A conversational recommender system for medical diagnosis using fuzzy logic

Example for Conversational Recommender using FCA

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Abstract

This file shows an extended toy example of the use of formal concept analysis and logic-based tools to make a conversational recommender system whose purpose is medical diagnosis.

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Contents

a	b	c	d	e	f	D1	D2
1.0	0.5	0.0	0.0	0.0	0.5	0	1
1.0	0.5	0.5	0.5	1.0	0.0	0	1
0.5	1.0	0.0	1.0	0.5	1.0	1	0
1.0	0.5	0.0	0.0	0.0	1.0	0	1
0.0	1.0	1.0	0.0	1.0	0.0	1	0
0.0	0.5	1.0	0.0	0.0	0.0	0	1
1.0	0.5	1.0	1.0	1.0	0.0	0	1
1.0	0.0	1.0	1.0	1.0	0.5	1	0

Table 1: Dataset used in the examples.

1 Introduction

In this file we present an example of the execution traces of the recommender system presented in the paper *A conversational recommender system for medical diagnosis using fuzzy logic*, which is under revision for its publication.

The aim is to present a guided and detailed trace of several of the features studied in the mentioned paper.

2 Dataset

In Table 1 we show the dataset used in these examples. The dataset simulate a set of 8 subjects and 6 symptom attributes with degrees ranging in $[0, 1]$. The symptoms are named a, b, c, d, e, f. Two possible diseases are denoted as D1 and D2.

3 Rule extraction

As in the paper, we use the NextClosure algorithm to extract a set of implications that comprises all the knowledge contained in the data table.

The resulting set of 21 implications is:

$$\{D2\} \Rightarrow \{^{0.5}/b\}$$
$$\{D1\} \Rightarrow \{^{0.5}/e\}$$
$$\{^{0.5}/f\} \Rightarrow \{^{0.5}/a\}$$
$$\{^{0.5}/d\} \Rightarrow \{^{0.5}/a,\ ^{0.5}/e\}$$
$$\{^{0.5}/c,\ ^{0.5}/e\} \Rightarrow \{e\}$$
$$\{^{0.5}/c,\ e,\ D1\} \Rightarrow \{c\}$$
$$\{^{0.5}/b,\ ^{0.5}/e,\ D2\} \Rightarrow \{a,\ ^{0.5}/c,\ ^{0.5}/d,\ e\}$$
$$\{^{0.5}/a,\ ^{0.5}/d,\ ^{0.5}/e,\ ^{0.5}/f\} \Rightarrow \{d,\ D1\}$$
$$\{^{0.5}/a,\ ^{0.5}/d,\ e\} \Rightarrow \{a\}$$
$$\{^{0.5}/a,\ ^{0.5}/c\} \Rightarrow \{a,\ ^{0.5}/d,\ e\}$$
$$\{^{0.5}/a,\ ^{0.5}/b,\ D2\} \Rightarrow \{a\}$$
$$\{^{0.5}/a,\ b,\ ^{0.5}/f\} \Rightarrow \{f\}$$
$$\{^{0.5}/a,\ b,\ ^{0.5}/d,\ ^{0.5}/e\} \Rightarrow \{d\}$$
$$\{a,\ ^{0.5}/d,\ ^{0.5}/e\} \Rightarrow \{e\}$$

$$\{^{0.5}/b,\ ^{0.5}/e,\ D1\} \Rightarrow \{b\} \qquad \{a,\ ^{0.5}/c,\ d,\ e\} \Rightarrow \{c\}$$
$$\{b,\ ^{0.5}/c\} \Rightarrow \{c\} \qquad \{a,\ c,\ ^{0.5}/d,\ e\} \Rightarrow \{d\}$$
$$\{^{0.5}/a,\ ^{0.5}/e\} \Rightarrow \{^{0.5}/d\} \qquad \{a,\ ^{0.5}/b,\ ^{0.5}/c,\ ^{0.5}/d,\ e\} \Rightarrow \{D2\}$$
$$\{^{0.5}/a,\ ^{0.5}/d,\ ^{0.5}/e,\ D1\} \Rightarrow \{d,\ ^{0.5}/f\}$$

This set of implications is used as the core knowledge of a Conversational recommender.

