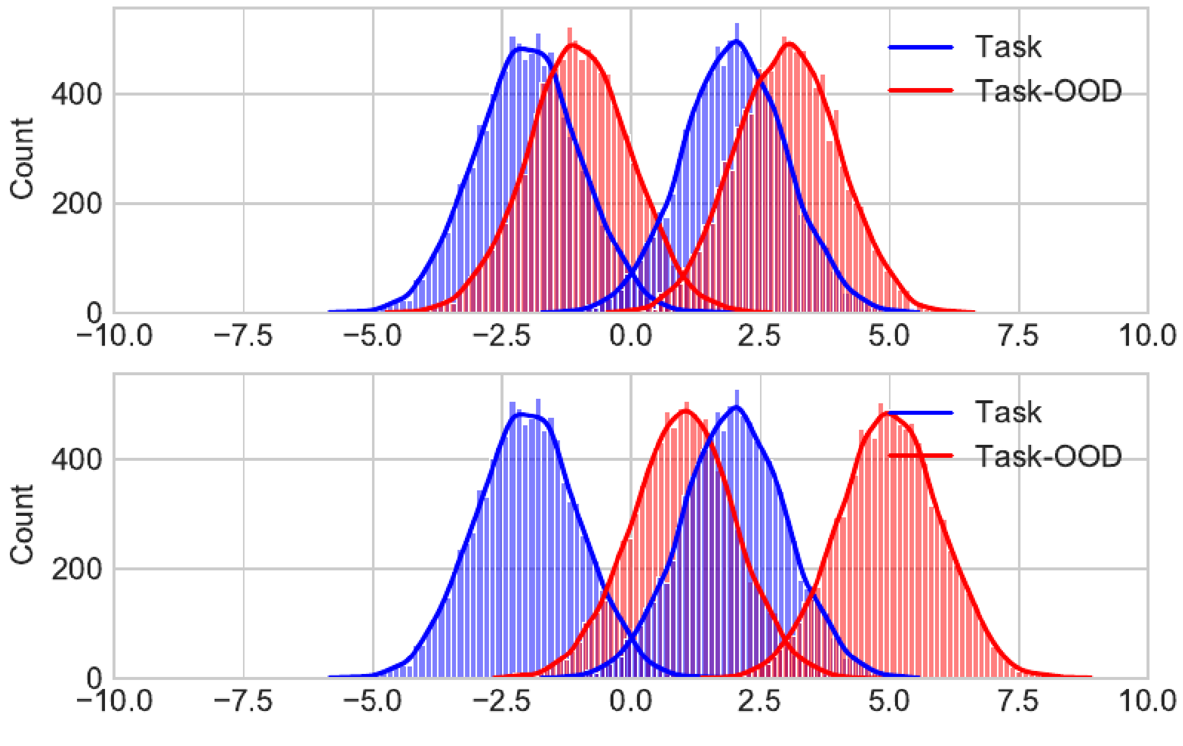
Gaussian Task

We consider a family of tasks

$$P(x, y) = \delta(y = 1)X_1 + \delta(y = -1)X_{-1}$$

where X_1 and X_2 are Gaussians with means $\pm\mu$



Error of Hypothesis

Consider random variable $\bar{h}$. If $\bar{h} = h$, then

$$e_t(h) = \frac{1}{2}\left((1 - \Phi(h+\mu) + \Phi(h-\mu)\right).$$

If we assume $\bar{h} \sim \mathcal{N}(\bar{\mu}, \bar{\sigma})$

$$\mathbb{E}[e_t(\bar{h})] = \frac{1}{2} - \frac{1}{2}\Phi\left(\frac{\bar{\mu}+\mu}{\sqrt{1+\bar{\sigma}^2}}\right) + \frac{1}{2}\Phi\left(\frac{\bar{\mu}-\mu}{\sqrt{1+\bar{\sigma}^2}}\right)$$

