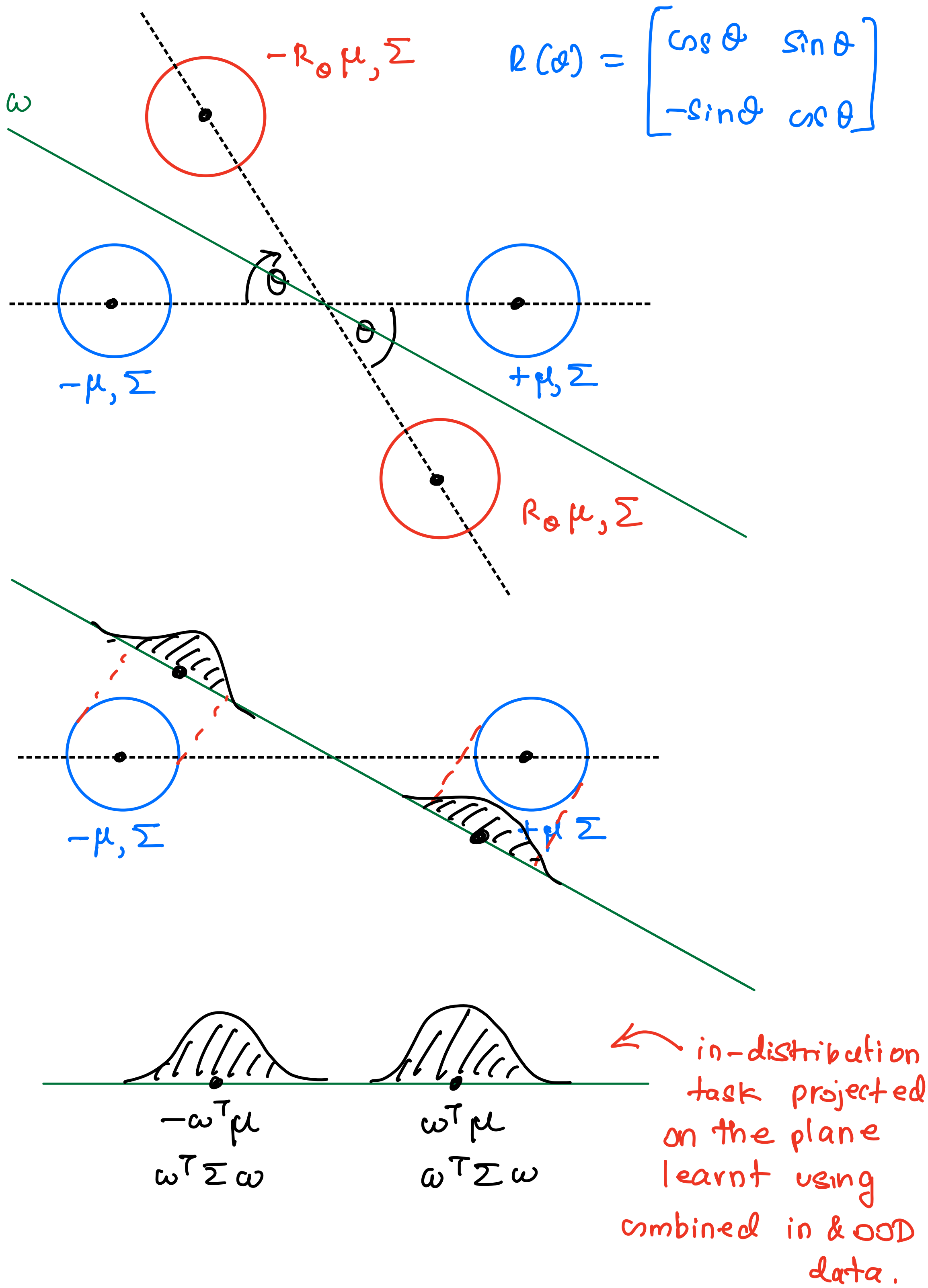


ω
$-R_\theta \mu, \Sigma$
$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
θ
θ
$-\mu, \Sigma$
$+\mu, \Sigma$
$R_\theta \mu, \Sigma$
$-\mu, \Sigma$
$+\mu, \Sigma$
$-\omega^T\mu$
$\omega^T \Sigma \omega$
$\omega^T\mu$
$\omega^T \Sigma \omega$
in-distribution task projected on the plane learnt using combined in & OOD data.

For simplicity, let's assume that projection vector ω is given by,

$$\omega = \hat{M}_1 - \hat{M}_0$$

where,

$$\hat{M}_1 \sim \mathcal{N}\left[\frac{(nI + mR_\theta)\mu}{n+m}, \frac{2}{n+m}\Sigma\right]$$

$$\hat{M}_0 \sim \mathcal{N}\left[\frac{-(nI + mR_\theta)\mu}{n+m}, \frac{2}{n+m}\Sigma\right]$$

$$\therefore \quad \omega \sim \mathcal{N}\left[\frac{(nI + mR_\theta)\mu}{n+m}, \frac{\Sigma}{n+m}\right]$$

The head pertaining to in-distribution is given by,

$$g_{in}(x) = \begin{cases} 1 & \omega^T x > 0 \\ 0 & \text{else} \end{cases}$$

Let $Y = \omega^T X$ where $X \sim \mathcal{N}(\pm\mu, \Sigma)$

$$\therefore \quad Y \sim \mathcal{N}(\pm\, \omega^T \mu, \; \omega^T \Sigma\, \omega)$$

(+)'ve $\Rightarrow$ class 1

(−)'ve $\Rightarrow$ class 0

$$\therefore \quad g_{in}(y) = \begin{cases} 1 & y > 0 \\ 0 & \text{else} \end{cases}$$

$$\therefore h(\omega) = \frac{1}{2}\left[1 - \Phi\left(\frac{\omega^T\mu}{\omega^T\Sigma\omega}\right) + \Phi\left(\frac{-\omega^T\mu}{\omega^T\Sigma\omega}\right)\right]$$

