Recommendations in CDSS using Fuzzy Formal Concept Analysis

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Clinical Decision Support Systems

Features

- A *Clinical Decision Support System* (CDSS) is any computer program designed to help health professionals make clinical decisions.
- In a sense, any computer system that deals with clinical data or medical knowledge is intended to provide support.
- Decision-support function varies from generalized to patient-specific.
- Uses:
  - Generating alerts and reminders.
  - **Diagnostic assistance**.
  - Therapy planning.
  - Image recognition and interpretation.

Clinical Decision Support Systems

Disadvantages

- Limiting professionals' possibilities for independent problem solving.
- Legal implications.
- Tremendous amount of data and rules must be incorporated into the system.
  - Currently, more and more data is available (open data movement).
  - **Rules** are hand-made, created by *experts* in each field.

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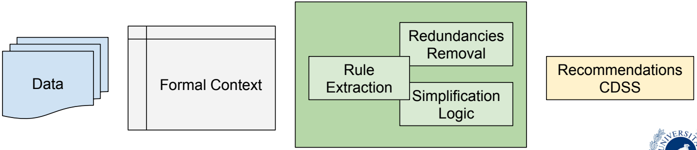
Objective

- Provide mathematical foundations for CDSSs.
- Develop algorithmic tools to generate an optimal set of rules covering known prior data.
- Use implications as the basis for a recommendation system.

The tool used to tackle this problem is **Fuzzy Formal Concept Analysis.**



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Fuzzy Formal Concept Analysis

Fuzzy FCA deals with fuzzy relationships between objects and their attributes.

Particularly, we'll deal with **graded attributes**, in the sense that the relationship between object g and attribute m is given by:

$$I(g,m) \in L = \{0 = a_0, a_1, \ldots, a_n = 1\}$$

where $a_0 < a_1 < \ldots < a_n$ and L is called the set of grades.

For example, $I(g,m) = \alpha$ means that object g possesses attribute m to degree α. $I(g,m) = 1$ means that the object fully possesses the attribute, and $I(g,m) = 0$ indicates the lack of attribute m in the object.

Actually, we have $\langle L, \vee, \wedge, \otimes, \rightarrow, \searrow, ^*, 0, 1\rangle$.

Graded attribute implications

Formulas

- We can define formulas $A \Rightarrow B$ where A and B are fuzzy sets over an alphabet Ω whose elements are named attributes.
- Informally: $\{a, {}^{0.5}/b\} \Rightarrow \{{}^{0.9}/c\}$ means every object that has attribute a to degree 1 (i.e. fully possesses a), and attribute b to degree 0.5, has attribute c to degree at least 0.9.

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Optimizations in computing the Duquenne–Guigues basis of implications

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Algorithm to extract implications

Algorithm 6 Canonical Basis(M, $''$), an optimized version

Input: A closure operator $X \mapsto X''$ on M, e.g., given by a formal context (G, M, I).
Output: The canonical basis for the closure operator.

```
A := ∅''
if A = ∅ then
    ℒ := ∅
else
    ℒ := {∅ → A}
i := the largest element of M

while A ≠ M do

    for all j ≤ i ∈ M in reverse order do
        if j ∈ A then
            A := A \ {j}
        else
            B := ℒ(A ∪ {j})
            if B \ A contains no element < j then
                A := B
                i := j
                exit for

    if A ≠ A'' then
        ℒ := ℒ ∪ {A → A''}
    if A'' \ A contains no element < i then
        A := A''
        i := the largest element of M
    else
        A := {m ∈ A | m ≤ i}

return ℒ
```

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Removing redundancy in graded attribute implications

- To obtain equivalent implicational sets with lower size (small number of implications) or with less attributes in the LHS or RHS.
- Widely studied in the classical setting.
- Approached by Vilem Vychodil in [IS2015].

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On minimal sets of graded attribute implications

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Removing redundancy in graded attribute implications

- We tackle this problem using the so-called *Fuzzy Attribute Simplification Logic*, **FASL**, which has been introduced in [Belohlavek 2016].
- This logic leads to the design of automatic reasoning methods for implications in data with grades.

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Automated prover for attribute dependencies in data with grades

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FASL

Axiomatic system

- Firstly, we proposed a new Simplification Rule adequate to remove redundancy in an automatic way.
- Simplification Rule turned into the *heart* of a novel logic: FASL - Fuzzy Attributes Simplification Logic.
- FASL becomes the *engine* of automated methods: redundancy removal, closure algorithm, etc.

