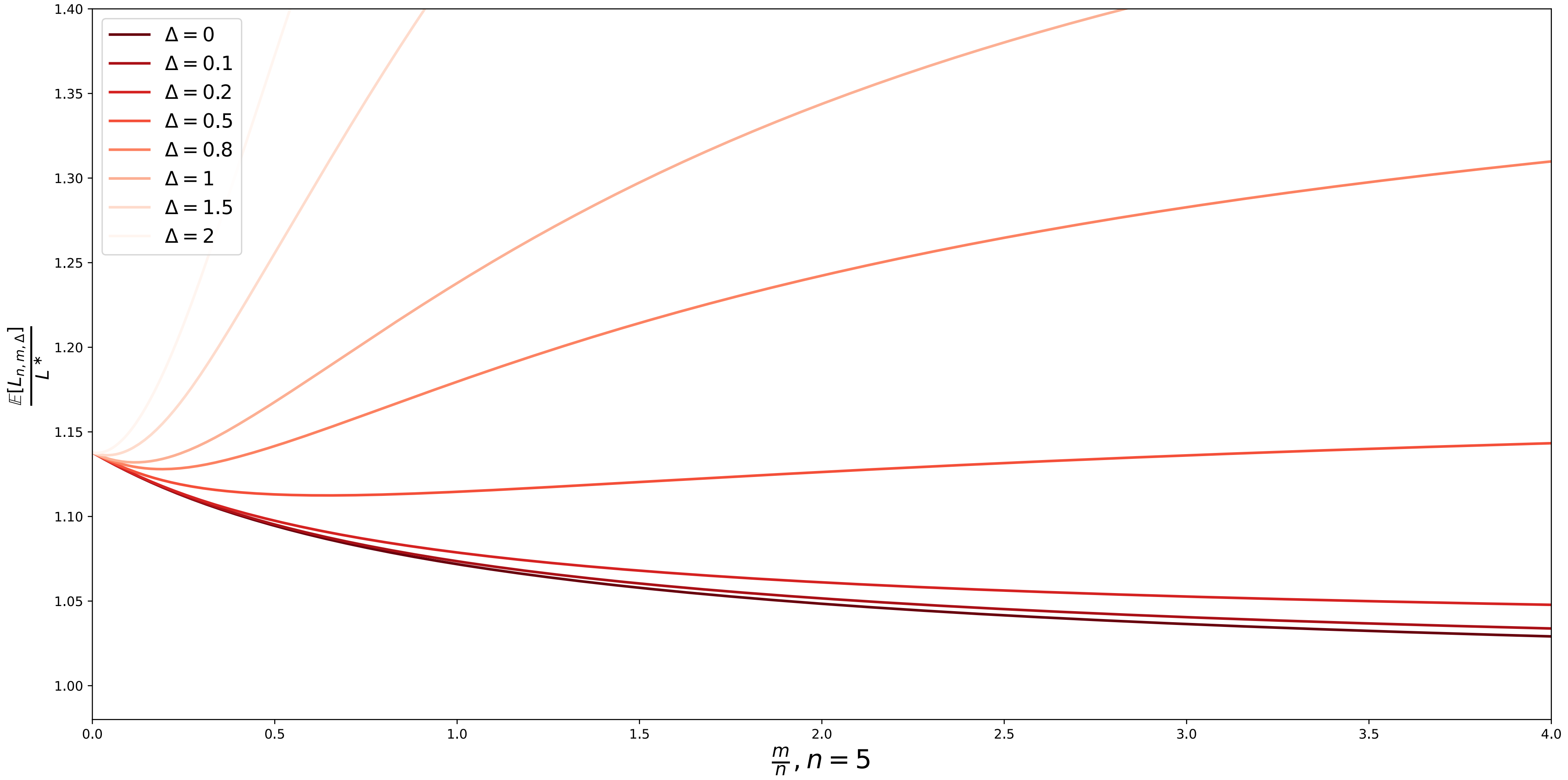
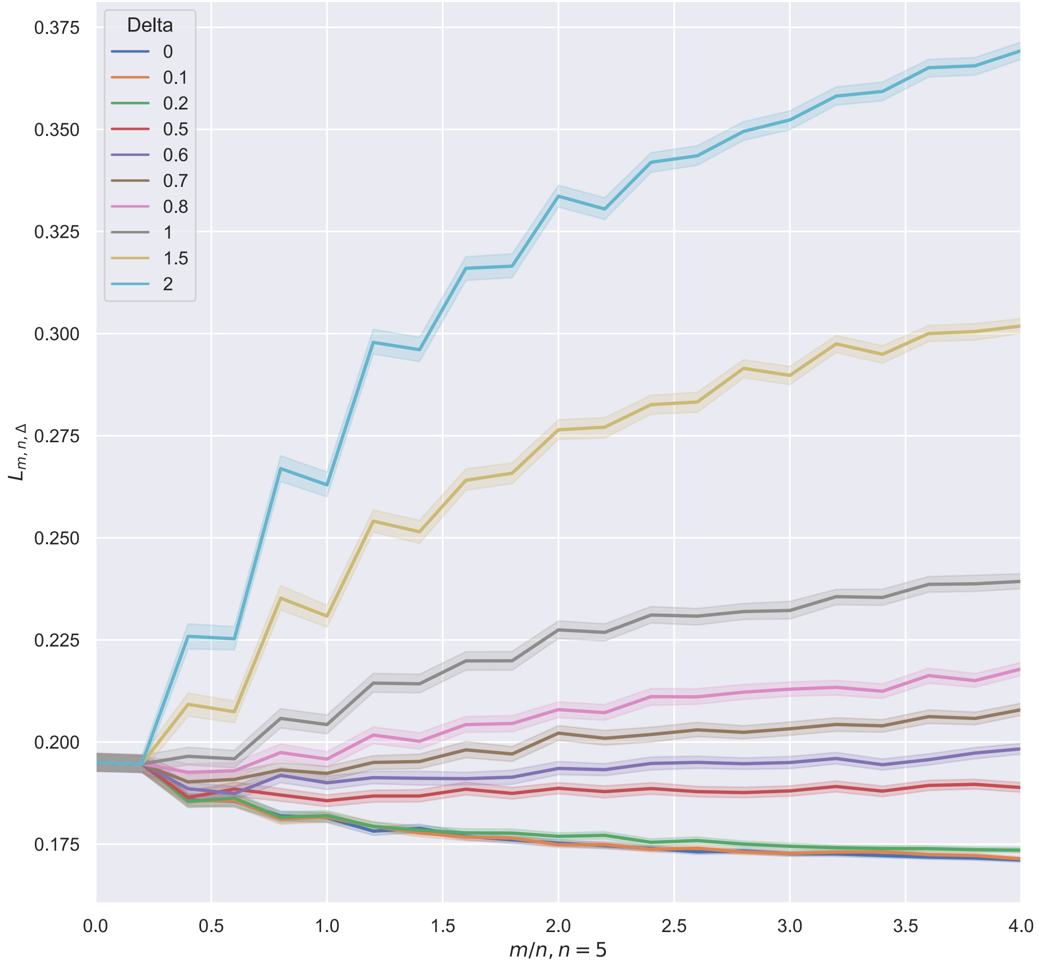
Out-of-Distribution Learning