Since $\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$ where $\omega_1 \sim \mathcal{N}\left(\frac{n+m\cos\theta}{n+m}, \frac{1}{n+m}\right)$

$\omega_2 \sim \mathcal{N}\left(\frac{-m\sin\theta}{n+m}, \frac{1}{n+m}\right)$

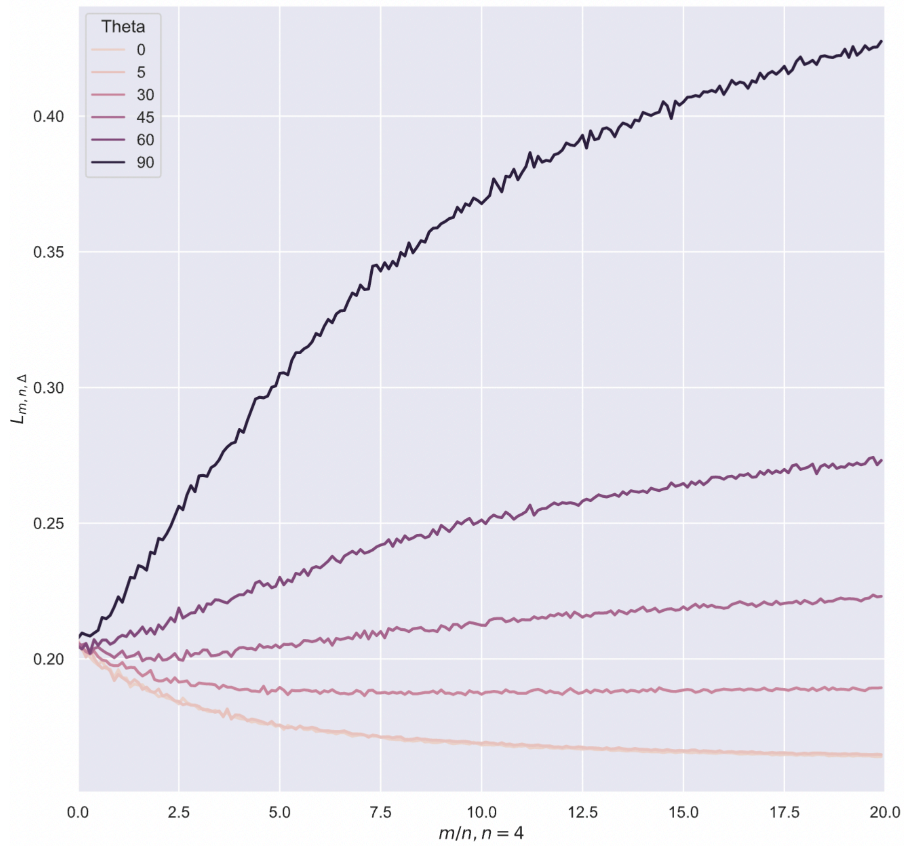
& $\mu = \begin{bmatrix} \mu \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

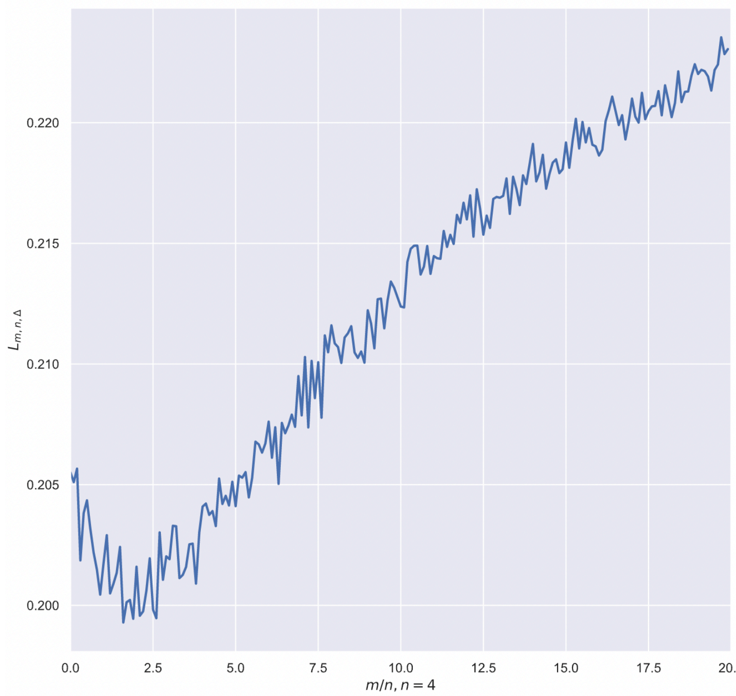
& $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$h(\omega_1, \omega_2) = \frac{1}{2}\left[1 - \Phi\left(\frac{\omega_1}{\omega_1^2 + \omega_2^2}\right) + \Phi\left(\frac{-\omega_1}{\omega_1^2 + \omega_2^2}\right)\right]$$

where, $\omega_1 \sim \mathcal{N}\left(\frac{n+m\cos\theta}{n+m}, \frac{1}{n+m}\right)$

$\omega_2 \sim \mathcal{N}\left(\frac{-m\sin\theta}{n+m}, \frac{1}{n+m}\right)$


Theta
0
5
30
45
60
90
0.40
0.35
0.30
0.25
0.20
$L_{m,n,\Delta}$
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
$m/n, n = 4$


0.220
0.215
0.210
0.205
0.200
$L_{m,n,\Delta}$
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
$m/n, n = 4$

$\theta = 45°$