We'll use the following subject data as starting point in the example conversations: $\{a,\ ^{0.5}/b,\ ^{0.5}/c,\ ^{0.5}/d,\ e,\ D2\}$

4 Examples

In this section, we present some exampes of executing the conversational recommender proposed in the mentioned paper with different configurations, depending on elicitation and critiquing strategies.

4.1 Example with random elicitation

The trace of the execution when the *random* elicitation is used is as follows:

Subject: $\{a,\ ^{0.5}/b,\ ^{0.5}/c,\ ^{0.5}/d,\ e,\ D2\}$

The elicitation preference computed for this subject is a, e, b, c, d.

Iteration 1:

In this iteration, the elicited attribute/value pair is $a = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a\}$

The computed closure is:

$\{a\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$
\begin{array}{rcl@{\qquad}rcl}
\{^{0.5}/d\} & \Rightarrow & \{^{0.5}/e\} & \{^{0.5}/c,\ d,\ e\} & \Rightarrow & \{c\} \\
\{^{0.5}/b,\ ^{0.5}/e,\ D2\} & \Rightarrow & \{^{0.5}/c,\ ^{0.5}/d,\ e\} & \{c,\ ^{0.5}/d,\ e\} & \Rightarrow & \{d\} \\
\{^{0.5}/e\} & \Rightarrow & \{^{0.5}/d\} & \{^{0.5}/b,\ ^{0.5}/c,\ ^{0.5}/d,\ e\} & \Rightarrow & \{D2\} \\
\{^{0.5}/d,\ ^{0.5}/e,\ D1\} & \Rightarrow & \{d,\ ^{0.5}/f\} & \{D2\} & \Rightarrow & \{^{0.5}/b\} \\
\{^{0.5}/d,\ ^{0.5}/e,\ ^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{D1\} & \Rightarrow & \{^{0.5}/e\} \\
\{^{0.5}/c\} & \Rightarrow & \{^{0.5}/d,\ e\} & \{^{0.5}/c,\ ^{0.5}/e\} & \Rightarrow & \{e\} \\
\{b,\ ^{0.5}/f\} & \Rightarrow & \{f\} & \{^{0.5}/c,\ e,\ D1\} & \Rightarrow & \{c\} \\
\{b,\ ^{0.5}/d,\ ^{0.5}/e\} & \Rightarrow & \{d\} & \{^{0.5}/b,\ ^{0.5}/e,\ D1\} & \Rightarrow & \{b\} \\
\{^{0.5}/d,\ ^{0.5}/e\} & \Rightarrow & \{e\} & \{b,\ ^{0.5}/c\} & \Rightarrow & \{c\}
\end{array}
$$

This gives a total of 18 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 2:

In this iteration, the elicited attribute/value pair is $e = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ e\}$

The computed closure is:

$\{a,\ ^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl@{\qquad}rcl} \{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} & \{{}^{0.5}/b,\ {}^{0.5}/c\} & \Rightarrow & \{D2\} \\ \{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{{}^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\ \{b\} & \Rightarrow & \{d\} & \{{}^{0.5}/b,\ D1\} & \Rightarrow & \{b\} \\ \{{}^{0.5}/c,\ d\} & \Rightarrow & \{c\} & \{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\ \{c\} & \Rightarrow & \{d\} & \{D2\} & \Rightarrow & \{{}^{0.5}/b\} \\ \{{}^{0.5}/b,\ D2\} & \Rightarrow & \{{}^{0.5}/c\} & \{b,\ {}^{0.5}/c\} & \Rightarrow & \{c\} \end{array}$$

This gives a total of 12 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 3:

In this iteration, the elicited attribute/value pair is $b = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl@{\qquad}rcl} \{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} & \{{}^{0.5}/c\} & \Rightarrow & \{D2\} \\ \{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{D1\} & \Rightarrow & \{b\} \\ \{b\} & \Rightarrow & \{d\} & \{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\ \{{}^{0.5}/c,\ d\} & \Rightarrow & \{c\} & \{b,\ {}^{0.5}/c\} & \Rightarrow & \{c\} \\ \{c\} & \Rightarrow & \{d\} & \{{}^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\ \{D2\} & \Rightarrow & \{{}^{0.5}/c\} & & & \end{array}$$

This gives a total of 11 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 4:

In this iteration, the elicited attribute/value pair is $c = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e,\ D2\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$
\begin{array}{rcl}
\{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} \\
\{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} \\
\{b\} & \Rightarrow & \{d\} \\
\{d\} & \Rightarrow & \{c\} \\
\{c\} & \Rightarrow & \{d\}
\end{array}
\qquad
\begin{array}{rcl}
\{D1\} & \Rightarrow & \{b\} \\
\{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\
\{b\} & \Rightarrow & \{c\} \\
\{D1\} & \Rightarrow & \{c\}
\end{array}
$$

This gives a total of 9 implications in this step, instead of the initial 21 implications. In this iteration, we have arrived at a classification/recommendation present in the current closure. Thus, there is no need to make any further iteration.