Gaussian Tasks Experiment



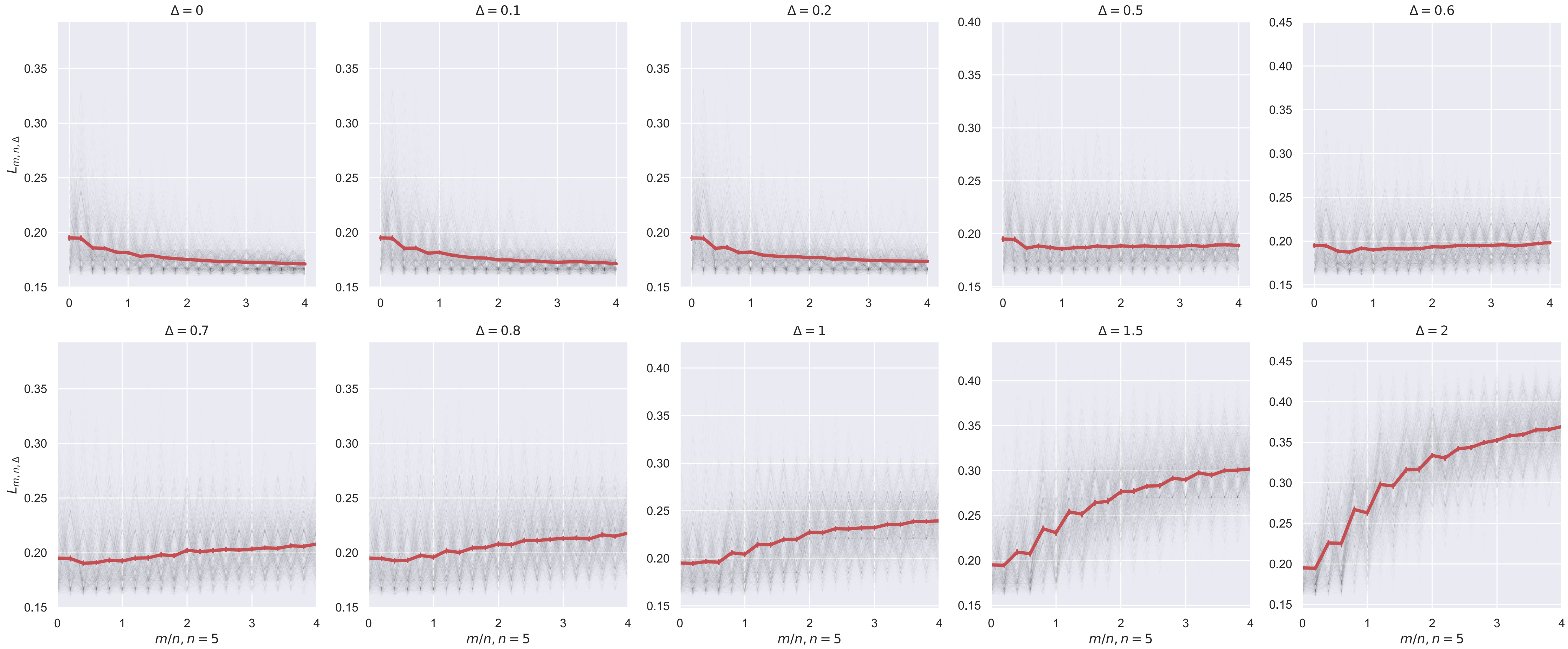
Gaussian Tasks Experiment

- Number of replicates: 1000



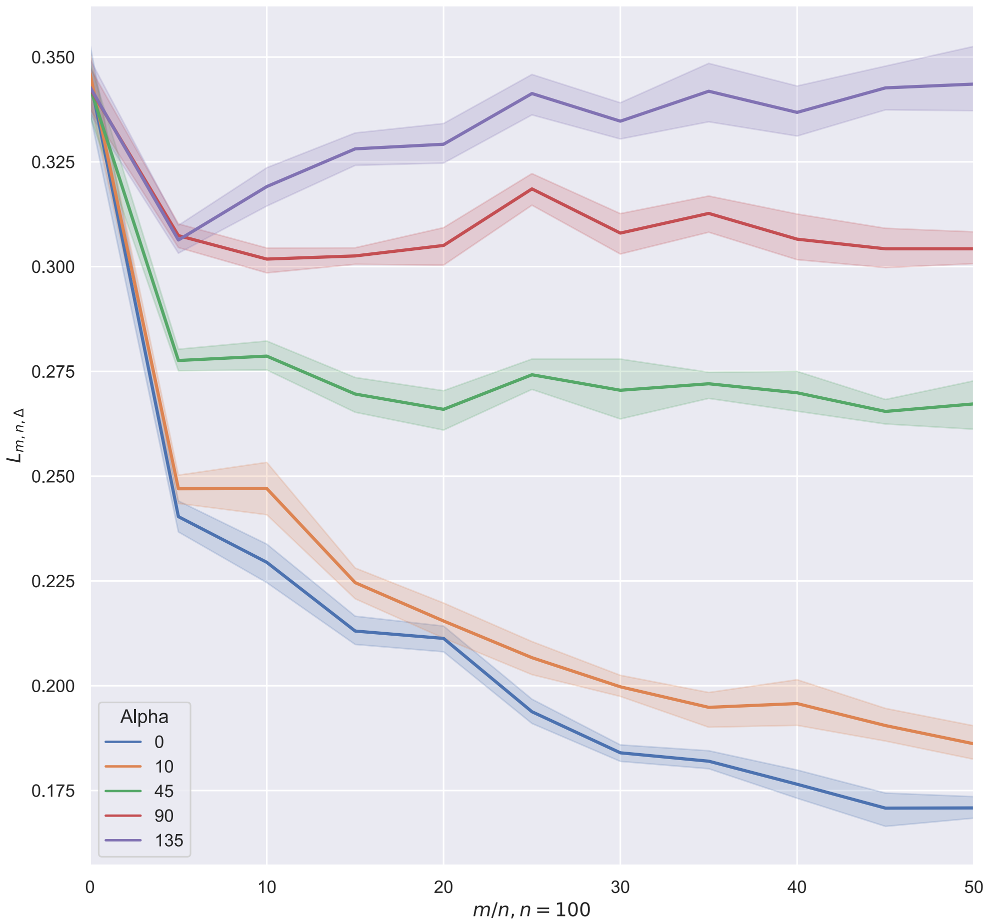
Gaussian Tasks Experiment

- Number of replicates: 1000