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FASL

Axiomatic system

The axiomatic system in FASL is defined as follows: for all $A, B, C, D \in L^{\Omega}$ and $c \in L$,

$[\mathtt{Ax}]$ infer $A \cup B \Rightarrow A$ (*Axiom*)

$[\mathtt{Mul}]$ from $A \Rightarrow B$ infer $c^{*} \otimes A \Rightarrow c^{*} \otimes B$ (*Multiplication*)

$[\mathtt{Sim}]$ from $A \Rightarrow B$ and $C \Rightarrow D$ infer $A \cup (C \smallsetminus B) \Rightarrow D$ (*Simplification*)

FASL

Equivalences

The following equivalences hold:

(DeEq) $\{A \Rightarrow B\} \equiv \{A \Rightarrow B \smallsetminus A\}$;

(UnEq) $\{A \Rightarrow B, A \Rightarrow C\} \equiv \{A \Rightarrow B \cup C\}$;

(SiEq) If $A \subseteq C$, $A \cap B = \emptyset$, then $\{A \Rightarrow B, C \Rightarrow D\} \equiv \{A \Rightarrow B, C \smallsetminus B \Rightarrow D \smallsetminus B\}$.

And

(Gen) If $A \subseteq C$, $D \subseteq B$, then $\{A \Rightarrow B, C \Rightarrow D\} \equiv \{A \Rightarrow B\}$

(UnEq) and **(Gen)** reduce the number of rules in the set, while **(DeEq)** and **(SiEq)** reduce the number of attributes in each rule.

Example of FASL application

```
data("Mushroom", package = "arules")

fc <- formal_context$new(I = Mushroom)

# run apriori with redudancies removal
fc$run_apriori()
(...)
writing ... [1799427 rule(s)] done [0.95s].
creating S4 object  ... done [0.92s].

# Cadinality
fc$implications$cardinality()
[1] 2002

# Rule size
sizes <- fc$implications$size()

# Mean size for LHS and RHS
colMeans(sizes)
     LHS      RHS
3.038462 1.000000

# Applying FASL rules
fc$implications$apply_rules(rules = c("composition",
                                      "simplification"))
Using parallel execution
Processing batch
--> composition : from 2002 to 961 in 0.995 secs.
--> simplification : from 961 to 961 in 5.957 secs.
Batch took 6.956 secs.

# Rule size
sizes <- fc$implications$size()

# Mean size for LHS and RHS
colMeans(sizes)
     LHS      RHS
3.122789 2.083247
```

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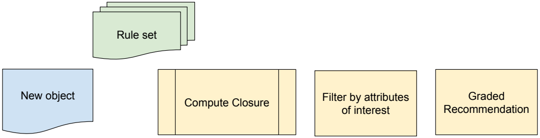
Generating recommendations




```
# Take one of the objects, without the class
attr <- fc$attributes[which(fc$I[, 1] > 0)[-1]]
S <- build_set(attrs = attr,
               values = rep(1, length(attr)),
               attributes = fc$attributes)
print_set(S, fc$attributes)
[1] "{CapShape=convex, CapSurf=smooth, CapColor=brown, Bruises=bruises,
    Odor=pungent, GillAttached=free, GillSpace=close, GillSize=narrow,
    GillColor=black, StalkShape=enlarging, StalkRoot=equal, SurfaceAboveRing=smooth,
    SurfaceBelowRing=smooth, ColorAboveRing=white, ColorBelowRing=white, VeilType=partial,
    VeilColor=white, RingNumber=one, RingType=pendant, Spore=black, Habitat=urban}"

# Compute its closure
cl <- fc$implications$compute_closure(S = S)
print_set(cl, fc$attributes)

[1] "{Class=poisonous, CapShape=convex, CapSurf=smooth, CapColor=brown,
    Bruises=bruises, Odor=pungent, GillAttached=free, GillSpace=close,
    GillSize=narrow, GillColor=black, StalkShape=enlarging, StalkRoot=equal,
    SurfaceAboveRing=smooth, SurfaceBelowRing=smooth, ColorAboveRing=white,
    ColorBelowRing=white, VeilType=partial, VeilColor=white, RingNumber=one,
    RingType=pendant, Spore=black, Habitat=urban}"

# Get recommendation for the class
fc$implications$recommend(S = S,
                          attribute_filter = "Class=poisonous")
[1] Class=poisonous
                  1
```

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ABIDE Dataset

ABIDE
Autism Brain Imaging Data Exchange
PREPROCESSED

ABIDE is a collaboration of 16 international imaging sites that have aggregated and are openly sharing neuroimaging data from 539 individuals suffering from ASD and 573 typical controls. These 1112 datasets are composed of structural MRI data along with an extensive array of phenotypic information (including diagnosis).