4.2 Example with z-score elicitation

The trace of the execution when the *zscore* elicitation is used is as follows:

Subject: $\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e,\ D2\}$

The elicitation preference computed for this subject is e, a, d, c, b.

Iteration 1:

In this iteration, the elicited attribute/value pair is $e = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{e\}$

The computed closure is:

$\{e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$
\begin{array}{rcl}
\{{}^{0.5}/d\} & \Rightarrow & \{{}^{0.5}/a\} \\
\{{}^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\
\{{}^{0.5}/b,\ D2\} & \Rightarrow & \{a,\ {}^{0.5}/c,\ {}^{0.5}/d\} \\
\{{}^{0.5}/b,\ D1\} & \Rightarrow & \{b\} \\
\{{}^{0.5}/a\} & \Rightarrow & \{{}^{0.5}/d\} \\
\{{}^{0.5}/a,\ {}^{0.5}/d,\ D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} \\
\{{}^{0.5}/a,\ {}^{0.5}/d,\ {}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} \\
\{{}^{0.5}/a,\ {}^{0.5}/d\} & \Rightarrow & \{a\} \\
\{{}^{0.5}/a,\ {}^{0.5}/c\} & \Rightarrow & \{a,\ {}^{0.5}/d\}
\end{array}
\qquad
\begin{array}{rcl}
\{{}^{0.5}/a,\ b,\ {}^{0.5}/d\} & \Rightarrow & \{d\} \\
\{a,\ {}^{0.5}/c,\ d\} & \Rightarrow & \{c\} \\
\{a,\ c,\ {}^{0.5}/d\} & \Rightarrow & \{d\} \\
\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d\} & \Rightarrow & \{D2\} \\
\{D2\} & \Rightarrow & \{{}^{0.5}/b\} \\
\{{}^{0.5}/f\} & \Rightarrow & \{{}^{0.5}/a\} \\
\{b,\ {}^{0.5}/c\} & \Rightarrow & \{c\} \\
\{{}^{0.5}/a,\ {}^{0.5}/b,\ D2\} & \Rightarrow & \{a\} \\
\{{}^{0.5}/a,\ b,\ {}^{0.5}/f\} & \Rightarrow & \{f\}
\end{array}
$$

This gives a total of 18 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 2:

In this iteration, the elicited attribute/value pair is $a = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ e\}$

The computed closure is:

$\{a,\ {}^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl@{\qquad}rcl}
\{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} & \{{}^{0.5}/b,\ {}^{0.5}/c\} & \Rightarrow & \{D2\} \\
\{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\
\{b\} & \Rightarrow & \{d\} & \{{}^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\
\{{}^{0.5}/c,\ d\} & \Rightarrow & \{c\} & \{{}^{0.5}/b,\ D1\} & \Rightarrow & \{b\} \\
\{c\} & \Rightarrow & \{d\} & \{D2\} & \Rightarrow & \{{}^{0.5}/b\} \\
\{{}^{0.5}/b,\ D2\} & \Rightarrow & \{{}^{0.5}/c\} & \{b,\ {}^{0.5}/c\} & \Rightarrow & \{c\}
\end{array}$$

This gives a total of 12 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 3:

In this iteration, the elicited attribute/value pair is $c = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl@{\qquad}rcl}
\{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} & \{D1\} & \Rightarrow & \{c\} \\
\{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{b\} & \Rightarrow & \{c\} \\
\{b\} & \Rightarrow & \{d\} & \{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\
\{d\} & \Rightarrow & \{c\} & \{{}^{0.5}/b,\ D1\} & \Rightarrow & \{b\} \\
\{c\} & \Rightarrow & \{d\} & \{D2\} & \Rightarrow & \{{}^{0.5}/b\} \\
\{{}^{0.5}/b\} & \Rightarrow & \{D2\} & & &
\end{array}$$