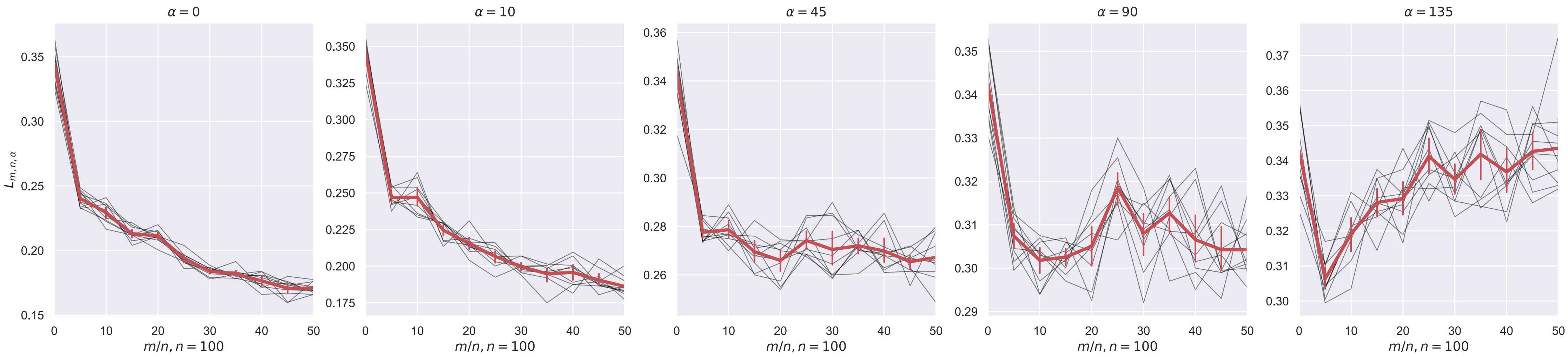
Bird vs. Cat & α-Rotated Bird vs. Cat (Single-Head Network)

- Number of replicates: 10, Network: SmallConv



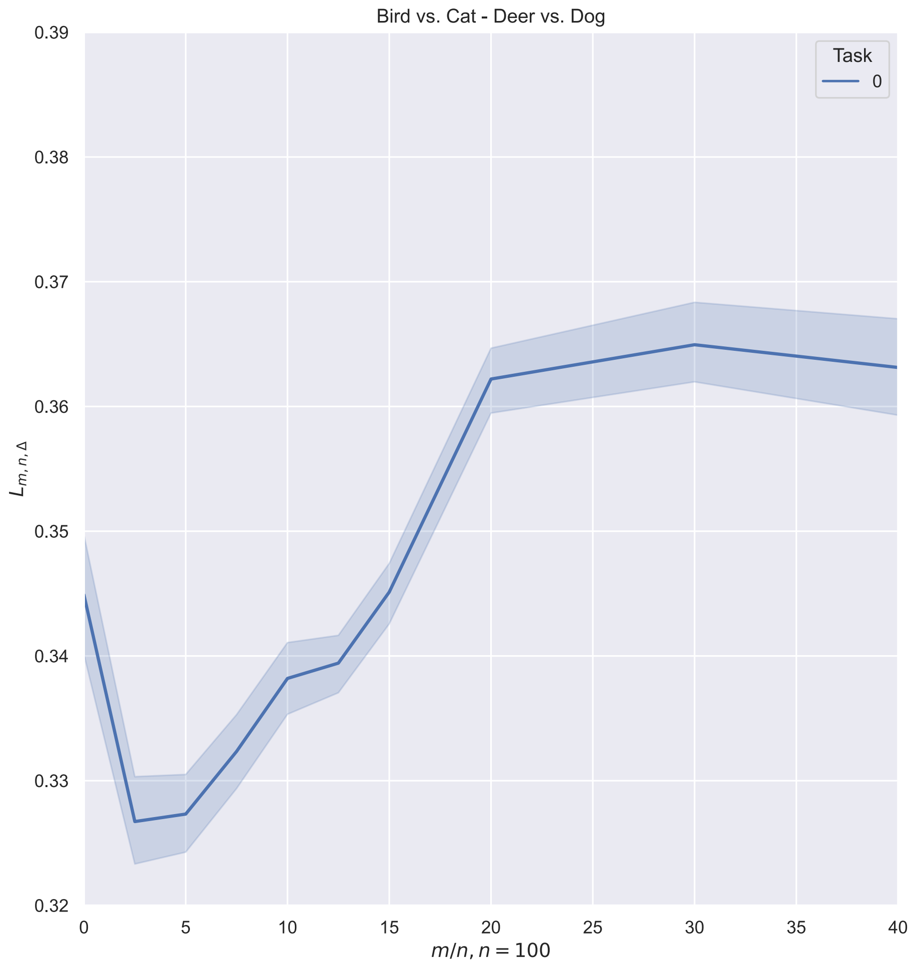
Bird vs. Cat & α-Rotated Bird vs. Cat (Single-Head Network)

