A Multivalued Neural Network for the Degree-Constrained Minimum Spanning Tree Problem

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Outline

1. The Problem
2. The Neural Model
3. Experimental Results
4. Conclusions and Future Work

Definition of the Problem I

Consider an undirected complete graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{n_1, \ldots, n_N\}$ is the set of N nodes (or vertices) and $\mathcal{E} = \{e_1, \ldots, e_m\}$ is the set of m arcs (or edges), with given costs c_ℓ for each $\ell \in \{1, \ldots, m\}$, and numbers $b_i \in \mathbb{Z}^+$.

A spanning tree of the connected graph $\mathcal{G}$ can be defined as a maximal subset of $\mathcal{E}$ (edges of $\mathcal{G}$) that contains no cycle. Equivalently, a spanning tree is a minimal set of edges that connect all vertices in the graph.

Definition of the Problem II

The DCMST problem on $\mathcal{G}$ is to find a spanning tree $\mathcal{T} = (\mathcal{V}, \mathcal{E}')$, with $\mathcal{E}' \subset \mathcal{E}$, such that the expression

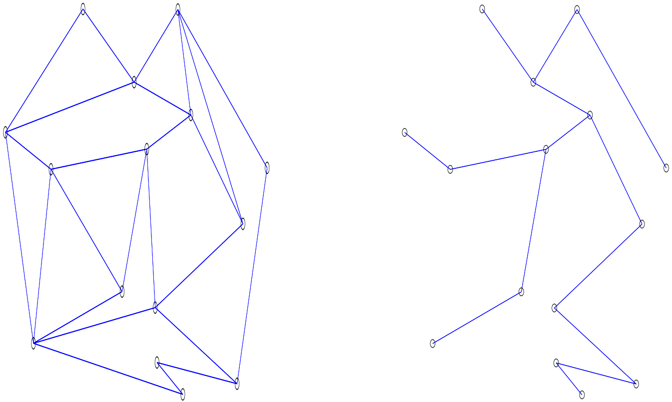
$$C(\mathcal{T}) = \sum_{\ell \in \mathcal{E}'} c_\ell \tag{1}$$

is minimum, subject to $d_i \leq b_i$ for all $i \in \mathcal{V}$, where d_i is the number of arcs incident at node i, that is, the degree of each node in the tree $\mathcal{T}$ is bounded by a positive constant b_i.

Applications:

- N terminals need to be connected by making use of a minimum length of wiring.
- Subproblem in the design of a centralized computer network.
- Design of a road system which has to serve a collection of cities and has an additional restriction that no more than b_i roads may meet at any crossing (node i).

Example



MREM

- N neurons may take any value in a finite set $\mathcal{M}$.
- State vector $\vec{V} = (v_1, v_2, \ldots, v_N) \in \mathcal{M}^N$ describes the network state.
- An energy function determines the behavior of the net:

$$E(\vec{V}) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,j} f(v_i, v_j) + \sum_{i=1}^{N} \theta_i(v_i)$$

  where $W = (w_{i,j})$ is a matrix, f is a similarity function and θ_i are the threshold functions.
- Generalization of Hopfield's model, and other multivalued models.

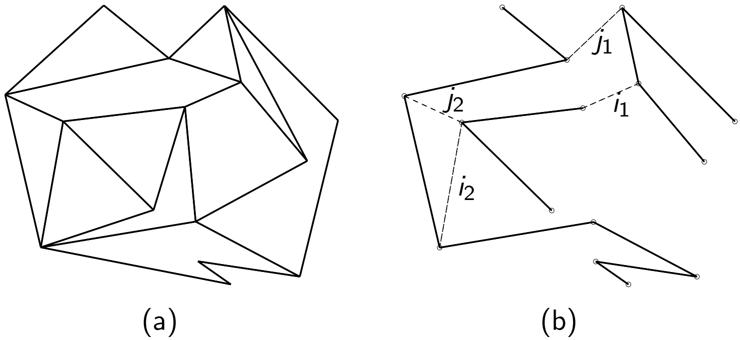
Computational Dynamics

Two MREM networks are considered:

- $\mathcal{H}_1$ solves the problem by using edge-exchange techniques (one neuron per edge). W, f, θ_i are defined in terms of the costs c_i.
- $\mathcal{H}_2$ is an auxiliary network (one neuron per node).