This gives a total of 11 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 4:

In this iteration, the elicited attribute/value pair is $b = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a, {}^{0.5}/b, {}^{0.5}/c, {}^{0.5}/d, e\}$

The computed closure is:

$\{a, {}^{0.5}/b, {}^{0.5}/c, {}^{0.5}/d, e, D2\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl} \{D1\} & \Rightarrow & \{d, {}^{0.5}/f\} \\ \{{}^{0.5}/f\} & \Rightarrow & \{d, D1\} \\ \{b\} & \Rightarrow & \{d\} \\ \{d\} & \Rightarrow & \{c\} \\ \{c\} & \Rightarrow & \{d\} \end{array} \qquad \begin{array}{rcl} \{D1\} & \Rightarrow & \{c\} \\ \{b\} & \Rightarrow & \{c\} \\ \{b, {}^{0.5}/f\} & \Rightarrow & \{f\} \\ \{D1\} & \Rightarrow & \{b\} \end{array}$$

This gives a total of 9 implications in this step, instead of the initial 21 implications. In this iteration, we have arrived at a classification/recommendation present in the current closure. Thus, there is no need to make any further iteration.

4.3 Example with maximum degree elicitation

The trace of the execution when the *maximum degree* elicitation is used is as follows:

Subject: $\{a, {}^{0.5}/b, {}^{0.5}/c, {}^{0.5}/d, e, D2\}$

The elicitation preference computed for this subject is a, e, b, c, d.

Iteration 1:

In this iteration, the elicited attribute/value pair is $a = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a\}$

The computed closure is:

$\{a\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl} \{{}^{0.5}/d\} & \Rightarrow & \{{}^{0.5}/e\} \\ \{{}^{0.5}/b, {}^{0.5}/e, D2\} & \Rightarrow & \{{}^{0.5}/c, {}^{0.5}/d, e\} \\ \{{}^{0.5}/e\} & \Rightarrow & \{{}^{0.5}/d\} \\ \{{}^{0.5}/d, {}^{0.5}/e, D1\} & \Rightarrow & \{d, {}^{0.5}/f\} \\ \{{}^{0.5}/d, {}^{0.5}/e, {}^{0.5}/f\} & \Rightarrow & \{d, D1\} \\ \{{}^{0.5}/c\} & \Rightarrow & \{{}^{0.5}/d, e\} \\ \{b, {}^{0.5}/f\} & \Rightarrow & \{f\} \\ \{b, {}^{0.5}/d, {}^{0.5}/e\} & \Rightarrow & \{d\} \\ \{{}^{0.5}/d, {}^{0.5}/e\} & \Rightarrow & \{e\} \end{array} \qquad \begin{array}{rcl} \{{}^{0.5}/c, d, e\} & \Rightarrow & \{c\} \\ \{c, {}^{0.5}/d, e\} & \Rightarrow & \{d\} \\ \{{}^{0.5}/b, {}^{0.5}/c, {}^{0.5}/d, e\} & \Rightarrow & \{D2\} \\ \{D2\} & \Rightarrow & \{{}^{0.5}/b\} \\ \{D1\} & \Rightarrow & \{{}^{0.5}/e\} \\ \{{}^{0.5}/c, {}^{0.5}/e\} & \Rightarrow & \{e\} \\ \{{}^{0.5}/c, e, D1\} & \Rightarrow & \{c\} \\ \{{}^{0.5}/b, {}^{0.5}/e, D1\} & \Rightarrow & \{b\} \\ \{b, {}^{0.5}/c\} & \Rightarrow & \{c\} \end{array}$$

This gives a total of 18 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 2:

In this iteration, the elicited attribute/value pair is $e = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ e\}$

The computed closure is:

$\{a,\ {}^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl@{\qquad}rcl}
\{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} & \{{}^{0.5}/b,\ {}^{0.5}/c\} & \Rightarrow & \{D2\} \\
\{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{{}^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\
\{b\} & \Rightarrow & \{d\} & \{{}^{0.5}/b,\ D1\} & \Rightarrow & \{b\} \\
\{{}^{0.5}/c,\ d\} & \Rightarrow & \{c\} & \{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\
\{c\} & \Rightarrow & \{d\} & \{D2\} & \Rightarrow & \{{}^{0.5}/b\} \\
\{{}^{0.5}/b,\ D2\} & \Rightarrow & \{{}^{0.5}/c\} & \{b,\ {}^{0.5}/c\} & \Rightarrow & \{c\}
\end{array}$$