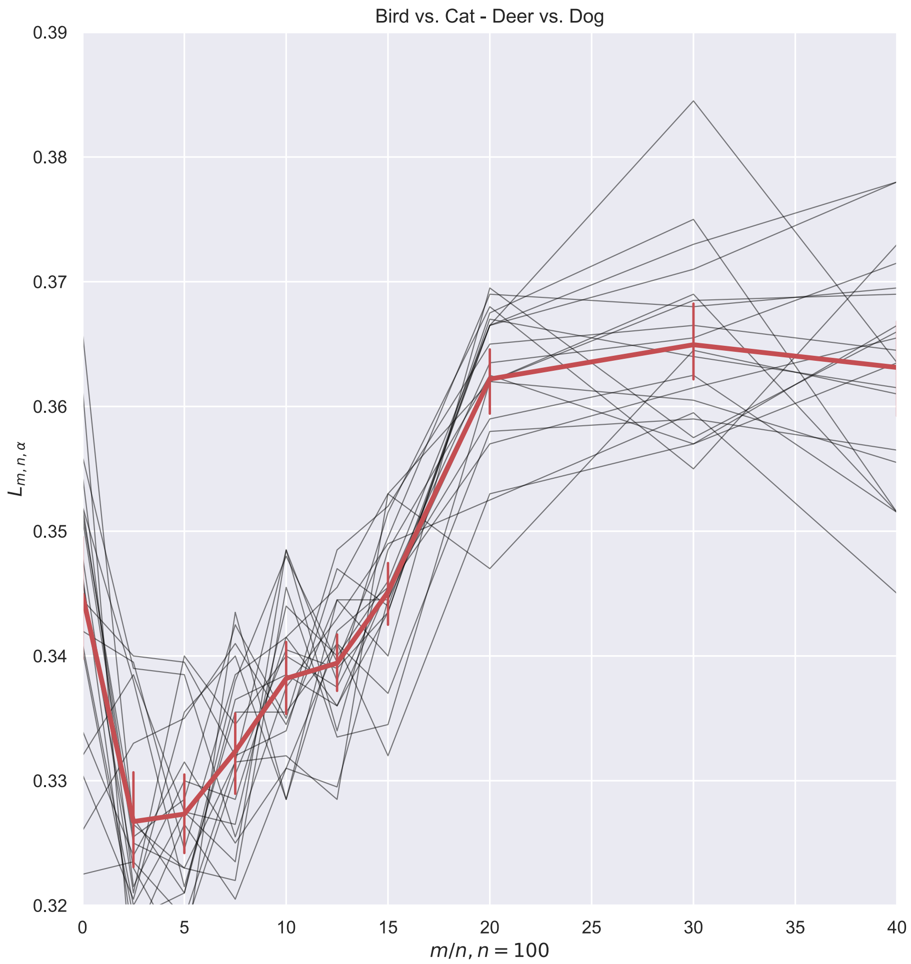
- Number of replicates: 10, Network: SmallConv



Task 2: Bird vs. Cat & Task 3: Deer vs. Dog (Single-Head Network)

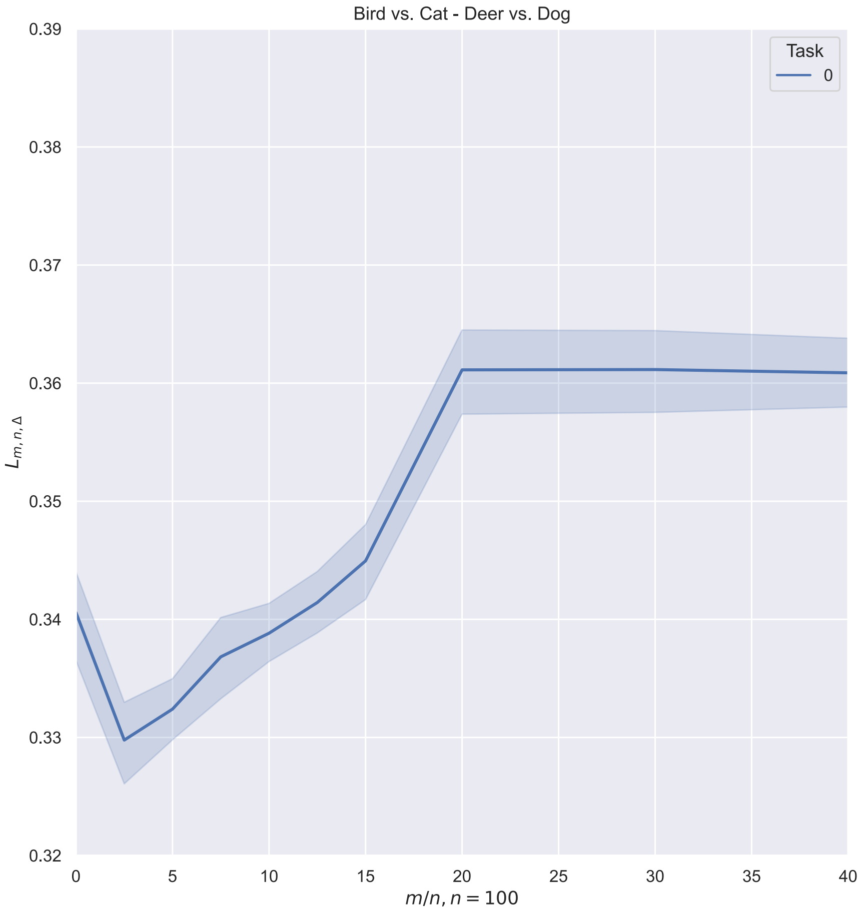
- Number of replicates: 20, Network: SmallConv