LDA - Single-head model

Consider the samples to be weighted as

$$S = \alpha S_t + (1 - alpha) S_{ood}$$

For LDA

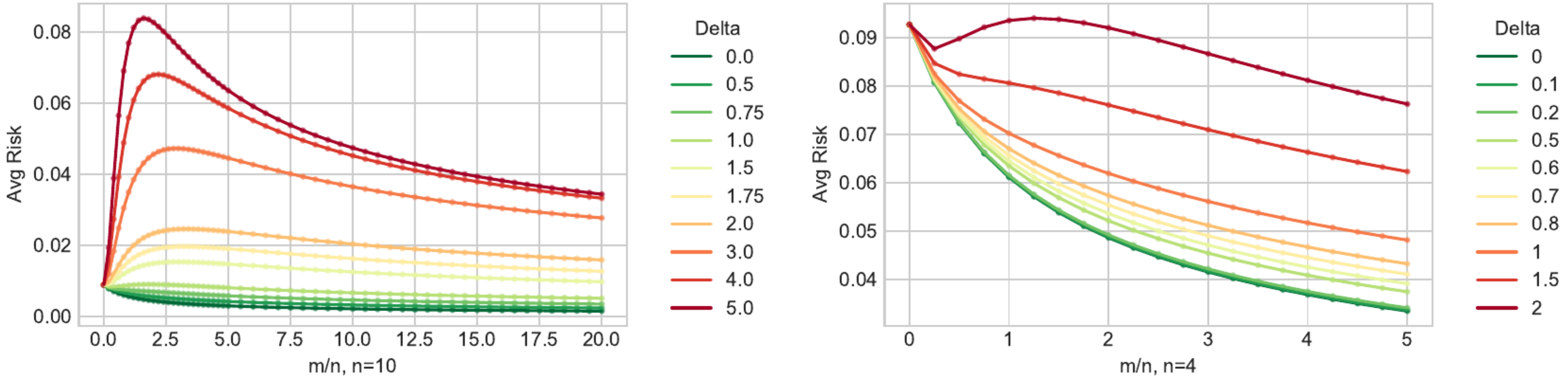
$$\bar{\mu} = \frac{(1-\alpha) m \Delta}{\alpha n + (1-\alpha) m}$$

and

$$\bar{\sigma}^2 = \frac{(1-\alpha)^2 m + \alpha^2 n}{(\alpha n + (1-\alpha) m)^2}$$

LDA with $\alpha = 0.5$

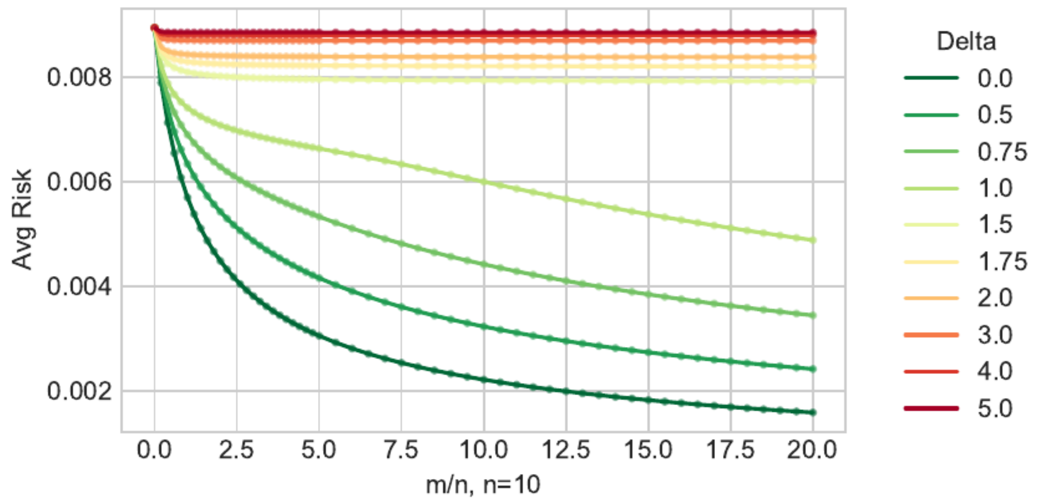
When both datasets are naively combined $(\alpha = 0.5)$

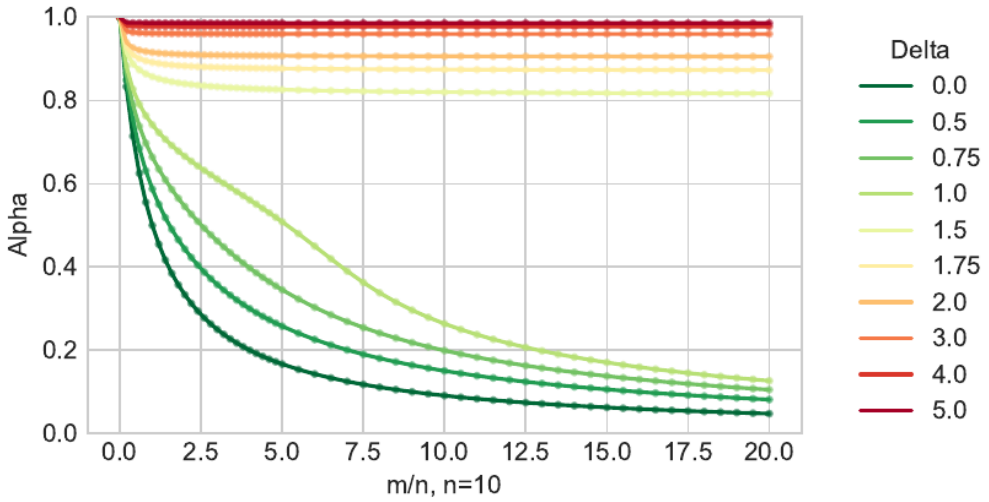


the loss increases/decreases depending on Δ.

LDA with optimized α

However if we optimize α, the loss is always better





Note on optimizing α

α is only usable if we can seperate out samples in S_{ood} and S_t. Otheriwse, we are forced to use $\alpha = 0.5$.