This gives a total of 12 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 3:

In this iteration, the elicited attribute/value pair is $b = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl@{\qquad}rcl}
\{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} & \{{}^{0.5}/c\} & \Rightarrow & \{D2\} \\
\{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{D1\} & \Rightarrow & \{b\} \\
\{b\} & \Rightarrow & \{d\} & \{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\
\{{}^{0.5}/c,\ d\} & \Rightarrow & \{c\} & \{b,\ {}^{0.5}/c\} & \Rightarrow & \{c\} \\
\{c\} & \Rightarrow & \{d\} & \{{}^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\
\{D2\} & \Rightarrow & \{{}^{0.5}/c\} & & &
\end{array}$$

This gives a total of 11 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next

iteration.

Iteration 4:

In this iteration, the elicited attribute/value pair is $c = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e,\ D2\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl} \{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} \\ \{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} \\ \{b\} & \Rightarrow & \{d\} \\ \{d\} & \Rightarrow & \{c\} \\ \{c\} & \Rightarrow & \{d\} \end{array} \qquad \begin{array}{rcl} \{D1\} & \Rightarrow & \{b\} \\ \{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\ \{b\} & \Rightarrow & \{c\} \\ \{D1\} & \Rightarrow & \{c\} \end{array}$$

This gives a total of 9 implications in this step, instead of the initial 21 implications. In this iteration, we have arrived at a classification/recommendation present in the current closure. Thus, there is no need to make any further iteration.

4.4 Example with variable importance elicitation

The trace of the execution when the *variable importance* elicitation is used is as follows:

Subject: $\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e,\ D2\}$

The elicitation preference computed for this subject is e, a, b, c, d.

Iteration 1:

In this iteration, the elicited attribute/value pair is $e = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{e\}$

The computed closure is:

$\{e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl} \{{}^{0.5}/d\} & \Rightarrow & \{{}^{0.5}/a\} \\ \{{}^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\ \{{}^{0.5}/b,\ D2\} & \Rightarrow & \{a,\ {}^{0.5}/c,\ {}^{0.5}/d\} \\ \{{}^{0.5}/b,\ D1\} & \Rightarrow & \{b\} \\ \{{}^{0.5}/a\} & \Rightarrow & \{{}^{0.5}/d\} \\ \{{}^{0.5}/a,\ {}^{0.5}/d,\ D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} \end{array} \qquad \begin{array}{rcl} \{{}^{0.5}/a,\ b,\ {}^{0.5}/d\} & \Rightarrow & \{d\} \\ \{a,\ {}^{0.5}/c,\ d\} & \Rightarrow & \{c\} \\ \{a,\ c,\ {}^{0.5}/d\} & \Rightarrow & \{d\} \\ \{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d\} & \Rightarrow & \{D2\} \\ \{D2\} & \Rightarrow & \{{}^{0.5}/b\} \\ \{{}^{0.5}/f\} & \Rightarrow & \{{}^{0.5}/a\} \end{array}$$

$$\begin{array}{rcl} \{^{0.5}/a,\ ^{0.5}/d,\ ^{0.5}/f\} & \Rightarrow & \{d,\ D1\} \\ \{^{0.5}/a,\ ^{0.5}/d\} & \Rightarrow & \{a\} \\ \{^{0.5}/a,\ ^{0.5}/c\} & \Rightarrow & \{a,\ ^{0.5}/d\} \end{array} \qquad \begin{array}{rcl} \{b,\ ^{0.5}/c\} & \Rightarrow & \{c\} \\ \{^{0.5}/a,\ ^{0.5}/b,\ D2\} & \Rightarrow & \{a\} \\ \{^{0.5}/a,\ b,\ ^{0.5}/f\} & \Rightarrow & \{f\} \end{array}$$

This gives a total of 18 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 2:

In this iteration, the elicited attribute/value pair is $a = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ e\}$

The computed closure is:

$\{a,\ ^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl} \{D1\} & \Rightarrow & \{d,\ ^{0.5}/f\} \\ \{^{0.5}/f\} & \Rightarrow & \{d,\ D1\} \\ \{b\} & \Rightarrow & \{d\} \\ \{^{0.5}/c,\ d\} & \Rightarrow & \{c\} \\ \{c\} & \Rightarrow & \{d\} \\ \{^{0.5}/b,\ D2\} & \Rightarrow & \{^{0.5}/c\} \end{array} \qquad \begin{array}{rcl} \{^{0.5}/b,\ ^{0.5}/c\} & \Rightarrow & \{D2\} \\ \{b,\ ^{0.5}/f\} & \Rightarrow & \{f\} \\ \{^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\ \{^{0.5}/b,\ D1\} & \Rightarrow & \{b\} \\ \{D2\} & \Rightarrow & \{^{0.5}/b\} \\ \{b,\ ^{0.5}/c\} & \Rightarrow & \{c\} \end{array}$$

This gives a total of 12 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 3:

In this iteration, the elicited attribute/value pair is $b = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ ^{0.5}/b,\ ^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ ^{0.5}/b,\ ^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl} \{D1\} & \Rightarrow & \{d,\ ^{0.5}/f\} \\ \{^{0.5}/f\} & \Rightarrow & \{d,\ D1\} \\ \{b\} & \Rightarrow & \{d\} \\ \{^{0.5}/c,\ d\} & \Rightarrow & \{c\} \\ \{c\} & \Rightarrow & \{d\} \end{array} \qquad \begin{array}{rcl} \{^{0.5}/c\} & \Rightarrow & \{D2\} \\ \{b,\ ^{0.5}/f\} & \Rightarrow & \{f\} \\ \{D1\} & \Rightarrow & \{b\} \\ \{b,\ ^{0.5}/c\} & \Rightarrow & \{c\} \\ \{^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \end{array}$$

$$\{D2\} \quad \Rightarrow \quad \{^{0.5}/c\}$$

This gives a total of 11 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 4:

In this iteration, the elicited attribute/value pair is $c = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ ^{0.5}/b,\ ^{0.5}/c,\ ^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ ^{0.5}/b,\ ^{0.5}/c,\ ^{0.5}/d,\ e,\ D2\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl@{\qquad}rcl} \{D1\} & \Rightarrow & \{d,\ ^{0.5}/f\} & \{b,\ ^{0.5}/f\} & \Rightarrow & \{f\} \\ \{^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{D1\} & \Rightarrow & \{b\} \\ \{b\} & \Rightarrow & \{d\} & \{b\} & \Rightarrow & \{c\} \\ \{d\} & \Rightarrow & \{c\} & \{D1\} & \Rightarrow & \{c\} \\ \{c\} & \Rightarrow & \{d\} & & & \end{array}$$

This gives a total of 9 implications in this step, instead of the initial 21 implications. In this iteration, we have arrived at a classification/recommendation present in the current closure. Thus, there is no need to make any further iteration.

4.5 Example with logistic coefficients elicitation

The trace of the execution when the elicitation based on *logistic regression coefficients* is used is as follows:

Subject: $\{a,\ ^{0.5}/b,\ ^{0.5}/c,\ ^{0.5}/d,\ e,\ D2\}$

The elicitation preference computed for this subject is a, e, b, c, d.

Iteration 1:

In this iteration, the elicited attribute/value pair is $a = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a\}$

The computed closure is:

$\{a\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\{^{0.5}/d\} \quad \Rightarrow \quad \{^{0.5}/e\} \qquad\qquad \{^{0.5}/c,\ d,\ e\} \quad \Rightarrow \quad \{c\}$$