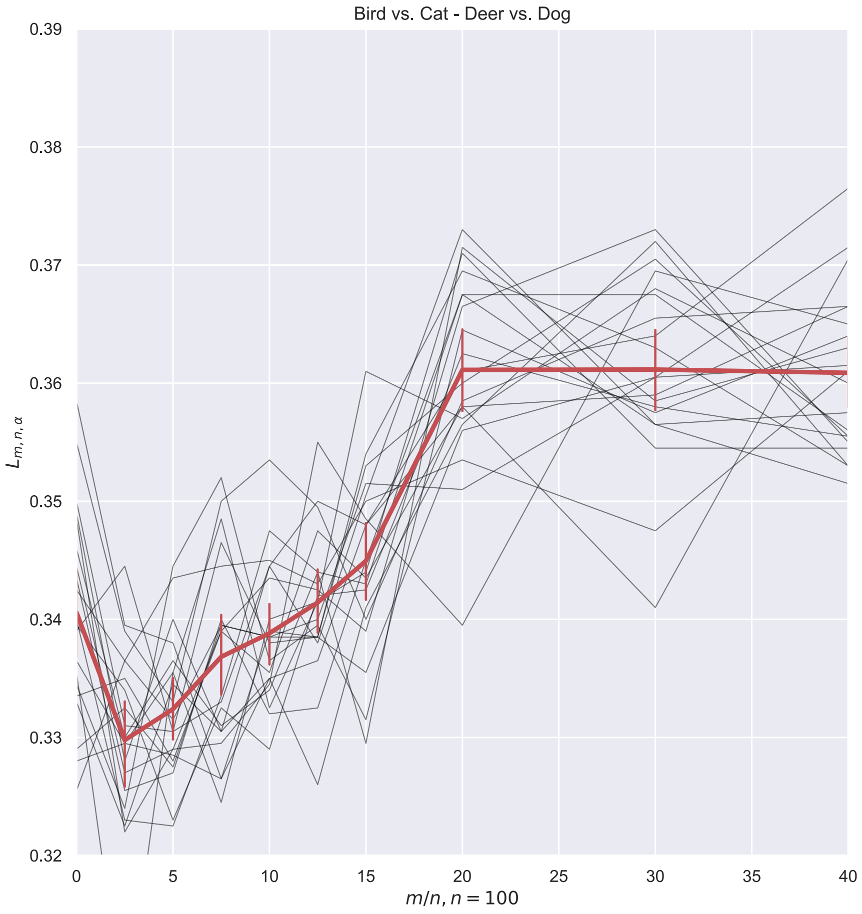




Task 2: Bird vs. Cat & Task 3: Deer vs. Dog (Single-Head Network)

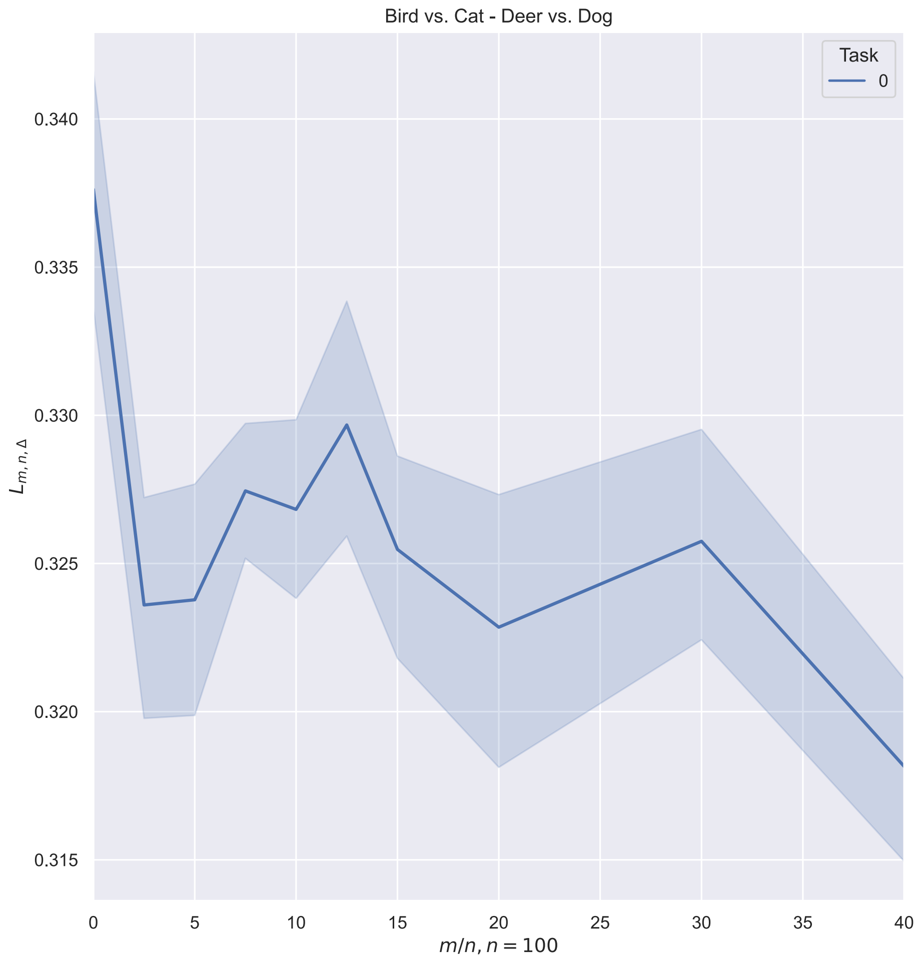
- Number of replicates: 20, Network: SmallConv, each model was trained for 100 epochs