1. First, an initial random state representing a feasible solution is considered in $\mathcal{H}_1$.
2. Repeat until convergence is detected:
    1. Two edges (i_1, i_2) are removed from the current spanning tree. The subgraph generated has 2 or 3 connected components.
    2. $\mathcal{H}_2$ assign a label to each node, denoting the corresponding connected component.
    3. $\mathcal{H}_1$ studies the possibility of replacing the two edges i_1, i_2 by new edges j_1, j_2 not violating the degree constraint.
    4. The best possible edge-exchange in made.

Graphical Representation of the Proposed Dynamics



Description of the Tests

SHRD Graphs: The first node is connected to all other nodes by an edge of length L, the second node is connected to all nodes but the first by an edge of length $2L$ and so on. As usual, the value for L is 20.

Previous works:

- Evolutionary Algorithms like F-EA, and P-EA (Krishnamoorthy,2001) (Prüfer number), and W-EA (Raidl,2000) (weight-coded, best results).
- Problem space search (PSS) (Krishnamoorthy,2001).
- Simulated Annealing (SA) (Krishnamoorthy,2001).
- Branch and Bound (B&B) (Krishnamoorthy,2001).

Values appearing in the Table are computed as follows:

$$\frac{C_{d-\text{Prim}} - C_{\text{alg}}}{C_{d-\text{Prim}}} \cdot 100$$

Results

		F-EA	P-EA	PSS	SA	B&B	W-EA	Prop.	
N	*b*	Avg.	Avg.	Avg.	Avg.	Avg.	Avg.	Avg.	t
15	3	13.66	15.07	16.62	14.93	18.03	14.20	**20.00**	1.03
	4	10.83	0.39	12.99	11.61	**14.76**	11.42	11.11	1.00
	5	4.00	-1.07	**9.60**	9.07	**9.60**	3.53	6.90	0.88
20	3	11.32	5.38	10.91	10.43	10.91	12.29	**23.07**	3.17
	4	6.82	0.80	7.05	5.57	7.05	8.50	**14.06**	3.46
	5	6.28	1.46	7.30	7.74	7.30	7.96	**9.80**	3.12
25	3	13.07	13.41	15.40	14.73	15.40	16.51	**25.45**	7.94
	4	4.84	1.59	6.79	5.56	6.79	6.83	**16.83**	9.12
	5	5.37	5.92	6.74	5.19	8.29	9.01	**11.39**	8.49
30	3	6.51	6.51	11.27	9.53	11.27	12.50	**26.40**	17.53
	4	7.30	3.79	10.58	8.45	10.58	11.76	**17.81**	19.76
	5	2.18	0.19	5.74	2.50	4.74	5.77	**12.39**	19.20
TOTAL		7.68	4.45	10.00	8.78	10.39	10.02	**16.27**	7.89

Comments to Results

Our model is able to outperform the other algorithms in most cases. It obtains much better results on average than the other approaches, and specially as the number of nodes increases. So, for larger problem instances, it is expected that our model achieve very good solutions.

In the last column, time values of our model are presented. It can be observed that our technique is not very time-consuming, being able to solve a large problem instance ($N = 30$), in less than 20 seconds.

Conclusions

- Application of the multivalued neural model MREM to the solution of the degree-constrained minimum spanning tree problem.
- With the use of two networks from this model, we are able to build a feasible solution to this problem, satisfying the degree constraints in every iteration, as well as to compute the connected components of the solution.
- Our method has proved to outperform other algorithms from the literature, including the algorithm achieving the best results up-to-date.

Future Works

Future research will cover aspects such as

- To use a parallel dynamics for this model, since computational times may be drastically reduced.
- To develop a method for solution improvement, based on stochastic dynamics of the network.
- To incorporate techniques to escape from local minima of the energy function of deterministic nature.