$$
\begin{array}{rcl}
\{^{0.5}/b,\ ^{0.5}/e,\ D2\} & \Rightarrow & \{^{0.5}/c,\ ^{0.5}/d,\ e\} \\
\{^{0.5}/e\} & \Rightarrow & \{^{0.5}/d\} \\
\{^{0.5}/d,\ ^{0.5}/e,\ D1\} & \Rightarrow & \{d,\ ^{0.5}/f\} \\
\{^{0.5}/d,\ ^{0.5}/e,\ ^{0.5}/f\} & \Rightarrow & \{d,\ D1\} \\
\{^{0.5}/c\} & \Rightarrow & \{^{0.5}/d,\ e\} \\
\{b,\ ^{0.5}/f\} & \Rightarrow & \{f\} \\
\{b,\ ^{0.5}/d,\ ^{0.5}/e\} & \Rightarrow & \{d\} \\
\{^{0.5}/d,\ ^{0.5}/e\} & \Rightarrow & \{e\}
\end{array}
\qquad
\begin{array}{rcl}
\{c,\ ^{0.5}/d,\ e\} & \Rightarrow & \{d\} \\
\{^{0.5}/b,\ ^{0.5}/c,\ ^{0.5}/d,\ e\} & \Rightarrow & \{D2\} \\
\{D2\} & \Rightarrow & \{^{0.5}/b\} \\
\{D1\} & \Rightarrow & \{^{0.5}/e\} \\
\{^{0.5}/c,\ ^{0.5}/e\} & \Rightarrow & \{e\} \\
\{^{0.5}/c,\ e,\ D1\} & \Rightarrow & \{c\} \\
\{^{0.5}/b,\ ^{0.5}/e,\ D1\} & \Rightarrow & \{b\} \\
\{b,\ ^{0.5}/c\} & \Rightarrow & \{c\}
\end{array}
$$

This gives a total of 18 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 2:

In this iteration, the elicited attribute/value pair is $e = 1$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ e\}$

The computed closure is:

$\{a,\ ^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$
\begin{array}{rcl}
\{D1\} & \Rightarrow & \{d,\ ^{0.5}/f\} \\
\{^{0.5}/f\} & \Rightarrow & \{d,\ D1\} \\
\{b\} & \Rightarrow & \{d\} \\
\{^{0.5}/c,\ d\} & \Rightarrow & \{c\} \\
\{c\} & \Rightarrow & \{d\} \\
\{^{0.5}/b,\ D2\} & \Rightarrow & \{^{0.5}/c\}
\end{array}
\qquad
\begin{array}{rcl}
\{^{0.5}/b,\ ^{0.5}/c\} & \Rightarrow & \{D2\} \\
\{^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\
\{^{0.5}/b,\ D1\} & \Rightarrow & \{b\} \\
\{b,\ ^{0.5}/f\} & \Rightarrow & \{f\} \\
\{D2\} & \Rightarrow & \{^{0.5}/b\} \\
\{b,\ ^{0.5}/c\} & \Rightarrow & \{c\}
\end{array}
$$

This gives a total of 12 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 3:

In this iteration, the elicited attribute/value pair is $b = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ ^{0.5}/b,\ ^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ ^{0.5}/b,\ ^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$
\begin{array}{rcl@{\qquad}rcl}
\{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} & \{{}^{0.5}/c\} & \Rightarrow & \{D2\} \\
\{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{D1\} & \Rightarrow & \{b\} \\
\{b\} & \Rightarrow & \{d\} & \{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\
\{{}^{0.5}/c,\ d\} & \Rightarrow & \{c\} & \{b,\ {}^{0.5}/c\} & \Rightarrow & \{c\} \\
\{c\} & \Rightarrow & \{d\} & \{{}^{0.5}/c,\ D1\} & \Rightarrow & \{c\} \\
\{D2\} & \Rightarrow & \{{}^{0.5}/c\} & & &
\end{array}
$$

This gives a total of 11 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 4:

In this iteration, the elicited attribute/value pair is $c = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e,\ D2\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$
\begin{array}{rcl@{\qquad}rcl}
\{D1\} & \Rightarrow & \{d,\ {}^{0.5}/f\} & \{D1\} & \Rightarrow & \{b\} \\
\{{}^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{b,\ {}^{0.5}/f\} & \Rightarrow & \{f\} \\
\{b\} & \Rightarrow & \{d\} & \{b\} & \Rightarrow & \{c\} \\
\{d\} & \Rightarrow & \{c\} & \{D1\} & \Rightarrow & \{c\} \\
\{c\} & \Rightarrow & \{d\} & & &
\end{array}
$$

This gives a total of 9 implications in this step, instead of the initial 21 implications. In this iteration, we have arrived at a classification/recommendation present in the current closure. Thus, there is no need to make any further iteration.

4.6 Example with critique

As commented in the paper, the selection of a critiquing strategy does not affect the course of a conversation with the recommendation system.

In this section, we provide an example of using *compound critiquing* with random elicitation.

Subject: $\{a,\ {}^{0.5}/b,\ {}^{0.5}/c,\ {}^{0.5}/d,\ e,\ D2\}$

The elicitation preference computed for this subject is d, b, c, e, a.