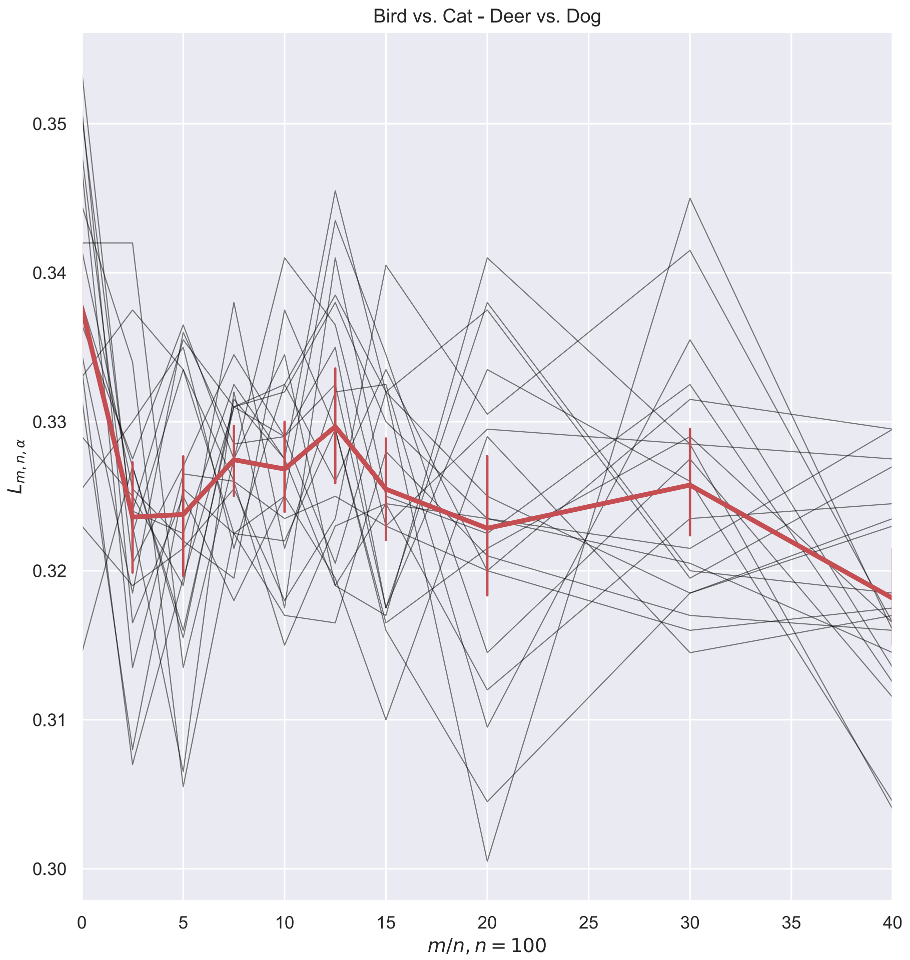




Task 2: Bird vs. Cat & Task 3: Deer vs. Dog (Multi-Head Network)

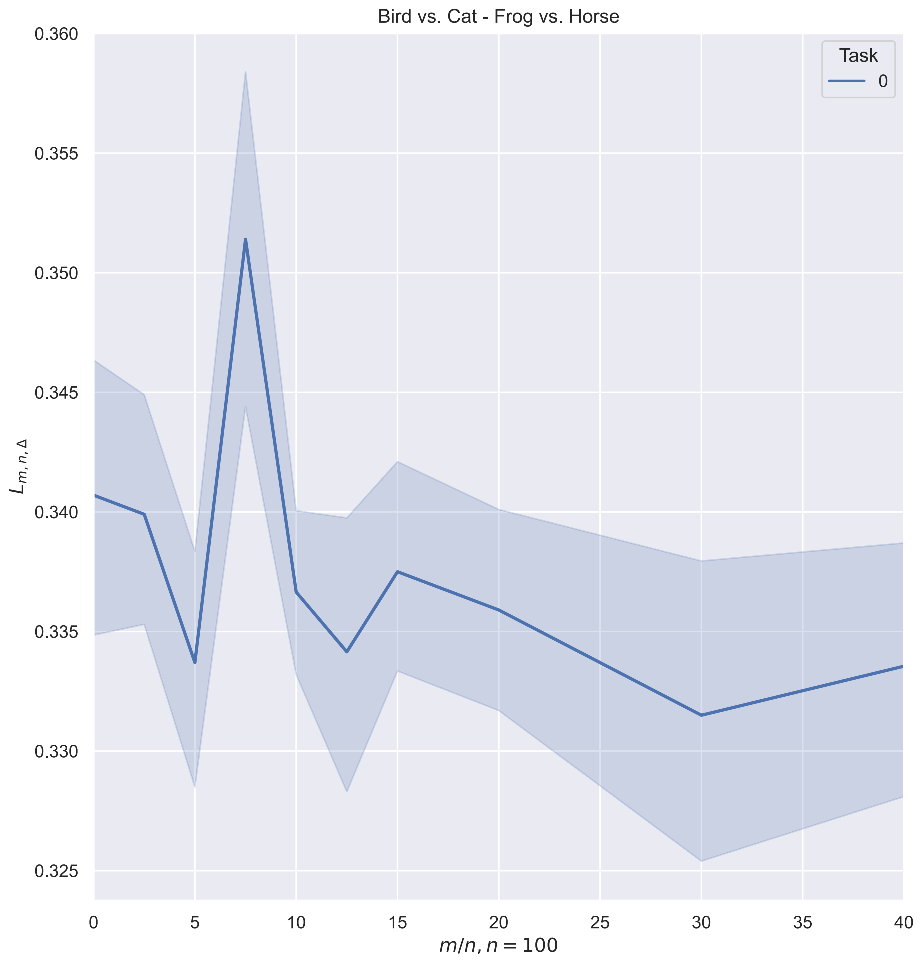
- Number of replicates: 20, Network: SmallConv