The aim of the recommendation system is to help clinicians get a diagnosis from data derived from MRI.

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Preprocessing

- Since all attributes are continuous, we'll transform them into graded.
- We are interested in the influence of regional brain atrophy (measured by volume and thickness changes in brain areas) on ASD diagnosis.
- For an object (individual) g, we define the presence of attribute m (degree of atrophy with respect to the population) as follows:

$$I(g,m) = \begin{cases} 0 & \text{if the value of } m \text{ does not represent atrophy} \\ 1/3 & \text{if it represents mild } (\leq 33\%) \text{ atrophy} \\ 2/3 & \text{if it represents medium } (33 < m \leq 66\%) \text{ atrophy} \\ 1 & \text{if it represents severe } (> 66\%) \text{ atrophy} \end{cases}$$

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Application of FASL

```
fc <- formal_context$new(abide_data)

fc$extract_implications()

# Take only implications with the class attribute in the RHS
diagn <- fc$implications$filter_by_rhs(attr_filter = "group", drop = TRUE)

# We have 1041 rules
diagn$cardinality()
[1] 1041

sizes <- diagn$size()
colMeans(sizes)
     LHS      RHS
1.697406 1.000000

# Apply FASL
diagn$apply_rules(rules = c("composition",
                            "generalization"))
Processing batch
--> composition : from 1041 to 911 in 0.303 secs.
--> generalization : from 911 to 311 in 0.114 secs.
Batch took 0.421 secs.

diagn$cardinality()
[1] 311

sizes <- diagn$size()
colMeans(sizes)
     LHS      RHS
1.577706 1.000000
```

Aid in Diagnosis

```
# Random object
v <- sample(c(0, grades_set), size = n_attributes, replace = TRUE,
            prob = c(0.8, rep(0.2 / length(grades_set), length(grades_set))))


S <- build_set(attrs = attributes, values = v, attributes = attributes)
print_set(S, attributes)
"{Left.Thalamus.Proper.volume, Right.Caudate.volume [0.333333333333333],
Supratentorial.volume [0.333333333333333], left.cerebellum.exterior.thickness,
left.thalamus.proper.thickness, left.caudate.thickness [0.666666666666667],
right.cerebellum.white.matter.thickness, right.thalamus.proper.thickness [0.666666666666667],
left.fusiform.thickness [0.333333333333333], left.pars.triangularis.thickness [0.333333333333333],
left.insula.thickness, right.caudal.anterior.cingulate.thickness [0.333333333333333],
right.caudal.middle.frontal.thickness [0.666666666666667],
right.middle.temporal.thickness [0.333333333333333],
right.superior.temporal.thickness [0.666666666666667]}"

# Recommend the class attribute
# (group == 1 -> ASD, group == 0 -> Control)
diagn$recommend(S, attribute_filter = "group")
group
    0
```

Some findings

```
1: {Intracranial.Volume [mild], left.pars.opercularis.thickness [medium]} -> {group}
2: {Intracranial.Volume [mild], left.pars.orbitalis.thickness [medium]} -> {group}
3: {Intracranial.Volume [mild], left.lateral.occipital.thickness [mild], left.parahippocampal.thickness [mild]} -> {group}
4: {Intracranial.Volume [mild], left.parahippocampal.thickness [mild], right.lateral.occipital.thickness [mild]} -> {group}
5: {Intracranial.Volume [mild], left.parahippocampal.thickness [mild], left.pars.opercularis.thickness [mild]} -> {group}
6: {Intracranial.Volume [mild], left.parahippocampal.thickness [mild], right.pars.orbitalis.thickness [mild]} -> {group}
7: {Intracranial.Volume [mild], left.parahippocampal.thickness [mild], right.transverse.temporal.thickness [mild]} -> {group}
8: {Intracranial.Volume [mild], left.insula.thickness [mild], right.pars.orbitalis.thickness [mild]} -> {group}
```

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Conclusions

- **FASL**: a logic for dependencies in data with grades.
- A method to remove redundancy, only applying the equivalence of FASL, is used to create a recommendation system for clinical decisions.
- The inferred ruleset can be thought of as a series of findings correlating MRI and ASD diagnosis.

Future work

- To obtain optimal basis for dependencies in data with grades, thus accelerating the computation of closures.
- To make a deeper comparative with other methods.
- To study positive and negative graded attributes, allowing not only to assess the presence of an attribute but also the degree to which an attribute is missing or not positive. In the case of this work, the analogous would be to consider not only brain atrophy, but also hypertrophy and its link to ASD.

Recommendations in CDSS using Fuzzy Formal Concept Analysis

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