Iteration 1:

In this iteration, the elicited attribute/value pair is $d = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{^{0.5}/d\}$

The computed closure is:

$\{^{0.5}/a,\ ^{0.5}/d,\ ^{0.5}/e\}$

Since, in the computed closure, there are attributes with degree lower than in the original subject, the user has performed 1 critique(s).

Critique number 1:

From the previous closure, the user has provided the following critique:

$\{^{0.5}/a,\ ^{0.5}/e\}$ is changed to $\{a,\ e\}$

Resulting in the following accumulated knowledge:

$\{a,\ ^{0.5}/d,\ e\}$

Performing the closure on the new information, we arrive at:

$\{a,\ ^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl@{\qquad}rcl}
\{D1\} & \Rightarrow & \{d,\ ^{0.5}/f\} & \{^{0.5}/b,\ D2\} & \Rightarrow & \{^{0.5}/c\} \\
\{^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{^{0.5}/b,\ ^{0.5}/c\} & \Rightarrow & \{D2\} \\
\{b\} & \Rightarrow & \{d\} & \{^{0.5}/b,\ D1\} & \Rightarrow & \{b\} \\
\{^{0.5}/c,\ d\} & \Rightarrow & \{c\} & \{b,\ ^{0.5}/f\} & \Rightarrow & \{f\} \\
\{c\} & \Rightarrow & \{d\} & \{D2\} & \Rightarrow & \{^{0.5}/b\} \\
\{^{0.5}/c,\ D1\} & \Rightarrow & \{c\} & \{b,\ ^{0.5}/c\} & \Rightarrow & \{c\}
\end{array}$$

This gives a total of 12 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 2:

In this iteration, the elicited attribute/value pair is $b = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\ ^{0.5}/b,\ ^{0.5}/d,\ e\}$

The computed closure is:

$\{a,\ ^{0.5}/b,\ ^{0.5}/d,\ e\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl@{\qquad}rcl}
\{D1\} & \Rightarrow & \{d,\ ^{0.5}/f\} & \{^{0.5}/c\} & \Rightarrow & \{D2\} \\
\{^{0.5}/f\} & \Rightarrow & \{d,\ D1\} & \{D1\} & \Rightarrow & \{b\} \\
\{b\} & \Rightarrow & \{d\} & \{b,\ ^{0.5}/f\} & \Rightarrow & \{f\}
\end{array}$$

$$\begin{array}{rcl} \{^{0.5}/c,\, d\} & \Rightarrow & \{c\} \\ \{c\} & \Rightarrow & \{d\} \\ \{D2\} & \Rightarrow & \{^{0.5}/c\} \end{array} \qquad \begin{array}{rcl} \{b,\, ^{0.5}/c\} & \Rightarrow & \{c\} \\ \{^{0.5}/c,\, D1\} & \Rightarrow & \{c\} \end{array}$$

This gives a total of 11 implications in this step, instead of the initial 21 implications. As we haven't arrived to a valid class (D1, D2) in the closure, we proceed with the next iteration.

Iteration 3:

In this iteration, the elicited attribute/value pair is $c = 0.5$.

The accumulated values up to this iteration, including the just elicited attribute is:

$\{a,\, ^{0.5}/b,\, ^{0.5}/c,\, ^{0.5}/d,\, e\}$

The computed closure is:

$\{a,\, ^{0.5}/b,\, ^{0.5}/c,\, ^{0.5}/d,\, e,\, D2\}$

After applying Simplification Logic, the resulting set of simplified implications is:

$$\begin{array}{rcl} \{D1\} & \Rightarrow & \{d,\, ^{0.5}/f\} \\ \{^{0.5}/f\} & \Rightarrow & \{d,\, D1\} \\ \{b\} & \Rightarrow & \{d\} \\ \{d\} & \Rightarrow & \{c\} \\ \{c\} & \Rightarrow & \{d\} \end{array} \qquad \begin{array}{rcl} \{D1\} & \Rightarrow & \{b\} \\ \{b,\, ^{0.5}/f\} & \Rightarrow & \{f\} \\ \{b\} & \Rightarrow & \{c\} \\ \{D1\} & \Rightarrow & \{c\} \end{array}$$

This gives a total of 9 implications in this step, instead of the initial 21 implications. In this iteration, we have arrived at a classification/recommendation present in the current closure. Thus, there is no need to make any further iteration.