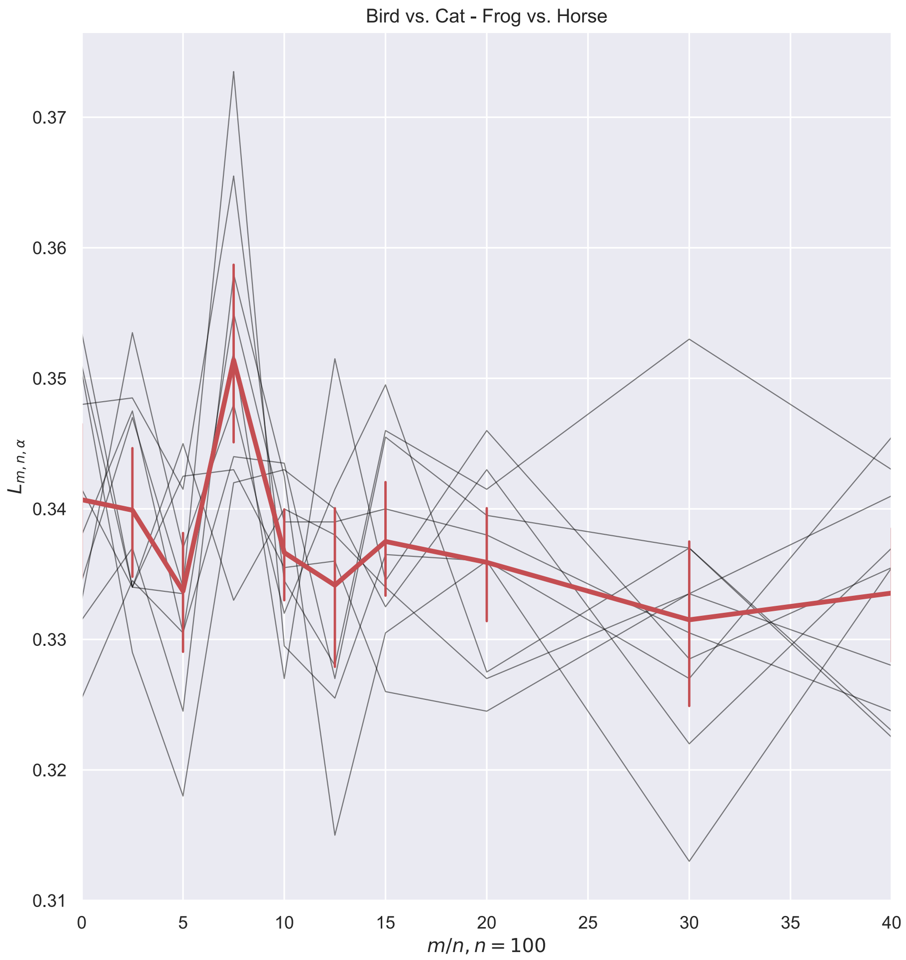




Task 2: Bird vs. Cat & Task 4: Frog vs. Horse (Multi-Head Network)

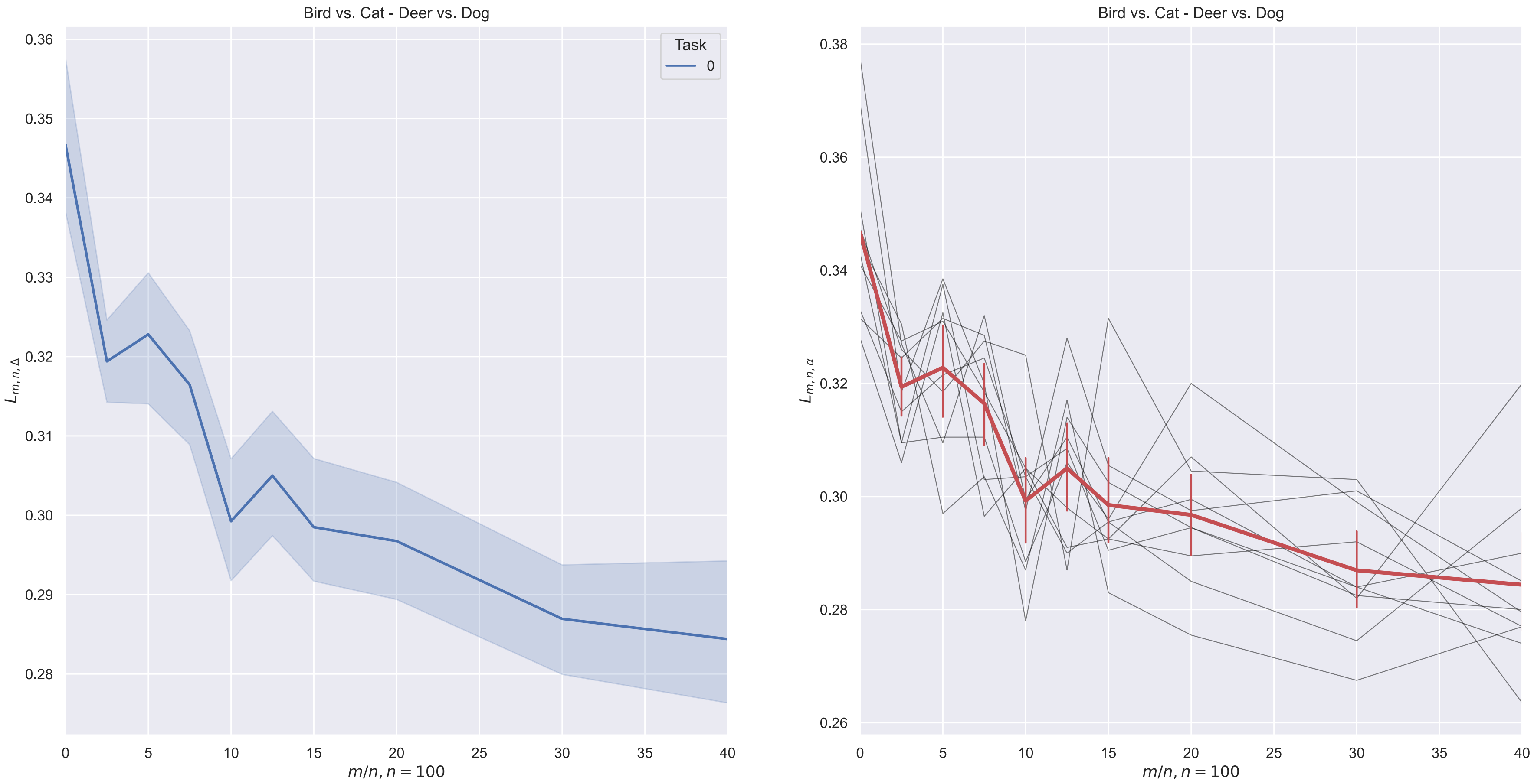
- Number of replicates: 20, Network: SmallConv



