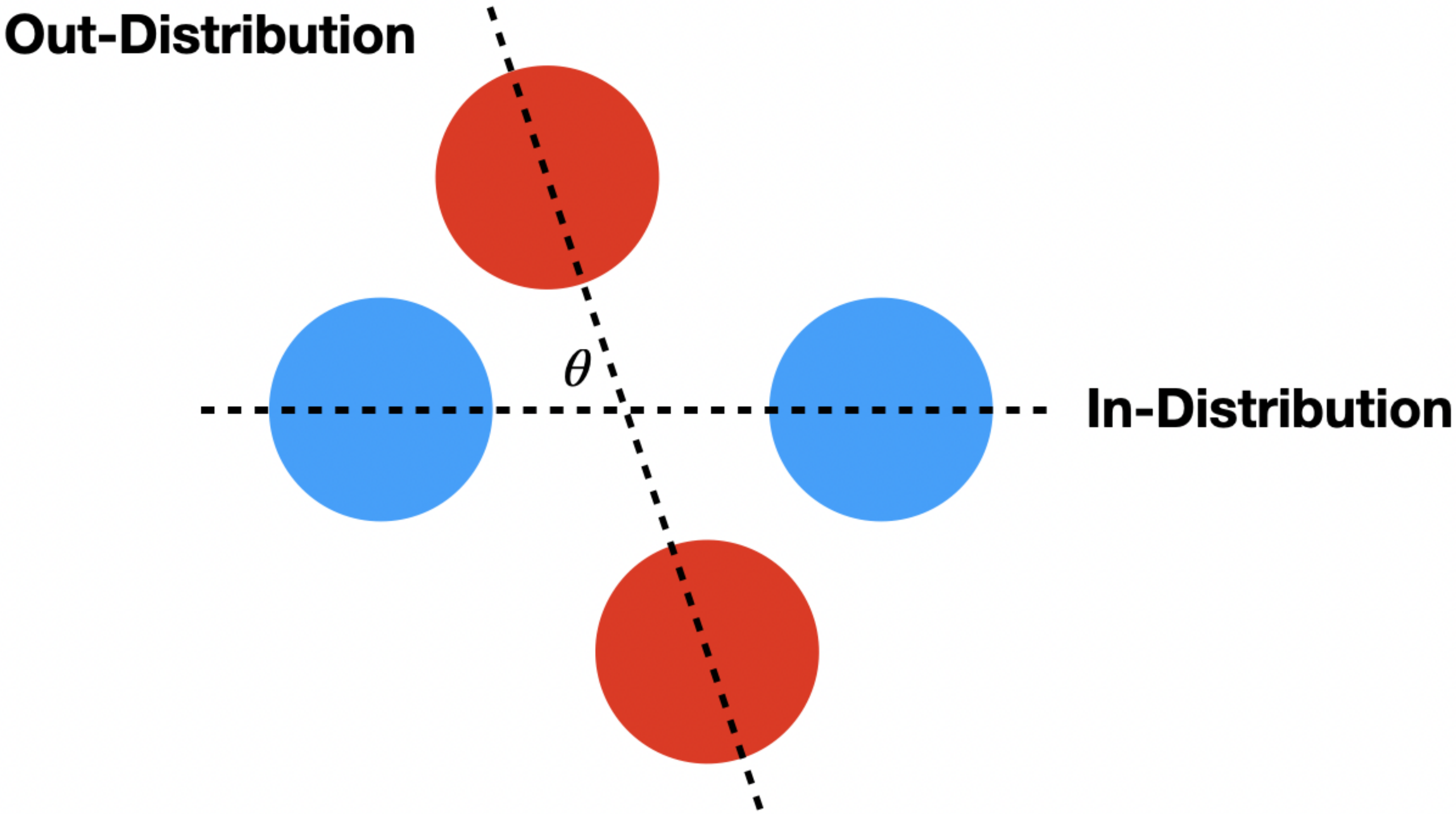
- Number of replicates: 10, Network: Wide Res-Net



CIFAR-10 Tasks (Multi-Head Network)



Bivariate LDA Problem



Bivariate LDA Problem

- $X|Y=-1 \sim \mathcal{N}(-\mu_0, \Sigma)$ and $X|Y=+1 \sim \mathcal{N}(\mu_0, \Sigma)$ consititute the in-distribution where $\mu_0 = [\mu, 0]^\top$
- $X|Y=-1 \sim \mathcal{N}(-\mu_\theta, \Sigma)$ and $X|Y=+1 \sim \mathcal{N}(\mu_\theta, \Sigma)$ consititute the out-of-distribution where $\mu_\theta = [\mu\cos\theta, -\mu\sin\theta]^\top$
- Then, the estimated class means $\hat{\mu}_{-1}$ and $\hat{\mu}_{+1}$ are given by,

$$\hat{\mu}_{-1} \sim \mathcal{N}\left(\left[\frac{-\mu(n+m\cos\theta)}{n+m}, \frac{\mu m\sin\theta}{n+m}\right]^\top, \frac{1}{n+m}\Sigma\right)$$

$$\hat{\mu}_{+1} = -\hat{\mu}_{-1} \sim \mathcal{N}\left(\left[\frac{\mu(n+m\cos\theta)}{n+m}, -\frac{\mu m\sin\theta}{n+m}\right]^\top, \frac{1}{n+m}\Sigma\right)$$

Bivariate LDA Problem

- The LDA's classification rule is given by,

$$g(x) = \text{sign}(w \cdot x > c)$$

  where,

$$w = \Sigma^{-1}(\hat{\mu}_{+1} - \hat{\mu}_{-1}) = 2\Sigma^{-1}\hat{\mu}_{+1}$$
$$c = \frac{1}{2}(\hat{\mu}_{+1} + \hat{\mu}_{-1}) = 0$$

- Therefore,

$$g(x) = \text{sign}(\hat{\mu}_{+1} \cdot x > 0)$$

Bivariate LDA Problem

- If $\mu = 1$ and $\Sigma = I$,

$$\hat{\mu}_{+1} \sim \mathcal{N}\left(\left[\frac{(n + m\cos\theta)}{n+m}, -\frac{m\sin\theta}{n+m}\right]^\top, \frac{1}{n+m}I\right)$$

$$x|y = -1 \sim f_{-1} = \mathcal{N}\left([-1, 0]^\top, \Sigma\right)$$

$$x|y = +1 \sim f_{+1} = \mathcal{N}\left([1, 0]^\top, \Sigma\right)$$

- Hence, the error $L(\hat{\mu}_{+1})$ is given by,

$$L(\hat{\mu}_{+1}) = \mathbb{P}_{x\sim f_{-1}}[\hat{\mu}_{+1} \cdot x > 0] + \mathbb{P}_{x\sim f_{+1}}[\hat{\mu}_{+1} \cdot x < 0]$$

- Therefore,

$$\mathbb{E}[L_{m,n,\theta}] = \mathbb{E}_{\hat{\mu}_{+1}}[L(\hat{\mu}_{+1})]$$