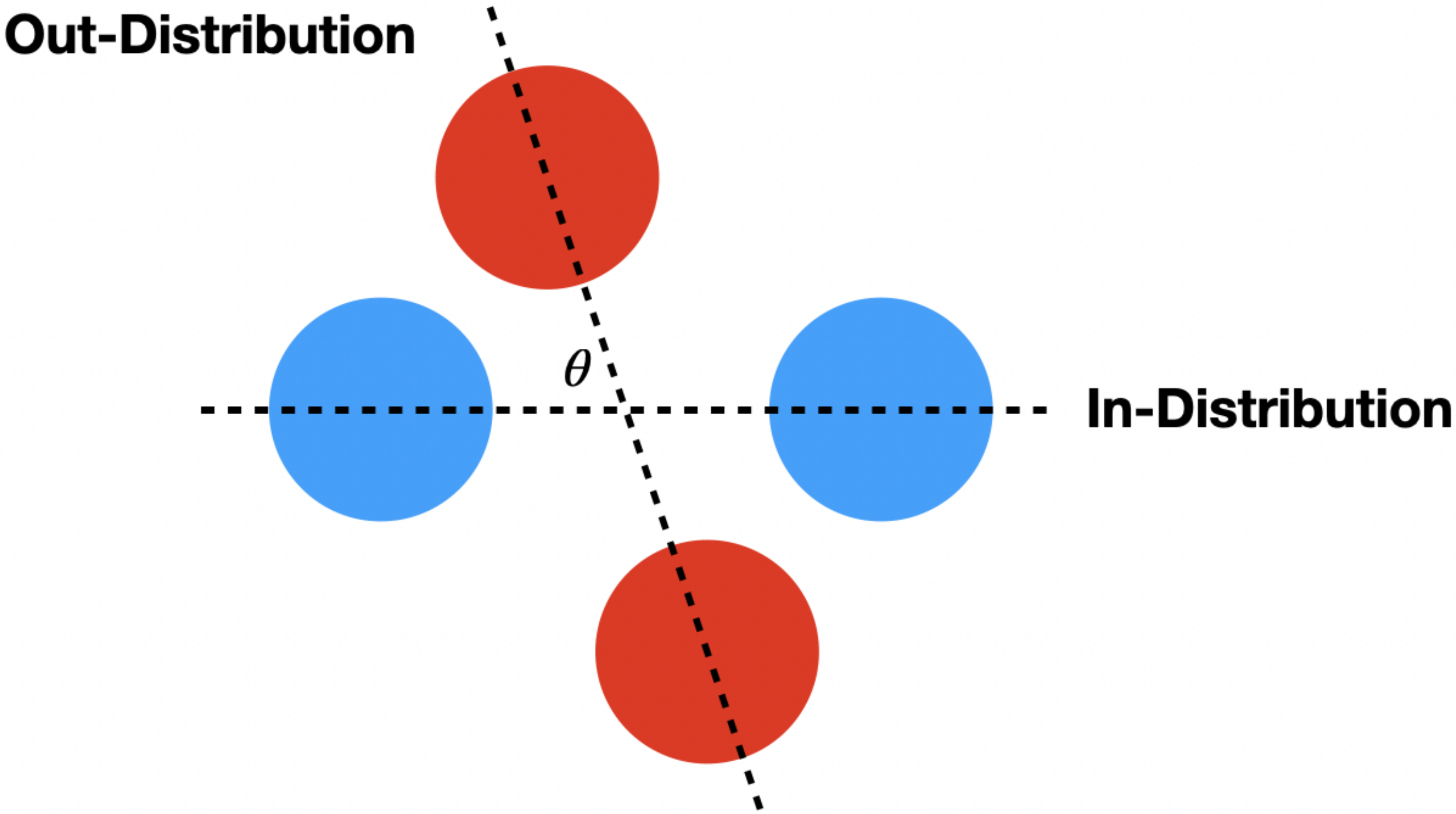


Task 2: Bird vs. Cat & Task 3: Deer vs. Dog (Multi-Head Network)

- Number of replicates: 10, Network: Wide Res-Net



Bivariate LDA Problem



Bivariate LDA Problem

- $X|Y=-1 \sim \mathcal{N}(-\mu_0, \Sigma)$ and $X|Y=+1 \sim \mathcal{N}(\mu_0, \Sigma)$ consititute the in-distribution where $\mu_0 = [\mu, 0]^\top$
- $X|Y=-1 \sim \mathcal{N}(-\mu_\theta, \Sigma)$ and $X|Y=+1 \sim \mathcal{N}(\mu_\theta, \Sigma)$ consititute the out-of-distribution where $\mu_\theta = [\mu\cos\theta, -\mu\sin\theta]^\top$
- Then, the estimated class means $\hat{\mu}_{-1}$ and $\hat{\mu}_{+1}$ are given by,

$$\hat{\mu}_{-1} \sim \mathcal{N}\left(\left[\frac{-\mu(n+m\cos\theta)}{n+m}, \frac{\mu m\sin\theta}{n+m}\right]^\top, \frac{1}{n+m}\Sigma\right)$$

$$\hat{\mu}_{+1} = -\hat{\mu}_{-1} \sim \mathcal{N}\left(\left[\frac{\mu(n+m\cos\theta)}{n+m}, -\frac{\mu m\sin\theta}{n+m}\right]^\top, \frac{1}{n+m}\Sigma\right)$$

Bivariate LDA Problem

- The LDA's classification rule is given by,

$$g(x) = \mathrm{sign}(w \cdot x > c)$$

  where,

$$w = \Sigma^{-1}\left(\hat{\mu}_{+1} - \hat{\mu}_{-1}\right) = 2\Sigma^{-1}\hat{\mu}_{+1}$$

$$c = \frac{1}{2}\left(\hat{\mu}_{+1} + \hat{\mu}_{-1}\right) = 0$$

- Therefore,

$$g(x) = \mathrm{sign}(\hat{\mu}_{+1} \cdot x > 0)$$

Bivariate LDA Problem

- If $\mu = 1$ and $\Sigma = I$,

$$\hat{\mu}_{+1} \sim \mathcal{N}\left(\left[\frac{(n + m\cos\theta)}{n+m}, -\frac{m\sin\theta}{n+m}\right]^\top, \frac{1}{n+m}I\right)$$

$$x|y = -1 \sim f_{-1} = \mathcal{N}\left([-1, 0]^\top, \Sigma)\right)$$

$$x|y = +1 \sim f_{+1} = \mathcal{N}\left([1, 0]^\top, \Sigma)\right)$$

- Hence, the error $L(\hat{\mu}_{+1})$ is given by,

$$L(\hat{\mu}_{+1}) = \mathbb{P}_{x\sim f_{-1}}[\hat{\mu}_{+1} \cdot x > 0] + \mathbb{P}_{x\sim f_{+1}}[\hat{\mu}_{+1} \cdot x < 0]$$

- Therefore,

$$\mathbb{E}[L_{m,n,\theta}] = \mathbb{E}_{\hat{\mu}_{+1}}[L(\hat{\mu}_{+1})]$$